Heterogeneous Behavioral Rules in the Oligopolistic Case[1]

Markus Pasche

Abstract: In a static symmetric duopoly the set of behavioral rules is extended to different types of markup pricing. Using an equilibrium concept suggested in Pasche (2001), it is shown that dependend on the markup neither pure Cournot nor pure Bertrand behavior is a behavioral equilibrium profile. Instead, there is a rationale for the usage of simple heuristics. The presence of markup rules leads to Stackelberg outcomes. Furthermore, pure markup behavior is more competitive than in Cournot case but less competitive than in Bertrand case. It is shown, that multiple behavioral equilibria and heterogeneous behavior may arise, where at least one player uses price setting strategies.

Keywords: oligopoly, markup rules, heterogenity, behavioral equilibrium.

JEL-Classification: L13, D21, D43

1 Introduction

Since the studies of Hall/Hitch (1939) it is occasionally claimed that the analysis of industrial organization should focus on empirically observable types of supply behavior. Some of the most prominent heuristics in oligopoly are different types of markup pricing: Since suppliers often have no idea about marginal cost and marginal revenues they try to acchieve a price as a fixed ratio (or a fixed amount) to the average cost. One major problem of such a heuristic behavior is that in general it is in contrast to the assumption of rational profit maximizing agents which is essential for microeconomic reasoning. Therefore there exist more empirical studies on markup-pricing than theoretical models which account for these heuristics. Grant/Quiggin (1994), for example, analyse the case of oligopolistic competition where the markups are chosen optimally. They show that the resulting equilibrium with markup strategies is more competitive than the Cournot solution.

However, it is by no means clear why such heuristics should be preferred to optimizing behavior, i.e. there is no economic *explanation* for the evidence of these rules of thumb. It seems plausible to argue that under the condition of competition heuristic rules will be ruled out in the long run by maximizing behavior. The usage of simple rules is then

[1]Working Paper Series B, No. 2002/01, University if Jena, Faculty of Economics

due to short run limitations of agent´s abilities or due to market imperfections. In this paper, however, it is shown that the opposite claim can be true: simple rules may outperform optimizing behavior and may constitute an equilibrium.

Things are getting more complicated if either the price or the quantity can be chosen as a strategic variable. It is well known that under rather general conditions pricing strategies will lead to more competitive outcomes than quantity strategies (Chang 1985, Vives 1985). Under certain conditions (like non-decreasing returns of scale and a very close substitution between goods) no economically reasonable equilibrium in pricing strategies exists. These arguments justify the assumption of quantity setting. However, one objection is that pure quantity setting behavior is empirically less relevant and the emergence of prices in markets remains “somewhat mysterious” (Shapiro 1989, p.343).

The paper shows for the case of a multi-stage symmetric duopoly game that there exists an incentive to use simple price or quantity setting rules. We use an equilibrium concept which does not only account for the chosen strategies but also for the *rules*, *how* these strategies are chosen (cf. Pasche 2001). Following Rubinstein (1998, pp. 3) this broader notion of equilibrium allows for the analysis of arbitrary types of boundedly rational decision behavior and may provide a rationale for some of them. We are interested in equilibria where the players behave differently even when the demand and cost functions are strictly symmetric. Furthermore it is analysed how the equilibrium structure depends on price or quantity strategies and how competitive equilibria with markup rules can be compared with Cournot and Bertrand solutions.

2 Behavioral Equilibria with Markup Rules

2.1 Mixing optimizing and markup behavior

In the following section a differentiated duopoly with complete information is considered. Both players have a quadratic cost function $C(q_i) = cq_i + dq_i^2, i = 1, 2$, with q_i as the quantity produced by firm i and $c, d \geq 0$ as parameters. The demand scheme is assumed to be linear with $p_i = a - bq_i + \delta bq_j,\ a > c, b > 0, \delta \in [0, 1], i = 1, 2, i \neq j$. The parameter δ captures the degree of substitution: For $\delta = 0$ the goods are completely differentiated and the players act as monopolists. With $\delta = 1$ the goods are perfect substitutes, i.e. they are homogeneous. Let $\sigma_i \in \{q_i, p_i\}$ be the strategy variable of player i. In case of $\sigma_i = p_i$ the linear demand scheme can simply be solved for q_i. If players are profit maximizers, the so-called reaction functions can easily be derived as

$$p_i^* = f^B(p_j) = \frac{1}{2}\frac{ba(1-\delta+\gamma)(1-\delta^2) + 2ad(1-\delta) + \delta(2d + b(1-\delta^2))p_j}{(1-\delta^2)b + d} \tag{1}$$

$$q_i^* = f^C(q_j) = \frac{1}{2}\frac{a - b\delta q_j - c}{b + d} \tag{2}$$

with $\gamma = c/a, i = 1, 2, i \neq j$ and $\gamma \in [0, 1)$ because of $0 \leq c < a$. The reaction functions f^B and f^C represent the *behavioral rule* of payoff maximization in the case of price competition (f^B for Bertrand behavior) and quantity competition (f^C for Cournot behavior). A vector (f^B, f^B) or (f^C, f^C) is called a *behavioral profile* in contrast to the *strategy profile* (p_1, p_2) or (q_1, q_2). The Cournot-Nash solution (q_1^*, q_2^*) can also be denoted as $(f^C(q_2^*), f^C(q_1^*))$ and the Bertrand-Nash solution (p_1^*, p_2^*) can be written as $(f^B(p_2^*), f^B(p_1^*))$. The corresponding equilibrium payoffs are

$$\pi_i^{BB} = \frac{Z_1(b(1-\delta^2) + d)}{(Z_2 + b\delta(1-\delta))^2} \tag{3}$$

$$\pi_i^{CC} = \frac{Z_1(b + d)}{(Z_2 + b\delta)^2} \tag{4}$$

with $Z_1 = a^2(\gamma - 1)^2 > 0$ and $Z_2 = 2(b + d) > 0$. The upper index BB (resp. CC) denotes the behavioral profile, where the first letter describes the own behavioral rule and the second letter the opponent´s rule. For homogeneous goods ($\delta = 1$) Bertrand competition leads to the competitive result, i.e. price equals marginal costs. Hence, if the cost function is linear ($d = 0$) profits are zero, while in the case of a quadratic cost function the payoffs are still positive. Comparing π^{CC} with π^{BB}, obviously $\pi^{CC} > \pi^{BB}$ for all $\delta > 0$, i.e. the Cournot scenario is less competitive.

In addition to these optimizing rules there are two heuristic rules of thumb, where the price is calculated by average cost and a markup. In case of the behavioral rule f^M, the price or quantity will be set in order to let $p_i/AC(q_i) = m_i$ constant, where $AC(q_i) = c + dq_i$ are the average costs and $m_i \in \mathbb{R}^+$ is the markup ratio. The profit per unit is then a fixed proportion of average cost. Let f^N denote another behavioral rule where the profit per unit is fixed by $p_i - AC(q_i) = m_i$ with $m_i \in \mathbb{R}^+$ as the markup amount. A game-theoretic analysis of these heuristics is not very common. Particularly, there is a lack of economic reasoning why these rules are used by (boundedly) rational players. To give a rationale for the usage of these rules we extend the equilibrium concept to deal with non-maximizing behavioral rules. For this reason let Ω be the *behavioral repertoire* of the players which contains different rules of decision making. In the present case we have $\Omega = \{f^C, f^M, f^N\}$ when $\sigma_i = \sigma_j = x$ and $\Omega = \{f^B, f^M, f^N\}$ when $\sigma_i = \sigma_j = p$. Since both markup rules can be played with price or quantity decisions, the rules f^M and f^N actually have to be indexed by the chosen strategy variable. In order to keep the notation simple we suppress this index. Since the achievement of the markup m_i depends also on the opponent´s decision, the corresponding reaction function can be derived. When $\sigma_i = p_i,\ i = 1,2$, the reaction function of both markup rules can be calculated by solving the demand function for q_i, inserting it into $p_i = m_i AC(q_i)$ (or $p_i = m_i + AC(q_i)$, resp.) and solving for p_i:

$$p_j^* = f^M(m_j, p_i) = \frac{m_j(bc(1-\delta^2) + ad(1-\delta) + d\delta p_i)}{b(1-\delta^2) + m_j d}, \tag{5}$$

$$p_j^* = f^N(m_j, p_i) = \frac{(m_j + c)(1-\delta^2)b + ad(1-\delta) + d\delta p_j}{b(1-\delta^2) + d}. \tag{6}$$

The profiles e.g. $(f^B(p_2^*), f^B(p_1^*)), (f^M(m_1, p_2^*), f^M(m_2, p_1^*))$ or $(f^B(p_2^*), f^N(m_2, p_1^*))$ are *behavioral equilibria*, since an equilibrium is a state where no player has a reason to select another strategy: in these behavioral equilibria the profit maximizer cannot achieve a higher payoff by changing the price and the markup agent realises the given m_i (cf. Pasche 2001). Cournot and Bertrand (or in general: Nash) solutions are obviously special cases of behavioral equilibria where the players are bounded to maximizing behavior. The existence of a behavioral equilibrium with markup rules requires that the markup m_i is chosen from a subset of $\mathbb{R}^+$ so that there exists at least one σ_i which realizes m_i in equilibrium. In other words: the markup player has to fix a *reasonable* markup. This condition is less demanding than the assumption that markups are chosen optimally. The latter case will be assumed later on. When all behavioral rules and markups are Common Knowledge, the agents are able to anticipate the prevailing behavioral equilibrium.

It is reasonable to argue that even non-maximizing (often called "boundedly rational") players will prefer high payoffs to lower payoffs, if they have the opportunity to acchieve them. Hence we assume that due to calculation or a learning process the markups are adjusted so that they maximize the payoff in equilibrium (Grant/Quiggin 1994 follow the same idea). This case will be called a behavioral equilibrium with a *balanced parametrization* (cf. Pasche 2001). In case of price competition we have five possible behavioral profiles (f^M, f^M), (f^N, f^N), (f^B, f^M), (f^B, f^N), (f^M, f^N) in addition to the Bertrand case (f^B, f^B), on which the selections of optimal markups depend. We assume that the behavioral rules are fixed. On the first stage of the game players select their (optimal) markups. On the second stage the usual oligopoly game is played.

First consider the behavioral profile (f^B, f^M), which leads to a behavioral equilibrium $(f^B(p_j^*), f^M(m_j, p_i^*))$ with the payoffs $\pi_i^{BM}(m_j)$ and $\pi_j^{MB}(m_j)$. The optimal parametrization of rule f^M is given by $m_j^* = \arg\max_{m_j} \pi_j^{MB}(m_j)$ which has a unique solution and yields the equilibrium payoffs

$$\pi_j^{MB}(m_j^*) = \frac{1}{4}\frac{Z_1(Z_2 - \delta b(1+\delta))^2}{(Z_2 - b\delta^2)(\frac{1}{2}Z_2^2 - b\delta^2(d+2b))} \tag{7}$$

$$\pi_i^{BM}(m_j^*) = \frac{1}{4}\frac{Z_1(b(1-\delta^2)+d)(b^2\delta(\delta^2-3\delta-2) - 2db\delta(1+\delta) + Z_2^2)^2}{(Z_2 - b\delta^2)^2(\frac{1}{2}Z_2^2 - b\delta^2(d+2b))^2} \tag{8}$$

Calculations show that $\pi_j^{MB} \geq \pi_j^{BB}$. Hence, a player can benefit from selecting rule f^M instead of f^B if the opponent is a profit maximizing player. The same result can be derived for the markup rule f^N. In this case the reaction function is linear in m_j which simplifies the calculation of an optimal markup. In case of an optimal m_j^* we have $\pi_i^{BN} = \pi_i^{BM}$ and $\pi_j^{NB} = \pi_j^{MB}$. The explanation is simple: In the (p_1, p_2)-space the markup m_j parametrizes the reaction function $f^M(m_j, p_i)$ (resp. $f^N(m_j, p_i)$) so that it intersects the Bertrand reaction function f^B at the point where the latter is a tangent to the highest possible iso-profit curve of player j. Markup agents with a fixed optimal markup then act like Bertrand-Stackelberg first mover, but in a static duopoly game. This can easily be proven by inserting the reaction function $p_i = f^B(p_j)$ into the profit equation of player i and calculating the optimal p_i^*.

Now let $\sigma_i = q_i$ $i = 1, 2$. The reaction functions of the markup rules are now defined in quantities and can be derived by inserting $p_i = m_i AC(q_i)$ (or $p_i = m_i + AC(q_i)$, resp.) into the demand scheme and solving for q_i:

$$q_j^* = f^M(m_j, q_i) = \frac{a - m_j c - b\delta q_i}{b + m_j d}, \tag{9}$$

$$q_j^* = N(m_j, q_i) = \frac{a - m_j - c - b\delta q_i}{b + d}. \tag{10}$$

The eqilibrium payoffs are also parametrized by m_j. We analyze the behavioral profile (f^C, f^M) with the corresponding equilibrium $(f^C(q_j^*), f^M(m_j, q_i^*))$. The unique optimal parametrization $m_j^* = \arg\max_{m_j} \pi_j^{MC}(m_j)$ leads to the equilibrium payoffs

$$\pi_j^{MC}(m_j^*) = \frac{1}{4}\frac{Z_1(b\delta - Z_2)^2}{Z_2(\frac{1}{2}Z_2^2 - b^2\delta^2)}, \tag{11}$$

$$\pi_i^{CM}(m_j^*) = \frac{1}{8}\frac{Z_1(b\delta(b\delta + Z_2) - Z_2^2)^2}{Z_2(\frac{1}{2}Z_2^2 - b^2\delta^2)^2}, \tag{12}$$

which is the Cournot-Stackelberg solution. Again, the markup agent takes the position of the first mover, and we have $\pi_i^{CN} = \pi_i^{CM}$ and $\pi_j^{NC} = \pi_j^{MC}$ in case of an optimal parametrization with m_j^*. The explanation is the same as in case of pricing strategies: in the (q_1, q_2)-space the markup m_j is chosen so that the reaction function $f^M(m_j, q_i)$ (resp. $f^N(m_j, q_i)$) intersects with the Cournot reaction function f^C in the point where the latter is a tangent to the highest possible iso-profit curve of player j.

Up to now we have assumed that the behavioral rules are exogenously given. Consider now that on the first stage of the game both players select a behavioral rule, i.e. they decide *how* to decide on prices or quantities (cf. Lipman 1991 on this idea). An equilibrium concept requires that no player can benefit from deviating from this behavioral rule, given the rules of the other players. On the second stage the (optimal) markups are chosen according to the selected behavioral profile on the first stage. In case of a non-observable choice of rules the players have to formulate consistent beliefs about the opponent´s rules. This case is not considered throughout this paper. A balanced parametrized behavioral profile where no player has an incentive to change to another rule is called a *balanced behavioral equilibrium profile* (see Pasche (2001) for details). With these definitions we can summarize the results discussed above:

Result 2.1. Consider a symmetric differentiated duopoly with a linear demand scheme and a quadratic cost function. Let the behavioral repertoire be $\Omega = \{f^C, f^M, f^N\}$ in case of quantity competition and $\Omega = \{f^B, f^M, f^N\}$ in case of price competition. Then we obtain the following results:

a) The balanced parametrized profiles (f^B, f^M) and (f^B, f^N) have Bertrand-Stackelberg equilibrium outcomes where the markup player has the first mover advantage. The balanced parametrized profiles (f^C, f^M) and (f^C, f^N) have the Cournot-Stackelberg equilibrium outcomes where the markup player has the first mover advantage.

b) The profiles (f^B, f^B) and (f^C, f^C) are not balanced behavioral equilibria profiles. $\diamond$

Proof:

a) The claim follows in a straightforward manner from computing the optimal m_j^* and the corresponding payoffs (cf. equations ,(7), (8), (11) and (12)).

b) The claim follows directly from a), since the Stackelberg first mover position implies $\pi^{NB} = \pi^{MB} > \pi^{BB}$ and $\pi^{NC} = \pi^{MC} > \pi^{CC}$. Hence there is an incentive to select rule f^M (or f^N) when the opponent is playing according to rule f^B or f^C. ◇

The incentive to change from Cournot or Bertrand behavior to a markup rule does not require that agents are able to compute the optimal markup exactly, even this is possible with the assumptions about the given information. For a behavioral rule which is often associated with "bounded rationality" it should be possible that the optimal or even a *sufficiently good* markup can be achieved by simple learning procedures. The set of all sufficiently good markups is defined by

$$\Phi_i^{MB} = \{m_i \in \mathbb{R}^+ | \pi_i^{MB}(f_i^M(m_i, p_j^*), f^B(p_i^*)) \geq \pi_i^{BB}(f^B(p_j^*), f^B(p_i^*))\} \quad (13)$$

$$\Phi_i^{MC} = \{m_i \in \mathbb{R}^+ | \pi_i^{MC}(f_i^M(m_i, q_j^*), f^C(q_i^*)) \geq \pi_i^{CC}(f^C(q_j^*), f^C(q_i^*))\} \quad (14)$$

Result 2.2. The set Φ_i^{MB} is convex and $m_i^* \in \Phi_i^{MB}$. The same holds true for Φ_i^{MC}. ◇

Proof: In the economically reasonable range $\mathbb{R}^+$ the profits π_i^{MB} (resp. π_i^{MC}) are strictly concave in m_i, which can easily be proven by calculating the second derivatives. The convexity of Φ_i^{MB} and Φ_i^{MC} then follows from the epigraph theorem in standard analysis (cf. Wolfstetter 1999, p. 326). The claim $m_i^* \in \Phi_i^{MB}$ (resp. $m_i^* \in \Phi_i^{MC}$) is trivial. ◇

In addition to the convexity of Φ_i^{MB} resp. Φ_i^{MC} these sets have the advantage that the player already knows one point on the boundary of the set: the markup m_i on variable cost which results from the behavioral profiles (f^B, f^B) and (f^C, f^C), i.e. if the players maximize profits. Making a (marginal) step into Φ_i^{MB} or Φ_i^{MC} then improves the performance. The ability to learn sufficiently good markups is important because in other scenarios (like in the (f^M, f^M) profile or in cases of demand uncertaity) the calculation of the optimal m_i^* is analytically complex. In these cases learning procedures or numerical recipes must be able to identify (almost) balanced parametrizations.

2.2 Mixing price and quantity decisions

Consider a two-stage game where on the first stage the strategy variable (price or quantity) is chosen and at the second stage the oligopoly game is conducted with payoff maximizing players. By rearranging the linear demand scheme it is possible to express the profits of player i as a function of q_i and p_j (resp. p_i and q_j). The corresponding reaction function in quantities then depends on rival´s price and vice versa. In case of maximizing agents we have

$$q_i^* = f^C(p_j) = \frac{1}{2}\frac{a - c + \delta p_j - \delta a}{b(1-\delta^2)+d}, \tag{15}$$

$$p_j^* = f^B(q_i) = \frac{1}{2}\frac{2ad + b(c+a) - (2db\delta + b^2\delta)q_i}{b+d}. \tag{16}$$

The corresponding payoffs are

$$\pi_i^{CB} = \frac{Z_1(b\delta - Z_2)^2(b(1-\delta^2)+d)}{(b\delta^2(3b+2d)-Z_2^2)^2}, \tag{17}$$

$$\pi_j^{BC} = \frac{Z_1(b\delta(1+\delta) - Z_2)^2(b+d)}{(b\delta^2(3b+2d)-Z_2^2)^2}. \tag{18}$$

Straightforward calculations lead to $\pi_i^{CB} \geq \pi_i^{BB}$ and $\pi_i^{CC} \geq \pi_i^{BC}$, i.e. choosing $\sigma_i = q_i, \imath = 1, 2$ as the strategic variable is the dominant solution. It can be expected that rational players will select the less competitive Cournot solution which depicts the known results in the industrial organization literature (e.g. Singh/Vives 1984 among others, cf. Kreps/Scheinkman 1983 for an alternative justification of Cournot behavior). In case of markup rules it is an open question which strategy variable σ_i will be selected. Although the term markup pricing suggests price setting this is not neccessary the case. For $\sigma_i \neq \sigma_j$ we have the reaction functions

$$q_i^* = f^M(p_j, m_i) = \frac{a(1-\delta) - m_i c + \delta p_j}{b(1-\delta^2)+dm_i}, \tag{19}$$

$$p_j^* = f^M(q_i, m_j) = \frac{m_j(da + cb - db\delta q_i)}{b + dm_j}, \tag{20}$$

$$q_i^* = N(p_j, m_i) = \frac{a(1-\delta) - m_i - c + \delta p_j}{b(1-\delta^2)+d}, \tag{21}$$

$$p_j^* = N(q_i, m_j) = \frac{da + cb + bm_j - db\delta q_i}{b+d}. \tag{22}$$

The resulting equilibrium payoffs depend on m_i, m_j. Again, consider the case in which a profit maximizing agent j plays against a markup agent i. Calculating the optimal markups yields the following result.

Result 2.3. With an optimal markup m_i^* it follows that the payoffs $\pi_i^{NB} = \pi_i^{NC} = \pi_i^{MB} = \pi_i^{MC}$ and $\pi_j^{BN} = \pi_j^{CN} = \pi_j^{BM} = \pi_j^{CM}$ represent the Cournot-Stackelberg solution in case of $\sigma_i = x_i$ and the Bertrand-Stackelberg solution in case of $\sigma_i = p_i$. In both cases the solution is independent from σ_j of the optimizing player. $\diamond$

Proof: The claim follows in a straightforward manner by computing the equilibrium payoffs (cf. (11), (12) in case of $\sigma_i = x_i$ and (7), (8) in case of $\sigma_i = p_i$). $\diamond$

Since the markup rule with balanced parametrization m_i^* is equal to the "first move" of the Stackelberg solution it is irrelevant for the optimzing agent whether he selects the price or the quantity in order to maximze profits.

2.3 Pure markup equilibria

For the reaction functions (5) and (6) in case of $\sigma_i = \sigma_j$ and (19) – (22) in case of $\sigma_i \neq \sigma_j$ ten different behavioral profiles with markup rules are possible. The resulting equilibrium payoffs are listed in appendix A. For arbitrarily parametrized behavioral equilibria we obtain the result that in case of pure markup profiles the outcome is completely independent from the chosen strategy variable.

Result 2.4. Let m_i, m_j be given. The following claims hold true:

- For $(f^N(q_j^*, m_i), f^N(q_i^*, m_j))$, $(f^N(p_j^*, m_i), f^N(q_i^*, m_j))$ and $(f^N(p_j^*, m_i), f^N(p_i^*, m_j))$ the payoffs $\pi_i^{NN} = \pi_j^{NN}$ are identical.
- For $(f^M(q_j^*, m_i), f^M(q_i^*, m_j))$, $(f^M(p_j^*, m_i), f^M(q_i^*, m_j))$ and $(f^M(p_j^*, m_i), f^M(p_i^*, m_j))$ the payoffs $\pi_i^{MM} = \pi_j^{MM}$ are identical.
- For $(f^M(q_j^*, m_i), f^N(p_i^*, m_j))$, $(f^M(p_j^*, m_i), N(q_i^*, m_j))$, $(f^M(p_j^*, m_i), f^N(p_i^*, m_j))$ and $(f^M(q_j^*, m_i), f^N(q_i^*, m_j))$ the payoffs π_i^{MN} and π_j^{NM} are identical. $\diamond$

Proof: The claims follows in a straightforward manner by computing the equilibrium payoffs (cf. appendix A). $\diamond$

With the exception of profile (f^N, f^N) with price and/or quantity strategies the calculation of optimal markups is analytically difficult because m_j^* is a nonlinear reaction function of m_i and vice versa. But it is possible to prove that in the economically reasonable part of the (m_1, m_2)-space all markup reaction functions are quasiconcave with a positive intersection with the ordinate and hence have a unique symmetric solution. For the profile (f^N, f^N) we have linear markup reaction functions

$$m_i^* = \frac{(a(d + b(1-\delta))(1-\gamma) + b\delta m_j}{Z_2}$$

and the balanced parametrized payoff is given by

$$\pi_i(m_i^*, m_j^*) = \frac{Z_1(b(1-\delta)+d)(b+d)}{(Z_2-b\delta)^2(b(1+\delta)+d)}$$

The following result is important to make some considerations about the degree of competition when the players use different behavioral rules:

Result 2.5. Assume a balanced parametrization for all behavioral equilibria. Then it follows that $\pi^{BB} \leq \pi^{NN} \leq \pi^{CC}$ and $\pi^{BB} \leq \pi^{MM} \leq \pi^{CC}$ independent from the chosen strategy variables for f^M, f^N. ◇

Proof: Result 2.4 states the independence from the strategy variable for all m_i, m_j and henceforth also for optimal m_i^*, m_j^*. In case of profile (f^N, f^N) the inequalities follow directly from computing the difference of the equilibrium payoffs:

$$\pi_i^{NN}(m_i^*, m_j^*) - \pi_i^{CC} = \frac{-Z_1 Z_2 b^3 \delta^3}{(b(1+\delta)+d)(Z_2-b\delta)^2(b\delta+Z_2)^2} \leq 0,$$
$$\pi_i^{NN}(m_i^*, m_j^*) - \pi_i^{BB} = \frac{Z_1 d b^2 \delta^3 (b\delta(1+d-\delta)+Z_2)}{(b(1+\delta)+d)(Z_2-b\delta)^2(b\delta(1-\delta)-Z_2)^2} \geq 0.$$

Since the equilibrium payoff $\pi_i^{MM}(m_i^*, m_j^*)$ is represented by an analytically exceedingly long expression, the claim is proven in another way (see appendix B). ◇

The result that markets with the behavioral profiles (f^M, f^M) and (f^N, f^N) are more competitive than Cournot and less competitive than Bertrand does not imply that these rules constitute balanced parametrized equilibrium profiles.

Result 2.6. Consider $\sigma_i = q_i$ and $\delta > 0$. Then (f^N, f^N) is not a balanced behavioral equilibrium profile. ◇

Proof: Straightforward computation of the balanced equilibrium payoffs yields $\pi_i^{CN} > \pi_i^{NN}$, i.e. if the opponent plays with rule f^N there is an incentive to select f^C. ◇

The results 2.1 and 2.6 imply that for a behavioral repertoire $\Omega = \{f^N, f^C\}$ only (f^N, f^C) and (f^C, f^N) are balanced parametrized equilibrium profiles. Henceforth the equilibrium behavior is heterogeneous even if demand and cost functions are strictly symmetric.

3 Endogenous Heterogeneity

The selected equilibrium profile and the associated outcomes depend on the repertoire Ω and the chosen strategy variables. This will be demonstrated with a numerical example.

Let $a = 30, b = 1, \delta = 0.5, c = 1, d = 1$. Matrix 1 and 2 contain the equilibrium payoffs for the cases $\sigma_1 = \sigma_2 = x$ with $\Omega = \{f^C, f^M, f^N\}$ and $\sigma_1 = \sigma_2 = p$ with $\Omega = \{f^B, f^M, f^N\}$. The matrix contains only the payoffs of the row player (for the column player the payoffs are given by the transformed matrix due to symmetry reasons). Each matrix has to be interpreted as a one-shot-game in which the rules are selected on the first stage, the optimal markups are chosen on the second stage and the market game is conducted on the third stage.

Matrix 1	f^C	f^M	f^N
f^C	83.062	82.727	82.727
f^M	83.083	82.719	82.693
f^N	83.083	82.404	82.384

Matrix 2	f^B	f^M	f^N
f^B	81.481	82.514	82.514
f^M	81.683	82.719	82.693
f^N	81.683	82.404	82.384

In case of quantity strategies (matrix 1) we have four balanced equilibrium profiles: (f^M, f^C), (f^N, f^C), (f^C, f^M) and (f^C, f^N), all of them representing the Cournot-Stackelberg solution and hence heterogeneous behavior of the two players. In case of price competition (matrix 2) we have (f^M, f^M) as a unique equilibrium profile. If one assumes one additional stage where the player first decide on the strategy variable, then on the behavioral rule, the parametrization and finally on the price or quantity itself, we obtain the one-shot-game denoted in matrix 3. The markup rules now have an index reflecting the stratgy variable chosen on the first stage.

Matrix 3	f^B	f^C	f_x^M	f_x^N	f_p^M	f_p^N
f^B	81.481	81.660	82.727	82.727	82.514	82.514
f^C	82.868	83.062	82.727	82.727	82.514	82.514
f_x^M	83.083	83.083	82.719	82.693	82.719	82.693
f_x^N	83.083	83.083	82.404	82.384	82.404	82.384
f_p^M	81.683	81.683	82.719	82.693	82.719	82.693
f_p^N	81.683	81.683	82.404	82.384	82.404	82.384

In this case we have nine balanced behavioral equilibrium profiles, namely all possible Cournot-Stackelberg solutions (f_x^M, f^C), (f_x^M, f^B), (f_x^N, f^C), (f_x^N, f^B), (f^C, f_x^M),

(f^C, f_x^N), (f^B, f_x^M), (f^B, f_x^N) and the profile (f_p^M, f_p^M). Optimizing behavior *can* but *need not* be part of a behavioral equilibrium profile. In contrast, the presence of a markup heuristic is neccessary. Note that the pure markup equilibrium profile (f_p^M, f_p^M) is Pareto-dominated by the other equilibrium profiles. But in (f_p^M, f_p^M) the Nash equilibrium condition is strictly fulfilled while in the Cournot-Stackelberg cases there is an indifference between f^B and f^C on the one hand and between f_x^M and f_x^N on the other had. Since the optimizer´s strategy variable is irrelevant in these cases it is possible to assume price setting behavior while the markup agent adjusts the quantity to acchieve the optimal markup. Not only in behavioral equilibria with *given* rules, but also in almost all balanced behavioral equilibrium profiles the behavior of the duopolists is different. This heterogeneity is not due to an asymmetric demand scheme, different production techniques or other exogenous reasons, but is a result of the strategic interdependence.

In contrast to the set of strategies and parameters the behavioral repertoire Ω cannot be considered as exogenously given to the players. The behavioral rules are rather *created* by experimentation, learning, habit formation and so on. While the choice of optimal strategies and markups can be modelled as an optimization problem, this is an artificial assumption in case of choosing the rules. Especially in case of applying new or modified behavioral rules it is not possible to explain this by optimizing behavior since optimizing presumes a closed set of well known alternatives. Instead, the set Ω is principally open. A closed and well defined set Ω, however, may be useful for analysing the comparative performance of different rules, detecting equilibria profiles and the conditions under which some rules are part of an equilibrium. If Ω is an open set there is no possibility to derive optimal parametrizations analytically. Then the behavior of oligopolistic players may be a *drift* in price-quantity space. Such a "market without equilibrium" (v. Stackelberg) can then be interpreted as a result of an agent´s search of rules which improve performance.

Do there exist behavioral rules which lead to perfect collusion? Rules which lead to higher payoffs than markup rules also have at least one free parameter which can be chosen optimally. In the (σ_i, σ_j)-space each balanced parametrized reaction function will lead to a tangential solution where the slope of j´s reaction function equals the slope of the highest possible iso-profit curve of player i. In the symmetric duopoly, perfect collusion implies that the slopes of the iso-profit curves are identical and equal to one, hence also the reaction functions must possess this slope. One may construct such functions even for the static game. But they are not plausible because the unique intersection point would be dynamically unstable (cf. Dixit 1986). For this reason it is not reasonable to expect collusion rules for static games. Since we have seen that multiple behavioral equilibrium profiles exist, one may think about trigger strategies in iterated games which lead to collusive behavior on stages $t < T$ (with T as the last stage of the game).

4 Discussion

In order to take simple but empirically relevant decision rules into account we employed two different markup heuristics in a symmetric duopoly model. It was shown that depending on the chosen markups the presence of these rules leads to Stackelberg outcomes when the markup rule has the first mover advantage. Hence there is an incentive for (at least) one player to deviate from optimizing and turning towards markup behavior. Pure Cournot or Bertrand behavior does not represent a behavioral equilibrium profile. Furthermore, it was shown that pure markup behavior is more competitive than Cournot and less competitive than Bertrand, independent of the chosen strategic variable (price or quantity). It turned out that in case of a combination of markup and optimizing behavior only the strategy variable of the markup agent determines the outcome.

It is questionable how it can be justified that payoff optimzing behavior is "full rational", like the decision theoretic position claims, while this behavior can be outperformed by simple rules and does not constitute a behavioral equilibrium profile. To what extent does it make sense to call agents only "boundedly rational", when they are able to anticipate equilibrium profiles and hence select best performing simple heuristics? Lipman (1991) argues that no logical inconsistency exists in modelling bounded rationality by "optimal" decision making about how to make decisions. The extended notion of (behavioral) equilibrium used in this paper requires also another notion of rationality in an explicative and a normative sense (cf. Güth/Kliemt 2001 on this topic).

Furthermore, the question arises whether the concept of a representative firm is valid since behavioral equilibrium profiles are constituted by heterogeneous rules. The idea of a representative firm requires that for *each* firm an incentive exists to behave in the same way like all other firms do. The concept of behavioral equilibrium profiles instead may imply (depending on Ω) that a firm behaves according to rule f^1 *because* the other firm decides according rule f^2 and vice versa even in case of completely symmetric conditions.

It seems promising to extend the present model in several ways: An extension to n players requires simpler demand and cost functions but may lead to more complex balanced behavioral equilibrium profiles. Furthermore the selection of rules and parametrizations can be analyzed under the assumption that the rival´s rule is not observable. The decision is then also based on expectations. In addition, it is reasonable to assume some uncertainty regarding the demand scheme and the cost function of other players. Finally, an extension to multi-period market games in order to study e.g. markup adaption procedures will be interesting. From a methodological point of view the relation between the concept of (balanced) behavioral equilibria profiles and the concepts of evolutionary game theory and Darwinian dynamics require further studies (cf. Rhode/Stegeman 2001, Qin/Stuart 1997).

Appendix A

The payoffs are calculated by solving the corresponding system of reaction functions (cf. eq. (19) – (22)). Let $Z_3 = (b+d)$, $Z_4 = (1-\delta)$, $Z_5 = (m_i - 1)$.

For $(f^M(q_j, m_i), f^M(q_i, m_j))$, $(f^M(p_j, m_i), f^M(q_i, m_j))$, $(f^M(p_j, m_i), f^M(p_i, m_j))$ we have

$$\pi_i^{MM} = \frac{a^2(bZ_4 + m_j d(1-\gamma m_i) + b\gamma(\delta m_j - m_i))(b\gamma(1+\delta) + d)(bZ_4 + m_j d)Z_5}{(b^2(1-\delta^2) + bd(m_i + m_j) + m_i m_j d^2)^2}$$

For $(f^N(q_j, m_i), f^N(q_i, m_j))$, $(f^N(p_j, m_i), f^N(q_i, m_j))$, $(f^N(p_j, m_i), f^N(p_i, m_j))$ we have

$$\pi_i^{NN} = \frac{((a(1-\gamma) - m_i)Z_3 + b\delta(a(1-\gamma) - m_j))m_i}{Z_3^2 - \delta^2 b^2}$$

For $(f^M(q_j, m_i), f^N(p_i, m_j))$, $(f^M(p_j, m_i), f^N(q_i, m_j))$, $(f^M(p_j, m_i), f^N(p_i, m_j))$, $(f^M(q_j, m_i), f^N(q_i, m_j))$ we have

$$\pi_i^{MN} = \frac{(a(1 - m_j\gamma)Z_3 + b\delta(a(1+\gamma) - m_i))Z_5(a(d + b\gamma)Z_3 - bd(\gamma a(Z_4 + bd) + \delta m_i))}{(Z_3(b + m_i d) - b^2\delta^2)^2} \tag{23}$$

$$\pi_j^{NM} = \frac{((a(1-\gamma) - m_j)(b + dm_i) - b\delta a(1 - \gamma m_i))m_j}{Z_3(b + m_i d) - b^2\delta^2} \tag{24}$$

Appendix B

Proof of $\pi_i^{BB} \leq \pi_i^{MM} \leq \pi_i^{CC}$ with arbitrary σ_i, σ_j for the markup rule and balanced parametrization: Note that π_i^{MM} is independent from σ_i, σ_j (result 2.4). Since a balanced parametrization is symmetric let $m_j = m_i$. In a behavioral equilibrium we have $q_i^*(m_i, m_j) = q_j^*(m_i, m_j) = q^*(m_i)$. Solving $q^*(m_i) = q_i^c$ to m_i, where q_i^c is the Cournot quantity, we have m^c as the markup which leads to $\pi_i^{MM}(m_i^c, m_j^c) = \pi_i^{CC}$. It follows that

$$\frac{\partial \pi_i^{MM}}{\partial m_i}(m_i^c, m_j^c) = \frac{a^2(1-\gamma)(d(1+\gamma)+\gamma b(2+\delta))^3 b^2 \delta^2}{(b\delta + Z_2)^3(b\gamma(1+\delta)+d)W_1} \leq 0$$

with $W_1 = bd(\delta - 2) + b^2\gamma(\delta^2 + \delta - 2) - 2d(b\gamma + d) < 0$. The partial derivative is negative (or zero). This indicates that m_i^c is not optimal and that there is an incentive for both players to decrease the markup. Since the markup reaction function is strictly quasiconcave and there is a unique solution $m_i^* = m_j^*$ it follows that $m_i^c \geq m_i^*$. Hence, in a balanced parametrized equilibrium a lower iso-payoff curve can be achieved as in the Cournot case. Solving $q^*(m_i) = q_i^b$ to m_i, where q_i^b is the Bertrand quantity, we have m^b as the markup which leads to $\pi_i^{MM}(m_i^b, m_j^b) = \pi_i^{BB}$. The partial derivative

$$\frac{\partial \pi_i^{MM}}{\partial m_i}(m_i^b, m_j^b) = \frac{a^2(1-\gamma)(\gamma b(\delta(1-\delta)-2) - d(1+\gamma))^2 db\delta^2 W_2}{(b\delta Z_4 + Z_2)^3(b\gamma(1+\delta)+d)W_3} \geq 0$$

with $W_2 = b(1 + \gamma(1+\delta) - \delta^2) + 2d > 0$ and $W_3 = \gamma b^2(2-\delta)(1-\delta^2) + ab(Z_4(2+\delta) + 2\gamma) + 2d^2 > 0$ is positive (or zero). This indicates that there is an incentive for both players to increase the markup. Hence, in a balanced parametrized equilibrium a higher iso-payoff curve can be achieved as in the Bertrand case. This completes the proof.

References

Chang, L. (1985), Comparing Bertrand and Cournot equilibria: A geometric approach. *Rand Journal of Economics* 16, 146-152.

Dixit, A.K. (1986), Comparative statics for oligopoly. *International Economic Review* 27, 107-122.

Grant, S., Quiggin, J. (1994), Nash equilibrium with mark-up-pricing oligopolists. *Economics Letters* 45, 245-251.

Güth, W., Kliemt, H. (2001), From full to bounded rationality. The limits of unlimited rationality. mimeo.

Hall, L.R., Hitch, C,J, (1939), Price theory and business behavior. *Oxford Economic Papers* 2, 12-45.

Kreps, D., Scheinkman, J. (1983), Quantity pre-commitment and Bertrand competition yield Cournot outcomes. *Bell Journal of Economics* 14, 326-337.

Lipman, B. (1991), How to decide how to decide how to...: modelling limited rationality. *Econometrica* 59, 1105-1125.

Pasche, M. (2001), Equilibrium Concepts for Boundedly Rational Behavior in Games. Working Paper Series B, No. 2001/03, University of Jena, Faculty of Economics.

Qin, C.-Z., Stuart, C. (1997), Are Cournot and Bertrand equilibria evolutionary stable strategies? *Journal of Evolutionary Economics* 7, 41-47.

Rhode, P., Stegeman, M. (2001), Non-Nash equilibria of Darwinian dynamics with applications to duopoly. *International Journal of Industrial Organization* 19, 415-453.

Rubinstein, A. (1998), *Modelling Bounded Rationality.* Cambridge, Mass.: MIT-Press.

Shapiro, C. (1989), Theories of Oligopoly Behavior. in: R.Schmalensee, R.D. Willig (eds,), *Handbook of Industrial Organization* Vol. 1. Amsterdam: North-Holland/Elsevier.

Singh, N., Vives, X. (1984), Price and quantity competition in differentiated oligopoly. *Rand Journal of Economics* 15, 546-554.

Vives, X. (1985), On the efiiciency of Cournot and Bertrand competition with product differentiation. *Journal of Economic Theory* 38(1), 166-175.

Wolfstetter, E. (1999), *Topics in Microeconomics.* Cambridge: Cambridge University Press.

The Maple code of all calculations can be obtained from the author:
m.pasche@wiwi.uni-jena.de.