Valuation of Sovereign Debt with Strategic Defaulting and Rescheduling

Michael WESTPHALEN

École des HEC, University of Lausanne and FAME

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Prof. Martin Hoesli
University of Geneva, HEC
40 bd du Pont d'Arve
1211 Genève 4
Tel: +41 (022) 705 8122
Martin.Hoesli@hec.unige.ch

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Michael Westphalen
École des HEC, University of Lausanne and Fame†

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Abstract

This paper provides a simple model of the rescheduling of debt following a sovereign default as a bond exchange. In case of default, the sovereign offers a new bond with lower coupon and principal. The debtors accept the offer if the value of the new bonds is higher than the proceedings of the litigation of the sovereign. Both the default decision of the sovereign as well as the exchange offer are modeled endogenously and in closed form. The resulting formulas for bond value and credit spreads are in closed form as well.

The analysis yields credit spread curves similar to corporate credit curves: For high risk issuers, i.e., sovereign with low country wealth relative to debt level, and high litigation costs, the credit spread curves are "hump"-shaped. Better quality issues exhibit increasing credit spread curves. The numerical analysis with reasonable parameters yields credit spreads of a size compatible to market spreads.

A comparison to corporate debt supports the stylized fact that, using the same parameters, corporate debt is less risky than sovereign debt since the threat of liquidation is stronger than the threat of litigation.

JEL Classification: F34, G13, G33
Keywords: Sovereign Debt, Debt pricing, Bond Exchange Offers

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†Contact Information: Michael Westphalen, HEC - Université de Lausanne, BFSH 1, 1015 Lausanne-Dorigny, Switzerland. Email: michael@westphalen.ch. Phone: +41 76 3656534

Valuation of Sovereign Debt with Strategic Defaulting and Rescheduling

Executive summary

Sovereign debt cannot be priced or analyzed using a model developed for corporate debt because sovereign debt is different from corporate debt: If a sovereign does not repay the amount which is specified in the debt contract it is not possible to initiate proceedings in a bankruptcy court which allow the lenders to seize all assets of the borrower. This implies that there are greater incentives for a sovereign to strategically default, i.e., to pay less than the contractual amount even if there are enough resources to fulfill the debt contract.

To take account of this behavior, my paper focuses rather on the country's willingness or incentives to fully honor its obligations than on its ability to do so. In this respect, the model differs from classical corporate credit risk models. It is not a drop of the company's asset value under the outstanding face value which triggers default, but rather a decision of the country whether to continue in respecting its commitments or whether to default. However, my model uses the framework of models of corporate credit risk, and extends it to incorporate features specific to sovereign credit risk and sovereign debt reorganization.

I directly model the dynamics of the country's wealth, i.e., I do not specify the production side of the sovereign's economy. Furthermore, I assume that there exists one utility function which describes the interests of the country. I do not allow, for instance, that the politicians in the government have interests different from the citizens of the country.

Default is designed such that the country does not continue to honor the original debt contract. Instead it offers a new contract with reduced coupon and principal repayments. After default, however, the sovereign has to face costs due to e.g. sanctions, disruptions in trade or loss of reputation, which result in reduced growth of its wealth. The recent defaults of Ecuador on its Brady bonds and some Eurobonds in 2000, Pakistan on three dollar denominated Eurobonds in 1999, the Ukraine on Eurobonds in 2000 and Russia in 2000 on the so called Prins and Ians support the modeling of a sovereign default as a bond exchange of the original bonds for new bonds with lower value.

The decision whether to default or not is made by comparing the expected terminal wealth, considering debt service and costs of default, under both scenarios. The level of country wealth at which the sovereign will be just indifferent between fulfilling its duty and defaulting, i.e., the level of wealth which triggers default, is named the *endogenous default boundary.*

Recent successful litigation of Peru shows that sovereigns face a litigation threat by the lenders. If the sovereign offers a bond exchange with too bad conditions, the lenders will try to seize foreign assets held by the sovereign. However, they are not able to recuperate the full principal. Part will be lost due to litigation costs, both explicit, i.e., lawyer fees etc., and implicit costs of defaulting: Because there is no formal bankruptcy court for sovereign debt, it is very difficult for the creditors to find a way to sue the sovereign, rendering a full recuperation of the principal impossible.

By assuming that the sovereign will always offer a new debt contract such that the lenders are indifferent between accepting and litigating, I endogenize the fraction of the original contractual obligations the sovereign continues to honor after default.

Even although the default boundary is time dependent, I find closed form solution for the default triggering wealth level, sovereign bond prices and credit spreads by applying an efficient approximation developed by Durbin (1992). Numerical analysis shows an important inverse relation between the credit spreads of sovereign debt and the amount that can be recuperated through litigation. The distance of the country's wealth to the default boundary influences the credit spreads inversely as well, while an increasing size of the coupon leads to higher spreads.

I compare sovereign debt to domestic corporate debt by analyzing the impact of the bankruptcy scheme: While a sovereign will be litigated for the principal, a corporation will be liquidated, i.e., all the assets of the corporation will be transferred to the lenders, minus the liquidation costs.

The stronger threat of liquidation leads corporates to wait longer until they default than sovereigns. This leads to considerably lower spreads of corporate bonds. If, in addition, we also assume lower liquidation costs for the corporate, the spreads decrease even more.

The behavior of the credit spread curves is similar to the credit spreads obtained by the classic models of corporate credit spreads such as Longstaff and Schwartz as well as Leland and Toft.

1 Introduction

While the literature of corporate credit risk is advancing at a rapid pace, the issue of sovereign credit risk is still in its early stages. Sovereign debt is different from corporate debt: If a sovereign does not repay the amount which is specified in the debt contract it is not possible to initiate proceedings in a bankruptcy court which allow the lenders to seize all assets of the borrower. This implies that there are greater incentives for a sovereign to strategically default, i.e., to pay less than the contractual amount even if there are enough resources to fulfill the debt contract.

To take account of these factors we will focus on the country's willingness or incentives to fully honor its obligations rather than on its ability to do so. The model differs from classical corporate credit risk models: It is not the dropping of the company's asset value under the outstanding face value which triggers default, but rather a decision of the country whether to continue in respecting its commitments or whether to default.

The benefits of defaulting are that the outstanding debt will be reorganized. The sovereign offers a bond exchange: Bond holders can change their defaulted bonds into new bonds with a lower coupon and principal.

The punishment will come in the shape reduced growth of country wealth. The reasons for the reduced growth are either direct trade sanctions in the Bulow and Rogoff sense, or the economic costs of lost reputation,in the debt market as well as spillover into other markets, i.e., the loss of the reputation as a trustworthy business partner. Furthermore, creditors can utilize the threat of litigation, i.e., the lenders will try to recuperate part of the principal by seizing assets of the sovereign held abroad.

The model itself is based on models of corporate credit risk, which are extended to incorporate features specific to sovereign credit risk and sovereign debt reorganization. It takes the idea of an endogenous bankruptcy rule as a flow condition from Kim, Ramaswamy and Sundaresan [27] whose model has corporations defaulting when the firm does not generate enough cash flow to meet contractual coupon payments.

We begin by modeling the dynamics of the country's wealth, incorporating debt service. Since it would be necessary to explicitly model the production side of the sovereign's economy to estimate the cost of the defaulting and the benefit of the debt relief in case of default, or to at least assume an ad-hoc 'utility' function for the country, we will assume, following Bulow and Rogoff [6], that the sovereign seeks to maximize terminal country wealth, i.e., that it has a linear utility function. This assumption is limiting in the sense that we assume that there exists one utility function which describes the interests of the country. We do not allow, for instance, that the politicians in the government have interests different from the citizens of the country.

Default is designed such that the country does not continue to honor the original debt contract. Instead it offers a new contract with reduced coupon and principal repayments. After default, however, the sovereign has to face costs due to e.g. sanctions, disruptions in trade or loss of reputation[1], which result in reduced growth of its wealth.

The recent defaults[2] of Ecuador on its Brady bonds and some Eurobonds in 2000, Pakistan on three dollar denominated Eurobonds in 1999, the Ukraine on Eurobonds in 2000 and Russia in 2000 on the so called Prins and Ians[3] support our modeling of a sovereign default as a bond exchange of the original bonds for new bonds with lower value. However, the reality is more complicated.

Our model assumes a simple exchange of the original bonds for new bonds with the same maturity date but a coupon rate and a principal which are reduced by the same proportion. For example, a sovereign bond with a 10% coupon on a principal of $ 100 would be exchanged for instance into a bond with 4% coupon on a principal of $ 40, with the same maturity.

Apart from the fact that coupon and principal are rarely reduced by the same proportion, sovereigns also often offer a "sweetener" to induce bond holders to tender: When Ecuador proposed its exchange offer in July 2000, it offered about $ 1 bn in cash to bond holders in addition to the new bonds. Russia's exchange offer for its Prins and Ians in February 2000 included an upgrade in obligor to the Russian Federation[4].

The most important feature of the mentioned bond exchanges however is that they proposed to exchange bonds close to maturity against bonds with longer maturity. For instance, Pakistan proposed in November 1999 to exchange bonds due in December 1999, February 2002 (with a put in February 2000) and May 2000 against a bond maturing in 2005. This delayed repayment of principal and interest of course constitutes a loss in present value terms to the bond holders. We do not explicitly model the later repayment but instead include it in the reduction of the principal and the coupon.

The decision whether to default or not is made by comparing the expected terminal wealth, considering debt service and costs of default, under both scenarios. The level of country wealth at which the sovereign will be just indifferent between fulfilling its duty and defaulting, i.e., the level of wealth which triggers default, will be named the *endogenous default boundary*.

Recent successful litigation of Peru[5] shows that sovereigns face a litigation threat by the lenders. If the sovereign offers a bond exchange with too bad conditions, the lenders will try to seize foreign assets held by the sovereign. However, they are not able to recuperate the full principal. Part will be lost due to litigation costs. By assuming that the sovereign will always offer a new debt contract such that the lenders are indifferent between accepting and litigating[6], we endogenize the fraction of the original contractual obligations the sovereign continues to honor after default.

The model provides a closed form solution for the default triggering wealth level, sovereign bond prices and credit spreads. However, the endogenous repayment fraction has to be solved numerically. Numerical analysis shows an important inverse relation between the credit spreads of sovereign debt and the amount that can be recuperated through litigation. The distance of the country's wealth to the default boundary influences the credit spreads inversely as well, while an increasing size of the coupon leads to higher spreads.

We compare sovereign debt to domestic corporate debt by analyzing the impact of the bankruptcy scheme: While a sovereign will be litigated for the principal, a corporation will be liquidated, i.e., all the assets of the corporation will be transferred to the lenders, minus the liquidation costs.

The stronger threat of liquidation leads corporates to wait longer until they default than sovereigns. This leads to considerably lower spreads of the corporate. If, in addition, we also assume lower liquidation costs for the corporate, the spreads decrease even more.

The behavior of the credit spread curves is similar to the credit spreads obtained by the classic models of corporate credit spreads such as Longstaff and Schwartz [32] and Leland and Toft [30].

The paper is organized as follows. In section 2 I provide a review of the existing literature on sovereign debt. Section 3 describes the setup of the model, while section 4 focuses on the determination of the endogenous default boundary. Section 6 gives numerical simulations of the endogenous default boundary and the resulting credit spreads. In section 7 we compare sovereign and corporate debt. Section 8 concludes and discusses the model's limitations and potential extensions of the model.

2 Literature review

While there exists an extensive literature of corporate bankruptcy and credit risk, sovereign credit risk is still a topic with rather little previous research.

The earliest models of sovereign debt focused on the role of debt as an acceleration for countries' economic growth, and sought to determine an optimal debt capacity for sovereigns. McDonald [33] provides a review of these so-called 'growth-cum-debt' models.

A second string in the sovereign debt literature considers the question why sovereigns repay their debt at all. Since liquidation of sovereigns is not enforceable, there is no direct way a lender can reclaim his assets in case of a default. This could tempt sovereigns just to keep the remaining outstanding debt service and hide behind their sovereignty. Reputation models let a borrower loose its good reputation as a creditworthy borrower if they fail to meet their obligations. A sovereign with bad reputation seeking to refinance its debt is repudiated by the lender banks.

Eaton and Gersowitz [14] present a model in which the sovereign uses debt to smooth its consumption by borrowing in bad states of nature and repaying debt in good states. Since a default would lead to a repudiation and thus the lack of consumption smoothing possibilities, Eaton and Gersowitz find a competitive equilibrium in which sovereigns honor their debt.

Bulow and Rogoff [6] show that debt is not the only way for a sovereign to smooth shocks in its production. If the sovereign has assets denominated in foreign currency, it can save and dissave to achieve the desired smoothing effect. They do not find equilibrium without additional penalties such as economic sanctions, because countries would not repay their debt if they do have enough foreign assets to ensure the hedging of its output. Furthermore, the competitive nature of the market for bank loans entails that the defaulted sovereign will get access to new credits fairly easily, making the threat of eternal repudiation fade.

In a second paper Bulow and Rogoff [7] demonstrate that in equilibrium lenders will always be willing to renegotiate the debt contract. Through the threat of sanctions leading to disruption of trade which is beneficial to the borrower, the lender is able to force the borrower to repay at all. However, the borrower will only use the fraction of its exports for debt repayments to achieve an optimum between gains from keeping back part of the due debt service and the cost of sanctions, and will constantly seek to recontract the loan. This property is in accord with observed frequent renegotiations of sovereign loan repayment programs.

Kulatilaka and Marcus [28] present a model that studies the timing decision of strategic defaults of sovereigns. They model a country's stock GDP as a continuous-time stochastic process. The burden of debt service induces a drag on GDP growth which increases with greater debt-service to GDP ratio, a relation characterized by an ad-hoc function. If the cost of the foregone GDP growth due to debt service gets higher than the cost of the sanctions, which come in form of a slump in stock GDP, the country chooses to default. Since Kulatilaka and Marcus assume that the sovereign faces repudiation risk, i.e., that it will not be able to take debt again after defaulting, which implies that default is a one-time option. The authors use contingent-claims analysis to numerically solve for the present value of consumption depending on the debt ratio, and for the responsiveness of the consumption gain induced by the loan to the risk premium. They find that the gain increases with the variance of the GDP process, and decreases with the default penalty on stock GDP and the GDP drag.

Gibson and Sundaresan [18] present a continuous-time model which starts with a country that uses borrowing to generate exports. In the event of default, the sovereign is punished by trade sanctions as well as the seizing of parts of the exports. The role of exports in the model is thus one of imperfect collateral. The lenders can improve the recovery rate by seizing exports. While sanctions only hurt the borrower without directly benefitting the lenders, the seized exports can be converted to money by the lenders. The authors derive an optimal voluntary default rule by looking at the Nash solution of a bargaining game where lenders and the borrower divide the country's wealth and future exports. Gibson and Sundaresan study the behavior of the sovereign's credit spreads over risk free treasury and find that the existence of a threat of sanctions and seizing of exports improves the value of sovereign debt.

There is little empirical evidence of sovereign defaults. While Moody's and Standard and Poor's rate sovereigns and include losses due to sovereign default in their loss statistics, they do not list them separately. Banks do possess loss statistics on their own loss experience with sovereign borrowers, but if this information gets outside it is so aggregated as to be useless. See for example Hurt and Felsovalyi [24].

While academia has focused on questions such as why sovereigns lend or repay money at all and tried to determine optimal debt capacities, practical research in the financial community was faced with the task of assessing a sovereign's credit quality. Popular ratios used for the rating of the credit quality of a sovereign include the debt-to-GDP ratio, the debt service-to-exports ratio and the import cover (official reserves divided by imports per month), as well as analyzing the macroeconomic foundations and the political environment of the country. Banks developed in-house rating systems which transform these ratios and economic indicators into a score, or a cardinal ranking, but still rely mainly on analysts' judgement and experience.

Edwards [16] examined the relation between the yield spread on sovereign debt on various macroeconomic variables to determine which best explain government payments crises. More recent work in the area includes Boehmer and Megginson [4]. Another approach consists in backing the dynamics of credit quality out of sovereign bond prices by assuming a specific stochastic process for the credit quality. Claessens and Pennacchi [8] derive the first of these models. They assume that credit quality is unobservable, but follows a continuous diffusion process in the spirit of the Longstaff and Schwartz [32] model for valuing corporate debt. They derive estimates of default probabilities of Brady bonds and examine the influence of economic time series on the default probabilities. Cumby and Evans [10] criticize that Claessens and Pennacchi impose too many restrictions on the stochastic process governing credit quality. Since more general stochastic processes are difficult to solve in closed form, Cumby and Evans move to discrete time and study three processes by conducting specification tests. Their results suggest that the added flexibility is important to fit the term structures of credit spreads.

Duffie et al. [12] construct a model to price Russian dollar-denominated bonds. They estimate the joint distribution of the term structure of LIBOR swap and Russian bond yields by extending the model of Duffie and Singleton [11]. Promised future cash flows are discounted using a default-adjusted rate equal to the risk free rate plus a credit spread which reflects the probability and magnitude of a credit event. They allow for different discount factors since sovereign bonds usually do not have cross-default clauses.

Harvey [22] [21] conducts studies of the returns of emerging market bonds, including sovereign bonds. Emerging market bond returns are highly variable over time, and highly correlated with equity market returns in crises.

Recently banks try to extract probabilities of default out of traded bond prices. However, lack of liquid markets for emerging market debt and heterogenous and intricate debt covenants complicate the undertaking. The Portfolio Research department of J.P.Morgan [36] developed a framework to price sovereign bonds by looking at the "credit fundamentals": Similarly to CreditMetrics, they use rating migration probabilities, recovery rates and default probabilities to determine the spread which investors require to be compensated for the default risk of sovereign bonds.

3 Specification of the model

In our analysis of sovereign behavior we will consider a sovereign which has written a bond denominated in foreign currency. We assume that the country's wealth follows a Geometric Brownian Motion, and define it as the market value of all assets either directly controlled by the sovereign's government, or indirectly controlled by the sovereign's ability to seize assets on its territory, that can be converted into hard currency, for instance the US dollar. This allows us to directly compare the country's wealth with the promised payments of the bond. The effects of exchange rate changes are included in the state variable determining country wealth and debt value. We will postulate that the country seeks to maximize its wealth at the end of the bond's life.

The holders of this bond are risk-neutral and identical investors. Their only requirement is a non-negative expected return on their investment, i.e., they only buy the bond if they at least break even.

The sovereign can choose whether to fully repay the bond, or whether to default: He will stop servicing the bond and offer a new bond in exchange which has a lower coupon and principal. The bond holders have the possibility to litigate the sovereign in order to try to seize at least part of the due principal. Since there is no formal bankruptcy court for sovereigns the litigation is difficult for the borrowers: They might try to claim assets held by the sovereign in a third country by battling in the courts of that country. This litigation process is costly not only in the explicit sense, i.e., high lawyer fees, but also in the implicit sense: The enforceability of the borrowers' claims is weak. This means that they will be able to claim only a small part of the principal. The sovereign will design its exchange offer just such that the bond holders will accept it.

Simultaneously with the default, the sovereign will incur costs of defaulting: These costs include reputational costs, losing market access for both trade and long term financing, and experiencing of output losses due to a lack of foreign capital for investments. However, we do not model these different costs of default separately but instead subsume them into a single parameter which reduces the country's growth rate of wealth. The costs of defaulting in our model are thus lost growth opportunities and hence lower expected wealth in the future.

By assumption, we model the relative change in the country's wealth R_t as a geometric Brownian motion.

$$dR_t = (\mu - \delta)\, R_t dt + \sigma R_t dW_t \tag{1}$$

μ is the growth rate of wealth. δ is the outflow, comprising consumption and debt service. $\sigma R_t dW_t$ is a diffusion term to depict changes in foreign currency denominated wealth due to shocks to the economy, either in the real economy or through the currency market.

The sovereign has a single bond outstanding with maturity T, principal P, paying a coupon rate c. The outflow rate δ includes the debt service payments cP as well as the consumption of the sovereign[7].

Default occurs when country wealth falls under a critical value K_t which we call the default boundary. If the country wealth decreases, the costs of default decrease as well since they are assumed to be proportional to the country's wealth. The benefits of defaulting, i.e., the reduced debt service payments, remain constant since they are linked to the principal of the bond and thus independent of the country's wealth. Hence, if country wealth falls enough such that the costs and benefits of defaulting are equal, the default boundary is reached at which the sovereign decides to default.

In this section we will first leave K_t as an exogenous, known constant (K_t will be endogenized in section 4). If the sovereign defaults at time t, it offers a new bond in exchange which promises to pay only a fraction α of the original contractual obligations[8]. This implies that total debt service from that moment on will only be αcP with a final principal repayment of αP[9]. However, in this first step we will keep α as a known constant. We will endogenize the exchange offer and the parameter α in section 5.

The negative consequences of default for the sovereign are that the country has to suffer a deduction $\lambda R_t dt$ in its foreign currency flow. I.e., from then on the country's wealth will switch to a lower growth rate[10]. An interpretation of λ is that the lender countries enact a trade embargo against the sovereign which curbs its exports, thus generating less foreign currency revenue. But we also have to think of "informal" trade sanctions in the sense of a loss of the country's reputation as a sound trade partner. For instance, international banks might be reluctant to finance trades between corporations after a default, rendering trade with the country more difficult.

The change in wealth after default is

$$dR_t^D = (\mu - \delta - \lambda)\, R_t^D dt + \sigma R_t^D dW_t \tag{2}$$

We can calculate the value of a single bond with coupon rate c, principal P and maturity T as follows, where $f(s; R_t, K)$ is the density of the first passage time s to the default boundary K[11].

$$\begin{aligned} b(T; R_t) \;\; = \;\; & \int_0^T e^{-rs} cP[1 - F(s; R_t, K)]ds \\ & + e^{-rT} P[1 - F(T; R_t, K)] \\ & + \int_0^T \left[\int_s^T e^{-ru} \alpha cP\, du + e^{-rT} \alpha P \right] f(s; R_t, K) ds. \end{aligned} \tag{3}$$

The first term is the expected value of the stream of discounted coupon payments given that the sovereign is solvent. The probability of this event is the complement of the probability that the country's wealth hits the reorganization boundary K over the life of the bond. The second term is the expected discounted principal repayment given that the sovereign remained solvent until maturity. The third term are the expected discounted coupon payments αc of the rescheduled debt after default, and the reduced expected discounted principal repayment αP in case of a default prior to maturity.

By integrating by parts we can simplify equation (3) to

$$\begin{aligned} b(T; R_t, K) &= \frac{cP}{r} - e^{-rT}\frac{cP}{r} + e^{-rT}\frac{cP}{r}F(T; R_t, K) \\ &\quad -\int_0^T e^{-rs}\frac{cP}{r}f(s; R_t, K)ds \\ &\quad +e^{-rT}P[1 - F(T; R_t, K)] \\ &\quad +\int_0^T \left[\frac{\alpha cP}{r}\left(e^{-rs} - e^{-rT}\right) + e^{-rT}\alpha P\right] \\ &\quad f(s; R_t, K)ds, \end{aligned}$$

which we can further simplify to find the value of the bond.

Proposition 1 *The value of a sovereign bond paying a coupon rate of c p.a. and a principal P at maturity T, and which, after country wealth falls below a constant default triggering country wealth level K, i.e., after default is exchanged into a bond paying a coupon rate of αc and a final repayment of αP at maturity, is at time t given by*

$$\begin{aligned} b(T; R_t, K) &= \frac{cP}{r} + e^{-rT}\left(P - \frac{cP}{r}\right)[1 - F(T; R_t, K)] \\ &\quad +\alpha e^{-rT)}\left(P - \frac{cP}{r}\right)F(T; R_t, K) \\ &\quad +(\alpha - 1)\frac{cP}{r}\int_0^T e^{-rs}f(s; R_t, K)ds. \end{aligned} \tag{4}$$

A solution for $F(T; R_t, K)$ can be found in Harrison [20], and a solution for $\int_0^T e^{-rs}f(s; R_t, K)ds$ is contained in Rubinstein and Reiner [38]. These solutions assume that K is a constant.

$$F(T; R_t, K) = N(h_{1,T}) + \left(\frac{R_t}{K}\right)^{-2a} N(h_{2,T}) \tag{5}$$

$$\int_0^T e^{-rs} f(s-t;R_t,K)ds = \left(\frac{R_t}{K}\right)^{-a+g} N(q_{1,T}) + \left(\frac{R_t}{K}\right)^{-a-g} N(q_{2,T}) \qquad (6)$$

with

$$\begin{array}{llllllll} a & = & \frac{\mu-\delta-(\sigma^2/2)}{\sigma^2}; & b & = & \ln\left(\frac{R_t}{K}\right); & g & = & \frac{\sqrt{(a\sigma^2)^2+2r\sigma^2}}{\sigma^2}; \\ h_{1,T} & = & \frac{-b-a\sigma^2 T}{\sigma\sqrt{T}}; & q_{1,T} & = & \frac{-b-g\sigma^2 T}{\sigma\sqrt{T}}; & & & \\ h_{2,T} & = & \frac{-b+a\sigma^2 T}{\sigma\sqrt{T}}; & q_{2,T} & = & \frac{-b+g\sigma^2 T}{\sigma\sqrt{T}}; & & & \end{array}$$

4 Endogenous default boundary

So far we have treated the default boundary K as exogenous and constant. Now we will think of the sovereign trying to maximize its wealth net of consumption and including the repayment of the principal at the end of the bond's life. This expected terminal wealth depends on the decisions of the sovereign: At each time s prior to maturity T the sovereign decides whether to default or not by comparing the value of the two strategies.

Denoting s as the hitting time of R_t to the boundary K_t, we define $V_t(s)$ as the country's expected wealth net of consumption, but including debt payments. Its dynamics depend on the sovereigns decision to default: Prior to default, the growth rate of $V_t(s)$ is the growth rate of country wealth μ minus the contractual coupon payments cP. After default, $V_t(s)$ grows with a reduced growth rate of $\mu - \lambda$ due to the damage of the trade disruptions, reputation loss etc. caused by the default, but only minus debt payments of αcP because of the smaller coupon. We assume that the sovereign remains unlevered after maturity of the bond, i.e., it does not have to service any coupons anymore, but if it defaulted, it will continue to suffer the damage λ. Assuming that the sovereign defaults at s, we thus define $V_t(s)$ as (To make clearer whether we mean the dynamics after default or after non-default we write $V_t^D(s)$ and $V_t^N(s)$ respectively):

$$dV_t(s) = \begin{cases} \left(\mu V_t^N - cP\right) dt + \sigma V_t^N dW_t & \text{if } t < s \text{ and } t < T \\ \left((\mu-\lambda) V_t^D - \alpha cP\right) dt + \sigma V_t^D dW_t & \text{if } t \geq s \text{ and } t < T \\ \mu V_t^N dt + \sigma V_t^N dW_t & \text{if } T < s \text{ and } t > T \\ (\mu-\lambda) V_t^D dt + \sigma V_t^D dW_t & \text{if } T > s \text{ and } t > T \end{cases} \qquad (7)$$

Furthermore, at T the sovereign repays the principal. We therefore define the net of consumption country wealth just after repayment of the principal as

$$V_{T^*}(s) = \begin{cases} V_t^N(s) - P & \text{if } T < s \\ V_t^D(s) - \alpha P & \text{if } T > s \end{cases} \tag{8}$$

denoting the time just after repayment of the principal as T^*. In other words, if the sovereign defaulted before maturity of the bond, it will only repay α of the principal P. If it did not default, it repays the full principal P.

The value of the strategy not to default, $U^N(s)$, is the expected country wealth, net of consumption, capitalized at T, after debt repayment and conditional on no default at s is

$$U^N(s) = \mathrm{E}_s \left[V_{T^*}(s) \middle| \, s > T \right] = \mathrm{E}_s \left[V_t^N \middle| \, s > T \right] - P. \tag{9}$$

If the country would decide to default at s, it would benefit of the lower coupon αc and principal αP of the exchanged bond, but its wealth would grow at the smaller growth rate given in (2) since it suffers the deduction due to the sanctions and worsened trade environment. This reduced growth rate continues also after the maturity of the bond. Hence, the sovereign must incorporate the future loss in growth after the maturity of the bond into its decision. As we assumed that the sovereign remains unlevered after maturity of the bond, the dynamics of $V_t^D(s)$, i.e., the country's wealth net of consumption are given by $dV_t^D(s) = (\mu - \lambda) V_t^D(s)dt + \sigma V_t^D(s) dW_t$, with $t > T$ and $s < T$. If the country had not defaulted at s, its wealth would evolve with $dV_t^N(s) = \mu V_t^N dt + \sigma V_t^N dW_t$ with $t > T$. The log-difference between the two trends is thus $d \ln V_t^D - d \ln V_t^N = -\lambda dt$. This allows us to calculate the expected discounted growth loss after maturity ΔV, capitalized at T, under risk neutrality, as

$$\begin{aligned}
\Delta V &= \mathrm{E}_s \left[V_{T^*}^D \int_T^\infty \lambda e^{-r(u-T)} \left(V_u^D(s) - V_u^N(s) \right) du \,\middle|\, V_s, s < T \right] \\
&= \lambda e^{rT} \mathrm{E}_s \left[V_{T^*}^D \int_T^\infty \exp\left\{ -ru \cdot \left(\ln V_t^D - d \ln V_t^N \right) \right\} du \,\middle|\, V_s, s < T \right] \\
&= \lambda \mathrm{E}_s \left[V_{T^*}^D(s) | V_s, s < T \right] \int_0^\infty \exp\left\{ -(r+\lambda)(u-T) \right\} du \\
&= \frac{\lambda}{\lambda + r} \mathrm{E}_s \left[V_{T^*}^D(s) | V_s, s < T \right]
\end{aligned} \tag{10}$$

where $\mathrm{E}_s\left[V_{T^*}^D(s)|s<T\right] = \mathrm{E}_s\left[V_t^D(s)|s<T\right] - \alpha P$ is the expected country wealth just *after* repayment of the principal. We can thus calculate $U^D(s)$, the expected value of the default strategy, capitalized at time T: It is the expected net of consumption country wealth at T, given that the sovereign defaulted at s, including the costs of default and the reduced debt service payments, minus the repayment of the principal and the expected discounted growth loss after maturity.

$$\begin{aligned}
U^D(s) &= \mathrm{E}_s\left[V_{T^*}^D\middle| s<T\right] - \frac{\lambda}{\lambda+r}\mathrm{E}_s\left[V_{T^*}^D(s)\middle| s<T\right] \\
&= \left(1-\frac{\lambda}{\lambda+r}\right)\left(\mathrm{E}_s\left[V_t^D\middle| s<T\right] - \alpha P\right) \\
&= \frac{r}{\lambda+r}\left(\mathrm{E}_s\left[V_t^D\middle| s<T\right] - \alpha P\right). \qquad (11)
\end{aligned}$$

It is optimal to default at time s when the expected value of defaulting, $U^D(s)$, exceeds the expected value of continuing to fully service the bond, $U^N(s)$, conditional on the information set at time s. At the default boundary $K(s,T,\alpha)$ those values are equal, and the sovereign is indifferent between defaulting and continuing to honor its contractual obligations. We can therefore determine $K(s,T,\alpha)$ as the value V_s^* which solves

$$\begin{aligned}
U^N(s) &= U^D(s) \\
\Rightarrow \quad \mathrm{E}_s\left[V_t^N\middle| s>T\right] - P &= \frac{r}{\lambda+r}\left(\mathrm{E}_s\left[V_t^D\middle| s<T\right] - \alpha P\right) \qquad (12)
\end{aligned}$$

To solve for $K(s,T,\alpha)$ we need to calculate the conditional expectation at time s of the country's net of consumption wealth at maturity of the bond which is

$$\mathrm{E}_s\left[V_t^D\middle| s<T\right] = V_s e^{(\mu-\lambda)(T-s)} + \frac{\alpha c P\left(1-e^{(\mu-\lambda)(T-s)}\right)}{\mu-\lambda} \qquad (13)$$

in case of default at time s and

$$\mathrm{E}_s\left[V_t^N\middle| s>T\right] = V_s e^{\mu(T-s)} + \frac{cP\left(1-e^{\mu(T-s)}\right)}{\mu} \qquad (14)$$

in case of no default. The derivations of equations (13) and (14) can be found in appendix B. We can therefore solve (12).

Proposition 2 *The sovereign will default, i.e., stop servicing the original bond and offer a new bond in exchange with a reduced coupon rate of αc and a principal of αP, when its wealth hits the endogenous default boundary $K(s,T,\alpha)$, which is defined as*

$$K(s,T,\alpha) = P\frac{1+\frac{c}{\mu}\left(e^{\mu(T-s)}-1\right)-\frac{r}{r+\lambda}\alpha\left(1+\frac{c}{\mu-\lambda}\left(e^{(\mu-\lambda)(T-s)}-1\right)\right)}{e^{\mu(T-s)}-\frac{r}{r+\lambda}e^{(\mu-\lambda)(T-s)}} \quad (15)$$

To fully understand the sovereign's decision to default, we have to realize that the debt service payments and thus the potential benefits of default are independent of the level of wealth. The cost of default is a function of the level of the country's wealth, however. Therefore, if wealth falls below the endogenous default boundary, the cost of default falls below the benefits of reduced coupon payments.

Furthermore, note that the sovereign will always default *before* its wealth reaches zero since $K(s,T,\alpha)$ is positive for any choice of parameters. In other words, the sovereign will never default due to *liquidity reasons*, since the endogenous default boundary where it *chooses* to default is always higher than zero, i.e., the point at which it *has to* default. A proof can be found in appendix C.

4.1 Valuation of the bond with time-dependent endogenous default boundary $K(s,T,\alpha)$

Since the endogenized default boundary $K(s,T,\alpha)$ is *time-dependent*, we cannot use the formulas in (5) and (6) to calculate the first-passage time densities in order to value the bond, since they require a *constant boundary.* The boundary $K(s,T,\alpha)$, however, is curved.

Still, by transforming the process of R_t we can use a result by Durbin [13] to compute the first-passage time densities.

Durbin shows that the first-passage density of a Brownian motion W_t to a curved boundary $a(s)$ at $t=s$ can be well approximated by

$$
\begin{aligned}
f(s,a(s)) \quad \approx \quad & \left[\frac{a(s)}{s} - a'(s)\right]\phi(s) \\
& -\int_0^s \left[\frac{a(u)}{u} - a'(u)\right]\left[\frac{a(s)-a(u)}{s-u} - a'(s)\right]\phi(u,s)du,
\end{aligned} \tag{16}
$$

where $a'(s)$ is the derivative of $a(s)$, $\phi(s)$ is the density of the Brownian motion W_t at time s, evaluated at $a(s)$, and $\phi(u,s)$ is the joint density of W_τ at times u and s, evaluated at $a(u)$ and $a(s)$.

In our model default occurs if the country wealth R_t hits the boundary K_t from above. We transform the problem into the first hitting time of a standard Brownian motion to a transformed boundary $A(s)$ defined in the interval $[0,T]$:

$$
A(s) = \frac{\ln\left(\frac{K(s,T,\alpha)}{R_t}\right) - \left(\mu - \delta - \frac{1}{2}\sigma^2\right)s}{\sigma} \tag{17}
$$

To calculate the expression in (16) we need the derivative of the boundary which is

$$
\begin{aligned}
a'(s) \quad = \quad & \frac{\partial A(s)}{\partial s} = \frac{1}{\sigma}\frac{\partial \ln K(s,T,\alpha)}{\partial s} - \frac{\mu - \delta - \frac{1}{2}\sigma^2}{\sigma} \\
= \quad & \left[\frac{\frac{r}{r+\lambda}\alpha c e^{(\mu-\lambda)(T-s)} - c e^{\mu(T-s)}}{K(s,T,\alpha)} P - \psi(s)\right]\sigma^{-1}\Psi(s)^{-1} \\
& - \frac{\mu - \delta - \frac{1}{2}\sigma^2}{\sigma}
\end{aligned} \tag{18}
$$

with

$$
\Psi(s) = e^{\mu(T-s)} - \frac{r}{r+\lambda}e^{(\mu-\lambda)(T-s)}
$$

and

$$
\psi(s) = \frac{\partial \Psi(s)}{\partial s} = \frac{r(\mu-\lambda)}{r+\lambda}e^{(\mu-\lambda)(T-s)} - \mu e^{\mu(T-s)}.
$$

Furthermore, we need the density of the Brownian motion W_t at time s and the joint density at times u and s, evaluated at $A(s)$ and $A(u)$, respectively:

$$
\begin{aligned}
\phi(s) &= \frac{1}{\sqrt{2\pi(s-t)}} \exp\left\{-\frac{1}{2}\frac{(A(s)-A(t))^2}{s-t}\right\} && (19)\\
\phi(u,s) &= \phi(s)\frac{1}{\sqrt{2\pi(u-s)}} \exp\left\{-\frac{1}{2}\frac{(A(u)-A(s))^2}{u-s}\right\} && (20)
\end{aligned}
$$

We can thus easily compute the cumulative probability of default $F(T-t;A(s)) = \int_t^T f(s,t,A(s))ds$ and the expression $\int_t^T e^{-rs} f(s,t,A(s))ds$ with $f(s,t,A(s))$ as defined by (16) and $A(s)$ as defined by (17). These results we enter into equation (4), yielding the value of the bond:

Proposition 3 *The value of a sovereign bond paying a coupon rate of c p.a. and a principal P at maturity T, and which, after country wealth falls below an endogenous default triggering country wealth level $K(s,T,\alpha)$, determined by the sovereign seeking to maximize terminal wealth, , i.e., after default is exchanged into a bond paying a coupon rate of αc and a final repayment of αP at maturity, is at time t approximatively given by*

$$
\begin{aligned}
b(t,T;R_t,\alpha,A(s)) &= \frac{cP}{r} + e^{-r(T-t)}\left(P-\frac{cP}{r}\right)[1-\int_t^T f(s,t,A(s))ds]\\
&\quad +\alpha e^{-r(T-t)}\left(P-\frac{cP}{r}\right)\int_t^T f(s,t,A(s))ds\\
&\quad +(\alpha-1)\frac{cP}{r}e^{rt}\int_t^T e^{-rs} f(s,t,A(s))ds. && (21)
\end{aligned}
$$

with

$$
\begin{aligned}
f(s,t,A(s)) &\approx \left[\frac{A(s)}{s} - A'(s)\right]\phi(s)\\
&\quad -\int_t^s \left[\frac{A(u)}{u} - A'(u)\right]\left[\frac{A(s)-A(u)}{s-u} - A'(s)\right]\phi(u,s)du,
\end{aligned}
$$

$$
\begin{aligned}
A(s) &= \frac{\ln\left(\frac{K(s,T,\alpha)}{R_t}\right) - \left(\mu - \delta - \frac{1}{2}\sigma^2\right)(s-t)}{\sigma} \\
K(s,T,\alpha) &= P\frac{1 + \frac{c}{\mu}\left(e^{\mu(T-s)} - 1\right) - \frac{r}{r+\lambda}\alpha\left(1 + \frac{c}{\mu-\lambda}\left(e^{(\mu-\lambda)(T-s)} - 1\right)\right)}{e^{\mu(T-s)} - \frac{r}{r+\lambda}e^{(\mu-\lambda)(T-s)}} \\
\phi(s) &= \frac{1}{\sqrt{2\pi(s-t)}}\exp\left\{-\frac{1}{2}\frac{(A(s)-A(t))^2}{s-t}\right\} \\
\phi(u,s) &= \phi(s)\frac{1}{\sqrt{2\pi(u-s)}}\exp\left\{-\frac{1}{2}\frac{(A(u)-A(s))^2}{u-s}\right\} \\
A'(s) &= \left[\frac{\frac{r}{r+\lambda}\alpha c e^{(\mu-\lambda)(T-s)} - c e^{\mu(T-s)}}{K(s,T,\alpha)}P - \psi(s)\right]\sigma^{-1}\Psi(s)^{-1} \\
&\quad - \frac{\mu - \delta - \frac{1}{2}\sigma^2}{\sigma} \\
\Psi(s) &= e^{\mu(T-s)} - \frac{r}{r+\lambda}e^{(\mu-\lambda)(T-s)} \\
\psi(s) &= \frac{\partial\Psi(s)}{\partial s} = \frac{r(\mu-\lambda)}{r+\lambda}e^{(\mu-\lambda)(T-s)} - \mu e^{\mu(T-s)}.
\end{aligned}
$$

4.2 Special Case: Infinite maturity sovereign bond

If we let the maturity of the sovereign bond go to infinity, the default boundary becomes constant:

$$K^\infty = \lim_{T\to\infty} K_s = \frac{cP}{\mu} \tag{22}$$

We can thus apply the valuation given in Proposition 1 to get the value of an infinite maturity sovereign bond.

$$b^\infty(t; R_t, K^\infty) = \frac{cP}{r} + (\alpha - 1)\frac{cP}{r}e^{rt}\left(\frac{R_t}{K^\infty}\right)^{-a-g} \tag{23}$$

with

$$a = \frac{\mu-\delta-(\sigma^2/2)}{\sigma^2}; \qquad g = \frac{\sqrt{(a\sigma^2)^2+2r\sigma^2}}{\sigma^2}$$

5 Endogenizing the repayment fraction α

When the sovereign reaches the default boundary $K(s,T,\alpha)$, it proposes to the lenders to exchange the current bond against a new bond with the same maturity, but paying only a fraction α of the originally promised coupon c and principal P. Assuming this happens at time s, the market value of this new bond would be $b(s,T;R_s,\alpha c,\alpha P,A(s))$.

The lenders can reject the offer, litigate the country and try to seize assets of the sovereign held abroad in order to recuperate their losses. They cannot seize the full principal, however, but only a fraction $(1-\gamma)P$[12]. The nature of these litigation costs is both explicit and implicit. Explicit costs of litigation are the costs of battling in court, and the costs of converting the seized assets in cash. For example, the lenders might be able to achieve the seizure of aircrafts belonging to the sovereign's national airline for which they will first have to find a buyer.

The implicit costs of litigation come from the weak enforceability of the debt contract. The borrowers are not able to claim the total principal in a formal bankruptcy court. Instead they have to try to lay their hands on assets of the sovereign held in third countries by claiming them in the courts of the third country. Most of the principal will be beyond their grasp, however, leading to a large litigation cost parameter γ. In fact, most of the litigation costs are implicit.

The lenders will accept the new bonds if their market value is not smaller than the proceedings from litigating the sovereign. Since the sovereign proposes the bond exchange, and litigation is costly, it will offer a repayment fraction α such that the lenders will not litigate.

Proposition 4 *The endogenous repayment fraction α is the α^* which solves the following equation:*

$$b(t,T;R_s,\alpha^*,A(s)) = (1-\gamma)P \tag{24}$$

i.e., the α^* which makes the market value of the new bonds with reduced coupon and principal payments equal to the gains from litigating the sovereign and recuperating part of the principal. Furthermore, with default at time s

we have $R_s = K(s, T, \alpha^*)$, *resulting in*

$$b(t, T; K(s, T, \alpha^*), \alpha^*, A(s)) = (1 - \gamma)P \tag{25}$$

Proposition 3 provided a closed form solution for the value of the bond. We use this formula to numerically solve for the α^* which makes the price of the renegotiated bond and the litigation value equal.

Inserting the obtained α^* into the equations given in proposition 3 yields the price of the sovereign bond today, i.e., $b(t, T; R_t, \alpha^*, A(s))$.

6 Numerical results

For the numerical analysis of the sensitivities of the endogenous repayment fraction α, the endogenous default boundary K_t and the sovereign credit spreads we need realistic estimates of the parameter values so that the obtained spreads will be close to the observed spreads of emerging market bonds.

We will use the parameter values for the interest rate $r = 0.05$, for the principal $P = 100$, for the coupon rate $c = 0.1$.

To obtain a order of magnitude for the litigation costs, which are difficult to estimate since defaults on public debt have been rare, we will follow a J.P. Morgan Research article [37] in noting that Ecuadorian, Russian and Ivory Coast PDIs have been trading around 20% of principal just before default or a bond exchange respectively. Since the default probability of these bonds was very near to 1 just prior to the default event, we can take their value as an indication of the recovery rate of sovereign bonds. We will therefore use an estimate of the litigation costs of $\gamma = 0.8$ for the analysis. This means that the bond holders could regain 20% of the principal through litigation.

For the volatility of country wealth we will assume a $\sigma = 60\%$ to be in line with the long-term average volatilities of emerging market stock markets.

The costs of default are difficult to estimate as well. They consist of the impact of direct sanctions, e.g., a trade embargo, on the growth of the country's wealth. But they also encompass indirect costs, i.e., a loss of reputation as a trustworthy partner in business, or the lost access to international financial markets. Bulow and Rogoff [6] cite studies which show that the cost of sanctions are not very high. Similarly, we will assume a yearly cost of $\lambda = 4\%$ in lost wealth for defaults.

With the exception of the analysis of the sensitivity to the time to maturity of the bond, we will use a maturity of $T = 10$ years.

For the valuation of the bond and the calculation of the credit spreads we need to assume the current level of country wealth. We will select $R_t = 200$ since this is close enough to the outstanding principal of the bond to ensure that the parameters' impact on the credit spreads are material.

By numerically solving for the endogenous repayment fraction given in equation (25) and inserting the solution for the endogenous default boundary given in equation (15) into the valuation formula of the bond (4, 5 and 6) we obtain the price of the bond, which can be used for calculating the spread of the credit risky sovereign bond over a comparable coupon bearing default free bond.

$$S_{t,T} = \frac{cP}{b(t,T;R_t,\alpha^*,A(s))} - \frac{cP}{\left(1-e^{-rT}\right)\frac{cP}{r} + Pe^{-rT}} \tag{26}$$

Figures 1 to 18 show the spreads which we obtain by the model. They show the same order of magnitude as the spreads of emerging market sovereigns: In Rappaport [37] we see that in October 2000, the market spread of sovereign bonds ranged from about 100 for BBB rated countries such as Korea and China to 2000 for countries near Default such as Ivory Coast, Nigeria, Ecuador and Russia, rated CCC. The numerical simulations show that the major impact in determining the spreads of the sovereign bonds is the distance to default, i.e., the ratio of country wealth to outstanding principal, and the litigation costs, i.e., how big a part principal could the bond holders seize by litigation in case of a complete default, i.e., if the sovereign would not propose a bond exchange.

6.1 Dependence on litigation costs γ

The litigation costs γ determine the recovery rate in the case of litigation. Part of the recovered principal will have to be used to pay for legal and lawyer fees, and part of the principal cannot be claimed due to the limited enforceability of the sovereign debt contract. Since the sovereign will propose a new debt contract such that the lenders will be indifferent between litigating and accepting the reduced payments, higher litigation costs mean that the sovereign can propose a less valuable bond in exchange to the original bond if litigation costs are high. Figure 2 shows this relation. A lower repayment fraction α in turn implies that defaulting is more attractive. We can see this in figure 3 by the relation between γ and the default boundary K_t: The higher

γ, the higher K_t, i.e., the sooner the sovereign will choose to default. These two effects together lead to rapidly rising credit spreads as the litigation costs increase, shown in figure 1.

6.2 Dependence on initial country wealth R_t

Obviously, the lower the initial country wealth, the higher the probability that the wealth will hit the default boundary K_t leading to the country's default. This effect is offset slightly by the cost of defaulting which decreases together with country wealth, since they come in a loss in wealth growth which is proportional to the level of wealth. This leads to a increasing default boundary with higher country wealth (Figure 6). However, this effect is negligible compared to the increased probability of default. Due to the higher probability of default, the new bonds offered by the sovereign in a bond exchange are less valuable, c.p. Hence the sovereign has to increase the promised coupon rate of the new bonds to induce the bond holders to accept the offer. Therefore, the endogenous repayment fraction α increases as country wealth decreases (Figure 5). But these two effects are dwarfed by the increased probability of default caused by a smaller distance to default: Figure 4 shows how the increasing default probability leads to higher spreads over credit risk-free bonds.

6.3 Dependence on sanction rate λ

The costlyness of defaulting, though e.g. direct sanctions, trade disruptions and reputation loss and spillovers, is what keeps sovereigns from defaulting right away in our model. The higher the growth rate loss λ, i.e., the higher the costs of sanctions in present value terms, the less attractive is the option to default. But since the costs of default are proportional to the country wealth level, whereas the benefits of defaulting are not, the sovereign will still decide to default, albeit only at lower levels of wealth, or, in other words, it will wait longer until it defaults. We can see how the endogenous default boundary K_t decreases as λ increases in figure 9. This in turn leads to lower probabilities of default and hence lower credit spreads (Figure 7). These lower probabilities of default means that the sovereign can propose a bond with a lower coupon, since the probability that it will default again on the new bonds is small. However, this effect on the endogenous repayment fraction α is small (Figure 8).

6.4 Dependence on country wealth volatility σ

The higher the volatility of the underlying country wealth process, the bigger the probability that the country wealth will fall below the endogenous default boundary K_t, triggering default. This is also true for the new bonds the sovereign offers in exchange for the original bond. As can be seen in figure 11, the conditions of the bond exchange have to be improved in order to ensure acceptance by the bond holders: The repayment fraction α increases with higher volatility. As the benefits of defaulting, i.e., the saved coupon payments, get smaller, the endogenous default boundary declines, as shown in figure 12: The sovereign will wait longer until it decides to default.

These two effects determine the low volatility end of the shape of the credit spreads: First, at low levels of country wealth volatility, the high default boundary K_t and hence high default probability, together with a less attractive exchange offer, lead to high credit spreads (Figure 10). As country wealth volatility increases, the better exchange offer together with the lower default boundary decrease credit spreads. At a level of about 30%, however, the direct effect takes over: The higher the volatility of country wealth, the higher the probability that the country wealth will hit the default boundary. This direct effect increases the credit spreads.

6.5 Dependence on coupon rate c

In figure 13 the relation between coupon rate c and the sovereign's credit spread is shown. It is more tempting to default on a bond with high coupon payments since this entails a greater saving for the sovereign. This leads to a higher rescheduling boundary K_t as the coupon rate c increases (Figure 15). This in turn compels the sovereign to offer better conditions in the bond exchange to ensure that the bond holders accept the offer, leading to an increasing coupon rate αc of the new bonds (Figure 14). Since the sovereign will default at higher levels of wealth, its default probability is higher and hence the credit spreads increase with the coupon rate (Figure 13).

6.6 Dependence on maturity of the bond T

Figure 16 shows the credit spread term structure. For short term bonds the probability that country wealth will fall under the default boundary is very low, and hence the debt is almost risk free. This probability rises very fast however, and we observe a characteristic "hump" at maturities around

2 years. With longer maturities, the spreads decrease again. The reasons for this behavior are the following:

While the costs of defaulting, i.e., the loss of country wealth growth through direct sanctions, reputation loss etc., are independent of the time to maturity of the bond, the benefits of defaulting are dependent on the time to maturity of the bond through two links:

1. Discounting: If a cash flow is further away in time, it will be discounted more. This means that it will be smaller in present value terms.

2. Lifetime: The sovereign pays the coupon continuously. Therefore the total coupon payments on a long-term bond are higher than the total coupon payments on bond with the same coupon rate but a shorter maturity.

These two effect explain the behavior of the default boundary in figure 18 and bond exchange in figure 17:

Defaulting on a bond shortly before it matures is attractive since one can save part of the principal. Therefore, the endogenous default boundary is high for short maturity bonds. As the maturity of the bond increases, however, this gain gets smaller in present value terms, making default less attractive: The default boundary drops with increasing maturity, as can be seen in figure 18.

The level of the default boundary influences the endogenous repayment fraction α, i.e., the conditions of the new bonds the sovereign proposes in exchange. In figure 18 we see that longer maturity bonds have higher endogenous default boundaries and thus lower probability of default than short maturity bonds. Their value is thus higher. Since, in the event of default, the sovereign proposes an exchange offer against bonds with the same maturity as the original bonds. It can therefore offer a lower coupon and principal for long maturity bonds and still induce the bondholders to tender.

Now we can explain the hump-shaped credit spread curve in figure 16: At maturities very close to zero, the probability that the country wealth will reach the level which triggers default is very small, and credit spreads are small as well. They increase rapidly with increasing maturities, however, since the probability that the country will default increases as well. For longer maturities the spreads decrease again because of the increasing coupon of the bond offered in the bond exchange during default. To recall, the increasing α is due to the fact that litigation yields $(1-\gamma)P$ in present value terms, while accepting the bond exchange promises $e^{-r(T-s)}\alpha P$ in present value terms, which decreases with increasing time to maturity.

Figure 19 shows the credit spread curves for differing levels of initial country wealth: The higher the initial level of country wealth, or, in other words, the higher the distance to default of the sovereign, the less pronounced is the hump shape of the credit curves.

The behavior of the credit spread curves is similar to the credit spreads obtained by the classic models of corporate credit spreads such as Longstaff and Schwartz [32] and Leland and Toft [30].

7 Comparison to otherwise equal domestic corporate bonds

In section 5 we determined the repayment fraction α by assuming that the sovereign will always propose a bond exchange against a bond with a coupon rate such that the value of the new bonds with the reduced payments will be equal to the gains the lenders could obtain from litigating the country. We furthermore assumed that the lenders cannot litigate for more than the principal of the outstanding bonds, and that they incur costs of γP should they litigate. This allowed us to write proposition 4.

If the defaulting party is a domestic corporation[13], however, it will be subject to a bankruptcy code. This means that the lenders are able to *liquidate* the company, and thus claim all assets of the borrower. In doing this they will again incur costs, which we will denote by γ_{corp} as well.

Proposition 5 *In our framework, if the counterparty is a corporation and thus subject to a bankruptcy code, the endogenous repayment fraction α is given by*

$$b(t, T; K(s, T, \alpha^*), \alpha^*, A(s)) = (1 - \gamma)K(s, T, \alpha^*) \tag{27}$$

The main difference to the sovereign case is that the recovery rate is now dependent on the value of assets of the borrower. If a sovereign borrower decides to default even though its wealth is high, the lenders cannot get more than the principal of the bond minus the litigation costs. If the borrower is a corporate, the lenders could threaten to liquidate[14], inducing the borrower to not default at high wealth levels. Furthermore, the existence of a bankruptcy code implies lower costs, so that we would assume that $\gamma_{sov} > \gamma_{corp}$.

In the numerical analysis given in figures 20 to 22, we first use the same parameters of $\gamma_{sov} = \gamma_{corp} = 0.8$ and $\sigma = 0.6$ for both corporate and sovereign, to isolate the impact of liquidating versus litigating. Then we also reduce the liquidation costs of the sovereign to see the further reduction in spreads. The graph for the default boundary in figure 22 shows that a corporation's default boundary is lower than a sovereign's: The threat to access the whole wealth through liquidation, in the corporate case, is stronger than the threat to only seize the principal through litigation, in the sovereign case.

This stronger threat in turn leads the corporate to offer a more attractive bond exchange than the sovereign. Together, the lower probability of default, and the more attractive exchange offer lead to considerably lower spreads of the corporate, as shown in figure 20. If, in addition, we also assume lower liquidation costs for the corporate, the spreads decrease even more.

8 Conclusion

This paper analyzed the decision of a sovereign whether to continue to honor its bond, or whether to default, i.e., whether to propose an exchange against a less valuable bond.

Numerical analysis showed that the most important parameters which influence the credit spreads of a sovereign are the fraction of principal that could be recuperated through litigation, the distance of the country's wealth to the default boundary, the size of the coupon and the volatility of country wealth. Using a set of reasonable parameters, we obtain credit spreads of a size compatible with spreads observed on sovereign bond markets.

The behavior of the credit spread curves is similar to the credit spreads obtained by the classic models of corporate credit spreads such as Longstaff and Schwartz [32] and Leland and Toft [30].

A comparison to corporate debt shows, consistent with the stylized fact that sovereign debt is more risky than equally rated corporate debt, that otherwise equal corporate bonds have a lower spread than sovereign bonds, due to the stronger threat of liquidation.

Our model explains another stylized fact of sovereign defaults: Sovereigns tend to default on bonds close to maturity[15]. Indeed our model shows that sovereign bonds with maturities between 6 months and 2 years have very high spreads, reflecting the high probability of default.

The model could be implemented to price sovereign debt. However, estimation of some of the parameters, such as, for example, the impact of economic sanctions or the amount which could be won through litigation, is difficult. This is a general problem of structural models of sovereign debt, as most of the variables are unobservable since the value of a sovereign is not traded, unlike the value of a corporation.

A problem in our model is that the sanctions imposed by the lenders in the case of default are *not negotiation proof*. If the sovereign, after default and the imposition of sanctions, offers a small amount of money to lenders if they stop the sanctions, the lenders would accept since they do not profit from the sanctions themselves. This is a general problem of sanction-based punishment in sovereign credit models, and can only be solved in a repeated game setting.

Sovereigns do not default on their total debt. In fact, Ecuador tried to circumvent the invoking of cross default clauses to prevent its Eurobond defaulting. In our model, where the sovereign has just one bond outstanding, however, it is not possible to analyze partial defaults. The extension into a setting where the sovereign has different kind of bonds outstanding with differing features would allow us to examine a sovereign's behavior during default.

Also we assume the lenders to be homogeneous: They either all tender or they all reject the offer. In reality however, we see a much richer process: The tender leaders try to convince the necessary majority to tender, and the majority and the sovereign try to protect themselves against litigation by the hold-outs.

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A Figures

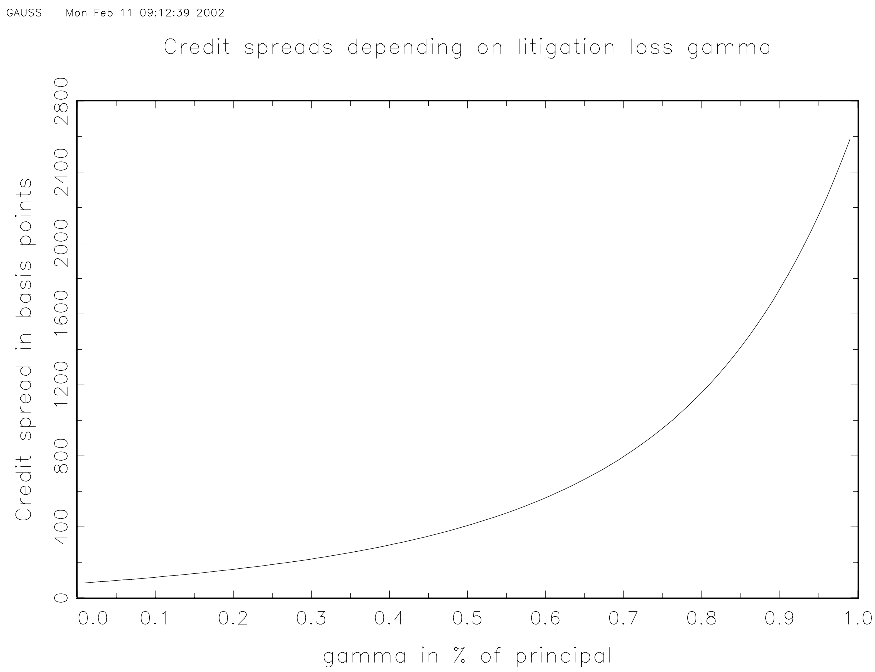

Figure 1: Credit spreads as a function of the litigation loss γ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\lambda = 0.04$ and $T = 10$.

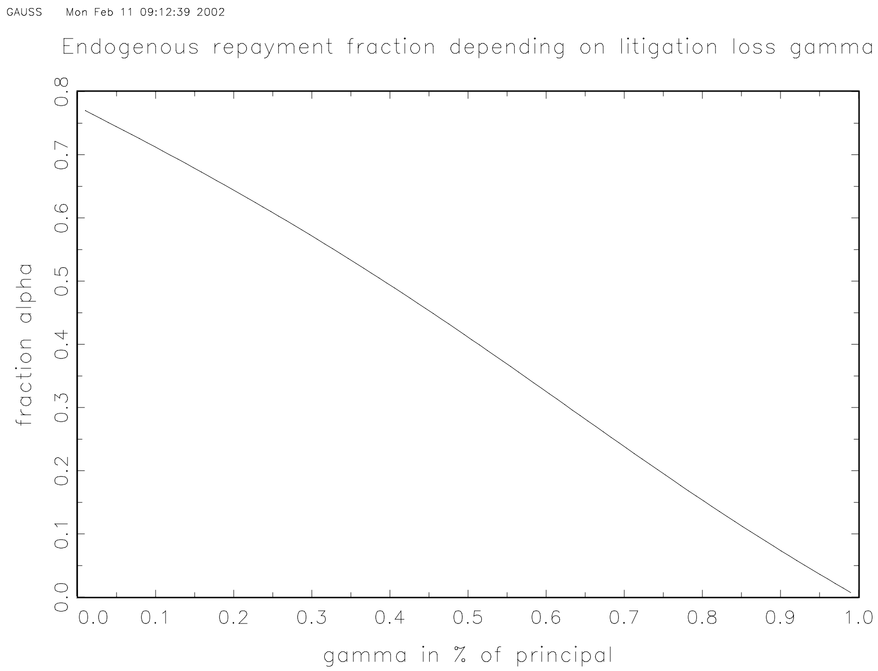

Figure 2: Endogenous repayment fraction α as a function of the litigation loss γ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\lambda = 0.04$ and $T = 10$.

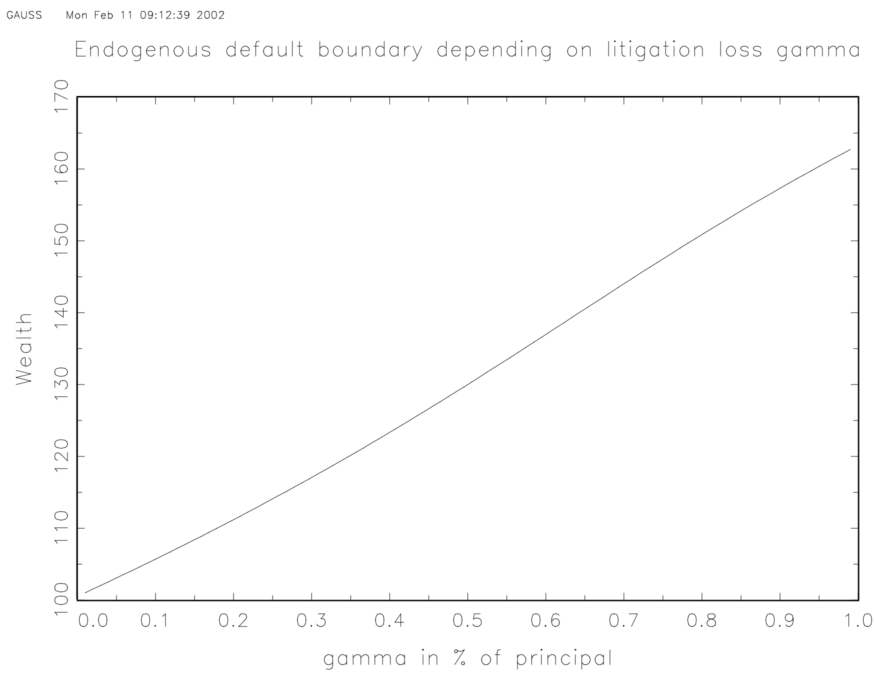

Figure 3: Endogenous default boundary K_t as a function of the litigation loss γ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\lambda = 0.04$ and $T = 10$.

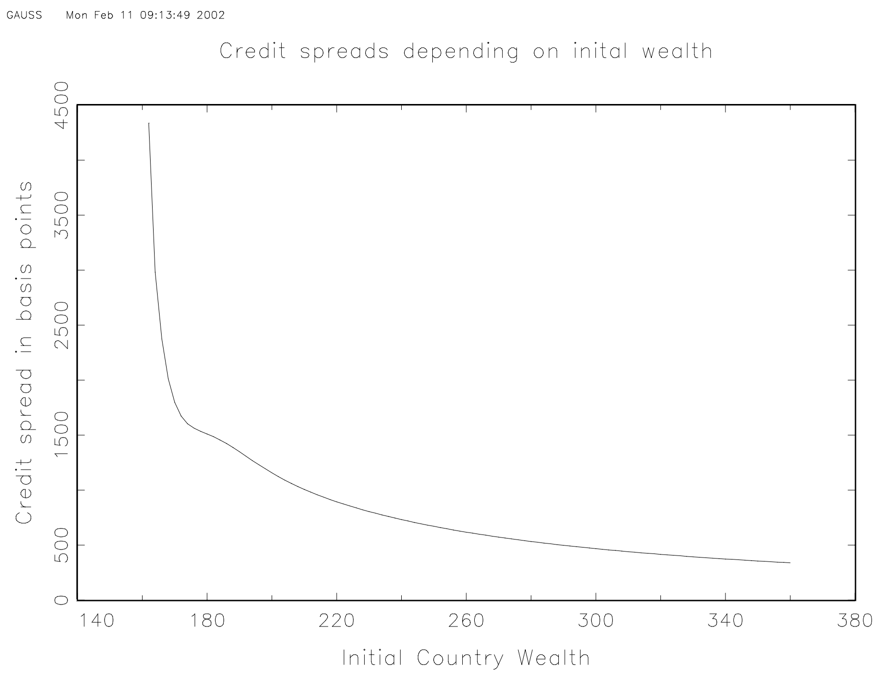

Figure 4: Credit spreads as a function of the initial country wealth R_t. Parameters are $r = 0.05$, $P = 100$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

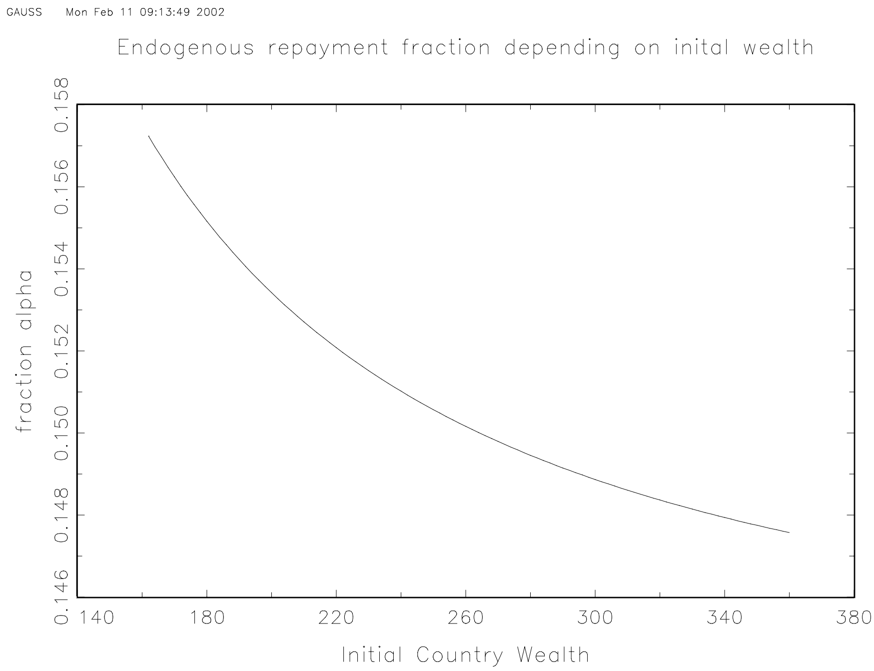

Figure 5: Endogenous repayment fraction α as a function of the initial country wealth R_t. Parameters are $r = 0.05$, $P = 100$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

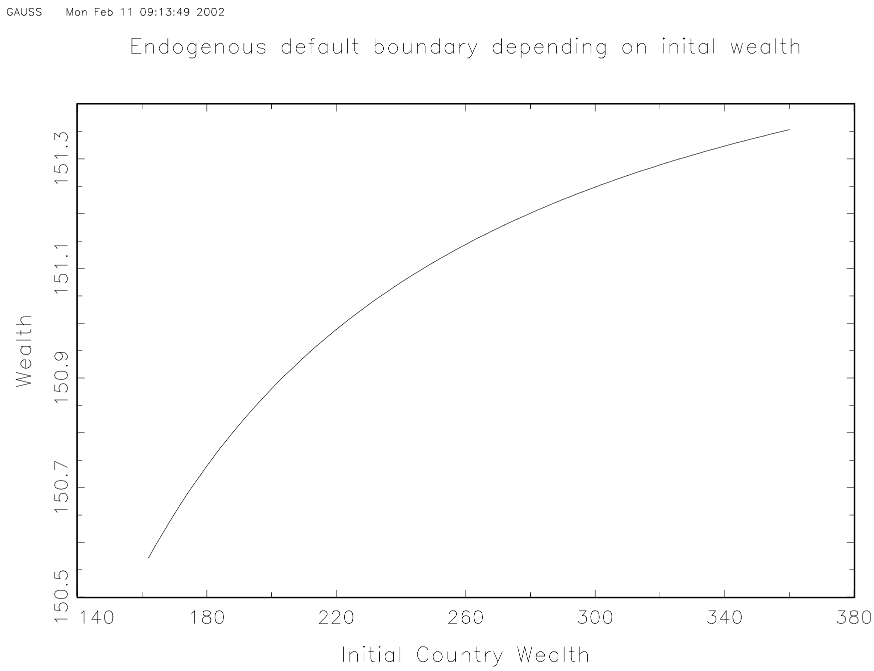

Figure 6: Endogenous default boundary K_t as a function of the initial country wealth R_t. Parameters are $r = 0.05$, $P = 100$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

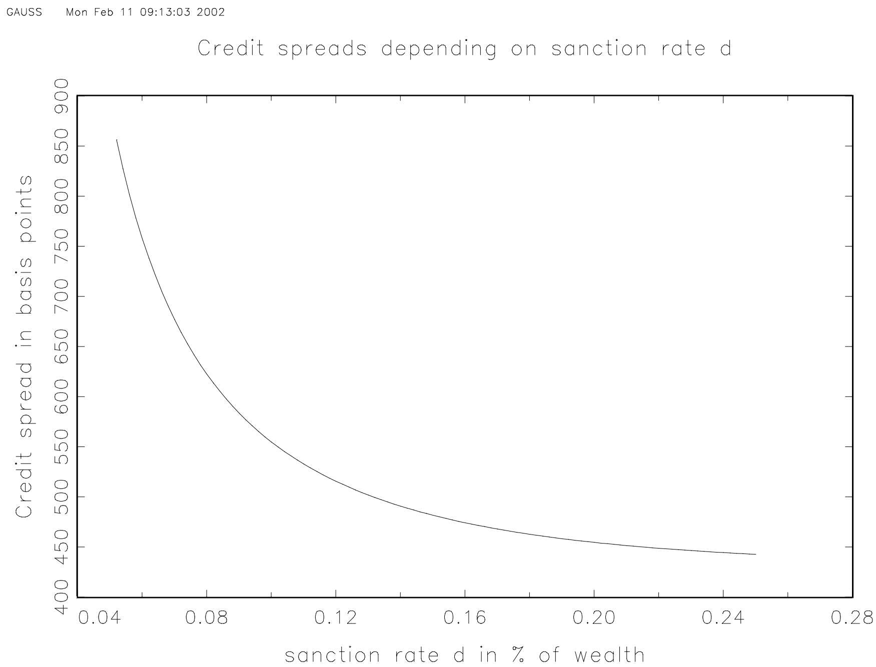

Figure 7: Credit spreads as a function of the sanction rate λ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$ and $T = 10$.

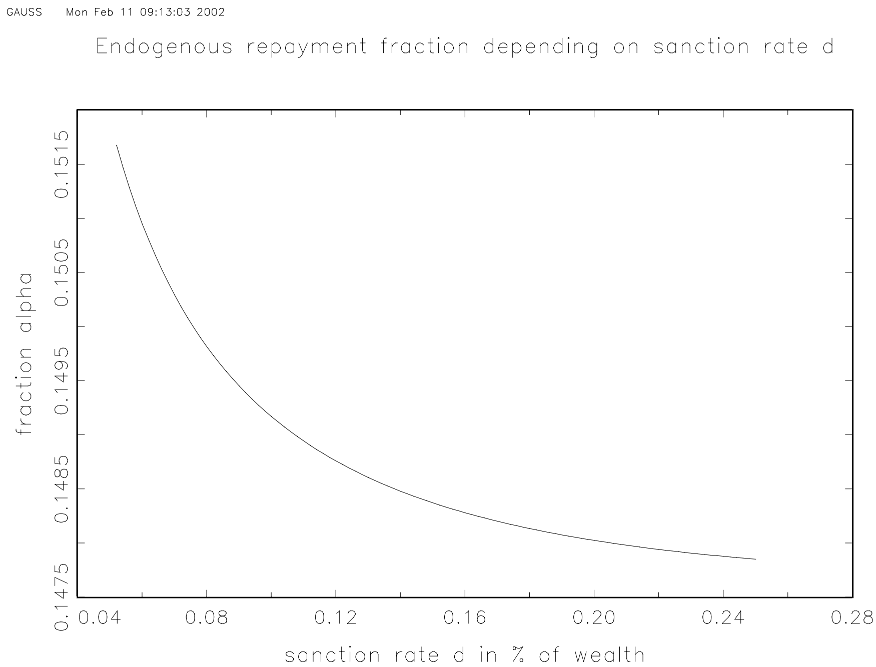

Figure 8: Endogenous repayment fraction α as a function of the sanction rate λ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$ and $T = 10$.

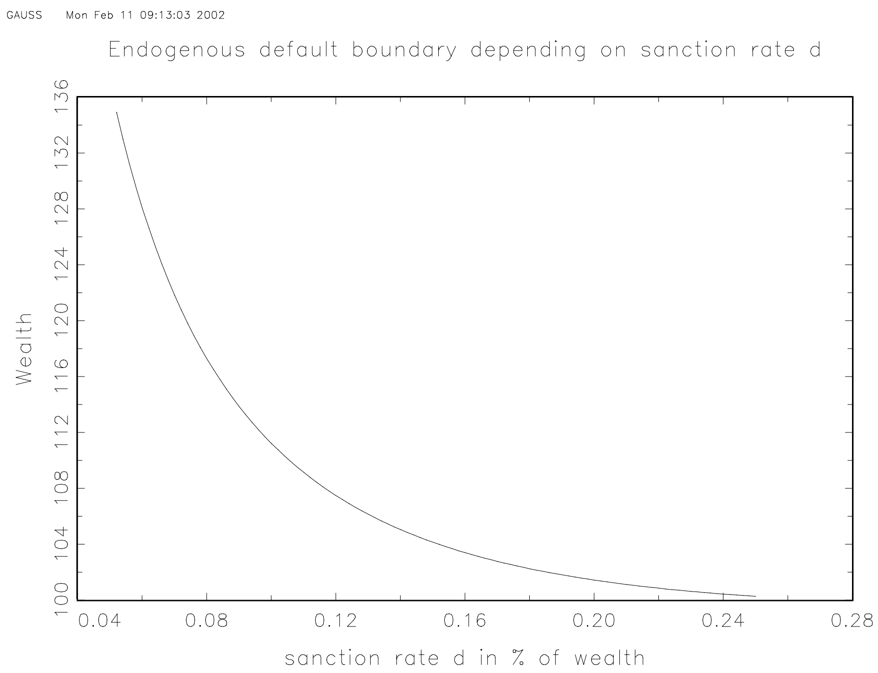

Figure 9: Endogenous default boundary K_t as a function of the sanction rate λ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\sigma = 0.6$, $\gamma = 0.8$ and $T = 10$.

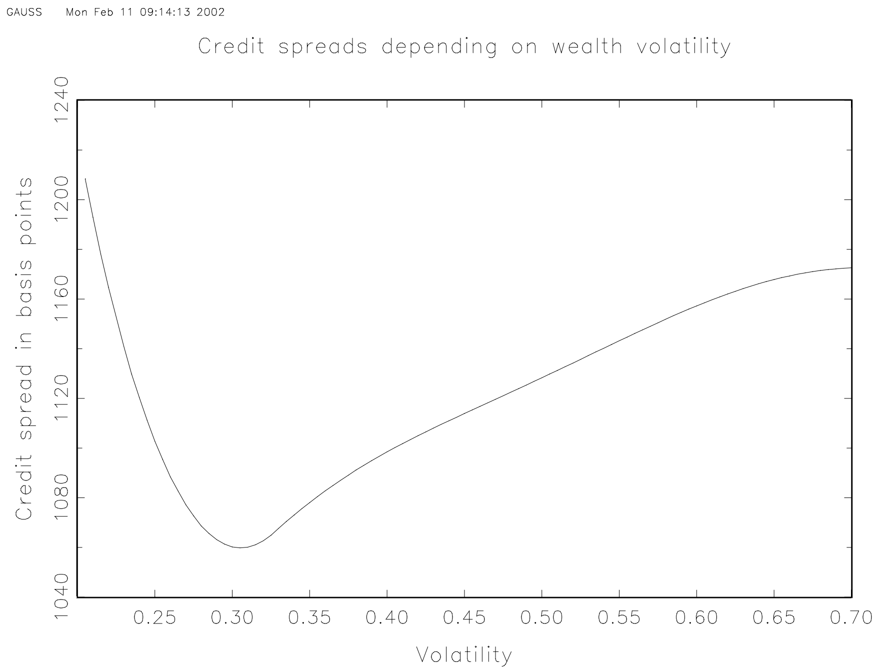

Figure 10: Credit spreads as a function of the country wealth volatility σ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

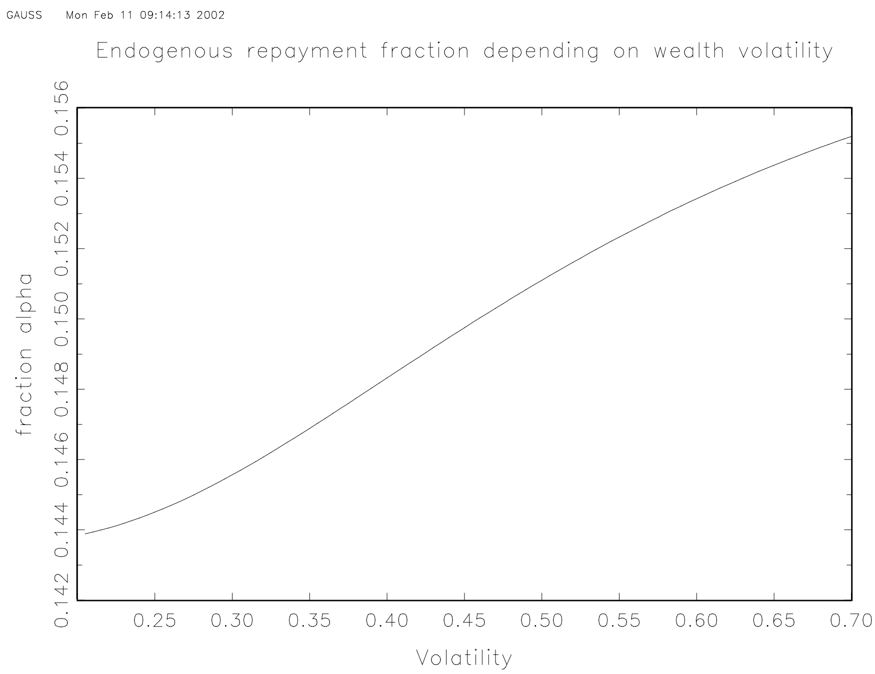

Figure 11: Endogenous repayment fraction α as a function of the country wealth volatility σ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

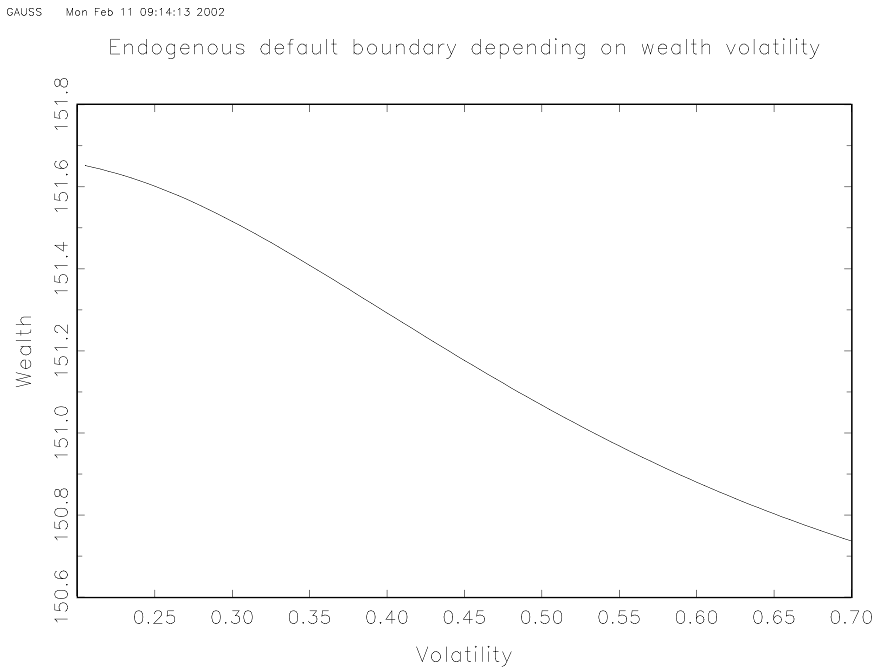

Figure 12: Endogenous default boundary K_t as a function of the country wealth volatility σ. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $c = 0.1$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

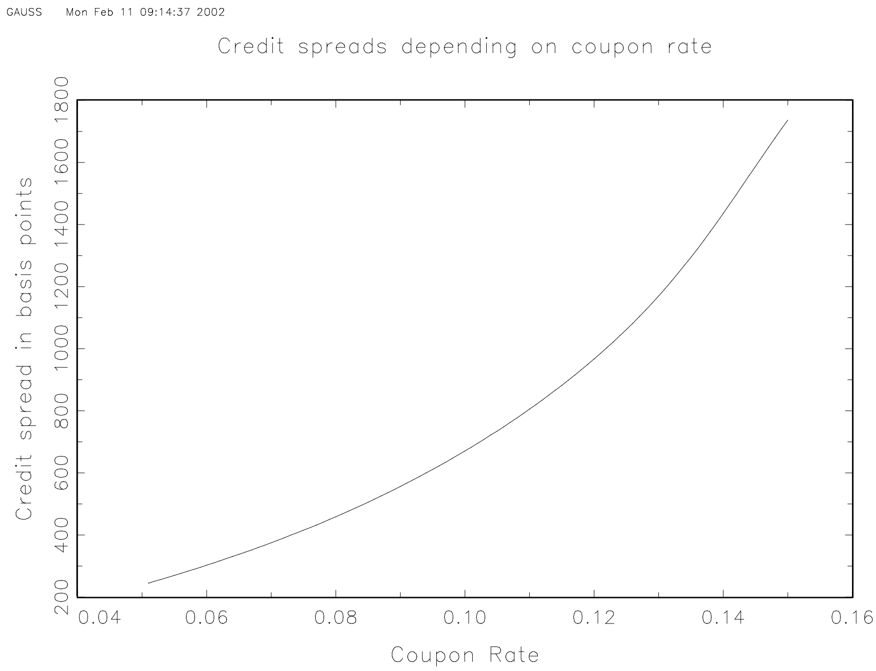

Figure 13: Credit spreads as a function of the coupon rate c. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

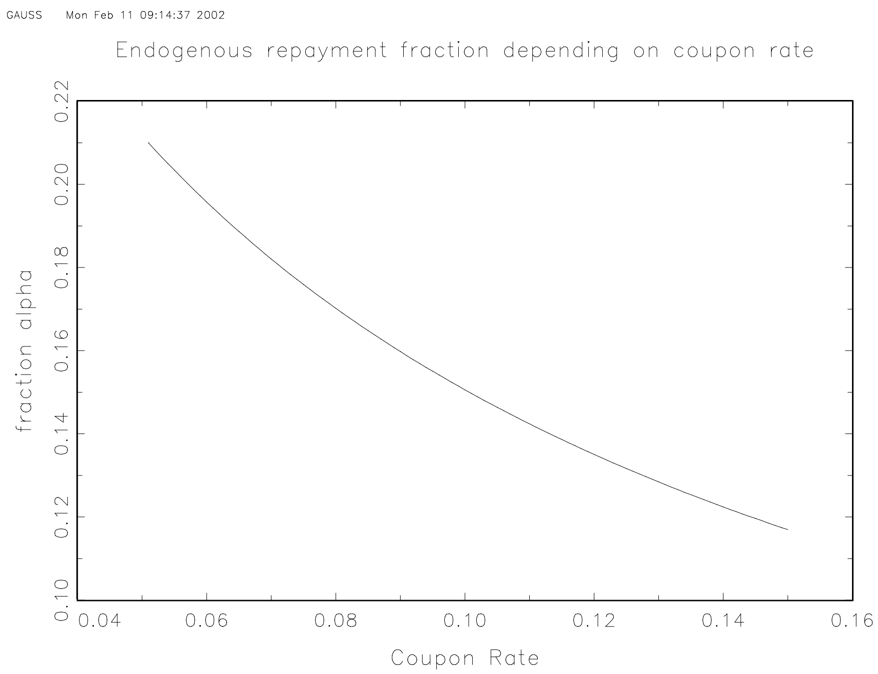

Figure 14: Endogenous repayment fraction α as a function of the coupon rate c. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

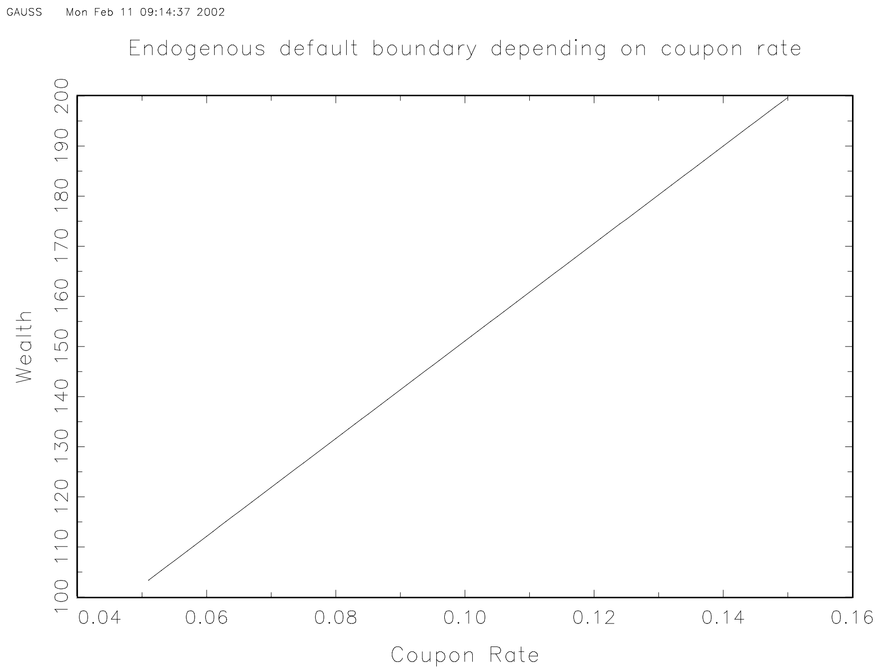

Figure 15: Endogenous default boundary K_t as a function of the coupon rate c. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $\gamma = 0.8$, $\lambda = 0.04$ and $T = 10$.

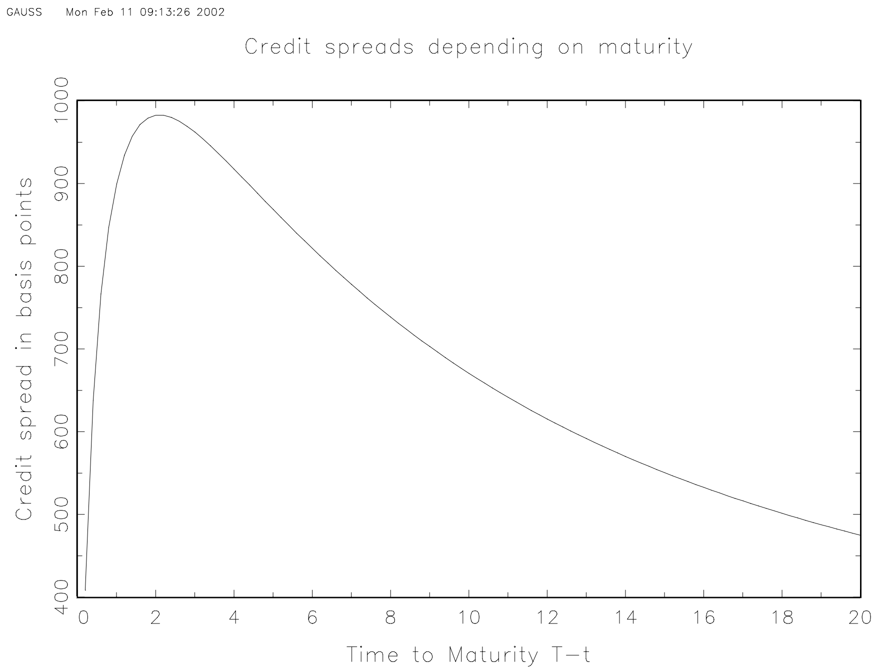

Figure 16: Credit spreads as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$.

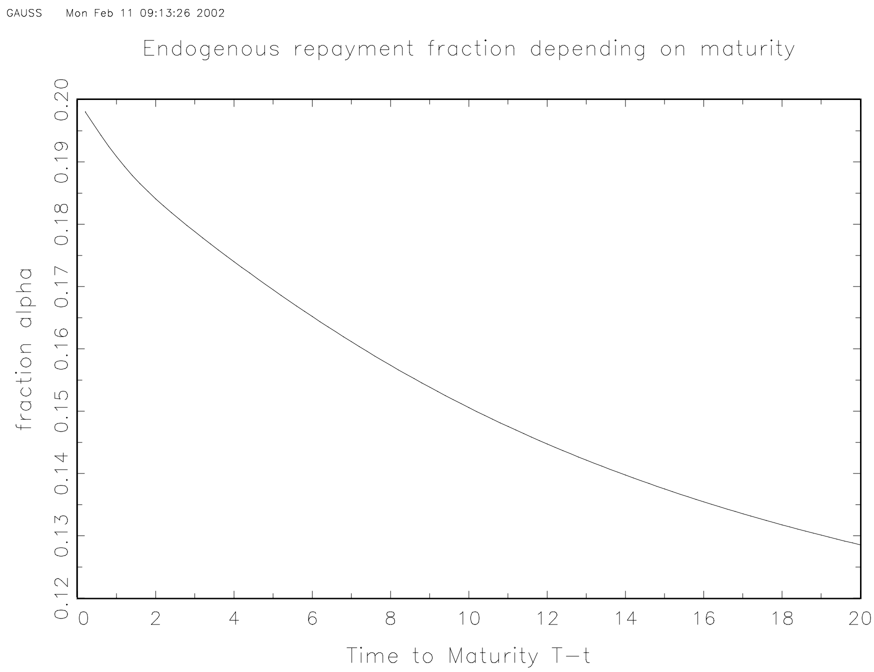

Figure 17: Endogenous repayment fraction α as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$.

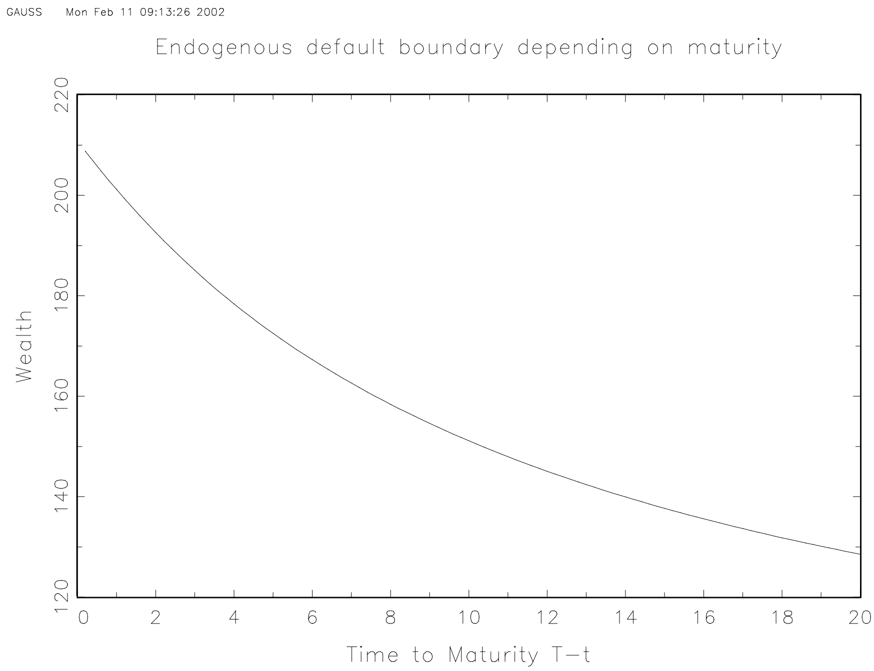

Figure 18: Endogenous default boundary K_t as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $R_t = 200$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$.

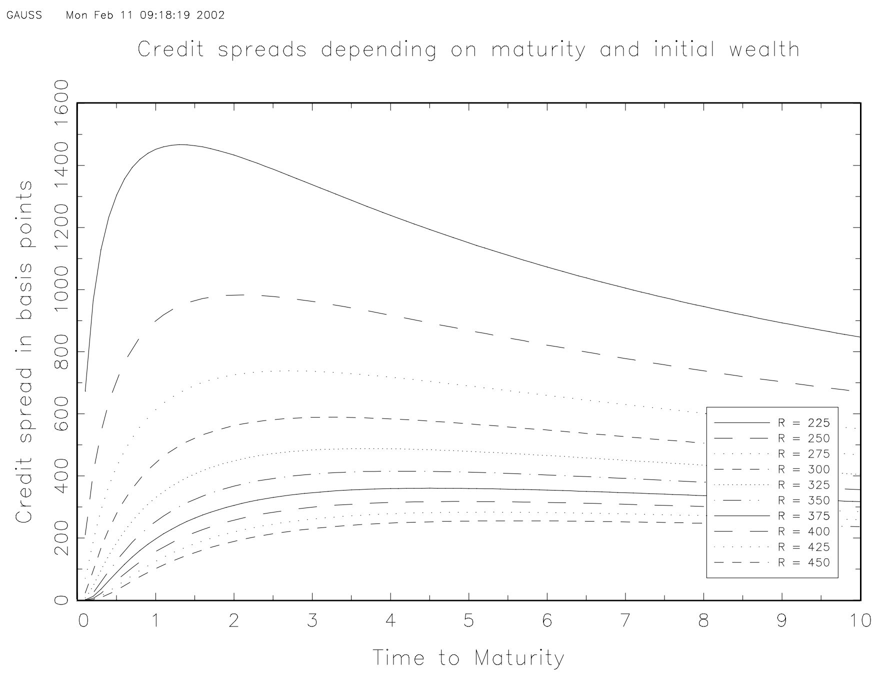

Figure 19: Credit spreads as a function of the maturity of the bond T for different levels of initial wealth R_t from 170 (highest spreads) to 260 (lowest). Parameters are $r = 0.05$, $P = 100$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$.

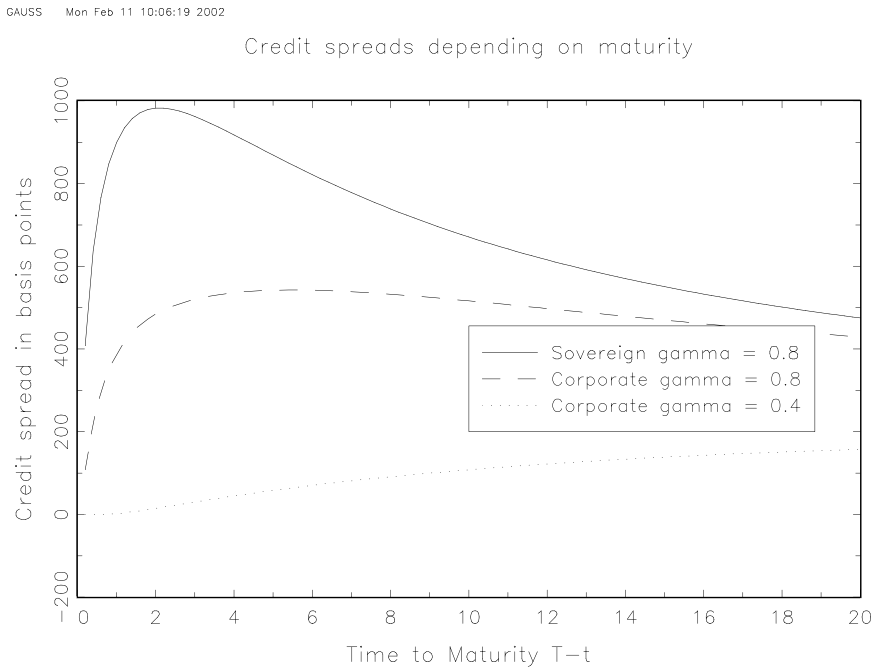

Figure 20: Credit spreads of sovereigns and corporates as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$. The solid line shows a sovereign bond's credit spreads and default boundary, respectively, while the dashed line shows a corporate bond.

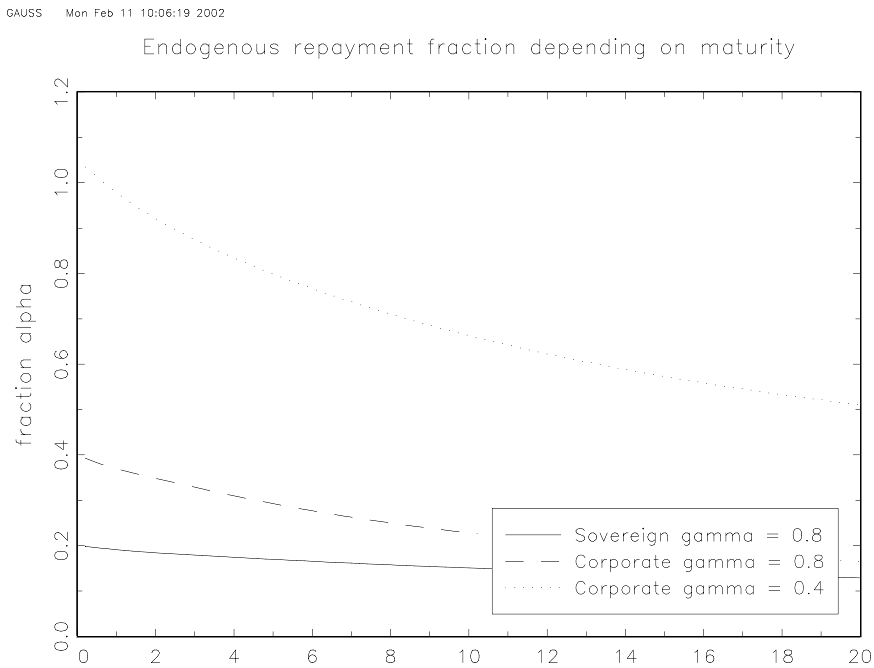

Figure 21: Endogenous repayment fraction α of sovereigns and corporates as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$. The solid line shows a sovereign bond's credit spreads and default boundary, respectively, while the dashed line shows a corporate bond.

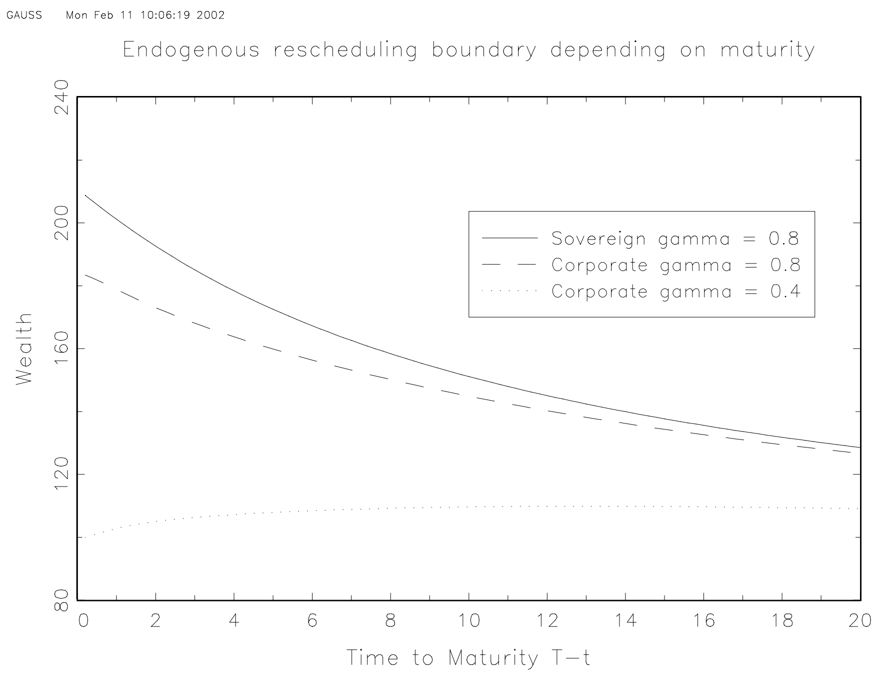

Figure 22: Endogenous default boundary K_t of sovereigns and corporates as a function of the maturity of the bond T. Parameters are $r = 0.05$, $P = 100$, $\sigma = 0.6$, $c = 0.1$, $\gamma = 0.8$ and $\lambda = 0.04$. The solid line shows a sovereign bond's credit spreads and default boundary, respectively, while the dashed line shows a corporate bond.

B Calculation of $\mathbf{E}_s\left[V_T|s<T\right]$

The dynamics of V_t follow

$$dV_t = \left(\left(\mu-\lambda\right)V_t - \alpha cP\right)dt + \sigma V_t dW_t$$

for $t<T$ and $s<T$, i.e., before maturity of the bond and given default at s. Karatzas and Shreve [26] provide a general solution for one-dimensional linear equations, with which we can calculate V_t, given default at s.[16].

$$V_t = e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)t+\sigma W_t}\left[A_0 + \int_0^t -\frac{\alpha cP}{e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)\tau+\sigma W_\tau}}d\tau\right] \tag{28}$$

Defining $Z_t = e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)t+\sigma W_t}$, we calculate the conditional expectation as

$$\begin{aligned}
\mathrm{E}_s\left[V_T|s<T\right] &= \mathrm{E}_s\left[V_0 Z_T\right] + \mathrm{E}_s\left[Z_T\int_0^T -\frac{\alpha cP}{Z_\tau}d\tau\right]\\
&= \mathrm{E}_s\left[V_0 Z_T + Z_T\int_0^s -\frac{\alpha cP}{Z_\tau}d\tau\right] + \mathrm{E}_s\left[Z_T\int_s^T -\frac{\alpha cP}{Z_\tau}d\tau\right]\\
&= \mathrm{E}_s\left[e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)(T-s)+\sigma\int_s^T dW_\tau}\underbrace{Z_s\left(V_0+\int_0^s -\frac{\alpha cP}{Z_\tau}d\tau\right)}_{V_s}\right]\\
&\quad -\alpha cP\mathrm{E}_s\left[\int_s^T \frac{Z_T}{Z_\tau}d\tau\right]\\
&= V_s e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)(T-s)}\mathrm{E}_s\left[e^{\sigma W_T-\sigma W_s}\right] - \alpha cP\mathrm{E}_s\left[\int_s^T \frac{Z_T}{Z_\tau}d\tau\right]
\end{aligned}$$

We can apply Fubini's theorem on the second term, since $Z_t = e^{\cdots} > 0\forall t$.

$$\begin{aligned}
&= V_s e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)(T-s)}e^{\sigma W_s}e^{\frac{1}{2}\sigma^2(T-s)}e^{-\sigma W_s} - \alpha cP\int_s^T \mathrm{E}_s\left[\frac{Z_T}{Z_\tau}\right]d\tau\\
&= V_s e^{(\mu-\lambda)(T-s)} - \alpha cP\int_s^T e^{\left(\mu-\lambda-\frac{1}{2}\sigma^2\right)(T-\tau)}\mathrm{E}_s\left[e^{\sigma W_T-\sigma W_\tau}\right]d\tau\\
&= V_s e^{(\mu-\lambda)(T-s)} - \alpha cP\int_s^T e^{(\mu-\lambda)(T-\tau)}d\tau
\end{aligned}$$

$$= \quad V_s e^{(\mu-\lambda)(T-s)} + \frac{\alpha c P\left(1 - e^{(\mu-\lambda)(T-s)}\right)}{\mu - \lambda} \tag{29}$$

C Proof that the endogenous default boundary $K(s, T, \alpha)$ is positive

We have derived $K(s, T, \alpha)$ as

$$K(s, T, \alpha) = P\frac{1 + \frac{c}{\mu}\left(e^{\mu(T-s)} - 1\right) - \frac{r}{r+\lambda}\alpha\left(1 + \frac{c}{\mu-\lambda}\left(e^{(\mu-\lambda)(T-s)} - 1\right)\right)}{e^{\mu(T-s)} - \frac{r}{r+\lambda}e^{(\mu-\lambda)(T-s)}},$$

Since the parameters P, c, r, μ, λ and $T - t$ are restricted to be positive, and $0 < \alpha < 1$, we can say that

$$K(s, T, \alpha) > 0,$$

because:

1. $$e^{\mu(T-t)} - \frac{r}{r+\lambda}e^{(\mu-\lambda)(T-t)} > 0,$$

   since $e^{\mu(T-t)} > e^{(\mu-\lambda)(T-t)}$ and $\frac{r}{r+\lambda} < 1$.

2. $$1 > \frac{r}{r+\lambda}\alpha,$$

   since $\frac{r}{r+\lambda} < 1$ and $0 < \alpha < 1$.

3. $$\frac{c}{\mu}\left(1 - e^{\mu(T-s)}\right) > \frac{r}{r+\lambda}\alpha\frac{c}{\mu-\lambda}\left(1 - e^{(\mu-\lambda)(T-s)}\right),$$

   since $\frac{r}{r+\lambda} < 1$, $0 < \alpha < 1$ and $\frac{e^{\mu(T-t)}-1}{\mu} > \frac{e^{(\mu-\lambda)(T-t)}-1}{\mu-\lambda}$.

Therefore, since $K(s, T, \alpha)$ is a division of two positive numbers, $K(s, T, \alpha)$ must be positive.

Notes

[1]Although we do not model reputation directly since the sovereign does not borrow repeatedly, we can think of a loss of reputation which decreases his future growth opportunities, leading to a lower growth rate of country wealth.

[2]See the IMF International Capital Markets Survey 2000 [17] for details.

[3]Bonds issued in 1997 in exchange for London Club principal (Prins) and for interest arrears (Ians).

[4]The old bonds were issued by Vnesheconombank and not guaranteed by Russia. The change in obligor was valuable since creditors had little legal recourse against Vnesheconombank.

[5]One of the major hindrances in the litigation of defaulted sovereigns by legal firms specialized in the investemt in distressed debt was the "Champerty" defense: Under New York law, a creditor is not allowed to buy debt with "the intent and sole purpose to sue". In October 1999, Elliot Associates, was able to revoke the use of the "Champerty" defense and laid claim assets in accounts at Chase Manhattan, a fiscal agent of Peru, wich were intended for a a coupon payment of a Brady bond issued by Peru. In order to prevent a default on the Brady bond the sovereign chose to pay the contested assets to Elliot. See Lindenbaum and Duran [31] for details.

[6]We assume that the lenders are identical and risk neutral. In particular, we do not consider the possibility that some lenders do not accept the

tender in order to litigate separately later on. Following this strategy, they would be able to recuperate a larger fraction of the principal, since they do not have to share the seized assets with the bond holders who accepted the exchange offer. To prevent this type of behavior, sovereign bond exchanges normally include collective action clauses which ensure that the majority of bond holders can legally bind the minority, or which ensure that the proceedings of litigation will also have to be shared with the bond holders which accepted the exchange. The exchange offer of Ecuador in July 2000 included a legal innovation: While changes to payment terms of a bond contract require unanimity of bond holders, changes to non-payment terms only require 51% to 75% of votes. The majority of the bond holders thus rendered the old bonds unattractive through e.g. stripping away of cross-default clauses, reducing the liquidity of the old bonds by delisting them and other clauses increasing bond value. See Lindenbaum and Duran [31] for details.

[7]We assume that the sovereign will always consume an amount such that the total of consumption and debt service will be equal to the outflow rate δ. In other words, we impose that the consumption is equal to $\delta - \frac{cP}{R_t}$, such that the total outflow is $\delta R_t dt = cPdt - \left(\delta - \frac{cP}{R_t}\right) R_t dt$.

[8]As stated in the introduction, sovereigns usually offer bonds with a longer maturity than the original bonds, a fact which we do not model directly. This prolongation of the maturity can be included easily into our model, however, by noting that a delaying of the payment of a cash flow causes a reduction in present value terms. For instance, the sovereign offers to repay the principal only at $T' > T$ instead at T, as originally promised. For the bond holders this implies a loss of $\left(1 - e^{-r(T'-T)}\right) P$ since they will receive the principal

$T' - T$ years later than promised. This would mean a reduction in principal of $\alpha = e^{-r(T'-T)}$.

[9]Contrary to Gibson and Sundaresan [18] I assume that it is not possible for the lenders to seize a fraction of the exports generated by the country. By litigating the lenders cannot recuperate more than the principal of the bond minus the litigation costs.

[10]This amounts to a reduction in the expected capitalized country wealth, as we will see later on.

[11]Note that this valuation of the bond only allows for a single default, i.e., a single reduction in the coupon and principal payments. The possibility of a second default on the new bonds is not considered.

[12]This is a major difference to Gibson and Sundaresan [18] who assume that the recovery value is the result of a dividing up of country wealth and future exports of the country. Here it is assumed that the lenders cannot seize more than the principal minus litigation costs, i.e., the recovery value is not a function of country wealth.

[13]We will compare sovereign debt to debt issued by domestic corporations. Specifically, we do not look at debt issued by foreign corporations which is subject to transfer risk and thus to sovereign risk as well.

[14]We still allow renegotiation of debt, i.e., the lenders only threaten to liquidate, and the corporation will propose a bond exchange such that the bond holders will not liquidate the company.

[15]See the IMF International Capital Markets Survey 2000 [17] for details.

[16]We will solve for $\mathrm{E}_s\left[V_T|s<T\right]$ since the solution for $\mathrm{E}_s\left[V_T|s>T\right]$ follows automatically by setting $\alpha=1$ and $\lambda=0$.

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