Research Paper Number 34

Variable Selection for Portfolio Choice

Winner of the 2001 FAME Research Prize

Authors:
Yacine AÏT-SAHALIA - Princeton University & NBER
Michael W. BRANDT - Wharton School, University of Pennsylvania & NBER

Date:
Feb. 2001

This paper has now been published and is no longer available as a part of our Research Paper Series. The reference to this paper is:

Aït-Sahalia, Y., Brandt, M.W. (2001): "Variable Selection for Portfolio Choice". Journal of Finance, 2001, 56, 1297-1351.

Abstract:
We study asset allocation when the conditional moments of returns are partly predictable. Rather than first model the return distribution and subsequently characterize the portfolio choice, we determine directly the dependence of the optimal portfolio weights on the predictive variables. We combine the predictors into a single index that best captures time-variations in investment opportunities. This index helps investors determine which economic variables they should track and, more importantly, in what combination. We consider investors with both expected utility (mean-variance and CRRA) and non-expected utility (ambiguity aversion and prospect theory) objectives and characterize their market-timing, horizon effects, and hedging demands.

Executive Summary:
There is by now ample evidence in the literature that the means, variances, covariances, and higher order moments of stock and bond returns are time-varying and predictable. However, it has proven difficult to translate this evidence of predictability into practical portfolio advice because the different moments of returns, which in turn determine the optimal portfolio weights, are predicted by different sets of economic variables. Perhaps because of this difficulty with modeling the conditional return distribution, most professional investment advice is given solely on the basis of variables that forecast expected returns, such as the dividend yield or the slope of

the term structure.

Looking beyond expected returns, it is difficult to decide which selection or combination of predictive variables the investor should focus on. This is true even in the few special cases where we have an explicit asset allocation formula, such as for mean-variance utility where the optimal allocation is proportional to the ratio of the conditional mean to the conditional variance of returns.

In this mean-variance case it is clear that we want to find variables that best predict the ratio of the first two conditional moments. Choosing variables that best predict the mean and variance separately is likely to be counter-productive. What should we do if a variable has a positive effect on both means (which the investor likes) and variances (which are detrimental to the investor)? What should we do if this variable is highly significant for one of the moments but less so for the other? How do we capture the relative importance that the investor's preferences place on the different moments? These questions all suggest that in a portfolio choice context we should select variables to directly predict optimal portfolio weights, rather than first select variables to predict separate features of the return distribution and then explore later their implications for asset allocation.

In this paper, we show how to select and combine variables to best predict an investor's optimal portfolio weights, both in single-period and multiperiod contexts. Rather than first model the various features of the conditional return distribution and subsequently characterize the portfolio choice, we focus directly on the dependence of the portfolio weights on the predictors. We do so by solving sample analogues of the conditional Euler equations that characterize the portfolio choice, as originally suggested by Brandt (1999). However, unlike the existing literature, we determine endogenously, for a given set of utility preferences, which of the candidate predictors are important for the optimal portfolio weights (rather than important for separate moments of the return distribution).

We form a linear combination or index of the conditioning variables that best predicts the investor's optimal portfolio weights and then judge the importance of each individual variable by the role it plays in this index. We make no further assumptions about the relationship between the optimal portfolio weights and the predictors for two reasons. First, the dependence of the portfolio choice on the predictors can be highly nonlinear, even when the conditional moments are approximately linear; and second, the particular form of the nonlinearities not only varies greatly with the investor's preferences but also cannot generally be determined explicitly. This leads us to a semiparametric approach, where the optimal portfolio weights depend nonparametrically on a parametric index of the predictors.
We study the portfolio choice of investors with both expected utility (mean-variance and CRRA)

and non-expected utility (ambiguity aversion and prospect theory) objectives in order to see how the optimal index composition depends on the characteristics of the investor's preferences. From a normative perspective, our results can help investors with any one of these preferences determine which economic variables they should track and, more importantly, in what single combination. Our index is a parsimonious way to describe the current state of the investor's investment opportunities, just as in different economic contexts indices summarize high-dimensional state vectors (the index of leading economic indicators, the business cycle index, the consumer confidence index, etc.).

Macroeconomic indices are country-specific, since countries have different characteristics, and for the same reason our investment opportunities index is investor-specific because investors have different preferences.

For the purpose of giving portfolio advice, one advantage of our index approach is that it helps investors understand their conditional asset allocation in a more intuitive manner. For instance, it delivers simple rules like \if the index increases, the allocation to stocks should increase." By contrast, it is generally difficult to represent graphically variables in more than two dimensions, let alone develop economic intuition about their interactions.

At least four stylized facts emerged from our empirical analysis:

- The slope of the term structure is an ubiquitous variable in our indices, appearing significantly across all preferences, investment horizons, and rebalancing frequencies. To a lesser extent, but fairly consistently, a S&P index momentum variable enters our indices at short horizons, while the dividend yield is the second most important variable at long horizons. The default risk premium generally records the lowest index loadings.
- All investors, when presented with their index of investment opportunities, find it optimal to engage in significant market timing.
- Horizon effects are most pronounced for prospect theory investors, who find stock losses at short horizons to be prohibitively costly. For investors who are not subject to loss aversion, the relative lack of returns autocorrelation translates into relatively small horizon effects.
- Hedging demands are weak and negative because stocks do not provide a good hedge for innovations in the index. However, the index coefficients vary with the horizons.