RAND Journal of Economics
Vol. 30, No. 4, Winter 1999
pp. 719–735

Pricing, product diversity, and search costs: a Bertrand-Chamberlin-Diamond model

Simon P. Anderson*

and

Régis Renault**

We study price competition in the presence of search costs and product differentiation. The limit cases of the model are the "Bertrand Paradox," the "Diamond Paradox," and Chamberlinian monopolistic competition. Market prices rise with search costs and decrease with the number of firms. Prices may initially fall with the degree of product differentiation because more diversity leads to more search and hence more competition. Equilibrium diversity rises with search costs, while the optimum level falls, so entry is excessive. The market failure is most pronounced for low preference for variety and high search costs.

1. Introduction

■ Why do consumers shop around before buying? One reason is to find a low price. Another is to find a product they like. There are many industries in which buyer search is an important feature of market interaction. Shopping for shoes is one example; another is a business that must decide between competing suppliers (e.g., builders choosing where to buy inputs). One would expect lower search costs to lead buyers to search more options before purchasing and hence lead to lower prices. Likewise, standard economic intuition suggests that prices would be lower if there were more firms because consumers can search across more options.

The consequences of buyer search on market performance are potentially severe. Diamond (1971) provides an extreme example, in what has been called the Diamond Paradox. There are two parts to the paradox. First, in Diamond's model the only equilibrium is for all firms to set the monopoly price regardless of the number of firms and the level of search costs (as long as these costs are positive). Second, in equilibrium, buyers do not search. The idea is loosely the following. Suppose there were an equilibrium at which some firm set a price below the monopoly price and (weakly) below

* University of Virginia; sa9w@virginia.edu.

** GEMMA, Université de Caen; regis.renault@econ.unicaen.fr.

We should like to thank Kyle Bagwell and an anonymous referee for useful suggestions as well as Winand Emons for providing us with the London funerals example. The first author gratefully acknowledges financial support from the Bankard Fund at the University of Virginia and from the NSF under grant no. SBR-9617784, and we are both grateful for travel funding from the CNRS and NSF under grant no. INT-9815703.

those set by all other firms. Then all consumers who go to the low-price firm would still buy there even if the firm raised its price by an amount less than the cost of searching another firm, since a consumer who searched again could not expect to gain enough to offset the search cost. This incentive to raise price means that all firms must charge the monopoly price in equilibrium. Since buyers rationally anticipate that the same price is charged by all firms in equilibrium, they have no reason to search. Hence this is a search model without search, and there is a large discontinuity between the market solution with search costs and the market outcomes traditionally analyzed by economists. In particular, the Bertrand (1883) model (where consumers are perfectly informed about prices) predicts marginal cost pricing independently of the number of firms. This latter result, which is also counter to common sense, has been termed the Bertrand Paradox (Tirole, 1988).

The only motive a consumer has to search in the Diamond model is to find a better price. As in the Bertrand analysis, products sold by different firms are implicitly assumed to be homogeneous, so that buyers know exactly what their consumer surplus is. This framework does not account for consumers searching for a product they like. To capture this idea, it is necessary to introduce heterogeneity across products. This article considers the effects of product heterogeneity on the performance of markets with consumer search costs.

The effects of product heterogeneity per se on market performance were described by Chamberlin (1933). He argued that prices would exceed marginal cost because firms have some market power even when there are many of them. Chamberlin's argument has since been investigated rigorously by many subsequent authors, in various frameworks. The one closest to our own is the model of Perloff and Salop (1985), which looks at monopolistic competition as the limit, as the number of firms gets large, of price competition in oligopoly with a discrete-choice model of differentiated products. These authors found that prices in the limit would exceed marginal cost only under some specifications of taste heterogeneity that they and other authors have found to be rather stringent.[1] This development led other authors to search for models that would generate ''true'' monopolistic competition. Most pertinent to our study is Wolinsky (1986). Wolinsky proposed an adroit model that appends consumer search to the Perloff-Salop framework and showed that price necessarily exceeds marginal cost in the limit of an infinite number of firms. Wolinsky also notes (1986) that the mechanism driving this result is different from the Diamond Paradox: in his model the limit price rises with search costs. We shall show here that the Wolinsky model yields the Diamond model as a special case—a feature Wolinsky overlooked—as well as generating such other models as standard Bertrand competition and the Perloff-Salop model as other special cases. The general model provides a rich framework that yields intuitive comparative statics results: the equilibrium price rises with search costs and falls with the number of firms. These comparative statics results constitute the building blocks for our major results, which address the effects of increased taste for variety on the equilibrium price and the comparison between equilibrium and optimum diversity.

In models of product differentiation without consumer search, equilibrium prices typically rise with the degree of consumer taste for diversity (see Anderson, de Palma, and Thisse (1992) and references therein). This is because greater taste for diversity imparts more market power through more intense preferences. The picture changes dramatically when there are consumer search costs. With low preference for diversity, the Diamond result prevails, since consumers have no incentive to search. As the

[1] Specifically, this result holds if the support of consumer tastes is infinite and the tails are not too thin, in a sense to be made precise below.

preference for diversity rises, some consumers will check out other firms if the product at the first firm sampled is not to their liking. This means that some consumers will actively search, bringing firms into direct competition, which in turn induces a lower equilibrium price. The greater the taste for variety, the more consumers search, and this intensified search activity increases the scope of competition. However, once taste for variety is high enough so that a sufficient number of consumers search, the situation is close to the case of perfectly informed consumers, and thereafter the equilibrium price rises with taste for variety for the standard reason. The argument above suggests that the equilibrium price should fall and then rise with taste for variety. We shall make clear in this article the conditions under which this intuition holds.

The other main contribution by Chamberlin was a discussion of equilibrium product diversity as compared to the optimal level, with Chamberlin suggesting that the outcome was ‘‘a sort of ideal.’’ We show that markets in which search costs are important may be particularly prone to excessive entry of firms in equilibrium, and the greater the search cost, the greater the extent of the market failure. This is essentially because higher search costs raise profits and attract entry, while the social optimum stipulates that fewer firms should serve the market when search costs are higher.

Section 2 presents the basic model and gives the conditions under which the Diamond Paradox arises. Section 3 discusses the comparative statics properties of the model and the limit cases of Bertrand, Diamond, and Chamberlin. In Section 4 we consider the effects of taste for product diversity on equilibrium price, with emphasis on its nonmonotonicity. Section 5 establishes the excess-entry result. Results are summed up in Section 6, which also provides a discussion of how the present analysis applies to actual market situations.

2. The model

■ There are n single-product firms, for which marginal production costs are zero. There is a population of L consumers. Each consumer $\ell = 1, \ldots, L$ has tastes described by a conditional utility function (net of any search cost) of the form

$$u_{\ell i}(p_i) = -p_i + \mu\epsilon_{\ell i} \qquad i = 1, \ldots, n \tag{1}$$

if she buys product i at price p_i. The parameter $\mu > 0$ is a scale parameter that captures the heterogeneity of consumer tastes and $\epsilon_{\ell i}$ is the realization of a random variable with distribution F and a continuously differentiable density f whose support is an interval $[a, b]$ of the extended real line. The term $\mu\epsilon_{\ell i}$ can be interpreted as a match value between consumer ℓ and product i, and these match values are assumed to be independent across consumers and products.

A consumer must incur a search cost c in order to learn the price charged by any particular firm as well as her match value for the product sold by that firm. Consumers search sequentially with costless recall. The utility of consumer $\ell = 1, \ldots, L$ is given by

$$u_{\ell i}(p_i) - kc,$$

if she buys product i at price p_i after visiting k firms. We only consider equilibria at which all firms charge the same price p^*. Thus a consumer expects all the firms that she has not yet visited to charge p^*.[2]

[2] In Anderson and Renault (forthcoming) we consider a duopoly version of the model to determine how the degree of consumer information affects equilibrium prices. We show that informed consumers impart a *negative* externality on uninformed ones, implying that the extent of investment in information may be too small.

Suppose that a consumer holds a best offer with utility $u_{\ell j}(p_j)$ (note that we allow p_j to differ from p^* in order to account for pricing off the equilibrium path). If the consumer samples another firm, firm i, at which she expects price p^*, she will prefer to buy its product if $-p^* + \mu\epsilon_{\ell i}$ exceeds $-p_j + \mu\epsilon_{\ell j}$, i.e., if $\epsilon_{\ell i}$ exceeds $x \equiv \epsilon_{\ell j} + (p^* - p_j)/\mu$. In that case the added utility is $p_j - p^* + \mu(\epsilon_{\ell i} - \epsilon_{\ell j}) = \mu(\epsilon_{\ell i} - x)$. Hence the expected incremental utility from searching one more firm is

$$\mu g(x) = \mu \int_x^\infty (\epsilon - x) f(\epsilon)\, d\epsilon. \tag{2}$$

It can be easily verified that g is strictly decreasing on $(-\infty, b]$ and goes from $+\infty$ to zero as x goes from $-\infty$ to b. Since $\mu > 0$, the expected incremental utility from a single search exceeds the search cost if and only if $x < \hat{x}$, where $\hat{x}$ is uniquely defined by

$$g(\hat{x}) = \frac{c}{\mu}. \tag{3}$$

If a single extra search yields a positive expected net benefit (i.e., if $x < \hat{x}$), then the consumer will clearly wish to search at least one more time if she has the option of further searches afterward. On the other hand, if a single extra search yields a negative expected net benefit (i.e., if $x > \hat{x}$), then with the option of further searches, the net expected benefit from searching would still be negative. To see this, suppose that the stopping rule described above (searching if and only if $x < \hat{x}$) is optimal when there are t firms left to search, and this is clearly true for $t = 1$. Consider now $t + 1$ remaining firms. If $x > \hat{x}$ and the consumer does search one more firm, then either a smaller value of x is revealed or a larger value of x is revealed. In any case, the best offer she holds corresponds to an x larger than $\hat{x}$. Since there are only t firms left to search and it was assumed that the stopping rule is optimal, the consumer would then stop searching. Thus, when there are $t + 1$ firms left and x exceeds $\hat{x}$, the consumer knows she would never search more than one additional firm, and since this yields a negative net benefit, she would rather not search at all. Hence if the stopping rule is optimal with t firms left, it is optimal with $t + 1$ firms left, for any t. Since it is optimal for $t = 1$, it is optimal for any t by induction.[3]

The reservation value $\hat{x}$ determines the probability that any given consumer goes on searching after visiting a firm. The larger it is, the more likely it is that she continues. Since g is strictly decreasing, $\hat{x}$ is a strictly decreasing function of the ratio c/μ, and it goes from b to $-\infty$ as c/μ goes from zero to $+\infty$. Thus the probability that a consumer stops her search at any firm i charging price p_i is an increasing function of the search cost and a decreasing function of product diversity measured by μ. Note that $\hat{x}$ cannot exceed the upper bound, b, of the support of f. It equals b if and only if $c = 0$: this corresponds to a situation in which (since search is costless) no consumer would make her purchase until she has searched all the firms.

The reservation value can, however, be less than the lower bound, a, of the support of f. Then we have $\mu g(\hat{x}) = \mu(E\epsilon - \hat{x}) > \mu(E\epsilon - a)$. Using (3), this implies that

$$c > \mu(E\epsilon - a), \tag{4}$$

[3] See Kohn and Shavell (1974) for a more formal argument.

which means that if a consumer expects all the firms to charge the same price, the expected incremental utility from further searches is less than the search cost, even if her current best offer involves the worst possible match. Then the only motive for search would be the expectation that subsequent firms charge prices lower than those already observed. In that sense, this situation is very similar to that analyzed by Diamond (1971): whatever the price charged by its competitors, some firm can increase its price (by some amount less than $c - \mu(E\epsilon - a)$) without losing any customer. Hence if the search cost, c, is large enough, or if the taste for variety, μ, is small enough (so that (4) holds), then the only equilibrium is that all firms set infinite prices, which is the analogue to the Diamond result in our context.[4] In the next section we analyze the more interesting case in which some consumers search in equilibrium.

3. Market equilibrium

■ Suppose all firms but firm i set price p^*. Then it is optimal for consumers, when sampling firm i, to use the search rule described in the previous section. Thus the probability of a consumer staying with firm i, given that i is sampled, is

$$\Pr(x > \hat{x}) = 1 - F(\hat{x} + \Delta),$$

where $\Delta \equiv (p_i - p^*)/\mu$ is the standardized price premium of firm i. Firm i is sampled first with probability $1/n$, second with probability $F(\hat{x})/n$ (since when another firm is sampled first, it is accepted with probability $1 - F(\hat{x})$), third with probability $F(\hat{x})^2/n$, etc. A consumer also purchases from firm i if she samples all the firms and i yields the highest utility. Then firm i's demand is the sum of the series that represents consumers who stop once they reach i plus the ‘‘comebacks’’ who sample all firms and then return to i:

$$D(p_i, p^*) = \frac{L}{n}[1 - F(\hat{x} + \Delta)]\left[\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})}\right] + L\int_{-\infty}^{\hat{x}+\Delta} F(\epsilon - \Delta)^{n-1} f(\epsilon)\, d\epsilon. \tag{5}$$

Assuming that $\hat{x} \in [a, b]$, we now turn to the characterization of equilibria in which all firms charge the same price. Note that $D(p^*, p^*) = L/n$. The derivative of firm i's demand with respect to p_i, evaluated at $p_i = p^*$, is

$$\frac{\partial D}{\partial p_i}(p^*, p^*) = \frac{L}{\mu}\left[-\frac{f(\hat{x})}{n}\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} + f(\hat{x})F(\hat{x})^{n-1} - \int_{-\infty}^{\hat{x}} (n - 1) f(\epsilon)^2 F(\epsilon)^{n-2}\, d\epsilon\right]. \tag{6}$$

The last two terms in the bracket can be written together as $\int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^{n-1}\, d\epsilon$. Hence the symmetric equilibrium price is[5]

$$p^* = \frac{-D(p^*, p^*)}{\frac{\partial D}{\partial p_i}(p^*, p^*)} = \frac{\mu}{\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} f(\hat{x}) - n\int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^{n-1}\, d\epsilon}. \tag{7}$$

The following proposition, proved in Appendix A, provides some comparative

[4] One can readily introduce finite reservation prices (see Anderson and Renault, forthcoming) to cap the infinite price equilibrium; we chose not to do so here to avoid a taxonomy that would detract from the main points.

[5] The first two terms in the bracket in (6) can be written together as $-f(\hat{x})/n \sum_{k=0}^{n-1} [F(\hat{x})^k - F(\hat{x})^{n-1}]$, which is negative. Thus $[\partial D/\partial p_i](p^*, p^*)$ is negative and, from (7), p^* is nonnegative.

statics results for this candidate equilibrium price under the assumption that $1 - F$ is logconcave on $[a, b]$. Logconcavity of $1 - F$, which is equivalent to an increasing hazard rate, is implied by logconcavity of f. We use logconcavity of f in Section 5 to show that the market always provides excessive variety. The logconcavity property, which means that the log of the function is concave, is weaker than concavity but stronger than quasi-concavity, and it holds for many common densities (see Caplin and Nalebuff (1991) for a list). It amounts to the density being well behaved.

Proposition 1. If $1 - F$ is logconcave on $[a, b]$, and $c < \mu(E\epsilon - a)$, p^* is an increasing function of the search cost, c, and a decreasing function of the number of firms, n. Furthermore,

$$\lim_{c\to 0} p^* = \frac{\mu}{n(n-1)\int_{-\infty}^{+\infty} f(\epsilon)^2 F(\epsilon)^{n-2}\, d\epsilon} \tag{8}$$

$$\lim_{n\to +\infty} p^* = \frac{\mu[1 - F(\hat{x})]}{f(\hat{x})}. \tag{9}$$

The proposition highlights how the introduction of product differentiation (via μ) smooths out the economic analysis of markets with search costs. It is intuitive that higher search costs should lead to higher prices and that if the cost of gathering information is nearly zero, the market outcome should be close to what would prevail if consumers were perfectly informed about prices and match values. The proposition says that this is what happens if there is enough heterogeneity in products (μ large enough so that (4) does not hold). In particular, the limit in (8) is the symmetric equilibrium price of the Perloff-Salop model where consumers are perfectly informed. This limit is attained smoothly as search costs go to zero. As we pointed out at the end of Section 2, if (4) holds, the situation is the analogue of the Diamond paradox. This does not happen for c small, but rather, for c large enough where the threshold value of search costs increases as product differentiation increases. Note that this situation actually never happens if the distribution of match values is unbounded from below (i.e., if a is minus infinity). Whether the Bertrand outcome may be obtained as a limit of this model depends on the order in which the limits are taken. Indeed, the Bertrand equilibrium price of zero (which is marginal cost by assumption) is obtained if the search cost, c, is taken to zero first and then the heterogeneity parameter, μ, is taken to zero. This is because taking c to zero with μ positive yields the Perloff-Salop equilibrium price (8) and then, taking μ to zero, yields marginal cost. On the other hand, as discussed in more detail in the next section, if μ is taken to zero first (for a given c), the price becomes infinite and would thus never reach zero if c is in turn taken to zero.

The other intuitive property given in the proposition is that the market price falls with the number of firms in the market. This is in contrast to both the Bertrand and Diamond cases, in which the price is unaffected by n. In the limit as the number of firms goes to infinity, as pointed out by Wolinsky (1986), the markup is positive, and this feature supports Wolinsky's claim to have proposed a model of ''true'' monopolistic competition. But for this phenomenon to be significant, it is crucial that search costs are not too small. To see this, suppose that we take c to zero in (9).[6] Then the equilibrium price tends to zero with the search cost if and only if

[6] It is easy to check that if $1 - F$ is logconcave, then the equilibrium price given by (9) is increasing in c.

$$\lim_{x \to b} \frac{[1 - F(x)]}{f(x)} = 0. \tag{10}$$

So for the Chamberlinian monopolistic competition markup to be significant for low search costs, it is necessary that (10) does not hold. It clearly holds if $f(b) > 0$. If $f(b) = 0$, then we can apply L'Hôpital's rule, since $1 - F(b) = 0$, and so the limit price is then zero if and only if

$$\lim_{x \to b} \frac{-f(x)}{f'(x)} = 0. \tag{11}$$

Considering (10) in conjunction with (11) shows there is a markup in the limit as both the number of firms goes to infinity and the search cost goes to zero under the (rather restrictive) condition that the support of the taste density has a thick enough upper tail. This condition on the density is the same as that found by Perloff and Salop to guarantee that the symmetric equilibrium price in their oligopoly model (without search) would go to zero as the number of firms becomes infinite. This equivalence is scarcely surprising, since the equilibrium price in their model is the limit of the price in ours as c goes to zero (and is given by (8)). In that sense, their result stems from taking c to zero and then n to infinity, whereas we have taken the limits in the reverse sequence.

The issue of existence of equilibrium is not our central concern and is discussed in some detail in Appendix B. There we show that equilibrium exists under monopolistic competition (with an infinite number of sellers). Furthermore, equilibrium can be shown to exist under oligopoly for any finite number of firms, if f is logconcave and nondecreasing on $[a, b]$ as well as for several other categories of distributions.

4. The effect of increased product diversity on prices

■ An increase in μ corresponds to an increase in product diversity because a higher μ means an increase in the variance of the match value between a consumer and a product. In the absence of search costs, as in Perloff and Salop (1985), an increase in product diversity would unambiguously *raise* prices (see (8)). This reflects the increase in market power of firms due to more intense preferences. In the setting of this article, however, since consumers must incur a cost to find out about product characteristics, an increase in product diversity implies more search by consumers trying to find better matches, which brings firms into more intense competition. This latter effect may cause a *drop* in the market price when product diversity is increased.

First, (7) shows that a change in μ affects p^* directly, as well as indirectly through $\hat{x}$. The direct affect is unambiguously positive. This is due to the increase in the firms' market power, keeping search behavior constant. On the other hand, it was shown in the proof of Proposition 1 that if $1 - F$ is logconcave on $[a, b]$, p^* is decreasing in $\hat{x}$. Since $\hat{x}$ is increasing in μ, the indirect effect of an increase in product diversity on the market price is negative. This is because more heterogeneity among firms leads to more search activity by consumers. As we show below, the effect of an increase in μ on p^* can be either positive or negative, depending on which effect dominates.

At the end of Section 2, it was pointed out that if (4) holds, we are in the Diamond case for which price is infinite. This will happen if μ is sufficiently small, more specifically if $\mu < \mu_0$, where μ_0 is defined from (4) as $c/(E\epsilon - a)$. The following proposition sums up some sufficient conditions under which the effect of an increased heterogeneity on price is unambiguous provided that $\mu > \mu_0$.

Proposition 2. Suppose f is logconcave on $[a, b]$ and $\mu > \mu_0$. Then

(i) For μ sufficiently large, p^* is increasing in μ.

(ii) On any interval such that $f'(\hat{x}) \leq 0$, p^* is increasing in μ.

(iii) If $f(a) = 0$ and $f'(a) > 0$, then p^* is decreasing in μ on some interval $(\mu_0, \mu_0 + \delta)$ with $\delta > 0$.

Proof. Using the definition of $\hat{x} > a$ from (3) (i.e., $\mu = c/g(\hat{x})$), the reciprocal of the equilibrium price can be written from (7) as

$$\frac{1}{p^*(\hat{x})} = \frac{g(\hat{x})k(\hat{x})}{c}, \tag{12}$$

where

$$k(\hat{x}) = \left[\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} f(\hat{x}) - nf(\hat{x})F(\hat{x})^{n-1} + n(n-1)\int_{-\infty}^{\hat{x}} f(\epsilon)^2 F(\epsilon)^{n-2}\, d\epsilon\right]$$

is just the denominator of (7). From (12) it is clear that the equilibrium price is increasing (decreasing) in μ if and only if the product gk is decreasing (increasing) in $\hat{x}$.

First note that logconcavity of g (which follows from the fact that g is the integral of the logconcave function $1 - F$, which follows from integrating (2) by parts) implies that $g/(1 - F)$ is decreasing.[7] Thus (from (12)) price will be increasing as long as $k(1 - F)$ is decreasing in $\hat{x}$ or

$$[1 - F(\hat{x})^n]f'(\hat{x}) - [1 - F(\hat{x})]nf'(\hat{x})F(\hat{x})^{n-1} - n(n-1)f(\hat{x})\int_{-\infty}^{\hat{x}} f(\epsilon)^2 F(\epsilon)^{n-2}\, d\epsilon \leq 0. \tag{13}$$

This inequality holds for $\hat{x}$ close enough to b, so price must be increasing for μ sufficiently large, which proves (i). Equation (13) may also be used to show that (ii) holds. The sum of the first two terms on the right-hand side of (13) has the same sign as $f'(\hat{x})$.[8] Hence the equilibrium price is always increasing with μ whenever $f'(\hat{x}) \leq 0$.

To determine whether price slopes down over some interval to the right of μ_0, we differentiate (12) above and evaluate it at $\hat{x} = a$ to yield

$$\operatorname{sgn}\frac{d(1/p^*)}{d\hat{x}}(a) = \operatorname{sgn}[[f(a)^2 + f'(a)](E\epsilon - a) - f(a)].$$

If $f(a) = 0$ and $f'(a) > 0$, price is therefore falling at first. *Q.E.D.*

Parts (i) and (ii) of Proposition 2 deal with cases where price increases with product heterogeneity. In particular, from condition (i), this happens for sufficiently high levels of product heterogeneity. This is because almost every consumer searches all firms so that the intuition from the Perloff-Salop case applies. When the density of match values is not monotonically increasing, condition (ii) provides a lower bound on product heterogeneity that ensures price is increasing. Since logconcavity of f implies f' cannot turn positive after it is negative, there is a minimal μ beyond which f' is negative so

[7] This follows since g logconcave implies that g'/g is decreasing, and $g' = -(1 - F)$.

[8] The two terms can be written together as $f'(\hat{x})(1 - F(\hat{x}))\sum_{k=0}^{n-1}[F(\hat{x})^k - F(\hat{x})^n]$, and the result follows since $F(\hat{x}) < 1$.

that price increases thereafter with μ. In particular, price increases with μ for all $\mu > \mu_0$ when $f'(a) \leq 0$.

On the other hand, for low levels of product heterogeneity (i.e., for μ in the neighborhood of μ_0), price is necessarily decreasing in μ. Since price is infinite for $\mu < \mu_0$ and finite for $\mu > \mu_0$, there is clearly a drop at μ_0. The way this drop actually occurs and the behavior of the price at μ_0 depend on the characteristics of f at a. To determine the behavior of the equilibrium price as μ tends to μ_0 from above, we take the limit as $\hat{x}$ tends to a in (12) to give

$$\lim_{\hat{x}\to a} p^* = \frac{c}{(E\epsilon - a)f(a)},$$

where we have used $g(a) = E\epsilon - a$. Hence if $f(a)$ is zero and $E\epsilon$ is finite, p^* goes to infinity as μ falls to its lowest value compatible with search activity by consumers (μ_0),[9] and therefore price must initially fall with μ. If $f(a)$ is not zero, the limit price is finite, and any price above that limit is an equilibrium price for $\mu = \mu_0$. Since consumers do not search, any price decrease will gain no consumers, but a price increase will lose consumers at a rate so fast that this is not profitable if the price exceeds the limit one.[10]

The conditions of Proposition 2 (iii) ensure that an increased heterogeneity causes a smooth drop in price for values of μ to the right of μ_0. For instance, these conditions hold if the distribution function is a power function, $F(x) = x^\gamma$, $1 < \gamma < 2$ (the first inequality being strict ensures a zero density at zero, while the second inequality ensures a strictly positive derivative of the density at zero), with support [0, 1]. The condition given in the proof can also be used to show that p^* is sloping down on some neighborhood to the right of μ_0 if f is logconcave and symmetric on its support [0, 1] with $f(0)$ small enough.

We have shown that the price starts by falling as product heterogeneity increases for low values of μ. Furthermore, it necessarily slopes up eventually as μ becomes large enough. We would expect that in many examples, the general shape is quasi-convex, with the price falling at first and then going up. This intuition is borne out in the case of monopolistic competition. Indeed, the equilibrium price is logconvex in $\hat{x}$ (and hence quasi-convex in μ) under the additional assumption that the hazard rate is logconcave. This is easily seen using logconcavity of g and taking the log of the right-hand side of (9), which is convex if the hazard rate is logconcave. For example, the logistic is a logconcave density that satisfies this property. Another example for which the equilibrium price exhibits a clear U-shape is for the distribution $F(x) = \exp[x]$, with $x \epsilon (-\infty, 0)$. Here $g(\hat{x}) = \exp[\hat{x}] - \hat{x} - 1$, and the price derivative with respect to $\hat{x}$ has the sign of $\exp[2\hat{x}] - 3\exp[\hat{x}] + \hat{x} + 2$. This latter function is first negative and then positive. The intuition for the U-shape is as follows. For low μ, there is very little consumer search and so very little competition among firms. As μ rises, consumers search more to get better matches, bringing more competition between firms for the itinerant consumers. However, past a point, enough consumers search so that the usual effect of higher taste heterogeneity kicks in, and the equilibrium price rises because of higher taste for particular products.

[9] The assumption that $E\epsilon$ is finite takes care of the case $a = -\infty$.

[10] Note that $\mu_0 = 0$ for $c = 0$. Then, as a special case, we have the result that (for no product differentiation and zero search costs), if consumers are imperfectly informed about prices, there is a range of equilibrium prices between marginal cost and the monopoly price. This point was suggested to us by Dale Stahl.

5. The optimum number of firms

■ Recent results comparing the equilibrium and optimum number of firms under monopolistic competition (and without search costs) suggest that the market system may tend to overprovide diversity, but not by much. For Chamberlinian models (with identically and independently distributed preferences, so that Chamberlin's symmetry assumption is verified), Deneckere and Rothschild (1992) show that the ratio of equilibrium to optimal numbers of firms tends to one as entry costs go to zero. Anderson, de Palma, and Nesterov (1995) show there is always overentry under oligopoly (for f logconcave), although the extent of overentry is frequently small: the largest degree of overentry found for monopolistic competition is 10%. Since the present article has the model of the latter article as a limit case when c goes to zero (Proposition 1), we can use their results to analyze the question of equilibrium versus optimal provision. The method we use affords a clear answer. Specifically, Proposition 1 shows that equilibrium prices increase with c and decrease with n. Since gross profit per firm is just p^*/n, the latter property implies that gross profits decrease with the number of firms, so there is a unique long-run equilibrium (at which gross profit equals the entry cost, K). The former property implies that the equilibrium number of firms rises with search costs because higher friction in the market facilitates higher prices. Intuitively, we would expect the optimal number of firms to be a *decreasing* function of c, because higher c implies less search activity so that there is less benefit to having many firms. We show that this property indeed holds. Thus the overentry found by Anderson, de Palma, and Nesterov (1995) is exacerbated by the introduction of search costs, and we have the following:

Proposition 3. For f logconcave, there are too many firms in the symmetric equilibrium.

Proof. The social benefit of an additional firm (firm $n + 1$) is the increase in surplus associated with it (given the optimal search behavior of consumers) minus the lump-sum cost, K, of setting up a new firm. Given that preferences are identically and independently distributed, the order of consumer search is irrelevant. For the social problem we can therefore assume, without loss of generality (and to simplify the derivation) that the added firm is searched last. Thus under the stopping rule (3) there are $LF(\hat{x})^n$ consumers who end up searching firm $n + 1$. The extra search costs are $LcF(\hat{x})^n$. The consumers who search firm $n + 1$ are those who would return to an earlier firm if there were only n firms, because they did not find an acceptable match (i.e., $\epsilon_i < \hat{x}$ for all $i = 1, \ldots, n$). The expected utility from returning after n searches is

$$\mu n L \int_{-\infty}^{\hat{x}} \epsilon f(\epsilon) F(\epsilon)^{n-1}\, d\epsilon.$$

With the additional $(n + 1)$th firm, a fraction $1 - F(\hat{x})$ of these consumers find $\epsilon_{n+1} > \hat{x}$, with associated benefit $\mu L \int_{\hat{x}}^{\infty} \epsilon f(\epsilon)\, d\epsilon$.

The others still return (possibly to firm $n + 1$), and the expected benefit on this account is

$$\mu(n + 1)L \int_{-\infty}^{\hat{x}} \epsilon f(\epsilon) F(\epsilon)^{n}\, d\epsilon.$$

The net surplus gain from the $(n + 1)$th firm is therefore

$$\Delta W = -LcF(\hat{x})^n + \mu LF(\hat{x})^n \int_{\hat{x}}^{\infty} \epsilon f(\epsilon)\, d\epsilon + \mu L(n+1) \int_{-\infty}^{\hat{x}} \epsilon f(\epsilon) F(\epsilon)^n\, d\epsilon$$
$$- \mu Ln \int_{-\infty}^{\hat{x}} \epsilon f(\epsilon) F(\epsilon)^{n-1}\, d\epsilon - K.$$

Integrating by parts, we have

$$\frac{\Delta W}{L\mu} = -\frac{c}{\mu}F(\hat{x})^n + F(\hat{x})^n \int_{\hat{x}}^{\infty} \epsilon f(\epsilon)\, d\epsilon + \hat{x}F(\hat{x})^n[F(\hat{x}) - 1] + \int_{-\infty}^{\hat{x}} F(\epsilon)^n[1 - F(\epsilon)]\, d\epsilon$$
$$- \frac{K}{L\mu}.$$

Recalling that, from (3), $\int_{\hat{x}}^{\infty} \epsilon f(\epsilon)\, d\epsilon - \hat{x}[1 - F(\hat{x})] = c/\mu$, this simplifies to

$$\frac{\Delta W}{L\mu} = \int_{-\infty}^{\hat{x}} F(\epsilon)^n[1 - F(\epsilon)]\, d\epsilon - \frac{K}{L\mu}. \tag{14}$$

The optimal number of firms is found by setting this last expression equal to zero: since the right-hand side is decreasing in n and increasing in $\hat{x}$, the optimal number of firms, n^*, is increasing in $\hat{x}$. By extension, n^* is *decreasing* in c. By Proposition 1, however, the equilibrium number is *increasing* in c. This means that there is necessarily overentry in equilibrium if there is not underentry for the model with $c = 0$. This latter property has been proved in Anderson, de Palma, and Nesterov (1995) for f logconcave. *Q.E.D.*

Wolinsky (1984) obtains a similar overentry result by introducing search costs in the circle model of Salop (1979). In Wolinsky's model the socially optimal number of firms is bounded above because of the search behavior of consumers (this bound being decreasing in the search cost). Then if the lump-sum cost of setting up a new firm, K, is small enough so that the number of firms in equilibrium is large enough, there is overentry. The reasoning behind Proposition 3 does not depend on there being an upper bound on the optimal number of firms. Indeed, as K goes to zero, for the integral in (14) to go to zero it is necessary that n goes to infinity, so there is no upper bound. An important difference between the two models is that in Wolinsky's model, if n is sufficiently large, all consumers find a brand that satisfies the sequential search stopping rule. Then the expected social benefit of an extra brand is zero, since the expected surplus of a consumer is the average surplus from a brand satisfying the stopping rule, the value of which is not affected by the number of firms. In our model, the benefit from an extra brand is that obtained by those consumers who buy the best brand they could find after searching all the firms. Adding an extra firm increases the expected surplus from that best brand (as in the model without search costs).

The analysis that leads to Proposition 3 stresses that an increase in consumer search costs exacerbates overentry. While higher search costs *increase* the equilibrium number of firms by raising profits, higher costs *reduce* the optimal number of firms because consumers search less and then benefit less from added variety. As an extreme example, consider the case where the Diamond Paradox holds (i.e., (4) holds and $\hat{x} \leq a$). There the equilibrium number of firms is infinite for any level of entry cost, whereas the optimal number of firms is one since the right-hand side of (14) is always negative.

These results highlight the fact that markets can perform very inefficiently in the presence of search costs.

6. Conclusion

■ Bertrand (1883) argued that price would be driven down to marginal cost even with only two firms in the market (although, as noted by Ekelund and Hebert (1990), Bertrand (1883) was not the first writer to think about an equilibrium in prices; the credit should go to Fauveau (1867)).[11] This result was termed the ''Bertrand Paradox'' by Tirole (1988). Chamberlin (1933), by introducing product differentiation, argued that price will exceed marginal cost even when there are many firms. Thus product differentiation resolves the Bertrand Paradox. Diamond (1971) argued that firms would set monopoly prices in the Bertrand context (i.e., with homogeneous products) if consumers face search costs, even arbitrarily small ones. Moreover, there is no search in equilibrium, since consumers rationally expect the same price to prevail at each firm and so have no reason to search beyond the first firm encountered. In this article we have elaborated upon how product differentiation resolves the ''Diamond Paradox.'' Indeed, as long as products are differentiated, prices fall as search costs fall and decrease as the number of firms rises; furthermore, consumers search in equilibrium. Diamond's prediction appears as a limit case when there is too little product differentiation or when search costs are too high. The cases of Chamberlinian and Bertrand competition also arise as limit cases of the present model: the former when the number of firms gets large, and the latter when search costs and product heterogeneity go to zero.

The model also has interesting comparative static properties with respect to the degree of product heterogeneity. In a standard model with perfectly informed consumers, market prices typically rise with the degree of product differentiation. But once we introduce consumer search costs, prices may fall with taste diversity, because more diversity engenders more search and hence more competition. This effect leads to perverse market incentives. In particular, a higher preference for variety may lead to a smaller range of products, although the social optimum prescribes a wider range. Nevertheless, since the equilibrium diversity rises with search costs while the optimal level falls, there is always excessive entry. The extent of the market failure is most pronounced for low preference for variety and for high search costs.

Actual market situations with high prices seem particularly puzzling when there is little product differentiation. Furthermore, if there are also many firms, it is hard to argue that the high observed prices arise from collusion among firms. ''Tourist traps'' (e.g., souvenir shops in Lourdes, Greek restaurants on the Paris Left Bank, seafood restaurants in the rue des Bouchers in Brussels, and countless others) provide a striking illustration. In such markets, the items offered are typically very close in characteristics, and prices are high despite the presence of a large number of small sellers. The results of the present article provide a simple interpretation. There is limited competition in such markets because buyers do not shop around, expecting high prices and similar products everywhere. This is reinforced by the high search costs for people on vacation, who would rather spend their time on other activities. Our results further suggest that there is a massive overentry problem. However, this problem is less acute the greater

[11] ''When price is the variable conjectured about, the non-collusive solution results in competitive quantity . . . This last argument, previously identified exclusively with Joseph Bertrand was actually advanced unambiguously by Fauveau sixteen years earlier.'' (Ekelund and Hebert, 1990, p. 145).

the degree of product differentiation, for example, stores in tourist areas that sell somewhat different goods. Thus we should expect some overentry and relatively high prices for stores selling skiing gear in ski resorts, or swimsuits in sea resorts, but exacerbated price and entry where restaurants specialize in a local speciality or where stores sell a local delicacy or regional novelty (like berets in the Basque country).

Although tourist markets fit particularly well into the present framework, there are other types of very different markets with a high markup in spite of little differentiation and a large number of sellers: Ausubel (1991) has documented (and casual empiricism confirms) that credit card interest rates are very sticky with respect to the underlying cost of funds. He explicitly refers to search costs (along with switching costs and consumer irrationality in imperfectly forecasting future borrowing) as a possible explanation but argues against it on the ground that these costs would have to be unrealistically large to explain the observed markups. However, search costs may cause major distortions in prices even if they are small (which is not the case for switching costs). Since Ausubel's study, credit card companies have advertised much more, even sending prefilled applications to prospective cardholders. The rates on these cards are very low, sometimes below 4% for an introductory period (the so-called teaser rates). This is consistent with a large decline in search costs due to the advertisements. However, the fact that the rates typically rise substantially after six months or so suggests a strong switching cost component.

As shown by the credit card example, the applicability of our analysis is limited by several restrictions that may be important in some contexts. First, we have not allowed firms to advertise the price or characteristics of their goods. In our ongoing research, we are looking at the incentives that firms have to make consumers aware of these features. Second, the model applies best under one-shot interactions, so there are no reputations nor repeat purchases. A dynamic version of the model with repeated interaction would be most welcome. Third, we have treated a parametric form of product differentiation: we are working on endogenous choice of differentiation in a spatially based model, with one driving force being the desire for little differentiation in order to raise prices, as we have seen in the current model.

Finally, the following quote from Ekelund and Hebert (1995, p. 217) provides a striking illustration of overentry resulting from search costs:

> Consider Chadwick's case for franchising funeral services for London, circa 1843. [See Edwin Chadwick, 1843.] He estimated that between 600 and 700 undertakers in London performed over 100 funerals per day. Therefore, about six undertakers competed for each funeral. Thus the market situation appeared to be noncollusive and roughly competitive; Chadwick alleged that the funeral suppliers acted like monopolies. They could charge exorbitant prices because demanders were faced with high information and search costs when shopping for a funeral supplier.

Appendix A

■ *Proof of Proposition 1.* We first show that p^* is increasing in c. Since $\hat{x}$ is decreasing in c, it suffices to show that (μ/p^*) is increasing in $\hat{x}$. To this end, rewrite (μ/p^*) as

$$\frac{\mu}{p^*} = \sum_{k=0}^{n-1} f(\hat{x})F(\hat{x})^k - n\int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^{n-1}\, d\epsilon.$$

Taking the derivative with respect to $\hat{x}$, we have

$$\left(\frac{\mu}{p^*}\right)'(\hat{x}) = \sum_{k=0}^{n-1} (f'(\hat{x})F(\hat{x})^k + kf(\hat{x})^2F(\hat{x})^{k-1}) - nf'(\hat{x})F(\hat{x})^{n-1}.$$

Now

$$\sum_{k=0}^{n-1} F(\hat{x})^k = \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} \quad \text{and} \quad \sum_{k=0}^{n-1} kF(\hat{x})^{k-1} = \frac{1 - F(\hat{x})^{n-1}}{[1 - F(\hat{x})]^2} - \frac{(n-1)F(\hat{x})^{n-1}}{1 - F(\hat{x})}.$$

Thus

$$\left(\frac{\mu}{p^*}\right)'(\hat{x}) = \frac{f'(\hat{x})[1 - F(\hat{x})] + f(\hat{x})^2}{1 - F(\hat{x})}\left[\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - nF(\hat{x})^{n-1}\right].$$

The first term is positive, since $1 - F$ is logconcave on $[a, b]$ and $\hat{x} \in [a, b]$. The second term is equal to $\sum_{k=0}^{n-1} F(\hat{x})^k - nF(\hat{x})^{n-1} = \sum_{k=0}^{n-1} [F(\hat{x})^k - F(\hat{x})^{n-1}]$, which is clearly positive since $0 \leq F(\hat{x}) \leq 1$. Thus p^* is increasing in c.

To show that p^* decreases with n, it suffices to show that

$$\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} f(\hat{x}) - n \int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^{n-1}\, d\epsilon \leq \frac{1 - F(\hat{x})^{n+1}}{1 - F(\hat{x})} f(\hat{x}) - (n+1) \int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^n\, d\epsilon.$$

Since

$$\int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^n\, d\epsilon = f(\hat{x})F(\hat{x})^n - n \int_{-\infty}^{\hat{x}} f(\epsilon)^2 F(\epsilon)^{n-1}\, d\epsilon,$$

it suffices to show

$$\begin{aligned} &\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} f(\hat{x}) - n \int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^{n-1}\, d\epsilon \\ &\quad \leq \frac{1 - F(\hat{x})^{n+1}}{1 - F(\hat{x})} f(\hat{x}) - n \int_{-\infty}^{\hat{x}} f'(\epsilon)F(\epsilon)^n\, d\epsilon - f(\hat{x})F(\hat{x})^n + n \int_{-\infty}^{\hat{x}} f(\epsilon)^2 F(\epsilon)^{n-1}\, d\epsilon. \end{aligned}$$

The above inequality can be rewritten as

$$0 \leq n \int_a^{\hat{x}} F(\epsilon)^{n-1}[f'(\epsilon)[1 - F(\epsilon)] + f(\epsilon)^2]\, d\epsilon,$$

which holds since $F(\epsilon) \geq 0$ and $1 - F$ is logconcave on $[a, b]$. Thus p^* is decreasing in n.

To prove (8), it is convenient to integrate by parts and rewrite p^* as

$$p^* = \frac{\mu}{\dfrac{1 - F(\hat{x})^n}{1 - F(\hat{x})} f(\hat{x}) - nf(\hat{x})F(\hat{x})^{n-1} + n(n-1) \displaystyle\int_{-\infty}^{\hat{x}} f(\epsilon)^2 F(\epsilon)^{n-2}\, d\epsilon}.$$

As c goes to zero, $\hat{x}$ goes to b. The first two terms in the denominator can be written together as $\sum_{k=0}^{n-1} [F(\hat{x})^k - F(\hat{x})^n]$, and since $F(\hat{x})$ tends to one as $\hat{x}$ tends to b, the result follows.

Finally, since $c > 0$, $\hat{x} < b$ and therefore, for all $\epsilon \in [-\infty, \hat{x}]$, $0 \leq F(\epsilon) < 1$ and $nF(\epsilon)^{n-1}$ tends to zero as n goes to infinity. Furthermore, for $n > [-1/\ln F(\hat{x})]$ and for all $\epsilon \in [-\infty, \hat{x})$, $(n+1)F(\epsilon)^n \leq nF(\epsilon)^{n-1}$. Thus, from Lebesgue's Monotone Convergence Theorem, the integral in the denominator of p^* tends to zero and the result follows. *Q.E.D.*

Appendix B

■ **On the existence of equilibrium.** (1) **Existence of equilibrium under monopolistic competition.**

Proposition B1. If $\hat{x} \geq a$, $1 - F$ is logconcave on $[a, b]$ and there is an infinite number of firms, then there exists an equilibrium in which all firms charge a price equal to

$$p^*_{mc} = \frac{\mu[1 - F(\hat{x})]}{f(\hat{x})}.$$

Proof. Since n is infinite, the probability that a consumer samples all the firms is zero. Thus, if all other firms are expected to charge some price p and firm i charges p_i, firm i's demand is given by

$$D(p_i, p) = \frac{\lambda}{1 - F(\hat{x})}[1 - F(\hat{x} + \Delta)],$$

where $\lambda = L/n$ is the average market share. Since $1 - F$ is logconcave on $[a, b]$, D is logconcave in p_i on $I = [p + \mu(a - \hat{x}), p + \mu(b - \hat{x})]$. For $p_i \leq p + \mu(a - \hat{x})$, firm i retains all consumers who sample it, so its demand is perfectly inelastic and firm i's profit is linearly increasing in p_i. For $p_i \geq p + \mu(b - \hat{x})$, no consumer stays with firm i and its profit is zero. Thus, if a price maximizes firm i's profit on I, it maximizes firm i's profit on $\mathbb{R}_+$. Since D is logconcave in p_i on I, firm i's profit is logconcave in p_i on I, and therefore its derivative is zero at some point only if that point corresponds to a maximum. Equation (9) is then obtained by setting the profit derivative to zero and imposing symmetry. *Q.E.D.*

□ (2) **Existence of equilibrium in oligopoly.**

Proposition B2. For f logconcave and either of the following conditions, p^* is an equilibrium price:

(i) $$2f(b) + f'(b)[(b - \hat{x}) + p^*/\mu] \geq 0;$$

(ii) $$\int_a^{\hat{x}} [1 - F(u + \tilde{\Delta}) + \frac{2f(\hat{x} + \tilde{\Delta})}{f'(\hat{x} + \tilde{\Delta})} f(u + \tilde{\Delta})] f(u)\, du \leq 0,$$

where $\tilde{\Delta}$ is the unique solution to $p^*/\mu + \tilde{\Delta} = [2f(\hat{x} + \tilde{\Delta})]/[f'(\hat{x} + \tilde{\Delta})]$.

Proof. Without loss of generality, set $L = 1$. First consider prices such that $\hat{x} + \Delta > b$. This means that p_i is so high that any consumer purchasing from firm i has sampled all the other firms. From (5), firm i's demand in this case is given by

$$D(p_i, p^*) = \int_a^b F(\epsilon - \Delta)^{n-1} f(\epsilon)\, d\epsilon.$$

Hence the demand function coincides with that of Perloff and Salop (where consumers are perfectly informed) for this range of prices. From the analysis in Caplin and Nalebuff (1991), the corresponding profit function is quasi-concave under logconcavity of f. Moreover, there exists a unique symmetric equilibrium price to the Perloff-Salop model. Since the best response to a common price charged by competitors is positive at zero and continuous, it must be below the 45-degree line for prices beyond the symmetric equilibrium price. By Proposition 1, the symmetric equilibrium price for the model with positive search costs exceeds the Perloff-Salop one, so profit is decreasing for prices p_i such that $\hat{x} + \Delta > b$. Hence, we need to show that profit is quasi-concave for $p_i < p^* + b - \hat{x}$.

Let us first define

$$D_A(p_i, p^*) = \frac{1}{n}[1 - F(\hat{x} + \Delta)]\left[\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - nF(\hat{x})^{n-1}\right]$$

and $D(p_i, p^*) = D_A(p_i, p^*) + D_B(p_i, p^*)$. Letting $u = \epsilon - \Delta$ and integrating (5) by parts yields

$$D_B(p_i, p^*) = F(a - \Delta)^{n-1} + \int_{a-\Delta}^{\hat{x}} [1 - F(u + \Delta)]\, dF(u)^{n-1}.$$

We now define $\pi_A(p_i, p^*) = p_i D_A(p_i, p^*)$ and $\pi_B(p_i, p^*) = p_i D_B(p_i, p^*)$, and we show that if π_A is increasing (concave), then π_B is increasing (concave).

First suppose π_A is increasing. Then $1 - F(\hat{x} + \Delta) - (p_i/\mu) f(\hat{x} + \Delta) \geq 0$. The derivative of π_B is $F(a - \Delta)^{n-1} + \int_{a-\Delta}^{\hat{x}} 1 - F(u + \Delta) - (p_i/\mu) f(u + \Delta)\, dF(u)^{n-1}$. By logconcavity of $1 - F$, $[1 - F\}/f$ is decreasing so that the integrand is positive when π_A is increasing. Thus π_B is increasing.

Now suppose π_A is concave, which implies that $-2f(\hat{x} + \Delta) - (p_i/\mu) f'(\hat{x} + \Delta) \leq 0$. The second derivative of π_B is proportional to

$$-p_i f(a)(n-1)f(a-\Delta)F(a-\Delta)^{n-2} - \int_{a-\Delta}^{\hat{x}} 2f(u+\Delta) + (p_i/\mu)f'(u+\Delta)\, dF(u)^{n-1}.$$

The first term is negative. By logconcavity of f, f'/f is decreasing, so that π_B is concave when π_A is concave. From these arguments, the profit function is concave as long as the last inequality holds (since profit is the sum of two concave functions, π_A and π_B). The left-hand side of the inequality has the sign of

$$-2 - \frac{p_i}{\mu}\frac{f'(\hat{x}+\Delta)}{f(\hat{x}+\Delta)},$$

which is negative if $f'(\hat{x}+\Delta) \geq 0$. For $f'(\hat{x}+\Delta) < 0$, this expression is increasing in p_i since f'/f is decreasing by logconcavity of f. Hence if condition (i) holds, profit is concave on $[0, b - \hat{x} + p^*]$ and is decreasing for higher prices, so that p^* is the best response to p^*.

Now suppose that (i) does not hold. Then there exists $\tilde{p} = \tilde{\Delta} + p^* \in [0, b - \hat{x} + p^*]$, where $\tilde{\Delta}$ is defined in the proposition. Note that π_A is concave for $p_i < \tilde{p}$ and is convex and decreasing beyond $\tilde{p}$. Since π_B is also concave for $p_i < \tilde{p}$, total profit is concave up to $\tilde{p}$ and is therefore quasi-concave on $[0, b - \hat{x} + p^*]$ as long as π_B is decreasing at $\tilde{p}$. This is equivalent to condition (ii) in the proposition. *Q.E.D.*

We give two corollaries to Proposition B2. The first follows from inspection of condition (i).

Corollary B1. For f logconcave with $f' \geq 0$ on $[a, b]$, p^* is the unique symmetric equilibrium price.
Corollary B2. For f logconcave and for $2f(a) \geq -f'(b)/f(b)$, p^* is the unique symmetric equilibrium price.

Proof. By logconcavity of $1 - F$, $1/f(a)$ is an upper bound on the first term of the integrand in condition (ii), while logconcavity of f implies that $2f(b)/f'(b)$ is an upper bound on the second term. Thus the condition in Corollary B2 guarantees that the integrand is always negative and therefore ensures condition (ii). *Q.E.D.*

Corollary B2 holds when f is exponentially decreasing. Another way to think about condition (ii) is that the hazard rate, $f/(1 - F)$, should not increase too quickly (it is constant for f exponentially decreasing). The condition of Corollary B2 holds with equality for $F(x) = [(1 - e^{-x})/(1 + e^{-x})]$ for $x \geq 0$, which is the logistic distribution truncated below zero. In this case $f(0) = ½$ while f'/f tends to -1 as x tends to infinity.

A family of distribution functions covered by Corollaries B1 and B2 is $F(x) = x^\gamma$, with $\gamma > 0$ and $[a, b] = [0, 1]$. The density functions for this family are logconcave since $f(x) = \gamma x^{\gamma-1}$ is loglinear. For $\gamma \geq 1$, f' is nonnegative on $[0, 1]$, so Corollary B1 applies. For $\gamma < 1$, $f(0)$ is infinite whereas $f'(1)/f(1)$ is finite, so Corollary B2 holds.

References

ANDERSON, S.P., DE PALMA, A., AND NESTEROV, Y. ''Oligopolistic Competition and the Optimal Provision of Products.'' *Econometrica,* Vol. 63 (1995), pp. 1281–1301.

———, ———, AND THISSE, J.-F. *Discrete Choice Theory of Product Differentiation.* Cambridge, Mass.: MIT Press, 1992.

——— AND RENAULT, R. ''Consumer Information and Firm Pricing: Negative Externalities from Improved Information.'' *International Economic Review,* forthcoming.

AUSUBEL, L.M. ''The Failure of Competition in the Credit Card Market.'' *American Economic Review,* Vol. 81 (1991), pp. 50–81.

BERTRAND, J.L.F. ''Théorie mathématique de la richesse sociale par Léon Walras: Recherches sur les principes mathématiques de la théorie des richesse par Augustin Cournot.'' *Journal des savants,* Vol. 67 (1883), pp. 499–508.

CAPLIN, A. AND NALEBUFF, B. ''Aggregation and Imperfect Competition: On the Existence of Equilibrium.'' *Econometrica,* Vol. 59 (1991), pp. 25–59.

CHADWICK, E. *Report on the Sanitary Conditions of the Laboring Population in Great Britain: A Supplementary Report on the Results of a Special Inquiry on the Practice of Interment in Towns.* London: W. Clowes and Sons, 1843.

CHAMBERLIN, E. *The Theory of Monopolistic Competition.* Cambridge, Mass.: Harvard University Press, 1933.

DENECKERE, R.J. AND ROTHSCHILD, M. ''Monopolistic Competition and Preference Diversity.'' *Review of Economic Studies,* Vol. 59 (1992), pp. 361–373.

DIAMOND, P.A. ''A Model of Price Adjustment.'' *Journal of Economic Theory,* Vol. 3 (1971), pp. 156–168.

EKELUND, JR., R.B. AND HEBERT, R.F. ''Cournot and His Contemporaries: Is an Obituary the Only Bad Review?'' *Southern Economic Review,* Vol. 57 (1990), pp. 139–149.

——— AND ———. *A History of Economic Theory and Method,* 4th ed. New York: McGraw Hill, 1995.

FAUVEAU, P.-G. ‘‘Considérations mathématiques sur la théorie de la valeur.’’ *Journal des Economistes,* Vol. 15 (1867), pp. 31–40.

KOHN, M.G. AND SHAVELL, S. ‘‘The Theory of Search.’’ *Journal of Economic Theory,* Vol. 9 (1974), pp. 93–123.

PERLOFF, J. AND SALOP, S. ‘‘Equilibrium with Product Differentiation.’’ *Review of Economic Studies,* Vol. 52 (1985), pp. 107–120.

SALOP, S. ‘‘Monopolistic Competition with Outside Goods.’’ *Bell Journal of Economics,* Vol. 10 (1979), pp. 141–156.

TIROLE, J. *The Theory of Industrial Organization.* Cambridge, Mass.: MIT Press, 1988.

WOLINSKY, A. ‘‘Product Differentiation with Imperfect Information.’’ *Review of Economic Studies,* Vol. 51 (1984), pp. 53–61.

———. ‘‘True Monopolistic Competition as a Result of Imperfect Information.’’ *Quarterly Journal of Economics,* Vol. 101 (1986), pp. 493–511.