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Werner Jammernegg und Peter Kischka
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Prof. Dr. Hans-Walter Lorenz h.w.lorenz@wiwi.uni-jena.de

# A newsvendor model with service and loss constraints 

Werner Jammernegg ${ }^{1)}$<br>Peter Kischka ${ }^{2}$<br>${ }^{1)}$ Vienna University of Economics and Business Institute of Production Management Nordbergstraße 15, 1090 Vienna, Austria<br>E-Mail: werner.jammernegg@wu-wien.ac.at<br>${ }^{2)}$ Friedrich Schiller University Jena<br>Department of Business Statistics<br>Carl-Zeiß-Str. 3, 07743 Jena, Germany<br>E-Mail: p.kischka@wiwi.uni-jena.de


#### Abstract

Actual performance measurement systems do not only consider financial measures like costs and profits but also non-financial indicators with respect customer service, quality and flexibility. Using the newsvendor model we explore the influence of possibly conflicting performance measures on important operations decisions like the order quantity and the selling price of a product. For price-independent as well as price-dependent demand distribution like in the classical newsvendor model the objective is to maximise the expected profit. But the optimal decisions are computed with respect to a service constraint - a lower bound for the level of product availability - and to a loss constraint - an upper bound for the probability of resulting in loss. For the price-independent model a condition for the existence of an optimal order quantity and its structure is presented. For the price-setting newsvendor the admissible region of the order quantity and the selling price is characterised for the additive and the multiplicative model. Furthermore, it is shown that higher variability of demand leads to a smaller admissible region of the decision variables thereby easing the computation of the optimal decisions.


Keywords: Constrained Newsvendor Model, Price-Setting Newsvendor
JEL Classification: C 44, M 11

## 1 Introduction

The performance of supply operations should not be evaluated just by a single measure. Using multiple performance measures often results in tradeoffs like the one between inventory costs and level of product availability. Also performance measurement systems try to reflect the interests of all stakeholders of a company. E.g. the balanced scorecard system considers not only the financial perspective mainly of interest to the managers and owners of a company but also the customer, supplier and employee perspectives. The like the top-level performance measures of the supply chain operations reference (SCOR) model are categorised into internal (costs, assets) and external (reliability, responsiveness, flexibility) measures (see www. supply-chain.org).

In this paper we use the newsvendor model to explore the influence of possibly conflicting performance measures on important operational measures like the order quantity and/or the selling price of a product. The newsvendor model is one of the fundamental operations models that is rich enough to gain important managerial insights (see Porteus (2008)). In the classical model for fixed price the order quantity for a selling season is based on the maximisation of the expected profit. It turns out that the optimal quantity solves the tradeoff resulting from the costs from ordering too many and the costs from ordering too few units of the product. The so-called critical ratio represents the profit value of the product and describes the cycle service level (in-stock probability) related to the optimal expected profit. Therefore, the order quantity for high-profit products is high and that of low-profit products is low. But these findings may not be desirable for high-profit and low-profit products in general as is indicated by experimental and empirical studies (see e.g. Schweitzer/Cachon (2000), Corbett/Fransoo (2007)).

There exists a large body of literature concerning extensions of the classical newsvendor model (see e.g. Khouja (1999)). Instead of maximising the expected profit there is a long history to maximise instead the probability of exceeding a specified minimum profit (see e.g. Lau (1980)). Parlar/Weng (2003) modified this so-called satisficing objective to maximise the probability of exceeding a moving profit target which they specify as the expected profit. In Gan et al. (2004) expected profit is maximised under a value at risk constraint. Another extension models the risk preferences of the newsvendor by using the expected utility framework (Eeckhoudt et al. (1995)), a loss-averse utility function (Wang/Webster (2009)) or the conditional value at risk (CVaR) (see e.g. Jammernegg/Kischka (2007)). With respect to customer-facing performance measures the order quantity is determined for a given level of product availability (fill rate, cycle service level) (see e.g. Cachon/Terwiesch (2009), section 11.6).

Another important stream of extensions allows the demand to be price dependent (see e.g. Petruzzi/Dada (1999), Yao et al. (2006)). There are also price-setting newsvendor models with other objectives than maximising expected profit. In Lau/Lau (1988) the objective is to maximise the probability of exceeding a specified target profit. For the additive and the multiplicative demand models Chen et al. (2008) give sufficient conditions for the existence of an optimal order quantity and an optimal selling price under the CVaR criterion.

We propose newsvendor models with price-independent as well as price-dependent demand distribution where the objective is to maximise the expected profit with respect to a lower bound for the level of product availability and an upper bound for the probability in resulting in loss. Instead of specifying shortage cost like in Wang/Webster (2009) in our approach the newsvendor must decide on a minimal cycle service level for the product. In this way external and internal performance measures are considered by adding two constraints to the classical newsvendor model.

The paper is organised as follows. The classical newsvendor model with service and loss constraints is presented and analysed in section 2 . A condition for the existence of an optimal order quantity is given and its structure is proven and discussed. In section 3 the price-setting newsvendor model with service and loss constraints is investigated. The admissible region for the order quantity and the selling price are characterized both for the multiplicative and the additive demand model. Moreover it is shown that "higher variability" of demand leads to a smaller admissible region of the decision variables thereby easing the computation of the optimal order quantity and the optimal selling price. Finally, section 4 presents the conclusions from the main results of the paper.

## 2 Classical Newsvendor with Service and Loss Constraints

### 2.1 Model

In this section we first introduce our notation for the classical newsvendor model and its performance measures and then we present our model.
The random demand X is characterized by the distribution function F . The purchase price per unit of the product is c . During the regular selling season, the product is sold to customers at a unit price p . Unsatisfied demand is lost, and leftover inventory of the product at the end of the selling season is sold in an other distribution channel at the salvage value per unit $\mathrm{z} . \mathrm{p}-\mathrm{c}$ describes the cost of understocking by one unit, whereas $\mathrm{c}-\mathrm{z}$ describes the cost of overstocking by one unit. It is assumed that $\mathrm{p}>\mathrm{c}>\mathrm{z}$ holds.
Let y denote the order quantity and g the profit. g depends on y and the stochastic demand X and is given by

$$
\begin{equation*}
g(y, X)=(p-c) y-(p-z)(y-X)^{+} \tag{1}
\end{equation*}
$$

with $(\mathrm{y}-\mathrm{X})^{+}=\max (0, \mathrm{y}-\mathrm{X})$.

In the classical newsvendor model the optimal order quantity $y^{*}$ is derived by maximizing the expected profit $\mathrm{E}(\mathrm{g}(\mathrm{y}, \mathrm{X})$ ). The optimality condition is given by (see, e.g., Cachon and Terwiesch 2009, section 11.4):

$$
\begin{equation*}
F\left(y^{*}\right)=(p-c) /(p-z) . \tag{2}
\end{equation*}
$$

Here and in the following, we assume that the distribution function F is strictly monotone increasing and continuous; then y * is defined uniquely by

$$
\begin{equation*}
y^{*}=F^{-1}\left(\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}}\right) . \tag{3}
\end{equation*}
$$

Now we can define the model with service and loss constraints:

$$
\begin{equation*}
\operatorname{Max}_{y \geq 0} E(g(y, X)) \tag{4a}
\end{equation*}
$$

s.t.

$$
\begin{equation*}
\mathrm{F}(\mathrm{y}) \geq \mathrm{CSL}, \tag{4b}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{P}(\mathrm{~g}(\mathrm{y}, \mathrm{X}) \leq 0) \leq \mathrm{PL} \tag{4c}
\end{equation*}
$$

In (4b) $\mathrm{F}(\mathrm{y})$ describes the probability that there is no stock-out during the selling season. It is called cycle service level. By CSL we denote a given lower bound for the cycle service level.

In (4c) the probability of loss is bounded from above by the specified value PL. The probability of loss is given by (cp. Lau (1980), Jammernegg/Kischka (2007)):

$$
\begin{equation*}
\mathrm{P}(\mathrm{~g}(\mathrm{y}, \mathrm{X}) \leq 0)=\mathrm{F}\left(\mathrm{y} \frac{\mathrm{c}-\mathrm{z}}{\mathrm{p}-\mathrm{z}}\right) . \tag{5}
\end{equation*}
$$

Using (5) we can rewrite the model (4) as follows:

$$
\begin{equation*}
\operatorname{Max} E(\mathrm{~g}(\mathrm{y}, \mathrm{X})) \tag{6a}
\end{equation*}
$$

s.t. $\quad F^{-1}(C S L) \leq y \leq F^{-1}(P L) \frac{p-z}{c-z}$.

For $\mathrm{PL} \geq \mathrm{CSL}$ an optimal order quantity exists in any case. For the reasonable case PL $<$ CSL an optimal order quantity y exists provided

$$
\begin{equation*}
\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}} \geq 1-\frac{\mathrm{F}^{-1}(\mathrm{PL})}{\mathrm{F}^{-1}(\mathrm{CSL})} \tag{7}
\end{equation*}
$$

The lhs of (7) represents the profit value of the product. Therefore, for high-profit products the existence of a solution is more likely than for low-profit products.

For PL $<$ CSL and $\mathrm{z} \geq 0$ we have the following sufficient condition for the existence of an optimal solution:

$$
\mathrm{c} \mathrm{~F}^{-1}(\mathrm{CSL}) \leq \mathrm{p} \mathrm{~F}^{-1}(\mathrm{PL})
$$

This is an easy consequence from (7).
For given demand distribution F and for given PL and CSL condition (7) can be used to describe the range of admissible price parameters $\mathrm{p}, \mathrm{c}$ and z .

Now we consider the influence of demand uncertainty on the existence of a solution. Let F and G be two distribution functions and assume that (7) is fulfilled for F .
Obviously if $\mathrm{G}^{-1}(\mathrm{PL})>\mathrm{F}^{-1}(\mathrm{PL})$ and $\mathrm{G}^{-1}(\mathrm{CSL})<\mathrm{F}^{-1}(\mathrm{CSL})$ then (7) is fulfilled for distribution G , too.

The previous condition holds in the following case:
Let $\mathrm{G}^{-1}(\mathrm{PL})<\mathrm{x}_{0}<\mathrm{G}^{-1}(\mathrm{CSL})$ for some $\mathrm{x}_{0}$ and

$$
\begin{array}{lll}
\mathrm{G}(\mathrm{x}) \leq \mathrm{F}(\mathrm{x}) & \text { for } & \mathrm{x} \leq \mathrm{x}_{0} \\
\mathrm{G}(\mathrm{x}) \geq \mathrm{F}(\mathrm{x}) & \text { for } & \mathrm{x} \geq \mathrm{x}_{0} . \tag{8}
\end{array}
$$

If a solution for distribution $F$ exists, then also a solution for distribution $G$ exists.
Figure 1 illustrates condition (8).
This condition is related to the stochastic order mean preserving spread (cp. Müller/Stoyan (2002), p. 28): Let $G$ be uniformly distributed over $[\mathrm{a}, \mathrm{b}]$ and F be uniformly distributed over [ $\mathrm{c}, \mathrm{d}]$ with $\mathrm{c} \leq \mathrm{a} \leq \mathrm{b} \leq \mathrm{d}$ and equal expected values; then F differs from G by a mean preserving spread. If $\mathrm{PL}<0.5<$ CSL then condition (8) is fulfilled.


Figure 1: Illustration of condition (8) for distribution functions F and G

### 2.2 Optimal Order Quantity

Now we characterize the optimal order quantity.
Proposition 1: If (7) is fulfilled the solution $y^{*}$ of model (6) is given by:
a) If $\mathrm{F}^{-1}\left(\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}}\right) \leq \mathrm{F}^{-1}(\mathrm{CSL})$
then $\mathrm{y}^{*}=\mathrm{F}^{-1}(\mathrm{CSL})$,
b) if $\mathrm{F}^{-1}(\mathrm{CSL}) \leq \mathrm{F}^{-1}\left(\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}}\right) \leq \mathrm{F}^{-1}($ PL $) \frac{\mathrm{p}-\mathrm{z}}{\mathrm{c}-\mathrm{z}}$
then $y^{*}=F^{-1}\left(\frac{p-c}{p-z}\right)$,
c) if $\mathrm{F}^{-1}\left(\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}}\right) \geq \mathrm{F}^{-1}($ PL $) \frac{\mathrm{p}-\mathrm{z}}{\mathrm{c}-\mathrm{z}}$
then $y^{*}=F^{-1}(P L) \frac{p-c}{p-z}$.

Proof. We know from the classical unconstrained newsvendor model that $\mathrm{E}(\mathrm{g}(\mathrm{y}, \mathrm{X}))$ is a concave function of $y$. Note that its optimal solution is given by (3).
With (7) it is guaranteed that an admissible solution (6b) exists. Part (a) describes the situation where the set of admissible solutions is right to the optimal unconstrained solution (3). Thus, the optimal solution $y^{*}$ is given by the lower bound of the admissible region, i. e. $\mathrm{F}^{-1}$ (CSL). Part b) and c) can be proved similarly.

If the demand distribution has bounded support, in case $c$ ) the optimal order quantity must not exceed the maximal demand.

In order to get further insight in the structure we consider uniformly distributed demand over $[\mathrm{a}, \mathrm{b}], 0 \leq \mathrm{a}<\mathrm{b}: \mathrm{F} \sim \operatorname{Unif}[\mathrm{a}, \mathrm{b}]$.

Then (7) is given by

$$
\begin{equation*}
\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}} \geq \frac{(\mathrm{CSL}-\mathrm{PL})(\mathrm{b}-\mathrm{a})}{\operatorname{CSL}(\mathrm{b}-\mathrm{a})+\mathrm{a}} . \tag{9}
\end{equation*}
$$

If $a=0$ the existence of an optimal order quantity is independent of the upper bound $b$.
Contrary, for fixed $a>0$ (9) is fulfilled for sufficiently small $b$. This means, the lower the demand variability the more likely an optimal order quantity exists.

Example:
Let $\quad \mathrm{CSL}=0,8, \quad \mathrm{PL}=0,1$ and let $\mathrm{F} \sim \operatorname{Unif}[\mathrm{a}, \mathrm{b}]$.
I) Low profit value: e.g. $\mathrm{p}=8, \quad \mathrm{c}=5, \quad \mathrm{z}=2$

From (9) we have: A solution exists if

$$
0,5 \geq \frac{(b-a) 0,7}{(b-a) 0,8+a}
$$

therefore, a solution exists for $0<a<b \leq 8 / 3 a$.

Since $\mathrm{F}^{-1}(\mathrm{CSL})=\mathrm{F}^{-1}(0,8)>\mathrm{F}^{-1}(0,5)=\mathrm{F}^{-1}\left(\frac{\mathrm{p}-\mathrm{c}}{\mathrm{p}-\mathrm{z}}\right)$ only case a) of proposition 1 is possible and we have for $0<a<b \leq 8 / 3 a$ :

$$
y^{*}=\mathrm{F}^{-1}(\mathrm{CSL})=0,8 \mathrm{~b}+0,2 \mathrm{a} .
$$

II) High profit value: e. g. $\mathrm{p}=8, \quad \mathrm{c}=1, \quad \mathrm{z}=0$

From (9) we have: A solution always exists.
Since $F^{-1}\left(\frac{p-c}{p-z}\right)=F^{-1}(0,875)>F^{-1}(0,8)$ case a) from proposition 1 is not possible.

Case b) is relevant, if $\mathrm{F}^{-1}(0,875) \leq \mathrm{F}^{-1}(0,1) \cdot 8$
This is equivalent to $\mathrm{b} \leq 7,93 \mathrm{a}$ and the solution is: $y^{*}=F^{-1}(0,875)=0,875 \mathrm{~b}+0,125 \mathrm{a}$.

Case c ) is relevant, if $\mathrm{F}^{-1}(0,1) \cdot 8 \leq \mathrm{F}^{-1}(0,875)$
and therefore if $b \geq 7,93 a$. The solution is:

$$
\mathrm{y}^{*}=\mathrm{F}^{-1}(0,1) \cdot 8=0,8 \mathrm{~b}+0,72 \mathrm{a} .
$$

Note that in case c) the coefficient of variation is larger than in case b).
For uniformly distributed demand over $[0, b]$ the optimal solution depends on the function CSL( $1-\mathrm{CSL}$ ). From Figure 2 it can be seen that for CSL in A loss aversion dominates and the optimal order quantity is given either by part b) or part c) of Proposition 1. Contrary, for CSL in B stockout aversion is dominating and the optimal solution in given either by part a) or by part b) of the Proposition 1.


Figure 2: Characterisation of the optimal solution for uniformly distributed demand over [0,b]

## 3 Price-Setting Newsvendor with Service and Loss Constraints

### 3.1 Model

Now we consider a model where the order quantity $y$ and also the selling price $p$ are decision variables. We denote by $X_{p}$ the random demand for price $p$ with distribution function $F_{p} . X_{p}$ is composed of a deterministic part and a stochastic part (cp. Petruzzi/Dada (1999)). The deterministic demand function $d(p)$ is decreasing in $p$; we assume $d(p)>0$ for $p<p_{0}$ and $\mathrm{d}\left(\mathrm{p}_{0}\right)=0$. The stochastic part is described by the random variable $\varepsilon$ with distribution function H .

The random profit now depends also on p and is given by (see (1))

$$
\begin{equation*}
g\left(y, p, X_{p}\right)=(p-c) y-(p-z)\left(y-X_{p}\right)^{+} . \tag{10}
\end{equation*}
$$

Generalising the model (4) we consider now

$$
\begin{align*}
& \max _{y, p} E\left(g\left(y, p, X_{p}\right)\right)  \tag{11a}\\
& y \geq 0, c<p<p_{0} \tag{11b}
\end{align*}
$$

$\begin{array}{ll} & y \geq 0, c<p<p_{0} \\ \text { s.t. } & F_{p}(y) \geq C S L\end{array}$
$\mathrm{P}\left(\mathrm{g}\left(\mathrm{y}, \mathrm{p}, \mathrm{X}_{\mathrm{p}}\right) \leq 0\right) \leq \mathrm{PL}$

According to (5) we have

$$
\begin{equation*}
\mathrm{P}\left(\mathrm{~g}\left(\mathrm{y}, \mathrm{p}, \mathrm{X}_{\mathrm{p}}\right) \leq 0\right)=\mathrm{F}_{\mathrm{p}}\left(\frac{\mathrm{c}-\mathrm{z}}{\mathrm{p}-\mathrm{z}} \mathrm{y}\right) \tag{12}
\end{equation*}
$$

We can rewrite model (11) as follows

$$
\begin{equation*}
\operatorname{Max} E\left(g\left(y, p, X_{p}\right)\right) \tag{13a}
\end{equation*}
$$

s.t. $\mathrm{y} \geq 0, \mathrm{c}<\mathrm{p}<\mathrm{p}_{0}$

$$
\begin{equation*}
\mathrm{F}_{\mathrm{p}}^{-1}(\mathrm{CSL}) \leq \mathrm{y} \leq \mathrm{F}_{\mathrm{p}}^{-1}(\mathrm{PL}) \frac{\mathrm{p}-\mathrm{z}}{\mathrm{c}-\mathrm{z}} \tag{13b}
\end{equation*}
$$

To gain further insight we consider two well known special cases for random demand $X_{p}$; the multiplicative and the additive model (see e. g. Petruzzi/Dada (1999)).

In the multiplicative model $\mathrm{X}_{\mathrm{p}}=\mathrm{d}(\mathrm{p}) \varepsilon$ we have

$$
\begin{equation*}
\mathrm{F}_{\mathrm{p}}(\mathrm{x})=\mathrm{H}\left(\frac{\mathrm{x}}{\mathrm{~d}(\mathrm{p})}\right) \text { for all } \mathrm{x}, \mathrm{p}<\mathrm{p}_{0} \tag{14}
\end{equation*}
$$

In the additive model $\mathrm{X}_{\mathrm{p}}=\mathrm{d}(\mathrm{p})+\varepsilon$ we have

$$
\begin{equation*}
\mathrm{F}_{\mathrm{p}}(\mathrm{x})=\mathrm{H}(\mathrm{x}-\mathrm{d}(\mathrm{p})) \text { for all } \mathrm{x}, \mathrm{p}<\mathrm{p}_{0} . \tag{15}
\end{equation*}
$$

### 3.2 Admissible solutions for multiplicative demand

First we characterise the admissible regions of the decision variables $p$ and $y$. It is the set of all ( $p, y$ ) fulfilling

$$
\begin{align*}
\mathrm{H}^{-1}(\mathrm{CSL}) \mathrm{d}(\mathrm{p}) & \leq \mathrm{y} \tag{16}
\end{align*} \leq \mathrm{H}^{-1}(\mathrm{PL}) \mathrm{d}(\mathrm{p}) \frac{\mathrm{p}-\mathrm{z}}{\mathrm{c}-\mathrm{z}}
$$

(16) follows immediately from (14) since $\mathrm{F}_{\mathrm{p}}^{-1}(\alpha)=\mathrm{H}^{-1}(\alpha) \mathrm{d}(\mathrm{p}), 0<\alpha<1$.

Proposition 2. For model (13) with multiplicative demand an admissible solution (p, y) exists if

$$
\begin{equation*}
\mathrm{z}+(\mathrm{c}-\mathrm{z}) \frac{\mathrm{H}^{-1}(\mathrm{CSL})}{\mathrm{H}^{-1}(\mathrm{PL})}<\mathrm{p}_{0}, \mathrm{PL}<\mathrm{CSL} . \tag{17}
\end{equation*}
$$

Proof. If PL $<$ CSL then the lhs of (17) is larger than c .
Let $\mathrm{p}=\mathrm{z}+(\mathrm{c}-\mathrm{z}) \frac{\mathrm{H}^{-1}(\mathrm{CSL})}{\mathrm{H}^{-1}(\mathrm{PL})}$ and define $\mathrm{y}=\mathrm{d}(\mathrm{p}) \mathrm{H}^{-1}(\mathrm{CSL})$, the lower bound in (16). Inserting p in the rhs of (16) shows that this order quantity y fulfills (16).
Moreover for $\mathrm{p}=\mathrm{z}+(\mathrm{c}-\mathrm{z}) \frac{\mathrm{H}^{-1}(\mathrm{CSL})}{\mathrm{H}^{-1}(\mathrm{PL})}<\mathrm{p}_{0}$ and $\mathrm{y}=\mathrm{H}^{-1}(\mathrm{CSL}) \mathrm{d}(\mathrm{p})$ the constraints in (11b) and (11c) are fulfilled as equality.

Like in the model with price independent demand we investigate the influence of demand uncertainty on the region of admissible prices and order quantities.

Proposition 3. Let H and K be two distribution functions for random variable $\varepsilon$ with

$$
\begin{equation*}
\mathrm{H}^{-1}(\mathrm{CSL}) \geq \mathrm{K}^{-1}(\mathrm{CSL}), \mathrm{H}^{-1}(\mathrm{PL}) \leq \mathrm{K}^{-1}(\mathrm{PL}) . \tag{18}
\end{equation*}
$$

Then the set of admissible prices and order quantities ( $\mathrm{p}, \mathrm{y}$ ) corresponding for H is a subset of the admissible set corresponding to K . This is obvious from (16). Therefore, if an optimal solution exists with respect to H there is also an optimal solution with respect to K .

We illustrate the structural properties by means of some examples.

Let $\varepsilon$ be uniformly distributed over $[1-\mathrm{a}, 1+\mathrm{a}], 0<\mathrm{a} \leq 1$. If $\mathrm{H} \sim \operatorname{Unif}(1-\mathrm{a}, 1+\mathrm{a})$ and $\mathrm{K} \sim \operatorname{Unif}\left(1-\mathrm{a}^{\prime}, 1+\mathrm{a}^{\prime}\right)$ then condition (18) is fulfilled for $\mathrm{a} \geq \mathrm{a}^{\prime}$ if $\mathrm{PL}<0.5<\mathrm{CSL}$.
Consider the following example with a linear deterministic demand function: $\mathrm{d}(\mathrm{p})=10-\mathrm{p}, \mathrm{c}=1, \mathrm{z}=0, \mathrm{CSL}=0.8, \mathrm{PL}=0.1, \varepsilon \sim \operatorname{Unif}(1-\mathrm{a}, 1+\mathrm{a})$.

| a | 0.5 | 0.8 | 1.0 |
| :---: | :---: | :---: | :---: |
| $\mathrm{p}^{*}$ | 5.75 | 5.79 | 8 |
| $\mathrm{y}^{*}$ | 6.38 | 6.44 | 3.2 |
| $\mathrm{CSL}\left(\mathrm{p}^{*}, \mathrm{y}^{*}\right)$ | 1 | 0.83 | 0.8 |
| $\mathrm{PL}\left(\mathrm{p}^{*}, \mathrm{y}^{*}\right)$ | 0 | 0.04 | 0.1 |
| $\mathrm{E}\left(\mathrm{g}\left(\mathrm{y}^{*}, \mathrm{p}^{*}, \mathrm{X}_{\mathrm{p}}{ }^{*}\right)\right)$ | 18.06 | 17.38 | 12.16 |

Table 1: Optimal decision variables and performance measures for different values of parameter a

In Table $1 \mathrm{p}^{*}$ and $\mathrm{y}^{*}$ denote the optimal selling price and the optimal order quantity. The performance measures in lines 4 to 6 correspond to these optimal values.

As parameter a increases the prescribed bounds for CSL and PL become more relevant. For $a=1$ the optimal solution is given by the specified $p$ and $y$ in the proof of Proposition 2. For the examples presented in Table 1 the set of admissible prices and order quantities is shown in Figure 3. The admissible region is given by the respective straight line as lower bound and parabola as upper bound. Figure 3 illustrates the statement of Proposition 3.


Figure 3: Admissible regions for the multiplicative model with uniform distribution Dotted lines: $\operatorname{Unif}(0.5,1.5)$, dashed lines: $\operatorname{Unif}(0.2,1.8)$, solid lines: $\operatorname{Unif}(0,2)$

It has to be noticed that the optimal order quantity cannot exceed the maximal demand for a given selling price. Therefore for distributions with bounded support $[\alpha, \beta]$ condition (16) can be replaced by

$$
\begin{equation*}
H^{-1}(C S L) d(p) \leq y \leq \operatorname{Min}\left\{H^{-1}(P L) d(p) \frac{p-z}{c-z}, d(p) \beta\right\} . \tag{19}
\end{equation*}
$$

In Figure 4 the relevant admissible region is shown for $\varepsilon \sim \operatorname{Unif}(0.5,1.5)$. It is characterized by the area bounded by the three plotted corners.


Figure 4: Relevant admissible region for the multiplicative model with uniform distribution
For less profitable products no solution may exist. E. g. this is the case for $\mathrm{d}(\mathrm{p})=10-\mathrm{p}, \mathrm{c}=5, \mathrm{z}=2, \mathrm{CSL}=0.8, \mathrm{PL}=0.1, \varepsilon \sim \operatorname{Unif}(0.3,1.7)$. Here condition $(17)$ is violated.

We present a second set of examples based on the Weibull distribution.
Let $\varepsilon$ be Weibull distributed with location parameter 1 and scale parameter $\gamma$, i. e.
$\mathrm{H}(\mathrm{v})=1-\mathrm{e}^{-\mathrm{v}^{\gamma}}$. If $\mathrm{H} \sim \operatorname{Weib}(\gamma)$ and $\mathrm{K} \sim \operatorname{Weib}(\delta)$ then condition (18) is fulfilled for $\gamma \leq \delta$ if $\mathrm{PL}<1-\frac{1}{\mathrm{e}}<\mathrm{CSL}$.

In Figure 5 the admissible regions for different scale parameters of the Weibull distribution are shown.


Figure 5: Admissible regions for the multiplicative model with Weibull distribution Dotted lines: Weib(3), dashed lines: Weib(2), solid lines: Weib(1)

### 3.3 Admissible solutions for additive demand

For the additive model the admissible region is the set of all (p, y) fulfilling

$$
\begin{align*}
& \mathrm{H}^{-1}(\mathrm{CSL})+\mathrm{d}(\mathrm{p}) \leq \mathrm{y} \leq\left(\mathrm{H}^{-1}(\mathrm{PL})+\mathrm{d}(\mathrm{p})\right) \frac{\mathrm{p}-\mathrm{z}}{\mathrm{c}-\mathrm{z}}  \tag{20}\\
& \mathrm{c}<\mathrm{p}<\mathrm{p}_{0} .
\end{align*}
$$

(20) follows immediately from (15) since $\mathrm{F}_{\mathrm{p}}^{-1}(\alpha)=\mathrm{H}^{-1}(\alpha)+\mathrm{d}(\mathrm{p}), 0<\alpha<1$.

Condition (20) is fulfilled if

$$
\mathrm{H}^{-1}(\mathrm{CSL})(\mathrm{c}-\mathrm{z})+\mathrm{H}^{-1}(\mathrm{PL}) \mathrm{z} \leq(\mathrm{p}-\mathrm{c}) \mathrm{d}(\mathrm{p})+\mathrm{H}^{-1}(\mathrm{PL}) \mathrm{p}
$$

and

$$
\mathrm{c}<\mathrm{p}<\mathrm{p}_{0}
$$

holds.
Like in the multiplicative model the qualitative properties presented in Proposition 3 hold.
Example
Let $\varepsilon$ be uniformly distributed over $[-\mathrm{a}, \mathrm{a}]$, $\mathrm{a}>0$. If $\mathrm{H} \sim \operatorname{Unif}(-\mathrm{a}, \mathrm{a})$ and $\mathrm{K} \sim \operatorname{Unif}\left(-\mathrm{a}^{\prime}, \mathrm{a}^{\prime}\right)$ then (18) is satisfied for $\mathrm{a} \geq \mathrm{a}^{\prime}$ if $\mathrm{PL}<0.5<\mathrm{CSL}$. Note that this is analogous to the condition in the multiplicative model.
In Figure 6 this is illustrated for a linear deterministic demand function. For $\varepsilon \sim \operatorname{Unif}(-5,5)$ no admissible solution exists.


Figure 6: Admissible regions for the additive model with uniform distribution Dotted lines: Unif( $-1,+1$ ), dashed lines: Unif( $(-3,+3)$, solid lines: Unif( $-5,+5$ )

## 4 Conclusions

In this paper we present a newsvendor model with the objective to maximise the expected profit with respect to a service constraint and a loss constraint. First we consider just the ordering decision. The condition for the existence of an optimal order quantity can be used to specify price parameters of the model. The structure of the optimal decision shows that the optimal order quantity for low profit products in general is higher than that of the classical newsvendor whereas the optimal ordering decision for high profit products is limited by the specified probability of loss.

In the second part of the optimal ordering and pricing decisions are investigated where the stochastic demand depends on the selling price. For the multiplicative and the additive model the admissible region of the decision variables is characterised. It turns out that for the multiplicative demand model the region of admissible solutions does not depend on the deterministic demand function.

For both models the influence of demand uncertainty on the region of the admissible decision variables is analysed. The proposed comparison of stochastic demand variables is related to the stochastic order mean preserving spread. For all models higher variability of demand leads to a smaller region of the admissible decision variables order quantity and selling price. The managerial implication of this finding is that the higher demand variability is the more the prescribed performance measures cycle service level and probability of loss determine the optimal decisions. The computational consequence is that for high variability the optimal solution is given by the model with equality constraints. This leads to a considerable reduction of the effort necessary for computing the optimal decisions. Moreover, if the demand variability is too high it is possible that no admissible order quantity and selling price exist.

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