Optimal International Diversification: Theory and Practice from a Swiss Investor's Perspective

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EXECUTIVE SUMMARY

This study investigates the issue of optimal international portfolio diversification. It considers the major economic regions as asset classes and the analysis is done within the well-known framework of the International Capital Asset Pricing Model (ICAPM). The purpose is to develop a model that is able to forecast in a dynamic way (at each period *t*, for the next period *t*+1):

- the *risk premiums* for the World Market risk and for the currency risks;
- the sensitivities (betas) of the portfolio to these premiums;
- the optimal asset allocation;
- the optimal currency hedge.

All of the above forecasts are made for the next period and tested "out-of-sample" over a ten year period. Our methodology significantly outperforms the MSCI World Index, with a higher average return and a lower standard deviation.

One of the key assumptions of the CAPM (either the domestic CAPM or the international version of the model) is that investors have to be remunerated for the risk they take on any investment. The price of any asset has to be such that the perceived return (that is, the "expected return"), in excess of the risk free rate, is positive and an increasing function of risk. How much return investors require per unit of risk is the *risk premium* ; the level of risk is measured by the beta of the asset or the asset class. For the traditional CAPM this yields the famous relation $E(R_i) - R_f = \beta_i \times [E(R_M) - R_f]$.

It is important to realize that the only reason why investors are willing to take risk is their perception of a positive expected return (in excess of the risk free rate). In an international framework, for instance, many investors do not have strong convictions about future currency movements. In other words, they do not have a positive expected return on any currency. If this is the case, currency movements induce additional risk in the portfolio that is not remunerated by a positive risk premium. Such risk should then be hedged.

This leads us to the concept of the International CAPM, which is used in this study. The first source of risk is the "World Market risk", for which investors anticipate a positive return. The main equity regions considered in this study react more or less to changes in this world market portfolio. This is measured by beta coefficients. The other sources of risk of an international investor are the above-mentioned currency risks. The investor's portfolio has, of course, exposures to these risk sources (measured by currency beta coefficients), and to each source of currency risk there is an associated *currency risk premium*.

In this paper we explore ways to forecast the risk premiums on the one hand, and the exposures of a portfolio to these risk sources on the other hand. This leads to a model that predicts the next period's expected returns (in excess of the risk free rate) on the world market portfolio and on each of the currencies involved. Typically, an investor with a US market exposure should hedge this position only if the anticipated risk premium on the US Dollar is insignificant or negative.

We investigate different models to forecast the risk premiums (for the world market risk and for the currency risks) and the associated beta coefficients. The risk premiums are modeled using "instruments" such as interest rate differentials, but also a technical "trend" indicator, and the beta coefficients are obtained from conditional covariances between regional equity returns and currency returns. We use "regime-switching" models, which consider different covariances (and therefore beta coefficients) and different risk premiums in different "states-of-the-world", also called "regimes". The idea is that in some regimes the correlations between currencies and equity markets are very different. It may model the well-known observation that during market crashes the correlation between assets dramatically increases, taking out much of the needed diversification effect. In each regime we also allow our selected "instruments" to influence differently the risk premium (for instance, an increase in interest rates may have a positive effect on the risk premium in some cases, while the effect may be opposite in other regimes).

We find that both the prices (risk premiums) and the quantities (beta coefficients) of risk are highly variable over time. Assuming them to be constant over time leads to severe misspecifications, and hence mispricing. We show that many of the tested specifications significantly outperform our benchmark, which is the World Market portfolio. This outperformance is due to higher average returns but also to a lower standard deviation of the simulated portfolio. The simulations are based on 10 years of "out-of-sample" data.

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Abstract

This paper reviews some recent developments in the area of optimal international portfolio diversification and investigates important issues for future research. In the latest models proposed in the financial literature that generate optimal holdings over time, both the quantities of risks (measured by the covariances with various risk factors) and the prices of risk (risk premiums) are time varying. The former are generally specified by some ARCH process, whereas the latter are estimated using instruments such as dividend yield or bond premiums. Available methodologies and the choice of the instruments are discussed in general terms, as well as the feasibility of active management with these models. I test a few of them by considering a Swiss investor who holds an internationally diversified portfolio including local stock indices, as well as an exposure to real estate, and who may hedge some or all of his currency risk. The empirical tests are performed using a very intuitive and powerful non-parametric threshold ARCH specification to model time-varying sources of risk. Risk premiums are estimated using simple and widely available instruments in the form of macroeconomic variables, but also indicators used in technical analysis. Both the in-sample and the out-of-sample results suggest that the proposed non-parametric approach is powerful and may constitute a valuable tool for international portfolio managers.

Keywords: Swiss institutional investors, mixed-asset portfolios, conditional asset allocation, hedging currency risk, QTARCH model, international portfolios, foreign exchange forecasting.

JEL Classification: G12.

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1. Introduction

International diversification is, today, one of the major topics in global asset management. The benefits from such diversification seem to be clear: the more assets with low correlation among each other are added to the portfolio, the lower the overall portfolio's variance. In practice, however, the portfolio manager is faced by a number of problems that I believe will be the major challenge both for academics and practitioners in the coming years. The crucial issues in this context and that are discussed here deal with the identification of the risk factors influencing international asset returns on one hand, and with the modelisation of how asset prices react to changes in these factors on the other. In particular, currency risk is a major source of risk for the internationally diversified investor and recent evidence suggests the importance of a joint modelisation of currencies and asset prices movements. Academics have already for a long time argued in favor of an integrated decision process both for currencies and assets, but practitioners often favor a separate one. Typically, a large institution's portfolio will be managed by multiple managers who follow the constraints imposed by the global investment committee. This committee decides on the allocations to each geographical area and to each currency and leaves only very little freedom to the individual manager. Currency managers will then be responsible for the portfolio's currency exposures. Asset managers, in turn, decide on the allocation to various asset classes or individual assets, generally following in-house analysts' recommendations.

The currency exposure is a complex issue. Corporations have individual policies regarding hedging or not of their exposure, and therefore a country's market index is a weighted average of corporations with different exposures. Although it can be argued that in the long run currencies follow a mean-reverting process and that Purchasing Power Parity (PPP) holds, short run deviations appear too important and to last for too long to accept a decision of not hedging currency exposure. Froot (1993), for instance, investigates 200 years of US and UK data on stocks and bonds and concludes that hedging reduces volatility in the short run (5 to 8 years), but increases volatility over longer holding periods. In other words, if the investor is only concerned with long term returns and the volatility of these returns, no hedging should be done. If short term volatility is important (for instance, for the annual performance statistics of the manager), currency risk should be hedged. The US financial analyst's organization AIMR suggests that the performance of international investors should be measured in local terms, fully hedged against currency risk. It considers that deviations from this rule involve active investment decisions that should be evaluated as such.

In practice, currency risk may be hedged in different manners. For example, the mandate of an active currency manager may consist in deciding on how much of the total currency exposure is to be hedged at any point in time. A good manager may generate profits larger than what would have been obtained both in the hedged and in the unhedged case, but a bad one may report substantial losses. Clearly, the success of a few active currency managers should not be seen as evidence that this is the optimal strategy to follow. There is a risk involved, as is the case for any other investment, and therefore the decision should be considered as an active investment decision, as suggested by the AIMR guidelines.

Active currency management should be evaluated as any other investment. The foreign exchange (FX) market is among the most liquid ones, expected to be highly efficient due to the very large number of traders and very low transaction costs. Although there is now increasing evidence, in particular from academic studies, that some technical trading rules may be profitable in some cases, forecasting first and second order moments of currency returns comes with estimation error problems [see, for instance, Gardner and Stone (1995) for a discussion]. Therefore, techniques that are based on these estimates to generate optimal hedging policies are often unstable over time, and unsuitable for out-of-sample forecasting. The "universal hedge ratio" proposed by Black (1989), for instance, involves only the two first order moments of the returns on the world market portfolio and on the currencies, but appears difficult to use in practice. In this study I will consider currencies as asset classes. Specific constraints on the weights of these assets in the portfolio are derived in order to remain in a framework that is acceptable to a typical global asset manager.

While currency exposure is an important aspect of the total allocation process, such a process is based on an asset pricing model. The CAPM, for instance, gives a simple linear relation between the expected return on an asset and the expected return on the "market portfolio". The relation can be seen as the product of the quantity of risk faced when investing in the particular asset (measured by the asset's covariance with the market portfolio) times the price of risk (that depends on the market risk premium). There is now ample academic support for asset pricing models that extend the traditional CAPM in three distinct ways: time-varying quantities of risk (covariances), time-varying prices of risk (risk premiums), and additional sources of risk over the market risk. The objective of this study is to discuss the progress that has been made in these areas and what we can expect from research in the near future. Furthermore, it will present and test some techniques that are believed valuable in these approaches.

The three main parts that follow deal respectively with a review of some of the most interesting and promising studies in the above areas (in Section 2), with the specification of a very general model for return and risk (in Section 3), and finally with an empirical application (in Section 4). The latter is an illustrative framework based on some recent econometric developments. It is applied to main stock indices as well as to an index of real estate funds, which is an asset class believed to play, at least potentially, an important role in the diversification process. Exchange rates are explicitly taken into account, as well as realistic constraints on portfolio weights. Switzerland is taken as country of reference.

2. Asset Pricing Models

2.1. The CAPM

The relation between expected or realized returns and sources of risk is central in all pricing theories. Clearly, higher risk has to be rewarded by higher expected returns, in order to induce investors to hold riskier assets. An asset with a too low expected return given its level of risk will drop in price, thereby increasing its expected return. In the general CAPM, the return-risk relation for any given asset i is written as:

$$E(R_i) - R_F = \beta_i[E(R_M) - R_F], \tag{2.1}$$

where R_F is the return on the risk free asset, $E(R_i)$ is the expected return on asset i, $E(R_M)$ is the expected return on the market portfolio containing all traded assets in the economy, and β_i is the beta coefficient for asset i, defined as:

$$\beta_i = \frac{COV(R_i, R_M)}{VAR(R_M)}. \tag{2.2}$$

The CAPM relation is not an intertemporal relation, but it may be used and tested in an intertemporal framework. We may introduce time indices and write the expected return for asset i at time t as:

$$E(R_{i,t}) - R_{F,t} = \widehat{\beta}_{i,t}[E(R_{M,t}) - R_{F,t}], \tag{2.3}$$
$$\text{with } \widehat{\beta}_{i,t} = \frac{COV_t(R_i, R_M|\phi_{t-1})}{VAR_t(R_M|\phi_{t-1})} \tag{2.4}$$

where $COV_t(R_i, R_M|\phi_{t-1})$ and $VAR_t(R_M|\phi_{t-1})$ are conditional moment estimates for time t given available information (ϕ_{t-1}) up to time $t-1$. The risk free rate $R_{F,t}$ is the return on a risk free zero coupon bond between time $t-1$ and t. It should be noted that equation 2.3 is the "asset pricing restriction", that is, the relation between return and risk that we wish to find. So far, nothing is said about the specification over time for the conditional variances and covariances.

If the strong assumption is made that variances and covariances remain constant over time, the best estimate for the return on any asset i as well as on the market portfolio M is nothing else than the historical mean. Similarly, the best estimate for the variances and covariances (and hence the beta coefficient) is simply the historical matrix of variances-covariances.

Different methodologies have been proposed in the financial literature in order to test the validity of beta as a measure of risk. None of them is without drawbacks, but the approach by Fama and MacBeth (1973) is the one that has been mostly used until today. The idea is to estimate the $\widehat{\beta}_{i,t}$ coefficients over historical data up to date $t-1$, and to investigate the return-risk relation at time t. This is done through a cross-sectional regression of all observed returns $R_{i,t}$ on their respective estimated beta coefficients $\widehat{\beta}_{i,t}$. Formally, we have for each period t:

$$R_{i,t} - R_{F,t} = \widehat{\gamma}_{0,t} + \widehat{\gamma}_{1,t}\widehat{\beta}_{i,t} + \varepsilon_{i,t}, \text{ for a given } t. \tag{2.5}$$

The above regression involves the N available assets in the economy at each time t. In order to validate the CAPM, the coefficient $\widehat{\gamma}_{0,t}$ should not be, on average, different from zero (otherwise $\widehat{\beta}_{i,t}$ is not the only source of risk) and $\widehat{\gamma}_{1,t}$ should be significantly positive (otherwise $\widehat{\beta}_{i,t}$ is not a significant measure of risk). Of course, as noted by Pettengill *et al.* (1997) and others, this positive relation between return and beta is what one expects on average. In expectations, the return on the market portfolio in excess of the risk free rate has to be positive, otherwise no investor would hold the risky market portfolio. In realizations, the market portfolio return is often below the return on the risk free rate and, in such periods, one can expect the return on high beta assets to be lower than returns on low beta assets. If the investor knew in advance the next period's market movement, he would select high beta stocks when an upmove is expected, and low beta stocks when a downmove is likely.

Once the above equation and the $\widehat{\gamma}_{0,t}$ and $\widehat{\gamma}_{1,t}$ coefficients estimated, the window is moved one period forward. Observations up to time t are now used to estimate the beta coefficients and observations at time $t+1$ are used to estimate $\widehat{\gamma}_{0,t+1}$ and $\widehat{\gamma}_{1,t+1}$. The validity of beta coefficients as a measure of an asset's risk is then measured by the significance of the average $\widehat{\gamma}_1$ coefficient.

The results reported in over 25 years of empirical financial studies show little evidence in favor of the CAPM. Most studies find either very weak evidence, or no evidence at all, of the validity of beta as an asset's measure of risk, while other variables, such as price-to-earnings ratios or the firm's size, are generally found to be more significant than beta. In a seminal paper, Fama and French (1992) concluded that the past 50 years of US data were unable to validate the CAPM, thus designating the today most widely used asset pricing model as "dead". Subsequent studies have tried to come up with explanations for this failure, which generally fall within one of the following three categories: a) Data issues: starting with Roll's (1977) critique, it has been argued that the CAPM cannot be tested because the composition of the "true" market portfolio is unknown. Furthermore, the data sets available to the researcher often suffer from biases, such as the survival bias; b) Testing issues: it has been argued that although the CAPM is a mathematically derived model that *has* to be true, the testing methodology is inadequate, but unfortunately there is no better one available; c) Specification related issues: here it is argued that the CAPM is not the right pricing model, or at least that other risk factors than the market risk alone drive returns. Such factors include the firm's size, accounting variables such as price earnings ratios, etc.

The latter issue is important. The CAPM is an equilibrium model mathematically derived from the standard assumptions underlying the Modern Portfolio Theory. Alternative pricing models have been proposed in the literature, in particular multi-factor models (where the APT is derived under specific non-arbitrage assumptions). In the case of a general K-factor model, we have the following relation at each time t:

$$E(R_{i,t}) = \lambda_{0,t} + \sum_{k=1}^{K} b_{i,k}\lambda_{k,t}, \tag{2.6}$$

where the coefficients $b_{i,k}$ are interpreted as the quantity of risk k involved in asset i and $\lambda_{k,t}$ as the price of risk k at time t. The term $\lambda_{0,t}$ is interpreted as the return on the risk free asset or, alternatively [see Black (1972)], as the return on a zero-beta portfolio.

2.2. International Asset Pricing Models

While the reasons for considering multiple risk sources may seem less obvious in a purely domestic economy, they become obvious in an international context, where, in addition to "market risk", the internationally diversified investor is also confronted with currency risks. These models were initially proposed by Solnik (1974), Merton (1980), Sercu (1980), and Adler and Dumas (1983), among others. It can be shown that when the set of available assets is constant and when all foreign investments are fully hedged for currency risk, all investors should hold a combination of the national risk free asset and a common portfolio that includes all available assets, including local risk free assets. This is often referred to as the pseudo-separation theorem.

The pricing models introduced by Sercu (1980) and Adler and Dumas (1983) remain the framework used in most recent studies today. The underlying assumption, necessary for the model to be derived, is that Purchasing Power Parity (PPP) does not hold, which is an assumption for which reasonable empirical evidence can be found. Under this assumption, investors in different countries perceive different rates of return for the same asset, measured in real terms. Therefore, optimal portfolios will differ among countries and it can be shown that, in equilibrium, the expected return on any asset in the economy, denominated in the investor's currency of reference, is equal to the return on the risk free domestic asset plus a series of premiums for exposure to global market risk and specific currency risks. If Switzerland is the country of reference, and CHF hereafter refers to the Swiss Franc, the following asset pricing equation is obtained for each asset in the economy and at each time t:

$$E(R_{i,t}^{CHF}) - R_{F,t}^{CHF} = \gamma_{0,t} COV_t(R_{i,t}^{CHF}, R_{Wrld,t}^{CHF}) + \sum_{k=1}^{K} \gamma_{k,t} COV_t(R_{i,t}^{CHF}, r_{k,t}^{CHF}), \tag{2.7}$$

where $R_{i,t}^{CHF}$, $R_{F,t}^{CHF}$ and $R_{Wrld,t}^{CHF}$ are, respectively, the returns at time t on asset i, on the risk free asset in Switzerland and on the world market portfolio, all measured in Swiss Francs.

The terms $r_{k,t}^{CHF}$ are the returns on the K currencies and the $\gamma_{k,t}$ coefficients measure the price of foreign exchange risk k.

3. A General Portfolio Allocation Model

3.1. Modeling Conditional Risk and Return

The CAPM, as well as its international extension presented above, are asset pricing restrictions, but do not specify how the risk premiums and risk factors are obtained for a given time t. Therefore, these models are not full models that may be used, for instance, for deriving optimal asset holdings. Various possibilities may be considered at this stage. Solnik (1993), for instance, uses unconditional covariances and the dynamics only come from time varying risk premiums. Risk premiums at time t are obtained through a linear combination of "instruments", such as the default premium, the world dividend yield, etc., observed at time $t-1$. A natural specification of the risk factors, through the covariances between returns, is an ARCH class of model. Bollerslev, Engle and Wooldridge (1988) suggest a multivariate CAPM with three asset classes and with constant prices of risk, but where the covariances between assets follow a multivariate ARCH process. The extension of such models to larger numbers of assets is difficult because of the increase in the number of parameters, in particular for the covariance matrices, but also for the risk premium estimates if the latter are based on instruments.

De Santis and Gérard (1997, 1998) have developed an interesting approach seeking to limit the number of parameters to be estimated and allowing larger multivariate specifications. In order to simplify the estimation procedure, the time-varying matrix of variances-covariances between assets is decomposed into a fixed (unconditional) component denoted H_0 and a time-varying (conditional) component denoted H_t:

$$H_t = H_0 * (\iota\iota' - \mathrm{aa}' - \mathrm{bb}') + \mathrm{aa}' * \varepsilon_{t-1}\varepsilon_{t-1}' + \mathrm{bb}' * H_{t-1}, \tag{3.1}$$

where a and b are N-dimensional vectors of parameters to be estimated, and $*$ denotes the Hadamard (element by element) matrix product. While such simplifications are an empirical necessity, they are less attractive from a theoretical and intuitive perspective. On the other hand, time-varying covariances between assets and asset classes is also an empirical evidence. The recent turmoil in financial markets around the world in the summer of 1998 has shown once more that during such periods the correlation between assets increases dramatically. At times when international diversification is most needed (when markets go down), this increase in correlations takes away much of the portfolio diversification effect.

Historical correlations between assets, measured over longer periods, can therefore be expected to be highly influenced by the periods of turmoil. When these historical correlations are used to derive optimal asset holdings, a sub-optimal allocation can be expected during "normal" market conditions when the actual correlation between assets is lower than the historical average correlation. This allocation is probably also sub-optimal in periods of turmoil, where the actual correlations are much higher. It would therefore be natural to consider a world with two regimes at least, one corresponding to normal market conditions, another to periods of turmoil.

Regime and threshold models have been developed, tested and improved since the late eighties [see in particular Hamilton (1990), Tong (1990), Teräsvirta and Anderson (1992), Gouriéroux and Monfort (1992) and Potter (1995)]. The difficulty, however, of implementing threshold models, and one of the reasons for their limited success, is that the partitioning method is mostly arbitrary and at the same time a crucial part of the specification. The threshold model proposed by Gouriéroux and Monfort (1992) is of particular interest because both the mean and the variance are conditional on a "state-of-the-world". Furthermore, the flexibility in the choice of both the partitioning variable and the thresholds turns the model into an ideal framework for representing the economic environment through "states-of-the-world" with very precise economic interpretations. It will be the basis for my empirical application described in the next sections. In this model, the conditional first and second order moments are functions of a partitioning variable Z through a set of thresholds. Let J be the number of partitions of $\mathbb{R}^n$. Each of these partitions is denoted A_j and is associated with a binary characteristic function I_{A_j}, which takes the value 1 if the partitioning variable Z_{t-1} belongs to A_j and 0 otherwise. Formally, the first and second order moments are expressed as:

$$
\begin{aligned}
E(R_t) &= \sum_{j=1}^{J} \alpha_j I_{R_{t-1} \in A_j}, \\
V(R_t) &= \sum_{j=1}^{J} \beta_j^2 I_{R_{t-1} \in A_j},
\end{aligned} \tag{3.2}
$$

where α_j $(j = 1, \ldots, J)$ are N-dimensional vectors and β_j^2 $(j = 1, \ldots, J)$ are symmetric positive definite matrices. The partitioning variable may be simply one of the time-series, or any other "constructed" series. In particular, Z_t may be some economic indicator, such as growth or inflation, but also some technical indicator such as one seeking to identify up or down trending markets. As the partition (also referred to as regime) at time t depends on the lagged value Z_{t-1}, the estimated moments are conditional on a "state-of-the-world" defined by growth or inflation during the previous period. For example, if Z is the GDP growth rate, two thresholds could be set at 0% and +2%. This way three "states-of-the-world" would be obtained. $E(R_t)$ and $V(R_t)$ then depend on whether Z_{t-1}, the GDP growth rate of the previous period, was negative, at a "normal" level (between 0% and +2%), or strongly positive.

The QTARCH methodology seems interesting from various perspectives. To begin with, a model based on a few regimes representing distinct "states-of-the-world" may, *a priori*, be more robust when applied out-of-sample than a model based on a traditional GARCH and with a large number of parameters. Furthermore, the smaller number of parameters to be estimated also allows larger multivariate frameworks. On the negative side, there is no straightforward way of defining the partitions, that is, selecting a partitioning variable and the corresponding thresholds. In the empirical section of this paper I will review a few possibilities that have proven successful in other QTARCH applications and also derive a new partitioning method based on cluster analysis.

3.2. Markowitz's Allocation Framework and Implementing Constraints

While the traditional CAPM has generally been tested out-of-sample using the Fama and MacBeth (1973) methodology, which consists in testing the relation between realized returns and estimated beta coefficients, international asset pricing models are generally evaluated in terms of performance of a simulated trading strategy. The vector of expected asset returns together with the matrix of variances-covariances of the model's residuals can be used in a Markowitz framework to derive optimal portfolio holdings for each point on the efficient frontier in mean-variance space. When the estimates of these two moments change over time, the resulting optimal portfolios will also change their composition over time.

Academics generally apply Markowitz's framework without much focus on additional constraints that may be added in the optimization process minimizing the portfolio variance for a given level of expected return. From a theoretic point of view this is correct, as markets are in equilibrium and all assets are mean-variance efficient: an asset with a lower expected return than any other asset that is at least as risky will drop in price until an equilibrium price is reached. In practice, however, things are very different. The structure of the vector of expected returns and of the variances-covariances, whether based on unconditional historical values or derived through a sophisticated econometric specification, may be such that degenerated solutions are found. Especially when the number of assets becomes large compared with the number of observations over time, a typical Markowitz framework will underestimate the true portfolio's variance. If the number of assets is equal to or larger than the number of observations over time, it will always be possible to find a linear combination of assets that is such that the expected risk is zero, while the expected return is positive.

Typically, a Markowitz framework will never be applied in practice without close monitoring, that is, without the asset manager imposing constraints on the possible outcomes of the model. In addition to a non-negativity constraint on all asset weights, managers will require, for instance, that no asset represents more than a given percentage of the total portfolio's wealth. I argue here that defining precisely all the portfolio manager's constraints is a crucial step in the whole allocation process when sophisticated models are used for estimating conditional returns and variances-covariances. Academic studies failing to take such constraints into account come sometimes forward with very unrealistic allocation suggestions. De Santis, Gérard and Hillion (1998), for instance, report that the maximum allocation to the Deutschmark has been 318% of the portfolio wealth.

In addition to realistic weights on the portfolio's individual holdings, the optimization procedure may also consider total investment issues. For instance, equity managers generally have to invest around 80% at least of the portfolio's value (no more than 20% may be kept in cash at any time). Limited borrowing is also often tolerated up to 20% of the portfolio's value, without the fund being considered as a "leveraged" fund. This would translate into the sum of weight being comprised between 0.8 and 1.2.

The currency allocation is another area to pay special attention to. Two situations should be distinguished. If a position in the foreign currency is taken as being an active investment decision, the currency may almost be considered as any other asset class, except for the fact that both long and short positions in currencies should be allowed. It should be incorrect, however, to assume that the value of a short position might be used to purchase other asset classes, such as stocks for instance. These assumptions would translate in the following set of constraints:

$$\sum |w_i| = 1, \tag{3.3}$$

together with a non-negativity constraint on all assets except currencies ($|w_i|$ represents the absolute value).

If positions in foreign currencies are taken for hedging purposes only, it will either be assumed that all positions are systematically hedged, in which case the currency weights do not enter the constraint on the sum of weights ($\sum w_i = 1$ for all assets except currencies) and all returns may simply be calculated taking into account the full hedge, or it may be assumed that the portfolio manager may decide on how much of the currency exposure is to be hedged, in which case the currency holdings should be an integral part of the optimization process. The latter is a very realistic assumption for global equity and equity related managers. The constraint on the sum of asset weights should, again, be $\sum w_i = 1$ for all assets except currencies, but additional constraints on the individual currencies should be added. Typically, the portfolio manager will hedge the foreign exposures through futures or forward contracts, for which no cash nor margin is required if the position is part of the total portfolio allocation. In order not to allow over-hedges (a hedge position in the currency that represents a larger part than the portfolio's actual exposure to this currency), the value of the short position in the foreign currency should not exceed the sum of asset weights exposed to the given currency. For instance, if 30% of the portfolio a Swiss investor holds is invested in US Dollar denominated assets, the constraint on the US Dollar would be $0 \leqslant -w_{US\$} \leqslant 0.3$.

These considerations on the constraints that should be put in place in the Markowitz optimization procedure seem to be overseen too often by academics who simulate trading strategies, resulting in unrealistic simulated portfolios. The set of constraints should be considered as an integral part of the allocation process, reflecting as much as possible the real constraints faced directly and indirectly by the portfolio manager.

3.3. Measuring Out-of-sample Performance of Dynamic Strategies

Whatever the specification used to model expected returns and variances-covariances, and the final allocation based on these estimates, the real test of the value of the model is obtained by comparing the out-of-sample returns generated by the model with some benchmark. This benchmark should be clearly defined and integrated in the optimization process. This is an area where theory (in academics) and practice (in the "real world") often diverge. Theory suggests that the efficient frontier the investor should consider is obtained by minimizing the expected portfolio's variance $w'Vw$, where w is the vector of asset weights and V the matrix of variances-covariances of returns, under the constraint $w'R$, where R is the vector of asset's expected returns, and other possible user-defined constraints. The implicit assumption in this optimization program is that the manager is measured in *absolute* terms. These returns may be adjusted for risk, and the return/risk profile of the manager (or the out-of-sample generated simulated trading results) may be compared to that obtained by some benchmark.

In practice, more and more managers are directly measured in terms of *deviations* compared to a benchmark, wheather these deviations are positive or negative. The appropriate measure of risk then is *tracking error*, which is defined by $(w' - W')V(w - W)$, where W is the vector of asset weights for the benchmark. If the manager's objective is to outperform the benchmark by $\overline{R}$%, the optimal weights are given by solving:

$$\min_{w} \ (w' - W')V(w - W), \text{ subject to } \{(w' - W')R = \overline{R} \ ; \ (w' - W')\iota = 0 \ ; \ S\}, \tag{3.4}$$

where S is a feasible set of constraints.

Tracking error is the indicated framework for index funds that have as aim to replicate (and not to outperform) a given index. Other fund managers, however, should not be evaluated in these terms. Consider the following example. Assume that, over a given period, the benchmark droped significantly (say, 30%) in value, before recovering and ending (say, 10%) higher than its level at the beginning of the period. Consider two managers who are evaluated in terms of tracking error compared to this benchmark. The first manager has perfectly tracked the benchmark and reports 10% return with a tracking error close to zero, but large intra-period variance of returns. The second manager, through active management, stayed out the of the market during the volatile period and therefore has not experienced the decline of 30% and the corresponding recovery. This manager also reports an end-of-period return of 10%, with a huge tracking error but low variance of intra-period returns. Who should be considered the best manager? Tracking error suggests the first manager, absolute risk adjusted returns the second manager! I therefore argue that tracking error is, in many cases, not the fair measure, althoug the fund's performance should be compared to that of a benchmark. Unless the objective is simply to replicate a given index, managers should be measured by risk adjusted average returns (the *Sharpe ratio*).

Defining the correct measure of performance and risk adjustment of returns is central in the selection process of the econometric model that will be used in practice by the portfolio manager. Adding multiple sources of risk to the model and optimizing on the variables included in the information set ϕ_{t-1} used for deriving moment estimates at time t will yield very good in-sample performances, but almost surely will turn out disastrous when used out-of-sample. In particular, if the ability of a sophisticated time-varying specification exists only in explaining relationships specific to an in-sample period, a simple unconditional model may be as good, if not better, for forecasting purposes. In order to assess the out-of-sample performance of any model, the full data set should be split into two parts, the first one for estimating the model's parameters, the second one for measuring the model's forecasting capabilities against the benchmark. Each observation t in the out-of-sample period will generally be obtained by re-estimating the full model using observations and information up to time $t-1$.

Even within this framework, there is still the problem of "data snooping", a problem well discussed in Sullivan *et al.* (1999). Considering numerous variables that may have some explanatory power as well as comparing various econometric specifications will result in a very high number (hundreds or even thousands) of truly out-of-sample series. Given this high number of series, one can expect some of them to do extremely well both in-sample and out-of-sample. In fact, even if these series were generated randomly, one would still expect a few series with very good in- and out-of-sample characteristics.

It is important that this problem be kept in mind, whatever the specification used for forecasting financial returns. There are a few precautions that the econometrician may take in order to limit as much as possible these potential problems. To begin with, I believe one should focus on a simple and robust specification. In this respect the QTARCH model presented earlier seems appealing because of the strong economic interpretation that may be given to each regime. With regard to the instruments used for estimating the risk premiums, various studies have reported that variables such as the world dividend yield in excess of the domestic yield, the change in the yield curve, etc., are "good" variables. As pointed out by Solnik (1993), many of these empirical studies by various authors have all selected the same variables not because of some fundamental economic reason, but rather because they had shown explanatory power in previous studies. In line with the critics by Sullivan *et al.* (1999), this raises the question of the validity of including such variables in models designed for forecasting purposes. We cannot implement a proper in-sample estimation procedure with out-of-sample results in order to asses the real performance of the model.

It therefore seems interesting to investigate a model's performance using a variety of data and instruments. For example, previous studies almost exclusively use monthly data and a good test may simply be to calculate monthly returns from using different days of the months, although the inherent non-simultaneity of the asset data with macro-economic variables may become an important problem. Also, depending on the data availability, one may look at the results obtained, for example, with weekly or two months' data. Another area worth investigating further is the instruments. There is now growing evidence from serious academic studies that technical analysis may be useful, in some cases, to forecast future returns. Technical indicators may be used as instruments to model risk premiums. To my knowledge, no study so far has investigated these issues.

While academic studies mostly use lagged values of these variables, which have shown to "work well" (dividend yield, default premium, etc.), banks and financial institutions typically have their own forecasts about such macro-economic values. These forecasts are generally unavailable to the academic researcher on a historical basis that is long enough. Nevertheless, I believe that the model's ability to determine the optimal future asset allocation may be significantly improved when next period's macro-economic estimates are used rather than simple lagged values. I think this should definitely be further investigated by institutions with interest in this area.

Keeping the data mining problem in mind, the methodology will generate a time series of out-of-sample returns which are true simulations of what the portfolio manager could have achieved, had he followed the optimal strategies given by the model. The first problem that arises is the choice of the benchmark against which the performance is compared. When the available assets are the components of an existing index, this index may be used, although a historical and unconditional analysis of the means and variances-covariances of these asset returns may, in some cases, reveal that the composition of the index was not efficient *ex post*. Therefore, comparing the performance of a dynamic allocation model with the existing index may be biased in favor of the model. An appropriate benchmark has also to be constructed when no reliable index is available. For example, when asset classes such as international equities and bonds are considered, no obvious index exists. Finally, even if a reliable index does exist, the general investment policy that the portfolio manager has to follow may impose constraints incompatible with the composition of this index.

The idea is then to compare the realized performances of two investors: one is uninformed or uses an "unconditional" model, the other is sophisticated and uses time-varying weights. Solnik (1993) discusses a methodology that assumes that each of these two investors derives optimal asset weights at each time t, based on their respective expectations regarding asset returns. The joint-normality assumption is necessary to conduct simple t-tests for the significance of the difference between the two investors' returns. Individual t-statistics are calculated for each out-of-sample period as the difference in returns between the two investors, divided by the unconditional portfolio's variance. The t-statistic that tests for the significance over the full period is obtained by summing the individual statistics and multiplying this by the square root of the total number of observations.

The performance evaluation of an informed investor's strategies against those of an uninformed one is clearly an area that deserves more attention from researchers in the future. The uninformed agent may be more or less sophisticated, adapting or not his expectations over time. The assumption of joint normality is convenient but not necessarily realistic. In practice, the fund manager will often be evaluated by means of other measures of return and risk. For example, calculating the "maximum drawdown" is very common among traders. It represents buying the asset or the portfolio at the "worst" possible moment and keeping it for the "worst" number of periods. It is measured as the difference between a local high and the next local low of the series of cumulated return. For example, the maximum drawdown on the S&P500 measured over the past decades was reached during the October 1987 crash. An investor who had bought the market a few days before the crash, when the market was at an all-time high, and who had sold it at the worst moment during the crash would have lost over 30%. Monthly data where returns are calculated, for instance, around the 15th of each month would show a much smaller loss for the period October 15-November 15, 1987. The concept of drawdown may also be applied in the context of the differences between a benchmark (or the performance of an uninformed investor) and an active strategy.

Academics are often scared by the way many finance practitioners continue to evaluate absolute or excess performance on a not fully or incorrectly risk adjusted basis. They should not loose sight, however, of the fact that, because of non-normality of asset and portfolio returns, individual preferences regarding risk aversion and other factors, academic measures are not always the best in practice, either.

4. Empirical Tests

In this section I provide a few empirical tests of some of the techniques described so far. As numerous studies [Solnik (1993), De Santis and Gérard (1997, 1998), De Santis, Gérard and Hillion (1998) among many others] have tested similar methodologies using similar data sets and instruments, I will take here a slightly different approach. The choice of the data will be described in Section 4.1. In Section 4.2 I present the details about regime-switching threshold specifications which are used for modeling conditional moments and which I believe may constitute a powerful tool in this framework. Section 4.3 discusses the results.

4.1. The Data Set, the Constraints and the Instruments

4.1.1. Asset Classes and Data Frequency

To begin with, the choice of monthly data found in most studies often seems a matter of convenience: to the practitioner it is intended to show that the reallocation processes would be feasible for a long term asset manager; to the econometrician that monthly data are considered to provide enough data points to (hopefully) obtain meaningful results. As argued before, for any optimal trading strategy to be useful to a "real-world" portfolio manager, the strategy should include very realistic constraints on the asset weights. Therefore, realistic constraints are of greater importance than the use of monthly (rather than, for example, weekly) data. Furthermore, long term global asset management requires not only long term tactical asset allocation decisions, but also short-term adjustments to "events". If advanced econometric specifications are able to detect and to react to these events, a long term manager should definitely take these short-term signals into account. In practice, this is often done intuitively: the long term tactical allocation is reviewed on a monthly or quarterly basis or when fundamental information arrives, but short run events are often dealt with through positions in the derivative markets. When the portfolio manager feels a given market is too high, he may sell a few call options or reduce temporarily his portfolio exposure to this market by selling some futures contracts. The size of these positions will usually remain relatively small, so as not to exceed the global portfolio's constraints.

The increasing integration of financial markets over the past decades makes it also questionable to use data from, say, 30 years back to derive next month's optimal allocation. The real test of any time-varying allocation strategy is based on comparing out-of-sample model's forecasts to some benchmark. In order to be meaningful, this out-of-sample period should not only be long enough in terms of number of observations, but should also cover different market conditions, such as, for example, trending and non-trending markets. For these reasons I have chosen to use weekly data from January 1984 to March 15, 1999, a total of 794 data points. The period until December 1990 is kept for the initial in-sample estimation, therefore out-of-sample forecasts will cover the period from January 1991 to March 15, 1999, a total of 430 points.

I consider six different equity and equity-related markets, measured by country indices and provided by Datastream. Taking into account more markets would allow a larger diversification effect. However, the estimation process, based on maximum likelihood optimization which is continuously updated along the out-of-sample periods, is very expensive in terms of computing time. Larger multivariate frameworks would therefore require simplifications in order to limit this computing time.[1]

The use of these Datastream indices is only partially a realistic assumption. While these may be reasonable approximations of the performances on well-diversified portfolios in the respective markets, a portfolio manager would typically adjust his portfolio weights using available futures contracts on market indices. Therefore, the implicit assumption I make is that the returns on the Datastream indices are close to those on available futures contracts on the same markets.

The markets I consider include some of the largest market capitalizations: the United States, the United Kingdom, Germany, Japan and Switzerland, the country of reference. The "World Market Index", as provided by Datastream, is also considered as a market. It is important to consider the latter as an "asset class", because it may turn out that the best *ex post* allocation is simply the composition of the world index. Finally, there is increasing evidence of the importance to include some form of real estate, usually real estate funds, in a diversified portfolio. Returns on real estate funds are generally found to have a low correlation with other equity markets, while their historical expected returns and standard deviations are lower than for equities. In order to take into account this asset class, I have also included the Datastream Property Index.

I will refer to these markets as *US* for the US equities, *UK* for UK equities, *D* for German equities, *J* for Japanese equities, and *CH* for equities in Switzerland, the country of reference). The World Property Index is referred to as *Prop*, and the World Market Index as *Wrld*.

4.1.2. Returns on Foreign Currencies

To a Swiss investor, these six markets (in addition to the world market portfolio) represent four different sources of currency risk: US Dollar, German Mark, British Pound and Japanese Yen. These enter the model's specifications for two reasons at least: because they represent a risk factor that might be priced, and because they represent an asset class used either as an "investment" (speculation is probably a more appropriate term in most cases), or as a hedge. I will only consider the latter. Currencies enter the full optimization process because I assume that the asset manager may choose to fully hedge, to partially hedge, or not to hedge at all any exposure to a foreign currency. The returns (in log terms) in Swiss Francs on foreign currency i at time t are calculated as:

$$R_{i,t}^{CHF} = \ln(S_{i,t}/S_{i,t-1}) + R_{F,t}^{i} - R_{F,t}, \tag{4.1}$$

where $S_{i,t}$ is the spot price of currency i at time t. $R_{F,t}^{i}$ represents the risk free rate of the country to which currency i pertains and $R_{F,t}$ the domestic risk free rate, both being continuously compounded. $R_{i,t}^{CHF}$ should be very close to the return realized on a futures contract on currency i.[2]

[1] The implementation seems quite possible in a framework containing a few more assets. Contrary to a GARCH specification where the number of parameters increases exponentially with the number of assets under consideration, QTARCH only sees a linear increase of this number.

[2] In fact, in order to have a short position in the US Dollar, the portfolio manager would probably buy futures contracts on the Swiss Franc, denominated in US Dollars. The International Monetary Market (IMM) Division of the Chicago Mercantile Exchange (CME) is the exchange where such contracts are traded. The liquidity in nearby contracts for the major currencies is extremely high, therefore it is reasonable to assume that portfolio managers can hedge currency risk at prices close to those reported by the exchanges.

Table 4.1: Portfolio's assets and constraints. The six markets are denoted *US*, *UK*, *D* (Germany), *J* (Japan), *CH* (Switzerland, country of reference), *Prop* (Worldwide property index); the World Market Portfolio is denoted *Wrld*.

Excess return on ...	Return	Weight constraint
US market in *CHF*	r_{US}^{CHF}	$0 \leqslant w_{US} \leqslant 1$
UK market in *CHF*	r_{UK}^{CHF}	$0 \leqslant w_{UK} \leqslant 1$
German market in *CHF*	r_{D}^{CHF}	$0 \leqslant w_{D} \leqslant 1$
Japanese market in *CHF*	r_{J}^{CHF}	$0 \leqslant w_{J} \leqslant 1$
Swiss market in *CHF*	r_{CH}^{CHF}	$0 \leqslant w_{CH} \leqslant 1$
Property in *CHF*	r_{Prop}^{CHF}	$0 \leqslant w_{Prop} \leqslant 1$
World index in *CHF*	r_{Wrld}^{CHF}	$0 \leqslant w_{Wrld} \leqslant 1$
US Dollars in *CHF*	r_{USD}^{CHF}	$-(w_{US} + w_{Prop} + w_{Wrld}) \leqslant w_{USD} \leqslant 0$
Deutschmarks in *CHF*	r_{DEM}^{CHF}	$-w_D \leqslant w_{DEM} \leqslant 0$
British Pounds in *CHF**	r_{GBP}^{CHF}	$w_{UK} \leqslant w_{GBP} \leqslant 0$
Japanese Yen in *CHF*	r_{JPY}^{CHF}	$-w_J \leqslant w_{JPY} \leqslant 0$

* The different constraint on the British Pound is due to the fact that this currency is quoted using the "indirect quotation".

Note: currencies enter the equations for hedging purposes only, therefore their weights are always negative. A currency exposure cannot exceed the total portfolio's exposure to assets denominated in that given currency.

4.1.3. Realistic Constraints on Portfolio's Weights

Table 4.1 introduces the notation I will use and summarizes the constraints that I set on these weights. In addition to these constraints on the asset classes and currencies involved, I adopt the following constraint on the overall portfolio weights:

$$w_{US} + w_{UK} + w_D + w_J + w_{CH} + w_{Prop} + w_{Wrld} = 1, \qquad (4.2)$$

in other words, all of the portfolio has to be invested and no leverage is allowed. Although this equality would typically be a little less strict for a real world portfolio manager, it allows easier comparison of the final results.

4.1.4. Choice of the Instruments

The previously presented methodology allows for instruments to explain the risk premiums. Rather than using instruments that were found successful in previous studies, I have chosen here to use three different ones: the US interest rate differential, the lagged return on the risk factor itself, and a technical indicator based on the returns of the risk factor. I have (arbitrarily) selected a trend indicator, MACD, which stands for Moving Average Convergence Divergence. This is a widely used indicator that measures in percentage the difference between a long moving average and a short moving average. I set, again arbitrarily, the values for the long and the short moving averages to 30 and 15 periods.

Again, much better alternatives should be available here to practitioners who would implement a similar strategy. They might have recourse to other technical indicators, especially those often used by FX traders or an institution's own forecasts of other variables at the macroeconomic level.

4.2. Further Details on the Econometric Specification

As mentioned previously, I believe that the econometric model for the time varying covariances and prices of risk has to be simple and robust, as it has to handle a large multivariate framework. The QTARCH of Gouriéroux and Monfort (1992) is used here, because the concept of various regimes for which an economic intuition can be found appears attractive. As the choice of the partitioning method is far from obvious but also crucial, I will present here results for various approaches. It is shown on one hand how the QTARCH is, in fact, a very general model that allows numerous variations through the partitioning method. On the other hand, the reader and the researcher will be reminded of the data snooping problem mentioned earlier: looking at enough possible partitioning methods will reveal a few that seem very appealing to potential investors, just like what would happen in the case of large numbers of randomly created returns!

Four broad classes of partitioning methods will be presented and applied to the data, some of them in more than one variation. The first is a single partition model that will be referred to as the "unconditional model". It is unconditional in the sense that the model's estimates do not depend on a "state-of-the-world" defined by a partitioning variable. Nevertheless, estimates at time t do depend on risk premiums calculated by using available instruments at time $t-1$. But the sources of risk, measured by the covariances, remain stable over time. These time-varying expected returns will therefore influence the optimal allocation at each time t. The derived optimal portfolios, however, can be expected to be much less volatile in terms of asset weights than with the following partitioning methods.

The second specification is referred to as "lagged market returns". The partitioning variable Z_t contains the lagged returns on the world market portfolio. I arbitrarily define two thresholds, -1% and $+1\%$, that create three partitions accounting for a declining market, a market with a close to zero return, and a climbing market.

The third model I consider is based on principal components analysis (hereafter, PCA). PCA has been successfully combined with QTARCH models, for instance by Fraser *et al.* (2000). The idea is to extract from all available returns the common factors that explain most of the total variations in returns. I report results based respectively on one, two and three factors, that are the most important. By construction, these factors are standardized and therefore a natural threshold is zero. Two, four and eight partitions are respectively created with this methodology.

I will present here a fourth partitioning method that I developed because of some encouraging initial results obtained by Fraser *et al.* (2000) who use a statistical method to derive the partitions. This statistical method is cluster analysis, a method of partitioning that attributes observed cases to relatively homogeneous groups in order to produce an operational classification. This method was initially developed by Johnson (1967). It has been used to classify stocks by, for example, Arnott (1980), Baer (1993), and Elton and Gruber (1971). Other sorts of observations have been classified with cluster analysis, such as towns and localities, by Andrews (1971), Giuliano and Small (1991), Grove and Roberts (1980), Varady (1991), and local property markets by Abraham, Goetzmann and Wachter (1994), Goetzmann and Wachter (1995). Van Poeck and Van Gompel (1994) have used principal component analysis and cluster analysis to group European countries according to inflation and interest rates.

Applying cluster analysis to a QTARCH model is interesting because the method is a statistical one, but also because in most cases a strong economic interpretation may still be found for each partition. For instance, one group (or partition) may contain all observations where European equities performed well while the US Dollar was rising (probably a pure US Dollar effect, because European companies often have better earning outlooks when the US Dollar is higher). In another group we may find all observations where, say, European equities drop despite a rise in the US Dollar. Rather than considering such cases *a priori*, cluster analysis will create groups based on a metric chosen to measure the similarity of observations between groups.

Different techniques to form clusters can be found in the literature. The two most common are referred to as k-means and *hierarchical*. Whereas k-means was used in connection with QTARCH before [Giliberto *et al.* (1999), for instance], I here use the hierarchical method, as it seems better suited for the proposed extension. The idea of hierarchical clustering is that each observation begins as a cluster and progressively one cluster is combined with another until all are in a single cluster. Once a cluster has been formed, it cannot be split; it can only be combined with another cluster. Although alternative measures of the distance between observations (or clusters) have been proposed in the literature, I will restrict myself in this study to the initially defined measure, which is the Euclidean distance. This is also known as the unweighted pairwise group method using arithmetic averages (UPGMA). It defines the distance between two clusters as the average of the distances between all pairs in which one member of the pair is in each of the clusters. A simple but subjective measure is mostly used to determine the appropriate number of clusters.

Figure 4.1 is an example of how 12 observations (which are in $\mathbb{R}^n$) are successively grouped into 12, 5, 4, 3, 2 and 1 cluster(s). There is, however, no straightforward theory regarding the number of necessary iterations, that is, after how many iterations the "right" number of partitions has been reached. Furthermore, some observations remain single, as is the case of observation X_{12} in the example below. This is clearly incompatible with the subsequent QTARCH methodology, where a "sufficient" number of observations is needed in each partition in order to calculate significant conditional moments. I will report the results for the first node (corresponds, for instance, to two clusters in Figure 4.1), the second node (3 clusters in the example) and the third node (4 clusters).

Before exposing how I will extend the traditional cluster analysis for forecasting purposes in a QTARCH context, I briefly review the principals of the cluster technique. Let X be the T by N-dimensional matrix of observations. Each of the J clusters is defined by a centroïd C_j ($j = 1, \ldots, J$) of dimension N. The membership M_t of each observation X_t is then determined by

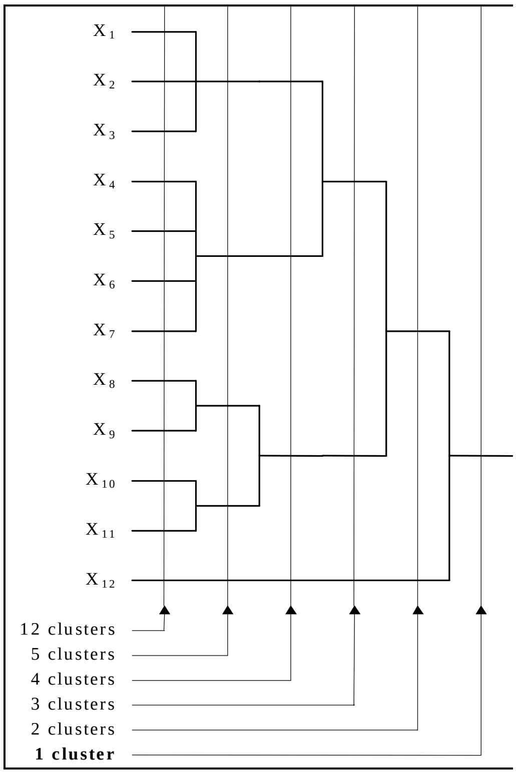
$$M_t = \arg\min_j D(X_t, C_j), \tag{4.3}$$

where $D(X_t, C_j)$ measures the distance between an observation X_t and centroïd C_j by some metric. If the Euclidean distance is used (as will be the case here), then:

$$D(X_t, C_j) = \sum_{i=1}^{N} (X_{t,i} - C_{j,i})^2, \tag{4.4}$$

where all variables should be standardized.

Figure 4.1: Example of hierarchical cluster analysis applied to 12 observations.



The main drawback of cluster analysis (whether k-means or hierarchical) applied to QTARCH is based on the assumption that knowing the membership of observations at time $t-1$ will be useful for a forecast at time t. In other words, the conditional moments of all observations that are such that the lagged observations have a given cluster membership should be significantly different from the conditional moments of all other observations. In fact, we may well obtain very distinct groups which yield very similar parameters for the next period. There may also be an attribution to wrong clusters of observations that yield very specific next period's estimates, simply because the next period's observations have not been taken into account.

In order to derive a general framework where the clusters are not necessarily only formed on the same variables used to derive the conditional moments, consider Z, a T by n-dimensional matrix of "instruments". We may have $Z_t = X_{t-1}$, in which case N is equal to n, but Z_t may also contain other informative variables observable up to time $t-1$. The attribution of observations to clusters and the conditional moment estimation is done in a two-step procedure. Assuming that all variables are standardized, the first step is to define for each t the membership M_t of observation X_t, which involves the distance to the centroïds C_j of the current observation X_t, but also of Z_t, which is based on observable information up to time $t-1$:

$$M_t = \arg\min_j D([\ \alpha Z_t \quad (1-\alpha)X_t\ ], C_j), \tag{4.5}$$

where C_j is of dimension $n+N$, and α $(0 < \alpha \leqslant 1)$ measures the importance given to the historical data contained in Z_t relative to the present data contained in X_t. The full data set X

Table 4.2: Definition of the models tested.

Model	Partitioning	Parameters
1	Unconditional	–
2	Historical Returns	Threshold 1 = −1%, Threshold 2 = 1%
3	PCA	1 Principal Component = 2 regimes
4		2 Principal Components = 4 regimes
5		3 Principal Components = 8 regimes
6	Enhanced Clustering	First node
7		Second node
8		Third node

may now be split into J smaller data sets, denoted X^j $(j = 1, \dots, J)$. Each data set X^j contains all observations X_t that are such that $M_t = j$. Within each regime a separate regression model may be applied:

$$X^j = \Lambda^j \widehat{\Gamma}^j + \varepsilon^j, \tag{4.6}$$

where Λ^j is a set of (lagged) explanatory variables and $\widehat{\Gamma}^j$ regression coefficients estimated through simple OLS for regime j.

In order to apply this framework for out-of-sample forecasting, say, at period $t + 1$, the membership M_{t+1} of observation $t + 1$ has to be determined. This is done by:

$$M_{t+1} = \arg\min_j D(\alpha Z_t, \overline{C}_j), \tag{4.7}$$

where $\overline{C}_j$ contains the n first observations of C_j. A point estimate for X_{t+1}, given that $M_{t+1} = j$, may now be given by:

$$\widehat{X}_{t+1} = \Lambda^j_{t+1} \widehat{\Gamma}^j, \tag{4.8}$$

where Λ^j_{t+1} is the explanatory variables used at time $t+1$ in regime j. Again, these variables are observable at time t.

4.3. Results

In this section I will present some results illustrating the various methodologies presented so far, in particular those obtained with the enhanced clustering method. All together, I will consider 8 sets of results, summarized in Table 4.2. The "unconditional" model is unconditional in the sense that a single partition is created. Expected returns and variances-covariances do change over time, because the risk premiums for the various sources of risk are linear combinations of time-varying instruments.

The focus will be on three types of results. First, I will present historical estimates based on the full sample of observations, showing in particular how the various risk premiums vary over time in the various regimes. The second series of results pertains to the last "rolling window" estimation. As the last observation in the data set is for March 15, 1999, the results are those for estimating the best asset allocation between this date and one week later, March 22nd, 1999. The third and last set of results deal with the analysis of true out-of-sample profits, that is, the results of a dynamic trading strategy where the portfolio is reallocated over time according to the various models.

The asset pricing model that I use is based on equation 2.7:

$$E(R_{i,t}^{CHF}) - R_{F,t}^{CHF} = \gamma_{0,t} COV_t(R_i^{CHF}, R_{Wrld}^{CHF}) + \sum_{k=1}^{4} \gamma_{k,t} COV_t(R_i^{CHF}, r_k^{CHF}), \quad (4.9)$$

$$\gamma_{k,t} = \sum_{j=1}^{J} Z'_{k,t} b_{k,j} I_{M_t=j} + \eta_{k,t}, \text{ for } k = 0, \dots, 4, \quad (4.10)$$

$$COV_t(R_A, R_B) = \sum_{j=1}^{J} COV_t^j(R_A, R_B) I_{M_t=j}, \quad (4.11)$$

where COV_t^j is the covariance of all observations belonging to rejime j. The returns on the equity indices are denoted R_i^{CHF} (i refers, respectively, to the World Property, US, UK, German, Japanese, Swiss and World equity indices) and those on the risk factors r_k^{CHF} (k refers, respectively, to the US Dollar, the British Pound, the Deutschmark, and the Japanese Yen). The binary variable $I_{M_t=j}$ takes the value 1 when M_t is equal to regime j, zero otherwise. The conditional covariance terms are obtained through one of the conditional specifications, that is, the QTARCH model with a given partitioning procedure. The time-varying risk premiums $\gamma_{k,t}$, the price of risk k at time t, are obtained through a regime-by-regime linear regression involving three instruments in addition to a constant term: the lagged return on the risk factor, the MACD(15,30) technical indicator and the interest rate term premium. These are included in the vectors $Z'_{k,t}$ which are of dimension 4. The vectors of coefficients $b_{k,j}$ measure the contribution of each instrument (plus the constant) to the total risk premium $\gamma_{k,t}$.

4.3.1. An Analysis of Historical Returns.

In this section I present some results for the time-varying risk premiums derived under the various conditional specifications. To begin with, Table 4.3 presents the average risk premium for the five sources of risk for each partitioning method and each regime. It clearly appears from the Table that very different premiums are obtained, on average, for the various regimes. While in the "unconditional" case the average risk premium is positive for the World Market (which is what one would expect), the premiums for all currencies, except the British Pound, are strongly negative. I also present the results in a graphical form in Table 4.4. The eight Panels refer to the partitioning methods, and each Panel shows the time-varying behavior of the price estimates. It can easily be seen that some regimes have a very specific behavior .

The World Market risk is generally positively priced, although the magnitude is very different across regimes. Two exceptions are the first regime derived for the "PCA-3 Factors" partitioning model and the fourth regime for the "Cluster-3rd Node" model. The World Market risk premium is strongly negative here. These two regimes are, however, very different from one another, as the risk premiums for the four other sources of risk are all of opposite signs. In the PCA case, the signs of these other risk sources are the same as for the unconditional partitioning method, while in the Cluster case, the risk source coefficients are of opposite sign. The other

Table 4.3: Mean Conditional Risk Premiums.

This Table reports the means of the conditional risk premiums calculated for the various partitioning methods and regimes. Significant premiums at the 95% (respectively, 99%) level are indicated by * (respectively, **). The risk premium for risk k at time t is obtained from the specification $\gamma_{k,t} = \sum_j Z'_{k,t} b_{k,j} I_{M_t=j}$, where $k =$ {US Dollar, British Pound, German DMark, Japanese Yen, World Market}, $Z'_{k,t}$ is a vector of instruments (a constant, the lagged return on the risk factor, the MACD(15,30) technical indicator and the interest rate term premium) available at time $t-1$. The $j = 1, ..., J$ regimes refer to the QTARCH specification.

Partitioning	Regime	N	US Dollar	Britsh Pound	German DMark	Japanese Yen	World Market
Unconditional	1	792	-4.33**	2.71**	-2.67**	-0.83**	3.50**
Historical Returns	1	230	-7.09**	4.15**	-8.21**	-7.94**	2.25**
	2	284	-3.90**	-3.79**	7.47**	7.54**	6.45**
	3	280	-2.36**	6.34**	-7.42**	2.05**	3.24**
PCA 1 Factor	1	368	-8.21**	-0.08	-2.01	-4.32**	3.20**
	2	425	-0.81**	5.05**	-3.02**	3.60**	3.91**
PCA 2 Factors	1	175	-11.14**	-3.55**	6.51**	-8.88**	0.96*
	2	194	-4.27**	4.79**	-19.97**	1.77	5.09**
	3	195	-3.77**	13.14**	-31.22**	9.96**	5.53**
	4	231	-1.77**	-6.87**	18.28**	-0.97*	4.66**
PCA 3 Factors	1	83	-22.24**	5.70*	-2.68	-13.66**	-3.41**
	2	93	-2.48	-5.52*	17.86**	-6.94**	1.25
	3	88	-12.03**	-1.52	-44.54**	-2.90	3.28*
	4	107	3.00*	9.39**	-5.30	3.54	7.76**
	5	96	-12.91**	1.79	-30.14**	-3.22	3.84**
	6	100	3.18*	23.44**	-36.90**	23.65**	8.70**
	7	108	3.04**	0.02	9.71	-11.04**	1.16
	8	124	-11.12**	-16.16**	23.93**	1.71	7.75**
Cluster 1st Node	1	497	-2.76**	1.18*	0.38	3.07**	3.04**
	2	296	-6.35**	4.90**	-7.76**	-4.08**	4.68**
Cluster 2nd Node	1	186	-6.90**	5.78**	-19.85**	-2.08**	3.82**
	2	283	1.80**	-1.82	2.99	8.06**	3.82**
	3	236	-9.51**	0.50	-12.06**	-6.87**	1.95**
	4	90	-7.67**	10.43**	35.02**	11.91**	21.80**
Cluster 3rd Node	1	124	0.46	8.91**	-10.77**	0.75	2.46*
	2	159	1.73	12.61**	-13.59**	5.64*	2.75**
	3	104	-10.36**	-3.48*	6.18	-3.97**	10.70**
	4	101	14.29**	-21.74**	25.57**	19.26**	-6.92**
	5	121	-11.94**	9.07**	-32.99**	-11.07**	2.78**
	6	114	-16.27**	-19.78**	4.11	-0.53*	2.01**
	7	75	-14.26**	6.96**	18.95**	10.29**	25.43**

risk premiums have very different sign patterns across partitioning methods and regimes. The "Historical Returns" as well as the "PCA-1 Factor" and "Cluster-1st Node" models all report average positive US Dollar risk premiums. In order to obtain regimes with positive US Dollar risk premiums, the partitioning method should generate a larger number of clusters. This is successfully done with the "PCA-3 Factors", "Cluster-2nd Node" and "Cluster-3rd Node". The "PCA-1 Factor" and "Cluster-1st Node" models clearly find opposite signs in the average risk premium for the Japanese Yen. Table 4.4 confirms some of these results. For some regimes, the behavior over time is strikingly different. Panel A pertaining to the "unconditional" case shows a high degree of variability of the Deutschmark risk premium, while the World Market and the US Dollar clearly have more stable prices of risk. The second partitioning method, based on "Historical Returns" and presented in Panel B, reveals some interesting patterns. The regimes are created on the previous week's behavior of the World Market. Though one might expect the price of the World Market risk to be influenced by the sign of the previous week's return on that source of risk, this does not seem to be much the case. However, the price of Japanese Yen risk is small when the previous week's World Market return was above 1% (regime 3). This premium seems of a large magnitude and unpredictable in the case of regime 2 (for small absolute values of the World Market return); in case of regime 1, after a negative return on the World Market portfolio, the Yen is either priced positively or negatively, but the price behaves in a more systematic way. For the Deutschmark, some persistence in the price of risk is observed for regimes 2 and 3. The British Pound risk premium is very smooth in regime 1, while its pattern seems chaotic in the two other regimes.

This visual analysis of the risk premiums in the various regimes shows very specific behaviors in some cases, and the persistence over time of most patterns suggests that a non-parametric approach, such as QTARCH, might be the ideal tool for modeling these risk premiums. Some patterns clearly seem predictable in-sample and therefore may be used for out-of-sample predictions as well.

The next section presents results pertaining to the last "rolling window". The parameter estimation is done using the full sample of observations and the optimal allocation for the first out-of-sample period, March 22nd, 1999, is derived.

4.3.2. Optimal Allocation Strategy for March 22nd, 1999

The allocation decision for March 22nd depends, for all but the "unconditional" specifications, on the valid regime for that date, which is determined by variables observable the week before. The expected excess returns on the assets, as well as the covariances of the residuals, may be estimated individually for all possible regimes, and separate efficient frontiers can be derived for each regime under the set of constraints. These are represented in Table 4.5. The eight Panels deal, again, with the eight partitioning methods. While the "unconditional" frontier shows a maximum expected return point on the frontier of 0.7% per week, a closer look at the frontiers obtained under the various regimes reveals much higher maximum expected return points. For instance, the "Historical Returns" method clearly suggests two regimes, one with close to zero expected returns, and one (in this case regime 1, corresponding to situations where previous week's World Market return was lower than -1%) where the expected return ranges between 0.5% (the minimum variance point) and 2.3% (the maximum return point). Again, these estimates are only valid at this particular point in time, as the return estimates are, through the conditional risk premiums, a function of the value of the instruments at this time. The PCA partitioning methods also reveal very different efficient frontiers. When 3 factors are considered, two frontiers with very high expected returns (around 5% for the maximum return points) are derived. The clustering method where the results are taken at the first node show

Table 4.4: Time-varying risk premiums based on the full sample of observations. All observations are ranked by regime membership.

two very similar frontiers, while at the second node one of the regimes yields a frontier clearly above all others. This is also the case for two regimes at the third node.

Each feasible point on the frontier is obtained by minimizing the portfolio's variance under the set of constraints on the weights:

$$\min_{w} w'Vw, \text{ subject to } \{w'R = \overline{R}\ ;\ S\}, \tag{4.12}$$

where R is the vector of expected returns, V is the matrix of variances-covariances, w is the vector of portfolio weights, and $\overline{R}$ is the level of expected return for which the minimum variance is to be determined. S is the set of general constraints on the weights that reflect the portfolio manager's objectives. The final point on the efficient frontier that will be selected depends on individual preferences. An often made assumption is that the investor has a relative risk aversion equal to twice his marginal rate of substitution of expected return for variance [see, for instance, Solnik (1993)]. In that case, the optimal asset holdings are obtained from the following optimization program:

$$\min_{w} w'Vw - wR, \text{ subject to } \{S\}. \tag{4.13}$$

Evidence from the empirical literature suggests that this value of 2 is a good estimation for the relative risk aversion coefficient of most investors. In addition to considering this optimal point on the frontier, I will also simulate two other investors who select their optimal portfolio, respectively based on the maximum return/risk ratio and on the minimum variance criterium. As all returns are in excess of the domestic risk free rate, this is simply the point of tangency of a line drawn from the origin with the efficient frontier. Mathematically, the holdings are obtained from:

$$\max_{w} \frac{wR}{w'Vw}, \text{ subject to } \{S\} \tag{4.14}$$

for the maximum return/risk ratio, and from:

$$\min_{w} w'Vw, \text{ subject to } \{S\}. \tag{4.15}$$

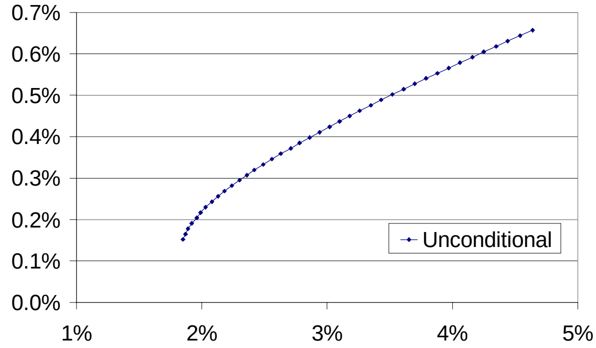
for the minimum variance criterium.

Tables 4.6, 4.7 and 4.8 report the compositions of the optimal portfolios obtained for each of these three investors. It should be noted that the figures are based on expected moment estimates for March 22$^{\text{nd}}$ and should not be interpreted as allocations resulting from historical constant estimates of these moments. The "unconditional" allocation method suggests an allocation of 21-79 percent to, respectively, US and Japanese equities for the investor with a relative risk aversion coefficient of 2. In addition, both currency risks should be fully hedged. Based on the maximum return/risk ratio, the portfolio should be fully invested in Japanese equities and the currency risk fully hedged.[3]

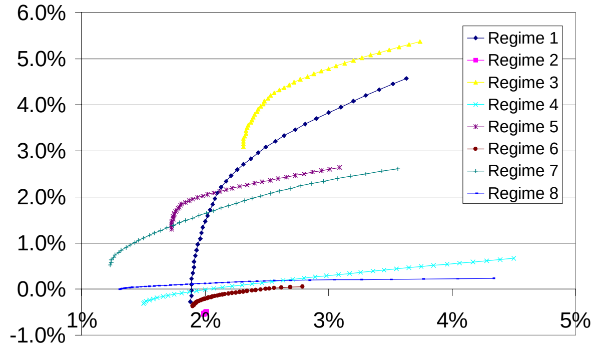
[3]These are, of course, irrealistic heavy weights to a single market, which no portfolio manager in practice would ever follow. It would be very easy to add constraints on the maximum allocation to each market a manager is allowed to; however, this would be so arbitrarly in nature, that I have chosen not to include these additional constraints in these simulation.

Table 4.5: Efficient frontiers based on expected returns and variances-covariances estimates for March 22nd, 1999.

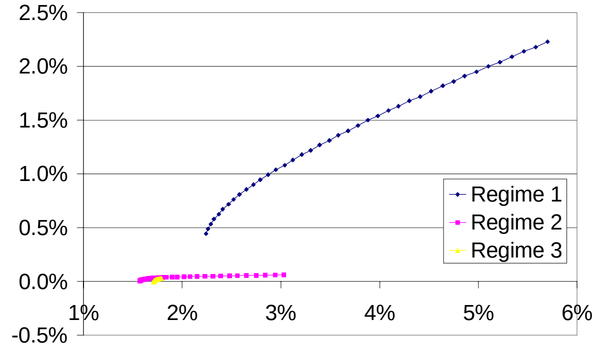
Panel A. Unconditional.



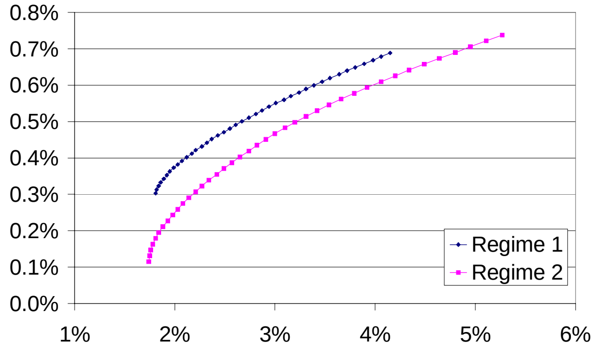
Panel E. PCA - 3 Factors-



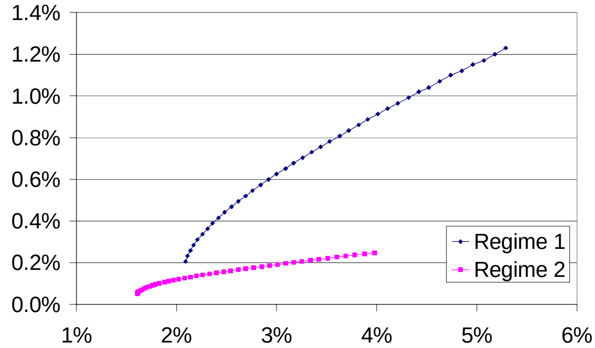
Panel B. Historical - 3 regimes.



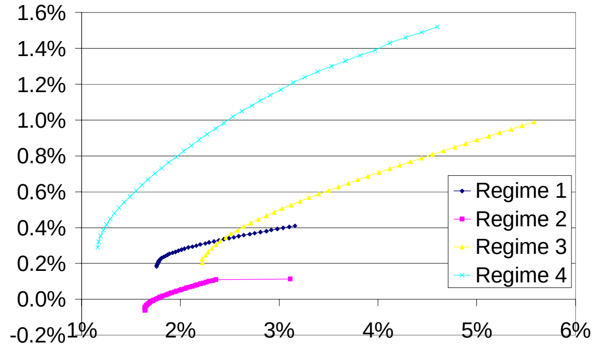
Panel F. Clusters-1st node.



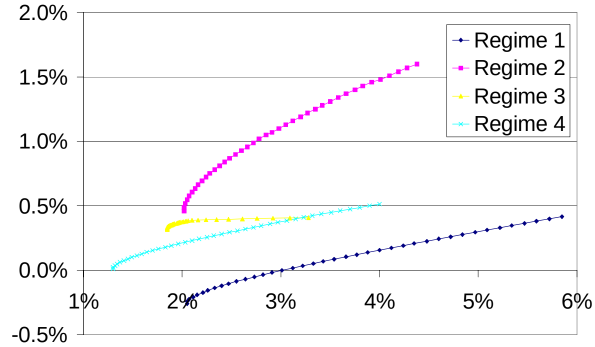
Panel C. PCA - 1 Factor.



Panel G. Clusters - 2nd node.



Panel D. PCA - 2 Factors.



Panel H. Clusters - 3rd node.

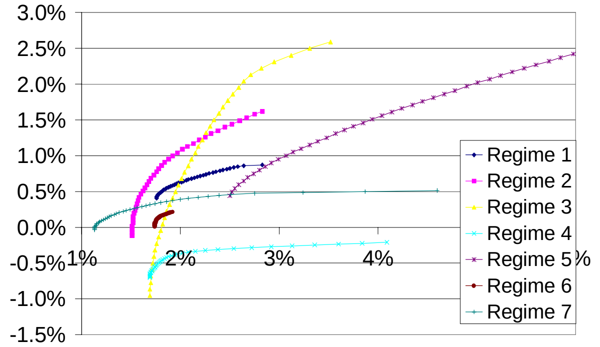


Table 4.6: Optimal weights for the portfolio on the efficient frontier with a risk-aversion coefficient of 2, estimated for the week ending March 22nd, 1999.

				Weights Market Indices							Weights Currencies			
Partitioning		$E(R_i)$	$\sigma(R_i)$	Prp	US	UK	D	J	CH	Wrld	USD	GBP	DEM	JPY
Unconditional*		0.54	3.79		21			79			-21			-79
Historical	1	1.93	4.92		15			85			-15			-85
Returns	2	0.04	2.07	37						64	-100			
	3*	0.03	1.79							100	-100			
PCA	1	1.23	5.29					100						-100
1 Factor	2*	0.18	2.86	12				58		29	-42			-58
PCA	1	0.42	5.85					100						-100
2 Factors	2	1.29	3.44	2	23	3		72			-25	-3		-72
	3	0.37	1.98	10	66				8	16	-92			
	4*	0.49	3.79	8				92			-8			-92
PCA	1	3.84	3.01	56						44	-100			
3 Factors	2	-0.49	2.01		1					99	-100			
	3	4.29	2.58	9		8		37		47	-55	-8		-37
	4	0.67	4.50					100						-100
	5	1.93	1.88	12						88	-100			
	6	0.05	2.79		100									
	7	1.91	2.31		29	17		50	4			-17		-50
	8*	0.18	2.52			67		33				-67		-33
Cluster	1*	0.43	2.27	3	11	8		30		48	-62	-8		-30
1st Node	2	0.50	3.18		52			44	4					-44
Cluster	1*	0.30	2.16		9		51			40	-49		-51	
2nd Node	2	0.11	2.37	55		45					-55	-45		
	3	0.87	4.87					83		17	-17			-83
	4	0.82	2.03		41			29	30					-15
Cluster	1*	0.82	2.48		48		27	18		7			-27	-6
3rd Node	2	1.56	2.72	92						8	-100			
	3	2.21	2.80		43	57						-57		
	4	-0.22	3.84					92	8					-92
	5	2.41	5.95		1			100			-1			-100
	6	0.21	1.92							100	-100			
	7	0.38	1.94		43			11	46					-11

Note: The Table reports the optimal holdings in percentage. The assets are, respectively, the World Property Index, the equity markets in the US, the UK, Germany, Japan and Switzerland, the US Dollar (USD), the British Pound (GBP), the Deutschmark (DEM), the Japanese Yen (JPY) and the World Market Index. The latter five are also the risk sources. The optimal points on the frontier are derived under the constraints that the sum asset weights (excluding currencies) is equal to one, and that currencies, when used, are for hedging purposes only. The * indicates which is the valid regime for the date of forecast (the other regimes are only reported for informational purposes).

Table 4.7: Optimal weights for the portfolio on the efficient frontier with the highest expected return/risk ratio, estimated for the week ending March 22nd, 1999.

				Weights Market Indices							Weights Currencies			
Partitioning		$E(R_i)$	$\sigma(R_i)$	Prp	US	UK	D	J	CH	Wrld	USD	GBP	DEM	JPY
Unconditional*		0.66	4.64					100						-100
Historical	1	2.23	5.70					100						-100
Returns	2	0.04	1.80	9						91	-100			
	3*	0.03	1.79							100	-100			
PCA	1	1.23	5.29					100						-100
1 Factor	2*	0.20	3.07	15				66		19	-35			-66
PCA	1	0.42	5.85					100						-100
2 Factors	2	1.60	4.38					100						-100
	3	0.39	2.12	29	71						-100			
	4*	0.51	4.00					100						-100
PCA	1	4.57	3.63	100							-100			
3 Factors	2	-0.49	2.01							100	-100			
	3	5.37	3.74					100						-100
	4	0.67	4.50					100						-100
	5	2.64	3.09	100							-100			
	6	0.05	2.79		100									
	7	2.61	3.56					100						-100
	8*	0.19	2.58			62		38				-62		-38
Cluster	1*	0.69	4.15					100						-100
1st Node	2	0.72	5.05		5			95						-95
Cluster	1*	0.41	3.16				100						-100	
2nd Node	2	0.11	2.37	54		47					-54	-47		
	3	0.99	5.58					100						-100
	4	1.52	4.60					100						-100
Cluster	1*	0.87	2.73		76		25						-25	
3rd Node	2	1.62	2.83	100							-100			
	3	2.59	3.52			100						-100		
	4	-0.27	2.97					64	36					-64
	5	2.42	5.98					100						-100
	6	0.21	1.92							100	-100			
	7	0.47	2.61		69			31						-31

Note: see Table 4.6.

Table 4.8: Optimal weights for the minimum variance portfolio on the efficient frontier, estimated for the week ending March 22nd, 1999.

				Weights Market Indices							Weights Currencies			
Partitioning		$E(R_i)$	$\sigma(R_i)$	Prp	US	UK	D	J	CH	Wrld	USD	GBP	DEM	JPY
Unconditional*		0.15	1.85		16	8	5		12	58	-74			
Historical	1	0.44	2.24		18	2	14		8	58	-76			
Returns	2	0.00	1.57		11	17			19	53	-64			
	3*	-0.01	1.71		19	6	9		5	61	-80			
PCA	1	0.21	2.09		16	4	6		15	59	-75			
1 Factor	2*	0.05	1.61		17	14	4		10	55	-73			
PCA	1	-0.26	2.05		37	6			10	48	-85			
2 Factors	2	0.46	2.02			5	19		7	68	-66			
	3	0.32	1.85		21	4	10			66	-87			
	4*	0.01	1.30		10	24			25	41	-51			
PCA	1	-0.27	1.88		42	18			23	17	-60			
3 Factors	2	-0.55	1.99		16	1				83	-99			
	3	3.09	2.31	2		7	21	23	3	44	-46			
	4	-0.31	1.50		39	9	2		12	38	-77			
	5	1.30	1.73		16		11			73	-87		-11	
	6	-0.37	1.90		26	9		1		64	-90			
	7	0.53	1.23			16		4	37	43	-34			
	8*	0.00	1.30	5	18	30			14	33	-56			
Cluster	1*	0.30	1.81			15			18	67	-67			
1st Node	2	0.12	1.74		53		12		6	30	-82			
Cluster	1*	0.18	1.76		9	9	4		7	72	-81			
2nd Node	2	-0.06	1.64		18	18			26	38	-56			
	3	0.21	2.22		13		12		3	71	-85			
	4	0.29	1.16		33	15	6		14	31	-65			
Cluster	1*	0.40	1.76		13	12	5		3	68	-81			
3rd Node	2	-0.12	1.51		9	7	2		39	43	-51		-2	
	3	-0.95	1.69		37	6			13	44	-80	-5		
	4	-0.70	1.69			31			15	54	-54			
	5	0.44	2.50		4		15			82	-85			
	6	0.01	1.74		40	11			5	44	-84			
	7	-0.03	1.13		29	13	9		12	37	-66			

Note: see Table 4.6.

For the investor with a relative risk aversion of 2 (see Table 4.6), the "Historical Returns" method suggests to be fully invested in the World Market portfolio with the currency exposure (the return on the World Market is measured in US Dollars) fully hedged. It is interesting to note here that for this partitioning method a large part of the portfolio had to be allocated to the Japanese market in regime 1, where the previous week's return on the World Market was below -1%. Regime 2, where this return was between -1% and $+1\%$, would suggest a 37% allocation to real estate funds. The current regime for the single factor PCA model also suggests a 58% allocation to Japan, the rest being allocated to property (12%) and the World Market (29%). All currencies are, again, fully hedged. For the two-factor PCA, Japan should represent 92% and property 8%. Only when three factors are considered within the PCA method is the Japan exposure reduced to 33%, while the UK equities now account for 67%. The conclusion so far is that whatever the partitioning method used (unconditional, based on historical returns or on any of the PCA analyzes), the suggested allocation is relatively homogeneous, attributing a large part to Japan with the currency exposure fully hedged. Even the Cluster method taken at the first node still allocates 30% to Japan, while the other two nodes respectively suggest 0% and 27% to this country. The cluster techniques therefore suggest less homogeneous results. From an intuitive point of view, it is reassuring when various partitioning methods suggest similar allocations. On the other hand, it might be investigated how a trading rule could be developed where trades are taken only when all methods suggest a similar allocation to a given asset. Whereas these are important issues for the every day trader, they are beyond the scope of this section.

Table 4.7 reports the results for portfolios allocated according to the highest Sharpe ratio. For this particular date t (and, hence, values of the instruments and risk factors), most conditional models yield "all" or "nothing" allocations. This is due to the form of the frontier in these cases, where the point of the highest Sharpe ratio corresponds to the extreme right point of the frontier (highest expected return).

Finally, Table 4.8 reports the suggested asset holdings when the selected portfolio is that with the minimum variance. As could be expected, less extreme weights are generally given to a few assets. In case of the unconditional model, only 12% should be allocated to domestic (Swiss) assets, while a large part is devoted to the World Market portfolio, with hedged currency risk. The other partitioning methods show similar results, in the sense that a relatively small proportion of the portfolio should be allocated to the domestic asset class. This is in line with the conclusion of most researchers in this field, who suggest that investors tend to over-weight their domestic asset class.[4] Interestingly, each of the 6 statistical partitioning methods (based on PCA and Cluster analysis) has one regime with a much higher domestic allocation. All regimes also suggest to allocate a high percentage to the World market portfolio, between 17% and 83%. Contrairly to what was suggested for the two previous points on the frontier, the allocation to the Japanese market remains marginal and is present for a few regimes only.

Property is the asset class that is almost never select by any conditional model, whatever the assumption about the investor's preferences. This result is very specific to the date for which the allocation is made. The optimal weights depend on the values of the instruments used to determine the risk premium for each of the factors, which are, in turn, conditional on a regime.

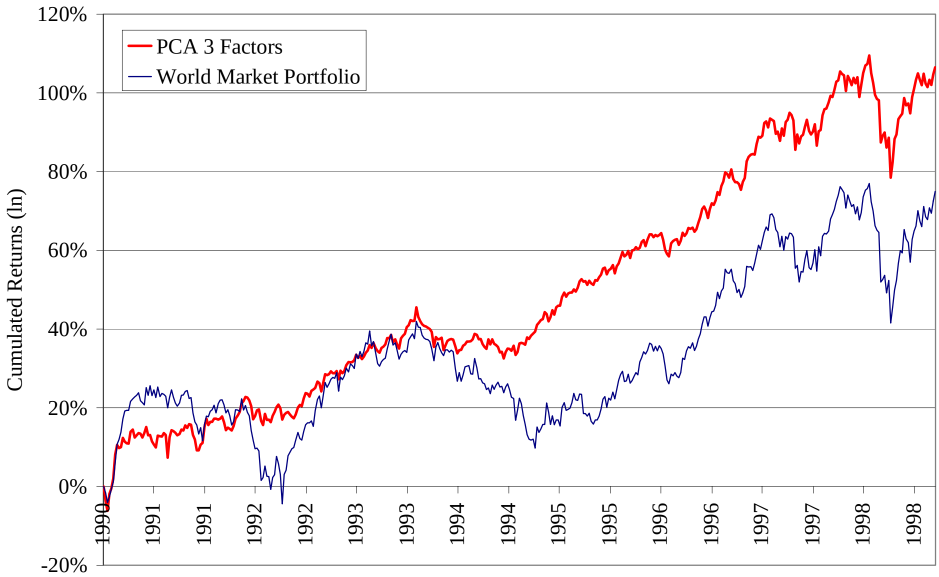
4.3.3. Trading Strategies Analysis

The real test of any econometric specification that is intended for forecasting purposes is the analysis of the out-of-sample simulated performance. In the present case, the optimal holidings

[4]See Solnik (1974), for instance.

at each point in time were derived from an optimization problem that minimized the portfolio's variance subject to a set of constraints. Consequently, the out-of-sample returns on the active trading strategy have to be compared to the returns on the benchmark (in terms of average returns, standard deviations of these returns, etc.).[5] The World Market portfolio is taken as the benchmark. The results for the nine models are presented in Table 4.9. The sample consists of 430 truly out-of-sample generated weekly portfolio returns, from January 1991 to March 1999. The World Market index yields an annualized excess return of 8.9% with an associated $t-$statistic of only 1.46. When the investor with a risk aversion coefficient of two is considered, only two specifications yield a better performance than the World Market in terms of both average returns and $t-$statistics: the unconditional model and the Cluster-2nd Node. All other strategies yield poorer performances than the World Market portfolio. The PCA model with 2 factors does well when the portfolio with the highest expected return/risk is selected: an average annual return of almost 6 percentage points higher than on the World market portfolio. The real interesting results come, however, from the minimum variance portfolios, which yield higher returns and higher $t-$statistics than the World Market, whatever the dynamic model under consideration. The best result is obtained with the PCA with 3 factors, for which the annualized return is 4 percentage points higher than for the benchmark, while the associated $t-$statistic is almost twice as large (2.80 instead of 1.46). For all minimum variance portfolios is the $t-$statistics above 2.

Figure 4.2: Cumulated returns (in log) obtained out-of-sample for the "PCA 3 Factors" strategy where the weights are those of the minimum variance portfolio, as well as for the market portfolio.



A graphic representation of the results is given in Figure 4.2, which shows the cumulated returns (in log) for the "PCA 3 Factors" conditional model. The weights are those from the minimum variance portfolio. The World market portfolio is also represented. In many cases of turmoil (large negative market returns) does the conditional asset allocation better than the

[5]If we had chosen to minimize the variance of the "active" returns (the returns on the portfolio minus the returns on the benchmark) subject to a set of constraints that includes the return by which the portfolio has to outperform the benchmark, then the performance of the trading strategy would be analyzed in terms of return and risk of the "active" returns.

Table 4.9: Out-of-sample annualized returns of the trading strategies, from January 1990 to March 1999.

Model	World Market	Risk Aversion =2	Max Return Risk	Minimum Variance
Unconditional	**8.9%**	9.8%*	4.9%	9.9%*
	(1.46)	(1.45)	(0.52)	(2.21)
Historical Returns		8.1%	3.6%	9.8%*
		(1.00)	(0.36)	(2.16)
PCA 1 Factor		6.7%	1.3%	9.9%*
		(1.02)	(0.13)	(2.20)
PCA 2 Factors		7.6%	14.7%*	10.8%*
		(1.07)	(1.54)	(2.39)
PCA 3 Factors		6.9%	12.2%	12.9%*
		(1.08)	(1.37)	(2.80)
Cluster 1^{st} Node		6.7%	6.5%	9.6%*
		(0.98)	(0.69)	(2.15)
Cluster 2^{nd} Node		12.4%*	9.6%	10.0%*
		(1.73)	(1.01)	(2.24)
Cluster 3^{rd} Node		4.8%	1.4%	9.2%*
		(0.77)	(0.17)	(2.01)

Note: $t-$statistics are given in brackets. The * indicates that the returns have both a higher mean and a higher $t-$statistics than for the World market index.

benchmark. An exception is the recent 1998 crisis when the correlation between all markets was very high, which makes any diversification between major equity market almost useless. I have represented here the best performing model, and the Sullivan *et al.* (1999) study on data-snooping should be kept in mind here. Nevertheless, it should be noted that all of the minimum variance portfolios performed better, in terms of mean returns and $t-$statistics than the benchmark. I explain the poorer performance of the other selected points on the frontier (risk aversion coefficient equal to two and the maximum return/risk point) by the fact that no maximum weight that could be attributed to a single asset was imposed to the optimization procedure. In too many cases over time the solution was a 100% allocation to a single asset, which would never be tolerated by a "real world" portfolio manager.

5. Conclusion

This paper has discussed various aspects of dynamic international portfolio allocation. Recent studies suggest that international asset returns, measured in the currency of the country of reference, can be explained by a multi-factor model where the investor is remunerated in terms of expected returns for bearing risk from various sources. These risks include a "market risk" (which is traditionally measured by the covariance of asset returns with a World Market Index), and the various currency risks (measured by the covariances between asset returns and returns on the foreign currencies). With each of these risk sources is associated a price of risk, or risk premium, which is interpreted as the remuneration for bearing the given risk. Such a framework is often referred to as the "asset pricing restrictions", because it states the linear relation between asset returns on one side, and prices and quantities of risk on the other side. A full model that can be used for forecasting purposes is obtained when additional hypotheses are made about the dynamics over time of the risk factors on one hand, and the prices of risk on the other. Recent literature suggests that sources of risk, in the form of covariances, may well be modeled by some ARCH process, while the prices of risk may well be explained by macro-economic variables such as, for instance, the world dividend yield. It is important to realize that the choice of these specifications is based on economic intuition and "good" empirical results obtained in many studies, but is not directly derived from an economic theory.

As the methodological choice made in many studies remains a little arbitrary in nature, it is important to investigate other approaches. After all, it may be that the relative "success" of the model is due to the fact that by chance a methodology was selected that is well adapted to a given historical data set. It is in this spirit that the empirical tests presented in this paper were performed. I took a non traditional approach to analyze and to forecast weekly data, a frequency generally left aside by researchers, but which has the advantage of providing a high number of recent observations. Weekly data were used from January 1984 to March 1999 (794 time observations) for returns on five major equity indices and a real estate index, as well as for the returns on the associated currencies. A non-parametric ARCH specification with thresholds was proposed to model conditional covariances, while technical indicators were introduced to model the respective risk premiums. In addition to an in-sample analysis, extensive out-of-sample tests were performed to measure the practical use of such a methodology. Practitioners are in general sceptical about allocation or trading models developed by academics, because important constraints faced by a portfolio manager are too often not taken into account. The out-of-sample tests are based on a long series of 430 data points that represent the returns on a simulated portfolio based on the model's forecasts of expected returns and covariances.

Both the variability of the results over the out-of-sample period of 430 observations and the different results obtained for the various models tested suggest the difficulty of forecasting assets' first and second order moments. Also, when the efficient frontier is derived based on next period's estimates of assets' expected returns and variances-covariances, a particular point on the frontier has to be selected that reflects the investor's attitude towards risk. The point with the highest return/risk ratio and the point which corresponds to the optimal portfolio for an investor with a risk aversion coefficient of two often generated worse out-of-sample results than, for example, the minimum variance point. These findings confirm the "data snooping" pitfall: when a large number of models or alternatives is examined there will always be some of them that perform very well, both in-sample and out-of-sample. I believe this issue, too often neglected, should be kept in mind in any research of the kind.

Nevertheless, a few interesting results have emerged from the tests. To begin with, the non-parametric approach presented seems powerful in creating regimes with different economic interpretations and in which very different parameter estimates are obtained. Furthermore, the long series of simulated returns suggest that the approach is also relatively robust out-of-sample, and therefore ideally suited for next period's optimal allocation applications. Selecting systematically the point on the efficient frontier that corresponds to the minimum variance portfolio provided economically significant improvements over the World Index, both in terms of achieved average returns (from 8.9% for the benchmark to between 9.2% and 12.9% for the different models) and in terms of risk, measured by the $t-$statistics of these returns (from 1.46 for the benchmark to between 2.01 and 2.80 for the conditional models).

These strong and robust results suggest that the various techniques presented in this study may constitute powerful tools in designing optimal portfolio allocation models based on time varying sources of risk and prices of risk. The framework seems ideally suited for deriving various non parametric specifications, where risk premiums may be estimated using instruments that include lagged values of macro-economic variables (as done in most previous studies), but also technical indicators (as done in this study and a companion paper). Good results may also be expected when the financial institution's own next period's estimates for macro-economic related variables enter the model, rather than simply lagged values.

Developing tools useful in determining the optimal and time varying portfolio allocation for an internationally global diversified investor is challenging, both for the practitioner and for the academic researcher. As this study has shown, it is an area of research with many pitfalls but also where tools can be developed that lead to significantly improved portfolio returns and risk characteristics. Future research should remain focused on simple approaches which are economically meaningful, so as to keep a good out-of-sample robustness, while it may include less "traditional" variables such as indicators derived from technical analysis.

6. References

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