Tax Incidence in Differentiated Product Oligopoly

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Abstract

We analyze the incidence of ad valorem and unit excise taxes in an oligopolistic industry with differentiated products and price-setting (Bertrand) firms. Both taxes may be passed on to consumers by more than 100 percent, and an increase in the tax rate can increase short run firm profits (and hence the long run number of firms). We provide summary conditions for these effects to arise. The conditions depend on demand curvatures and are written in elasticity form. Surprisingly, the analysis largely corroborates Cournot results with homogeneous demand.

Keywords: Excise tax, unit tax, specific tax, ad valorem tax, imperfect competition, product differentiation, Bertrand, oligopoly, tax incidence, discrete choice models.

JEL Classification #s: D43, H21, H22, L13

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"EXCISE – A hateful tax levied on commodities, and adjudged not by the common judges of property, but wretches hired by those to whom excise is paid."

— Samuel Johnson's *Dictionary of the English Language* (1755)

1. Introduction

Historically, selective sales taxes (excises) have been among the most controversial and despised forms of taxation in America, largely because of their ability to discriminate against particular classes of producers and consumers. Indeed, such taxes are widely considered to have been catalysts for the American Revolution, and the first U.S. excise tax enacted under President Washington, a specific tax levied on distilled spirits, incited the Whiskey Rebellion of 1794 – an uprising that ended any substantive government interest in excise taxation until late into the next century. In 1862, when the nation faced its first fiscal crisis during the Civil War, Union officials created a special agency to oversee commodity taxation on virtually all consumer goods in the North.[1] Soon after, excise taxes accounted for half of all federal revenue and became a permanent fixture in the U.S. tax system.

Selective sales taxes are still used extensively at both the federal and state levels. In 1995, federal and state excises raised $59 billion and $63 billion, respectively, with the latter figure comprising a third of total state sales tax revenues.[2] Excises can be levied at either unit (specific) or ad valorem (percentage) rates. About half of federal excise revenue is provided by the unit gasoline tax. Major state excises include unit levies on motor fuels, tobacco products, and alcoholic beverages (39 percent of state excise revenue, 11 percent, and 6 percent, respectively), with ad valorem levies imposed on public utilities (14 percent).[3]

It is well known that unit taxes are equivalent to (appropriately calculated) ad valorem taxes under perfect competition and so have the same economic impacts,

[1] The first commissioner of this new *Office of the Commissioner of Internal Revenue* characterized it as "the largest Government department ever organized." (Brownlee, 1996, p. 25)

[2] Federal data taken from "Federal Excise Taxes, Fiscal Years 1994 and 1995," *Statistics of Income Bulletin,* IRS, Fall 1996, Figure B. State data taken from *The Book of the States*, The Council of State Governments, Vol. 31, 1996-97, Table 6.30.

[3] Excise taxes appeal to governments for various reasons. Since the taxes are typically included in sales prices, they are often invisible to consumers. They may also be relatively efficient methods of raising revenue if targeted toward inelastic commodities. (A tax on salt, essential for the preservation of food before refrigeration, provided a dependable source of revenue in the Middle Ages.) Excises on tobacco and alcoholic beverages, which were originally imposed as luxury-consumption taxes, persist in part to discourage use of these products.

but this equivalence does not hold under different market structures. The relatively small amount of work that has analyzed excise taxation under imperfect competition (e.g., Seade, 1987; Stern, 1987; Besley, 1989; Delipalla and Keen, 1992; Skeath and Trandel, 1994; and Hamilton, 1999) has usually considered a homogeneous product with a corresponding single market demand curve (see also the survey paper by Keen, 1998). It has also typically been assumed that firm interaction is described by Cournot competition,[4] a description that is losing popularity in modern Industrial Organization. One difficulty with the Cournot assumption is that price is determined only indirectly from the market demand: firms do not directly choose their prices.[5] It seems preferable to model firms as setting prices and supplying the amount of goods and services consumers desire at those prices. "After all, firms almost always compete in prices." (Tirole, 1988, p. 224).

While it may seem eminently reasonable to model markets as firms selling differentiated products and competing in prices, there are various technical problems (e.g., proving the existence of a pure-strategy equilibrium under product differentiation) that have only recently been satisfactorily resolved. State-of-the-art empirical work has successfully applied the Bertrand differentiated products model to the auto industry (Berry, Levinsohn, and Pakes, 1995; Goldberg, 1995).

We show in this paper that the particular type of excise can also have different implications for economic incidence under price setting oligopoly and product differentiation. Clearly, given the major presence of selective sales taxes in most tax systems, and given that the predominant market form is oligopoly, it is important for policymakers to have a sound understanding of how the *forms* of excise taxes, as well as their levels, affect economic outcomes. To this end, we derive the comparative statics results for the two different tax types in both the short and long term. Despite the difference in context, we find support for previous conclusions that ad valorem taxation is associated with lower consumer prices and lower firm profits. In particular, the conditions under which taxes are overshifted, derived for

[4] Two exceptions are Kay and Keen (1983) and Delipalla and Keen (1992). The former study considers differentiated products under monopolistic competition with fixed aggregate demand. The latter study uses a conjectural variations approach (with a homogeneous product). While this approach is largely discredited by theorists, it is sometimes felt to constitute a useful shorthand way of describing market conduct. When we refer to their results below, we take the case of Cournot competition.

[5] While the Cournot model can be rationalized as the outcome of a two stage game in which firms first choose capacities and then prices (Kreps and Scheinkman, 1983), results are sensitive to the rationing rule assumed and are not readily applicable when products are differentiated.

a Cournot model with homogeneous products (henceforth the "Cournot model"), extend nicely to our framework. This may be surprising to readers accustomed to seeing Cournot and Bertrand frameworks lead to opposite market outcomes:[6] the robustness of the distributional conclusions for selective sales taxes is quite unexpected.

In the next section, we develop our basic model of Bertrand oligopoly with differentiated products. Using this model, Section 3 provides a comparison of the distributional effects of unit and ad valorem taxes in the short run (for a fixed number of firms). Section 4 then analyzes incidence consequences for the long run with free entry. Section 5 concludes.

2. The Model

Consider an industry comprised of n firms in which each produces a variant of a differentiated product at constant marginal cost c. In the short term, fixed cost K is sunk and so does not affect the short run analysis. We consider symmetric differentiated demand systems in which demand for firm i's product can be written as continuously differentiable $D(p_i; p_{-i})$, which is decreasing in Firm i's consumer price, p_i, and increasing in the common price charged by other firms, p_{-i}. Note that the demand addressed to each firm is the same when all prices are the same ($p_i = p_{-i} = p$). We present our main analysis using the general symmetric demand system (which incorporates as special cases the Chamberlinian symmetry formulation whereby all cross-price effects are symmetric, as well as extensions of Salop's 1979 circle model). We illustrate our results with a generalized CES-Logit model (described below) and a linear demand model.

Under a unit (specific) tax t, Firm i's producer price is $p_i - t$. Its profit can then be written as

$$\pi_{Ui} = (p_i - \widetilde{c}_U)D(p_i; p_{-i}), \tag{2.1}$$

with effective cost $\widetilde{c}_U \equiv c + t$. Under an ad valorem tax applied on all firms at rate s, the firm's producer price is $\frac{p_i}{1+s}$. Letting $\tau \equiv \frac{s}{1+s}$, the producer price can

[6] For example, the Cournot "merger paradox" – that merging firms' profits decrease – does not hold for Bertrand competition (Deneckere and Davidson, 1985). Also, firms selling differentiated substitute products have a first-mover advantage under Cournot competition but a second-mover advantage under Bertrand competition (Gal-Or, 1985), and firms agglomerate in space under spatial Cournot competition but maintain separation under Bertrand competition (Anderson and Neven, 1991).

be written equivalently as $(1-\tau)\,p_i$ and we shall use this formulation in what follows. Firm i's profit under the ad valorem tax is then given by

$$\pi_{Ai} = (1-\tau)\,(p_i - \widetilde{c}_A)\,D(p_i;\,p_{-i}), \tag{2.2}$$

with effective cost $\widetilde{c}_A \equiv \frac{c}{1-\tau}$. As noted in Anderson, de Palma, and Kreider (2000), when tax rates are set such that $\widetilde{c}_A = \widetilde{c}_U = \widetilde{c}$, i.e., when $t = \frac{\tau}{1-\tau}c$, the two profit functions are identical except for the $1-\tau$ term pre-multiplying the gross effective markup in (2.2). This term acts like a pure profits tax and is neutral in the short run since it does not affect firm behavior. Thus, when the two taxes are set so that $t = \frac{\tau}{1-\tau}c$, consumer prices and output levels will be the same in the short run under the two taxes, and this benchmark allows us to compare incidence across the taxes.

The strategic interaction between firms is described by a Bertrand-Nash game. At an interior solution, a firm's best-reply price when all other firms choose a common price p_{-i} is characterized by the first order condition

$$(p_i - \widetilde{c})\frac{\partial D(p_i;\,p_{-i})}{\partial p_i} + D(p_i;\,p_{-i}) = 0. \tag{2.3}$$

Denote the common equilibrium price and quantity under the unit tax by p^* and $D(p^*;p^*)$, respectively. In what follows, D and its derivatives are evaluated at the equilibrium price unless otherwise indicated. Chamberlin's (1933) *dd* curve traces quantity demanded for Firm i's product through the equilibrium when Firm i's price alone changes. Then $\frac{\partial D}{\partial p_i}$ is the slope of this curve. Further, Chamberlin's DD curve traces quantity demanded for Firm i's product when *all* prices, p_i as well as the prices of the other firms, p_{-i}, move in tandem. The slope of this curve is denoted $\frac{\partial D}{\partial p} = \frac{\partial D}{\partial p_i} + \frac{\partial D}{\partial p_{-i}}$.

We assume that the corresponding second order condition for a maximum is met,[7] and moreover that the equilibrium is stable, which means that the reaction function should have a slope less than one in the neighborhood of a symmetric candidate. The reaction function slope is given by applying the implicit function theorem to (2.3):

$$\left.\frac{dp_i}{dp_{-i}}\right|_{R_i} = \frac{\frac{\partial D}{\partial p_{-i}} + (p_i - \widetilde{c})\frac{\partial^2 D}{\partial p_i \partial p_{-i}}}{-\left(2\frac{\partial D}{\partial p_i} + (p_i - \widetilde{c})\frac{\partial^2 D}{\partial p_i^2}\right)}, \tag{2.4}$$

[7] This condition can be verified for the examples we use.

where the denominator is positive by the second order condition. The condition that the slope be less than unity, as required for stability, is then

$$2\left(\frac{\partial D}{\partial p_i}\right)^2 - D\frac{\partial^2 D}{\partial p_i^2} + \frac{\partial D}{\partial p_i}\frac{\partial D}{\partial p_{-i}} - D\frac{\partial^2 D}{\partial p_i \partial p_{-i}} > 0, \tag{2.5}$$

where we have substituted $p_i - \widetilde{c}$ from the first order condition (2.3). This inequality can be written equivalently as

$$\left(\frac{\partial D}{\partial p_i}\right)^2 + \frac{\partial D}{\partial p_i}\frac{\partial D}{\partial p} - D\frac{\partial\left(\frac{\partial D}{\partial p_i}\right)}{\partial p} > 0, \tag{2.6}$$

which is a condition on the slopes of the dd and DD curves, along with how the slope of the dd curve changes with respect to a common price change.

It will be useful to work with elasticities in what follows. The elasticity of the dd curve at $p_i = p^*$ is given by $\varepsilon_{dd} \equiv \frac{\partial D}{\partial p_i}\frac{p^*}{D}$. The DD curve's elasticity at the equilibrium price is given by $\varepsilon_{DD} \equiv \frac{\partial D}{\partial p}\frac{p^*}{D}$. Assuming that the goods are substitutes, we have

$$\varepsilon_{DD} > \varepsilon_{dd}. \tag{2.7}$$

(Imagine the two demand curves intersecting at price p^* with DD steeper than dd; a firm's output is more sensitive to a change in its own price alone than to a simultaneous change in all firms' prices.) Also important to our analysis is the elasticity of the *slope* of the dd curve with respect to the common price p:

$$\varepsilon_m \equiv \frac{\partial}{\partial p}\left(\frac{\partial D}{\partial p_i}\right)\frac{p^*}{\frac{\partial D}{\partial p_i}}.$$

As discussed below, Seade (1987) found a similar construction useful for the case of homogeneous products under Cournot competition. We can now write the stability condition (2.6) succinctly as

$$\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m < 0. \tag{2.8}$$

The next results follow directly from the elasticity definitions:

Lemma 1. $\frac{\partial}{\partial p}\left[D/\frac{\partial D}{\partial p_i}\right] = \left(\varepsilon_{DD} - \varepsilon_m\right)/\varepsilon_{dd}$.

Lemma 2. $\frac{\partial}{\partial p}\left[D^2/\frac{\partial D}{\partial p_i}\right] = D\left(2\varepsilon_{DD} - \varepsilon_m\right)/\varepsilon_{dd}$.

The equilibrium is stable when the first expression exceeds -1.

3. Tax Incidence in the Short Run

We now investigate the effects of these taxes on short run equilibrium prices and profits. Our first result is that both types of tax raise consumer prices. We then confirm that producer prices and even profits can also rise with the tax rates.

3.1. The effects on prices in the short run

The first proposition parallels a result in Delipalla and Keen (1992) for the case of Cournot competition with homogeneous products. The underlying result driving this finding is that cost increases raise consumer prices in oligopoly models with stable equilibria. Both taxes increase effective costs.

Proposition 1. Consumer prices rise with both the unit and ad valorem tax rate.

Proof. At a symmetric equilibrium, (2.3) holds with $p_i = p_{-i} = p^* = \widetilde{c} - D/\frac{\partial D}{\partial p_i}$. For the unit tax, differentiating yields

$$\frac{dp^*}{dt} = \left[1 + \frac{\partial}{\partial p^*}\left(D/\frac{\partial D}{\partial p_i}\right)\right]^{-1} = \frac{\varepsilon_{dd}}{\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m} > 0, \tag{3.1}$$

where we have used Lemma 1, and the inequality follows from (2.8).

Similarly, for the ad valorem case we have

$$\frac{dp^*}{d\tau} = \frac{c}{(1-\tau)^2}\left[1 + \frac{\partial}{\partial p^*}\left(D/\frac{\partial D}{\partial p_i}\right)\right]^{-1} = \frac{c}{(1-\tau)^2}\frac{dp^*}{dt} > 0. \tag{3.2}$$

□

Next, we turn to the effects of these taxes on short run producer prices. In perfectly competitive industries, producer prices either fall with commodity taxes or remain unchanged. In imperfectly competitive environments, however, taxes may be overshifted. For Cournot competition, Seade (1987) provides criteria for assessing the effects of taxes on prices and profits that involve the elasticity of the demand curve slope $E = -P_{xx}X/P_x$, where X is total output and $P(X)$ is inverse demand. Although our demand specification is quite different because of product heterogeneity, we provide a similar construction $\widetilde{E} \equiv \frac{\varepsilon_m}{\varepsilon_{DD}}$ representing the elasticity of the dd curve slope normalized by the elasticity of the DD curve.[8] We find that taxes are overshifted if $\widetilde{E}$ is sufficiently large, with the unit tax more likely to be overshifted than the ad valorem tax:

Proposition 2. An increase in the unit tax raises producer prices (overshifting) if and only if $\widetilde{E} > 1$. *An increase in the ad valorem tax raises producer prices if and only if* $\widetilde{E} > 1 - \frac{1}{\varepsilon_{DD}} > 1$. *Hence, overshifting of the ad valorem tax implies overshifting of the unit tax.*

Proof. For the unit case, we have

$$\frac{d(p^* - t)}{dt} = \frac{dp^*}{dt} - 1 = \frac{\varepsilon_m - \varepsilon_{DD}}{\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m} \overset{s}{=} \widetilde{E} - 1 \tag{3.3}$$

using (2.8) and the definition of $\widetilde{E}$.[9] For the ad valorem case,

$$\begin{aligned}\frac{d\left[(1-\tau)p^*\right]}{d\tau} &= -p^* + (1-\tau)\frac{dp^*}{d\tau} \\ &= -p^* + \widetilde{c}_A / \left[1 + \frac{\partial}{\partial p}\left(D / \frac{\partial D}{\partial p_i}\right)\right],\end{aligned}$$

where the denominator in the second term is positive by (3.1). Hence,

$$\begin{aligned}\frac{d\left[(1-\tau)p^*\right]}{d\tau} &\overset{s}{=} -\left(p^* - \widetilde{c}_A\right) - p^* \frac{\partial}{\partial p}\left(D / \frac{\partial D}{\partial p_i}\right) \\ &\overset{s}{=} \frac{1 - \varepsilon_{DD} + \varepsilon_m}{\varepsilon_{dd}} \overset{s}{=} \widetilde{E} - 1 + \frac{1}{\varepsilon_{DD}}\end{aligned} \tag{3.4}$$

using the first order condition (2.3), Lemma 1, and the elasticity definitions. □

Indeed, the unit tax entails a higher producer price than the ad valorem tax when the taxes are set so as to equate the effective costs (i.e., $c + t = c/(1-\tau)$).

[8] The definition of E involves an elasticity with respect to output while $\widetilde{E}$ involves elasticities with respect to price. This is to be expected from the respective Cournot and Bertrand roots, although the difference might appear disquieting. To allay suspicion that E and $\widetilde{E}$ might be very different, consider the monopoly case. Then there is no distinction between the dd and DD curves, and $\widetilde{E} = D(p)\frac{\partial^2 D}{\partial p^2} / \left(\frac{\partial D}{\partial p}\right)^2$. Inverting the various functions so $D(p) = X$, $\frac{\partial D}{\partial p} = 1/P_x$, and $\frac{\partial^2 D}{\partial p^2} = -P_{xx}/\left(P_x\right)^3$ shows that $E = \widetilde{E}$ for the monopoly case.

[9] The symbol "$\overset{s}{=}$" means "has the same sign as."

Then consumer prices and outputs are the same. However, tax revenue is higher under the ad valorem tax (Anderson, de Palma, and Kreider, 2000; Proposition 1) so the producer price is lower. The ad valorem tax thus has a greater incidence on producers than that of the unit tax.

Our results are consistent with previous results for the Cournot model. In particular, our $\widetilde{E} > 1$ elasticity condition is analogous to Seade's (1987) finding that the unit tax is overshifted whenever $E > 1$. The intuition for this result is easily illustrated for monopoly. Imposing a unit tax increases effective marginal cost from c to $c+t$. Then the equilibrium marginal revenue will also increase by t. If demand is steeper than marginal revenue, i.e. if $P_x < (P + XP_x)' = 2P_x + XP_{xx}$ or equivalently if $E \equiv -P_{xx}X/P_x > 1$, then the equilibrium price rises by more than t and thus the tax is overshifted.[10] The oligopoly case is a clean extension of this result.

Likewise, our conclusion that overshifting of the unit tax is necessary for overshifting of the ad valorem tax is consistent with Delipalla and Keen (1992). To see why, consider the contrapositive statement that undershifting of the unit tax implies undershifting of the ad valorem tax. If the unit tax is undershifted, then $dp^* < d\widetilde{c}$, so the consumer price rises by less than the increase in effective cost. If the ad valorem tax is to induce an equivalent increase in effective cost, then $d\widetilde{c} = \frac{c}{(1-\tau)^2}d\tau$. Hence $dp^* < \frac{c}{(1-\tau)^2}d\tau$. For an ad valorem tax to reduce the producer price, $d[(1-\tau)p^*] = -p^*d\tau + (1-\tau)dp^* < 0$: given the above inequality, the producer price falls under the ad valorem tax if $-(p^* - \widetilde{c})d\tau < 0$, which is necessarily true since $p^* > \widetilde{c}$ (the mark-up is positive).

Empirically, Delipalla and O'Donnell (1998) analyze the incidence of cigarette taxes in Europe and find support for the theoretical proposition that the unit tax has a larger effect on price. Poterba (1996) provides empirical evidence on price responses to changes in state and local sales taxes by exploiting variation in tax policy across cities and across time. Using postwar quarterly price data on clothing and personal care items, he cannot reject the hypothesis that consumer prices adjust one-for-one with tax changes; his analysis of pre-war data points to some undershifting. Besley and Rosen (1998), in contrast, find evidence of substantial overshifting for more than half of their (quite disaggregated) commodities such as bananas, bread, milk, and shampoo.

That the tax can be passed on by more than 100 percent is illustrated for the case of a duopoly in Figure 1 (using a special case of the model presented in Example 1 below). The initial equilibrium is at the intersection of the lower

[10] The usual textbook diagram has marginal revenue steeper than demand ($E < 1$).

pair of reaction functions. It is useful to think of the tax (here 25 cents) as being imposed first on Firm 1, then on Firm 2. With the first tax, the "impact effect" – that is, holding Firm 2's price fixed – is that Firm 1 raises its consumer price (here by 14 cents), although typically by less than the tax increase. Firm 2 then raises its price since the goods sold are substitutes, which elicits a further increase by Firm 1, etc., leading to a form of multiplier effect so that the new equilibrium (at the intersection of Firm 1's $t = .25$ reaction function and Firm 2's $t = 0$ reaction function) involves substantially higher prices for both firms (here Firm 1's price increases by 19 cents and Firm 2's price increases by 9 cents). This is because reaction functions slope up in prices and a tax shifts up the reaction function of the firm facing the tax. When we then impose the tax on the other firm too, the tax can be overshifted (the final price is 28 cents higher here).

At first glance, this argument would suggest that this result is specific to Bertrand competition. More surprising perhaps is that it can also hold in the Cournot model, especially since one is used to Cournot and Bertrand competition giving opposite results. In the Cournot case, the thought experiment above is applied to outputs rather than prices. The impact effect of the tax on one firm is that it cuts output, raising the (market) consumer price. The response of the rival firm depends on the slope of its reaction function. If the output reaction functions slope down, the response is a smaller output rise from the rival firm, which then causes the firm to cut back more. If the reaction functions slope up,[11] the rival firm also cuts back its output, further raising the market price. In either case, the new equilibrium involves lower total output and so a higher consumer price. Again, imposing then the tax on the other firm too can yield overshifting, and necessarily does so under the unit tax when reaction functions slope up.

Example 1. Generalized CES-Logit model.

In this model, each consumer chooses one of the n firms to buy from and then chooses the amount to purchase.[12] Consumer j's indirect utility associated with buying from Firm i is given by

$$V_{ij} = y_j + v(p_i) + \mu e_{ij},$$

where y_j is income (assumed not to be binding), $v(p_i) = \left(1 - p_i^{1-\alpha}\right)/(1-\alpha)$ with $\alpha \geq 0$ is consumer surplus from buying product i at price p_i, and e_{ij} are

[11] Upward-sloping reaction functions arise for Cournot duopoly in the neighborhood of the equilibrium if $E > 2$.

[12] The model is described more fully in Anderson and de Palma (2000).

independent double exponentially distributed taste values. The parameter $\mu > 0$ measures the intensity of taste heterogeneity; a large value of μ corresponds to strong ties between consumers and particular firms.

Roy's Identity yields a conditional demand function with constant elasticity of demand α conditional on purchasing good i. Hence Firm i's demand is $D(p_i, p_{-i}) = p_i^{-\alpha}\mathbb{P}_i$, where $\mathbb{P}_i$ is the fraction of consumers who choose to purchase from Firm i. Under the double exponential distribution,[13] the fraction of customers purchasing good i can be derived as a logit form

$$\begin{aligned}\mathbb{P}_i &= \Pr(V_i > V_k,\, k = 1, ..., n,\, i \neq k) \\ &= \frac{\exp\left(v(p_i)/\mu\right)}{(n-1)\exp\left(v(p_{-i})/\mu\right) + \exp\left(v(p_i)/\mu\right)}.\end{aligned}$$

The demand facing a firm is given by the logit model when $\alpha = 0$, with each consumer purchasing one unit of his or her preferred good. It is given by the CES model when $\alpha \to 1$.

It is now straightforward to calculate $\varepsilon_{dd} = -\left[\alpha + p^{1-\alpha}(1-\mathbb{P})\right]$, $\varepsilon_{DD} = -\alpha$, $\varepsilon_m = -\frac{\alpha(\alpha+1)+2\alpha p^{1-\alpha}(1-\mathbb{P})}{\alpha+p^{1-\alpha}(1-\mathbb{P})}$, and $\widetilde{E} = \frac{\alpha+1+2p^{1-\alpha}(1-\mathbb{P})}{\alpha+p^{1-\alpha}(1-\mathbb{P})}$. By Proposition 1, prices are overshifted under the unit tax if $\alpha > 0$ since then $\widetilde{E} > 1$. The price equilibrium solves the implicit equation $1 = (p - \widetilde{c})\left(\frac{\alpha}{p} + p^{-\alpha}\frac{n-1}{\mu n}\right)$ (see our working paper, Equation 6). The unit tax has no effect on producer prices if $\alpha = 0$ since then the equilibrium price is simply $p^* = \widetilde{c} + \mu n/(n-1)$. For the ad valorem case (see (3.4)), we have $\varepsilon_{DD} - \varepsilon_m - 1 = -\alpha + \frac{\alpha(\alpha+1)+2\alpha p^{1-\alpha}(1-\mathbb{P})}{\alpha+p^{1-\alpha}(1-\mathbb{P})} - 1$. This value is zero for $\alpha = 1$, in which case the tax is fully passed on to the consumer. We find undershifting of the tax for $\alpha < 1$ and overshifting for $\alpha > 1$. Hence, taxes are overshifted under an ad valorem tax if and only if demand is elastic.[14]

The constant elasticity analogue for Cournot competition products involves the inverse demand specification $p(X) = X^{-\frac{1}{\alpha}}$, which leads to $E = 1 + \frac{1}{\alpha}$. Parallel to our finding for the unit tax, $\alpha > 0$ implies $E > 1$ and thus overshifting of this tax (Seade, 1987). (The $\alpha = 0$ case is undefined under Cournot competition.) For the ad valorem tax under Cournot competition, summing the output first order

[13] Our working paper presents a more general model with logconcave taste distributions (log-concavity guarantees the existence of a Bertrand Nash equilibrium). The same qualitative results hold.

[14] It is readily verified that this result holds for the special case of a monopolist facing an isoelastic demand curve with $\alpha > 1$. (Note that the monopoly problem is undefined for $\alpha < 1$ since in that case the monopolist's profit strictly decreases with output for any positive output.)

conditions and using the isoelastic demand specification yields $(1-\tau)p = \frac{\alpha n c}{\alpha n - 1}$, implying that the producer price is invariant to changes in the ad valorem tax rate.[15]

In summary, the unit tax is always overshifted under this specification for both Bertrand competition with differentiated products and the Cournot model. The ad valorem tax can be either overshifted ($\alpha > 1$) or undershifted ($\alpha < 1$) under Bertrand competition, while there is always 100 percent pass-on under Cournot competition for constant elasticity demand.

Example 2. Linear demand.

Overshifting does not hold for all reasonable specifications of product differentiation. The common linear demand specification implies $\widetilde{E} = 0$ so that both taxes are undershifted by firms. To confirm this for a duopoly, let a firm's demand be given by $D_i = A(n) - p_i + \phi(n)p_{-i}$ with $\phi(n) \in (0,1]$, which yields a symmetric Bertrand equilibrium consumer price $p^* = [A(n) + \widetilde{c}] / [2 - \phi(n)]$. The pass-on rate for the producer price is $\frac{d(p^*-t)}{dt} = -\frac{1-\phi}{2-\phi} < 0$ for the unit tax and $\frac{d[(1-\tau)p^*]}{d\tau} = -\frac{A(n)}{2-\phi} < 0$ for the ad valorem tax.[16] With a homogeneous product linear demand and under Cournot competition, undershifting can readily be shown.

The linear demand example underscores the limitations of the constant elasticity approach used in the first example – while it encompasses some commonly used models, it is nevertheless restrictive. This latter point can be important in empirical calibration: if one were to estimate such a generalized discrete choice model, one would necessarily build in the tax-overshifting property for the unit tax which could lead to misleading policy prescriptions.

3.2. The effects on profits in the short run

The possibility of excess pass-on in Proposition 2 raises the possibility that taxes may even raise profits, a result that has also been found for the Cournot model (see especially Seade, 1987). We find an analogous condition for our analysis of Bertrand competition with differentiated products. Proposition 3 shows that

[15] A sufficient condition that ensures that this is an equilibrium for oligopoly is $\alpha \geq 1$ since then profit functions are concave. For α small, $\left(\alpha < \frac{1}{2n-1}\right)$ there can be no (symmetric) equilibrium because the profit function is strictly convex in the neighborhood of the candidate solution.

[16] We are grateful to an anonymous referee for pointing out this example.

overshifting is a requisite for higher profits. This is readily apparent by examining the profit function. A higher tax rate increases the consumer price (Proposition 1) and hence decreases demand. For profits to rise, the after-tax markup must rise. Recall that the unit tax is overshifted if $\widetilde{E} > 1$. We now show that profits increase if $\widetilde{E} > 2$ and that profits are more likely to rise under unit taxation than under ad valorem taxation.

Proposition 3. Profits rise with the unit tax if and only if $\widetilde{E} > 2$, and profits rise with the ad valorem tax if and only if $\widetilde{E} > 2 + \frac{1}{\varepsilon_{dd}} - \frac{1}{\varepsilon_{DD}} > 2$. A profit increase under the unit tax is a necessary condition for profits to rise under the ad valorem tax. Tax overshifting is necessary for a profit increase.

Proof. The effect of an increase in the unit tax on equilibrium profits, π_U, can be decomposed as

$$\frac{d\pi_U}{dt} = \frac{d(p^* - \widetilde{c}_U)}{dt} D + (p^* - \widetilde{c}_U)\frac{\partial D}{\partial p^*}\frac{dp^*}{dt}.$$

The second term is negative since the effective markup $(p^* - \widetilde{c}_U)$ is positive, $\frac{\partial D}{\partial p^*} < 0$, and $\frac{dp^*}{dt} > 0$ (Proposition 1).

For the unit case, the profit equation (2.1) and first order condition (2.3) also imply $\pi_U = -D^2/\frac{\partial D}{\partial p_i}$. The effect of the tax on profits is then

$$\begin{aligned} \frac{d\pi_U}{dt} &= -\frac{dp^*}{dt}\frac{\partial}{\partial p}\left(D^2/\frac{\partial D}{\partial p_i}\right) \stackrel{s}{=} -\frac{\partial}{\partial p}\left(D^2/\frac{\partial D}{\partial p_i}\right) \qquad (3.5) \\ &= -D\left(\frac{2\varepsilon_{DD} - \varepsilon_m}{\varepsilon_{dd}}\right) \stackrel{s}{=} \widetilde{E} - 2 \end{aligned}$$

using $\frac{dp^*}{dt} > 0$ from Proposition 1 and Lemma 2.

For the ad valorem case, equilibrium profit is $\pi_A = -(1-\tau)D^2/\frac{\partial D}{\partial p_i}$ and

$$\begin{aligned} \frac{d\pi_A}{d\tau} &= D^2/\frac{\partial D}{\partial p_i} - (1-\tau)\frac{dp^*}{d\tau}\frac{\partial}{\partial p}\left(D^2/\frac{\partial D}{\partial p_i}\right) \\ &= \frac{Dp^*}{\varepsilon_{dd}} - \widetilde{c}_A \frac{2\varepsilon_{DD} - \varepsilon_m}{\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m} D, \end{aligned}$$

where we have used (3.2) and Lemma 2. Substituting in $\widetilde{c}_A = p^*\left(1 + \frac{1}{\varepsilon_{dd}}\right)$ from the first order condition (2.3) yields

$$\frac{d\pi_A}{d\tau} \stackrel{s}{=} \frac{1}{\varepsilon_{dd}} - \left(\frac{1}{\varepsilon_{dd}} + 1\right)\left(\frac{2\varepsilon_{DD} - \varepsilon_m}{\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m}\right),$$

where $\varepsilon_{dd} + \varepsilon_{DD} - \varepsilon_m < 0$ by the stability condition. Further manipulation gives

$$\begin{aligned}\frac{d\pi_A}{d\tau} &\overset{s}{=} (\varepsilon_{dd} - \varepsilon_{DD}) - \varepsilon_{dd}(2\varepsilon_{DD} - \varepsilon_m) \\ &\overset{s}{=} \widetilde{E} - 2 + \left(\frac{1}{\varepsilon_{dd}} - \frac{1}{\varepsilon_{DD}}\right).\end{aligned} \tag{3.6}$$

The last term in parentheses is negative by (2.7) and $\frac{d\pi_A}{dt} = \widetilde{E} - 2$ from above. Hence, $\frac{d\pi_A}{d\tau} > 0$ implies $\frac{d\pi_U}{dt} > 0$. □

Our result for the unit tax is parallel to Seade's (1987) finding that profits rise with the unit tax whenever $E > 2$. Similarly, our result that profits cannot rise under the ad valorem tax unless they rise under the unit tax parallels a finding by Delipalla and Keen (1992) for Cournot competition.

It is nevertheless noteworthy that demand curves for which $E > 2$ are "highly convex" and as such are rarely considered in the literature on Cournot competition.[17] Indeed, the condition $E > 2$ implies that the marginal revenue curve to the industry demand curve slopes *up*. For monopoly, this means the solution with marginal revenue equals marginal cost constitutes a local minimum. For Cournot duopoly, upward-sloping reaction functions arise in the neighborhood of the candidate equilibrium if $E > 2$. While not inadmissable, this is rather unusual. While the shape of the demand function is an empirical matter, we just wish to point out that the cases when taxes increase profits (and, to a lesser extent, when there is overshifting) are usually ruled out by assumption when economists write down simple Cournot models.[18] In Bertrand models with differentiated products, the possibility of overshifting and profit-increasing taxes is prevalent even in some of the most common formulations, as the next example illustrates.

Example 3. Returning to the CES-Logit model of Example 1, we find that profits fall with the unit tax for $\alpha > 1$, stay the same for $\alpha = 0$ or $\alpha = 1$, and rise for $\alpha \in (0, 1)$. Hence, profits rise with the unit tax when demand is inelastic,[19] a result that also holds for the Cournot model and isoelastic demand. Using the expression for ε_{dd} in equation (2.3), we find the standard CES result that equilibrium price

[17] One problem is that the existence of an equilibrium can be jeopardized.

[18] A standard assumption is that $E < 0$, or that demand is concave; sometimes this is weakened to $E < 1$, which is associated with logconcave demand.

[19] In the example of Figure 1, profits rise (from 93 cents to 94 cents) because $\alpha = .1$.

is proportional to effective cost in the limit case of unit elasticity ($\alpha = 1$). Hence, with unit demand elasticity a one percent increase in marginal cost induces a one percent increase in the equilibrium consumer price, so both total costs and total revenue are unaltered. In the special case $\alpha = 0$, a one dollar increase in t leads to a one dollar increase in p^* so again profits are unchanged because aggregate demand is totally inelastic. Between these limits, the revenue increase exceeds the cost increase, while for $\alpha > 1$ the increase in producer price is insufficient to offset the demand decline.

Profits always fall under this specification for the ad valorem tax. This can be shown directly for $\alpha \geq 1$ by calculating $\frac{d\pi_A}{d\tau}$ and checking its sign. It was shown in Example 1 that the producer price falls with τ for $\alpha < 1$, which implies that profits always fall by Proposition 3. This result also holds for Cournot competition with isoelastic demand. As argued in Example 1, the producer price is independent of τ in the Cournot model, so a higher τ increases the consumer price and decreases profits by reducing demand.

Example 4. In the linear demand model, profits cannot rise with taxes since we saw previously that the taxes are never overshifted under this specification. This result also holds under Cournot competition.

With the short run results in hand for a fixed number of firms, we turn to an analysis of the long run effects of taxes when the number of firms is endogenous.

4. Tax Incidence in the Long Run

For the long run analysis, we allow the number of firms to adjust to the tax policy through the process of entry and exit, with profits driven to zero. Write $D(p_i, p_{-i}; n)$ as the demand addressed to an individual firm, which will be denoted $D(p^*, p^*; n)$ in equilibrium. We assume the demand system satisfies properties that ensure that increasing the number of firms decreases gross profit and decreases consumer and producer prices (as would be expected from increased competition).We first assume that

$$\varepsilon_n \equiv \frac{\partial D}{\partial n}\frac{n}{D} < 0 \tag{4.1}$$

so that adding firms does not increase demand per firm (with prices constant). This condition is satisfied for the CES-Logit specification. We also assume that

$$\varepsilon_n < \varepsilon_q, \tag{4.2}$$

where $\varepsilon_q \equiv \frac{\partial}{\partial n}\left(\frac{\partial D}{\partial p_i}\right)\left(n/\frac{\partial D}{\partial p_i}\right)$ is the elasticity of the dd curve slope with respect to n.[20] Given (4.1), we show that this is equivalent to a condition that prices decline with the number of firms. Indeed, from the pricing condition (2.3) we find

$$\frac{dp^*}{dn} = -\frac{\partial}{\partial n}\left(D/\frac{\partial D}{\partial p_i}\right) / \left[1 + \frac{\partial}{\partial p^*}\left(D/\frac{\partial D}{\partial p_i}\right)\right]. \tag{4.3}$$

Since the denominator is positive by the stability condition (2.6), we have

$$\frac{dp^*}{dn} \overset{s}{=} -\frac{\partial}{\partial n}\left(D/\frac{\partial D}{\partial p_i}\right) \overset{s}{=} \frac{\partial D}{\partial n}\frac{n}{D} - \frac{\partial}{\partial n}\left(\frac{\partial D}{\partial p_i}\right) n \left(\frac{\partial D}{\partial p_i}\right)^{-1} = \varepsilon_n - \varepsilon_q < 0. \tag{4.4}$$

Condition (4.2) then implies that $\frac{dp^*}{dn} < 0$: more competition lowers prices.

The long run equilibrium is defined by the equilibrium pricing condition (2.3) and a zero-profit condition. Letting K denote a fixed entry cost, profits are zero under the unit and ad valorem tax, respectively, when

$$-D^2/\frac{\partial D}{\partial p_i} - K = 0 \tag{4.5}$$

and

$$-(1-\tau)D^2/\frac{\partial D}{\partial p_i} - K = 0. \tag{4.6}$$

Profits fall with the number of firms under our assumptions since both prices and demand fall with the number of firms. Note also that an increase in short run profits leads to an increase in the long run number of firms.

4.1. Long run consumer price

In the short run, the equilibrium consumer price always increases with either the unit or ad valorem tax by Proposition 1. It must also rise in the long run if profits fall with the tax since then the number of firms declines and less competition exacerbates the short run rise in price. More striking is that the long run price rises even if profits increase; the additional entry is not enough to completely erode the higher prices. This result is stated in the next proposition:

[20] This inequality is satisfied for the CES-Logit model in which case $\varepsilon_n = -1$ and $\varepsilon_q = -\frac{n\alpha+(n-2)p^{1-\alpha}}{n\alpha+(n-1)p^{1-\alpha}}$.

Proposition 4. The long run equilibrium consumer price rises with both the unit tax and the ad valorem tax.

Proof. For the unit tax, the first order condition (2.3) and zero profit condition (4.5) yield the comparative statics

$$\begin{pmatrix} 1+\frac{\partial}{\partial p}\left(D/\frac{\partial D}{\partial p_i}\right) & \frac{\partial}{\partial n}\left(D/\frac{\partial D}{\partial p_i}\right) \\ \frac{\partial}{\partial p}\left(D^2/\frac{\partial D}{\partial p_i}\right) & \frac{\partial}{\partial n}\left(D^2/\frac{\partial D}{\partial p_i}\right) \end{pmatrix} \begin{pmatrix} \frac{dp^*}{dt} \\ \frac{dn^*}{dt} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{4.7}$$

which can be written in elasticity form as

$$\begin{pmatrix} 1+\frac{\varepsilon_{DD}-\varepsilon_m}{\varepsilon_{dd}} & \frac{p^*}{n^*}\frac{\varepsilon_n-\varepsilon_q}{\varepsilon_{dd}} \\ 2\varepsilon_{DD}-\varepsilon_m & \frac{p^*}{n^*}\left(2\varepsilon_n-\varepsilon_q\right) \end{pmatrix} \begin{pmatrix} \frac{dp^*}{dt} \\ \frac{dn^*}{dt} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{4.8}$$

Then

$$\frac{dp^*}{dt} = \frac{p^*}{n^*}\left(2\varepsilon_n-\varepsilon_q\right)/\Delta \tag{4.9}$$

where $\Delta \equiv \frac{p^*}{n^*\varepsilon_{dd}}\left[\varepsilon_q\left(\varepsilon_{DD}-\varepsilon_{dd}\right)+\varepsilon_n\left(2\varepsilon_{dd}-\varepsilon_m\right)\right]$. The numerator in (4.9) is negative by (4.1) and (4.2). Some manipulation yields $\Delta \overset{s}{=} -\left[\varepsilon_q\left(\varepsilon_{DD}-\varepsilon_{dd}\right)+\varepsilon_n\left(2\varepsilon_{dd}-\varepsilon_m\right)\right] < -\left[\varepsilon_n\left(\varepsilon_{DD}-\varepsilon_{dd}\right)+\varepsilon_n\left(2\varepsilon_{dd}-\varepsilon_m\right)\right] = -\varepsilon_n\left(\varepsilon_{dd}+\varepsilon_{DD}-\varepsilon_m\right) < 0$ using $\varepsilon_{DD} > \varepsilon_{dd}$ and $\varepsilon_n < \varepsilon_q$ for the first inequality; the second inequality follows from the stability condition (2.8) and $\varepsilon_n < 0$. This establishes a positive relationship between the unit tax rate and the long run equilibrium consumer price.

A parallel result holds for the ad valorem tax. In that case, the comparative statics in (4.8) are replaced by

$$\begin{pmatrix} 1+\frac{\varepsilon_{DD}-\varepsilon_m}{\varepsilon_{dd}} & \frac{p^*}{n^*}\frac{\varepsilon_n-\varepsilon_q}{\varepsilon_{dd}} \\ 2\varepsilon_{DD}-\varepsilon_m & \frac{p^*}{n^*}\left(2\varepsilon_n-\varepsilon_q\right) \end{pmatrix} \begin{pmatrix} \frac{dp^*}{d\tau} \\ \frac{dn}{d\tau} \end{pmatrix} = \begin{pmatrix} \frac{c}{(1-\tau)^2} \\ \frac{p^*}{1-\tau} \end{pmatrix}, \tag{4.10}$$

which gives

$$\frac{dp^*}{d\tau} = \frac{p^*}{(1-\tau)\,n}\left[\frac{c}{1-\tau}\left(2\varepsilon_n-\varepsilon_q\right)-p^*\frac{\varepsilon_n-\varepsilon_q}{\varepsilon_{dd}}\right]/\Delta > 0. \tag{4.11}$$

□

The comparative statics also confirm that higher tax rates increase the number of firms whenever they increase short term profits.[21]

[21] For the unit tax, the system of equations yields $\frac{dn^*}{dt} \overset{s}{=} 2\varepsilon_{DD}-\varepsilon_m \overset{s}{=} \widetilde{E}-2$, which is the same condition as (3.5). The analogous condition for the ad valorem tax is $\frac{dn^*}{d\tau} \overset{s}{=} \left(\varepsilon_{dd}-\varepsilon_{DD}\right) - \varepsilon_{dd}\left(2\varepsilon_{DD}-\varepsilon_m\right)$, which corresponds to equation (3.6).

4.2. Long run producer price

We now examine the effect of taxation on long run producer prices. By equation (3.3), the producer price increases with the unit tax in the short run if and only if $\widetilde{E} > 1$. If profit falls with the unit tax, then exit occurs and the consumer price increases. Since profits fall when $\widetilde{E} < 2$ by equation (3.5), it follows that the producer price must increase with this tax if $\widetilde{E} \in (1, 2)$ and in this case the exit effect reinforces the short-run rise. As shown next, however, a rising short run price under the unit tax guarantees a rising long run price, regardless of the effect of the tax on the number of firms.

Proposition 5. The long run equilibrium producer price rises with the unit tax if and only if $\varepsilon_n \varepsilon_m - \varepsilon_{DD} \varepsilon_q > 0$, which holds if $\widetilde{E} > 1$. This price rises with the ad valorem tax if and only if $\varepsilon_n \varepsilon_m - \varepsilon_{DD} \varepsilon_q > -\varepsilon_n > 0$. Hence, the producer price rises with the unit tax if it rises with the ad valorem tax.

Proof. From (4.9), the impact of the unit tax on the producer price is given by

$$\begin{aligned} \frac{d\left(p^* - t\right)}{dt} &= \frac{p^*}{n}\left(2\varepsilon_n - \varepsilon_q\right)/\Delta - 1 \overset{s}{=} \varepsilon_n \varepsilon_m - \varepsilon_q \varepsilon_{DD} \\ \overset{s}{=} \widetilde{E} - \frac{\varepsilon_q}{\varepsilon_n} &< \widetilde{E} - 1. \end{aligned}$$

For the ad valorem case, (4.11) implies

$$\begin{aligned} \frac{d\left[(1-\tau)p^*\right]}{d\tau} &= (1-\tau)\frac{dp^*}{d\tau} - p^* \\ &= \frac{p^*}{n}\left[\frac{c}{1-\tau}\left(2\varepsilon_n - \varepsilon_q\right) - p^* \frac{\varepsilon_n - \varepsilon_q}{\varepsilon_{dd}}\right]/\Delta - p^*. \end{aligned}$$

Substituting $\frac{c}{1-\tau} = p^*\left(\frac{1+\varepsilon_{dd}}{\varepsilon_{dd}}\right)$ from the first order condition (2.3) and manipulating obtains

$$\begin{aligned} \frac{d\left[(1-\tau)p^*\right]}{d\tau} &= \frac{p^*}{n}\left[p^*\left(\frac{1+\varepsilon_{dd}}{\varepsilon_{dd}}\right)\left(2\varepsilon_n - \varepsilon_q\right) - p^* \frac{\varepsilon_n - \varepsilon_q}{\varepsilon_{dd}}\right]/\Delta - p^* \\ &\overset{s}{=} \varepsilon_n\left(\varepsilon_m + 1\right) - \varepsilon_{DD}\varepsilon_q. \end{aligned}$$

Since $\varepsilon_n < 0$, the producer price clearly rises with the unit tax whenever it rises with the ad valorem tax. □

Corollary. The long run equilibrium producer price rises with either tax in the long run if it rises in the short run.

Proof. By inspection of the conditions in Propositions 2 and 5: for the unit tax, $p^* - t$ rises with t in the short run if and only if $\widetilde{E} > 1$, while it rises with t in the long run if and only if $\widetilde{E} > \frac{\varepsilon_q}{\varepsilon_n}$, where $\frac{\varepsilon_q}{\varepsilon_n} < 1$. Producer prices rise under the ad valorem tax in the short run if and only if $\widetilde{E} > 1 - \frac{1}{\varepsilon_{DD}}$, while they rise in the long run if and only if $\widetilde{E} > \frac{\varepsilon_q}{\varepsilon_n} - \frac{1}{\varepsilon_{DD}}$.

This result is consistent with Besley's (1989) Cournot conclusion that an increase in the unit tax is more likely to increase producer prices in the long run than in the short run (the unit tax increases long run producer prices whenever $E > 0$) When $\widetilde{E} > 2$, the unit tax leads to a smaller increase in the long run price than the short run price because higher profits induce firm entry, thus dampening the price increase. The opposite is true when $\widetilde{E} < 2$ since then profits fall with the tax; firms exit and prices are driven still higher.

For the CES-Logit demand specification, the long run producer price always rises with the unit tax for $\alpha > 0$ since it satisfies the sufficient condition $\widetilde{E} > 1$ in Proposition 5 (see Example 1). For $\alpha = 0$, the short run producer price is independent of t, and, since total demand is perfectly inelastic, so is profit. There is no entry or exit, so the long run producer price is independent of t. For the ad valorem tax, we have $\varepsilon_n (\varepsilon_m + 1) - \varepsilon_{DD}\varepsilon_q \overset{s}{=} n(\alpha - 1) + 1$,[22] which implies from Proposition 5 that the producer price rises with that tax for $\alpha \geq 1$ and falls for $\alpha = 0$ (and for other sufficiently low values of α). For the linear demand specification $D_i = A(n) - p_i + \phi(n)p_{-i}$ (and with $A'(n), \phi'(n) \leq 0$ so that (4.2) is satisfied) we have $\varepsilon_n < \varepsilon_m = \varepsilon_q = 0$. This implies that the long run producer price is unaffected by the unit tax, meaning that the short run price decrease (see Example 2) is exactly offset by the increase resulting from the induced exit. For the ad valorem tax, the short run decrease is not recovered despite exit.

5. Conclusion

We find that conclusions in the literature concerning the economic incidence of indirect taxes in Cournot oligopoly settings with homogeneous products extend

[22] This follows since for the CES-Logit model we have $\varepsilon_n = -1$, $\varepsilon_m = -\frac{n\alpha(\alpha+1)+2\alpha(n-1)p^{1-\alpha}}{n\alpha+(n-1)p^{1-\alpha}}$, $\varepsilon_{DD} = -\alpha$, and $\varepsilon_q = -\frac{n\alpha+(n-2)p^{1-\alpha}}{n\alpha+(n-1)p^{1-\alpha}}$.

nicely to arguably more realistic Bertrand-Nash environments with differentiated products. In particular, our results tend to amplify Delipalla and Keen's (1992) conclusions about the differential effects of ad valorem and unit excises taxes on prices and profits in the short run, and on prices and numbers of firms in the long run. The direct correspondences between most of our results may come as a surprise to readers accustomed to seeing these quite different market mechanisms lead to disparate conclusions. Despite very different underlying models, our results for the unit tax are strikingly similar to those for Cournot competition, with the analogous parameter values for demand elasticity determining the crucial cutoff values for different regimes. We also find some strong similarities for the ad valorem tax, though some of the correspondences are different in terms of critical parameter values. Our companion paper (Anderson, de Palma, and Kreider, 2000) shows that the social welfare dominance of ad valorem taxes in Cournot settings, established in the previous literature, can be overturned in both the short run and the long run when firms compete in prices and sell differentiated products.

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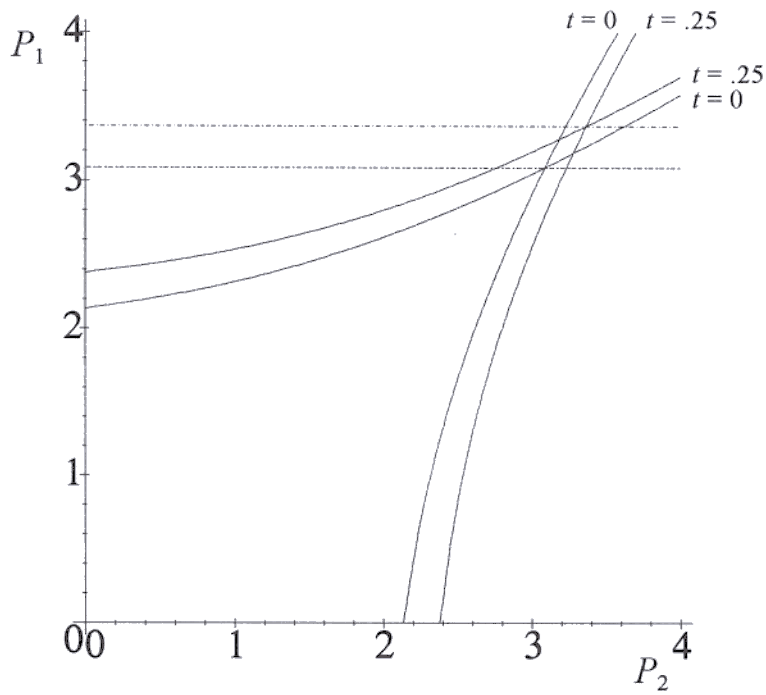
Figure 1



CES-Logit ($\alpha = .1$, $c = 1$, $t = .25$)

The 25 cent unit tax is overshifted as the equilibrium price rises 28 cents from \$3.09 to \$3.37.