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Abstract

This paper studies the quantitative importance of precautionary wealth accumulation relative to life—cycle saving for retirement. Section 1 examines panel data on earnings from the PSID. Using a bivariate normal model of random effects, we find that second—period—of—life earnings are strongly positively correlated with initial earnings but have a higher variance. Section 2 studies the consequences for life—cycle saving. Households know their youthful earning power as they enter the labor market, but only in midlife do they learn their actual second—period earning ability. For plausible calibrations, precautionary saving only adds 5—6% to aggregative life—cycle wealth accumulation. Nevertheless, we find that, given borrowing constraints on households' behavior, the variety of earning profiles that our bivariate normal model generates itself stimulates more than twice as much extra wealth accumulation as precautionary saving.

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Precautionary Saving over the Life Cycle

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Two principal models that economists use to describe private saving behavior are the life–cycle, or "overlapping generations," model and the dynastic model. In each, an agent's current flow of utility depends upon his flow of consumption (and leisure) and the flow utility function is concave. The concavity makes the agent desire a smooth, as opposed to choppy, time path of consumption. The original life–cycle model stressed the natural unevenness of lifetime earnings — rising in youth and middle age, and disappearing at retirement. In that context, households should save in earning years and dissave in retirement to attain an even lifetime profile of consumption. Alternative life–cycle formulations incorporate year–to–year fluctuations in earnings due to erratic promotions, business cycles, etc. Households might want to save extra relatively early in life to accumulate a stock of wealth, which we might call a "precautionary" stock, as a reserve to buffer such high frequency fluctuations. In the second basic model, the dynastic model, a household with exceptionally high earnings may accumulate wealth to build an estate, through which it can share its good luck with its descendants. We can think of buffer–stock behavior as saving predicated on a very short time horizon, traditional life–cycle wealth accumulation (and decumulation) as behavior predicated on the time horizon of one life span, and estate building as behavior based upon an intergenerational time horizon. Laitner [2001, 2002, 2003] argues that the latter may be especially important in explaining the substantial empirical wealth disparities among U.S. households; Barro [1974] shows that dynastic behavior may enormously influence policy implications.[1] The purpose of the present paper is to formulate, and to calibrate, a life–cycle model with both saving for retirement and precautionary saving — with the ultimate goal of developing a well–specified component for a compound model with both life–cycle and dynastic behavior.

There are at least two types of lifetime uncertainty of potential interest. One includes aggregative shocks from, for example, business cycle fluctuations. Aiyagari [1994] argues that these may not have a quantitatively large effect on household saving — though results tend to be very sensitive to the way one models the stochastic process of the shocks.[2] A second arises from the heterogeneity of earnings among individual households. The latter is the focus of the present paper. There is a distribution of starting wages and salaries, and we assume that each household quickly realizes its initial position; nevertheless, the distribution tends to fan out with age and relative positions change. We assume that a young household is unsure about its eventual luck, and the effect on saving of uncertainty about the evolution of one's earnings later in life is this paper's topic.

This paper finds, strictly speaking, a relatively small role for precautionary saving. In contrast, it finds that differences in lifetime earning profiles across individuals can affect aggregative saving to a quantitatively important degree regardless of whether the

[1] See, for instance, the discussion in Laitner [2001]. See also Altig et al. [2001], Gokhale et al. [2001], and others.

[2] See also, for instance, Zeldes [1989], Caballero [1990], and Deaton [1991]. Many such papers comment on the substantial role of idiosyncratic heterogeneity — see below.

differences are predictable or not. In other words, in this paper uncertainty per se turns out not to be as important as heterogeneity of lifetime earning profile shapes. Analyses that overlook uncertainty tend to assume uniformity of earning profiles, and we find that it is the latter assumption that may generate misleading results.

1. Lifetime Earnings

We begin by examining lifetime earnings profiles for men from the Panel Study of Income Dynamics (PSID).

Data. Table 1 presents information on the subsamples that we employ. We use male earnings histories from 1967–1994. We separate the sample into four education categories: less than high school, high school, some college, and college or more. We do not use the so–called poverty sample in the PSID. We use only ages less than or equal to 60 and greater than or equal to the larger of years of education plus 6 and 16.

Table 1 shows that our panel is unbalanced: for a minority of men, we have 28 consecutive earnings figures; for most, we have far fewer. The total number of observations in every education category is, however, over 8,000. Although in 1983 PSID earnings were top coded at $99,999, the data shows this is a relatively minor issue. Some men work part time. When hours were less than 1750 hours per year, we compute the wage rate and adjust earnings upward to 1750 hours. (Figures above 1750 hours/year receive no correction.) For men who desired part time work, this adjustment seems appropriate to make their earnings reflect their potential. Similarly for the case of insured health leaves. In the case of involuntary and uninsured unemployment, on the other hand, the adjustment causes us to understate earnings uncertainty, making our results below conservative. Table 1 shows that the adjustment of hours affects more than 1 in 7 earnings figures. With the same reasoning, we drop observations with 0 hours. Table 1 records drops preceded and followed by positive hours (e.g., a zero in 1984 for a man who had positive hours in 1983 and 1985 is recorded). About one tenth of the potential observations were zero.

Ordinary Least Squares. Economists have long used so–called "earnings dynamics" models to characterize the life course of an individual's earnings (e.g., Lillard and Weiss [1979] and Abowd and Card [1989]). Such a model usually has the following form: we regress the logarithm of an individual's earnings at each age on a (low order) polynomial of age and a system of yearly dummy variables. The polynomial should show earnings rising with age until the mid forties to mid fifties, and then beginning a slow decline; the time dummies should show the influence of technological progress, with earnings generally rising over time, and business cycle peaks and troughs, with earnings growth flat or even negative in the troughs. The idea of the age–dependent part of the earnings dynamics model is that on–the–job training and experiential human capital accumulation should increase a worker's earning ability through middle age, but subsequently depreciation of skills may well predominate.

Table A1 in the Appendix to this paper presents OLS regression results for the simple (but standard) model

$$\ln(y_{it}) = \alpha_0 + \alpha_1 \cdot z_{it} + \alpha_2 \cdot [z_{it}]^2/100 + \sum_{j=1967}^{1994} \beta_j \cdot D_j(t) + \epsilon_{it} \,, \tag{1}$$

where y_{it} is the earnings of male i at time t, z_{it} is the male's age at time t, $D_j(t)$ is a dummy variable which is 1 if $t = j$ and 0 otherwise, and ϵ_{it} is a regression error (capturing measurement error in $ln(y_{it})$ and omitted explanatory variables orthogonal to the included regressors). We omit a dummy for 1984, so that remaining betas measure the effect of time relative to 1984.

The estimates in Table A1 conform with our anticipations. Omitting the influence of technological progress, earnings peak in the age range 45–49.[3] If we compare peak earnings with earnings at say age 25, the ratio is about 1.5 for the lowest education group and about 2.2 for the highest. Looking at the table for all education groups together, the time dummies show strong annual growth from technological change from 1967–1978; after that there is very little growth, and business cycle dips induce declines in the early 1980s and early 1990s. The slow growth in the second half of the period is consistent with the general slowdown in the rate of technological progress after the early 1970s, which economists have frequently noted.[4]

Table 2 is particularly important for this paper's model: for individual ages, Table 2 presents weighted–average estimates of the variance of the residual from equation (1). The table omits the youngest workers — for whom labor market participation is especially erratic. Each column except the first then reveals a clear pattern: the variance of the regression error rises with age. The increase for the first column is miniscule. For column 2, however, between ages 25–39 and 46–60 the increase is 25%; for column 3, it is 62%; for college graduates, it is 65%; and for the sample as a whole, it is 44%.

<u>Maximum Likelihood.</u> As explained in the introduction, we assume that workers understand their initial earning differences but that young workers are unsure about how they will fair relative to their peers as the differences reshuffle and grow with age. The purpose of this paper is to study the consequences for saving behavior of the resulting uncertainty for individual households. The key to our analysis is the variance pattern in Table 2. To proceed, we examine the error term of equation (1) in detail.

A common approach in the earnings dynamics literature is to specify the regression error as the sum of two components:

$$\epsilon_{it} = \mu_i + \eta_{it} \,, \tag{2}$$

with μ an individual–specific characteristic, and η an independent, idiosyncratic error. This assumes that individuals have differences in life–long earning ability, which the life–long component of their regression error, μ_i, captures. Typically, one would assume that

[3] Positive technological progress will increase the age at which earnings actually peak — the faster the technological change, the later the peak.

[4] It is also true that the PSID data on earnings corresponds to take home pay — it omits "benefits" such as employer contributions to social security, to private pensions, and for medical insurance. To the extent that benefits have risen relative to wages and salaries in the recent past, the coefficients on the dummy variables are biased downward.

μ and η are independently normally distributed. This is the "random effects" model of ϵ. The random effects model by itself, however, will not explain the pattern of rising variances in Table 2.

Although one might guess that cross–sectional differences in earnings vary from year to year, that presumably does not lead to the variance pattern of Table 2. Earnings will tend to be high in general in years of business–cycle prosperity, and low in troughs, but our time dummies should capture such phenomena. Although conceivably cross–sectional variation is higher in some years than others, the PSID attempts to represent the entire population at each date: as the original respondents (from 1968) aged and died, the PSID replaced them with young households. Thus, the fraction of, say, 50 year olds in the sample should match the U.S. population as a whole in every year. Sample weights should correct for minor problems of representativeness, and all of our regressions use weights. Since the sample then represents all ages in every year, year to year cycles should not affect the pattern of variances by age in Table 2.

One possible hypothesis, say, H_0, that could explain the pattern in Table 2 is that older workers endure larger idiosyncratic shocks. In other words, perhaps the variance of η_{it} in (2) rises with age. For example, upward steps in earnings typically follow promotions, and for young workers promotions may be frequent and small, but for older workers promotions may be infrequent and sizable. Letting z_{it} be the age of worker i at time t, a simple specification would then be

$$\epsilon_{it} = \begin{cases} \mu_i + \eta_{it}, & \text{for } z_{it} \leq 45, \\ \mu_i + \eta^*_{it}, & \text{for } z_{it} > 45, \end{cases} \tag{3}$$

where age 45 is the middle of a working life, μ_i and η_{it} and η^*_{it} are independent normal random variables with zero mean, and the variance of η^* is larger than the variance of η.

A second hypothesis, say, H_{00}, is that μ_i changes over a worker's life span. One story could be as follows. In youth, a worker does "technical" tasks — assembly line jobs, assigned research work, etc. In the second half of a career, a worker may rise to a managerial position in which he is directing younger workers. If a worker does assume managerial responsibilities, his earnings trajectory takes an upward step; if not, his earnings may be level or even erode as his technical skills become obsolete. Another story could be that some workers experience health problems in old age, and their earnings suffer, while others do not. A third possibility is that in youth, a worker trains for a career involving a particular technology or product; over time, the technology or product may grow in importance and the worker may prosper, or a new technology or product may arrive and make the worker's training obsolete. A simple formulation is

$$\epsilon_{it} = \begin{cases} \mu_i + \eta_{it}, & \text{for } z_{it} \leq 45, \\ \mu^*_i + \eta_{it}, & \text{for } z_{it} > 45, \end{cases} \tag{4}$$

where μ and μ^* and η are normal random variables with zero means; η is independent of the other two; and μ and μ^* are distributed bivariate normal with the marginal distribution of the latter having a higher variance. We might expect the correlation coefficient for μ and μ^* to be positive but less than one.

The procedure that we employ nests (3)–(4): we assume a components of error formulation

$$\epsilon_{it} = \begin{cases} \mu_i + \eta_{it}, & \text{for } z_{it} \leq 45, \\ \mu_i^* + \eta_{it}^*, & \text{for } z_{it} > 45, \end{cases} \tag{5}$$

where μ and μ^* and η and η^* are normal random variables with zero means; η and η^* are each independent of the other three, and have variance σ_η and $\sigma_{\eta*}$, respectively; and μ and μ^* are distributed bivariate normal with marginal variances σ_μ and $\sigma_{\mu*}$ and correlation $\rho \in (-1, 1)$.

Consider a household with index i. Let the vector θ include α and β from (1) and the variances and correlation from (5). Letting x_{it} be the vector of covariates for household i at time t, use the notation

$$e_{it} = e(y_{it}, x_{it}, \theta) \equiv \ln(y_{it}) - \alpha_0 - \alpha_1 \cdot z_{it} - \alpha_2 \cdot [z_{it}]^2/100 - \sum_{j=1967}^{1994} \beta_j \cdot D_j(t) \,. \tag{6}$$

Let times before the household is age 45 be indexed with s; let the times after age 45 be indexed with t. Let the normal density function for a variable z with mean 0 and standard deviation σ be $\phi(z \,|\, \sigma)$. Then the likelihood function for the household if none of its observations are top coded is

$$L_i(\theta) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\mu_i, \mu_i^* \,|\, \sigma_\mu, \sigma_{\mu^*}, \rho) \cdot \prod_s \phi(e_{is} - \mu_i \,|\, \sigma_\eta) \cdot \prod_t \phi(e_{it} - \mu_i^* \,|\, \sigma_{\eta^*}) \, d\mu_i \, d\mu_i^* \,. \tag{7}$$

The likelihood function for top coded households is only slightly different. Top coding can only occur in 1983. Suppose household i is top coded at age $\bar{s} < 45$. Then the likelihood function for the household's observations is

$$\bar{L}_i(\theta) \equiv \int_{e_{i\,\bar{s}}}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\mu_i, \mu_i^* \,|\, \sigma_\mu, \sigma_{\mu^*}, \rho) \cdot \phi(e - \mu_i \,|\, \sigma_\eta) \cdot \prod_s \phi(e_{is} - \mu_i \,|\, \sigma_\eta) \cdot \\ \prod_t \phi(e_{it} - \mu_i^* \,|\, \sigma_{\eta^*}) \, d\mu_i \, d\mu_i^* \, de \,. \tag{8}$$

Similarly if the top coding occurs after age 45.

If I is the set of non-top coded households and $\bar{I}$ the set of top coded households, then maximum likelihood estimation determines θ from

$$\theta = \arg\max_{\theta_0} \prod_{i \in I} L_i(\theta_0) \cdot \prod_{\bar{i} \in \bar{I}} \bar{L}_{\bar{i}}(\theta_0) \,. \tag{9}$$

(Table 1 shows the number of top coded households is small.)

Table 2A in the Appendix exhibits maximum likelihood estimates of α and β. Rather than force the changes to take place instantly, we exclude observations for ages 3 years before and after age 45. The results resemble those from OLS in Table A1. This is not surprising: other than top coded observations, OLS should provide consistent estimates.

Table 3 presents our estimates of the precisions $h_\eta = 1/\sigma_\eta$, $h_{\eta*} = 1/\sigma_{\eta*}$, $h_\mu = 1/\sigma_\mu$, and $h_{\mu*} = 1/\sigma_{\mu*}$ and of the correlation coefficient ρ. Under H_0, since $\sigma_\eta < \sigma_{\eta*}$, we would have

$$h_\eta > h_{\eta*} \text{ but } h_\mu \approx h_{\mu*} ;$$

under H_{00}, since $\sigma_\mu < \sigma_{\mu*}$, we would have

$$h_\eta \approx h_{\eta*} \text{ but } h_\mu > h_{\mu*} .$$

Table 3 strongly favors H_{00}. In every column, $h_\mu > h_{\mu*}$: in columns 1–5, respectively, $h_{\mu*}$ is 82% as large as h_μ, 76%, 77%, 48%, and 68% as large. In most cases h_η and $h_{\eta*}$ are almost the same. The one anomaly is column 1, where $h_{\eta*}$ is 18% larger than h_η — and even then the inequality is in the opposite direction from what H_0 predicts.

The next section assumes H_{00} and turns to a model of household saving.

2. Life Cycle Saving

This section lays out a traditional life cycle model emphasizing saving for retirement. Then it adds the precautionary saving that is this paper's focus.

Saving for Retirement. We begin with a traditional life–cycle model emphasizing saving in youth and middle age and dissaving in old age (e.g., Modigliani [1986]).

Let the number of "equivalent adults" per household be n_s. Let a household's head constitute 1 "equivalent adult." For a married household, let the spouse constitute ξ^S additional equivalent adults. Although ξ^S might be 1, it could also be substantially less if there are scale economies to household size. If at age s the household head has a spouse, set $n_s^S = 1$; otherwise, set $n_s^S = 0$. Similarly, let n_s^C be the number of children in a household when the head's age is s, and let ξ^C be the adult equivalency of each child. A recent literature (e.g., Banks et al. [1998], Bernheim et al. [2001], Hurd and Rohwedder [2003]) identifies an empirical drop in consumption at retirement; Laitner [2003] associates this with the increase in leisure time. Let ξ^R be the drop at retirement. Then if R is the age of retirement, let

$$n_s = \begin{cases} 1 + \xi^S \cdot n_s^S + \xi^C \cdot n_s^C , & \text{if } s < R, \\ \xi^R \cdot (1 + \xi^S \cdot n_s^S) , & \text{if } s \geq R. \end{cases} \qquad (10)$$

We follow Tobin [1967], who suggests a utility–flow model

$$n_s \cdot u(\frac{c_s}{n_s}) .$$

The idea is that a single–member household with consumption c^1 and the same household at a different age with n equivalent adults and consumption c^n achieve the same per capita current utility flow when $c^n = n \cdot c^1$, and that a household weights $u(.)$ with n because the household values the per capita utility flows of all members equally.

This paper's life–cycle maximization model is then as follows: for household i, born at t, and retiring at age R, we solve for consumption c_{its} at each age s

$$\max_{c_{its}} \int_0^T e^{-\delta \cdot s} \cdot q_s \cdot n_{is} \cdot u\Big(\frac{c_{its}}{n_{is}}\Big)\, ds \tag{11}$$

$$\text{subject to:} \quad \frac{\partial a_{its}}{\partial s} = r_s \cdot a_{its} + \psi_{is} \cdot w \cdot (1-\tau) \cdot e^{g\cdot(t+s)} + SS_{its} - c_{its} \,,$$

$$a_{it0} = 0 = a_{itT} \,,$$

$$a_{its} \geq 0 \text{ all } s \,,$$

where δ is the subjective discount rate; equivalent adults, n_{is}, come from (10); and, a_{its} is the household's net worth (e.g., net asset) position at age s. We assume that financial institutions do not allow borrowing without collateral; hence, the household's net worth can never be negative. As is common in the literature, we assume $u(.)$ is isoelastic:[5]

$$u(x) = \begin{cases} \frac{x^\gamma}{\gamma}\,, & \text{with } \gamma < 1 \text{ and } \gamma \neq 0; \\ \ln(x)\,, & \text{otherwise.} \end{cases} \tag{12}$$

The maximal life span is T years.

The household supplies ψ_{is} "effective hours" in the labor market per hour of work time; thus, if $w \cdot e^{g\cdot(t+s)}$, where $g > 0$ is the rate of labor augmenting technological progress, is the economy wide average wage rate, the household's pretax earnings are $\psi_{is} \cdot w \cdot e^{g\cdot(t+s)}$ per hour at age s. We assume a proportional income tax with rate τ. Life spans are uncertain. Let q_s be the probability of surviving through age s. To simplify, we average male and female survival rates and assume a husband and wife die together. Aftertax earnings at age s are

$$\psi_{is} \cdot w \cdot e^{g\cdot(t+s)} \cdot (1-\tau)\,.$$

We assume that markets offer actuarially fair annuities and that all households take advantage of them. The underlying interest rate is r. At age s, an annuity pays

$$r - \frac{\dot{q}_s}{q_s}\,.$$

This exceeds r since q_s is a declining function of s. The aftertax rate of return on savings is

$$r_s \equiv \big(r - \frac{\dot{q}_s}{q_s}\big) \cdot (1-\tau)\,. \tag{13}$$

[5] This is the only additively separable case with homotheticity. The latter is virtually essential if we are to allow technological progress in a model economy over time.

Economists have long realized that Social Security benefits reduce households' needs for life–cycle wealth. The term SS_{its} in the budget constraint of (11) reflects Social Security taxes in youth and benefits in old age. The Social Security tax is proportional up to a cap; benefits vary with lifetime earnings and a progressive structure of brackets. This paper assumes that over time the cap and the benefit brackets move proportionately to the level $e^{g \cdot t}$ of technology, which preserves the homothetic structure of (11).

Precautionary Saving. This subsection modifies the framework above to incorporate uncertainty about lifetime earnings. Although in our framework markets provide securities (i.e., annuities) that protect a household against mortality risk, we assume that problems stemming from moral hazard preclude market insurance against earnings uncertainty. Households respond with self–insurance efforts. We call the additional wealth that self–insurance stimulates "precautionary saving."

The earnings dynamics analysis of Section 1 provides the template. Each household's age–trajectory of "effective hours" is a quadratic function of age:

$$Q(\text{age}) \equiv \widehat{\alpha_0} + \widehat{\alpha_1} \cdot \text{age} + \widehat{\alpha_2} \cdot \frac{\text{age}^2}{100}\,, \tag{14}$$

with $\widehat{\alpha_i}$ as estimated — see Table A2 in the Appendix. Each household is born with a different earning ability — which Section 1's individual effect μ captures. We assume that a household discovers its μ as it begins work. Nevertheless, in midlife the household's individual effect changes to μ^*, and we assume that though the household knows the distribution from which μ^* will emerge, it only learns its actual realization from the distribution at age 45. Section 1 posits a bivariate normal distribution for (μ, μ^*) pairs in the population as a whole, with zero means and parameters σ_μ, $\sigma_{\mu*}$, and ρ. For consistency with the regression model, we assume that condition on its beginning individual effect μ, a young household perceives that it faces a normal distribution for μ^* with[6]

$$\mu^* \sim N\big(\rho \cdot (\sigma_{\mu*}/\sigma_\mu) \cdot \mu,\ \sigma^2_{\mu*} \cdot (1-\rho^2)\big)\,. \tag{15}$$

If $Q(.)$ is as in (14) and household i is age s, and if M is midlife ($M = 45$ in this paper), we then have

$$\psi_{is} \equiv \psi(\mu_i, \mu_i^*, s) = \begin{cases} \mu_i \cdot Q(s)\,, & \text{if } s \le M, \\ \mu_i^* \cdot Q(s)\,, & \text{if } M < s \le R, \\ 0\,, & \text{if } s > R, \end{cases} \tag{16}$$

where R is the age of retirement. This paper treats R as exogenously given.[7] From this point forward, we ignore the error components η and η^* from our likelihood function: think of them as characterizing measurement error. Section 1 provides a possible story for the change from μ_i to μ_i^* in midlife.

[6] The first argument in $N(.,.)$ below is the mean, and the second is the variance. Note that the mean in this case is the mean conditional on individual effect μ in the first period of life; the unconditional mean, as stated, is zero by assumption.

[7] In contrast, see, for example, Laitner [2003].

Aggregate Household Life–Cycle Net Worth. Let A_t be aggregate household life–cycle net worth at time t if households face lifetime earning uncertainty, and let A^c_t be the same in the certainty case.

Although our focus is precautionary saving, we develop a control, or comparison case as follows. Suppose household i, born at t, learns its μ_i and its μ^*_i at its inception. Such households have no lifetime uncertainty; hence, they have no need for precautionary wealth accumulation. In this case, given (16), we can solve (11).[8] Call the resulting net worth at age s for a household born at time s and having earning abilities μ and μ^*

$$a^c(\mu, \mu^*, t, s) .$$

Let Section 1's bivariate normal density for (μ, μ^*) be

$$\phi(\mu, \mu^* \mid \sigma_\mu, \sigma_{\mu^*}, \rho) ,$$

recalling that the population means for μ and for μ^* are zero. Then average household assets at time t are

$$\int_0^T \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\mu, \mu^* \mid \sigma_\mu, \sigma_{\mu *}, \rho) \cdot q_s \cdot a^c(\mu, \mu^*, t - s, s)\, d\mu\, d\mu^*\, ds . \tag{17}$$

Similarly, average gross–of–tax earnings are

$$w \cdot e^{g \cdot t} \cdot \int_0^R \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\mu, \mu^* \mid \sigma_\mu, \sigma_{\mu *}, \rho) \cdot q_s \cdot \psi(\mu, \mu^*, s) \cdot e^{g \cdot s}\, d\mu\, d\mu^*\, ds . \tag{18}$$

Call the integral in (18) E. If we multiply (17) by the population of the economy, we have A^c_t; if we multiply (18) by the population, we have the economy's gross–of–tax wage bill. A ratio of the two is independent of the population's absolute size. Furthermore, an important consequence of homothetic preferences is that technological change and the wage w each affect (17) and (18) strictly proportionately. Thus,

$$\frac{A^c_t}{w \cdot e^{g \cdot t} \cdot E} = \frac{A^c_0}{w \cdot E} , \tag{19}$$

with the ratio independent of t, w, and the population.[9]

If there is lifetime earning uncertainty, the analysis is slightly more complicated. As before, let M be the age at midlife. Let $J(a_M, \mu, \mu^*, t)$ be second–period–of–life utility for a household born at t, entering its second period with net worth a_M, having second–period–of–life earning ability μ^*, and having first–period ability μ. Then

[8] Problem (11) is a standard optimal control problem — except for the constraint $a_{its} \geq 0$. All of this paper's computations employ Mariger's [1987] algorithm for dealing with the constraint.

[9] To be more precise, the fact that ratio (19) is independent of time and w reflects the homotheticity of preferences, our assumptions about the Social Security system, and our assumptions that the underlying interest rate is fixed and the wage grows only with technology. The last assumptions mean that we are studying the household sector of an economy that has reached a so-called steady–state equilibrium.

$$J(a_M, \mu, \mu^*, t) \equiv \max_{c_{ts}} \int_M^T e^{-\delta \cdot s} \cdot q_s \cdot n_s \cdot u(\frac{c_{ts}}{n_s})\, ds \tag{20}$$

$$\text{subject to:} \quad \frac{\partial a_{ts}}{\partial s} = r_s \cdot a_{ts} + \psi(\mu, \mu^*, s) \cdot w \cdot (1-\tau) \cdot e^{g \cdot (t+s)} + SS(\mu, \mu^*)_{ts} - c_{ts}\,,$$

$$a_{tM} = a_M$$

$$a_{tT} = 0\,,$$

$$a_{ts} \geq 0 \text{ all } s \in [M, T]\,.$$

(Notice that $J(.)$ depends on μ because a household's Social Security benefits depend on its lifetime earnings — though $\psi(\mu, \mu^*, s)$ for $s \geq M$ does not dependent on μ.) Similarly, let $I(a_M, \mu, t)$ be first–period–of–life utility if the household has earning ability μ and ends its first period with net worth a_M. Since Social Security taxes depend only on current earnings, we can write $SS(\mu, .)_{ts}$ for $s \leq M$. Then

$$I(a_M, \mu, t) \equiv \max_{c_{ts}} \int_0^M e^{-\delta \cdot s} \cdot q_s \cdot n_s \cdot u(\frac{c_{ts}}{n_s})\, ds \tag{21}$$

$$\text{subject to:} \quad \frac{\partial a_{ts}}{\partial s} = r_s \cdot a_{ts} + \psi(\mu, \mu^*, s) \cdot w \cdot (1-\tau) \cdot e^{g \cdot (t+s)} + SS(\mu, .)_{ts} - c_{ts}\,,$$

$$a_{t0} = 0$$

$$a_{tM} = a_M\,,$$

$$a_{ts} \geq 0 \text{ all } s \in [0, M]\,.$$

For a given μ and t, we can solve for a_M from

$$a_M = a_M(\mu, t) = \arg\max_a \{I(a, \mu, t) + E_{\mu* \mid \mu}\big[J(a, \mu, \mu^*, t)\big]\}\,, \tag{22}$$

where the density for μ^* conditional on μ comes from (15).

For the model with uncertainty, our procedure is as follows: determine a_M from (22); then determine assets $a(\mu, \mu^*, t-s, s)$ from (20)–(21); and, then substitute the latter into (17) in place of $a^c(.)$. Line (18) remains as before. As in (19), our isoelastic preferences enable us to derive

$$\frac{A_t}{w \cdot e^{g \cdot t} \cdot E} = \frac{A_0}{w \cdot E}\,, \tag{23}$$

with E as above, and with the last ratio independent of time, w, and the economy's population.

3. Simulations

We want to compare (19) and (23) to find the quantitative importance of precautionary saving. Laitner [2001] suggests the empirical ratio for 1995 of private net worth to gross–of–tax labor earnings for the U.S. was about 4.61.[10] Because this paper's model omits estate building, we do not necessarily expect our simulations to produce ratios as high as the empirical one.

Calibration. Although early life–cycle analyses calibrated their parameters in part on the basis of author introspection (e.g., Tobin [1967]), this paper relies heavily on recent estimates from empirical studies.

The child, spouse, and retirement adult–equivalency weights in (10) are potentially important determinants of life–cycle saving — high relative weights for children, for example, front load household consumption and can drastically reduce total life–cycle wealth accumulation (e.g., Auerbach and Kotlikoff [1987, ch.11]). Many authors set ξ^C in the range of .30–.50 and ξ^S equal to 1.00 (e.g., Mariger [1987]). Using U.S. Consumer Expenditure Survey data from 1984–2000, Laitner [2003] finds support for values $\xi^S \leq .50$ and $\xi^C \leq .25$. These presumably reflect returns to scale for larger households. In fact, $\xi^S = .50$ would be consistent with U.S. Social Security benefits to couples, and a low ξ^C perhaps implies that parents reduce their own consumption in years in which they have children at home. For our base case, we set $\xi^S = .50$ and $\xi^C = .25$. Banks et al. [1998], Bernheim et al. [2001], and Laitner [2003] find a consumption reduction of 10–20% or more upon retirement. For our base case, we set $\xi^R = .85$.

Typical values of the household subjective discount rate δ are .00–.02, reflecting households' impatience to consumer sooner rather than later. Laitner [2003] finds that a household's consumption per capita seems to grow on average about 2%/year with age. With this rate of growth, a household's consumption is roughly 2.2 times as high at age 65 as at age 25. Our simulations assume such a growth rate, and derive the δ in each case consistent with it.

The isoelastic parameter γ determines households' degree of risk aversion: if γ is near 1, utility is almost linear and households are quite comfortable with substantial year–to–year consumption unevenness; if γ is small, very negative in particular, utility is sharply concave and households are very averse to consumption fluctuations, hence they are very risk averse. Estimates in the literature range from $\gamma = 0$ to -4. For instance, Auerbach and Kotlikoff [1987] use $\gamma = -3$, Cooley and Prescott [1995] use 0, Rust and Phelan [1997] estimate -.072.[11] On the basis of the size of the consumption decline at retirement, Laitner [2003] estimates $\gamma = -1$ to -1.5.

We use a standard mortality table for 1995, averaging mortality rates for men and

[10] The figure is based mainly on U.S. Flow of Funds data. Private net worth does include the capitalized value of private pension rights, but it does not include Social Security benefits (which receive separate treatment in our analysis). The denominator of the ratio is GDP times labor's share. Labor's share is .7015 (which we determine from wages and salaries as a share of corporate output).

[11] See also Barsky et al. [1997].

women. The average life expectancy is 77 years. For simplicity, we assume that a husband and wife die together.

For earnings, we use our estimates for the whole PSID sample of α_i from Table A2 in the Appendix. Our base–case estimates of h_μ, $h_{\mu *}$, and ρ are described in Section 1 and presented in column 5 of Table 3.

We use the U.S. Social Security System 1995 proportional tax rate, .1052, on earnings; the System's earnings cap ($61,200/year for 1995); and its 1995 benefit formula. U.S. National Income and Product Account government spending (Federal and state and local) on goods and services suggests $\tau = .231$. Based on the slow rate of technological progress after 1970, we set $g = .01$.

For our base case, we set $r = .05$. This is derived as follows. The ratio of ratio of corporate wages and salaries to corporate output is about .2985.[12] Multiplying this times GDP and subtracting total depreciation, we have return to capital net of depreciation. We further subtract the cost to households of financial services (e.g., brokerage fees and financial counseling, service charges of financial intermediaries, and handling expenses for life insurance and pension plans).[13] Then we divide by the sum of the current–cost nonresidential private capital stock, the residential private capital stock, the government fixed capital stock, and the stock of business inventories. The ratio is the average rate of return on capital; under marginal cost pricing and constant returns to scale, this is also the marginal return. The average return 1951–2001 is .055; the 1995 return is .051. Our net–of–tax return for households is $r \cdot (1 - \tau)$. Other calculations are, of course, possible. If we exclude residential housing services from GDP, exclude depreciation on residential housing from total depreciation, and omit the stock of residential housing from our denominator above, the average (gross of tax) rate of return is .081, and the 1995 value is .076. Conversely, Laitner and Stolyarov [2003] argue that intangible capital may be 50 percent as large as the nonresidential capital stock, and with such a correction the average rate of return (reinstating residential capital) falls to .050 and the 1995 rate to .046.

There is no need to set w — homotheticity makes the numerators of (19) and (23) linear in w just as the denominator is, so the wage cancels out of the ratio in each case.

Table 4 summarizes our base–case parameter choices.

Simulations. Table 5 presents three sets of simulations. The first, see row 1, generates aggregative ratios $A/(w \cdot E)$ for our specification with uncertainty over earnings in the second half of life. A household resolves the uncertainty at age 45 — see (20)–(22). The second–row specification eliminates uncertainty, fixing, past age 45, mean earnings conditional on initial earnings. Economic theory shows that row 2 ratios will be smaller than row 1. The third row of Table 5 follows all possible first and second period of life outcomes, with a household knowing its μ and μ^* as it begins work — see (19). We take row 1 minus row 3 as our measure of precautionary wealth accumulation. (Note that there is no theoretical reason that this measure must always be positive.)

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[12] All of the U.S. National Income and Product Accounts data comes from
http://www.bea.doc.go/bea/nd1.ham,
either the interactive "NIPA tables" or the interactive "fixed asset tables."

[13] See lines 87–90 of interactive NIPA Table 2.4.5. In general, these "personal business charges" reduce the net rate of return by almost 1.5%/year.

Table 5 presents results for values of γ between 0 and -4. As we would expect, a lower γ, implying more curvature in the utility function $u(.)$, leads to higher precautionary wealth accumulation. For $\gamma = 0$, our measure of precautionary wealth, the difference between row 1 and row 3, is slightly negative. For $\gamma = -1$, precautionary saving increases national wealth by 5.3%; for $\gamma = -1.5$, the increase is 8.3%; for $\gamma = -2$, it is 11.1%; and, for $\gamma = -4$, the increase is 20.5%. Since the empirical ratio $A/(w \cdot E)$ is about 4.61, for $\gamma = -1$ life–cycle saving including precautionary wealth accumulation explains about 73% of U.S. wealth. With $\gamma = -2$, the explained fraction rises to 77%; with $\gamma = -4$, it is 84%. For comparison, Modigliani [1986] argues that the life–cycle model can account for roughly 80% of U.S. net worth.

The last row of Table 5 suggests a problem with very low values of γ: for γ less than -1, the corresponding value of the subjective discount rate δ is negative — yet we explained above that values $\delta \in [0, .02]$, reflecting some impatience on the part of households, seem the most plausible. One possibility is that the empirical analysis yielding our base–case calibration of consumption growth did not include households' uncertainty about their earnings — see, for example, Caballero [1990].

A second possibility is that slight changes in our calibrations would help. Mathematically, if $\hat{c}_t$ is the percentage growth rate of a household's consumption per capita over ages in which the household's composition is not changing and in which new information about future earnings is not becoming available, we have[14]

$$\hat{c}_t = \frac{r_t \cdot (1 - \tau) - \delta}{1 - \gamma} \,. \tag{24}$$

Our base case sets $\hat{c}_t = .02$. For a given γ, however, we can see that δ can be larger if r_t is higher or if $\hat{c}_t$ is lower. The lowest estimate of $\hat{c}_t$ in Laitner [2003] is .0176. Table 6 considers .015 — a rate of growth at which a household's consumption per capita would rise by a factor of about 1.8 over 40 years. If we exclude residential capital (and its service flow and depreciation), we argued above that we might set $r_t = .076$. Table 6 considers this as well.

For either $\hat{c}_t = .015$ or $r_t = .076$, Table 6 shows that a non-negative δ emerges for γ as low as -1.5. Precautionary saving then augments life–cycle wealth accumulation by 6–7%. In both cases, the percent of U.S. net worth accounted for is smaller, however, than when $\gamma = -1$ in Table 5.

A third possibility is that our utility function — although very standard in the economics literature — is too restrictive.[15]

We proceed assuming that values of γ much below -1 yield implausible implications for δ.

Returning to Table 5, the large difference between rows 2 and 3 is a surprise. Consider the column with $\gamma = -1$. In the certainty–equivalent case, each household finishes life with average earnings conditional on its starting earnings. Total life–cycle saving is only 58%

[14] In fact, our numerical calculations assume discrete time — providing an approximation to (24).

[15] See, for example, Weil [1990].

of empirical national net worth. In row 3, initial earnings are the same, but though there is a distribution of second–stage–of–life earnings, each household knows its second–stage realization as it begins adulthood.[16] A household expecting a low second–period realization will save extra in youth; a household expecting a high second–period realization will save less. The reactions will be asymmetric, however: the liquidity constraint $a_{ts} \geq 0$ puts a restriction on the reduction in saving for a household anticipating high earnings late in life, but there is no corresponding limitation for the increase in saving for a household that is pessimistic about its future earnings. Row 3 generates 69–70% of empirical net worth. Making second–period–of–life earnings uncertain until age 45 — see row 1 — only increases life–cycle net worth to 73% of the empirical total. Although a full recognition of the uninsurable earning uncertainty that households face is appealing from the point of view of realism, in practice the step from row 2 to row 3 is much larger than the step from row 3 to row 1.

Sensitivity Analysis. Table 7 considers alternative child and retirement weights. In all cases the subjective discount rate remains as in Table 5.

Suppose $\gamma = -1$. With $\xi^C = .50$, a value consistent with Mariger [1987] and others, the role of precautionary saving virtually disappears (uncertainty actually lowers aggregative life–cycle net worth slightly). As we would expect, higher consumption for children substantially lowers the fraction of empirical net worth that the model can explain — from 73 percent in Table 5 to 60 percent in Table 7.

With $\xi^C = .25$ as in Table 5, changes in the fall in consumption at retirement have little effect on the role of precautionary saving — as in Table 5, precautionary wealth accumulation is about 5 percent of the life–cycle total. As one would expect, if the weight on retirement consumption is higher, young households save more and aggregative life–cycle net worth is higher. In Table 5, life–cycle saving explains 75 percent of 1995 empirical net worth when $\gamma = -1$; in Table 7 it explains 78 percent with $\xi^R = .90$, but only 69 percent with $\xi^R = .80$.

Table 8 summarizes our last experiment. It employs h_μ, $h_{\mu*}$, and ρ from column 4 (i.e., college graduates) in Table 3. The second–period–of–life standard deviation is noticeably higher, and the correlation ρ lower, than for other education categories. The college educated group makes up about one–quarter of the whole sample (by sampling weight).

Precautionary saving increases life–cycle accumulation by 7.4% in column 2, Table 8 — up from 5.3% in the same column of Table 5. Perhaps more surprising, $A^c/(w \cdot E)$ for the certainty case is 12% larger than Table 5. Again, asymmetric responses to increases and decreases of second–period earnings seem quantitatively more important to total wealth accumulation than uncertainty about second–period–of–life earnings.

[16] In row 1, at age 22 a household knows its μ and the conditional distribution for its μ^*; the household learns its actual μ^* at age 45. In row 2, at age 22 a household learns μ and μ^*, with the latter equaling its conditional mean from row 1. In row 3, at age 22 a household learns both μ and μ^*; μ^* can take any of the values possible in row 1.

4. Conclusion

This paper studies the quantitative importance of precautionary wealth accumulation relative to life–cycle saving for retirement. The first section examines panel data on earnings from the PSID. We find that the cross–sectional variance of earnings within a cohort rises with age. Using a bivariate normal model of random effects, we find that second–period–of–life earnings are strongly positively correlated with initial earnings but indeed have a higher variance. The paper's next section studies the consequences for life–cycle saving. It assumes that households know their youthful earning power as they enter the labor market but that they know only the conditional distribution of their second–period–of–life earnings. Only in midlife do they learn their actual second–period earning ability.

For our most plausible calibrations, precautionary saving only adds 5–6% to aggregative life–cycle wealth accumulation. Nevertheless, our earnings model emerges as quite important: even if second–period–of–life earning changes are fully predictable from youth, so that precautionary saving (i.e., responsiveness to uncertainty) plays no role, the variety of earning profiles that our bivariate normal model generates itself stimulates enough extra wealth accumulation to merit careful consideration. In the presence of liquidity constraints, predictions of rising earnings decrease youthful saving less than anticipations of falling earnings raise it. In the end, heterogeneity of earning profiles, even without uncertainty, tends to increase aggregative life–cycle wealth accumulation.

Appendix

Table A1. Weighted Ordinary Least Squares PSID 1967–94: Less Than High School Education			
Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	8.1418	0.0907	89.7724
AGE	0.0719	0.0043	16.7510
AGE**2/100	-0.0767	0.0052	-14.7021
DUM 67	-0.0210	0.0462	-0.4544
DUM 68	0.0084	0.0475	0.1767
DUM 69	0.0333	0.0477	0.6980
DUM 70	0.0204	0.0483	0.4228
DUM 71	0.0445	0.0487	0.9156
DUM 72	0.1004	0.0490	2.0464
DUM 73	0.1500	0.0492	3.0478
DUM 74	0.1156	0.0499	2.3176
DUM 75	0.0607	0.0503	1.2062
DUM 76	0.1100	0.0506	2.1751
DUM 77	0.1400	0.0512	2.7337
DUM 78	0.1534	0.0520	2.9527
DUM 79	0.1438	0.0521	2.7599
DUM 80	0.0900	0.0525	1.7139
DUM 81	0.0756	0.0537	1.4091
DUM 82	-0.0352	0.0545	-0.6460
DUM 83	-0.0251	0.0551	-0.4559
DUM 85	-0.0248	0.0568	-0.4358
DUM 86	-0.1150	0.0569	-2.0201
DUM 87	-0.0953	0.0582	-1.6385
DUM 88	-0.0630	0.0594	-1.0593
DUM 89	-0.1400	0.0607	-2.3055
DUM 90	-0.1214	0.0625	-1.9435
DUM 91	-0.2209	0.0629	-3.5140
DUM 92	-0.0565	0.0647	-0.8732
DUM 93	-0.1080	0.0698	-1.5483
DUM 94	0.0460	0.0689	0.6674
Observations	8048		
Error Mean Square	.3597		
R^2	.0695		
Addendum:			
Age of maximum earnings[a]	46.8940		
Maximum Earnings ÷ Earnings at Age 25	1.4444		

a. Age of maximum earnings omitting technological progress.

Table A1 (cont.). Weighted Ordinary Least Squares PSID 1967–94: High School Education

Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	8.2095	0.0604	135.8710
AGE	0.0807	0.0030	26.8944
AGE**2/100	-0.0859	0.0038	-22.7431
DUM 67	0.0326	0.0330	0.9867
DUM 68	0.0449	0.0338	1.3302
DUM 69	0.0760	0.0335	2.2656
DUM 70	0.0568	0.0335	1.6984
DUM 71	0.0587	0.0332	1.7675
DUM 72	0.1291	0.0329	3.9184
DUM 73	0.1512	0.0329	4.5963
DUM 74	0.0995	0.0328	3.0352
DUM 75	0.0642	0.0329	1.9498
DUM 76	0.0805	0.0330	2.4407
DUM 77	0.1081	0.0330	3.2806
DUM 78	0.1411	0.0328	4.2984
DUM 79	0.1160	0.0327	3.5511
DUM 80	0.0672	0.0328	2.0454
DUM 81	0.0323	0.0329	0.9832
DUM 82	-0.0322	0.0330	-0.9760
DUM 83	-0.0373	0.0331	-1.1275
DUM 85	-0.0153	0.0332	-0.4612
DUM 86	-0.0172	0.0332	-0.5185
DUM 87	-0.0613	0.0331	-1.8507
DUM 88	-0.0423	0.0333	-1.2680
DUM 89	-0.0653	0.0335	-1.9505
DUM 90	-0.1152	0.0338	-3.4123
DUM 91	-0.1289	0.0337	-3.8196
DUM 92	-0.0624	0.0345	-1.8098
DUM 93	0.0077	0.0350	0.2192
DUM 94	0.0101	0.0347	0.2916
Observations	15030		
Error Mean Square	.3121		
R^2	.0815		
Addendum:			
Age of maximum earnings[a]	46.9510		
Maximum Earnings ÷ Earnings at Age 25	1.5127		

a. Age of maximum earnings omitting technological progress.

Table A1 (cont.). Weighted Ordinary Least Squares PSID 1967–94: Some College Education			
Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	7.6807	0.0923	83.2571
AGE	0.1084	0.0047	23.2817
AGE**2/100	-0.1151	0.0059	-19.3969
DUM 67	0.0731	0.0490	1.4932
DUM 68	0.1081	0.0504	2.1460
DUM 69	0.1079	0.0494	2.1831
DUM 70	0.1139	0.0482	2.3618
DUM 71	0.0866	0.0477	1.8177
DUM 72	0.1529	0.0468	3.2679
DUM 73	0.1705	0.0464	3.6741
DUM 74	0.1672	0.0457	3.6580
DUM 75	0.1020	0.0453	2.2543
DUM 76	0.1549	0.0450	3.4418
DUM 77	0.1290	0.0447	2.8860
DUM 78	0.1462	0.0443	3.2999
DUM 79	0.1406	0.0442	3.1797
DUM 80	0.1284	0.0442	2.9071
DUM 81	0.0796	0.0442	1.8031
DUM 82	-0.0117	0.0440	-0.2663
DUM 83	-0.0090	0.0442	-0.2034
DUM 85	-0.0143	0.0438	-0.3268
DUM 86	0.0096	0.0441	0.2177
DUM 87	-0.0268	0.0442	-0.6065
DUM 88	-0.0368	0.0442	-0.8321
DUM 89	-0.0487	0.0442	-1.1033
DUM 90	-0.0611	0.0444	-1.3772
DUM 91	-0.0945	0.0443	-2.1341
DUM 92	0.0061	0.0447	0.1371
DUM 93	0.0833	0.0459	1.8142
DUM 94	-0.0071	0.0463	-0.1532
Observations	8402		
Error Mean Square	.3395		
R^2	.1204		
Addendum:			
Age of maximum earnings[a]	47.0866		
Maximum Earnings ÷ Earnings at Age 25	1.7530		

a. Age of maximum earnings omitting technological progress.

Table A1 (cont.). Weighted Ordinary Least Squares PSID 1967–94: College/more Education

Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	7.3573	0.0949	77.5567
AGE	0.1335	0.0047	28.5248
AGE**2/100	-0.1370	0.0058	-23.7165
DUM 67	0.0366	0.0464	0.7879
DUM 68	0.0603	0.0460	1.3104
DUM 69	0.0725	0.0450	1.6103
DUM 70	0.0539	0.0444	1.2118
DUM 71	0.0508	0.0439	1.1577
DUM 72	0.0745	0.0435	1.7135
DUM 73	0.0578	0.0426	1.3574
DUM 74	0.0444	0.0420	1.0565
DUM 75	0.0019	0.0419	0.0443
DUM 76	0.0305	0.0417	0.7320
DUM 77	0.0652	0.0414	1.5745
DUM 78	0.0509	0.0412	1.2341
DUM 79	0.0287	0.0412	0.6960
DUM 80	0.0032	0.0410	0.0771
DUM 81	-0.0370	0.0408	-0.9088
DUM 82	-0.0447	0.0406	-1.1001
DUM 83	-0.0471	0.0406	-1.1598
DUM 85	0.0189	0.0406	0.4655
DUM 86	0.0483	0.0405	1.1939
DUM 87	0.0411	0.0405	1.0155
DUM 88	0.0030	0.0403	0.0748
DUM 89	0.0153	0.0405	0.3786
DUM 90	0.0196	0.0406	0.4832
DUM 91	-0.0304	0.0405	-0.7502
DUM 92	-0.0200	0.0409	-0.4890
DUM 93	0.1375	0.0421	3.2662
DUM 94	0.0521	0.0423	1.2320
Observations	12052		
Error Mean Square	.4068		
R^2	.1346		
Addendum:			
Age of maximum earnings[a]	48.7223		
Maximum Earnings ÷ Earnings at Age 25	2.1613		

a. Age of maximum earnings omitting technological progress.

Table A1 (cont.). Weighted Ordinary Least Squares PSID 1967–94: All Education Groups			
Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	7.7214	0.0430	179.6805
AGE	0.1116	0.0021	52.7909
AGE**2/100	-0.1233	0.0026	-46.8739
DUM 67	-0.0932	0.0220	-4.2419
DUM 68	-0.0525	0.0225	-2.3307
DUM 69	-0.0237	0.0224	-1.0602
DUM 70	-0.0280	0.0223	-1.2540
DUM 71	-0.0183	0.0223	-0.8212
DUM 72	0.0439	0.0221	1.9836
DUM 73	0.0683	0.0220	3.0983
DUM 74	0.0478	0.0220	2.1752
DUM 75	0.0081	0.0220	0.3670
DUM 76	0.0458	0.0220	2.0849
DUM 77	0.0722	0.0220	3.2818
DUM 78	0.0901	0.0220	4.1013
DUM 79	0.0745	0.0219	3.3959
DUM 80	0.0433	0.0220	1.9706
DUM 81	0.0137	0.0220	0.6214
DUM 82	-0.0400	0.0220	-1.8153
DUM 83	-0.0373	0.0221	-1.6862
DUM 85	-0.0023	0.0222	-0.1019
DUM 86	0.0031	0.0222	0.1397
DUM 87	-0.0156	0.0222	-0.7005
DUM 88	-0.0152	0.0223	-0.6816
DUM 89	-0.0267	0.0224	-1.1890
DUM 90	-0.0396	0.0226	-1.7521
DUM 91	-0.0740	0.0226	-3.2804
DUM 92	-0.0059	0.0229	-0.2582
DUM 93	0.0889	0.0235	3.7779
DUM 94	0.0498	0.0235	2.1175
Observations	43532		
Error Mean Square	.4105		
R^2	.0715		
Addendum:			
Age of maximum earnings[a]	45.2511		
Maximum Earnings ÷ Earnings at Age 25	1.6582		

a. Age of maximum earnings omitting technological progress.

Table A2. Weighted Maximum Likelihood PSID 1967–94: Less Than High School Education

Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	8.279941	0.096553	85.7553
AGE	0.066201	0.004187	15.8112
AGE**2/100	-0.072579	0.004948	-14.6690
DUM 67	-0.007690	0.037809	-0.2034
DUM 68	0.000855	0.038217	0.0224
DUM 69	0.041521	0.037923	1.0949
DUM 70	0.016641	0.037884	0.4393
DUM 71	0.014586	0.037706	0.3868
DUM 72	0.088604	0.037620	2.3552
DUM 73	0.125054	0.036977	3.3820
DUM 74	0.101226	0.037021	2.7343
DUM 75	0.028708	0.036697	0.7823
DUM 76	0.071199	0.036390	1.9566
DUM 77	0.069135	0.036201	1.9098
DUM 78	0.118541	0.036450	3.2521
DUM 79	0.098236	0.036104	2.7209
DUM 80	0.034143	0.036306	0.9404
DUM 81	0.025908	0.036723	0.7055
DUM 82	-0.073257	0.036943	-1.9830
DUM 83	-0.066577	0.037259	-1.7869
DUM 85	-0.058271	0.038735	-1.5043
DUM 86	-0.154304	0.039222	-3.9342
DUM 87	-0.084447	0.040748	-2.0724
DUM 88	-0.032542	0.042085	-0.7732
DUM 89	-0.067380	0.043294	-1.5563
DUM 90	-0.107726	0.045134	-2.3868
DUM 91	-0.199414	0.045936	-4.3411
DUM 92	-0.064014	0.047054	-1.3604
DUM 93	-0.056603	0.051189	-1.1058
DUM 94	0.065458	0.051374	1.2741
Observations	6680		
-Log(likelihood)	4193.8052		
Addendum:			
Age of maximum earnings[a]	45.6061		
Maximum Earnings ÷ Earnings at Age 25	1.3609		

a. Age of maximum earnings omitting technological progress.

Table A2 (cont.). Weighted Maximum Likelihood PSID 1967–94: High School Education

Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	8.266227	0.064765	127.6346
AGE	0.074114	0.003050	24.3015
AGE**2/100	-0.076620	0.003852	-19.8916
DUM 67	0.062837	0.028903	2.1740
DUM 68	0.069526	0.028893	2.4063
DUM 69	0.102014	0.028243	3.6120
DUM 70	0.079769	0.027831	2.8662
DUM 71	0.088426	0.027269	3.2427
DUM 72	0.155841	0.026895	5.7944
DUM 73	0.169386	0.026485	6.3955
DUM 74	0.119724	0.026145	4.5792
DUM 75	0.080904	0.025907	3.1228
DUM 76	0.101325	0.025630	3.9534
DUM 77	0.127049	0.025389	5.0040
DUM 78	0.165184	0.024896	6.6348
DUM 79	0.140383	0.024553	5.7176
DUM 80	0.079933	0.024429	3.2721
DUM 81	0.037083	0.024260	1.5285
DUM 82	-0.027064	0.024263	-1.1155
DUM 83	-0.040539	0.024132	-1.6799
DUM 85	-0.004354	0.024343	-0.1788
DUM 86	-0.003415	0.024432	-0.1398
DUM 87	-0.049937	0.024609	-2.0293
DUM 88	-0.031488	0.025010	-1.2590
DUM 89	-0.058021	0.025364	-2.2875
DUM 90	-0.098311	0.025812	-3.8088
DUM 91	-0.124475	0.025981	-4.7910
DUM 92	-0.047823	0.026874	-1.7795
DUM 93	0.011274	0.027594	0.4086
DUM 94	0.014965	0.027966	0.5351
Observations	13048		
-Log(likelihood)	7742.7954		
Addendum:			
Age of maximum earnings[a]	48.3652		
Maximum Earnings ÷ Earnings at Age 25	1.5194		

a. Age of maximum earnings omitting technological progress.

Table A2 (cont.). Weighted Maximum Likelihood PSID 1967–94: Some College Education

Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	7.431462	0.097467	76.2456
AGE	0.122543	0.004756	25.7669
AGE**2/100	-0.134016	0.006190	-21.6507
DUM 67	0.065335	0.041389	1.5785
DUM 68	0.112989	0.041993	2.6906
DUM 69	0.092816	0.040595	2.2864
DUM 70	0.116975	0.039107	2.9911
DUM 71	0.096178	0.038230	2.5158
DUM 72	0.156968	0.037106	4.2302
DUM 73	0.175721	0.036523	4.8112
DUM 74	0.156599	0.035501	4.4112
DUM 75	0.099345	0.034537	2.8765
DUM 76	0.146510	0.033782	4.3369
DUM 77	0.108987	0.032958	3.3069
DUM 78	0.151929	0.032231	4.7138
DUM 79	0.133734	0.031810	4.2041
DUM 80	0.120133	0.031566	3.8058
DUM 81	0.045373	0.031392	1.4454
DUM 82	-0.032855	0.031136	-1.0552
DUM 83	-0.013388	0.031454	-0.4256
DUM 85	-0.025243	0.031135	-0.8108
DUM 86	-0.011238	0.031401	-0.3579
DUM 87	-0.036138	0.031769	-1.1375
DUM 88	-0.045722	0.032189	-1.4204
DUM 89	-0.071995	0.032765	-2.1973
DUM 90	-0.088465	0.033623	-2.6311
DUM 91	-0.090339	0.034360	-2.6292
DUM 92	-0.042244	0.035487	-1.1904
DUM 93	0.052715	0.036875	1.4296
DUM 94	0.032529	0.037858	0.8592
Observations	7288		
-Log(likelihood)	4286.6292		
Addendum:			
Age of maximum earnings[a]	45.7197		
Maximum Earnings ÷ Earnings at Age 25	1.7777		

a. Age of maximum earnings omitting technological progress.

Table A2 (cont.). Weighted Maximum Likelihood PSID 1967–94: College/more Education			
Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	6.973176	0.092855	75.0972
AGE	0.154507	0.004376	35.3092
AGE**2/100	-0.165338	0.005729	-28.8596
DUM 67	0.044327	0.039269	1.1288
DUM 68	0.084895	0.038292	2.2170
DUM 69	0.097997	0.037075	2.6432
DUM 70	0.100493	0.036134	2.7811
DUM 71	0.076964	0.035095	2.1930
DUM 72	0.095185	0.034172	2.7855
DUM 73	0.085333	0.032917	2.5924
DUM 74	0.072009	0.031862	2.2600
DUM 75	0.021892	0.031270	0.7001
DUM 76	0.045858	0.030509	1.5031
DUM 77	0.057084	0.029784	1.9166
DUM 78	0.052023	0.029156	1.7843
DUM 79	0.033307	0.028818	1.1558
DUM 80	0.018452	0.028327	0.6514
DUM 81	-0.031345	0.027743	-1.1299
DUM 82	-0.049523	0.027355	-1.8104
DUM 83	-0.051585	0.027198	-1.8966
DUM 85	0.007380	0.027244	0.2709
DUM 86	0.046630	0.027381	1.7030
DUM 87	0.029964	0.027658	1.0834
DUM 88	0.011126	0.028157	0.3952
DUM 89	0.014515	0.028916	0.5020
DUM 90	0.019387	0.029682	0.6532
DUM 91	0.013864	0.030547	0.4539
DUM 92	0.011311	0.031695	0.3569
DUM 93	0.161604	0.033437	4.8331
DUM 94	0.088158	0.034594	2.5484
Observations	10231		
-Log(likelihood)	6589.3093		
Addendum:			
Age of maximum earnings[a]	46.7246		
Maximum Earnings ÷ Earnings at Age 25	2.1822		

a. Age of maximum earnings omitting technological progress.

Table A2 (cont.). Weighted Maximum Likelihood PSID 1967–94: All Education Groups			
Variable	Coefficient Estimate	Standard Error	T-Statistic
CONSTANT	7.903786	0.042979	183.9001
AGE	0.099501	0.001965	50.6303
AGE**2/100	-0.109248	0.002461	-44.4001
DUM 67	-0.026447	0.018027	-1.4671
DUM 68	-0.000070	0.018042	-0.0039
DUM 69	0.024524	0.017640	1.3902
DUM 70	0.019041	0.017343	1.0979
DUM 71	0.022134	0.017006	1.3016
DUM 72	0.076102	0.016713	4.5534
DUM 73	0.091683	0.016352	5.6067
DUM 74	0.065794	0.016056	4.0978
DUM 75	0.020552	0.015811	1.2999
DUM 76	0.057397	0.015549	3.6913
DUM 77	0.067513	0.015297	4.4135
DUM 78	0.098813	0.015060	6.5615
DUM 79	0.084902	0.014864	5.7119
DUM 80	0.050613	0.014757	3.4297
DUM 81	0.007465	0.014641	0.5098
DUM 82	-0.045980	0.014566	-3.1566
DUM 83	-0.039966	0.014559	-2.7452
DUM 85	0.001325	0.014645	0.0905
DUM 86	0.011166	0.014737	0.7577
DUM 87	-0.002808	0.014917	-0.1883
DUM 88	0.004032	0.015177	0.2657
DUM 89	-0.012602	0.015486	-0.8137
DUM 90	-0.029247	0.015867	-1.8432
DUM 91	-0.053140	0.016153	-3.2899
DUM 92	0.004083	0.016713	0.2443
DUM 93	0.096477	0.017436	5.5333
DUM 94	0.081996	0.017805	4.6051
Observations	37247		
-Log(likelihood)	24013.7078		
Addendum:			
Age of maximum earnings[a]	45.5391		
Maximum Earnings ÷ Earnings at Age 25	1.5854		

a. Age of maximum earnings omitting technological progress.

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Table 1. Panel Study of Income Dynamics Sample 1967–1994[a]

Panel Length:	less than HS	high school	some college	college or more	all
1–5	329	278	103	108	818
6–10	161	214	104	99	578
11–15	114	184	87	129	514
16–20	99	165	95	130	489
21–25	63	175	102	167	507
26–28	47	121	79	114	361
Total House–holds:	813	1137	570	747	3267
Total Obs.	8,048	15,030	8,402	12,052	43,532
Obs. with Censured Earnings	1	4	4	59	68
Obs. with Adjusted Hours:[b]	1,594	2,155	1,164	1,609	6,522
Obs. Dropped Zero Hrs:	98	122	107	117	444
Average Earnings[c]	$19,264	$23,954	$27,076	$38,723	$27,778

a. Men only; no poverty sample; ages $\max\{education + 6, 16\}$ to 60.
b. Work hours adjusted upward to minimum 1750 hours/year.
c. Arithmetic average; 1984 dollars; NIPA consumption deflator.

Table 2. Estimated Variance of Residual from Weighted OLS:

PSID Sample 1967–1994[a]

Age	less than HS	high school	some college	college or more	all
25	0.311984	0.398072	0.212237	0.300392	0.329970
26	0.262474	0.354262	0.254899	0.264066	0.313142
27	0.260435	0.210086	0.290596	0.258597	0.266634
28	0.395252	0.194260	0.281459	0.245593	0.282218
29	0.260636	0.207678	0.217061	0.315352	0.277497
30	0.365447	0.266462	0.341118	0.279861	0.335799
31	0.431972	0.290941	0.267845	0.260505	0.327952
32	0.215023	0.293851	0.277570	0.274925	0.311889
33	0.274406	0.324884	0.271092	0.288077	0.332506
34	0.385872	0.248387	0.290102	0.295364	0.346200
35	0.379533	0.279906	0.286025	0.393355	0.388124
36	0.400243	0.269780	0.335601	0.305611	0.373906
37	0.446240	0.243054	0.250060	0.341668	0.369742
38	0.446234	0.340350	0.286133	0.386692	0.416816
39	0.295150	0.318533	0.254221	0.534404	0.434403
40	0.267611	0.292404	0.299539	0.395925	0.387590
41	0.306671	0.308220	0.246820	0.420451	0.392671
42	0.316903	0.278106	0.261265	0.340663	0.363474
43	0.280287	0.260034	0.239901	0.571841	0.413591
44	0.259459	0.263336	0.419063	0.440348	0.409213
45	0.318383	0.258221	0.400519	0.788129	0.519896
46	0.311886	0.246555	0.348500	0.319797	0.380750
47	0.274505	0.327321	1.054556	0.415975	0.538985
48	0.297978	0.248192	0.536444	0.368542	0.411322
49	0.326165	0.313183	0.269459	0.875103	0.525585
50	0.313074	0.306346	0.325590	0.421587	0.418631
51	0.327585	0.344874	0.292986	0.413396	0.425698
52	0.347307	0.380379	0.411005	0.383428	0.456921
53	0.276323	0.301929	0.416874	0.489554	0.436239
54	0.353970	0.433464	0.355427	0.441288	0.475476
55	0.312131	0.463306	0.417202	0.508088	0.508746
56	0.353727	0.367588	0.392770	0.654748	0.505648
57	0.440915	0.424415	0.331049	0.649548	0.558536
58	0.402615	0.387770	0.404991	0.557092	0.522803
59	0.372167	0.387991	0.513761	0.728239	0.551823
60	0.568806	0.349382	0.576206	0.731860	0.629284

Table 2 (cont.). Estimated Variance of Residual from Weighted OLS:

PSID Sample 1967–1994[a]

Aver–age for Age	less than HS	high school	some college	college or more	all
25–39	0.342060	0.282701	0.274401	0.316297	0.340453
46–60	0.351944	0.352180	0.443121	0.530550	0.489763

a. As in Table 1: men only; no poverty sample; ages $\max\{education + 6, 16\}$ to 60; work hours adjusted upward to minimum 1750 hours/year.

Table 3. Maximum Likelihood Precision Estimates (Standard Deviation): PSID Sample 1967–1994[a]

Para– meter	less than HS	high school	some college	college or more	all
h_{μ}	2.3882 (0.0963)	2.6407 (0.0707)	2.6428 (0.0947)	2.4991 (0.0762)	2.2968 (0.0359)
h_{η}	2.4195 (0.0300)	2.5894 (0.0191)	2.6326 (0.0246)	2.4747 (0.0201)	2.4952 (0.0109)
$h_{\mu *}$	1.9724 (0.0690)	2.0141 (0.0751)	2.0213 (0.1260)	1.1856 (0.0573)	1.5614 (0.0324)
$h_{\eta *}$	2.8461 (0.0383)	2.6717 (0.0349)	2.4546 (0.0526)	2.4879 (0.0412)	2.6286 (0.0202)
ρ	0.7955 (0.0405)	0.7165 (0.0390)	0.6471 (0.0671)	0.5749 (0.0530)	0.7009 (0.0217)

a. As in Table 1: men only; no poverty sample; ages $\max\{education + 6, 16\}$ to 60; work hours adjusted upward to minimum 1750 hours/year.

Table 4. Parameter Values and Empirical Ratios	
Name	Value
Parameter	
ξ^C	.25
ξ^S	.50
ξ^R	.85
consumption growth with age	.02
τ	.231
g	.01
Social Security tax rate	.1052
r	.05
h_μ	2.2968
$h_{\mu *}$	1.5614
ρ	.7009
Age	
adulthood	22
child bearing	24
M	45
retirement	64
T	90
Ratio	
aggregate net worth/labor earnings	4.610

Source: see text.

Table 5. Simulation Results: Ratio $\frac{A}{w \cdot E}$ for Base–Case Calibration				
$\gamma = 0$	$\gamma = -1$	$\gamma = -1.5$	$\gamma = -2$	$\gamma = -4$
Uncertainty Case				
3.167	3.380	3.475	3.564	3.868
Certainty–equivalent Case				
2.679	2.679	2.679	2.679	2.679
Certainty Case				
3.209	3.209	3.209	3.209	3.209
[Row 1 - Row 3] ÷ Row 3				
-.013	.053	.083	.111	.205
Uncertainty–Case $A/(w \cdot E)$ ÷ empirical ratio				
.687	.733	.754	.773	.839
Implied Subjective Discount Rate δ				
.018	-.002	-.012	-.022	-.063

Source: see text.

Table 6. Simulation Results: Ratio $\frac{A}{w \cdot E}$ for Alternative Calibrations

$\gamma = 0$	$\gamma = -1$	$\gamma = -1.5$	$\gamma = -2$	$\gamma = -4$
Calibration with $\hat{c}_t = .015$				
Uncertainty Case				
2.514	2.704	2.791	2.873	3.152
Certainty–equivalent Case				
2.057	2.057	2.057	2.057	2.057
Certainty Case				
2.633	2.633	2.633	2.633	2.633
[Row 1 - Row 3] $\div$ Row 3				
-.045	.027	.060	.091	.197
Uncertainty–Case $A/(w \cdot E) \div$ empirical ratio				
.545	.587	.605	.623	.684
Implied Subjective Discount Rate δ				
.023	.008	.001	-.007	-.037
Calibration with $r_t = .076$				
Uncertainty Case				
2.804	3.021	3.117	3.208	3.510
Certainty–equivalent Case				
2.306	2.306	2.306	2.306	2.306
Certainty Case				
2.909	2.909	2.909	2.909	2.909
[Row 1 - Row 3] $\div$ Row 3				
-.036	.039	.072	.103	.207
Uncertainty–Case $A/(w \cdot E) \div$ empirical ratio				
.608	655	.676	.696	.761
Implied Subjective Discount Rate δ				
.037	.018	.008	-.002	-.042

Source: see text.

Table 7. Simulation Results: Ratio $\frac{A}{w \cdot E}$ for Alternative Values of ξ^C and ξ^R				
$\gamma = 0$	$\gamma = -1$	$\gamma = -1.5$	$\gamma = -2$	$\gamma = -4$
Calibration with $\xi^C = .50$				
Uncertainty Case				
2.562	2.768	2.863	2.952	3.257
Certainty Case				
2.795	2.795	2.795	2.795	2.795
[Row 1 - Row 2] ÷ Row 2				
-.083	-.010	.024	.056	.165
Uncertainty–Case $A/(w \cdot E)$ ÷ empirical ratio				
.556	.600	.621	.640	.707
Calibration with $\xi^R = .80$				
Uncertainty Case				
2.961	3.171	3.265	3.353	3.652
Certainty Case				
3.019	3.019	3.019	3.019	3.019
[Row 1 - Row 2] ÷ Row 2				
-.019	.050	.081	.111	.210
Uncertainty–Case $A/(w \cdot E)$ ÷ empirical ratio				
.642	.688	.708	.727	.792
Calibration with $\xi^R = .90$				
Uncertainty Case				
3.370	3.585	3.682	3.772	4.087
Certainty Case				
3.397	3.397	3.397	3.397	3.397
[Row 1 - Row 2] ÷ Row 2				
-.008	.055	.084	.110	.203
Uncertainty–Case $A/(w \cdot E)$ ÷ empirical ratio				
.731	.778	.799	.818	.887

Source: see text.

Table 8. Simulation Results: Ratio $\frac{A}{w \cdot E}$ for Table 3, column 4, Precisions and Correlation Coefficient

$\gamma = 0$	$\gamma = -1$	$\gamma = -1.5$	$\gamma = -2$	$\gamma = -4$
Uncertainty Case				
3.520	3.871	4.017	4.153	4.587
Certainty–equivalent Case				
2.445	2.445	2.445	2.445	2.445
Certainty Case				
3.604	3.604	3.604	3.604	3.604
[Row 1 - Row 3] ÷ Row 3				
-.023	.074	.115	.152	.273
Uncertainty–Case $A/(w \cdot E)$ ÷ empirical ratio				
.763	.840	.871	.901	.995
Implied Subjective Discount Rate δ				
.018	-.002	-.012	-.022	-.063

Source: see text.