Labor Market Regulation and Retirement Age

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Abstract

Pension system and labor market reforms are widely debated issues in all industrialized countries and especially in Europe; actual economic and demographic trends indeed, urge politicians to revise Social Security System setup. A preminent role in particular, is played by employment protection regulation and by mandatory retirement age; this is the case for the potential consequences on the functioning of the whole economy of any change over these two aspects of the welfare state. Present paper considers the political economy of such social policies jointly focusing on the effects on total employment turnover that derive either by temporary, selective exits due to the dynamic of the labor market or by permanent, non-selective exits due to retirements. The degree of employment protection and the mandatory retirement age emerge in this setting, as a result of a political process that involves three social groups: young, high and low productivity old; the productivity gap that distinguish different workers is the crucial variable that drives social consensus towards a specific setup of the Social Security System.

1 Introduction

Pension system and labor market reforms are widely debated issues in all industrialized countries and specially in Europe; the demographic trend and the recent slowdown of economic growth in the EU indeed, urge politicians to revise the structure of Social Security System. The relevance of these issues is further enhanced by the fact that any change in the status quo is likely to affect heavily the functioning of the whole economy and the welfare of large sections of the population.

A prominent role in this debate is played by employment protection regulation and by mandatory retirement age. Indeed a reform of the labor market involving a reduction in employment protection is considered an important tool to promote economic growth; an increase in retirement age moreover, is a key element to guarantee the viability of the pension system in the long run.

Present work focus on the political economy of such social policies jointly and considers the interaction between the choice over the protection of employees in the labor market and over retirement age.

The approach is twofold: first a theoretical model is built to analyze the determinants of social consensus over different setups of the Social Security System; the second step is to test some hypothesis and results of the model through the use of microdata.

The analysis considers Social Security from a wide point of view and includes in the picture mutual connections between labor market regulation and the structure of the pension system; this allows to look more accurately at the reasons why some institutional setups obtain enough support and are implemented while other don't.

What turns out to be crucial in this setting is total turnover on the labor market that is driven by two main factors: temporary exits due to workers selection at the firm level and permanent non-selective exits due to retirement; the confrontation in the political system of insiders and outsiders of the labor market results into different combinations of these two elements.

In particular the old generation representing the insiders supports high degrees of employment protection; this is especially true for low-skill old workers that are affected in the first instance by any selection process.

On the other side of the labor market, though, high employment protection limits the access of the young generation that looks for jobs; therefore, a social conflict arises mainly determined by the quest for employment.

The pension system in this scenario represents for young workers, a useful tool to increase outsiders' entries into the labor market through mandatory retirement; old workers instead, might consider retirement as an insurance device to limit income uncertainty, if employment protection decreases.

In the end, both the degree of employment protection and the mandatory retirement age result from a political process involving three social groups: the young, low-skill and high-skill old; two main setups get enough consensus and are implemented: a first one where employment protection is high and old workers retire early, and a second where instead there is low protection and late retirement.

The main elements driving the result of the political process are two: the productivity gap between high and low productivity workers and the size of the pension paid to retired workers; when productivity differs a lot among workers the first outcome is more easily observed. On the other hand high levels of the replacement rate tend to favor the second setup for the Social Security System.

2 Related Literature

A number of papers have studied the relationship between public pension and other aspects of the Social Security System in a political economy framework.

Conde-Ruiz and Galasso (2002) for instance, analyze the links between public pension and retirement age; they find that in equilibrium, the political support for a large Social Security System relies on elderly with incomplete working history and low-ability young that expect to retire early.

In Galasso and Conde-Ruiz (2001) instead, pension and redistribution systems are considered jointly; in equilibrium a welfare state characterized by generous public pensions and a high level of income redistribution is sustained by a coalition composed by elderly and low-income young. Similar results are found also in Lambertini and Azariadis (1998) where unskilled workers and retirees form the winning coalition.

Another strain of literature did consider mutual interactions between labor market and the setup of the Social Security System.

Boeri, Conde Ruiz and Galasso (2003) focus on the protection against labor market risks considering in particular, the trade-off between employment protection legislation and unemployment benefits; they show that two main outcomes emerge from the political process. The first one is characterized by low unemployment benefits and high employment protection and arises if low-skill insiders are a majority; the second one instead, includes high unemployment benefits and low employment protection.

Recently, Brugiavini, Conde Ruiz and Galasso (2003) built a model that includes also transfers from parents to kids; the tax rate financing the Social Security System, in this setting, depends on the number of unemployed workers in the economy.

When the unemployment rate is high a large number of old parents are recipient of the welfare state; on the other hand kids are less likely to find a job and must rely heavily on the transfers they receive from their parents. Thus a coalition between old recipient of the welfare state and non-emancipated kids supports a large Social Security System; the opposite happens when instead unemployment rate is low.

Present work contributes to this debate considering employment protection legislation and retirement age as the outcomes of a unique political process; other papers analyzed such aspects of the Social Security System, but not jointly.

The main assumptions about agents' behavior introduced in the paper moreover, are tested using microdata; the dataset used for the econometrics includes the informations collected in two different surveys by Fondazione Rodolfo De Benedetti on the themes of pension system and labor market reforms.

The empirical section of this work follows mainly the approach of Brugiavini, Conde Ruiz and Galasso (2003) and of Boeri, Borsch-Supan and Tabellini (2001, 2002) that used the same survey on the pension system.

The paper is organized as follows: section 2 describes the model, section 3 defines the political equilibrium and section 4 includes the empirical analysis. The last section concludes.

3 The Model

3.1 The Economy

Consider an economy where each agent lives for two periods and is young in the first and old in the second; both old and young workers are present at the same time on the labor market.

There is no population growth and each generation counts the same number of individuals; the size of one cohort is normalized to one.

Agents differ for their labor productivity: in each generation, half are high productivity workers and half are low productivity. At time t old individuals' type is common knowledge; the productivity of the young instead, is observed only when they start looking for a job on the labor market.

Agents can be classified initially, in three different groups according to their age and to their productivity: high productivity old (HP), low productivity old (LP) and young workers (Y).

Utility is a function $\nu(c_t, c_{t+1})$ of present, c_t, and future consumption, c_{t+1}; in particular it is the case that:

$$\nu(c_t, c_{t+1}) = u(c_t) + \frac{1}{2} \cdot u(c_{t+1})$$

where $\frac{1}{2}$ is the intertemporal discount factor.

There are no saving means and output is not storable; as a consequence all that is produced within a period is also consumed; each representative agent thus, face a budget constraint equal to his labor income.

Notice that utility is influenced exclusively by own consumption; both intra-generational (between high and low productivity old) and inter-generational (between old and young workers) altruism are absent.

One representative firm produces a unique good that is also the numeraire; the market for it is competitive and any quantity can be sold at the equilibrium price.

The production function uses only labor as input and is defined as:

$$Y_t = \left(a \cdot L_t^{HP} + L_t^{LP}\right)^{\alpha}$$

where L_t^{HP} is the number of high productivity agents and L_t^{LP} is the number of low productivity agents employed at time t; the parameters $a \in [1,2]$ and $\alpha \in [0,1]$ describe respectively the productivity gap between the two types of workers and the technology in use.

Since young agents reveal their type by entering in the labor market, the representative firm can sort high and low productivity individuals. Different workers though, are perfect substitutes; the only difference among them is labor endowment: 1 for LP agents and a for HP agents.

The legislation sets two minimum wage levels: one for low productivity workers

$$\underline{\mathrm{W}}^{LP} = \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}}$$

and one for high productivity workers

$$\underline{\mathrm{W}}^{HP} = \frac{a \cdot \alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}}$$

i.e. $\underline{W}^{HP} = a \cdot \underline{W}^{LP}$.

Except for these constraints, labor market is competitive; therefore it is possible to consider a unique market where unit of efficient labor are exchanged and where each of this unit cannot be paid less than $\underline{W}^{LP}$.

The representative firm chooses the quantity of labor, x_t that maximizes its profits Π_t, defined as:

$$\Pi_t = Y_t - W_t^{LP} \cdot x_t$$

Despite the fact that one unit of labor has the same price no matter if the supplier is a high or a low productivity worker, the firm hires high productivity workers first; this is the case for instance, because they can improve the technology in use through the introduction of more effective routines in the productive process[1].

All agents have no disutility from working and labor supply is perfectly rigid for any positive wage.

Old agents supply work on the labor market for a fraction $\frac{1+\theta}{2}$ of the unitarian period and retire in the last fraction $\frac{1-\theta}{2}$ of it; the young generation instead, remains on the labor market for the whole period.

Unemployed workers do not receive any benefits and simply get nothing if they do not work.

The Social Security System awards to each retired old a fraction $p \in [0, 1]$ of his last wage. Pensions are financed via a lump sum tax over labor income; the tax rate, τ_t, must guarantee a balanced budget.

3.2 Timing and Structure of the Game

The unitary period is divided into 2 sub-periods of the same size; there are four stages in the game, two for each subperiod.

The following sections describe both the subperiods and the subgames.

3.2.1 First Sub-period

At time 0 the first stage begins and the political process takes place; the degree of protection of insider workers, η, and the mandatory retirement age, θ are chosen, once and for all, after that.

The first variable sets the fraction of low productivity workers that the representative firm is allowed to fire each period; the second one defines the fraction of the period that an old worker will actually work before retirement.

The choice over these two dimensions of the Social Security System is discrete; in particular, it is the case that $\eta \in \{0; 1\}$ and $\theta \in \left\{0; \frac{1}{2}; 1\right\}$[2].

After the political process, the second stage starts; the young start looking for job and reveal their type (suppose for instance, they completed a schooling

[1] Representative firm preference for high productivity worker is not explicitly modeled in this setting; a formal treatment of this issue though can be found in the appendix.

[2] Using continuous variables does not change the qualitative results obtained in the paper and is completely irrelevant in the case of η.

path that is a perfect signal of their productivity) while a stochastic displacement process hits the insiders.

This causes a half of employed old workers to leave the job for reasons that are not covered by employment protection such as: bankruptcy of the firm, illness, need to move to other places, serious mistakes or fraud, voluntary unemployment or maternity etc.. Since the displacement process is stochastic, it affects in the same measure high and low productivity agents.

The representative firm then, must choose the quantity, x_0^1, of input to bring in the production process to offset the outflow due to displacement. In hiring new workers, high productivity individuals are chosen first; the number of incoming workers is such that, given the composition of the pool between high and low productivity unemployed, the quantity of labor equals x_0^1.

3.2.2 Second Sub-period

In the second sub-period, at date $\frac{1}{2}$, the third stage begins; a selection process takes place where, according to the results of the political process, a fraction η of low productivity workers is fired.

Old and young low productivity agents are randomly chosen so that both are fired in the same proportion, η.

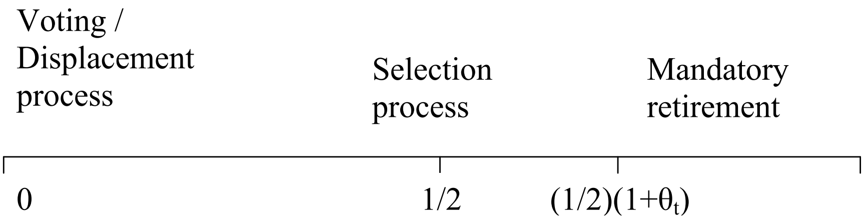
The representative firm then, decides the quantity of labor, x_0^2, to get from the market to offset the reduction in input due to the selection process; again high productivity individuals are hired first and the number of workers involved depends on the composition of the unemployment pool.

After a fraction θ of this sub-period the old retire; at date $\frac{1+\theta}{2}$ thus, the stock of labor employed in the production decreases again.

In the last stage of the game then, the representative firm must choose the quantity, x_0^3, of input to get from the market; the same procedure described before defines the hiring process.

The following graph displays the timing of the model:

Graph 1: Timing of the Model



3.2.3 Subgames Description

In the first stage workers turn to vote and choose their preferred pair (θ, η).

The degree of employment protection and the retirement age are set in a pure majority voting stage where electoral platforms including both the elements characterizing the Social Security System are compared pairwise.

In this setting a strategy for agent i, Σ_i, is a function of his type, ξ, of the replacement rate, of the technology parameter and of the productivity gap between high and low productivity workers.

$\Sigma_i(\xi, p, \alpha, a) : \{HP; LP; Y\} \times [0,1] \times [0,1] \times [0, +\infty) \to \left\{0; \frac{1}{2}; 1\right\} \times \{0; 1\}$

Right after the political process, the representative firm chooses the quantity x_0^1; a strategy for the firm Σ_F^1 is a function of its initial stock of labor, L_{-1}, of the decrease in input due to displacement, γ_0^1 and of the technology parameter:

$\Sigma_F^1\left(L_0^1, \gamma_0^1, \alpha\right) : \left[0, \frac{1}{2}(a+1)\right] \times \left[0, \frac{1}{2}(a+1)\right] \times [0,1] \to [0, (a+1)]$

Notice that both L_{-1} and γ_0^1 never exceed $\frac{1}{2}(a+1)$, i.e. total labor provided by one generation, since at the beginning of each period only old agents are employed.

In the third stage, the representative firm chooses x_0^2; a strategy, Σ_F^2, in this setting, is a function of previous subperiod stock of labor, L_0^1, of input reduction due to labor selection, γ_0^2, and of the technology parameter:

$\Sigma_F^2\left(L_0^2, \gamma_0^2, \alpha\right) : [0, (a+1)] \times [0,1] \times [0,1] \to [0, (a+1)]$

When all old workers retire, input shrinks again and the representative firm gets from the market the quantity x_0^3 of labor; its strategy Σ_F^3, is a function of the stock of input deriving from the period going from date $\frac{1}{2}$ to date $\frac{1-\theta}{2}$,

L_0^2, of labor decrease due to old workers retirement, γ_0^3, and of the technology parameter:

$\Sigma^3\left(L_0^3, \gamma_0^3, \alpha\right) : [0,(a+1)] \times \left[0, \frac{1}{2}(a+1)\right] \times [0,1] \rightarrow [0,(a+1)]$

4 Equilibrium Analysis

The game is sequential and requires backward induction to be solved; the equilibrium analysis then, starts from the maximization problem of the representative firm.

4.1 The Problem of the Firm

A preliminary observation is worth to be stressed before proceeding with the equilibrium analysis of the last three stages of the game: representative firm hiring decisions, change in each period, the composition of the employment and unemployment pools across time. This happens both with respect to agents' type (high or low productivity) and to agent's age (old or young agents).

As a consequence employment decisions influence also γ_0^1, γ_0^2 and γ_0^3; nonetheless the representative firm does not care about the composition and the size of labor flows going in and out, since there are no costs for hiring and firing workers. In order to simplify the analysis then, it is possible to consider these quantities as exogenous parameters.

4.1.1 Retirement

At time $\frac{1+\theta}{2}$ the representative firm chooses the quantity x_0^3 of labor to get from the market when all old workers retire; its maximization problem is the following:

$$V_0^3 = Max_{x_0^3}\left(L_0^3 - \gamma_0^3 + x_0^3\right)^\alpha - W_0^3\left(L_0^2 - \gamma_0^3 + x_0^3\right) + \frac{1}{2} \cdot V_1^1\left[L_0^3\left(x_0^3\right), x_1^1\right]$$

where W_0^3 is current price of one unit of efficient labor, equal to its marginal productivity and L_0^2 is defined as follows:

$$L_0^3 = L_0^2 - \gamma_0^2 + x_0^2$$

Notice that future profits are discounted at the same rate used in the workers' problem.

The first order condition with respect to x_0^3 is:

$$\frac{\delta V_0^3}{\delta x_0^3} = \frac{\alpha}{\left(L_0^3 - \gamma_0^3 + x_0^3\right)^{1-\alpha}} - W_0^3 + \frac{1}{2} \cdot \frac{\delta V_1^1}{\delta x_0^3} = 0$$

4.1.2 Selection

At time $\frac{1}{2}$ the political process has already defined the fraction η of low productivity workers that the representative firm can fire; this amounts for a total γ_0^2 decrease in the stock of labor.

The maximization problem for the representative firm, then is the following:

$$V_0^2 = Max_{x_0^2} \left(L_0^2 - \gamma_0^2 + x_0^2\right)^{\alpha} - W_0^2 \left(L_0^1 - \gamma_0^2 + x_0^2\right) + V_0^3 \left[L_0^2 \left(x_0^2\right), x_0^3\right]$$

where W_0^2 is current price of one unit of efficient labor, equal to its marginal productivity and L_0^1 is:

$$L_0^2 = L_0^1 - \gamma_0^1 + x_0^1$$

The first order condition with respect to x_0^2 is:

$$\frac{\delta V_0^2}{\delta x_0^2} = \frac{\alpha}{\left(L_0^2 - \gamma_0^2 + x_0^2\right)^{1-\alpha}} - W_0^2 + \frac{\delta V_0^3}{\delta x_0^2} = 0$$

From the envelope theorem it is the case that:

$$\frac{\delta V_0^3}{\delta x_0^2} = \frac{\alpha}{\left(L_0^2 - \gamma_0^2 + x_0^2\right)^{1-\alpha}} - W_0^3$$

Substituting the above equation in the first order condition gives:

$$\frac{\delta V_0^2}{\delta x_0^2} = \frac{\alpha}{\left(L_0^2 - \gamma_0^2 + x_0^2\right)^{1-\alpha}} - W_0^2 + \frac{\alpha}{\left(L_0^3 - \gamma_0^3 + x_0^3\right)^{1-\alpha}} - W_0^3 = 0$$

4.1.3 Displacement

At time 0 the displacement process takes place and causes a reduction in labor input amounting to γ_0^1; the representative firm solves then, the following maximization problem:

$$V_0^1 = Max_{x_0^1} \left(L_0^1 - \gamma_0^1 + x_0^1\right)^{\alpha} - W_0^1 \left(L_0^1 - \gamma_0^1 + x_0^1\right) + V_0^2 \left[L_0^2 \left(x_0^1\right), x_0^2\right]$$

where W_0^1 is the current price of one unit of efficient labor equal to its marginal productivity and L_0^1 is defined as:

$$L_0^1 = L_{-1}^3 - \gamma_{-1}^3 + x_{-1}^3$$

The first order condition with respect to x_0^1 is:

$$\frac{\delta V_0^1}{\delta x_0^1} = \frac{\alpha}{\left(L_{-1}^3 - \gamma_0^1 + x_0^1\right)^{1-\alpha}} - W_0^1 + \frac{\delta V_0^2}{\delta x_0^1} = 0$$

From the envelope theorem is:

$$\frac{\delta V_0^2}{\delta x_0^1} = \frac{\alpha}{\left(L_0^2 - \gamma_0^2 + x_0^2\right)^{1-\alpha}} - W_0^2$$

Substituting the above equation in the first order condition gives:

$$\frac{\delta V_0^1}{\delta x_0^1} = \frac{\alpha}{\left(L_0^1 - \gamma_0^1 + x_0^1\right)^{1-\alpha}} - W_0^1 + \frac{\alpha}{\left(L_0^2 - \gamma_0^2 + x_0^2\right)^{1-\alpha}} - W_0^2 = 0$$

4.1.4 Equilibrium

From the maximization problem of the displacement stage is possible to derive, through the envelope theorem, the quantity $\frac{\delta V_1^1}{\delta x_0^3}$; consider indeed

$$\frac{\delta V_0^1}{\delta x_{-1}^3} = \frac{\alpha}{\left(L_0^1 - \gamma_0^1 + x_0^1\right)^{1-\alpha}} - W_0^1$$

Update the above equation one period ahead:

$$\frac{\delta V_1^1}{\delta x_0^3} = \frac{\alpha}{\left(L_1^1 - \gamma_1^1 + x_1^1\right)^{1-\alpha}} - W_1^1$$

and rewrite the first order condition relative to the retirement stage as:

$$\frac{\delta V_0^3}{\delta x_0^3} = \frac{\alpha}{\left(L_0^3 - \gamma_0^3 + x_0^3\right)^{1-\alpha}} - W_0^3 + \frac{1}{2} \cdot \left[\frac{\alpha}{\left(L_1^1 - \gamma_1^1 + x_1^1\right)^{1-\alpha}} - W_1^1\right] = 0$$

Proposition 1 *In equilibrium, the representative firm held constant the level of labor input at* $\frac{1}{2}(a+1)$ *and each unit of efficient labor is paid* $\underline{W}^{LP}$. *Labor market then, is either in equilibrium or faces excess supply.*

Proof. *Guess that in facts, the amount of labor employed in the production process is held constant at* $\frac{1}{2}(a+1)$ *and that every subperiod, the price for each unit of efficient labor is* $\underline{W}^{LP}$.

Verify now, that the first order conditions are verified; start from the last stage to get:

$$\begin{aligned}\frac{\delta V_0^3}{\delta x_0^3} &= \\ & \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} + \\ & \frac{1}{2} \cdot \left\{\frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}}\right\} \\ &= 0\end{aligned}$$

while for the remaining two holds:

$$\begin{aligned}\frac{\delta V_0^1}{\delta x_0^1} &= \frac{\delta V_0^2}{\delta x_0^2} = \\ & \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} + \\ & \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} \\ &= 0\end{aligned}$$

Check then, that on the labor market, there is either excess supply or equilibrium, if an efficient unit of labor is paid $\underline{W}^{LP}$.

Notice that when all old workers retire, a complete turnover happens and the whole young generation gets hired by the representative firm; therefore at the beginning of every period, all the old are employed while all the young are unemployed. Total unemployment is zero at the end of the period and amounts to a whole generation at the beginning of the next one.

The displacement process causes a reduction in the stock of labor amounting to:

$$\gamma_t^1 = \frac{1}{4}(a+1)$$

this is the case because the employment pool includes in the same proportion high and low productivity workers. Moreover, since the representative firms keeps constant the level of input, it is the case that:

$$x_t^1 = \gamma_t^1$$

The unemployment pool includes, at that time, the whole young generation so that the representative firm can hire only high productivity workers to fill the gap caused by displacement.

The productivity of such workers is a *so that the number of people that get a job is* $\frac{1}{4}\left(\frac{a+1}{a}\right)$. *Unemployment increases at* $\frac{5a-1}{4a} \geq 1$. *The composition of the unemployment pool changes; high and low productivity workers amount respectively to* $\frac{2a-1}{4a}$ *and* $\frac{3}{4}$.

The probability that a high productivity young agent is hired at time t *is then* $\frac{1}{2}\left(\frac{a+1}{a}\right)$.

When the selection process takes place, the number of low productivity workers employed in the representative firm is $\frac{1}{4}$; *this means that labor input is reduced of the quantity:*

$$\gamma_t^1 = \frac{\eta}{4} = x_t^2$$

Also in this case there are enough high productivity individuals in the unemployment pool to fill the decrease in labor input; the number of them that gets a job is $\frac{\eta}{4a}$ *and the probability to be hired that these agents, either young or old, face amounts to* $\frac{\eta}{2a-1}$.

Total unemployment increases to $\frac{5a-1}{4a}+\frac{\eta(a-1)}{4a}$ *while the composition of the pool is such that high productivity workers are* $\frac{2a-1-\eta}{4a}$ *and low productivity ones are* $\frac{3+\eta}{4}$.

In the last stage the old retire and the representative firm has an expected reduction in its labor input amounting to:

$$\gamma_t^3=\frac{a}{4}+\frac{1}{4}\left(1-\frac{\eta}{2}\right)+\frac{\eta}{4}=x_t^3$$

Since the representative firm is an approximation for the whole economy, many workers are involved in the process; the law of large numbers then, guarantees that actual and expected values of γ_t^3 *coincide.*

After that all the unemployed young are hired.

As it is required for the initial guess to be an equilibrium, labor market is either in equilibrium or faces excess supply and the price for one unit of efficient labor is fixed at $\underline{W}^{LP}$*; finally also the first order conditions for profit maximization are fulfilled each subperiod.* ■

Given the equilibrium dynamic of labor market, the balanced budget condition requires:

$$\tau\left[\frac{1}{2}(a+1)\right]^{\alpha}=p\cdot\frac{1-\theta}{2}\cdot\left[\frac{1}{2}(a+1)\right]^{\alpha}$$

and the tax rate then, must be set at:

$$\tau=p\cdot\frac{1-\theta}{2}$$

4.2 The Political Process

The political process is an open agenda pure majority voting over a set of policy alternatives; the platforms include the degree of employment protection and retirement age. They are compared pairwise and the preferred pair $(\eta;\theta)$ is implemented.

Since the electorate is a continuum, each voter has zero mass and cannot affect the result of the election. Strategic voting thus, is excluded and it is possible to assume that each voter expresses his preferences sincerely.

Consider now the political preferences of the agents, given the equilibrium strategy of the representative firm.

4.2.1 Political Preferences of High Productivity Old

The maximization problem of a high productivity old worker (HP) is given by:

$$
\begin{aligned}
& MAX_{\eta,\theta} U_0^{HP}(\eta,\theta) \\
& with \\
U_0^{HP}(\eta,\theta) \quad = \quad & \frac{1}{2}\cdot v\left[\frac{(1+\theta)}{2}\cdot a\cdot \underline{\mathrm{W}}^{LP}(1-\tau)+\frac{(1-\theta)}{2}p\cdot a\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\frac{1}{2}\cdot\frac{\eta}{2a-1}\cdot v\left[\frac{\theta}{2}\cdot a\cdot \underline{\mathrm{W}}^{LP}(1-\tau)+\frac{(1-\theta)}{2}p\cdot a\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\frac{1}{2}\left(1-\frac{\eta}{2a-1}\right)v\left[\frac{(1-\theta)}{2}p\cdot a\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\lambda_1\cdot\theta+\lambda_2(1-\theta)+\mu_1\cdot\eta+\mu_2(1-\eta)
\end{aligned}
$$

Expected utility is the summation of three elements; each refers to a specific working history of the agent.

The first one describes the event in which an high productivity worker is not displaced in the first sub-period and works until retires.

The second element refers to the situation where the agent is displaced in the first sub-period, but hired after the selection process; as a consequence he works for a fraction θ of the second sub-period and then retires.

The last one corresponds to the case where the worker is displaced in the first sub-period and never hired again; he gets only the pension for the fraction $(1-\theta)$ of the second sub-period.

4.2.2 Political Preferences of Low Productivity Old

The maximization problem of a low productive old worker (LP) is the following:

$$
\begin{aligned}
& MAX_{\eta,\theta} U_0^{LP}(\eta,\theta) \\
& with \\
U_0^{LP}(\eta,\theta) \quad = \quad & \frac{1}{2}(1-\eta)\,v\left[\frac{(1+\theta)}{2}\cdot \underline{\mathrm{W}}^{LP}(1-\tau)+\frac{(1-\theta)}{2}p\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\frac{\eta}{2}\cdot\nu\left[\frac{1}{2}\cdot \underline{\mathrm{W}}^{LP}(1-\tau)+\frac{(1-\theta)}{2}p\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\frac{1}{2}\cdot\nu\left[\frac{(1-\theta)}{2}\cdot p\cdot \underline{\mathrm{W}}^{LP}\right]+ \\
& +\lambda_1\cdot\theta+\lambda_2(1-\theta)+\mu_1\cdot\eta+\mu_2(1-\eta)
\end{aligned}
$$

Again expected utility is the result of agents' working history; a low productivity worker may face three different situations corresponding to the elements of the summation in the above equation.

The first situation corresponds to the case where he is not displaced in the first sub-period and further is not fired in the second sub-period; the agent works until retires.

In the second case the worker is not displaced in the first sub-period, but is fired in the second one; as a consequence, he works only in the first sub-period and then is unemployed until he retires.

In the third situation initial displacement happens and the agent is never hired again; therefore, he remains unemployed until retirement.

4.2.3 Political Preferences of the Young

The young maximize expected utility across two periods and solve the following optimization problem:

$$
\begin{aligned}
MAX_{\eta,\theta} U_0^Y(\eta,\theta) &= U_0^Y(\eta,\theta) + \frac{1}{4} \cdot U_1^{HP}(\eta,\theta) + \frac{1}{4} \cdot U_1^{LP}(\eta,\theta) \\
&\quad + \lambda_1 \cdot \theta + \lambda_2 (1-\theta) + \mu_1 \cdot \eta + \mu_2 (1-\eta).
\end{aligned}
$$

where

$$
\begin{aligned}
U_0^Y(\eta,\theta) &= \frac{1}{2}\nu \left[\frac{(1-\theta)}{2} \cdot \underline{W}^{LP}(1-\tau)\right] + \frac{a+1}{4a} \cdot \nu \left[a \cdot \underline{W}^{LP}(1-\tau)\right] + \\
&\quad + \frac{1}{2}\left(1 - \frac{a+1}{2a}\right)\left(\frac{\eta}{2a-1}\right) \cdot \nu \left[\frac{a}{2} \cdot \underline{W}^{LP}(1-\tau)\right] + \\
&\quad + \frac{1}{2}\left(1 - \frac{a+1}{2a}\right)\left(1 - \frac{\eta}{2a-1}\right) \nu \left[\frac{(1-\theta)}{2} \cdot a \cdot \underline{W}^{LP}(1-\tau)\right]
\end{aligned}
$$

Notice that since saving means are absent and output is not storable, no transfers between periods are possible; each agent relies on current income to finance his consumption.

Expected utility in 0, $U_0^Y(\eta,\theta)$, results from the summation of four elements.

The first of them describes the situations where the worker happens to be a low productivity type: he is hired only when all old agents retire and works for a fraction $\frac{(1-\theta)}{2}$ of the whole period.

The remaining elements refer to the case where the agent is a high productivity worker; in particular the second of them describes the situation where the agent is hired in the first sub-period, after displacement. He works during the whole period.

The third one corresponds to the case where the worker is hired only at the beginning of the second sub-period due to the selection process.

The last element is equivalent to the first one and the worker is hired only after retirement of old agents.

Different wages correspond to states of the world where he is a high productivity or a low productivity type.

4.2.4 The Voting Game

This section characterizes the outcome of the political process leading to the choice of mandatory retirement age θ, and of the degree of employment protection η.

The analysis goes through the steps reported below.

First a simplified setting where each representative agent is risk neutral and has a linear utility function is considered; in the second step risk aversion is introduced.

The political process with risk neutrality In this section the preferences of each representative agent are derived under the simplifying hypothesis of risk neutrality.

High productivity old workers Consider the problem of a risk neutral, high productivity old worker that chooses the value of the parameters η and θ in order to maximize his expected utility; the maximization problem is the following:

$$MAX_{\eta,\theta} \frac{a \cdot \underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta_t}{2}\right)\left[1 + \theta\left(1 + \frac{\eta}{2a-1}\right)\right] +$$
$$+ p \cdot a \cdot \underline{\mathrm{W}}^{LP} \cdot \frac{1-\theta}{2} + \lambda_1 \cdot \theta + \lambda_2 (1-\theta) + \mu_1 \cdot \eta + \mu_2 (1-\eta)$$

Proposition 2 *High productivity old agents' preferred solution is:*

- *No employment protection, $\eta = 1$, and no retirement, $\theta = 1$ if $\frac{4}{3} \cdot \frac{a}{2a-1} \geq p$.*
- *No employment protection, $\eta = 1$, and early retirement, $\theta = 0$ if $1 \geq p > \frac{4}{3} \cdot \frac{a}{2a-1}$.*

Proof. Consider the Kuhn-Tucker conditions for the problem of a high productivity old worker; look first at the condition related to η.

$$\frac{a \cdot \underline{\mathrm{W}}^{LP}}{4} \cdot \frac{\theta}{2a-1}\left(1 - p \cdot \frac{1-\theta}{2}\right) + \mu_1 - \mu_2 = 0$$

Since is $p \leq 1$, it is always the case that $\frac{\delta U_0^{HP}(\eta,\theta)}{\delta \eta} \geq 0$; the optimal choice for this type of agent then, always entails $\mu_1 = 0$, $\mu_2 > 0$, and $\eta = 1$.

Look now at the K-T condition with respect to θ_t:

$$\begin{aligned} & \frac{a \cdot \underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta}{2}\right)\left(1 + \frac{\eta}{2a-1}\right) - \frac{p}{2} \cdot a \cdot \underline{\mathrm{W}}^{LP} + \\ & + \frac{p}{4} \cdot \frac{a \cdot \underline{\mathrm{W}}^{LP}}{2}\left[1 + \theta\left(1 + \frac{\eta}{2a-1}\right)\right] \\ +\lambda_1 - \lambda_2 &= 0 \end{aligned}$$

Substitute for $\eta = 1$ to have:

$$\begin{aligned} & \frac{a \cdot \underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta}{2}\right)\left(\frac{2a}{2a-1}\right) - \frac{p}{2} \cdot a \cdot \underline{\mathrm{W}}^{LP} + \\ & + \frac{p}{4} \cdot \frac{a \cdot \underline{\mathrm{W}}^{LP}}{2}\left[1 + \theta\left(\frac{2a}{2a-1}\right)\right] \\ +\lambda_1 - \lambda_2 &= 0 \end{aligned}$$

and check the second order conditions to have a point of maximum:

$$\frac{\delta'' U_0^{HP}}{\delta\theta}|_{\eta=1} = \frac{p \cdot a \cdot \underline{\mathrm{W}}^{LP}}{2} \cdot \left(\frac{a}{2a-1}\right) \geq 0$$

It is the case then, that the equation defining high productivity old workers utility has a unique point of minimum for a value of θ such that $\frac{\delta U_0^{HP}}{\delta\theta}|_{\eta=1} = 0$; this means further that optimal choice for this type of agent is either $\theta = 0$ or $\theta = 1$. In order to see which one is the solution for the maximization problem consider when is $U_0^{HP}(1,1) - U_0^{HP}(1,0) \geq 0$:

$$\frac{a \cdot \underline{\mathrm{W}}^{LP}}{4}\frac{4a-1}{2a-1} - \frac{a \cdot \underline{\mathrm{W}}^{LP}}{4}\left(1 - \frac{p}{2}\right) - p \cdot \frac{a \cdot \underline{\mathrm{W}}^{LP}}{2} \geq 0$$

Solving for p gives:

$$\frac{4}{3} \cdot \frac{a}{2a-1} \geq p$$

therefore high productivity old workers choose $\theta = 1$ if $\frac{4}{3} \cdot \frac{a}{2a-1} \geq p$ and $\theta = 0$ if $1 \geq p > \frac{4}{3} \cdot \frac{a}{2a-1}$. ■

Low productivity old workers The maximization problem faced by a low productivity old worker is the following:

$$\begin{aligned} & MAX_{\eta,\theta} \frac{\underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta}{2}\right)\left[1 + \theta(1-\eta)\right] + p \cdot \underline{\mathrm{W}}^{LP} \cdot \frac{1-\theta}{2} + \\ & + \lambda_1 \cdot \theta + \lambda_2(1-\theta) + \mu_1 \cdot \eta + \mu_2(1-\eta) \end{aligned}$$

Proposition 3 *Low productivity old agents' preferred solution is:*

- *Complete employment protection, $\eta = 0$, and no retirement, $\theta = 1$ if $\frac{2}{3} \geq p$.*
- *Complete employment protection, $\eta = 0$, and early retirement, $\theta = 0$ if $1 \geq p > \frac{2}{3}$.*

Proof. Consider the K - T conditions for the maximization problem of this kind of agent starting from that with respect to η:

$$-\frac{\underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta}{2}\right)\theta \cdot \eta + \mu_1 - \mu_2 = 0$$

$\eta = 0$, $\mu_1 > 0$ and $\mu_2 = 0$ is always the preferred solution for low productivity old workers.

Look now at the K - T condition with respect to θ:

$$\frac{\underline{\mathrm{W}}^{LP}}{4} \cdot \frac{p}{2}\left[1 + \theta\left(1 - \eta\right)\right] + \frac{\underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1-\theta}{2}\right)\left(1 - \eta\right) - p \cdot \frac{\underline{\mathrm{W}}^{LP}}{2} + \lambda_1 - \lambda_2 = 0$$

Check now for an internal solution where $\lambda_1 = \lambda_2 = 0$; substituting for $\eta = 0$ in the previous equation and simplifying, gives:

$$\theta = 2 - \frac{1}{p}$$

Notice that the second order conditions are not fulfilled in this case since is:

$$\frac{\delta'' U_0^{LP}}{\delta\theta}|_{\eta=0} = \frac{\underline{\mathrm{W}}^{LP}}{4} \cdot p \geq 0$$

moreover there is only one point of minimum for LP workers' utility function corresponding to the value of θ reported above.

The optimal choice for this kind of agents thus, is either $\theta = 1$ or $\theta = 0$; in particular it is optimal for low productivity old workers to choose no retirement when is $U_0^{LP}(0,1) - U_0^{HP}(0,0) \geq 0$, i.e.:

$$\frac{\underline{\mathrm{W}}^{LP}}{2} \geq \frac{\underline{\mathrm{W}}^{LP}}{4}\left(1 - p \cdot \frac{1}{2}\right) + \frac{p}{2} \cdot \underline{\mathrm{W}}^{LP}$$

Solving for p gives:

$$p \leq \frac{2}{3}$$

therefore low productivity old workers choose $\theta = 1$ if $\frac{2}{3} \geq p$ and $\theta = 0$ if $1 \geq p > \frac{2}{3}$. ■

Young workers The maximization problem for a young worker is the following:

$$MAX_{\eta,\theta}\frac{\underline{\mathrm{W}}^{LP}}{8}\left\{(a+1)\left[3+\frac{1}{2}-\frac{3}{2}\cdot p\left(\frac{1-\theta}{2}\right)\right]-\frac{\theta}{4}\left(a+1-\eta+\eta\cdot\frac{a}{2a-1}\right)\right\}+$$
$$+\lambda_1\cdot\theta+\lambda_2\left(1-\theta\right)+\mu_1\cdot\eta+\mu_2\left(1-\eta\right)$$

Proposition 4 *Young agents' preferred solution is:*

- $\eta=1$ *and* $\theta=1$ *if* $p>\frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}$.
- $\eta=1$ *and* $\theta=0$ *if* $p\leq\frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}$.

Proof. Consider the K - T condition with respect to η:

$$\frac{\underline{\mathrm{W}}^{LP}}{8}\cdot\frac{\theta}{4}\cdot\frac{a-1}{2a-1}+\mu_1-\mu_2=0$$

The left hand side of the previous equation is always positive for $a>1$; the optimal choice for this type of agent then is $\eta=1$.

Look now at the K - T condition with respect to θ:

$$\begin{aligned}&\frac{\underline{\mathrm{W}}^{LP}}{16}\left[\frac{3}{2}p\left(a+1\right)-\left(a+1-\eta+\eta\cdot\frac{a}{2a-1}\right)\right]\\+\lambda_1-\lambda_2\quad&=\quad 0\end{aligned}$$

The above quantity is either positive or negative and its sign depends upon the agent's choice on η and on the value of the parameters; therefore a solution for the maximization problem always entails a corner solution for θ. In particular when $a>1$ and $\eta=1$, optimal choice for a young agent is no retirement ($\theta=1$) if:

$$p>\frac{2}{3}\frac{2a^2}{(2a-1)\left(a+1\right)}$$

and $\theta=0$ when instead, this is not the case. ■

Voting game equilibrium The set-up that emerge from the political process is characterized by no employment protection; the choice over θ depends upon the level of the replacement rate.

Proposition 5 *If* $a>1$ *then:*

- $\eta^*=1$, $\theta^*=1$ *is a Condorcet winner in the voting game if* $p\in\left[\frac{4}{3}\cdot\frac{a}{2a-1};\frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}\right]$.

- $\eta^* = 1$, $\theta^* = 0$ *is a Condorcet winner in the voting game if* $p \notin \left[\frac{4}{3}\cdot\frac{a}{2a-1};\frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}\right]$.

Proof. *In this case there is always a majority of voters that sustains the introduction of* $\eta = 1$*; both high productivity old and young workers indeed, prefer no employment protection, no matter what the equilibrium value of* θ *is. The same coalition moreover, sustains* $\theta = 1$ *when* $p \in \left[\frac{4}{3}\cdot\frac{a}{2a-1};\frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}\right]$ *as it is clear from the description of the preferences of these type of agents.*

When instead, the replacement rate lies outside the above interval a majority sustaining $\theta = 0$ *forms that includes low productivity old workers and either young agents (if* $p \leq \frac{2}{3}\frac{2a^2}{(2a-1)(a+1)}$*) or high productivity old (if* $p > \frac{4}{3}\cdot\frac{a}{2a-1}$*); this is the case since LP agents' marginal utility of* θ *when there is no employment protection, is:*

$$\frac{\underline{W}^{LP}}{4}\cdot\frac{p}{2} - p\cdot\frac{\underline{W}^{LP}}{2} \leq 0$$

so that optimal choice for old low productivity workers is to retire at the beginning of the second sub-period. ■

The political process with risk aversion Introducing risk aversion makes the analysis much more complicated; agents' maximization problems indeed, are not easily solved analytically. Nonetheless assuming that workers' preferences are logarithmic in income permits to get some results.

In order to study this problem, it is useful to go through two steps: in the first one the Kuhn-Tucker conditions are derived for each agent type, in the second one the main outcomes of the analysis are derived.

Consider first the case of high productivity workers.

High productivity old workers The maximization problem of these agents is the following:

$$MAX_{\eta,\theta}\log\left(\frac{a\cdot\underline{W}^{LP}}{2}\right)+\frac{1}{2}\log\left[1+\theta+\frac{p}{2}(1-\theta)^2\right]+$$
$$+\frac{1}{2}\left(\frac{\eta}{2a-1}\right)\log\left[\theta+\frac{p}{2}(1-\theta)(2-\theta)\right]+\frac{1}{2}\left(1-\frac{\eta}{2a-1}\right)\log\left[p(1-\theta)\right]+$$
$$+\lambda_1\cdot\theta+\lambda_2(1-\theta)+\mu_1\cdot\eta+\mu_2(1-\eta)$$

The corresponding Kuhn-Tucker condition with respect to η is:

$$\frac{1}{2}\left(\frac{1}{2a-1}\right)\log\left[1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}\right]+\mu_1-\mu_2=0$$

so that $\mu_1 = 0$, $\mu_2 \geq 0$ and $\eta = 1$ is a solution for the maximization problem. The condition with respect to θ is:

$$\begin{aligned}
& \frac{1}{2}\cdot\frac{1-p(1-\theta)}{1+\theta+\frac{p}{2}(1-\theta)^2}+\frac{1}{2}\left(\frac{\eta}{2a-1}\right)\frac{1-\frac{p}{2}(3-2\theta)}{\theta+\frac{p}{2}(1-\theta)(2-\theta)}+\\
& -\frac{1}{2}\left(1-\frac{\eta}{2a-1}\right)\frac{1}{1-\theta}+\\
+\lambda_1-\lambda_2 &= 0
\end{aligned}$$

and substitute for $\eta = 1$ to have:

$$\begin{aligned}
& \frac{1}{2}\cdot\frac{1-p(1-\theta)}{1+\theta+\frac{p}{2}(1-\theta)^2}+\frac{1}{2}\left(\frac{1}{2a-1}\right)\frac{1-\frac{p}{2}(3-2\theta)}{\theta+\frac{p}{2}(1-\theta)(2-\theta)}+\\
& -\left(\frac{a-1}{2a-1}\right)\frac{1}{1-\theta}+\\
+\lambda_1-\lambda_2 &= 0
\end{aligned}$$

The above equation cannot be solved for θ and it is not possible to derive the optimal choice with respect to retirement age for this type of agents; notice though that it can never be the case that $\theta = 1$ is an optimal choice for these agents since $U_0^{HP}|_{\theta=\eta=1} = -\infty$

Given the above described preferences it is possible to exclude that the platforms $(0,0)$, $\left(0,\frac{1}{2}\right)$, $(0,1)$ and $(1,1)$ represent an optimal choice for high productivity workers.

Look now at the situation where these agents choose no employment protection and late retirement; it must be the case that:

$$U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)=\frac{1}{2}\left(\frac{1}{2a-1}\right)\cdot\log\left(\frac{1}{p}+\frac{3}{4}\right)-\frac{1}{2}\cdot\log\left(\frac{4+2p}{3+\frac{p}{4}}\right)\geq 0$$

The above inequality holds only for some values of the parameters a and p; in particular notice that as a increases $U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)$ decreases and is also:

$$\lim_{a\to+\infty}\left[U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)\right]=-\frac{1}{2}\cdot\log\left(\frac{4+2p}{3+\frac{p}{4}}\right)<0$$

Therefore for every value of $p\in[0,1]$ it is possible to find a value of the productivity gap such that $U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)<0$.

On the other hand since $\frac{\partial\left[U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)\right]}{\delta p}<0$ and since:

$$\lim_{p\to 0}\left[U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)\right]=+\infty$$

for every a finite is possible to find a level for the replacement rate such that $U_0^{HP}\left(1,\frac{1}{2}\right)-U_0^{HP}(1,0)\geq 0$.

Low productivity old workers The maximization problem faced by a low productivity old worker with logarithmic utility is:

$$MAX_{\eta,\theta} \log\left(\frac{\underline{W}^{LP}}{2}\right) + \frac{1-\eta}{2}\log\left[1+\theta+\frac{p}{2}(1-\theta)^2\right] +$$
$$+\frac{\eta}{2}\log\left[1+\frac{p}{2}(1-\theta)\right] + \frac{1}{2}\log\left[p(1-\theta)\right] +$$
$$+\lambda_1\cdot\theta + \lambda_2(1-\theta) + \mu_1\cdot\eta + \mu_2(1-\eta)$$

Look first at the K-T condition relative to the choice over employment protection:

$$-\frac{1}{2}\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right] + \mu_1 - \mu_2 = 0$$

Since marginal utility of η is always negative a solution for the maximization problem requires that $\mu_1 \geq 0$, $\mu_2 = 0$ and $\eta = 0$.

Look now at the choice over retirement age and the corresponding K-T condition:

$$\left(\frac{1-\eta}{2}\right)\frac{1-p(1-\theta)}{1+\theta+\frac{p}{2}(1-\theta)^2} - \left(\frac{\eta}{2}\right)\frac{\frac{p}{2}}{1+\frac{p}{2}(1-\theta)} - \frac{1}{2}\cdot\frac{1}{1-\theta} + \lambda_1 - \lambda_2 = 0$$

Substituting for $\eta = 0$ gives:

$$\frac{1}{2}\cdot\frac{1-p(1-\theta)}{1+\theta+\frac{p}{2}(1-\theta)^2} - \frac{1}{2}\cdot\frac{1}{1-\theta} + \lambda_1 - \lambda_2 = 0$$

since $p \leq 1$it is the case that $\lambda_1 \geq 0$, $\lambda_2 = 0$ and $\theta = 0$.

Old low productivity workers prefer then, complete employment protection and early retirement.

Young workers The maximization problem for a young worker is the following:

$$
\begin{aligned}
&MAX_{\eta,\theta} \log\left(\frac{\mathrm{W}^{LP}}{2}\right)+\frac{1}{2}\log a+\log\left[1-\frac{p}{2}(1-\theta)\right]+\\
&+\frac{1}{2}\left[1+\left(\frac{a-1}{2a}\right)\left(1-\frac{\eta}{2a-1}\right)\right]\log(1-\theta)+\frac{1}{4}\cdot\frac{a+1}{a}\log 2+\\
&+\frac{1}{4}\left\{\begin{array}{c}\log\left(\frac{a\cdot\mathrm{W}^{LP}}{2}\right)+\frac{1}{2}\log\left[1+\theta+\frac{p}{2}(1-\theta)^2\right]+\\ \frac{\eta}{2}\left(\frac{1}{2a-1}\right)\log\left[\theta+\frac{p}{2}(1-\theta)(2-\theta)\right]+\frac{a-1}{2a-1}\log\left[p(1-\theta)\right]\end{array}\right\}+\\
&+\frac{1}{4}\left\{\log\left(\frac{\mathrm{W}^{LP}}{2}\right)+\frac{1-\eta}{2}\log\left[1+\theta+\frac{p}{2}(1-\theta)^2\right]+\frac{\eta}{2}\log\left[1+\frac{p}{2}(1-\theta)\right]+\frac{1}{2}\log\left[p(1-\theta)\right]\right\}\\
&+\lambda_1\cdot\theta+\lambda_2(1-\theta)+\mu_1\cdot\eta+\mu_2(1-\eta)
\end{aligned}
$$

The choice over employment protection is characterized by the following K-T condition:

$$
\begin{aligned}
&\frac{1}{4}\left(\frac{a-1}{2a}\right)\left(\frac{1}{2a-1}\right)\log\left(\frac{1}{1-\theta}\right)+\\
&+\frac{1}{8}\left(\frac{1}{2a-1}\right)\log\left[1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}\right]+\\
&-\frac{1}{8}\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]+\\
&+\mu_1-\mu_2=0
\end{aligned}
$$

while that over θ is:

$$
\begin{aligned}
&\frac{\frac{p}{2}}{1-\frac{p}{2}(1-\theta)}-\frac{1}{2(1-\theta)}\left[2-\frac{\eta}{2}\left(\frac{1}{2a-1}\right)-\left(\frac{a+1}{2a}\right)+\frac{1}{4}\left(2-\frac{\eta}{2a-1}\right)\right]+\\
&+\frac{1}{4}\left(\frac{2-\eta}{2}\right)\frac{1-p(1-\theta)}{1+\theta+\frac{p}{2}(1-\theta)^2}+\\
&+\frac{1}{4}\cdot\frac{\eta}{2}\left[\left(\frac{1}{2a-1}\right)\frac{1-\frac{p}{2}(3-2\theta)}{\theta+\frac{p}{2}(1-\theta)(2-\theta)}-\frac{\frac{p}{2}}{1+\frac{p}{2}(1-\theta)}\right]\\
&+\lambda_1-\lambda_2 \quad = \quad 0
\end{aligned}
$$

No analytical solution is possible for the system of equations defining young agents' preferences over the set-up for the Social Security System; also for these agents though, it is possible to exclude that $\theta = 1$ is an optimal choice since $U_0^Y|_{\theta=1} = -\infty$.

Nonetheless there are two things that are worth to be noticed with respect to the optimal choice of young workers over employment protection.

The first one is relative to the second order conditions for the maximization problem; since $\frac{\delta'' U_0^Y}{\delta\eta} = 0$ the optimal choice for this type of agents entails a corner solution, i.e. is either $\eta = 0$ or $\eta = 1$.

The second one is reported in the proposition that follows:

Proposition 6 *If retirement age is set at $\theta = 0$, young agents preferred choice with respect to employment protection is $\eta = 0$.*

Proof. *Consider the K-T condition with respect to employment protection if $\mu_1 \geq 0$ and $\mu_2 = \eta = 0$; it must be the case that :*

$$\begin{aligned}
& -\frac{a^2}{2}\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]+ \\
& +\frac{a}{4}\left\{\log\left[\frac{1}{1-\theta}+\frac{\theta}{1-\theta}\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]+\log\left[1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}\right]\right\}+ \\
& -\frac{1}{2}\log\left(\frac{1}{1-\theta}\right) \\
\leq\ & 0
\end{aligned}$$

Notice that for $a \to +\infty$ the above condition is always fulfilled while for $a = 1$ is instead:

$$\frac{1}{16}\log\left[\frac{1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}}{1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}}\right]\geq 0$$

there is then an interval $[1, a(\theta)]$ such that for every $a \geq a(\theta)$ is $\frac{\delta' U_0^Y}{\delta\eta} \leq 0$[3] *Solve now for $a(\theta)$ to have:*

$$\begin{aligned}
a(\theta) = & \frac{\frac{1}{2}\left\{\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]+\log\left(\frac{1}{1-\theta}\right)+\log\left[1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}\right]\right\}}{\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]}+ \\
& +\frac{\sqrt[2]{\begin{array}{c}\frac{1}{16}\left\{\log\left[\frac{1}{1-\theta}+\frac{\theta}{1-\theta}\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]+\log\left[1-\frac{\theta}{2}+\frac{\theta}{p(1-\theta)}\right]\right\}^2 \\ -\log\left(\frac{1}{1-\theta}\right)\cdot\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]\end{array}}}{\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]}
\end{aligned}$$

and look at what happens as θ approaches 0:

$$\lim_{\theta\to 0} a(\theta) = 1$$

Since is $a(0) = 1$, and $a > 1$, it is the case that $\frac{\partial U^Y}{\delta\eta}$ approaches zero from below as retirement age gets closer to the beginning of the second subperiod; it is

[3] Notice that such value always exists since is

$$\lim_{a\to+\infty}\frac{\delta' U_0^Y}{\delta\eta} = -\frac{1}{8}\log\left[1+\theta\cdot\frac{1-\frac{p}{2}(1-\theta)}{1+\frac{p}{2}(1-\theta)}\right]\leq 0$$

reasonable to assume then, that when $\theta = 0$ preferred choice of young workers is $\eta = 0$ ■

As a consequence these agents prefer the platform $(0, 0)$ to $(1, 0)$.

Consider now the comparison among $\left(0, \frac{1}{2}\right)$ and $(0, 0)$; notice that complete employment protection and early retirement is always preferred to the alternative since:

$$U_0^Y(0,0) - U_0^Y\left(0, \frac{1}{2}\right) = \frac{3a-1}{4a}\log 2 + \frac{1}{4}\cdot\log\left[\frac{(2+p)^2}{3+\frac{p}{4}}\right] - \frac{1}{2}\cdot\log\left(\frac{4-p}{4-2p}\right) \geq 0$$

Optimal choice for young agents is either $(0, 0)$ or $\left(1, \frac{1}{2}\right)$; indeed young workers preferences are such that the platform including complete employment protection and early retirement is preferred to both $\left(0, \frac{1}{2}\right)$ and $(1, 0)$ so that when is

$$U_0^Y(0,0) \geq U_0^Y\left(0, \frac{1}{2}\right)$$

$(0, 0)$ is the optimal choice for young agents.

If instead, it is the case that:

$$U_0^Y(0,0) \leq U_0^Y\left(1, \frac{1}{2}\right)$$

must hold also:

$$U_0^Y\left(0, \frac{1}{2}\right) \leq U_0^Y\left(1, \frac{1}{2}\right)$$

and

$$U_0^Y(1,0) \leq U_0^Y\left(1, \frac{1}{2}\right)$$

No employment protection and late retirement then is young agents' preferred setup for the Social Security System.

Analyze now the conditions that drive the choice of young workers and in particular look at the following inequality:

$$\begin{aligned} & U_0^Y(0,0) - U_0^Y\left(1, \frac{1}{2}\right) = \\ & \frac{1}{2}\left[1 + \frac{(a-1)^2}{a(2a-1)}\right]\log 2 - \log\left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right) + \\ & +\frac{1}{8}\left[\log\left(\frac{4+2p}{3+\frac{p}{4}}\right) - \left(\frac{1}{2a-1}\right)\log\left(\frac{1}{p}+\frac{3}{4}\right) + \log\left(\frac{2+p}{1+\frac{p}{4}}\right)\right] \\ \geq\ & 0 \end{aligned}$$

Consider the role played by the productivity gap in the definition of the outcomes of the political process: a low level of a favors the implementation of the platform $\left(1, \frac{1}{2}\right)$; deriving the above quantity for a indeed, gives:

$$\begin{aligned}
& \frac{\delta\left[U_0^Y(0,0)-U_0^Y\left(1, \frac{1}{2}\right)\right]}{\delta a} = \\
& \frac{1}{2}\left(\frac{a-1}{a}\right)^2 \cdot\left(\frac{1}{2 a-1}\right)^2 \cdot \log 2+\left(\frac{1}{2 a-1}\right)^2 \log \left(\frac{1}{p}+\frac{3}{4}\right) \\
\geq\ & 0
\end{aligned}$$

Moreover for every $p \in[0,1]$ it is always possible to find a productivity gap wide enough that the pair $(0,0)$ is preferred over the alternative; indeed the limit for a that goes to infinity is:

$$\begin{aligned}
& \lim_{a \rightarrow+\infty}\left[U_0^Y(0,0)-U_0^Y\left(1, \frac{1}{2}\right)\right]= \\
& \frac{3}{4} \log 2-\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)+\frac{1}{8} \log \left[\frac{2(2+p)^2}{3+p+\frac{p^2}{16}}\right] \\
\geq\ & 0
\end{aligned}$$

Solve now the previous inequality for a^* such that for every $a \geq a^*(\theta)$ is $U_0^Y(0,0)-U_0^Y\left(1, \frac{1}{2}\right) \geq 0$ to have:

$$\begin{aligned}
a^*(\theta) \quad = \quad & \frac{\left\{\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)^2-\frac{3}{2} \log (2)-\frac{1}{8} \log \left[\frac{(2+p)^2}{3+p+\left(\frac{p}{4}\right)^2} \cdot \frac{4+3 p}{p}\right]\right\}}{2\left\{\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)^2-\frac{3}{2} \log (2)-\frac{1}{4} \log \left[\frac{2(2+p)^2}{3+p+\left(\frac{p}{4}\right)^2}\right]\right\}}+ \\
& + \frac{\sqrt[2]{\begin{array}{c}\left\{\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)^2-\frac{3}{2} \log (2)-\frac{1}{8} \log \left[\frac{(2+p)^2}{3+p+\left(\frac{p}{4}\right)^2} \cdot \frac{4+3 p}{p}\right]\right\}^2 \\ +4 \cdot \log (2)\left\{\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)^2-\frac{3}{2} \log (2)-\frac{1}{4} \log \left[\frac{2(2+p)^2}{3+p+\left(\frac{p}{4}\right)^2}\right]\right\}\end{array}}}{2\left\{\log \left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right)^2-\frac{3}{2} \log (2)-\frac{1}{4} \log \left[\frac{2(2+p)^2}{3+p+\left(\frac{p}{4}\right)^2}\right]\right\}}
\end{aligned}$$

A further observation is relative to the replacement rate p; it is the case that for high levels of p there are no values $a>1$ such that:

$$U_0^Y(0,0)-U_0^Y\left(1, \frac{1}{2}\right) \leq 0$$

To see why this is the case consider that if $a=1$ and $p=1$ is :

$$\left[U_0^Y(0,0)-U_0^Y\left(1, \frac{1}{2}\right)\right]|_{a=1, p=1}=-\log \left(\frac{9}{8}\right)+\frac{1}{4}\left[\frac{1}{2} \log \left(\frac{24}{13}\right)-\frac{1}{2} \log \left(\frac{7}{4}\right)+\frac{1}{2} \log \left(\frac{12}{5}\right)\right] \geq 0$$

Voting game equilibrium Two main set-ups emerge from the political process: one where there is complete employment protection ($\eta_t = 0$) and early retirement ($\theta_t = 0$), and one where instead there is no employment protection and late retirement ($\eta_t = 1$, $\theta_t = \frac{1}{2}$). The variables that drive agents' choice are the productivity gap and the replacement rate.

Proposition 7 *The political process has two possible outcomes namely:*

- *The platform* $(0, 0)$ *is a Condorcet winner for the voting game if* $a > a^*$
- *The platform* $\left(1, \frac{1}{2}\right)$ *is a Condorcet winner for the voting game if* $1 < a \leq a^*$

Proof. *Consider first that no majority can sustain a platform that includes no retirement; for all agents indeed, expected utility approaches* $-\infty$ *as* θ *approaches* 1.

Notice then that the platform $\left(0, \frac{1}{2}\right)$ *is never preferred to* $(0, 0)$.

When there is complete employment protection every difference among old workers disappears and both have the same preferences; given that if $\eta = 0$, *is* $\frac{\delta' U_0^{LP}}{\delta\theta}|_{\eta=0} \leq 0$, *these agents prefer* $(0, 0)$. *Also for young workers moreover, this platform represent an optimal choice.*

Therefore all agents choose complete employment protection and early retirement if the alternative is $\left(0, \frac{1}{2}\right)$.

The platform $(1, 0)$ *never gets a majority versus* $(0, 0)$*; being low productivity old workers preferred solution, the platform* $(0, 0)$ *is always sustained by this type of agents; moreover since it is the case that* $a > 1$ *and* $\lim_{\theta \to 0} a(\theta) = 1$ *also young workers prefer* $(0, 0)$ *to* $(1, 0)$.

Consider now the comparison between the platforms $(0, 0)$ *and* $\left(1, \frac{1}{2}\right)$*; notice that if complete employment protection and early retirement is preferred to no employment protection and late retirement, then* $(0, 0)$ *is a Condorcet winner for the voting game.*

Low productivity old workers always choose the platform $(0, 0)$ *since:*

$$U_0^{LP}(0, 0) - U_0^{LP}\left(1, \frac{1}{2}\right) = \frac{1}{2}\log\left(\frac{2 + p}{1 + \frac{p}{4}}\right) > 0$$

The choice of high productivity old and young workers instead depends upon the values of the parameters defining the productivity gap and the replacement rate; the conditions for $(0, 0)$ *to be preferred to* $\left(1, \frac{1}{2}\right)$ *are respectively:*

$$U_0^{HP}(0, 0) - U_0^{HP}\left(1, \frac{1}{2}\right) = \frac{1}{2}\log\left(\frac{4 + 2p}{3 + \frac{p}{4}}\right) - \frac{1}{2}\left(\frac{1}{2a - 1}\right)\log\left(\frac{1}{p} + \frac{3}{4}\right)$$

and

$$U_0^Y(0,0) - U_0^Y\left(1,\frac{1}{2}\right) = \frac{1}{2}\left[1+\frac{(a-1)^2}{a(2a-1)}\right]\log 2 - \log\left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right) + \\ +\frac{1}{4}\left\{\left[U_0^{HP}(0,0) - U_0^{HP}\left(1,\frac{1}{2}\right)\right] + \left[U_0^{LP}(0,0) - U_0^{LP}\left(1,\frac{1}{2}\right)\right]\right\}$$

The above equalities imply that whenever the platform $(0,0)$ *is chosen by high productivity old workers, it is the case that also young agents choose it since is:*

$$\frac{1}{2}\left[1+\frac{(a-1)^2}{a(2a-1)}\right]\log 2 - \log\left(\frac{1-\frac{p}{4}}{1-\frac{p}{2}}\right) + \frac{1}{4}\left[U_0^{LP}(0,0) - U_0^{LP}\left(1,\frac{1}{2}\right)\right] > 0$$

Young agents then are pivotal in the voting process.

From the analysis of the preferences of these agents emerges that for every level of $p \in [0,1]$ *there is a value of the productivity gap* a^* *such that for every* $a \geq a^*$ *is* $U_0^Y(0,0) \geq U_0^Y\left(1,\frac{1}{2}\right)$. *Therefore the platform* $(0,0)$ *is a Condorcet winner for the voting game if the previous condition is fulfilled.*

Notice now that if instead $a < a^*$ *and* $U_0^Y(0,0) < U_0^Y\left(1,\frac{1}{2}\right)$, *it is also* $U_0^{HP}(0,0) < U_0^{HP}\left(1,\frac{1}{2}\right)$; *the platform including no employment protection and late retirement then prevails. Moreover when this is the case* $\left(1,\frac{1}{2}\right)$ *is also a Condorcet winner of the voting game.*

Consider indeed that since is all agents prefer $(0,0)$ *to* $\left(0,\frac{1}{2}\right)$, *young workers and high productivity old prefer* $\left(1,\frac{1}{2}\right)$ *to* $\left(0,\frac{1}{2}\right)$; *furthermore also the platform* $(1,0)$ *doesn't get a majority versus* $\left(1,\frac{1}{2}\right)$ *since is* $U_0^Y(1,0) = U_0^Y(0,0) < U_0^Y\left(1,\frac{1}{2}\right)$ *and also* $U_0^{HP}(1,0) = U_0^{HP}(0,0) < U_0^{HP}\left(1,\frac{1}{2}\right)$.

Being preferred to all other alternatives, no employment protection and late retirement is the outcome of the political process and a Condorcet winner in the voting game. ■

5 Empirical Analysis

The model presented in the previous section provides a description of the reasons why a specific Social Security System setup is chosen and consequently may be difficult to reform; it gives also some inside on the different social groups that support its implementation.

The simple insider-outsider framework adopted, describes the behavior of each type of agent; the final results of the analysis rely heavily on such preferences so that it seems useful to test wether or not they give a correct representation of what happens in a real economy.

Present section tries to answer this question using microdata; the empirical evidence comes from two surveys conducted in Italy and in Germany that focus on citizens opinions over different reforms of the welfare state.

Unfortunately the questionnaires were not tailored on the particular purposes of current analysis and do not directly address the specific issues considered. Informations then, must be gathered from the attitude of interviewed persons over slightly different arguments.

Three main aspects of the model have been tested and namely:

1. The existence of altruism among different generations.
2. Agents preferences toward retirement age.
3. Agents preferences toward different degrees of employment protection.

5.1 Data Description

Fondazione Rodolfo De Benedetti provided data sets reporting the results of two surveys on pension system and labor market reforms.

In particular, one data set included 1000 records from a questionnaire about two possible scenarios for a reformed labor market; the observations are the results of a survey conducted in Italy in the year 2000 on a representative sample of the population aged 14-80. The questionnaire was administrated to the interviewed persons directly at home.

This information has been used to characterize workers' actual preferences over employment protection.

A second dataset reports the informations gathered from a survey about pensions conducted in Germany (2500 records) and Italy (2000 records) in the year 2001.

A particular issue arises with respect to the interviews collected between September and October 2001 in Italy; some of them happened to be recorded in the very same day or immediately after the terrorist attack of New York. Due to that 480 individuals were contacted again and asked the same questions; an additional 500 people moreover, were added to the survey in order to have observations not compromised by the above mentioned event.

The whole database including the 500 new records has been used by Boeri et al. (2002); the analysis that follows instead, relies on the original data set including 2000 observations. In order to account for the bias in people attitude deriving from the terrorist attacks, a dummy variable with value zero for the interviews recorded before September 11th 2001, has been used in the regressions.

Interviews were carried out through a Computer Assisted Telephone Interview (CATI) technique on a representative sample of the population aged 16-80; the questionnaire was part of a wider omnibus survey.

These data allowed to disentangle the effects of personal characteristics of the interviewed person in determining the support to different setups of the pension system; in particular the analysis focus on the effects of age and education on intergenerational altruism and on preferences over retirement age.

5.2 Methodology

A first methodological issue is posed by the selection of the questions included in the survey; the main criterium is obviously the bearing on the subject at a stake. A second less obvious criterium prescribes to choose questions that do not involve variations in the quantities that in the model are exogenously set (as for instance, the replacement rate paid by the pension system).

Different methodologies have been used in the empirical analysis; each of the above mentioned issues, indeed, required a specific treatment of the data from the surveys.

The test for the existence of intergenerational altruism requires only descriptive statistics; probit and ordered probit estimation techniques instead, have been used to study the preference over employment protection and retirement age.

Socio-economics variables such as sex, geographic provenience, family size, marital and professional status were used in all estimations.

Available informations about family income moreover were included among the regressors when data from the labor market survey are considered; the estimations based on the pension survey finally, used also some variables about the respondent status with respect to the person with the highest contribution to familiar income, about the respondent opinion over the social security system and her political orientation.

Non respondents raise in this framework a methodological issue due to the self-selection bias affecting the sample; in order to cope with this problem the following approach has been adopted.

Records that are lacking some informations on both personal characteristics of the interviewed person and her opinion over the Social Security System are expunged from the sample. Instead a profile of those people that simply refused to answer the specific question at a stake is reported at the beginning of each regression; in this way it is possible to describe the causes of self-selection.

The same procedure obviously, cannot be adopted for those records where personal characteristics of the respondent are missing; in this case the bias in the sample remains unexplained mainly due to lack of information.

A second problem posed by non respondents involves the trade-off between robustness of the results and the size of the self-selection bias; expanding the set of regressors indeed, helps to test the robustness of the relevant variables but requires, at the same time, to expunge a larger number of records from the sample. This is especially true when the regressors include some sensible characteristics of the interviewed person as for instance, income level or political orientation.

In order to control for this problem two rounds of estimations have been conducted; in the first one only personal characteristics of the respondent that are recorded in both surveys were used as regressors; this represents the minimum set of independent variables. In the second round instead, regressors varied across different estimations and included all the informations available for each survey.

A posterior check revealed no significant discrepancies among the two rounds; the results reported in the following section then are those obtained using the largest set of variables.

5.3 Results

5.3.1 Altruism

In the model social groups and individual agents are completely selfish; in other words there is no intergenerational nor intragenerational altruism. This obviously affects significantly the political process.

Unfortunately no questions could be found providing informations on intragenerational altruism; thus the analysis is limited to the existence of some preference for redistribution among different generations.

The test over this issue relies on the following questions, included in the survey on the pension system:

- Would you be willing to shoulder during a transition period of 10 years higher contributions to the pension system if this were to prevent that your children and grandchildren must pay even higher contributions?

- How much of a higher contribution rate?

  1) yes, up to 1 percent points;

  2) yes, up to 3 percent points;

  3) yes, up to 5 percent points;

  4) yes, up to 10 percent points;

  5) no, not at all;

Both fit pretty well to the aim of the analysis and allow to verify directly the presence of intergenerational altruism.

The pool of the respondent included individuals with different characteristics depending on the country where the interview was conducted.

In particular the first question was asked to all currently employed in Italy; in Germany instead currently employed that were also self employed or employee in familiar enterprises, and the civil servants were excluded from the respondents pool. These two sets jointly amount to a 45.2% of the whole sample.

The Italian survey moreover, did not use the first question as a filter for the second one while in Germany this was the case; therefore only people declaring to be willing to pay higher contributions during the transition period were also asked about the size of the increase.

Consider now the evidence derived from the first question.

Non respondents amount to less than 1% of the total; the self-selection bias thus, is very limited. These people are both males and females, mainly middle-aged (between 35 and 54 years old), from Germany (especially western Germans), non-single, employed as white collars, informed over the pension reform issues and fearing a future crisis of the pension system; all of them moreover were interviewed after September 11^{th}. Since such individuals are those potentially more hurt by this kind of reform, it may be the case that self-selection bias derive from the fact that the question was perceived as disturbing.

A majority of the interviewed refused to bear the costs of a transition period that could alleviate the costs for the next generations; the following table displays the results of the survey:

Table 1: Intergenerational Altruism

	% Total	% Valid
No	*57.53%*	*57.83%*
Yes	*41.95%*	*42.17%*
No Answer	*0.53%*	
	100.00%	*100.00%*

Source: Author's calculation based on German-Italian survey

This outcome represents a first evidence that altruism between members of different generations is in facts, quite weak.

A further confirm comes from the second question.

Since the German survey used the answer to the previous question as a filter for the next one while the Italian did not, the observations are not homogeneous between countries; a problem arises then on how to define the degree of intergenerational altruism.

The first step of the analysis showed that intergenerational altruism is shared only by a minority of the population, therefore it seems interesting to limit the analysis of the second question to these people; in this way it is possible to assess more accurately how strong are the preferences for intergenerational redistribution of the "altruists".

The pool of potential respondent amounts only to 19% of the sample while non respondent are a negligible fraction.

Also in this case the interviewed people show a low degree of intergenerational altruism; looking at the descriptive statistics reveals that those willing to increase their contributions of more than a 3% represent slightly more than 1/3 of the respondents (see table 2).

Table 2: Degree of Intergenerational Altruism

	% Total	% Cumulate Total	% Valid	% Cumulate Valid
0	5.23%	5.23%	5.24%	5.24%
1	22.79%	28.02%	22.86%	28.11%
2	36.40%	64.43%	36.51%	64.61%
3	26.38%	90.81%	26.46%	91.07%
4	8.90%	99.71%	8.93%	100.00%
No Answer	0.29%	100.00%	--	

Source: Author's calculation based on German-Italian survey

This result is obviously reinforced when the methodology adopted in the Italian survey is used and those among the Germans that refused to shoulder the transition burden are included in the first class of table 2 (labeled "0"); in this case people that are willing to pay higher contributions in excess of 3% fall to about 17% of the respondents set.

The assumption of substantial egoism of each generation then, is not too far from what is observed in the data.

5.3.2 Preference towards retirement age

Consider now agents preferences toward retirement age.

One question included in the survey over pension system addresses directly the issue and asks what follows:

- A possibility to stabilize future contribution rates is an increase in the retirement age. Would you accept an increase in the retirement age if this would mean that the future contributions to public pensions could remain constant?

  1) yes, 1 years later;

  2) yes, 2 years later;

  3) yes, 3 years later;

  4) yes, up to 5 years later;

  5) yes, even if more than 5 years later;

  6) no;

The set of people interviewed over this issue is slightly different in the two surveys; in particular while in Germany all employed in the private sector that are not self-employed are included, in Italy the respondents are all currently employed, no matter in which sector nor if self-employed.

Respondents amount to 30.84% of total sample; a negligible fraction of them (0.11%) did not answer the question and also in this case there is little room to give an explanation of the self-selection bias[4].

Possible answers are classified according to a scale ranging from 0 to 5 where zero corresponds to the answer "no" and 5 to the answer "yes, even if more than 5 years later"; an ordered probit estimation technique is used to find the most important variables affecting the choice over this issue.

Almost all the regressors are dummies describing the marital status (single, couple, widow), the job (bluecollar, whitecollar) and the relationship with the person that mostly contributes to family income (myself, sondaughter, partner, parent).

Unionization, information about current debate on the pension system and respondent's opinion on this issue are reported in ad hoc dummy variables (union, informed, crisisfear, futurereform); political orientation is measured on a scale ranging from 0 (extreme left) to 10 (extreme right).

The place of residence is defined through dummy variables as well; in particular, Italy has been divided in three areas, north, centre and south, while for Germany only the distinction between east and west has been considered.

The variables "age" and "family size" are not dummies; the numbers reported by the survey are used.

The regressor "education" is classified according to a scale where 1 identifies the attainment of primary education or no formal education, 2 identifies the attainment of a secondary degree of education, and 3 identifies people with a university degree or a higher level of education.

The results of the estimate are reported below

[4] The self-selection bias is particularly limited in this case because the use of a large set of variables required to expunge many observations due to some missing records (26.14% of the set of potential respondent); by considering the smallest set of variable it emerges that non-respondents are mainly middle-aged eastern Germans and count for less than 1% of potential respondents.

Table 3: Preference for Late Retirement

Ordered probit estimates		Number of obs =	1423
		Wald chi2(24) =	151.09
		Prob > chi2 =	0.0000
Log likelihood =	-1811.2287	Pseudo R2 =	0.0454

retirement	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
september11	-.1997759	.2576442	-0.78	0.438	-.7047493	.3051974
north	.0762154	.1143855	0.67	0.505	-.1479761	.3004069
south	.099154	.1283722	0.77	0.440	-.152451	.3507589
westernger~y	-.3557767	.2712482	-1.31	0.190	-.8874133	.17586
easternger~y	-.4053732	.2763061	-1.47	0.142	-.9469231	.1361768
male	.1685696	.0791462	2.13	0.033	.0134458	.3236934
age	-.0091891	.0040272	-2.28	0.023	-.0170822	-.0012959
single	-.0594758	.1499606	-0.40	0.692	-.3533933	.2344416
coupled	-.142699	.138645	-1.03	0.303	-.4144383	.1290402
widow	.5439055	.2995812	1.82	0.069	-.0432628	1.131074
familysize	-.020112	.0399209	-0.50	0.614	-.0983555	.0581316
education	.1384044	.06397	2.16	0.030	.0130255	.2637833
bluecollar	-.1491776	.3461817	-0.43	0.667	-.8276812	.5293261
whitecollar	-.1633749	.3397764	-0.48	0.631	-.8293245	.5025747
selfemployed	.0576724	.3468286	0.17	0.868	-.6220992	.7374441
myself	-.4784354	.3087018	-1.55	0.121	-1.08348	.1266091
sondaughter	-.7373946	.3426416	-2.15	0.031	-1.40896	-.0658293
partner	-.3296414	.3185863	-1.03	0.301	-.9540591	.2947762
parent	-.7454531	.3655345	-2.04	0.041	-1.461888	-.0290187
informed	.036143	.0748298	0.48	0.629	-.1105206	.1828067
crisisfear	-.0651803	.1064457	-0.61	0.540	-.27381	.1434495
futurereform	.0029417	.0971139	0.03	0.976	-.187398	.1932814
union	-.1077952	.0788574	-1.37	0.172	-.2623528	.0467625
politicalp~n	.0111959	.018143	0.62	0.537	-.0243637	.0467555

Ordered probit analysis points out that age and level of education are statistically significant for a 5% confidence interval; both aspects are relevant in the description of agents' choice made in the previous section (education can be considered a proxy for labor productivity) but their effects are partially at odd with the assumption of the model.

A problem indeed, arises with respect to the variable age that is significant and has negative impact on people preferences toward retirement age; the model predicts that where the Social Security System is characterized by early retirement and high employment protection, as it is the case for Italy and Germany, the young should support this set-up.

A substantial argument that can give reason of these results is found in the paper by Boeri et al. (2002); using the same data set indeed, the authors show that young and more educated individuals are those that mostly favor a reform of the pension system.

This is coherent with the hypothesis that a consensus toward a change of regime is forming in the new generations and among the most productive fraction of the labor market; the wave of reforms introduced in recent years both in Italy and Germany moreover, may represent a further signal of an uprising trend in this direction.

Not surprisingly also the variable education is highly significant in the estimation and has a positive effect on individuals preferences over late retirement; people with higher education are more likely to accept the proposal to retire later in order to stabilize current contribution. Interpreting schooling as a proxy for labor productivity then allows to say that high productivity and young workers preferences converges toward an increase of mandatory retirement age.

This result is robust to different definition of the set of independent variable; moreover it seems to confirm the hypothesis that since the establishment of current pension system, the mismatch in the labor market has decreased thus provoking a shifts in young agents preference from early to late retirement.

5.3.3 Preference towards employment protection

This last section considers agents preferences towards employment protection using data from the Italian survey on labor market; a question included in it indeed, very much fits the aim of the analysis and asks what follows:

- Which of the two following labor markets do you prefer?

1) a labor market where is very difficult to find a job, but once found it is hard to lose it;

2) a labor market where finding a job is easy, but losing it is easy too.

The dependent variable in this case is a dummy with value 1 in correspondence of the first answer and 0 otherwise.

The set of potential respondents includes the whole sample; a fraction of about 21% of it though, did not answer the question. The self-selection problem is relevant and requires a careful description of non-respondents.

From the descriptive statistics emerges the profile of a person in a non working status (mainly retired people or housewives but also students), generally middle aged or old (more than 55 years of age), with a low level of education and income. These people are both male and female, with a slight majority of women, and are evenly distributed in all geographic areas; they live in family with less than 4 members where there are no children and with the same probability are either the main contributors to family income or not.

No records were dropped due to lack of some basic informations; the self-selection process thus depends exclusively upon the fact that the interviewed couldn't answer to the question or simply refused to do that.

Given the above description of non-respondents a plausible explanations of why some people did not answer the question is because they really did not have an opinion on this specific issue; most of them indeed were not in the labor market at the time of the interview and as a consequence they could feel little concern about employment protection.

The analysis involves a probit estimation where a preference for a rigid labor market is regressed against the main characteristics of the interviewed person;

the set of regressors includes some of the variables used also in the previous estimations (i.e. "family size", "education" and the dummies relative to the job of the interviewed and of the place where he lives) and some different ones. The composition of the sample in particular, required to introduce a set of dummies relative to the non-professional status of the individuals("retired", "unemployed", "student").

Additional informations reported in the survey are also used, i.e. those relative to the fact that the respondent is the one that mostly contributes to household income (dummy "breadwinner"), that is responsible for family expenses (dummy "expenses"), and those relative to the presence of children living with her (dummy "children"). A further variable describes the size of the town where people live, ranging from value 1 for those living in towns with up to 30000 inhabitants, to value 3 for those living in towns with more than 100.000 inhabitants.

Finally since informations on personal income were available, a further variable, named "incomeclass", has been included; individuals are classified along the scale reported below:

0) no income;
1) less than 516 euro;
2) from 516 to 775 euro;
3) from 775 to 1033 euro;
4) from 1033 to 1549 euro;
5) from 1549 to 2066 euro;
6) more than 2066 euro.

The table that follows reports the main result of the estimation.

Table 4: Preference over employment protection.

```
Probit estimates                                  Number of obs   =        788
                                                    Wald chi2(17)   =      54.85
                                                    Prob > chi2     =     0.0000
Log likelihood =  -442.99675                        Pseudo R2       =     0.0633
```

rigid	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0015545	.055436	0.03	0.978	-.1070981	.1102071
male	-.13894	.1462124	-0.95	0.342	-.4255111	.147631
north	.1350242	.1379946	0.98	0.328	-.1354402	.4054886
south	.3581812	.1493125	2.40	0.016	.0655341	.6508282
towndimesion	.0496875	.0584584	0.85	0.395	-.0648888	.1642638
breadwinner	.1388223	.1648055	0.84	0.400	-.1841906	.4618352
expenses	.0539145	.1446325	0.37	0.709	-.22956	.337389
education	-.397002	.125155	-3.17	0.002	-.6423013	-.1517027
bluecollar	.1610882	.2486406	0.65	0.517	-.3262385	.6484148
whitecollar	.149249	.2344237	0.64	0.524	-.3102131	.6087111
selfemployed	-.4080166	.2454982	-1.66	0.097	-.8891842	.073151
retired	.0221417	.2365973	0.09	0.925	-.4415805	.4858639
student	-.176763	.2563568	-0.69	0.490	-.6792131	.325687
unemployed	-.5768671	.2796288	-2.06	0.039	-1.124929	-.0288047
familysize	.1188685	.0551253	2.16	0.031	.0108249	.2269121
children	.0005088	.1417935	0.00	0.997	-.2774014	.278419
incomeclass	-.0320017	.0470471	-0.68	0.496	-.1242123	.0602089
_cons	.8037835	.4562434	1.76	0.078	-.0904372	1.698004

Also in this case, the variable "education" is statistically significant for a 5% confidence interval and has a negative effect on the probability that an individual prefers a rigid labor market, as it is predicted by the theoretical model. In other words more educated people choose the scenario where it is easy to get a job but it is also easy to lose it; workers with lower degree of education prefer instead, a labor market where there is high employment protection.

The variable age is not statistically significant; the insider/outsider status of the respondents prevails over this aspect. It is the case indeed, that the variable unemployed has some statistical relevance (in particular is significant for an interval of confidence of 5%) and a negative impact on the preference for a rigid labor market.

Also the localization in southern Italy of the interviewed person has a sizeable effect over the probability that she prefers a rigid labor market.

It may be the case then, that the variable unemployment represents the preferences of young outsiders for less employment protection. The variable south on the other hand, may capture the effects of the endemic unemployment in the area hitting mainly low productivity workers; these individuals and especially the old ones that are less likely to move to other places to find a job, prefer in facts more protection.

In this sense the outcomes of the analysis can be brought back to the model predictions and signal the existence of a mounting consensus toward a flexibilization of the labor market, shared by young outsiders and high productivity insiders.

The empirical evidence then, seems to support the hypothesis that from the initial support to a setup with early retirement and high employment protection, social preferences are shifting to a system with late retirement and low employment protection; this change is lead by the same groups singled out in the model, i.e. high productivity workers and young.

6 Final Remarks

Present analysis is intended to contribute to current debate on Social Security System reforms by stressing the mutual interdependence between labor market and pension system; the crucial element in particular, is total turnover between different generations of workers.

Young agents indeed, want to ease their entry in the labor market either by decreasing employment protection or by reducing the mandatory retirement age; both choice are costly for them either in terms of present and future risk, specially in case they reveal to be low productivity type, or in terms of taxation.

Old agents instead are mainly concerned by the reduction of current income variability and they choose different tools to cope with that depending on their type.

High productivity old workers, use employment protection and retirement against the risk of displacement; low productivity types on the other hand face also an unemployment risk and thus must dispose of the same instruments to reduce the effects of both these events.

Agents preferences are mainly driven by the size of the productivity gap existing between different workers types; as a consequence two main setups result from the political process.

The first of them entails high employment protection and early retirement and is sustained by the social groups of the young and of low productivity old workers.

The second setup instead, includes low employment protection and late retirement; it emerges in the political process when young and high productivity old workers form a coalition.

This prediction finds a partial confirm in the empirical evidence; the surveys conducted in Italy and Germany reveal that the waves of reforms that are changing the setups of the Social Security Systems in these countries are backed by the consensus of the social groups indicated in the model. Indeed the shift from high levels of employment protection and low mandatory retirement age to lower protection and later retirement that took place during the 90's is particularly supported by the young and by highly educated people.

In this framework then, reforms that modify the setup of the Social Security System require in order to be successful to provide a wide spectrum change that includes in the same picture both the pension system and the labor market; in this way indeed it is possible to calibrate the generational turnover to a level that gathers enough consensus among the different groups of the economy.

7 Appendix: Representative Firm Preference for High Productivity Workers

This section is aimed to justify in a formal framework, the assumption of a representative firm preference for high productivity workers.

Consider a setup where hiring a high productivity workers causes a reduction in production costs; this may be the case for instance because they allow to use more efficient routines or because they learn faster how to do their job.

Define representative firm profits as follows:

$$\Pi = Y - W^{LP} \cdot LP - W^{HP} \cdot HP + i \cdot HP$$

where the parameter i describes the decrease in costs generated by each HP worker.

Assume now that the size of i is such that:

$$i \leq \frac{\alpha \cdot a}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} - \alpha \cdot a^{\alpha} \cdot \left(\frac{4}{3}\right)^{1-\alpha}$$

so that when the wage for high productivity workers is set at $\underline{\mathrm{W}}^{HP}$ labor demand never exceeds the number of these workers in the unemployment pool[5].

As a consequence when old agents are still working, labor market faces always excess supply; each unit of efficient labor provided by HP and LP workers then, is paid in the same way ($\underline{\mathrm{W}}^{LP}$) and hiring high productivity individuals is always beneficial due to the reduction in costs that they can guarantee.

This justifies firm preferences when displacement and selection take place.

A different issue is the decision at time when old agents retire since labor market faces a supply shrink that causes an excess demand for high productivity workers.

The representative firm then must decide wether or not to hire LP workers and in case it does how many of them to get from the market.

Hiring low productivity workers is beneficial both because it increases overall production and because reduces HP workers marginal productivity; since labor market is competitive indeed, when demand exceeds supply, high productivity workers wage must be equal to their marginal productivity.

On the other hand each LP agent employed diminishes future profits since limits the possibility to exploit the reduction in costs deriving from HP workers

[5] Notice indeed, that even if all old high productivity workers are employed at time 0, so that half of them are displaced and cannot be hired immediately, labor demand does not exceed $\frac{3}{4} \cdot a$ i.e. the number of HP workers that are either already in the firm or in the unemployment pool.

up until the selection process (at least for those individuals that are not hit by the displacement process).

Suppose now that when retirement age is set at $\theta = 1$, old individuals do not work until they die but there is a period of length ε where the old generation exits from the labor market; the hiring decision of the firm after retirement in this way, is not equivalent to that taken after displacement.

Given that it is possible to show that if i is small enough, the representative firm chooses to hire all the low productivity workers when the old generation retires.

In order to simplify the analysis, a "worst case scenario" is considered where LP individuals employed at the beginning of the period, work in the representative firm until they retire at the mandatory age $\theta = 1$; this is equivalent to say that there is no displacement and that no selection take place at half of the period. In other words at time 0, the firm adjusts the level of labor input derived from the previous period and maintains the same labor force (in terms of size and composition) until retirement.

The maximization problem of the firm when the old generation retires is the following:

$$Max_{LP,HP}\ \varepsilon \cdot \left\{ \left(\frac{1}{2} \cdot a + LP\right)^{\alpha} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} \cdot LP - \frac{a}{2} \cdot \frac{\alpha}{\left(\frac{1}{2} \cdot a + LP\right)^{1-\alpha}} \right\} +$$
$$+\frac{1}{2}\left\{ \left(\frac{1}{2} \cdot a + LP + a \cdot HP\right)^{\alpha} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} \cdot LP - \frac{\alpha \cdot a}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} \cdot HP + i \cdot HP \right\}$$

Notice that the initial level of labor input is set at $\frac{1}{2} \cdot a$ since all high productivity workers are hired after retirement; this implies further that maximization problem can be treated as a static one

Consider the first order conditions for the maximization problem and start with that referred to HP:

$$\frac{1}{2}\left\{ \frac{\alpha \cdot a}{\left(\frac{1}{2} \cdot a + LP + a \cdot HP\right)^{1-\alpha}} - \frac{\alpha \cdot a}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} + i \right\} = 0$$

Deriving with respect to LP gives:

$$\begin{aligned} &\varepsilon \left\{ \frac{\alpha}{\left(\frac{1}{2} \cdot a + LP\right)^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} + \frac{a}{2} \cdot \frac{\alpha(1-\alpha)}{\left(\frac{1}{2} \cdot a + LP\right)^{2(1-\alpha)}} \right\} \\ &+\frac{1}{2}\left\{ \frac{\alpha}{\left(\frac{1}{2} \cdot a + LP + a \cdot HP\right)^{1-\alpha}} - \frac{\alpha}{\left[\frac{1}{2}(a+1)\right]^{1-\alpha}} \right\} \\ = \ \ & 0 \end{aligned}$$

Notice now that from the first equation, it is the case that:

$$\frac{\alpha}{\left(\frac{1}{2}\cdot a+LP+a\cdot HP\right)^{1-\alpha}}-\frac{\alpha}{\left[\frac{1}{2}\left(a+1\right)\right]^{1-\alpha}}=-i$$

Substituting this result in the second equation gives:

$$\begin{aligned}&\varepsilon\left\{\frac{\alpha}{\left(\frac{1}{2}\cdot a+LP\right)^{1-\alpha}}-\frac{\alpha}{\left[\frac{1}{2}\left(a+1\right)\right]^{1-\alpha}}-\frac{a}{2}\cdot\frac{\alpha\left(1-\alpha\right)}{\left(\frac{1}{2}\cdot a+LP\right)^{2(1-\alpha)}}\right\}-\frac{1}{2}\cdot i\\ =&\quad 0\end{aligned}$$

Consider now which are the conditions to have a complete turnover when old workers retire, i.e. to have $LP=\frac{1}{2}$:

$$\varepsilon\cdot\frac{a}{2}\cdot\frac{\alpha\left(1-\alpha\right)}{\left[\frac{1}{2}\left(a+1\right)\right]^{2(1-\alpha)}}-\frac{1}{2}\cdot i\geq 0$$

Notice that as i approaches zero the above condition is always true; moreover since this result holds when no substitution of LP workers is possible and retirement age is set at the maximum, it must hold *a fortiori* when instead, displacement and selection allow the replacement of low productivity individuals with high productivity ones and $\theta<1$.

The original setting then, can be thought as an approximation for the model introduced above where the benefit associated to the hiring of HP workers is very small (in the limit null) and the period in which old individuals quit working without retiring is negligible.

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