Conditional Dependency of Financial Series: The Copula-GARCH Model

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Research Paper N° 69
December 2002

FAME - International Center for Financial Asset Management and Engineering

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INTERNATIONAL CENTER FOR
FINANCIAL ASSET MANAGEMENT AND ENGINEERING

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Conditional Dependency of Financial Series: The Copula-GARCH Model

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February 2001 (Revised: December 2002)

Abstract

We develop a new methodology to measure conditional dependency between time series each driven by complicated marginal distributions. We achieve this by using copula functions that link marginal distributions, and by expressing the parameter of the copula as a function of predetermined variables. The marginal model is an autoregressive version of Hansen's (1994) GARCH-type model with time-varying skewness and kurtosis. Here, we extend, to a dynamic setting, the research that focuses on asymmetries in correlation during extreme events. We show that, for many market indices, dependency increases subsequent to large extreme realizations. Furthermore, for several index pairs, this increase is stronger after crashes. Our model has many potential applications such as VaR measurement and portfolio allocation in non-gaussian environments.

Keywords: International correlation, Stock indices, Skewed Student-t distribution.

JEL classification: C51, F37, G11

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1 Introduction

The measurement of dependency is a rather difficult task, for instance, the usual Pearson correlation may be too restrictive a criterion as shown by Embrechts, Lindskog, and McNeil (2001). The aim of this paper is to present a new methodology to measure conditional dependency. This methodological framework can be applied to various problems that arise in finance. Our methodology builds on so-called "copula" functions. These functions provide an interesting tool to link univariate models. The insight of this research is that by expressing the parameter of the copula as a function of other variables as well as of its own lagged values, one obtains a model with conditional dependency. We illustrate how our methodology can be applied by investigating the impact of certain joint stock-return realizations on the subsequent dependency of international markets.

Our research considers a univariate model for each stock index and joins these models via a copula function into a conditional multivariate framework. Presently, we wish to relate the various building blocks of this research to the existing literature. First, our univariate model builds on Hansen's (1994) seminal paper. In that paper, a so-called skewed Student-t distribution is derived. This distribution, while retaining the desired property of having a zero mean and unit variance, has two additional parameters controlling asymmetry and fat-tailedness. By rendering these parameters conditional, it is possible to obtain time-varying higher moments.[1] This model, therefore, extends Engle's (1982) ARCH and Bollerslev's (1986) GARCH model. In an extension to Hansen (1994), Jondeau and Rockinger (2002a, b) determine the expression of skewness and kurtosis of the skewed Student-t distribution, show how the cumulative distribution function (cdf) and its inverse can be computed, as well as how to simulate associated data. They also discuss how the skewed Student-t distribution should get parametrized. A number of studies have considered alternative skewed Student-t distributions. Harvey and Siddique (1999) model the conditional skewness with a non-central Student-t distribution. Recent work is by Adcock (2002) and Lambert and Laurent (2002).

Second, we use a copula function to link univariate models. Copulas have been introduced to model a multivariate distribution when only marginal distributions are known. Such an approach is particularly useful in situations where multivariate normality does not hold.[2] Given that most copula functions introduce an explicit parameter that may be interpreted, intuitively, as a correlation, it is easy to render this parameter conditional. In other words, our model allows marginal distributions to be conditionally dependent. Our model, thus, provides an alternative approach to multivariate GARCH models. Some recent papers focus on the multivariate skewed distributions, and in particular on the skewed Student-t distribution (Sahu, Dey, and Branco, 2001, and Bauwens and Laurent, 2002). Copulas, however, offer the advantage of allowing more flexible marginal dynamics and they also allow to model the dependence parameter rather easily.

Third, we apply our framework to investigate how, subsequent to joint realizations of stock returns, dependency varies. An abundant literature has addressed the issue how

[1] Higher moments refer to the standardized third and fourth central moments.

[2] See Joe (1997) and Nelsen (1999) at textbook level.

correlation between stock-market returns varies when markets become agitated. A first strand of the literature focuses on the equality of linear correlation coefficients computed over periods before and after a crash. For instance, Kaplanis (1988) and Ratner (1992) cannot reject the assumption of the constancy of the correlation matrix. In contrast, Koch and Koch (1991) find that correlation increases through time, while King, Sentana, and Wadhwani (1994) object that this result is due to the 1987 stock-market crash. However, Boyer, Gibson, and Loretan (1997) as well as Forbes and Rigobon (1999) have shown that the correlation coefficient between two series is biased, when it is computed conditionally on one of the series exceeding a threshold. Consequently, studies in which correlation is computed on a subsample where one of the series exceeds a given level will find an artificially high correlation.

On the other hand, in a multivariate GARCH framework, Hamao, Masulis, and Ng (1990), Susmel and Engle (1994), and Bekaert and Harvey (1995) measure the interdependence of returns and volatilities across stock markets. More specifically, Longin and Solnik (1995) test the hypothesis of a constant conditional correlation between a large number of stock markets. They find that correlation generally increases in periods of high volatility of the U.S. market. In addition, in such a context, tests of a constant correlation have been proposed by Bera and Kim (1996) and Tse (2000). Recent contributions by Kroner and Ng (1998) as well as Engle and Shepard (2001) develop GARCH models capable of estimating and testing hypotheses of time-varying covariance matrices. Ang and Chen (2002) document that dependency between U.S. stocks and the aggregate U.S. market increases more during downside movements than during upside movements

As an alternative approach, Ramchand and Susmel (1998) and Ang and Bekaert (1999) estimate a multivariate Markov-switching model and test the hypothesis of a constant international conditional correlation between stock markets. They obtain that correlation is generally higher in the high-volatility regime than in the low-volatility regime. These papers, however, assume a joint normal distribution. Chesnay and Jondeau (2001) also test for a constant correlation between stock returns in a Markov-switching context, but while allowing for non-Gaussian innovations.

Some papers also consider how correlation varies when stock-market indices are simultaneously affected by very large (positive or negative) fluctuations. Engle and Manganelli (1999) focus on the modelling of large realizations using quantile regressions. Longin and Solnik (2001), using extreme value theory, find that dependency increases more during downside movements than during upside movements. Poon, Rockinger, and Tawn (2000) provide an alternative statistical framework to test conditional dependency between extreme returns

Our research is strongly related to the persistence in the dependency between two series. A few studies have proposed to model the correlation coefficient in a way similar to the GARCH model for volatility. See, for instance, Kroner and Ng (1998), Engle (2002), Engle and Sheppard (2001), Tse and Tsui (2002).

In the empirical part of this paper, we show that dependency between daily index returns of major stock markets varies after movements in returns. More specifically, dependency increases subsequently to a large joint stock-market movement. We also provide evidence that, for European countries, dependency increases over time. For the other markets, there is some evidence that dependency was maximal somewhere during the mid-80s beginning 90s.

In the next section, we introduce copula functions and describe the three copula functions used in the empirical application: Plackett's (1965), the Gaussian, and the Student-t copulas. In section 3, we first introduce our univariate model which allows volatility, skewness, and kurtosis to vary over time. Then, we show how to link the univariate models using a copula function. In section 4, we describe the data and discuss our results. Section 5 contains a conclusion and some guidelines for further research. Our model is very general and the idea of capturing conditional dependency within the proposed framework can be applied to many situations.

2 Copula distribution functions

2.1 Generalities

As mentioned by Nelsen (1999, p. 1), the study of copulas is quite a recent phenomenon in statistics. Hence, it is not astonishing that copulas have only recently found their way into empirical finance. In order to understand the usefulness of copulas, consider two random variables X and Y with marginal distributions, or *margins*, $F(x) = \Pr[X \leq x]$ and $G(y) = \Pr[Y \leq y]$. In this paper, we assume that the cumulative distribution functions (cdf) are continuous. The random variables may also have joint distribution function, $H(x, y) = \Pr[X \leq x, Y \leq y]$. All the distribution functions, $F(\cdot)$, $G(\cdot)$, and $H(\cdot, \cdot)$ have as range the interval $[0, 1]$. In some cases, a multivariate distribution exists, so that the function $H(\cdot, \cdot)$ has an explicit expression. One such case is the multivariate normal distribution. In many cases, however, a description of the margins $F(\cdot)$ and $G(\cdot)$ is relatively easy to obtain, whereas an explicit expression of the joint distribution $H(\cdot, \cdot)$ may be difficult to obtain. This is where copulas are useful since they link margins into a multivariate distribution function.

We now define copulas more formally and describe how to construct the Plackett's copula, which is useful for many applications in finance. We would like to emphasize from the onset that many results developed in this paper extend to a higher dimensional framework. Some of the results, however, hold in the bivariate framework only. In particular, the ease of interpretation of the Plackett's copula does not hold if there are more than two margins.[3]

Definition 1 *A two-dimensional copula is a function* $C : [0, 1]^2 \to [0, 1]$ *with the three following properties:*

1. $C(u, v)$ *is increasing in* u *and* v.
2. $C(0, v) = C(u, 0) = 0$, $C(1, v) = v$, $C(u, 1) = u$.

[3] The following definition and proposition may be found in Nelsen (1999, p. 8).

3. $\forall u_1, u_2, v_1, v_2$ *in* $[0,1]$ *such that* $u_1 < u_2$ *and* $v_1 < v_2$*, we have* $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$.

Point 1 states that, when one marginal distribution is constant, the joint probability will increase provided that the other marginal distribution increases. Point 2 states that if one margin has zero probability to occur then it must be the same for the joint occurrence. Also, if on the contrary, one margin is certain to occur, then the probability of a joint occurrence is determined by the remaining margin probability. Property 3 indicates that, if u and v both increase, then the joint probability also increases. This property is, therefore, a multivariate extension of the condition that a cdf is increasing. Another important property of the copula function is that, since the margins are the cdf F and G, it is defined over variables uniformly distributed over $[0, 1]$.

Furthermore, if we set $u = F(x)$ and $v = G(y)$, then $C(F(x), G(y))$ yields a description of the joint distribution of X and Y. Having obtained this intuitive definition, we can now propose the two following properties.

Proposition 2 *If* u *and* v *are independent, then* $C(u, v) = uv$.

Proof. The proof of this property follows immediately from the definition of independent random variables. ■

Proposition 3 *(Sklar's Theorem for continuous distributions). Let* H *be a joint distribution function with margins* F *and* G*. Then, there exists a copula* C *such that, for all real numbers* x, y*, one has the equality*

$$H(x, y) = C(F(x), G(y)). \tag{1}$$

Furthermore, if F *and* G *are continuous, then* C *is unique. Conversely, if* F *and* G *are distributions, then the function* H *defined by equation (1) is a joint distribution function with margins* F *and* G*.*

Proof. The proof of this theorem first appeared in Sklar (1959). A relatively simple proof may be found in Schweizer and Sklar (1974). ■

This theorem justifies the importance of copulas for empirical research. In this work, we use the "conversely" part of the proposition and construct a multivariate density from marginal ones. Now, we show how to obtain a copula that is relevant for finance.

2.2 Construction of the estimated copula functions

An abundant taxonomy of copula functions has emerged in the literature, in order to fit most situations that can be encountered in practice. In our case, we do not want to make hypotheses on the dependence between variables. In particular, it is necessary to express a positive and negative dependence between variables. Therefore, we construct a copula which allows marginal distributions to be either positively or negatively dependent. Another important issue concerns the dependency of the copula in the tails of the distribution.

We focus on how the dependency of international markets varies after some joint realizations. Yet, we do not want to put particular emphasis on extreme events. Such an issue has been already addressed, using an alternative methodology, by Longin and Solnik (2001). Therefore, we consider copula functions which have different characteristics in terms of tail dependence. The Plackett's and Gaussian copulas do not have tail dependence, while the Student-t copula has such a tail dependence (see, for instance, Embrechts, Lindskog, and McNeil, 2001). In addition, graphical evidence indicates that the Gaussian copula displays more dependence for large joint realizations than the Plackett's one. Finally, note that the three copula functions are symmetric. Therefore, when the dependency parameter is assumed to be constant, large joint positive realizations have the same probability of occurrence than large joint negative realizations. In section 3.4, we relax this assumption by allowing the dependency parameter to be conditional on past realizations. We begin with a brief description of how the Plackett's copula is constructed.[4]

Consider Figure 1, where we assume that we have two random variables X and Y. Both variables may take two discrete states, say high and low. As indicated in the figure, we associate probabilities a, b, c, and d to the various simultaneous realizations. Intuitively, if the probabilities are high along the 45^o diagonal, then we would have a positive dependence between the two random variables. Indeed, if one state is high, the other state will be high as well. In contrast, if there are as many observations along the (a, b) diagonal as along the (c, d) diagonal, then the random variables may be considered independent.

These observations suggest to define $\theta = ab/cd$ as a natural measure of dependency. If $\theta = 1$, there will be independence; if $\theta < 1$, dependence will be negative; and if $\theta > 1$, dependence will be positive. Plackett (1965) then associated with the states 'Low' the marginal cdf $F(x)$ and $G(y)$ in $[0, 1]$. Assuming that θ does not depend on x and y yields the following expression for the joint cdf of X and Y

$$C_\theta(u,v) = \begin{cases} \frac{1}{2(\theta-1)}\left[1+(\theta-1)(u+v)-\sqrt{[1+(\theta-1)(u+v)]^2-4uv\theta(\theta-1)}\right] & \text{if } \theta \neq 1, \\ uv & \text{if } \theta = 1, \end{cases}$$

defined for $\theta > 0$. It is easy to establish the density of a Plackett's copula as

$$c_\theta(u,v) \equiv \frac{\partial^2 C_\theta(u,v)}{\partial u \partial v} = \frac{\theta[1+(u-2uv+v)(\theta-1)]}{\left([1+(\theta-1)(u+v)]^2-4uv\theta(\theta-1)\right)^{\frac{3}{2}}}.$$

It is worth noticing that θ is only defined for positive values. In numerical applications, this restriction is easily implemented by using a logarithmic transform of θ. In this case, independency corresponds to a value of $\ln(\theta) = 0$. When $\ln(\theta)$ is positive (negative), we have positive (negative) dependency.

The Gaussian copula is defined by the following cdf and density

$$C_\rho(u,v) = \Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$$

and

$$c_\rho(u,v) = \frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\psi'\left(\Omega^{-1}-I_2\right)\psi\right)$$

[4]We follow the derivation of Nelsen (1999, p. 79–89).

where $\psi = \left(\Phi^{-1}(u), \Phi^{-1}(v)\right)'$ and Ω is the $(2,2)$ correlation matrix with ρ as correlation between u and v. Φ_ρ is the bivariate standardized Gaussian cdf with correlation ρ.

Similarly, the Student-t copula is defined by

$$C_\rho(u,v) = T_{\rho,n}\left(t_n^{-1}(u), t_n^{-1}(v)\right)$$

and

$$c_\rho(u,v) = \frac{1}{\sqrt{1-\rho^2}} \frac{\Gamma\left(\frac{n+2}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\left(\Gamma\left(\frac{n+1}{2}\right)\right)^2} \frac{\left(1+\frac{1}{n}\psi'\Omega^{-1}\psi\right)^{-\frac{n+2}{2}}}{\prod_{i=1}^{2}\left(1+\frac{1}{n}\psi_i^2\right)^{-\frac{n+1}{2}}}$$

where $\psi = \left(t_n^{-1}(u), t_n^{-1}(v)\right)'$. $T_{\rho,n}$ is the bivariate Student-t cdf with n degrees of freedom and correlation ρ.

In Figure 2, we display examples of contour plots associated with the density of the Plackett's, Gaussian, and Student-t copula functions for the case of positive dependency ($\rho = 0.5$) and of corresponding negative dependency ($\rho = -0.5$).

2.3 Conditional dependency

In practical bivariate situations, we observe a sample (x_t, y_t), $t = 1, \cdots, T$. It is assumed that x_t gets generated by a continuous marginal distribution $F(\cdot, w_x)$, where w_x represents a vector of parameters. Similarly, y_t is generated by a continuous distribution, $G(\cdot, w_y)$, where w_y is a parameter vector. For instance, F could represent the cdf of a residual, x_t, of a GARCH model.

For notational convenience, we set $u_t \equiv F(x_t, w_x)$ and $v_t \equiv G(y_t, w_y)$. We denote by γ the dependency parameter. This is θ in the case of the Plackett's copula, and ρ in the cases of the Gaussian and Student-t copulas. The key observation of this research is that the copula depends on an explicit parameter γ that can be easily conditioned. We define γ_t as the value taken by the dependency parameter at time t. The conditioning can, thus, be achieved by expressing γ_t as a function of explanatory variables, for instance lagged values of u_t and v_t, or some other predetermined variable z_t and even time, t, itself. The most general specification for γ_t is

$$\gamma_t = \Gamma(\underline{u}_{t-1}, \underline{v}_{t-1}, \underline{z}_{t-1}, \underline{\gamma}_{t-1}; w_\gamma),$$

where Γ is a function depending on the parameter vector w_γ and $\underline{u}_{t-1}$ denotes $\{u_{t-1}, u_{t-2}, ...\}$.[5]

Note that various measures of dependence can be easily computed in terms of the copula. In particular, as shown by Schweizer and Wolff (1981), the Kendall's tau,

$$\tau = 4 \int\int_{[0,1]^2} C(u,v)\, \mathrm{d}C(u,v) - 1.$$

In addition, the Spearman's rho, which corresponds to the correlation coefficient between margins, is shown to be equal to

$$\rho = 12 \int\int_{[0,1]^2} u\, v\, \mathrm{d}C(u,v) - 3.$$

[5] By using a Taylor-series expansion, it is also possible to introduce a large degree of non-linearity.

Nelsen (1999) has shown that Kendall's tau and Spearman's rho satisfy conditions required to be concordance measures. In contrast, the Pearson's correlation coefficient cannot be used, in general, to measure dependence.

Note that the Spearman's ρ of the Plackett's copula is simply derived from the dependence parameter θ as

$$\rho_\theta = \frac{\theta+1}{\theta-1} - \frac{2\theta}{(\theta-1)^2 \ln(\theta)}. \tag{2}$$

This relationship is used in section 3.4 to derive an alternative dynamics for the dependency parameter.

2.4 Estimation of the model

In this section, we assume that $\gamma_t = \Gamma(u_{t-1}, v_{t-1}; w_\gamma)$. By, writing f and g as the marginal densities, the joint density of an observation (x_t, y_t) is

$$l(x_t, y_t; w_x, w_y, w_\gamma) = c_{\Theta(F(x_{t-1},w_x),G(y_{t-1},w_y);w_\gamma)}(F(x_t, w_x), G(y_t, w_y))\, f(x_t, w_x)\; g(y_t, w_y).$$

As a consequence, the log-likelihood of a sample becomes

$$\begin{aligned} L(w_x, w_y, w_\gamma) &= \sum_{t=1}^{T} \left(\ln\left[c_{\Gamma(F(x_{t-1},w_x),G(y_{t-1},w_y);w_\gamma)}(F(x_t, w_x), G(y_t, w_y))\right]\right. \\ &\quad \left. + \ln\left[f(x_t, w_x)\right] + \ln\left[g(y_t, w_y)\right]\right). \end{aligned} \tag{3}$$

Ideally, one would like to maximize the likelihood simultaneously over all the parameters, yielding the parameter estimates written as $\hat{w}_x$, $\hat{w}_y$, and $\hat{w}_\gamma$. In practical applications, this estimation may be difficult. First, the dimension of the problem can be large. In such a case, it may be necessary to help the estimation by providing starting values obtained from the marginal estimations

$$\tilde{w}_x \in \operatorname{argmax} \sum_{t=1}^{T} \ln[f(x_t, w_x)], \tag{4}$$

$$\tilde{w}_y \in \operatorname{argmax} \sum_{t=1}^{T} \ln[g(y_t, w_y)]. \tag{5}$$

Second, the dependency parameter of the copula function may be a convoluted expression of the parameters. In such a case, an analytical expression of the gradient of the likelihood might not exist. Therefore, only numerical gradients may be computable, implying a slowing down of the numerical procedure.

For complicated situations, it is therefore recommended to use the set $(\tilde{w}_x, \tilde{w}_y)$ obtained by estimating in a first step (4) and (5) before solving for

$$\tilde{w}_\gamma \in \operatorname{argmax}\; L((x_t, y_t), t = 1, \cdots, T; \tilde{w}_x, \tilde{w}_y, w_\gamma).$$

Patton (2001) shows in his Ph.D. thesis that this two step estimation yields asymptotically efficient estimates.

3 A model for the marginal distributions

Our margin model builds on Hansen (1994).[6] It is well known that the residuals obtained for a GARCH model are generally non-normal. This observation has led to the introduction of fat-tailed distributions for innovations. Nelson (1991) considers the generalized error distribution, while Bollerslev and Wooldridge (1992) focus on Student-t innovations.[7] Engle and Gonzalez-Rivera (1991) model residuals non-parametrically. Even though these contributions recognize the fact that errors have fat tails, they generally do not render higher moments time-varying, i.e. the parameters of the error distribution are assumed to be constant over time.

3.1 Hansen's skewed Student-t distribution

Hansen (1994) is the first to propose a GARCH model, in which the first four moments are conditional and, therefore, time-varying. He achieves this by introducing a generalization of the Student-t distribution that allows the distribution to be asymmetric while maintaining the assumption of a zero mean and unit variance. The conditioning is obtained by defining parameters as functions of past realizations. Some extensions to this seminal contribution may be found in Theodossiou (1998) and Jondeau and Rockinger (2002a).[8]

Hansen's skewed Student-t distribution is defined by

$$d(z|\eta,\lambda)=\begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } z<-a/b, \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } z\geq -a/b \end{cases} \tag{6}$$

where

$$a\equiv 4\,\lambda\, c\frac{\eta-2}{\eta-1},\qquad b^2\equiv 1+3\lambda^2-a^2,\qquad c\equiv\frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi\,(\eta-2)}\,\Gamma\left(\frac{\eta}{2}\right)}.$$

If a random variable Z has the density $d(z|\eta,\lambda)$, we will write $Z\sim ST(z|\eta,\lambda)$. Inspection of the various formulas reveals that this density is defined for $2<\eta<\infty$ and $-1<\lambda<1$. Furthermore, it encompasses a large set of conventional densities. For instance, if $\lambda=0$, Hansen's distribution is reduced to the traditional Student-t distribution, which is not skewed. If, in addition, $\eta=\infty$, the Student-t distribution collapses to the normal density.

[6] The literature concerning GARCH models is huge. Several reviews of the literature are available, e.g., Bollerslev, Chou, and Kroner (1992), as well as Bollerslev, Engle, and Nelson (1994).

[7] For a definition of the traditional Student-t distribution, see, for instance, Mood, Graybill, and Boes (1982).

[8] Harvey and Siddique (1999) have proposed an alternative specification, based on a non-central Student-t distribution, which allows higher moments to vary over time. This distribution is designed so that skewness depends on the non-centrality parameter and the degree-of-freedom parameter. However, differences between the two models are noteworthy. On one hand, Hansen's distribution has a zero mean and unit variance, and the two parameters controlling asymmetry and fat-tailedness are allowed to vary over time. On the other hand, in Harvey and Siddique, innovations are non-standardized, skewness is directly rendered conditional and is therefore time-varying, while kurtosis is not modeled. Note also that the specification of the skewed Student-t distribution adopted by Bauwens and Laurent (2002) corresponds to the distribution proposed by Hansen, in which asymmetry is differently parameterized.

It is well known that a traditional Student-t distribution with η degrees of freedom allows for the existence of all moments up to the ηth. Therefore, given the restriction $\eta > 2$, Hansen's skewed Student-t distribution is well defined and its second moment exists. The higher moments are not directly given by the parameter η, although formulas exist for these moments.

Proposition 4 *If $Z \sim ST(z|\eta, \lambda)$, then Z has zero mean and unit variance.*

Proof. See Hansen (1994). ■

Proposition 5 *Introduce $m_2 = 1 + 3\lambda^2$, $m_3 = 16c\,\lambda(1+\lambda^2)(\eta-2)^2/[(\eta-1)(\eta-3)]$, defined if $\eta > 3$, and $m_4 = 3(\eta-2)(1+10\lambda^2+5\lambda^4)/(\eta-4)$, defined if $\eta > 4$. The higher moments of Z are given by:*

$$E[Z^3] = [m_3 - 3a\ m_2 + 2a^3]/b^3, \tag{7}$$

$$E[Z^4] = [m_4 - 4a\ m_3 + 6a^2\, m_2 - 3a^4]/b^4. \tag{8}$$

Proof. See Jondeau and Rockinger (2002a). ■

Since Z has zero mean and unit variance, we obtain that skewness (Sk) and kurtosis (Ku) are directly related to the third and fourth moments: $\mathrm{Sk}[Z] = \mathrm{E}[Z^3]$ and $\mathrm{Ku}[Z] = \mathrm{E}[Z^4]$.

We emphasize that the density and the various moments do not exist for all parameters. Given the way asymmetry is introduced, we must have $-1 < \lambda < 1$. As already mentioned, the distribution is meaningful only if $\eta > 2$. Furthermore, careful scrutiny of the algebra yielding equation (7) shows that skewness exists if $\eta > 3$. Last, kurtosis in equation (8) is well defined if $\eta > 4$.[9]

In the continuous-time finance literature, asset prices are often assumed to follow a Brownian motion combined with jumps. This translates into returns data with occasionally very large realizations. Our model captures such instances since, if η is small, e.g. close to 2, not even skewness exists.

3.2 The cdf of the skewed Student-t distribution

The copula involves marginal cumulative distributions rather than densities. For this reason, we now derive the cumulative distribution function (cdf) of Hansen's skewed Student-t distribution. To do so, we recall that the conventional Student-t distribution is defined by

$$f(x) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})} \frac{1}{\sqrt{\pi\, n}} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

where n is the degree-of-freedom parameter. Numerical evaluation of the cdf of the conventional Student-t is well known and procedures are provided in most software packages. We write the cdf of a Student-t with n degrees of freedom as

$$A(t; n) = \int_{-\infty}^{t} f(x)\, \mathrm{d}x.$$

[9] In the empirical application, we only impose that $\eta > 2$ and let the data decide for itself if, for a given time period, a specific moment exists or not.

The following proposition presents the cdf of the skewed Student-t distribution.

Proposition 6 *Defining $GT(t) = Pr[Z < t]$, where Z follows the density (6), yields*

$$GT(t) = \begin{cases} (1-\lambda)A\left(\frac{bt+a}{1-\lambda}\sqrt{\frac{\eta}{\eta-2}}, \eta\right) & \text{if } t < -a/b \\ \\ (1+\lambda)A\left(\frac{bt+a}{1+\lambda}\sqrt{\frac{\eta}{\eta-2}}, \eta\right) - \lambda & \text{if } t \geq -a/b. \end{cases}$$

Proof. Set $w/\sqrt{\eta} = (bz + a)/[(1-\lambda)\sqrt{\eta - 2}]$. The result follows then from the change of variable in equation (6) of z into w. ■

3.3 A GARCH model allowing for conditional skewness and kurtosis

Let r_t, for $t = 1, \cdots, T$, be the returns of a given series. Hansen's margin model, that allows volatility, skewness, and kurtosis to vary over time is defined by

$$r_t = \mu_t + z_t, \qquad z_t = \sigma_t \varepsilon_t, \tag{9}$$

$$\sigma_t^2 = a_0 + b_0^+ (z_{t-1}^+)^2 + b_0^- (z_{t-1}^-)^2 + c_0\, \sigma_{t-1}^2, \tag{10}$$

$$\varepsilon_t \sim GT(\varepsilon_t | \eta_t, \lambda_t). \tag{11}$$

Equation (9) decomposes the return of time t into a conditional mean, μ_t, and an innovation, z_t. The conditional mean can be modelled as involving past returns and day of the week dummies. Equation (9) then defines this innovation as the product between conditional volatility, σ_t, and a residual, ε_t. The next equation (10) determines the dynamics of volatility. We use the notation $z^+ = \max(z, 0)$ and $z^- = \max(-z, 0)$. Such a specification has been suggested by Glosten, Jagannathan, and Runkle (1993) and by Zakoïan (1994). In a similar spirit, one may mention Campbell and Hentschel (1992), Gourieroux and Monfort (1992), or Engle and Ng (1993). Equation (11) specifies that residuals follow a skewed Student-t distribution with time-varying parameters η_t and λ_t.

Many specifications could be used for η_t and λ_t. To ensure that η_t and λ_t remain within their authorized range, we consider an unrestricted dynamic that we constrain via a logistic map.[10] A discussion what type of functional specification should be retained is provided by Jondeau and Rockinger (2002a). The general unrestricted model that we estimate is given by

$$\tilde{\eta}_t = a_1 + b_1^+ \, z_{t-1}^+ + b_1^- \, z_{t-1}^- + c_1 \, \tilde{\eta}_{t-1}, \tag{12}$$

$$\tilde{\lambda}_t = a_2 + b_2^+ \, z_{t-1}^+ + b_2^- \, z_{t-1}^- + c_2 \, \tilde{\lambda}_{t-1}. \tag{13}$$

Various restrictions of these specifications will also be considered in the empirical section of the paper.

[10] The logistic map, $g_{]L,U[}(x) = L + (U - L)(1 + e^{-x})^{-1}$ maps $\mathcal{R}$ into the interval $]L, U[$. For practical purposes, we use for η the constaints $L = 2$, $U = 30$ and for λ we use $L = -1, U = 1$.

3.4 Restrictions of the general model

This general model encompasses several models. A model obtained under the assumption of a constant volatility could be obtained as in

$$\begin{aligned} r_t &= \mu_t + \sigma\varepsilon_t, \quad \varepsilon_t \sim ST(\varepsilon_t|\eta = \infty, \lambda = 0), \qquad \text{Normality}, & (14)\\ r_t &= \mu_t + \sigma\varepsilon_t, \quad \varepsilon_t \sim ST(\varepsilon_t|\eta, \lambda = 0), \qquad \text{Student-t}, & (15)\\ r_t &= \mu_t + \sigma\varepsilon_t, \quad \varepsilon_t \sim ST(\varepsilon_t|\eta, \lambda), \qquad \text{Skewed Student-t}. & (16)\end{aligned}$$

In the conditional mean, we include 10 lags of r_t and day-of-the-week dummies. If the model is well specified, we would expect that

$$u_t = ST\left(\frac{r_t - \mu_t}{\sigma}|\eta, \lambda\right)$$

is uniform iid. This assumption may be tested using the methodology developed by Diebold, Gunther, and Tay (1998). An extension to the general model, allowing for a skewed Student-t distribution with time-varying parameters may be tested in the same way by replacing the (η, λ) pair by (η_t, λ_t) and also by allowing for a time-varying volatility equation. In that case we obtain the model

$$r_t = \mu_t + \sigma_t\varepsilon_t, \quad \varepsilon_t \sim ST\left(\varepsilon_t|\eta_t, \lambda_t\right) \qquad \text{Time-varying skewed Student-t}. \tag{17}$$

3.5 Alternative specifications for the conditional dependency parameter

As suggested above, the dependency parameter γ may be rendered time-varying. Several studies have shown the correlation parameter to vary over time (Longin and Solnik, 1995, Ramchand and Susmel, 1996) in models where the dynamic of returns is simpler than ours. Many different specifications of the dependency parameter are possible in our context. As a first approach, we follow Gourieroux and Monfort (1992) and adopt a specification in which γ_t depends on the position of past joint realizations in the unit square. This means that we decompose the unit square of joint past realizations into a grid. The parameter γ_t will be constant for each element of the grid. More precisely, our basic model is

$$\ln(\gamma_t) = \sum_{j=1}^{16} d_j \, I[(u_{t-1}, v_{t-1}) \in \mathcal{A}_j], \tag{18}$$

where $\mathcal{A}_j$ is the jth element of the unit-square grid. To each parameter d_j, an area $\mathcal{A}_j$ is associated.[11] For instance, $\mathcal{A}_1 = [0, p_1[\times[0, q_1[$ and $\mathcal{A}_2 = [p_1, p_2[\times[0, q_1[$.[12] The choice of 16

[11] Figure 6 illustrates the position of the areas d_j. How the figure is constructed is discussed in detail below.

[12] In the figures, we have set equally spaced threshold levels, i.e. p_1, p_2, and p_3 take the values 0.25, 0.5, and 0.75. The same for q_1, q_2, and q_3. In the empirical part of the paper, we will use as thresholds the values 0.15, 0.5, and 0.85. The reason for this choice is that we want to focus on rather extreme values. If we had used 0.25, 0.5, and 0.75, the results would have been rather similar.

subintervals is somewhat arbitrary. This choice of parameterization has the advantage to provide an easy testing of several conjectures concerning the impact of past joint returns on subsequent dependency while still allowing for a large number of observations per area. In the empirical section, we test several specification hypotheses for the dependency parameter.

It should be recognized that this specification does not allow the measurement of persistence in γ. The difficulty is to derive an adequate model to capture the dynamic of the dependency parameter. We adopt an approach close to the one proposed by Tse and Tsui (2000) in their modelling of the Pearson's correlation in a GARCH context. We assume that the dynamic of the Spearman's rho is given by

$$\rho_t = (1 - \alpha - \beta)\, \rho + \alpha\, \psi_{t-1} + \beta\, \rho_{t-1} \tag{19}$$

where $\psi_t = \left(\sum_{i=0}^{m-1} u_{t-i} v_{t-i}\right) / \left(\sum_{i=0}^{m-1} u_{t-i}^2 \sum_{i=0}^{m-1} v_{t-i}^2\right)^{1/2}$ represents the correlation between the margins over the recent period. We impose that $0 \leq \alpha, \beta \leq 1$ and $\alpha + \beta \leq 1$. In the empirical application, we set $m = 5$, so that the correlation is computed over one week of data. For Plackett's copula, once a time series of ρ_t is obtained, it is possible to compute the dependency parameter θ_t by solving equation (2) numerically. Therefore, this approach provides an alternative to model dependency while focusing on persistence.[13] The null hypothesis $\alpha = \beta = 0$ can be tested using a standard Wald statistic.

4 Empirical Results

4.1 The data

We investigate the interactions between five major stock indices. The labels are SP for the S&P 500, NIK for the Nikkei stock index, FTSE for the Financial Times stock index, DAX for the Deutsche Aktien Index, and CAC for the French Cotation Automatique Continue index. Our sample covers the period from January 1st, 1980 to December 31st, 1999.

All the data are from Datastream, sampled at a daily frequency. To eliminate spurious correlation generated by holidays, we eliminated from the database those observations when a holiday occurred at least for one country. This reduced the sample from 5479 observations to 4578. Note that such an observation would not affect the dependency between stock markets during extreme events. Yet, it would affect the estimation of the return marginal distribution and, subsequently, the estimation of the distribution of the copula. In particular, the estimation of the copula would be distorted to account for the excessive occurrence of null returns in the distribution. To take into account the fact that international markets have different trading hours, we use once lagged U.S. returns. This does not affect the correlation with European markets significantly (because trading times are partially overlapping), but increases the correlation between the S&P and the Nikkei from 0.1 to 0.26. Preliminary estimations also revealed that the crash of October 1987 was of such importance that the dynamics of our model would be very much influenced by

[13] We did not obtain satisfactory results when we combined the square with its 16 areas and persistence.

this event. For the S&P, on that date, the index dropped by -22%. The second largest drop was -9% only. For this reason, we eliminated the data between October 17th and 24th. This reduces the sample by 6 observations to a total of 4572 observations.

Table 1 provides summary statistics on market-index returns. Returns are defined as $100 \times \ln (P_t/P_{t-1})$, where P_t is the value of the index at time t. Statistics are computed after holidays have been removed from the time series. Therefore, the number of observations is the same one for all markets, and the series do not contain days when the market was closed. We begin with the serial dependency of returns. The LM(K) statistic tests whether the squared return is serially correlated up to lag K. This statistic clearly indicates that ARCH effect are likely to be found in all market returns. Also, when considering the Ljung-Box statistic, $QW(K)$, after correction for heteroskedasticity, we obtain that in most cases returns are serially correlated. We obtain clear indication of such autocorrelation for the SP, the FTSE, and the CAC.

Presently, we consider the unconditional moments of the various series. All the standard errors have been computed with the GMM procedure. We notice that for all series, except the Nikkei, that skewness is negative. Moreover, considering excess kurtosis, XKu, we observe a significant parameter for all the series. This indicates that all the series display fatter tails than the Gaussian distribution. The Wald statistics of the joint test of significance of skewness and excess kurtosis corroborates this finding.[14]

Finally, the unconditional correlation matrix indicates that rather large dependency is likely to be found between market returns. The correlation is the smallest between the Nikkei and the CAC, and the largest between the DAX and the CAC.

4.2 Estimation of the marginal model

In a preliminary step, we consider several restrictions of the general model, see (14) to (17), as possible candidates for adjusting the empirical return distribution. Table 2 reports the test of goodness of fit for these distributions. We follow Diebold, Gunther, and Tay (1998), DGT, who suggested that, if the marginal distributions are correctly specified, the margins u_t and v_t should be iid Uniform$(0,1)$. The test is performed in two steps. First, we evaluate whether u_t and v_t are autocorrelated. For this purpose, we examine the autocorrelations of $(u_t - \bar{u})^i$, for $i = 1, ..., 4$.[15] We thus regress $(u_t - \bar{u})^i$ on 20 lags of the variable. The LM test statistic is defined as $(T-20)\,R^2$, where R^2 is the coefficient of determination, and is distributed, under the null, as a χ^2 with 20 degrees of freedom. We find that the LM tests for autocorrelation of margins generally do not reject the null hypothesis of no autocorrelation. In particular, even if the general model is restricted to model (14), i.e. residuals are supposed to be Gaussian, the first four moments are found to be non-autocorrelated for the SP, the Nikkei, and the FTSE. For the DAX, we reject the non-autocorrelation of the first moment, while for the CAC we reject the non-

[14] When the 1987 crash is not removed, the SP distribution is characterized by a very strong asymmetry (with a skewness equal to -2.55) and fat tails (with an excess kurtosis as high as 57). Yet, due to uncertainty around higher-moment point estimates, the Wald test does not reject normality.

[15] Zero correlation is equivalent to independence, only under gaussianity. The correlogram is, therefore, only suggestif of possible independence.

autocorrelation of the second moment. Note that the non-autocorrelation of the moments mainly comes from the introduction of lagged returns in the mean equation.

Second, we test the null that u_t and v_t are Uniform$(0,1)$. For this, we cut the empirical and theoretical distributions into N slices and test whether the two distributions significantly differ on each slice. An advantage of the approach suggested by DGT is that it permits a graphical representation which can be used to identify areas where the theoretical distribution fails to fit the data. The test statistic is distributed as a χ^2 with $N-1$ degrees of freedom. Table 2 reports the test statistic for various distributions as well as the p-value computed with $N-1$ degrees of freedom. We consider the case where $N=20$ bins. We notice that the normal distribution is strongly rejected for all markets, at any significance level. The standard Student-t distribution is not rejected for the SP and the CAC, suggesting that asymmetry may not be a major feature for these indices. When we consider the skewed Student-t distribution, we obtain that it allows to fit the data, except for some numbers of bins, for the DAX. Finally, when skewness and kurtosis are allowed to vary over time, we only reject the null hypothesis that the theoretical distribution provides a good fit of the empirical distribution for the DAX return, and this only marginally, at the 10% level.

Table 3 presents the results of the general model in which asymmetries in the impact of past good and bad news on conditional volatility are allowed and skewness and kurtosis are time varying.[16]

We can summarize our empirical evidence for margins as follows. First, a negative return has a stronger effect on subsequent volatility than a positive return of the same magnitude. This is the well-known leverage effect, documented by Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), as well as Zakoïan (1994).[17]

Second, the impact of extreme returns on the subsequent distribution is measured via λ_t and η_t. The unrestricted dynamics of $\tilde{\lambda}_t$ and $\tilde{\eta}_t$ gets mapped into λ_t and η_t with the logistic map. The estimations show that there is quite a large persistence in the dynamics of the degree-of-freedom parameter η_t. For most markets, we obtain an estimate of the persistence parameter c_1 ranging between 0.4 and 0.65, but the FTSE. The negative sign of b_1^+ suggests that subsequent to large positive realizations, tails thin down. In contrast, we do not obtain significant estimates of b_1^-, although the point estimate is generally positive.

The asymmetric impact of extremes on returns is measured by the dynamics of λ_t. We find that, in general, past positive returns enlarge the right tail while past negative returns enlarge the left tail. The effect of positive returns is slightly larger than the effect of negative returns, although not always significantly. Furthermore, for the FTSE, the DAX, and the CAC, we find persistence in the asymmetry parameter.

Figures 3 and 4 display the evolution of the η_t and λ_t parameters for the SP and the CAC, respectively. As far as the asymmetry parameter, λ_t, is concerned, we recall that λ_t

[16] See Jondeau and Rockinger (2002b) for more details on the estimation method.

[17] These findings cannot be directly compared with Harvey and Siddique (1999). Their model strongly differs from ours in the choice of error distribution and of model specification. They also use data that differs from ours. In our estimations, the introduction of time-varying skewness does not alter the asymmetry of news on volatility, while in Harvey and Siddique (1999), the result depends on the series used.

is constrained to the range -1 and 1. We observe that the distribution of the SP return is characterized by large movements in the asymmetry. The estimate of the parameter λ_t ranges between -0.4 and 0.5 for the SP, while it ranges between -0.3 and 0.25 for the CAC.

4.3 Estimation of the multivariate model

4.3.1 The model with constant dependency parameter...

Now, we present for each stock-index pair the estimates of the multivariate model. Firstly, we present in Figures 5a and 5b scatterplots of the marginal cumulative distribution functions u_t and v_t for the SP-NIK and for the FTSE-CAC respectively. We notice that except for the regions where one margin is large and the other small, the unit square is rather uniformly filled with realizations. From a modeling point of view, these scatterplots suggest that, even if we introduce 16 areas for the conditioning, each one will contain enough observations so as to yield good estimates. In both figures, there is, in addition, a higher concentration in the corners, along the diagonal. This clustering corresponds to the observation that correlation is higher in the tails. Some studies have focused on the strength of correlation in the tails, see Longin and Solnik (2001), or Ang and Chen (2002). These figures corroborate such studies. This is not, however, the scope of this research. We will investigate whether, subsequent to some joint realization, a similar joint realization can be expected. It should be emphasized that these scatterplots are unable to tell anything about temporal dependency. To establish whether a temporal dependency exists, it is necessary to estimate a dynamic model as given by equations (18) or (19).

In Table 4, Panel A, we report several statistics on the estimation of the copula with constant dependency parameter. We compare three copula functions: The Plackett's, the Gaussian, and the Student-t copulas. First, we report the parameter estimates of the copula function: $\ln(\theta)$ and the associated implied Spearman's rho for the Plackett's copula.[18] We also report the Spearman's ρ for the Gaussian copula, for the Student-t copula we report in addition the degree-of-freedom parameter ν. For all market pairs, the estimate of the dependency parameter is found to be positive and significant. For the Plackett's copula, this result is confirmed by the value of the implied Spearman's rho obtained from equation (2). It can be compared with the empirical value of the correlation between margins reported in the last row of the table. The two estimates of the Spearman's rho are very close one to the other, suggesting that the copulas chosen provide a rather good description of the dependency between markets under study.

To provide further insight on the ability of the chosen copulas to fit the data, we report the log-likelihood, the AIC and SIC information criteria. We also present the LRT statistic for the null hypothesis that the degree-of-freedom parameter of the Student-t copula is infinite, so that the Student-t copula reduces to the Gaussian copula. For all market pairs, we obtain that the log-likelihood of the Gaussian copula is larger than the one of the Plackett's copula. Since the two functions have the same number of parameters to be estimated, the Gaussian copula would be selected on the basis of information criteria. As

[18] The standard error of the implied rho is computed with the delta method, using the relation (2).

regards the Student-t copula, comparison with the Gaussian one can be based both on the information criteria and on the LR test. In the two cases, we select the Student-t copula. As will be shown later on, this result is consistent with the finding that the dependency is stronger in the tails of the distribution than in the middle of the distribution.

Since the Gaussian copula is better than the Plackett one, and the Student-t better than the Gaussian one, we focus from now on only on the Student-t copula. Notice that we performed the same estimations as the ones reported with the other copulas under investigation.[19]

4.3.2 ...and the model with conditional dependency parameter

Parameter estimates. We first turn to the discussion of the estimation of the model in which the Spearman's rho ρ is rendered conditional on past realizations. The conditional model (18) is used. Due to the large number of parameters, we do not report the estimates for all market pairs. Instead, we display in Figures 6 and 7 the unit-square with parameter estimates of the various d_j and their standard errors, for the SP-NIK as well as the FTSE-CAC. These two pairs can be viewed as two polar cases. The first pair has a very low dependency parameter ($\rho = 0.25$), while the second one has the largest Spearman's rho ($\rho = 0.49$). Inspection of the figures indicates that the extreme diagonal elements for the FTSE-CAC take the values 0.638 and 0.535 that compare with 0.340 and 0.250 for the SP-NIK. Inspection of the figures, and comparison with the off-diagonal elements shows that, subsequent to dissimilar events, i.e. one market goes up and the other down, the likelihood to find a similar event is small. This observation holds also for most of the country pairs under investigation.

Formal tests of conditional dependency. In Table 5, we report the results of the tests for conditional dependency. The top line reminds the value of the unconditional dependency measure ρ. We next present those parameters which are located along the diagonal, corresponding to the level of the Spearman's rho when lagged realizations of both markets belong to the same quartile. Observation of these estimates reveals that the dependency involving two European indices behaves differently than estimates where one index is either the SP or the Nik. Indeed, for the last three columns of the table, we observe that the estimates, along the diagonal, tend to take higher values than the unconditional ρ presented in the first row. For instance, for the DAX/CAC pair, the unconditional ρ takes the value 0.465 whereas the elements along the diagonal take the values 0.519, 0.576, 0.525, and 0.530. This suggest that subsequent to similar realizations, e.g., both markets crash, in Europe, one may expect further joint large negative realizations. The row labeled $H_{0,1}$ presents a formal test if the elements along the diagonal tend to be larger than the off diagonal ones. The hypothesis tested is $H_{0,1} : d_1 = d_6 = d_{11} = d_{16} = d_{13} = d_9 = d_{14} = d_3 = d_4 = d_8$ versus $d_1 = d_6 = d_{11} = d_{16} > d_{13} = d_9 = d_{14} = d_3 = d_4 = d_8$. This formal test confirms our intuition and reveals also that the SP/NIK and the NIK/DAX pairs display this feature of greater dependency subsequent to similar realizations.

[19] Results are available upon request from the authors.

Next, we consider a test of asymmetry in the persistence of extreme events. Whereas Longin and Solnik (2001) and Ang and Chen (2002) focus on the contemporaneous correlations in the tails, finding that correlation is stronger in downside markets than in upside markets, we question whether the dependence between markets is stronger subsequent to downside markets than subsequent to upside markets. Therefore, we compare the magnitude of parameters d_1 and d_{16}. The value d_1 measures the dependence subsequent to downside markets, while d_{16} measures the dependence subsequent to upside markets. We notice first that, for most pairs, the difference $d_1 - d_{16}$ takes a positive value, meaning that joint downside movements create stronger dependence than corresponding upside movements. A formal test of the null hypothesis, $H_{0,2}$, that $d_1 = d_{16}$ versus $d_1 > d_{16}$, reveals that the difference is significant for four market pairs only, the NIK-FTSE, NIK-DAX, NIK-CAC, and the FTSE-CAC. Therefore, we find that a crash or a boom of a similar magnitude have generally a similar effect on subsequent correlation. In a conditional setting the asymmetry between negative and positive realizations appears to be weakened. These findings extend the results of Longin ad Solnik (1995) Ramchand and Susmel (1996), as well as Chesnay and Jondeau (2001).

4.3.3 The model with time-varying Spearman's rho

The last issue we address in this paper is the persistence of the dependency parameter. Estimations presented above have shown that, in many circumstances, past joint realizations affect the international dependency. We now measure the extent to which the persistence in dependency is likely to attenuate this link. We, thus, estimate relation (19) for the Student-t copula. Results are reported, in Table 6, with $m = 5$ lags in the computation of ψ_t. Notice that the results are not altered when we select $m = 10$ or 20. We notice that the persistence parameter, β, ranges between 0.44 for the NIK/FTSE and 0.99 for the SP/CAC. Persistence is very strong for all market pairs except those involving the NIK and an European index. The effect of the past short-term correlation between margins, measured by α, is in general significantly positive. Inspection of the persistence measure $(\alpha + \beta)$ suggests that persistence in dependency is large between European stock markets, but also between the SP and other markets. In Figure 8, we display the evolution of the parameter ρ_t for the SP-NIK and the FTSE-CAC pairs. These figures suggest that the persistence is much more pronounced for the latter than for the former. We therefore conclude that dependency is not only strongest but is also the most persistent between European markets.

The LR test for the null hypothesis that α and β are jointly equal to zero (so that the model with time-varying Spearman's rho reduces to the model with constant rho) is also reported in the table. The model with constant dependency parameter is rejected for all market pairs, except for the SP-FTSE, the NIK-FTSE, and the NIK-CAC. This confirms our previous result that dependency depends on lagged realizations.

5 Further research topics

In this paper, we developed a framework, based on copulas, with conditional dependency. We have also shown how this model can be used to measure dependency subsequent to certain types of events. Presently, we wish to discuss other fields where our model can be useful. First, this framework may be used to investigate the spillover of large realizations in emerging markets. The volatility spillovers among such markets have been investigated for instance in Bekaert and Harvey (1995) and in Rockinger and Urga (2001). The focus on extreme realizations may provide further insights.

Another application of our model is the conditional asset allocation in a non-gaussian framework. Such a model has been developed by Rubinstein (1973). Kraus and Litzenberger (1976) provide a first empirical test of this model. Barone-Adesi (1985) shows how a model involving higher moments can be obtained with the assumption of a quadratic market model. Ingersoll (1990) treats the topic at textbook level. Harvey and Siddique (2000) provide tests of these models. Further theoretical elements are brought forward by Jurczenko and Maillet (2001). To implement asset allocation, in a non-gaussian world, it is necessary to compute expressions involving higher moments. Such expressions will typically involve computations such as

$$m_{i,j,t} = \int_{x_t \in \mathcal{R}} \int_{y_t \in \mathcal{R}} x_t^i y_t^j \, c_{\theta_t}(F(x_t, w_x), G(y_t, w_y)) f(x_t, w_x) \, g(y_t, w_y) \, dx_t \, dy_t.$$

Such integrals may be efficiently evaluated using a change in variables $u_t = F(x_t, w_x)$, $v_t = G(y_t, w_y)$. With this change, we get

$$m_{i,j,t} = \int_{u_t \in [0,1]} \int_{y_t \in [0,1]} (F^{-1}(u_t))^i (G^{-1}(v_t))^j c_{\theta_t}(u_t, v_t) du_t \, dv_t.$$

Once the model is estimated, these moments can be computed.

Still another application may be found in Value-at-Risk applications. There it is necessary to compute the probability that a portfolio exceeds a given threshold. Again, once the marginal models are known, the exceedance probability may be numerically computed as a simple integration, using the fact that, if the pair (X_t, Y_t) has some joint distribution function $C(F(x_t, w_x), G(y_t, w_y))$, then

$$\Pr[\delta X_t + (1-\delta) Y_t > \gamma] = \int_{\delta x_t + (1-\delta) y_t > \gamma} dC_{\theta_t}(F(x_t, w_x), G(y_t, w_y)).$$

Again, this expression is easy to implement numerically. Similarly, one could compute expected shortfall.

Furthermore, a straightforward extension of our framework could yield a model for the joint distribution of returns, volume, and duration between transactions. For instance, Marsh and Wagner (2000) investigate the return-volume dependence when extreme events occur. For this purpose, one could use a trivariate copula or proceed in successive steps: First, one could model the dependency between volume and duration using a first copula. Then, in a second step, one could link this copula to the return series through another copula. Hence, our model may be adapted to settings where the data of each margin is not of the same nature.

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Captions

Table 1: This table reports summary statistics on market returns. Mean, Std, Sk, and XKu denote the mean, the standard deviation, the skewness, and the excess kurtosis of returns, respectively. Standard errors are computed with GMM. Wald Stat. is the Wald statistic which tests the null hypothesis that skewness and excess kurtosis are jointly equal to zero. It is distributed, under the null, as a χ^2 with 2 degrees of freedom. Min and Max represent the minimum and maximum of centered and reduced returns, while $q1$, $q5$, $q95$, and $q99$ represent the 1, 5, 95, and 99 percentiles. The 1%, 5%, 95%, and 99% percentiles for a normal distribution are -2.3263, -1.6449, 1.6449, and 2.3263. The LM(K) statistic for heteroskedasticity is obtained by regressing squared returns on K lags. QW(K) is the Box-Ljung statistic for serial correlation, corrected for heteroskedasticity, computed with K lags. Since international markets have different trading hours, we have used once lagged U.S. returns to compute the correlation matrix. Significance is denoted by superscripts at the 1% (^{a}), 5% (^{b}), and 10% (^{c}) levels.

Table 2: This table reports goodness-of-fit statistics for several marginal restrictions of the general univariate model. These restrictions are given by models (14) to (17). The first part of each panel contains the LM test statistic for the null of no auto-correlation of moments of the u_t. It is defined as $(T-20)\,R^2$, where R^2 is the coefficient of determination of the regression of $(u_t-\bar{u})^i$ on 20 of its lags, for $i=1,...,4$. Under the null, the statistic is distributed as a χ^2 with 20 degrees of freedom. Following Diebold, Gunther, and Tay (1998), DGT, the table also reports the Kolmogorov-Smirnov test statistic for the test that the cdf of residuals is Uniform$(0,1)$. Under the null, the statistic is distributed as a χ^2 with 20 degrees of freedom. Finally, the table presents the log-likelihood (lnL).

Table 3: This table reports parameter estimates and residuals summary statistics for the model with a skewed Student-t distribution and time-varying higher moments. Parameters are those given by equations (10), (12), and (13). Summary statistics include the LM(K) statistic for heteroskedasticity, obtained by regressing squared returns on K lags, and the QW(K) statistic for serial correlation, corrected for heteroskedasticity, computed with K lags. lnL is the sample log-likelihood of the model.

Table 4: This table reports parameter estimates for the copula functions when the dependency parameter is assumed to be constant over time. Parameters are $\ln(\theta)$ for the Plackett's, ρ for the Gaussian, and ρ and ν for the Student-t copula. We also report the log-likelihood (lnL) as well as the AIC and SIC information criteria (divided by T). For the Student-t copula, LRT1 is the LRT statistic for the null hypothesis that $1/\nu=0$. Finally, empirical ρ is the sample correlation between the margins.

Table 5: This table reports parameter estimates and test statistics for the Student-t copula when the Spearman's rho depends on the position of past joint realizations in the unit square. ρ is the Spearman's rho under constancy. d_1, d_6, d_{11}, and d_{16} correspond to the Spearman's rho when u_t and v_t belong to the same quartile along the diagonal.

Table 6: This table reports parameter estimates for the Student-t copula when the Spearman's rho is allowed to be time varying. Parameters are those given by equations (19). We also report the log-likelihood statistics LRT2 for the null hypothesis that $\alpha = \beta = 0$. It is distributed as a χ^2 with 2 degrees of freedom.

Figure 1: This figure displays a $(2, 2)$ contingency table.

Figure 2: This figure displays contour plots of Plackett's, Gaussian, and Student-t copula functions for the case of positive dependency ($\rho = 0.5$) and of corresponding negative dependency ($\rho = -0.5$). In all instances, the marginal distributions are assumed to be $N(0, 1)$.

Figure 3: This figure displays the evolution of the degree-of-freedom parameter η_t and the asymmetry parameter λ_t for the SP.

Figure 4: This figure displays the evolution of the degree-of-freedom parameter η_t and the asymmetry parameter λ_t for the CAC.

Figure 5: This figure displays scatterplots of the marginal cumulative distribution functions u_t and v_t for the SP-NIK and for the FT-CAC respectively.

Figure 6: This figure displays the unit-square with parameter estimates of the various d_j and their standard errors, for the SP-NIK pair.

Figure 7: This figure displays the unit-square with parameter estimates of the various d_j and their standard errors, for the FTSE-CAC pair.

Figure 8: This figure displays the evolution of the parameter ρ_t for the SP-NIK and the FTSE-CAC pairs as estimated by the model (19).

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The University of Geneva

The University of Geneva, originally known as the Academy of Geneva, was founded in 1559 by Jean Calvin and Theodore de Beze. In 1873, The Academy of Geneva became the University of Geneva with the creation of a medical school. The Faculty of Economic and Social Sciences was created in 1915. The university is now composed of seven faculties of science; medicine; arts; law; economic and social sciences; psychology; education, and theology. It also includes a school of translation and interpretation; an institute of architecture; seven interdisciplinary centers and six associated institutes.

More than 13'000 students, the majority being foreigners, are enrolled in the various programs from the licence to high-level doctorates. A staff of more than 2'500 persons (professors, lecturers and assistants) is dedicated to the transmission and advancement of scientific knowledge through teaching as well as fundamental and applied research. The University of Geneva has been able to preserve the ancient European tradition of an academic community located in the heart of the city. This favors not only interaction between students, but also their integration in the population and in their participation of the particularly rich artistic and cultural life. ***http://www.unige.ch***

The University of Lausanne

Founded as an academy in 1537, the University of Lausanne (UNIL) is a modern institution of higher education and advanced research. Together with the neighboring Federal Polytechnic Institute of Lausanne, it comprises vast facilities and extends its influence beyond the city and the canton into regional, national, and international spheres.

Lausanne is a comprehensive university composed of seven Schools and Faculties: religious studies; law; arts; social and political sciences; business; science and medicine. With its 9'000 students, it is a medium-sized institution able to foster contact between students and professors as well as to encourage interdisciplinary work. The five humanities faculties and the science faculty are situated on the shores of Lake Leman in the Dorigny plains, a magnificent area of forest and fields that may have inspired the landscape depicted in Brueghel the Elder's masterpiece, the Harvesters. The institutes and various centers of the School of Medicine are grouped around the hospitals in the center of Lausanne. The Institute of Biochemistry is located in Epalinges, in the northern hills overlooking the city. ***http://www.unil.ch***

The Graduate Institute of International Studies

The Graduate Institute of International Studies is a teaching and research institution devoted to the study of international relations at the graduate level. It was founded in 1927 by Professor William Rappard to contribute through scholarships to the experience of international co-operation which the establishment of the League of Nations in Geneva represented at that time. The Institute is a self-governing foundation closely connected with, but independent of, the University of Geneva.

The Institute attempts to be both international and pluridisciplinary. The subjects in its curriculum, the composition of its teaching staff and the diversity of origin of its student body, confer upon it its international character. Professors teaching at the Institute come from all regions of the world, and the approximately 650 students arrive from some 60 different countries. Its international character is further emphasized by the use of both English and French as working languages. Its pluralistic approach - which draws upon the methods of economics, history, law, and political science -reflects its aim to provide a broad approach and in-depth understanding of international relations in general. ***http://heiwww.unige.ch***

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UNIVERSITÉ DE LAUSANNE