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The Allocation of Assets Under Higher Moments

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Abstract:
We evaluate how deviations from normality may a affect the allocation of assets. A Taylor expansion of expected utility allows us to focus on certain moments and to compute numerically the optimal portfolio allocation. A decisive advantage of our approach is that it remains operational even if a large number of assets is involved. We obtain that for small values of the risk-aversion parameter, non-normality does not alter significantly the optimal allocation. In contrast, when the investor is strongly risk averse and restricted to invest in risky assets, we also obtain significant changes in portfolio weights.

Executive Summary:
The non-normality of asset returns is a well known empirical regularity. Many reasons can be provided why the distribution of returns is non-normal. For instance, because volatility changes over time, or because rare yet extreme events occur. These extreme events generate higher moments that are different from the higher moments one would obtain with a Gaussian distribution. One may expect that an investor facing the small but non-zero probability of an extreme realization will not allocate his wealth in the same manner as an investor who only cares about the first and second moment.
In this contribution, we investigate how an investor will change his portfolio allocation when he cares not only about mean and variance but also about the third and fourth moment. To do so, we start with a traditional constant relative risk aversion utility function. This type of utility function

is a standard criterion for choices under uncertainty. This function is expanded in a Taylor approximation up to the fourth order. The first two moments correspond to mean and variance. The third and fourth moment correspond to a directional measure of extreme events and to a symmetric measure respectively. A negative third moment indicates that there are more extreme negative realizations than there are positive ones. The fourth moment measures how the tails of the return distribution compare with the tails of the Gaussian distribution.

From a technical point of view, in our paper, a numerical optimization takes place where the allocation involves higher moments. We build on recent advances by Athayde and Flores, to express these moments and the gradients thereof in a compact manner that can be easily programmed.
In our empirical implementation, we use the total return indices from Morgan Stanley Capital International for the period from January 1976 through December 2001 at a weekly frequency. The indices concern several large geographic areas. As such, these series may be viewed as representing very well diversified portfolios. Also, for such large portfolios, one may expect that extreme realizations of individual stocks are largely diluted. More pronounced results are to be expected in the case one considers portfolios of individual stocks. Our findings are the following:

a) When an investor may allocate his wealth to the indices and to the risk-free asset, then the weights corresponding to the risky assets are essentially unaffected by the introduction of a concern for the third or fourth moments.

b) If an investor is limited to invest in the risky assets only, then higher moments may come to play a role. This happens, however, only for very high values of the parameter of risk aversion.

c) As the third moment comes to play a role, Asia (except Japan) gets less weighted because it contains large negative returns, whereas the importance of Japan is increased. The stronger weighting of Japan comes from the fact that the Japanese returns contain several very large positive outliers generating a positive third moment.

The implication of this research is that the traditional utility functions or expansions thereof may not sufficiently weigh realizations of extreme nature. This brings up the question how to adapt a utility function so that more weight is given to extreme realizations. A further question is how the allocation would change in a conditional setting, or with individual stocks rather than with portfolios, since in such circumstances, one may expect that higher moments take larger values than in the present setting. These questions are left for future research.