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Capital Gains Taxes, Irreversible Investment, and Capital Structure

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Abstract

This paper studies the corporate policy distortions caused by realization-based capital gains taxation at the personal level in a dynamic trade-off theory model. The lock-in effect of embedded capital gains creates severe conflicts of interest between incumbent and new investors. The firm's optimal policy exhibits path-dependency and non-stationarity, since the tax basis of the firm's owners is a valuable conditioning variable for corporate decisions. Ex-ante identical firms follow very different investment and financing policies depending on their stock price evolution. Firms delay irreversible investment further the lower the tax basis of their owners falls. The reason is the investment hedge provided by personal tax loss offsets weakens as investors reset their basis. Capital gains taxation also creates incentives to time equity issues. Firms employ more equity in their capital structure the higher the stock price-to-basis ratio, since locked-in investors with out-of-the-money tax timing options value the firm less than the market. The value gain from conditioning on the owners' tax basis is substantial. Using simulated data I show the combined effects are consistent with recent empirical evidence on the relation between leverage, Tobin's Q, and past performance.

JEL Classification: G31, G32, H24, H32.

Keywords: Capital Gains Taxation, Real Options, Capital Structure, Trade-off Theory, Market Timing.

1 Introduction

One of the most vigorous debates among finance scholars and policymakers for the past decades has been to which extent personal taxation distorts corporate investment and financial policy. Yet, our knowledge is limited due to the inherent intractability of models with a realization-based capital gains tax system.[1] The optionality in the timing of realization-based taxation "locks in" investors and alters security valuations. Two investors with the same statutory tax rate can have different "effective" tax rates simply because they have acquired the security at different prices. Inevitably, the Modigliani-Miller (1958, 1963) and Miller (1977) irrelevancy theorems break down.[2]

Traditional investment and financing theories have since limited the analysis to accrual-based capital gains taxation, which captures the deferred taxation but neglects the inherent optionality.[3] Most of these models, however, lack explanatory power for observed patterns in external financing, capital structure, and investment.[4] Baker and Wurgler (2002) provide compelling evidence that the history of a firm plays a pivotal role in determining capital structure, and they conclude "capital structure is the cumulative outcome of a series of market-timing-motivated financing decisions." Polk and Sapienza (2004), Baker, Stein and Wurgler (2003), and Gilchrist, Himmelberg and Huberman (2004) show stock prices influence corporate investment, and argue for behavioral explanations.

This paper develops a tractable capital budgeting model under realization-based capital gains taxation that can account for many of the features observed in the data. The lock-in effect of embedded capital gains causes severe conflicts of interest between the current owners of the firm and new investors. The firm's optimal policy exhibits path-dependency and non-stationarity,

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[1] A large body of literature studies the valuation effects of capital gains taxes (see Viard (2000), Klein (2001)), and the portfolio and consumption implications (see Dammon, Spatt and Zhang (2001)).

[2] See Schneller (1980), Constantinides (1983), Balcer and Judd (1987), Lewellen and Mauer (1988).

[3] See Fisher, Heinkel and Zechner (1989), Mello and Parsons (1992), Goldstein, Ju and Leland (2001). Novel approaches include Hennessy and Whited (2004), Hackbarth, Miao and Morellec (2004), Titman and Tsyplakov (2005). Auerbach (2001, p. 4) justifies the reduced-form approach as follows: "[I]ncorporating a realization-based capital gains tax would complicate the present analysis greatly, and is not as important [...]."

[4] Equity offerings have been shown to be clustered in time, cyclical, and positively related to stock market performance (Masulis and Korwar (1986), Asquith and Mullins (1986), Jung, Kim and Stulz (1996), Bayless and Chaplinsky (1996), Lowry and Schwert (2002), Benninga, Helmantel and Sarig (2003)). Choe, Nanda and Masulis (1993) and Korajczyk and Levy (2003) show that capital structure varies with macroeconomic conditions. Welch (2004) argues capital structure dynamics are dominated by corporate inertia. Other empirical studies provide evidence that capital structures only slowly revert back to optimal leverage ratios (see Hovakimian, Opler and Titman (2001), Alti (2004), Hovakimian (2004), Huang and Ritter (2004), Kayhan and Titman (2004), Leary and Roberts (2004)).

since corporate managers with aligned incentives condition their decisions on the tax basis of the firm's owners. Capital gains taxation further creates incentives to time equity issues, since locked-in shareholders with out-of-the-money tax timing options value the firm less than the market. Empirical proxies for a firm's historical performance are therefore likely to have explanatory power in cross-sectional regressions.

The main insight offered by the paper is that the presence of a tax timing option at the personal level can have a dramatic impact on corporate policy. There are two main effects. Personal taxation distorts the intertemporal link between uncertainty and irreversible investment, and it alters the debt-equity trade-off. The **timing effect** is that firms delay investment further, the lower the tax basis of their owners falls. The **financing effect** is that firms employ more equity financing, the higher the stock price-to-basis ratio of their owners.

Figure 1 illustrates the two effects. The graph plots a sample path for the firm's operating income or, equivalently, the stock price. The top line corresponds to the level of operating income that triggers additional capital investment in the firm. The bottom line corresponds to the relevant bankruptcy trigger. Whenever shareholders realize tax losses due to a drop in the stock price below their current basis, the tax basis as illustrated by the middle line gets reset downward. Simultaneously, the firm optimally shifts the investment trigger (top line) upward, thus delaying investment further. In this event the firm uses more equity to fund the project than in case the owners had not reset. The interaction with the firm's financial policy can even lead to underinvestment, or "excessive" delay, relative to the case without capital gains taxation.

The basic intuition for the timing effect of capital gains taxes follows the classical insight in real options theory that "the greater the uncertainty over future cash flows, the larger is the excess return the firm demands before making the irreversible investment," Dixit and Pindyck (1994). Through asymmetric taxation of gains versus losses the government shares investment risk disproportionately and encourages "risk-loving" corporate behavior. Capital loss offsets at the personal level reduce investors' uncertainty about after-tax payoffs in down-states. Since after-tax returns are relevant for corporate decision making, the concavity in personal tax liabilities diminishes the value of the firm's real option to delay and *a priori* speeds up irreversible investment at the corporate level. The investment stimulus is, however, only transitory. Incumbent investors optimally reset their tax basis whenever the stock price has fallen below the basis, such that the inherent tax asymmetry gradually
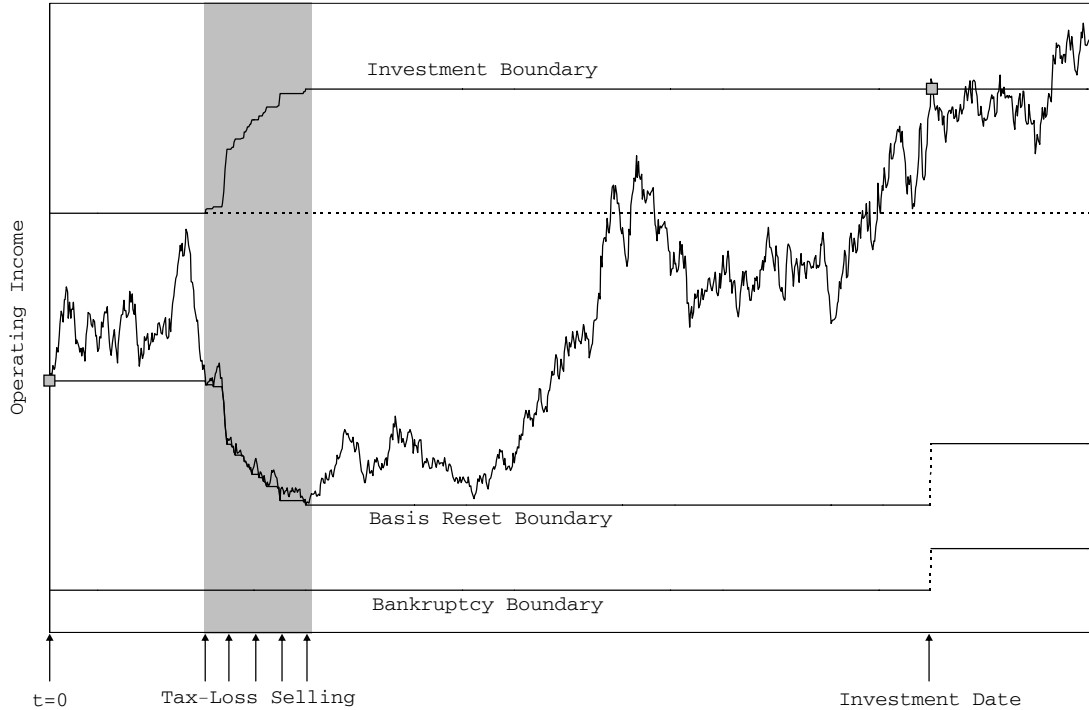
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Figure 1. Path-Dependency in the Firm's Policy.

disappears. The firm's management rationally responds by raising the investment threshold any time the shareholders reset. Analog to a depletable resource, capital loss offsets exploited before real option exercise cannot again be utilized in the future when the new project performs poorly.

The financing effect of capital gains taxes results from asymmetric valuations of equity and debt across the firm's incumbent owners and new investors. Realization-based taxation creates an embedded tax timing option with a value that is higher, the larger the tax basis. This drives a wedge between the private valuations of locked-in incumbents and the market price. Corporate managers acting in the interests of the firm's current owners exploit the market's valuation premium by issuing more equity relative to debt, the larger the stock price-to-basis ratio of the owners. The ownership dilution associated with equity issuance creates a surplus for incumbents with *out-of-the-money* tax options, since every newly issued share entitles taxable investors to a new *at-the-money* tax option granted by the government. Locked-in shareholders also prefer ownership dilution through issuance of new shares to personal sales of existing shares, since the latter are accompanied by realization of taxable gains. Across tax basis values, more locked-in (low-basis) shareholders have

lower reservation values, or private valuations, than less locked-in (high-basis) shareholders. The surplus extracted by the current shareholders from the dilution of ownership is therefore relatively bigger, the larger the stock price-to-basis ratio.

A simple example illustrates the basic intuition behind the financing effect. Assume capital gains are taxed upon realization at rate $\tau > 0$, and financial markets are competitive and frictionless as in Constantinides (1983). There are no corporate or ordinary income taxes. For simplicity, investors have no incentive for trade other than tax timing. The firm has assets in place that generate a stochastic stream of dividends with value X. Valuations then consist of two parts—the asset value and a tax timing option. If dividends follow a geometric Brownian motion process, the value of the tax-loss selling option compounded into the stock price is a fraction $(\beta - 1) > 0$ of the asset value X, with $1 < \beta < 1/(1 - \tau)$ as in Constantinides (1983). The share price thus equals $p^0 = \beta X$. Conversely, the present value of future loss offsets is zero for incumbent shareholders with negligible tax basis. Hence, their private valuation of the firm is $v^0 = X$, which is strictly less than the share price. Note that if they sold their shares, they would receive only $(1 - \tau)p^0$ after taxes, which is less than v^0 since $\beta < 1/(1 - \tau)$. This implies they are "locked-in," or $p^0 > v^0 > (1 - \tau)p^0$. The firm now gets the one-time opportunity to double its capacity to $2X$ by investing I. What is the investment rule and what is the optimal project financing? If the project is equity-financed with $n^1 - 1$ new shares, the budget constraint $p^1(n^1 - 1) = I$ implies the share price after investment equals $p^1 = \beta 2X/n^1 = \beta 2X - I$ since $n^1 = \beta 2X/(\beta 2X - I)$. The private valuation of the incumbent shareholders is $v^1 = 2X/n^1 = 2X - \beta^{-1}I$. Hence, the firm makes the investment if and only if $X \geq \beta^{-1}I$ ($\Leftrightarrow p^1 \geq p^0 \Leftrightarrow v^1 \geq v^0$). What happens in case the project is financed with riskless debt $D^1 = I$? The share price, again, equals $p^1 = \beta 2X - I$, and $p^1 \geq p^0 \Leftrightarrow X \geq \beta^{-1}I$. In contrast, the valuation of the incumbent shareholders is $v^1 = 2X - I$, and thus lower than in the case of equity financing. In summary, both incumbent and new investors agree on the investment rule *Invest if and only if $X \geq \beta^{-1}I$*, but they have conflicting goals about the optimal financing. New shareholders are indifferent between equity- and debt-financing but incumbents strictly prefer equity.

The predictions of the model for the cross-section and time-series of external financing, capital structure and investment patterns have strong empirical support. Path-dependency, non-stationarity, and market timing have all been shown to be features of actual corporate behavior.

More generally, the model predicts that ex-ante identical firms follow different investment and financial policies depending on their stock price evolution. In time-series, firms delay investment after temporary stock price declines that lead to tax-loss selling, e.g., in a recession. In cross-section, mature firms with low-basis owners have lower investment hazards than high-basis startups. The external financing mix and capital structure dynamics also depend on the firm's past performance. Firms rationally time the market by issuing more equity and targeting lower leverage ratios, the higher the stock price and the lower the basis of their owners. The resulting capital structure dynamics are characterized by moving target leverage ratios which invalidates the stationarity assumption in most empirical capital structure studies.[5] The combination of the timing and financing effects illustrates it is jointly optimal to invest at high market-to-book and raise equity. This prediction is consistent with cross-sectional evidence that high Tobin's Q firms use more equity financing and have lower leverage than low Q firms. More generally, the paper emphasizes the important role ownership structure plays in corporation finance. Even absent agency conflicts between management and owners or conflicts of interest between bond- and stockholders, the composition of ownership is crucial purely for personal tax reasons in explaining corporate behavior.[6]

Numerical parameterizations show the distortions of personal taxes in corporate policy are economically significant. Firms optimally employ up to ten percentage points more equity in their capital structure, the lower the basis-to-price ratio of their owners. The investment thresholds can vary by more than 200% across firms with different investor tax basis, and the expected investment dates can be years apart. In cross-sectional leverage regressions using simulated data, the coefficients on the external financing-weighted market-to-book ratio are similar to the empirical estimates in Baker and Wurgler (2002)—both for book and market leverage. Numerical calculations illustrate that firms could profit significantly from pursuing a state-dependent policy that takes into account the evolution of the owners' tax basis. The value gain from switching to the state-dependent policy is substantial and ranges from four to seven percent depending on the parametrization.

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[5]Most of the empirical tests of the trade-off theory against the pecking-order theory rely on stationarity assumptions (see Shyam-Sunder and Myers (1999), Fama and French (2002)).

[6]The catering theory of Allen, Bernardo and Welch (2000) complements this paper by stressing that clientele effects due to tax heterogeneity can contribute to explaining corporate payout policy.

The results in this paper do not rely on market inefficiency or market frictions, they hold in perfect capital markets. Nonetheless, the qualitative predictions are robust to various imperfections. The real-life reset behavior of investors differs from the full exploit predicted by Constantinides (1983) for a number of reasons including transaction costs, short-sale restrictions, wash-sale rules, and behavioral biases (see Odean (1998)). The magnitude of the distortions determined in this paper represent upper bounds for the ones likely to be observed in practice. The stock price relative to the personal tax basis of the firm's owners, or some weighted-average of them, should in any case be a valuable conditioning variable for corporate managers acting in the interests of their taxable shareholders.[7]

1.1 Related Literature

The paper follows a long tradition of studies on the link between taxes and the amount of risk-taking. Domar and Musgrave (1944) were first to note that the government shares private risks through taxation. Gordon (1985) established that taxes do not distort investment only if the reduction in expected returns is compensated by an equiproportional reduction in systematic risk. The neutrality result breaks down with an asymmetric tax system, and nonlinearities in the tax system can have important welfare effects. Limited offset provisions for corporate tax losses are one example, realization-based capital gains taxes are another. Green and Talmor (1985), MacKenzie (1994) and Faig and Shum (1996)—although in different setups—all find that imperfect corporate tax loss offsets lead to underinvestment, since the convexity in the tax liability induces "risk-averse" behavior and discourages investment. In this paper the reverse effect occurs due to the concavity in personal taxes induced by realization-based capital gains taxation.[8]

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[7]See Lewellen and Lewellen (2004) for an empirical investigation of this effect.

[8]MacKie-Mason (1990) shows that non-convexities in corporate taxation may have "perverse" effects on investment. Tax rate hikes may encourage, and subsidies may discourage investment. Mayer (1986) finds that financing flexibility may dampen the adverse effect of corporate tax asymmetry on investment. This paper looks at similar issues related to realization-based capital gains taxation. Fazzari and Herzon (1995) argue that lowering the capital gains tax rate discourages incremental investment in the presence of undiversified risk. The prerequisite—as discussed by Haliassos and Lyon (1994)—is that the government can efficiently redistribute the systematic risks and diversify the idiosyncratic risks it takes on through risk-sharing with investors. Haliassos and Lyon (1994) further show the risk-sharing effects of capital gains taxes are important for encouraging stockholding, and that their excess burden is negative. Closely related is the literature around Mello and Parsons (1992), Mauer and Triantis (1994), Parrino and Weisbach (1999), Mauer and Sarkar (2003), Titman and Tsyplakov (2005) on dynamic investment and financing distortions due to agency conflicts between debt- and equityholders. The main focus in this paper is on conflicts of interest between incumbent shareholders and outsiders due to the lock-in effect of personal taxes.

In the recent corporate finance literature there are several competing rationales for the behavioral evidence in Baker and Wurgler (2002). Barclay, Morellec and Smith (2003) show that poor corporate governance in conjunction with cross-sectional variation in growth options can lead to the negative empirical relationship between book leverage and Tobin's Q. Strebulaev (2004) limits attention to market leverage and argues its empirical features arise mechanically in a standard trade-off theory model along the lines of Fisher, Heinkel and Zechner (1989) or Goldstein, Ju and Leland (2001). Hennessy and Whited (2004) abstract from Miller's (1977) clientele equilibrium effects and develop a dynamic capital budgeting model with time-varying financing margins that is able to generate debt hysteresis and slow mean-reversion in the firm's capital structure. This paper produces predictions resembling the stylized facts about both market and book leverage without introducing behavioral biases or cross-sectional variation in investment opportunities.

Plenty of support for the economic relevance of the effects studied in this paper are provided in the empirical taxation literature. Capital gains taxes have been found to be important determinants both for investor behavior and for securities valuations. Evidence for tax-induced trading can be found, for instance, in Dyl (1977), Lakonishok and Smidt (1986), Badrinath and Lewellen (1991), Odean (1998), Grinblatt and Keloharju (2000), and Ivković, Poterba and Weisbenner (2004). Jin (2004) shows the lock-in effect influences institutional trading and price reactions to earnings announcements. Ayers, Lefanowicz and Robinson (2003) find acquisition premia for taxable acquisitions to be consistent with a lock-in effect. Shackelford (2000) provides evidence that stock prices capitalize capital gains taxes. Guenther and Willenborg (1999) show that capital gains taxes are compounded into the offer prices of small business IPOs.

The remainder is organized as follows: Section 2 lays out the model and details the economic assumptions. Section 3 discusses the implications of the model for corporate investment, capital structure, and bankruptcy decisions. Subsequent Monte-Carlo simulations and regression results illustrate the patterns in external financing and capital structure that emerge based on a dynamic trade-off theory model with personal taxes. Section 4 summarizes and concludes the paper.

2 The Model

In the following I set up the model, introduce the assumptions sufficient for retaining tractability, and describe the solution approach. I start with the opportunity set of the firm, the tax code and the economic environment. Then I discuss the issue of unanimity and define the objective function for the firm. Last, I set up the firm's optimization problem and provide solutions to the private and public valuations of debt and equity.

2.1 The Assumptions

The firm goes public at $t = 0$ and has assets in place that generate pre-tax operating income πX, $\pi \in [0, 1]$. There is a risk-neutral measure $\mathbb{Q}$ under which $X = (X_t)_{t \geq 0}$ is governed by a geometric Wiener process with coefficients (μ, σ). The after-tax risk-free rate is $r > 0$ and $\mu < r$, $\sigma > 0$ are given. That is

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \qquad X_0 > 0, \tag{1}$$

where $W = (W_t)_{t \geq 0}$ is a standard Wiener process on the probability space $(\Omega, \mathcal{F}, \mathbb{Q})$.

The firm owns the perpetual rights to an irreversible investment project. At each date, the firm can exercise the rights and step up capacity from πX to full scale X. The capital expenditure required equals $I > 0$. The parameter π determines the growth potential of the firm, and will be useful for calibration in Section 3.4. The two extreme values for π illustrate the nature of the problem:

- $\pi = 0$: The firm owns the right to an investment project and has no assets in place.

- $\pi = 1$: The firm has no growth potential but is given the opportunity to reorganize its capital structure. In this case, one can interpret I as underwriting and other issuance fees.

The initial capital structure consists of perpetual non-callable debt with promised coupon flow c^0, and the number of shares outstanding is $n^0 = 1$. The firm sells new securities—an optimal mix of debt and equity—to fund the project. The number of shares after investment is denoted n^1, and c^1 is the aggregate debt coupon after the second round of financing.[9] For simplicity, the old

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[9]Intermediate capital restructurings besides bankruptcy are excluded to retain tractability. The capital structure remains unchanged between the date of the first and the second round of financing, and after the investment date until bankruptcy. Incorporating repeated capital restructurings into the model adds little qualitative insight about

and new debt are issued *pari passu*.[10] The firm's management acting in favor of the shareholders decides when to enter bankruptcy.[11] Direct bankruptcy costs $\omega \in (0,1]$ consume a fraction of the firm's capital stock as in Goldstein, Ju and Leland (2001). The bankruptcy procedure is designed to minimize overall bankruptcy costs as described in Appendix A.

The tax code provisions and the financial market structure resemble Constantinides (1983). These assumptions capture the key aspects of realization-based capital gains taxation while preserving consumption-portfolio separation. Financial markets are frictionless and competitive, and collateralized short-sales are feasible as in Constantinides (1983). Investors are rational and taxable, and have full use of capital loss offsets.[12] Capital gains are taxed upon realization at statutory rate τ. The provisions for ordinary income taxation are standard. The personal tax rates on interest and dividend income are τ^p and τ^d, respectively. The firm is subject to corporate income taxation at marginal rate τ^c, and it distributes its net income as dividends since the double-taxation imposes a prohibitive cost on internal funds.[13] Debt services are tax-deductible with full loss offsets. The effective tax rate on ordinary income from equity is

$$\tau^e = 1 - (1 - \tau^c)(1 - \tau^d), \tag{2}$$

and the double-taxation of internal funds is[14]

$$\phi = (1 - \tau^e)/(1 - \tau^p). \tag{3}$$

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the effect of capital gains taxes but complicates the analysis considerably. See Leland (1994) for a model with no restructurings other than bankruptcy.   In the presence of deadweight external financing costs external financing events for restructuring purposes occur infrequently. See Goldstein, Ju and Leland (2001) for a model that relates the frequency of external financing events to the magnitude of refinancing costs. In practice external financing events do occur infrequently.

[10]This assumption is not crucial but simplifies the valuation of debt, since all debt claimants have the same recovery ratio in the event of bankruptcy. In addition, it allows the analysis of how capital gains taxation affects financial risk-shifting incentives, that is the firm may exploit existing debtholders by issuing new risky debt.

[11]Intermediate funding shortfalls are met through rights issues to existing equityholders, where management has leeway in setting the subscription price to keep the aggregate tax basis unaffected.   In effect this resembles the negative dividends or deep pockets assumption in Fischer, Heinkel and Zechner (1989) and Leland (1994).

[12]Currently, deductibility of capital losses from ordinary income is limited to $3000 per annum. Capital losses that are not accounted for in a given calendar year may be deducted from the investors' taxable income in consecutive years.   In the model shorting-against-the-box does not evade capital gains taxation, since net short-sale proceeds depend as in Constantinides (1983) on the basis of the particular share being borrowed.

[13]For tractability I omit the choice between dividends and share repurchases (see Green and Hollifield (2003)).

[14]Miller (1977) argued in a static setting without lock-in effect that tax clienteles lead to $\phi = 1$ for the marginal income tax bracket. Setting $0 < \phi < (1 - \tau)$ in this paper can be supported with the empirical findings in Graham (1999, 2000) and the fact that the actual tax code is realization-based—creating a lock-in effect.

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2.2 The Investors' Tax Timing Problem

The assumptions in Section 2.1 guarantee all investors pursue the same simple tax timing policy.

Proposition 1 *Investors' optimal trading strategy is deferment of capital gains and immediate realization of capital losses until the firm declares bankruptcy. Investors are "locked-in" whenever the market price exceeds their basis. By the time of bankruptcy, all equityholders have reset their tax basis to zero. The bankruptcy decision is therefore supported unanimously by all equityholders.*

Proof. *See Appendix.* ∎

The intuition for the second part of Proposition 1 is that the share price is falling as the firm approaches bankruptcy. While they may have heterogeneous basis values, all shareholders optimally realize losses and reset their basis values on the way down. By the time the equity value reaches zero, the heterogeneity will have been eliminated since all tax bases are nil.

The lock-in effect described in Proposition 1 drives a wedge between the private valuations of incumbent investors and the market price. The optionality inherent in the tax code generates an embedded tax timing option. The value of this tax-loss selling option is larger the higher the tax basis, since the strike of the embedded put option equals the basis. Private valuations, therefore, differ across investors with different basis values—notwithstanding the fact that all investors receive the same amount of dividends or coupon payments, respectively.

I denote by M_t the minimum pre-tax earnings until date $t \leq \kappa_u$, where $\kappa_u \in \mathcal{T}$ is the stopping time for real option exercise and $\mathcal{T}$ is the set of all stopping times adapted to the filtration $\mathcal{F}$ generated by X. That is I associate with X the stopped minimum process $M = (M_t)_{t \geq 0}$,

$$M_t = \inf_{0 \leq s \leq \min(t, \kappa_u)} X_s. \tag{4}$$

Proposition 1 implies that in perfect capital markets the tax basis of the initial shareholders equals the historical minimum stock price at all times starting with the IPO, since they engage in tax-loss selling whenever the stock price falls to a new low. Since the stock price is monotonic in X, there is a one-to-one mapping between historical stock price lows and the process M. Thus, M_t coincides at any date $t \leq \kappa_u$ with the optimal basis reset threshold for the initial shareholders.

Let (x, m) denote realizations of (X, M) in the state space $\mathcal{S} = \{(x, m) \in \mathbb{R}^+ \times \mathbb{R}^+ : x \geq m\}$, and define

$$
\begin{aligned}
v^i(x, m; B) &\quad : \quad \text{Private equity valuation of a share with tax basis } B, \\
v^i(x, m) &\quad : \quad \text{Private equity valuation of the initial shareholders,} \\
p^i(x, m) &\quad : \quad \text{Stock price,} \\
D^i(x, m) &\quad : \quad \text{Market price of the aggregate debt outstanding.} \\
i = 0, 1 &\quad : \quad \text{before (0) or after (1) investment takes place.}
\end{aligned}
$$

The *competitive clearing condition* (cf. Williams (1985), Dammon and Spatt (1996)) requires that the market price is set by new investors. Since they enter with a basis equal to the market price, the market price compounds an *at-the-money* tax timing option. This creates the following fixed-point problem

$$p^i(x, m) = v^i(x, m; p^i(x, m)), \ \ i = 0, 1, \ (x, m) \in \mathcal{S}. \tag{5}$$

Proposition 1 has a number of implications for both investor and corporate behavior. Investors' optimal strategy is one of instantaneous control by keeping the basis-to-price ratio from exceeding unity. Denote by $b^i(m; B)$ the value of pre-tax income X that triggers tax-loss selling of a share with basis B. Proposition 1 thus implies $v^i(x, m; B) \leqq p^i(x, m)$ for $x \geqq b^i(m; B)$, $i = 0, 1$, and

$$p^i(b^i(m; B), m) = B. \tag{6}$$

Hence, private valuations of incumbents are (weakly) lower than the market price since investors with a basis above the market price immediately reset. Finally, Proposition 1 and condition (5) imply that the private valuations of the initial owners satisfy

$$v^i(x, m) = v^i(x, m; p^i(m, m)), \ \ i = 0, 1. \tag{7}$$

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2.3 The Firm's Capital Budgeting Problem and Valuation

Proposition 1 further carries important implications for the firm's decision problem. Investors with different basis values generally disagree about value maximization.[15] The Modigliani and Miller (1958, 1963) and Miller (1977) irrelevancy theorems break down, since investors cannot self-select themselves into clienteles once they are locked-in. Stock price maximization is therefore not a time-consistent firm objective.

A natural choice for the firm's objective is to assume management commits to maximizing the private valuations of the initial shareholders—given the large portion of family and top management allocations documented in IPOs (see Mikkelson, Partch and Shah (1997)). This objective is well-defined, since collateral requirements preclude tax-arbitrage and align the incentives of continuing investors with those of exiting ones. New investors, although they disagree, are not being expropriated since they rationally anticipate the firm's policy and pay a fair price when they enter.

The firm's decisions concern when to exercise the growth option, how to optimally fund the investment with debt and equity, and when to declare bankruptcy. In the interests of the initial shareholders the firm's policy must take their current tax basis and its future evolution into account. A simple time-independent trigger strategy as in McDonald and Siegel (1986) is not optimal. Yet, it is sufficient to know the current value of operating income $X_t = x$ and its historical minimum $M_t = m$, since Proposition 1 implies the initial shareholders realize losses whenever the stock price, or equivalently X, falls to a new low. The firm's decision problem is therefore Markov in $(x, m) \in \mathcal{S}$.

The policy functions, $u(m), c^1(m), n^1(m), l^0, l^1(m)$, defined on $\mathcal{M} = \{(m) \in \mathbb{R}^+ : \underline{m} \leq m \leq \overline{m}\}$ with $\underline{m} = l^0$ and $\overline{m} = \sup(m \in \mathbb{R}^+ : u(m) \geq m)$ represent

$$
\begin{aligned}
u(m) \quad &: \quad \text{Investment boundary,} \\
c^1(m) \quad &: \quad \text{Aggregate debt coupon flow after the second round of financing,} \\
n^1(m) \quad &: \quad \text{Number of shares outstanding after the second round of financing,} \\
l^0, l^1(m) \quad &: \quad \text{Bankruptcy thresholds before and after investment.}
\end{aligned}
\tag{8}
$$

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[15]See Schneller (1980) and Lewellen and Mauer (1988). In this paper the lock-in effect of capital gains taxes precludes investor unanimity about the financing mix, the investment timing, and between debt- and equityholders also about the bankruptcy date. Maximizing their private valuations is in the interests of locked-in investors, whereas new investors in financial markets perceive market-value maximization as optimal.

All policy functions are smooth in $m \in \mathcal{M}$, since M has continuous paths and the decision problem is well-behaved (cf. Pedersen (2000), Guo and Shepp (2001), and Peskir (2001)). The optimal default trigger l^0 before investment is independent of m, since shareholders vote unanimously on the optimal bankruptcy policy. The corresponding hitting times (whichever comes first) are

$$\text{Investment date} \quad : \quad \kappa_u \equiv \inf(t \geq 0 : X_t \geq u(\inf_{0 \leq s \leq t} X_s)), \tag{9}$$

$$\text{Bankruptcy date} \quad : \quad \kappa_l^0 \equiv \inf(t \geq 0 : X_t \leq l^0). \tag{10}$$

The firm's optimization problem amounts to optimal stopping and control on the boundary of the continuation region $\mathcal{C} = \{(x, m) \in \mathcal{S} : m \leq x \leq u(m), m \in \mathcal{M}\}$. If $x > \overline{m}$ the firm invests immediately, and if $x < \underline{m}$ the firm immediately declares bankruptcy. Figure 2 illustrates the bankruptcy region ($\mathcal{L}$), the continuation region ($\mathcal{C}$), and the investment region ($\mathcal{U}$) in the state space $\mathcal{S}$. The arrows indicate the feasible direction of movement in the state space for the cases $x > m$ and $x = m$, respectively. More formally, the optimization problem is

$$\sup_{u(m), c^1(m), n^1(m), l^0, l^1(m)} v^0(x, m) \tag{11}$$

subject to the budget constraint

$$p^1(u(m), m)[n^1(m) - 1] + D^1(u(m), m) - D^0(u(m), m) = I, \tag{12}$$

and (c^0, π, I) are given, and $m \in \mathcal{M}$.

The objective function in (11) represents the private equity valuation of a representative investor with basis equal to the historical minimum stock price. The constraint (12) says the proceeds from issuing $n^1(m) - 1$ new shares and $D^1(\cdot) - D^0(\cdot)$ in new debt suffice to cover the capital expenditure $I > 0$. In case $n^1(m) < 1$, the firm repurchases shares through brokers. Competitiveness of financial markets requires that the offer price of new securities equals their market price after issuance and there is no price reaction, or $p^0(u(m)-, m) = p^1(u(m)+, m)$.

The Hamilton-Jacoby-Bellman equation corresponding to (11)-(12) gives the following set of

conditions

$$\frac{\sigma^2}{2}x^2 v_{xx}^0(x,m) + \mu x v_x^0(x,m) - r v^0(x,m) + (\pi x - c^0)(1-\tau^e) = 0 \quad \text{for } (x,m) \in \mathcal{C}, \quad (13)$$

with the value-matching and smooth-fit conditions

$$
\begin{array}{llll}
v^0(x,m) = v^1(x,m), & v_x^0(x,m) = v_x^1(x,m) & \text{for } x = u(m), & \\
v^0(x,m) = 0, & v_x^0(x,m) = 0 & \text{for } x = m = l^0, & (14) \\
v^0(x,m) = p^0(m,m), & v_m^0(x,m) = \frac{\tau}{1-\tau}v_x^0(x,m) & \text{for } x = m. &
\end{array}
$$

The solution to $v^1(x,m) = \sup_{c^1,n^1} v^1(x,m;p^0(m,m)|c^1,n^1)$ subject to (12) determines $(c^1(m), n^1(m))$ for all $m \in \mathcal{M}$.

The first line in (14) determines the optimal investment boundary $u(m)$. The firm's optimal investment policy takes a particularly simple form while $M = (M_t)_{t\geq0}$ is constant. Conditional on $M_t = m$, the decision rule is a time-homogeneous trigger strategy as in McDonald and Siegel (1986). On the boundary $\{(x) : x = u(m), m \in \mathcal{M}\}$ it satisfies the smooth-fit principle (see Pedersen (2000), and Peskir (2001)). The limited liability of shareholders leads in (14) to the second pair of value-matching and smooth-fit conditions. They determine the bankruptcy trigger l^0. The last two conditions in (14) come out of the tax timing problem of the initial shareholders (cf. Constantinides (1983) and Williams (1985)).[16] In more detail, using (6, 7) the optimality conditions for tax-loss selling in the special case $B = p^0(m,m)$ are

$$v^0(x,m) = (1-\tau)p^0(x,m) + \tau p^0(m,m), \quad v_x^0(x,m) = (1-\tau)[p_x^0(x,m) + p_m^0(x,m)] \quad \text{for } x = m. \quad (15)$$

The functional form for $p^0(x,m)$ at times when X falls to a new low, $p^0(m) \equiv p^0(m,m)$, and similarly for $p^0(x,m)$ at $x > m$, can be determined by using the identity (6) and rewriting the optimality conditions (15) as a differential equation for $p^0(m)$. One obtains the following conditions: $p^0(m) = v^0(m,m)$ and $p_m^0(m) = \frac{1}{1-\tau}v_x^0(m,m)$ for $l^0 < m \leq u(m)$, and $p^0(m) = p_m^0(m) = 0$ for $m = l^0$.

The private valuation functions $v^1(x,m)$, $v^i(x,m;B)$, $i = 0,1$, and the prices of debt, $D^0(\cdot)$

---

[16]The last condition in (14) represents "normal reflection" at the diagonal. See Grigelionis and Shiryaev (1966), Dubins, Shepp and Shiryaev (1993).

and $D^1(\cdot)$, satisfy similar conditions that are relegated to Appendix C. The next proposition summarizes the solutions.

Proposition 2 *The private equity valuation of a share associated with basis B equals*

$$
\begin{aligned}
v^0(x, m; B) &= \lambda \pi x - \delta c^0 + \varphi_b^0(x, m; B)[B - (\lambda \pi b^0(m; B) - \delta c^0)] \\
&\quad + \varphi_u^0(x, m; B)[v^1(u(m), m; B) - (\lambda \pi u(m) - \delta c^0)], \qquad (16) \\
v^1(x, m; B) &= (\lambda x - \delta c^1)/n^1 + \varphi_b^1(x, m; B)[B - (\lambda b^1(m; B) - \delta c^1)/n^1], \qquad (17)
\end{aligned}
$$

where

$$
\begin{aligned}
\beta &= (1 + \gamma)/(1 + \gamma - \tau), \\
\delta &= (1 - \tau^e)/r, \\
\lambda &= (1 - \tau^e)/(r - \mu), \\
\gamma &= \frac{1}{2}\left(\frac{\mu}{\sigma^2/2} - 1 + \sqrt{\left(\frac{\mu}{\sigma^2/2} - 1\right)^2 + 4\frac{r}{\sigma^2/2}}\right), \\
\eta &= \frac{1}{2}\left(\frac{\mu}{\sigma^2/2} - 1 - \sqrt{\left(\frac{\mu}{\sigma^2/2} - 1\right)^2 + 4\frac{r}{\sigma^2/2}}\right).
\end{aligned}
$$

The optimal tax loss selling thresholds $b^i = b^i(m; B)$, $i = 0, 1$, satisfy (6). The expressions for the state-contingent claims $\varphi_b^0(\cdot)$, $\varphi_u^0(\cdot)$, $\varphi_b^1(\cdot)$, $\varphi_l^1(\cdot)$ are

$$
\begin{aligned}
\varphi_b^0(x, m; B) &= (x^{-\gamma} u(m)^{-\eta} - u(m)^{-\gamma} x^{-\eta})/((b^0)^{-\gamma} u(m)^{-\eta} - u(m)^{-\gamma}(b^0)^{-\eta}), \qquad (18) \\
\varphi_u^0(x, m; B) &= ((b^0)^{-\gamma} x^{-\eta} - x^{-\gamma}(b^0)^{-\eta})/((b^0)^{-\gamma} u(m)^{-\eta} - u(m)^{-\gamma}(b^0)^{-\eta}), \qquad (19) \\
\varphi_b^1(x, m; B) &= (b^1)^{\gamma} x^{-\gamma}, \qquad (20) \\
\varphi_l^1(x, m) &= (l^1)^{\gamma} x^{-\gamma}. \qquad (21)
\end{aligned}
$$

The stock price before and after investment, respectively, equals

$$
\begin{aligned}
p^0(x, m) &= \frac{1}{1 - \tau}[f(x, m)\int_{l^0}^m h(m, z)dz + \int_m^x g(x, m, z)dz], \qquad (22) \\
p^1(x, m) &= (\beta \lambda x - \delta c^1)/n^1 - \varphi_l^1(x, m)^{\frac{1}{1-\tau}}(\beta \lambda l^1 - \delta c^1)/n^1, \qquad (23)
\end{aligned}
$$

with $f(x, m)$, $g(x, m, z)$ and $h(x, z)$ given in the Appendix.

The market value of the aggregate debt after investment (and similarly of the old and new debt issues with c^1 replaced by c^0 and $c^1 - c^0$, respectively) is

$$D^1(x, m) = c^1(1 - \tau^p) / \rho^1(x, m) . \tag{24}$$

The after-tax yield is the same for both the old and the new debt issue, since they have equal priority:

$$\rho^1(x, m) = r/(1 - \alpha \varphi_l^1(x, m)^{\frac{1}{1-\tau}}). \tag{25}$$

The coefficient

$$\alpha = 1 - (1 - \omega)\zeta\xi\phi \tag{26}$$

measures the total bankruptcy cost and is decreasing in τ. In (26) the constant coefficient $\zeta \equiv V^*(l^1)/(\lambda l^1)$ represents the ex-ante gains to leverage (where $V^*(x)$ is the market value of the optimally recapitalized firm), and $\xi \equiv (\lambda l^1)/(\delta c^1)$. Both ζ and ξ are given in the Appendix.

Proof. See Appendix. ∎

Some Special Cases. In the special case $\tau = 0$, expressions (23) and (24) for the time after investment collapse to the perpetual debt model in Leland (1994). The value of the embedded tax option vanishes and private valuations (16) and (17) coincide with the share prices (22) and (23). The optimal investment trigger $u(m)$ becomes a time-independent trigger u as in McDonald and Siegel (1986). The stock price is independent of m and equals

$$
\begin{aligned}
p^0(x, m) &= \pi\lambda x - \delta c^0 - \frac{x^{-\gamma}u^{-\eta} - u^{-\gamma}x^{-\eta}}{(l^0)^{-\gamma}u^{-\eta} - u^{-\gamma}(l^0)^{-\eta}}(\pi\lambda l^0 - \delta c^0) \\
&\quad + \frac{(l^0)^{-\gamma}x^{-\eta} - x^{-\gamma}(l^0)^{-\eta}}{(l^0)^{-\gamma}u^{-\eta} - u^{-\gamma}(l^0)^{-\eta}}[p^1(u, m) - (\pi\lambda u - \delta c^0)], \tag{27} \\
p^1(x, m) &= (\lambda x - \delta c^1)/n^1 - (l^1)^\gamma x^{-\gamma}(\lambda l^1 - \delta c^1)/n^1. \tag{28}
\end{aligned}
$$

In the special case $c^1 = 0$ ($n^1 = 1$) and $\tau > 0$, the valuations in period 1 coincide with the equations in Constantinides (1983, footnote 6) for the value of an unlevered firm. The basis reset

16

trigger, $b^1(m; B)$, equals $B/(\beta\lambda)$ and

$$v^1(x, m; B) = \beta^{-1}p^1(x, m) + (1 - \beta^{-1})B^{1+\gamma}p^1(x, m)^{-\gamma}, \qquad (29)$$

$$p^1(x, m) = \beta\lambda x. \qquad (30)$$

A comparison of the two special cases to the general solution (16)-(26) illustrates the effect of personal taxes on private valuations, market prices, and bankruptcy costs:

The General Case. Private equity valuations (16, 17) consist of two components—the present value of dividend distributions after ordinary income taxes and the present value of the stream of capital loss offsets. Shareholders receive after-tax dividends $(\pi X_t - c^0)(1 - \tau^e)$ before investment and, respectively, $(X_t - c^1)(1 - \tau^e)/n^1$ per share after investment. Accordingly, the first terms in (16) and (17) are the after income-tax values of the perpetual dividend flow. Once operating income X_t falls to $b^i(M_t; B)$, $i = 0, 1$, investors with basis B sell their shares. They receive after-tax proceeds equal to B per share $(= (1 - \tau)p^i(b^i(M_t; B), M_t) + \tau B)$. The second terms in (16) and (17) represent the value of this timing option including the value of limited liability. The third term in (16) reflects the value of the real option to invest and restructure. Finally, the terms φ_b^0 and φ_u^0 are the pre-tax values of state-contingent claims that pay either a single unit when X_t hits the basis-reset threshold $b^0(M_t; B)$ before the investment boundary $u(M_t)$ (subscript b) or, respectively, when X_t hits $u(M_t)$ before $b^0(M_t; B)$ (subscript u).

The effect of capital gains taxes on prices becomes apparent in (23). The coefficient β captures the personal tax benefit associated with volatility. Note that β is monotonically increasing in σ and τ, and it satisfies $1 \leq \beta < (1 - \tau)^{-1}$ as in Constantinides (1983). Riskless coupon flows c^1, on the other hand, are not associated with a tax option premium in (23). The price of a bankruptcy contingent claim, however, is discounted due to the fact that investors collect capital loss offsets on the path to bankruptcy. This is captured by the exponent $\frac{1}{1-\tau}$ on φ_l^1 in (23). In (22), the second integral represents the after-tax value of dividend income plus tax loss offsets until operating income falls to m. The first term captures the risk-adjusted present value of after-tax cash flows starting at the latter date and ending once shareholders make use of their limited liability.

3 The Effect on Corporate Policy and the Empirical Implications

In the following, I discuss the main predictions of the model. Section 3.1 outlines qualitative properties of the firm's optimal bankruptcy policy. In section 3.2, I decompose the effects of realization-based capital gains taxation into financing and timing effects. I calculate the value of switching to a state-dependent policy in Section 3.3. Section 3.4 provides simulation results, and section 3.5 briefly discusses the robustness of the predictions to capital market imperfections.

3.1 The Optimal Bankruptcy Policy

The first result is an irrelevancy theorem for the firm's bankruptcy decision under realization-based capital gains taxation. The result follows intuitively from Proposition 1 and allows an explicit solution to the bankruptcy problem after investment.

Proposition 3 *The shareholder value-maximizing bankruptcy trigger after investment does not depend on the capital gains tax rate τ. The optimal bankruptcy threshold l^1 is proportional to the face value of debt, and equals*

$$l^1 = \frac{\gamma}{1+\gamma} \frac{\delta}{\lambda} c^1. \tag{31}$$

    Proof. *See Appendix.* ∎

Proposition 3 implies that the outcome of a one-time capital structure decision under realization-based capital gains taxation is observationally equivalent to the one under accrual-based taxation (see, e.g., Leland (1994)). The reason is that—as shown in Proposition 1 in Section 2—by the time of bankruptcy all shareholders have reset their basis values to nil. Once the basis is zero private equity valuations are independent of the capital gains tax rate—which can be seen by setting $B = 0$ in (17). As a result, the optimal bankruptcy trigger l^1 is independent of τ. Nonetheless, realization-based capital gains taxation has testable implications for dynamic corporate policy, since the locked-in effect distorts investment and funding decisions.

Irrespective of the irrelevance of capital gains taxation for the timing of bankruptcy, capital gains taxes affect debt valuations through the magnitude of expected bankruptcy costs. The total

bankruptcy costs incurred at the bankruptcy date are $\alpha \frac{\delta}{\phi} c^1$. They represent a constant fraction α of the default-free debt value, since both pre- and post-restructuring debt coupons are proportional to l^1. The trade-offs debtholders face in bankruptcy are altered by the presence of capital gains taxation since α varies with τ.[17]

3.2 The Optimal Investment and Financial Policy

The next result characterizes how the initial shareholders' tax basis, or alternatively m, affects the optimal mix of debt and equity financing, $c^1(m), n^1(m)$.

Proposition 4 *Firms use more equity to fund their investment project, the lower the basis (i.e., the larger the embedded capital gains) of their shareholders. That is $\frac{\partial c^1(x|B)}{\partial B} \geq 0$ given x and c^0. The optimal capital structure is path-dependent on the stock price evolution prior to restructuring, since the stock price path determines the owners' basis.*

In the limit as the basis-price ratio approaches one and $c^0 = 0$, the target leverage ratio after the second round of financing equals the ex-ante optimal static leverage ratio. The latter is increasing in τ and equals

$$\frac{(1-\omega)}{(1-\alpha)}(1 - \alpha \vartheta^{\frac{\gamma}{1-\tau}})\vartheta. \tag{32}$$

The constant coefficient ϑ is given in the Appendix.

Proof. *See Appendix.* ■

The second part of Proposition 4 provides sufficient conditions for the *ex-ante* and *ex-post* financing problems to have identical outcomes. The term *ex-post* refers to the situation in which a tax basis has already been assigned and is predetermined, whereas in the *ex-ante* case the tax basis is endogenous. The solutions to the *ex-ante* and *ex-post* financing problems generally differ. Proposition 4 establishes that the *ex-ante* optimum coincides with the *ex-post* optimal capital structure if there are no conflicts of interest among the various stakeholders. If the firm is unlevered ($c^0 = 0$), debt-equityholder conflicts trivially disappear. In the limit as the basis-to-price ratio approaches unity there is unanimity among incumbent shareholders and outsiders.

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[17]Given the assumption of perfect capital markets α is decreasing in τ as shown in Proposition 2. The reason is that through the embedded tax timing option capital gains taxes positively affect the price at which new securities can be issued.

The intuition for the first part of Proposition 4 is simple but requires some care, since several effects offset each other. In the following I decompose the effects of realization-based capital gains taxation on financing and investment. I also provide guidance on the economic magnitude of the effects. I start by quantifying the variation in capital structure ratios and investment rates, and show how the size of the investment project affects the value of the option to delay. I determine the value of conditioning on the owners' tax basis in Section 3.3. In Section 3.4, I run capital structure regressions on simulated data to assess the empirical relevance of the effects.

3.2.1 The Financing Effect

The financing effect described in Proposition 4 is the result of asymmetric valuations of debt and equity across investors with different tax basis. The trade-offs in the *ex-ante* decision problem are between interest tax shields due to corporate taxation, bankruptcy costs, and the income effect of capital gains taxation.

The *income effect* of realization-based capital gains taxes is that potential tax payments, loss offsets, and tax credits are all compounded into pre-tax valuations. In particular, the government provides investors through capital loss offsets and tax credits with a hedge in the event the share price drops. The resulting tax timing option compounded into the market price is always *at-the-money*. The marginal investor who determines the market price is subject to just this income effect.[18]

The *arbitrage effect* of realization-based capital gains taxation is driven by a wedge between the private valuations of incumbent investors and the market price. Since the value of the embedded tax timing option is larger the higher the tax basis, incumbent investors have lower private valuations than new investors whenever the market price exceeds their basis (see Section 2.2). This valuation differential is crucial when raising external financing. Issuing equity is a local tax arbitrage opportunity for incumbent shareholders—similar to issuing debt for interest tax shield purposes.[19]

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[18]In bankruptcy debtholders benefit from the income effect of capital gains taxes. The initial offer price of reissued capital compounds potential future loss offsets. In the model, the income effect increases residual firm value, and lowers the overall bankruptcy costs α. The reason is that market prices increase in the capital gains tax rate τ, since the discounted value of future capital loss offsets is strictly positive and increasing in τ.

[19]See Ross (1987). The arbitrage effect is, in general, dependent on the use of the financing proceeds. The value of the tax timing option is an increasing function of the volatility of the combined cash flow stream from current operations and new investments. Positive correlation between the firm's current and future business is advantageous,

Every newly issued share entitles new investors to an *at-the-money* tax option provided by the government—given the investor is taxable. The ownership dilution associated with new equity issues creates a surplus equal in magnitude to the difference between the reservation values of the new investors (i.e., the competitive market price) and of the incumbent shareholders (i.e., their private valuations). In competitive markets, incumbent owners can capture the entire surplus created by issuing equity. For low-basis owners, since their reservation values are lower, the surplus is larger in relative terms than for high-basis owners—although the ownership dilution and the amount of funds raised per share are the same. The firm's management takes this into account and optimally issues more equity, the larger the stock price-to-basis ratio.

The *hedging effect* is a positive externality of riskless coupon payments on private equity valuations. In general, the source of funds affects the volatility of flows to equity, thereby altering the value of existing tax timing options. Additional debt payments raise the volatility of the residual equity claim. This represents a tax-advantage for incumbent shareholders. The result is an increase in the value of the equity-embedded tax option and, in turn, in private equity valuations. The hedging effect shifts the optimal financing mix towards debt.

The magnitudes of both the arbitrage effect and the hedging effect depend on the tax basis. The arbitrage effect (favoring equity) is stronger, the smaller the basis-to-price ratio. The reason is private valuations (17) are increasing in B and, thus, valuation differentials between incumbent and new investors are decreasing in B. The hedging externality of coupon payments on the value of existing tax timing options (favoring debt) is larger, the larger the basis-to-price ratio. The reason is that the sensitivity of the tax option value with respect to c^1 in (17) is increasing in absolute terms with the tax basis B (and τ). Although the two effects work in opposite directions, i.e., the arbitrage effect represents a tax-advantage to equity and the hedging effect represents a

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since it increases the overall volatility and diminishes the adverse effect of Jensen's inequality on the value of embedded tax timing options. The "effective" tax rate on equity is a decreasing function of volatility and of signed correlation. As a result, hedging is non-neutral. The intuition follows from the standard result in financial option pricing theory that an option's vega is always positive, i.e., the value of an option increases with volatility. The tax timing decision of equityholders corresponds to an embedded American option pricing problem. Therefore given an arbitrary tax basis, or tax option strike price, the tax option value increases with the operational and financial risk characteristics of the firm. As a result, private equity valuations fall if the proceeds from external financing are invested risk-free, since it reduces the volatility of future earnings. Investing the proceeds in less than perfectly correlated financial instruments is disadvantageous for the same reason. In the model, the new funds are used for a perfectly correlated expansion project, which entirely eliminates the effect of Jensen's inequality. I leave for future work the study of less than perfectly correlated investment projects, mergers and takeovers, spinoffs, etc.

tax-advantage to debt, across tax bases they have the same impact. Both imply that low-basis shareholders find equity more and debt less attractive than high-basis shareholders. Proposition 4 shows that as a result the target leverage ratio depends positively on the basis-to-price ratio.

3.2.2 How large are the financing distortions?

Table 2 quantifies the resulting variation in the optimal financing mix and in both the ex-ante and ex-post optimal capital structure. I report both target leverage ratios and credit spreads at the issuance date. I vary the basis-to-price ratio at the investment date between 0% and 100% by conditioning on the historical minimum, $m \in [\underline{m}, \overline{m}]$, relative to where it falls in between the bankruptcy trigger $\underline{m} = l^0$ and the value corresponding to immediate investment, $\overline{m} = \sup(m \in \mathbb{R}^+ : u(m) \geq m)$. The base parameterizations are summarized in Table 1 and $c^0 = 0$, $\pi = 100\%$.

The first panel in Table 2 shows that in the base parametrization with $\tau = 25\%$, the cross-sectional dispersion in target leverage ratios ranges up to 9.2 percentage points. Firms with shareholders that have a negligible tax basis (due to prior loss realizations) have a target leverage of 60.7 percent compared to 69.9 percent for firms with a basis-price ratio of one at the restructuring date. As shown in the second panel of Table 2, this translates into a debt yield for the zero-basis firms that is 45 bp lower than for the highest basis firms. In case the basis-to-price ratio at the restructuring date is 100% (0%), leverage in the base parametrization with $\tau = 25\%$ is 3.5 points higher (5.7 points lower) than in the case without capital gains taxation. The effect of capital gains taxes on aggregate leverage is thus not uniform across basis values. Last, the ex-ante optimal static leverage ratio is equal to the value in the column corresponds to a basis-to-price ratio of one. The results in the first and second panel, respectively, confirm that the ex-ante optimal leverage ratio increases and that the ex-ante target credit spread decreases with the capital gains tax rate.

The remaining rows in both panels of Table 2 demonstrate that the predictions are robust to different parameterizations. The cross-sectional variation in target leverage is larger, the smaller the direct bankruptcy costs ($\Delta = 9.9\%$/ 55 bp if $\omega = 25\%$). Similarly, a reduction in the double-taxation of corporate income increases the dispersion in ex-post optimal leverage ratios. For instance if ϕ equals 75%, the target leverage of a firm with negligible tax basis is 53.5 percent compared to 65.3 percent if the basis-price ratio is one—a difference of close to twelve points.

3.2.3 The Timing Effect

The basic intuition for the effect of realization-based capital gains taxes on the timing of corporate investment can be gained from the standard result in real options theory that "the greater the uncertainty over future cash flows, the larger is the excess return the firm demands before making the irreversible investment," Dixit and Pindyck (1994). Realization-based capital gains taxation distorts the relation between irreversible investment and uncertainty by altering the risk characteristics of after-tax cash flows. The government shares investment risk disproportionately through asymmetric taxation of gains and losses. Tax credits and offset provisions in loss-states create concave tax liabilities and induce "risk-loving" corporate behavior. Therefore, realization-based capital gains taxation *a priori* diminishes the value of the option to delay investment and lowers the critical threshold for irreversible investment. However, the concavity in the personal tax liabilities vanishes endogenously, i.e., the kink in the effective tax schedule moves down, if investors reset their tax basis before the firm's real option exercise. The investment stimulus due to capital gains taxes is thus only transitory. The firm's management rationally anticipates that the owners can deduct fewer capital losses in the future when the investment project performs poorly, since capital loss tax shields have already been used up before investment has taken place. As a result, the critical threshold for investment shifts up whenever shareholders realize tax losses.[20]

3.2.4 How large are the investment distortions?

Table 3 reports comparative statics for the investment policy $u(m)$ as a function of the running minimum m in the case $\pi = 1$. The effect of the parameter π is illustrated in Table 4. Each line in Table 3 represents an alteration of the base parametrization from Table 1. Separate results

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[20]In more detail, both the marginal value of exercising the growth option and the marginal value of waiting depend on the tax basis. The propensity to keep the firm's capacity low and save the option to expand for later use increases as the tax basis B decreases, i.e., $\frac{\partial^2}{\partial x \partial B} v^0(\cdot) \leq 0$. The marginal value of investing also grows as B falls, i.e., $\frac{\partial^2}{\partial x \partial B} v^1(\cdot) \leq 0$, since the option delta of the embedded tax loss put option becomes smaller in magnitude as B decreases. At the optimal investment date the marginal value of exercising the growth option and the marginal value of waiting must be equal. Inevitably, the investment threshold depends on the tax basis of the firm's owners. How the basis affects the investment threshold, however, depends on the financial policy. Optimal financing of the project is sufficient for the investment boundary to be monotonically decreasing in B. The predictions are different under exogenous financing. Equity financing is a tax arbitrage for locked-in shareholders. In case of exogenous pure equity-financing, the arbitrage effect is so strong that low-basis firms will take advantage of the tax arbitrage sooner than high-basis firms. The investment trigger becomes a monotonically increasing function of the tax basis. The opposite occurs with exogenous debt financing. In extensive simulations under exogenous financing I find the investment boundary to be increasing in m only under close to pure equity financing.

for capital gains tax rates varying between 0% and 35% are reported for the base parametrization. In the different columns, m varies within $[\underline{m}, \overline{m}]$ which corresponds to basis-to-price ratios (BP_u-ratio) between zero and one at the investment date. The second panel of Table 3 characterizes the investment policy in terms of the market equity value-to-investment ratio at the investment date.

The common feature in all parameterizations is that the optimal investment trigger $u(m)$ shifts upward as m falls. In the base parametrization with $\tau = 25\%$, the threshold for a BP_u-ratio of zero is 178 percent higher than for a BP_u-ratio of one. Compared to an otherwise identical economy without realization-based capital gains taxation, real option exercise is accelerated only for BP_u-ratios above approximately 30%. The second panel in Table 3 shows that the net effect on the market value of equity at the restructuring date is even more dramatic. In the base parametrization, the lock-in effect creates variation in the market value of the firm's equity at $t = \kappa_u$ of up to 260%.

The predictions for corporate investment from these comparative statics results are distinct, testable patterns in both the cross-section and time-series. In general, events that cause the stock price to fall and shareholders to realize losses, such as poor firm performance or negative macroeconomic shocks, have long-term impact on capital budgeting decisions. Short-term stock price fluctuations lead to long-term reductions in investment rates and leverage ratios. Corporate inertia as documented in Welch (2004) is more pronounced after poor performance associated with stock price declines.

The remaining lines in Table 3 show that the magnitude of the timing effect increases with σ, μ, ϕ, and $(1 - \omega)$. The path-dependency and non-stationarity of investment and capital structure should therefore be more pronounced in industries with high earnings volatility and little systematic risk, in growth sectors, in firms with little specificity of physical or human capital, in firms with low direct bankruptcy costs, with low marginal corporate tax rates, and during regimes with less double-taxation of corporate income.

3.2.5 How does personal taxation affect the interaction between corporate investment and financial policy?

Under realization-based capital gains taxation the firm's investment and financial policy interact. The basis-to-price ratio at the investment date κ_u (BP_u-ratio) or, equivalently, the value of the state variable $M_{\kappa_u} = m$ determine the optimal financing mix for the investment project. Proposition 4 shows that the investment project is financed with more equity the higher the BP_u-ratio, or the smaller m. The financial policy, in turn, affects the optimal timing of corporate investment through its impact on shareholders' tax timing option as shown in Section 3.2.3. As shareholders realize losses, the BP_u-ratio drops for two reasons. The tax basis declines and the optimal investment trigger shifts upwards, since shareholders with low basis are willing to wait longer until real option exercise than high basis shareholders. Low basis shareholders also prefer to raise equity. More equity financing, in turn, alters the option value to wait. The optimal investment trigger given m, $u(m|c^1)$, is a U-shaped function of the debt coupon c^1. Hence, the direction of the interaction depends on whether the optimal debt coupon lies on the down- or on the upward sloping part of the functional $c^1 \mapsto u(m|c^1)$.

The size of the investment project is an important determinant of the direction of the interaction. The project size relative to existing operations is governed by the coefficient π. Depending on π, financial flexibility either dampens or exacerbates the timing effect:

- For small project sizes (π large), the optimal investment trigger is monotonically decreasing in c^1. In this case, more equity financing as m falls increases the option value to wait. The longer wait, in turn, increases the BP_u-ratio further, and the firm's management responds by issuing ever more equity. Optimal financing thus reinforces investment delay.

- For large investment projects (π small), optimal financing dampens the deferral effect. In this case, equity financing decreases the option value to wait, since the optimal c^1 lies on the upward-sloping portion of $c^1 \mapsto u(m|c^1)$.

Table 4 confirms that the investment and financing patterns are qualitatively similar irrespective of the project size. Across different values for π, however, the cross-sectional range in both the threshold Q and the target leverage ratio increase with π. The difference in investment thresholds between $\underline{m}$ (BP_u-ratio $=0$) and $\overline{m}$ (BP_u-ratio $=1$) increases from 7 percent for $\pi = 0\%$ to 178

percent for $\pi = 100\%$. Similarly, the dispersion in target leverage ratios increases from 5.6 to 9.2 percentage points.

Investment is not always accelerated compared to an economy without capital gains taxation. Investment is accelerated for all $m \in [\underline{m}, \overline{m}]$ only if the size of the expansion project exceeds existing operations by a factor of more than two ($\pi \leq 1/3$) in the base parametrization. For $\pi > 1/3$, there are firms postponing investment "excessively" after a sufficiently large slump in operating profits. In this case, the critical threshold at which investment commences is for small enough m larger than if capital gains were not taxed. The reason for this excessive delay, or underinvestment, is the interaction between investment and financing under realization-based capital gains taxation. For small and medium-sized projects, more equity financing leads to additional delay in investment and, thus, reinforces the dependence on m of the investment trigger $u(m)$. As a result, investment is being delayed beyond the point at which it occurred if capital gains remained untaxed. For large projects the interaction effect is small and does not cause excessive delay.

3.3 How important are personal taxes for corporate capital budgeting?

To answer the question about the economic relevance of the effects studied in the previous sections I determine the gain in firm value that is being created by switching from a time-independent investment and financial policy—similar to McDonald and Siegel (1988) and Mauer and Sarkar (2003)—to a state-dependent policy. More specifically, I first determine the market-value maximizing investment threshold. Then I ask by how much the private valuation of the incumbent shareholders—as a function of their tax basis—increases if the firm switches to a path-dependent policy once the market-value maximizing constant threshold is being hit. In general, the value gain depends on both the owners' basis and the current share price. The relative gain before reaching the market-value maximizing trigger is even larger than the numbers reported in Table 5.

Table 5 summarizes the results. Using the base parametrization summarized in Table 1, the benefit from switching to the optimal policy ranges between 0% and 5.6% of firm value depending on the owners' tax basis. For a basis equal to the stock price the two policies are identical, thus offering no gains. For a basis of nil they differ the most, thus offering the highest gains. In the remaining parameterizations the increase in firm value ranges from 3.9% to 6.9% for a basis-to-price

ratio of zero. These numbers are lower bounds in the sense that the relative gains are larger when the stock price is below the myopic investment trigger. In particular, bankruptcy occurs sooner under the myopic policy than under the state-dependent policy.

3.4 The Cross-Sectional Capital Structure Implications

The model makes strong predictions about external financing and investment patterns in the cross-section and time-series, in particular the cross-sectional relation between Tobin's Q and leverage in IPO time. To illustrate the basic economic relationships I simulate data generated from the model using Monte-Carlo methods and then estimate cross-sectional capital structure regressions on the simulated data similar to the empirical setup in Baker and Wurgler (2002) (BW, henceforth). I include a measure of external financing-weighted market-to-book, MB^{efwa}, in a standard capital structure regression and check if the measure has explanatory power for leverage in IPO time. BW use this measure as a proxy for market-timing. If their measure is significant in the simulated data, it provides evidence against MB^{efwa} being a good proxy for behavioral market timing.

The basic cross-sectional regression estimated in IPO time is

$$L_t = a_0 + a_1 MB_{t-1}^{efwa} + a_2 MB_{t-1} + f(\mathcal{F}_t) + \varepsilon_t, \tag{33}$$

where MB_{t-1}^{efwa} is the lagged external-financing weighted average market-to-book ratio, MB_{t-1} is the lagged market-to-book ratio, and $f(\mathcal{F}_t)$ is an additive function of control variables measurable with respect to the filtration $\mathcal{F}_t$. The market timing measure is defined as

$$MB_{t-1}^{efwa} = w_{t-1} MB_0 + (1 - w_{t-1}) MB_u. \tag{34}$$

The weight w_{t-1} equals 1 if the firm has not invested by date $t - 1$, and otherwise it is $w_{t-1} = p_{t=0}^0 / (p_{t=0}^0 + I)$ with $p_{t=0}^0$ denoting the IPO proceeds.

The book value of the firm remains to be specified. I assume the change in book value at the investment date κ_u equals the capital expenditure I as in Barclay et al. (2003). That is after date $t = \kappa_u$ the book value of the firm equals $B_0 + I$ and remains constant thereafter, which neglects write-offs. The initial book value B_0 is defined as the market value of a firm with operations of

size π, coupon flows c^0, the ability to restructure at the original bankruptcy trigger l^0, but with no growth potential:

$$B_0 \equiv \beta\lambda\pi X_0 + (\phi^{-1} - 1)\delta c^0 + (l^0/X_0)^{\frac{\gamma}{1-\tau}}\{[(1 - \omega)\zeta - \beta]\lambda\pi l^0 - (\phi^{-1} - 1)\delta c^0\}, \qquad (35)$$

where X_0 is the operating income level at $t = 0$ and ζ is given in Appendix C. This definition allows disentangling the value of the growth opportunity from the value of existing operations—including tax shields and loss offsets.

The simulation procedure and the basic setup are described in Appendix B. Appendix B also discusses the choice of parameter values. The base parametrization is summarized in Table 1. Descriptive statistics for the cross-sectional distribution of the resulting patterns in capital structure and investment rates for years 1-10 after the IPO are reported in Table 6.

The first panel of Table 6 shows that the market-to-book ratio in the IPO year—which is not calibrated to the data—matches the empirical data remarkably well. Empirically it is 2.29 compared to 2.33 in the simulated data. This provides support for using the book value definition (35). The dynamic patterns of leverage ratios and asset growth rates are also similar to the COM-PUSTAT data used in BW. Leverage ratios are low just after the IPO and increase subsequently. Investment rates drop sharply after the peak in the second year following the IPO. The second panel of Table 6 shows that in the simulated data the majority of firms invest soon after the IPO. Their option value to wait is small, and therefore the investment threshold is reached sooner. The remaining firms must have performed poorly for some time. Their optimal investment trigger has shifted upwards which causes their investment hazard rates to drop.

For robustness, I report estimation results from three separate regression setups. The basic setup (BW1) follows BW most closely. The controls $f(\cdot)$ include X_t and $\ln(X_t)$. Operating income X_t controls for cross-sectional variation in operating performance, and $\ln(X_t)$ is a proxy for the firm size variable in BW, the log of sales.[21] In a second set of regressions (BW2), I include investment fixed effect dummies. The third specification (BW3) differs from (BW1) in that I

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[21]Alternatively, I have run regressions that control for lagged values of the state variable x, i.e., x_{t-1} and $\ln(x_{t-1})$, and/or the time elapsed since the investment date κ_u. The results are similar and omitted. Coefficient estimates, however, are more noisy. For robustness, I have also run censored and truncated tobit regressions in which the group of bankrupt firms enter with a leverage ratio of one. The coefficient estimates are similar and omitted.

exclude the subsample of firms that have not yet invested by the specified time in order to control for investment fixed effects.

Table 7 reports the estimation results. In the first panel I report regression coefficients on book leverage and, respectively, in the second panel on market leverage. The most important commonality is that in both panels the coefficient a_1 on the external-financing weighted average historical market-to-book ratio is negative in all time periods; the coefficient a_2 on lagged market-to-book is close to zero, and even changes signs. In model (BW1), the coefficient estimates are increasing in IPO time. The same features have been observed empirically by BW.

The magnitude of the coefficients is the main aspect in which the data differs from the model. In setup (BW1), the coefficient a_1 is 26 to 40 times larger than observed in the data. Yet, the economic order of magnitude is about the same, since the standard deviation of MB_{t-1}^{efwa} is 13 to 19 times larger empirically than in the simulated data. The reason is that the market-to-book ratio at the IPO has a cross-sectional standard deviation in the data of approximately 140% whereas in the simulated data there is by construction no variation. In the simulations I did not introduce heterogeneity in the initial market-to-book in order to be able to separate endogenous variation in the investment policy from differences in investment opportunities. As a result, the cross-sectional variation in the market-to-book ratio after the IPO is by construction smaller in the simulated data than in the actual data. In regression setup (BW2) I include a fixed effects dummy for investment. The estimates for a_1 are now smaller in magnitude and closer to the actual data—though still higher by a factor of $3 - 7$.

Regression setup (BW3) controls for investment fixed effects by excluding the subset of firms that have not yet invested. This setup reveals another feature of the simulated data. The third column in the body of Table 7 shows the coefficient estimates for setup (BW3) are decreasing in IPO time rather than increasing as documented by BW. The reason is that the cross-sectional relation between the optimal investment threshold and the target leverage ratio, that results from varying $m \in [\underline{m}, \overline{m}]$, is decreasing and convex. In order to generate an increasing coefficient pattern, however, this relationship would have to be concave. Right after the IPO, only high target leverage firms appear in the subsample of firms that already have invested. Firms with low target leverage exercise their real option on average later. Thus the larger IPO+t the wider is the range of the mapping between the threshold market-to-book and the target leverage that gets

traced out in the simulated data. As a result, the coefficient estimates change monotonically, but in a direction opposite to the data.

Finally, Table 8 provides evidence that the model can match the persistence of market timing-motivated external financing on corporate leverage as documented in BW. In Table 8 I estimate regression (33) with longer lag lengths, denoted ι. I report the estimates for the regressions on leverage L_t at $t = 10$, i.e., ten years after the IPO. On the different lines I vary the lags on $MB_{t-\iota}^{efwa}$ and $MB_{t-\iota}$ between 1 year and 9 years. The coefficient estimates show that the effect of $MB_{t-\iota}^{efwa}$ on leverage is very persistent—both in terms of book and market leverage. Even for the largest lag length of $\iota = 9$ years, the coefficient a_1 is negative and large.[22]

In summary, the main patterns in BW are present in the simulated data. The coefficient estimates on the external financing-weighted market-to-book ratio are negative, large and robust to the lag length. The coefficients on lagged market-to-book change signs and are negligible.

3.5 The Robustness of the Optimal Policy to Capital Market Imperfections

The assumption of perfect capital markets in Section 2.1 leads to consumption-portfolio separation and allows a tractable solution to the model. Yet, in practice the value of the tax timing option is limited. Investors trade for reasons other than tax timing and realize losses as well as gains. Transaction costs, short-sale constraints, and wash-sale restrictions preclude investors from implementing the optimal strategy prescribed by Proposition 1. Liquidity shocks force capital gains realizations of investors without access to sophisticated tax avoidance schemes.

The results in this paper are robust to trading frictions since the economic benefits documented in Section 3.3 outweigh real-life transaction costs by orders of magnitudes. The predictions of the model are also qualitatively robust to exogenous liquidity shocks, since although the latter reduce the value of the embedded tax timing option they do not entirely eliminate the tax asymmetry at the personal level that is necessary for distortions in corporate policy. Plenty of empirical support shows that investors recognize the value of tax timing and optimize given the practical constraints.

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[22]This result is not driven by the one-time nature of capital restructuring that is imposed in this paper for tractability. I have simulated a model along the lines of Goldstein, Ju and Leland (2001) with repeated capital restructurings, callable (riskless) debt, and realization-based capital gains taxation. The resulting capital structure patterns are qualitatively similar to the ones in the model with a single opportunity to refinance.

4 Conclusion

The paper incorporates the lock-in effect of realization-based capital gains taxation in a dynamic capital budgeting problem. The model provides novel testable predictions about the cross-section and time-series of external financing, capital structure, and corporate investment. The firm's optimal policy is non-stationary and path-dependent on past firm performance, since the endogenous evolution of the owners' capital gains tax basis affects corporate decision making. Capital gains taxation *a priori* encourages irreversible investment, since capital loss offsets at the personal level reduce investment risk in loss-states. This investment stimulus is, however, only transitory and vanishes if investors realize capital losses before investment takes place. Ex-ante identical firms thus follow very different policies depending on their stock price evolution. Firms delay irreversible investment further the lower the tax basis of their owners falls. The difference in threshold Q's can exceed 200%, and the expected investment dates of otherwise identical firms can be years apart. In addition, capital gains taxation creates incentives to time external financing. Firms use up to ten percentage points more equity in their capital structure, the higher the stock price-to-basis ratio of their owners. The interaction with the firm's financial policy can even lead to underinvestment, or "excessive" delay, relative to the case without capital gains taxation. The value gain from conditioning on the owners' tax basis exceeds four to seven percent.

The model is able to match recent empirical evidence on the cross-section of capital structures for IPO firms. Firms that raise external financing when their valuations are high have low target leverage ratios. Given the evidence in the literature that the marginal investor is taxed, it is thus not surprising that proxies for a firm's history have explanatory power in leverage and investment regressions. Empirical evidence of path-dependency in investment and financing, therefore, do not lend conclusive support for behavioral theories. In order to discriminate between dynamic trade-off theories and alternative explanations, more suitable empirical proxies are needed.

The model has a variety of untested predictions that may allow constructing new tests of personal tax effects on corporate capital budgeting. Important extensions of the model left for future work include the incorporation of repeated investment and capital restructuring, the analysis of share repurchases, and the implications for governmental tax policy.

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A Bankruptcy Procedure

The bankruptcy procedure is an important determinant of the costs and benefits of debt financing. Endogenizing the bankruptcy decision precludes potential distortions from inefficient bankruptcy that are not the focus of this paper. The bankruptcy procedure is designed as follows: Debtholders take over the assets of the firm in the event of bankruptcy. They then relever the firm to the *ex-ante* optimum by issuing new debt and equity securities. Finally they pay the restructuring expenses which consume a constant fraction $\omega \in (0,1]$ of the firm's capital stock. This sequence of events minimizes overall bankruptcy costs, since the basis is endogenous in the *ex-ante* decision problem. Indirect bankruptcy costs arise from lower interest tax shields (due to a reduction in the face value of debt during reorganization) but are partially offset by higher personal tax-loss shields associated with the newly issued claims. The overall costs of bankruptcy, denoted α with $\omega < \alpha \leq 1$, require the solution to a fixed-point problem that takes into account possible bankruptcies in the future.

B Monte-Carlo Simulations

The artificial data set is constructed as follows: I take a sample of $100,000$ ex-ante identical, mutually independent firms. They have the same initial operating income, face value of debt, firm size, and investment opportunity set. For each firm, I simulate 10 years of operating performance X and construct the minimum process M. I discretize time into calendar weeks, $\Delta t = 1/52$. Using a numerical solution for the firm's financing, investment, and bankruptcy policy, I determine the bankruptcy date during period 0 or 1, and the investment date if no prior default happens. I exclude observations where the firm defaults prior to investment. At the end of each calendar year, I calculate the ratios of market-to-book (MB_t), book leverage (L_t^B), and market leverage (L_t) for the subsample of firms still alive. For all firms in period 1 I also record the investment date κ_u, and at the investment date the market-to-book (MB_u), book leverage (L_u^B), and market leverage (L_u).

The numerical values for the parameters r, μ, σ, ω, τ, τ^c, τ^d, τ^p, I are summarized in Table 1. They are a consensus of values found in the literature and in the tax code. I calibrate the remaining parameters to match various features of the data reported in Baker and Wurgler (2001, 2002). I choose the initial operating income πX_0 such that the peak of investment is in the second year after the IPO. The resulting investment pattern is very similar to asset growth rates reported in Table 1 of Baker and Wurgler (2001). The calibrated ratio of X_0 to $u(max(m : m \leq u(m)))$ is approximately 69.2 percent. Given X_0, I pick the coupon flow c^0 of the initial debt outstanding as to maximize the firm's ex-ante market value. I obtain this value by numerical optimization and using Monte-Carlo simulations for calculating the initial debt value D_0^0 in each iteration. The optimal value equals $c^0 = 6.0$, and the resulting optimal ex-ante leverage ratio is approximately 39 percent. Finally, I calibrate the size of the expansion project, $1/\pi$, to match the average asset growth rate in years IPO+1 and IPO+2, i.e., the two years of maximum growth in the data (see Table 2 in Baker and Wurgler (2001) for comparison).

C Proofs

Proposition 1: The Optimal Tax Timing Policy.

Constantinides (1983, Theorem 1, p. 617) proves the special case with no leverage. Similarly, in this model assumption (1) and time-homogeneity imply that both the optimal tax timing and bankruptcy policy are trigger strategies. For brevity I suppress the dependence on m, and denote by $b(B)$ the tax loss selling

trigger for basis B, and by l the bankruptcy trigger. The stock price is denoted $p(x)$, and the equity valuation of shareholders with tax basis B is $v(x; B)$.

Using Theorem 1 in Constantinides (1983) it remains to be shown that

$$l = b(0) \le b(B) \text{ for all } B \ge 0.$$

Equityholders may realize capital gains or losses at any time prior to default by selling their shares in return for after-tax proceeds $(1 - \tau)p(x) + \tau B$. The smooth-pasting condition for tax-loss selling at time t when $X_t = b(B)$ is

$$v_x(b(B); B) = (1 - \tau)p_x(b(B)). \tag{36}$$

Conversely, if shareholders do not engage in tax loss selling before the firm declares bankruptcy, they are left only with a tax credit τB for realized capital losses. The necessary condition for optimality when the basis equals B is

$$v_x(l; B) = 0. \tag{37}$$

Absence of arbitrage requires $p_x(x) \ge 0$ for all feasible x. The result now follows immediately from a comparison of (36) and (37) since $v_x(x; B)$ is continuous in x and the process X has no jumps. Shareholders' optimal trading strategy is thus one of instantaneous control by keeping the basis-to-price ratio from exceeding unity. They defer capital gains, instantaneously recognize all capital losses, and keep resetting all shares as long as the stock price is positive. This implies $l = b(0)$. ∎

Proposition 2 and 3: The Firm's Valuation and Bankruptcy Policy.

The Firm's Valuation After Investment In the following, I take m and the firm's policy $(u(m), c^1(m), n^1(m), l^0, l^1(m))$ as given, and drop the dependence of m where obvious. I proceed in several steps:

a) All equityholders receive after-income tax per-share cash flows $(X_t - c^1)(1 - \tau^e)/n^1$ while the firm is solvent. Equityholders may also realize capital gains or losses at any time prior to default by selling their shares in return for after-tax proceeds $(1 - \tau)p^1(X_t, m) + \tau B$, where $p^1(\cdot)$ denotes the stock price. Time-homogeneity implies the optimal tax timing strategy can be described by a trigger function $b^1(m; B)$ for $B \ge 0$. Proposition 1 implies that the optimal trigger satisfies

$$p^1(b^1(m; B), m) = B. \tag{38}$$

The optimal bankruptcy policy can also be identified with a trigger, denoted $l^1(m)$. I define the corresponding stopping times by

$$
\begin{aligned}
\kappa_b^1(m; B) &\equiv \inf(t \ge \kappa_u : X_t \le b^1(m; B)), \\
\kappa_l^1(m) &\equiv \inf(t \ge \kappa_u : X_t \le l^1(m)).
\end{aligned}
$$

The private equity valuation function given $l^1(m)$ equals

$$
v^1(X_t, m; B) = \sup_{\kappa_b^1} \mathbf{E}_t \Big[\int_t^{\kappa_b^1 \wedge \kappa_l^1} e^{-rs}(1 - \tau^e)(X_s - c^1)ds
$$

$$
+ \mathbf{1}_{\{\kappa_b^1 \le \kappa_l^1\}} e^{-r\kappa_b^1} \{(1 - \tau)p^1(b^1(m; B), m) + \tau B\} + \mathbf{1}_{\{\kappa_b^1 > \kappa_l^1\}} e^{-r\kappa_l^1}(\tau B) \Big],
$$

where the last term drops out since $l^1(m) = b^1(m; 0) \le b^1(m; B)$ for all $B \ge 0$. The solution is (17), and

the values of basis-reset- and bankruptcy-contingent claims (subscripts b and l, respectively) are

$$\varphi_b^1(x, m; B) \equiv \mathbf{E}_{x,m}[e^{-r\kappa_b^1(m;B)}] = (b^1(m; B))^\gamma (x)^{-\gamma}, \tag{39}$$

$$\varphi_l^1(x, m) \equiv \mathbf{E}_{x,m}[e^{-r\kappa_l^1(m)}] = (l^1(m))^\gamma (x)^{-\gamma}. \tag{40}$$

b) Using (38), the value-matching and smooth-pasting conditions for optimal tax-loss selling can be used to infer the stock price $p^1(x, m)$. For $B \geq 0$ (see Constantinides (1983) and Williams (1985)),

$$v^1(b^1(m; B), m; B) = (1-\tau)p^1(b^1(m; B), m) + \tau B, \tag{41}$$

$$v_x^1(b^1(m; B), m; B) = (1-\tau)p_x^1(b^1(m; B), m). \tag{42}$$

Since (42) holds for arbitrary basis $B \geq 0$ and (38) implies there is a one-to-one match between B and $b^1(m; B)$, (42) holds for arbitrary x with $B = p^1(x, m)$. Hence, $p^1(x, m)$ solves the following first-order differential equation in terms of x and given $l^1 = l^1(m)$:

$$p_x^1(x, m) = \frac{1}{1-\tau} v_x^1(x, m; p^1(x, m)), \ x > l^1,$$
$$p^1(l^1, m) = 0, \tag{43}$$
$$p_x^1(l^1, m) = 0.$$

The last two conditions correspond to limited liability and equity value-maximizing bankruptcy. The solution is (23).

c) Since all debt issues have, by assumption, the same priority and recovery rate in the event of bankruptcy, it is sufficient to value the aggregate debt. Debtholders in aggregate receive after-tax coupon flows $c^1(1 - \tau^p)$ while the firm is solvent, and $(1 - \omega)$ percent of the optimally recapitalized market value $V^*(X_t)$ when the firm goes bankrupt. Debtholders can also claim capital loss offsets in the same way as equityholders, since the debt is risky. The solution to the private debt valuation given basis D is

$$D^1(X_t, m; D) = \frac{\delta}{\phi} c^1 + \varphi_d^1(x, m; D)[D - \frac{\delta}{\phi} c^1]. \tag{44}$$

where $\varphi_d^1(x, m; D) = d^1(m; D)^\gamma x^{-\gamma}$ is the value of a basis-reset contingent claim, and $d^1(m; D)$ is the basis-reset trigger. The market value of the aggregate debt is

$$D^1(X_t, m) = D^0(X_t, m) + dD(X_t, m),$$

where $D^0(X_t, m)$ and $dD(X_t, m)$ are the market prices of the old and the new debt issue, respectively. $D^1(x, m)$ satisfies a differential equation similar to (43), though with different boundary conditions:

$$D_x^1(x, m) = \frac{1}{1-\tau} D_x^1(x, m; D^1(x, m)), \ x > l^1,$$
$$D^1(l^1, m) = (1-\omega)V^*(l^1), \tag{45}$$
$$\lim_{x \to \infty} D^1(x, m) = \frac{\delta}{\phi} c^1.$$

The last condition is that debt becomes riskless as $x \to \infty$.

d) The market value $V^*(l^1)$ reflects the rational expectation that the firm may go bankrupt again in the future. Thus, $V^*(\cdot)$ is the solution to a fixed-point problem taking repeated future bankruptcies *ad infinitum* into account. I conjecture that the market value of the optimally relevered firm (excluding direct

bankruptcy expenses) takes the simple affine form

$$V^*(X_t) = \zeta \lambda X_t,$$

where ζ is a constant. Then the solution to (45) is

$$
\begin{aligned}
D^1(x,m) &= \frac{\delta}{\phi}c^1 + \varphi_l^1(x,m)^{\frac{1}{1-\tau}}[(1-\omega)V^*(l^1) - \frac{\delta}{\phi}c^1] \\
&= [1 - \alpha\varphi_l^1(x,m)^{\frac{1}{1-\tau}}]\frac{\delta}{\phi}c^1,
\end{aligned}
\tag{46}
$$

where the value $\varphi_l^1(x,m)$ of a bankruptcy-state contingent claim (excluding capital loss credits) is given by (40) and

$$\alpha = 1 - (1-\omega)\zeta\xi\phi \qquad (>\omega). \tag{47}$$

e) The optimality conditions for equity value-maximizing default at $X_t = l^1$ are

$$
\begin{aligned}
p^1(l^1, m) &= v^1(l^1, m; 0) = 0, & \text{(value-matching)} & \tag{48} \\
p_x^1(l^1, m) &= v_x^1(l^1, m; 0) = 0. & \text{(smooth-pasting)} & \tag{49}
\end{aligned}
$$

The solution is

$$l^1 = \xi\frac{\delta}{\lambda}c^1 \tag{50}$$

with

$$\xi = \frac{\gamma}{1+\gamma}, \tag{51}$$

since $v_x^1(x,m;0) = \lambda(1 + \gamma\varphi_l^1(x,m)(\frac{\lambda l^1 - \delta c^1}{\lambda x}))/n^1$. The bankruptcy threshold is thus unaffected by the capital gains tax rate.

f) It remains to determine the coefficient ζ. The market value of the optimally recapitalized firm (with endogenous basis) is

$$V^*(X_t) \equiv p^*(X_t, X_t) + D^*(X_t, X_t), \tag{52}$$

where the functional forms of $p^*(\cdot)$ and $D^*(\cdot)$ are given by (23, 24). Expression (52) is maximized for a coupon flow of

$$c^*(X_t) = \vartheta\frac{\lambda}{\xi\delta}X_t, \tag{53}$$

where ϑ is a constant. The first-order condition gives ϑ implicitly as the solution to the algebraic equation

$$
\begin{aligned}
&(\phi^{-1} - 1) - (1 + \frac{\gamma}{1-\tau})\vartheta^{\frac{\gamma}{1-\tau}}[(\beta\xi - 1) + \alpha\phi^{-1}] = 0 \\
\Leftrightarrow \quad &(\phi^{-1} - 1) - (1 + \frac{\gamma}{1-\tau})\vartheta^{\frac{\gamma}{1-\tau}}[(\phi^{-1} - 1) + \beta\xi\{\omega - (1-\omega)(\phi^{-1} - 1)\vartheta\}] = 0.
\end{aligned}
\tag{54}
$$

After substitution, the coefficient ζ is indeed constant, and equals

$$\zeta = \beta[1 + (\phi^{-1} - 1)\vartheta]. \tag{55}$$

As a result, the ex-ante optimal static leverage ratio is

$$D^*(X_t, X_t)/V^*(X_t) = \frac{(1-\omega)}{(1-\alpha)}(1 - \alpha\vartheta^{\frac{\gamma}{1-\tau}})\vartheta. \tag{56}$$

The expected time until bankruptcy, $E(\kappa_l^*)$, is $\ln(\vartheta)/(\mu - \frac{\sigma^2}{2})$, the debt recovery ratio $(1-\omega)V^*(l^*)/D^*(X_t, X_t)$

equals $(1-\alpha)/(1-\alpha\vartheta^{\frac{\gamma}{1-\tau}})$, and the value of a bankruptcy-state contingent claim (excluding capital gains taxes and loss offsets) is $\varphi_l^*(X_t, X_t) = \vartheta^\gamma$. ∎

The Firm's Valuation Before Investment   Again I take the firm's policy $(u(m),c^1(m),n^1(m),l^0,l^1(m))$, $m \in \mathcal{M}$, as given. I start by valuing equity at $t \leq \kappa_u \wedge \kappa_l^0$. Private valuations before investment depend on both the investor's own basis $B \in \mathcal{B}^0(m)$ and the current basis of the firm's initial shareholders, $p^0(m, m)$. The latter determines the optimal investment trigger $u(m)$ and the financing mix $(c^1(m), n^1(m))$. The set of feasible tax bases in period 0 is $\mathcal{B}^0(m) = \{(B) : p^0(m, m) \leq B \leq \max_{x \geq m} p^0(u(x), x)\}$, $m \in \mathcal{M}$, since there exist no shares with $B < p^0(m, m)$. I proceed, again, in several steps:

a) Shareholders receive pre-tax cash flows $\pi X_t - c^0$ while the firm takes no action (default or restructuring). Once operating income X_t falls to $b^0(M_t; B)$, shareholders sell all shares with basis B and receive after-tax proceeds per share of $(1-\tau)p^0(b^0(M_t; B), M_t) + \tau B$. When the firm exercises its real option, equity is worth $v^1(u(M_t), M_t; B)$, where $v^1(\cdot)$ is given by (17). The state-contingent claims φ_b^0 and φ_u^0 pay either a single unit when X_t hits the basis-reset threshold $b^0(M_t; B)$ before the investment boundary $u(M_t)$ (subscript b) or *vice versa* (subscript u):

$$\varphi_b^0(x, m; B) \equiv \mathbf{1}_{\{\kappa_b^0(m;B)<\kappa_u(m)\}}\mathbf{E}_{x,m}[e^{-r\kappa_b^0(m;B)}],$$
$$\varphi_u^0(x, m; B) \equiv \mathbf{1}_{\{\kappa_b^0(m;B)\geq\kappa_u(m)\}}\mathbf{E}_{x,m}[e^{-r\kappa_u(m)}].$$

Using (13) and the appropriate boundary conditions (see Goldstein et al. (2001)), φ_b^0 and φ_u^0 equal (18) and (19), respectively, and the functional form of $v^0(\cdot)$ is (16).

b) The stock price before investment, $p^0(x, m)$, can again be determined using the optimality conditions for tax-loss selling. The only caveat is that the conditions differ dependent on whether the stock price is at a historical low or not. In the former case, the relevant optimality conditions are those of the initial shareholders. I start with the simpler case that $X_t > M_t$. In that case, the tax timing problem of an investors with arbitrary tax basis B (larger than the current basis of the initial shareholders) is straightforward, since m and the firm's optimal policy $(u(m), c^1(m), n^1(m))$ are constant. Therefore, m can be treated as a parameter. The optimality conditions for tax-loss selling are identical to (41, 42). After substitution of (6), $p^0(x, m)$ solves the following differential equation given m:

$$p_x^0(x, m) = \frac{1}{1-\tau}v_x^0(x, m; p^0(x, m)), \ m < x < u(m),$$
$$p^0(u(m), m) = p^1(u(m), m), \tag{57}$$
$$p^0(m, m) = p^0(m),$$

where $p^0(x) \equiv p^0(x, x)$ is the stock price functional at the diagonal (i.e., when the stock price is at a historical minimum). Note that (57) has a singularity at $u(m)$, and that the stock price is continuous but not smooth across the investment boundary, i.e., $p_x^0(u(m), m) \neq p_x^1(u(m), m)$.

c) It remains to determine $p^0(m)$ for $m \in \mathcal{M}$. The representative initial shareholder must take into consideration that the firm's policy may depend on her tax timing decision. The state variable m for the initial shareholders' basis-reset trigger is a decision variable, and only at the optimum does it coincide with the running minimum of X. A shareholder with tax basis $B = p^0(m)$ resets her basis whenever X_t hits m, such that

$$v^0(m, m; p^0(m)) = p^0(m), \qquad \text{(value-matching)} \tag{58}$$
$$v_x^0(m, m; p^0(m)) = (1-\tau)p_x^0(m). \qquad \text{(smooth-pasting)} \tag{59}$$

40

Last, bankruptcy is declared in period 0 when

$$v^0(l^0, l^0; 0) = p^0(l^0) \quad = \quad 0, \qquad \text{(value-matching)} \tag{60}$$
$$v_x^0(l^0, l^0; 0) = p_x^0(l^0) \quad = \quad 0. \qquad \text{(smooth-pasting)} \tag{61}$$

Again, there exists a one-to-one mapping in the set $\{(x, m) : x = m\}$ between tax timing triggers and tax bases. Collecting equations (59, 60, 61), one obtains the following free-boundary problem for l^0 and $p^0(x)$:

$$
\begin{aligned}
p_x^0(x) &= \frac{1}{1 - \tau} v_x^0(x, x; p^0(x)), \ \ l^0 < x < u(x), \\
p^0(x) &= p^1(x, x), \ \ x = u(x), \\
p^0(l^0) &= 0, \\
p_x^0(l^0) &= 0.
\end{aligned}
\tag{62}
$$

d) The solution to (57, 62) is a second-order Volterra integral equation for $p^0(x, m)$:

$$p^0(x, m) = p^0(m) + \int_m^x \mathbf{f}(z, m)dz \tag{63}$$

and

$$p^0(x) = \int_{l^0}^x \mathbf{g}(z)dz,$$

where from (57, 62),

$$
\begin{aligned}
\mathbf{f}(x, m) &\equiv \frac{1}{1 - \tau} v_x^0(x, m; p^0(x, m)) \\
&= \ \{\lambda\pi + \Phi_b(x, m)[p^0(x, m) - (\lambda\pi x - \delta c^0)] \\
&\quad + \Phi_u(x, m)[v^1(u(m), m; p^0(x, m)) - (\lambda\pi u(m) - \delta c^0)]\} / (1 - \tau), \\
\mathbf{g}(x) &\equiv \frac{1}{1 - \tau} v_x^0(x, x; p^0(x)) = \mathbf{f}(x, x).
\end{aligned}
$$

By introducing the function $b^1(x, m) \equiv b^1(m; p^0(x, m))$, which determines how tax timing thresholds shift after investment, (63) and thus (57, 62) can be reduced to the first-order integral equation

$$p^0(x, m) = f(x, m)p^0(m) + \frac{1}{1 - \tau} \int_m^x g(x, m, z)dz \ , \ \ m \le x \le u(m), \tag{64}$$

with

$$p^0(x) = \frac{1}{1 - \tau} \int_{l^0}^x h(x, z)dz \ , \ \ l^0 \le x \le u(x),$$

where

$$
\begin{aligned}
f(x, m) &\equiv \ e^{\frac{1}{1-\tau} \int_m^x \Phi_u(s,m)\Psi_1(s,m)ds} \Psi_0(x, m, m), \\
g(x, m, z) &\equiv \ e^{\frac{1}{1-\tau} \int_z^x \Phi_u(s,m)\Psi_1(s,m)ds} \Psi_0(x, m, z)\Psi_2(z, m), \\
h(x, z) &\equiv \ e^{\frac{1}{1-\tau} \int_z^x (\Phi_b(s,s)+\Phi_u(s,s)\Psi_1(s,s))ds} \Psi_2(z, z),
\end{aligned}
$$

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The functions $\Phi_b(x,m)$, $\Phi_u(x,m)$, $\Psi_0(x,m,z)$, $\Psi_1(x,m)$, $\Psi_2(x,m)$ equal

$$
\Phi_b(x,m) \equiv \frac{\partial\varphi_b^0(x,m;p^0(x,m))}{\partial x} = \frac{\eta\mathbf{x}^\gamma - \gamma\mathbf{x}^\eta}{\mathbf{x}^\eta - \mathbf{x}^\gamma}x^{-1},
$$

$$
\Phi_u(x,m) \equiv \frac{\partial\varphi_u^0(x,m;p^0(x,m))}{\partial x} = \frac{\gamma - \eta}{\mathbf{x}^\eta - \mathbf{x}^\gamma}x^{-1},
$$

with $\mathbf{x} \equiv x/u(m)$ and

$$
\Psi_0(x,m,z) \equiv e^{\frac{1}{1-\tau}\int_z^x \Phi_b(s,m)ds} = \varphi_b^0(x,m;p^0(z,m))^{\frac{1}{1-\tau}} = [\frac{\mathbf{x}^\eta - \mathbf{x}^\gamma}{(x/z)^\gamma\mathbf{x}^\eta - \mathbf{x}^\gamma(x/z)^\eta}]^{\frac{1}{1-\tau}},
$$

$$
\Psi_1(x,m) \equiv \varphi_b^1(u(m),m;p^0(x,m)),
$$

$$
\Psi_2(x,m) \equiv \{\pi\lambda - \Phi_b(x,m)(\pi\lambda x - \delta c^0) + \Phi_u(x,m)[(1/n^1 - \pi)\lambda u(m)
$$
$$
-\delta(c^1/n^1 - c^0) - \Psi_1(x,m)(\lambda b^1(x,m) - \delta c^1)/n^1]\}.
$$

■

The Optimal Investment and Financing Policy

In the following, I start by taking m and the investment trigger $u(m)$ as given and solve for the optimal financing mix $c^1 = c^1(u(m),m)$ and $n^1 = n^1(u(m),m)$. Then, I determine the investment trigger $u(m)$ for each $m \in \mathcal{M}$ while imposing optimal financing. The optimal bankruptcy policy $l^1 = l^1(c^1)$ is given by (50).

The Budget Constraint At the investment date, the firm has demand $I > 0$ for external financing. The budget constraint is

$$
p^1(u(m),m)[n^1(u(m),m) - 1] + dD(u(m),m) - I = 0. \tag{65}
$$

Eq. (65) says that if the firm decides to raise an amount $dD(u(m),m)$ through new debt, the remainder $I - dD(u(m),m)$ must be financed by issuing $(n^1(u(m),m) - 1)$ new shares. Hence,

$$
n^1(u(m),m) = 1 + (I - dD(u(m),m))/p^1(u(m),m). \tag{66}
$$

From (65), the competitive equity offer price is

$$
p^1(u(m),m) = E^1(u(m),m) + dD(u(m),m) - I, \tag{67}
$$

where $E^1(\cdot)$ denotes the firm's equity market value

$$
E^1(x,m) \equiv p^1(x,m)n^1(x,m)
$$
$$
= \beta\lambda x - \delta c^1 - \varphi_l^1(x,m)^{\frac{1}{1-\tau}}(\beta\lambda l^1 - \delta c^1). \tag{68}
$$

Using (68), (65) can be rewritten in terms of (c^1, n^1) as $\mathbf{F}(u(m),m|c^1,n^1) = 0$, where

$$
\mathbf{F}(x,m|c^1,n^1) \equiv E^1(x,m|c^1)(1 - \frac{1}{n^1}) + dD(x,m|c^1) - I.
$$

The Implicit Function Theorem (IFT) guarantees the existence of a policy function $n^1(x, m|c^1)$ linking the total debt coupon to the size of the equity offering, and

$$\frac{\partial}{\partial c^1} n^1(x, m|c^1)^{-1} = -\frac{\partial}{\partial c^1}\mathbf{F}(x, m|c^1, n^1) \bigg/ \frac{\partial}{\partial(1/n^1)}\mathbf{F}(x, m|c^1, n^1)$$

$$= \left[\frac{\partial}{\partial c^1}E^1(x, m|c^1)(1 - \frac{1}{n^1}) + \frac{\partial}{\partial c^1}dD(x, m|c^1)\right] \bigg/ E^1(x, m|c^1). \quad (69)$$

Since the optimal bankruptcy trigger $l^1 = l^1(c^1)$ is affine in c^1,

$$\frac{\partial}{\partial c^1}E^1(x, m|c^1) = -\delta[1 - \varphi_l^1(x, m|c^1)^{\frac{1}{1-\tau}}],$$

$$\frac{\partial}{\partial c^1}dD(x, m|c^1) = \frac{\delta}{\phi}[1 - \alpha\varphi_l^1(x, m|c^1)^{\frac{1}{1-\tau}}\{1 + \frac{\gamma}{1-\tau}(1 - \frac{c^0}{c^1})\}].$$

Note that $\frac{\partial}{\partial l^1}E^1(x, m|c^1) = 0$ by optimality of l^1.

The Optimal Financing Given (x, m), the optimal financing mix can be characterized by functions $c^1(x, m)$ and $n^1(x, m|c^1)$. The tax basis of the firm's owners is $B = p^0(m, m)$. Optimal financing at the investment trigger $x = u(m)$, thus, requires the following necessary and sufficient first-order condition to hold for $c^1 \geq 0$:

$$\frac{\partial}{\partial c^1}v^1(u(m), m; p^0(m, m)|c^1, n^1(u(m), m|c^1)) = 0 \quad (70)$$

The solution to (70) is a policy function $m \mapsto c^1(m)$ for all feasible $m \in \mathcal{M}$. In (70),

$$\frac{\partial}{\partial c^1}v^1(x, m; B|c^1, n^1(x, m|c^1)) = (\lambda x - \delta c^1)\frac{\partial n^1(x, m|c^1)^{-1}}{\partial c^1} - \frac{\delta}{n^1}$$

$$+ \varphi_b^1(x, m; B)[(1 - \tau)\frac{\partial p^1(b^1, m|c^1, n^1)}{\partial c^1}$$

$$- (\lambda b^1 - \delta c^1)\frac{\partial n^1(x, m|c^1)^{-1}}{\partial c^1} + \frac{\delta}{n^1}],$$

$$\frac{\partial}{\partial c^1}p^1(x, m|c^1, n^1(x, m|c^1)) = E^1(x, m|c^1)\frac{\partial n^1(x, m|c^1)^{-1}}{\partial c^1} + (\frac{1}{n^1})\frac{\partial E^1(x, m|c^1)}{\partial c^1}. \quad (71)$$

Note that $\frac{\partial}{\partial b^1}v^1(x, m; B|c^1, n^1) = \frac{\partial}{\partial l^1}v^1(x, m; B|c^1, n^1) = 0$ by optimality of l^1 and $b^1 = b^1(m; B)$, respectively.

The Investment Threshold The optimal investment boundary $m \mapsto u(m)$ depends on the firm's financial policy $(c^1(x, m), n^1(x, m|c^1))$ *et vice versa*. Given m, optimality of the investment threshold $u(m)$ requires the value-matching and smooth-pasting conditions to be satisfied for a representative shareholder with basis $p^0(m, m)$:

$$v^0(x, m; p^0(m, m)) = v^1(x, m; p^0(m, m)|c^1(x, m), n^1(x, m|c^1)) \quad , x = u(m) \quad (72)$$

$$\frac{\partial}{\partial x}v^0(x, m; p^0(m, m)) = \frac{\partial}{\partial x}v^1(x, m; p^0(m, m)|c^1(x, m), n^1(x, m|c^1)) \quad , x = u(m). \quad (73)$$

The r.h.s of (73) equals

$$\frac{\partial}{\partial x}v^1(x,m;B|c^1(x,m),n^1(x,m|c^1)) = (\lambda x - \delta c^1)\frac{\partial n^1(x,m|c^1)^{-1}}{\partial x} + \frac{\lambda}{n^1}$$
$$+ \varphi_b^1(x,m;B)[(1-\tau)\frac{\partial p^1(b^1,m|c^1(x,m),n^1(x,m|c^1))}{\partial x}$$
$$- (\lambda b^1 - \delta c^1)\frac{\partial n^1(x,m|c^1)^{-1}}{\partial x} - \gamma\frac{1}{x}(B - \frac{\lambda b^1 - \delta c^1}{n^1})],$$

where $b^1 = b^1(m;B)$ and c^1 can be treated as constants, since optimal financing implies $\frac{\partial}{\partial c^1}v^1(x,m;B|\ c^1,$ $n^1(x,m|c^1)) = 0$. Applying the IFT to (65) gives

$$\frac{\partial}{\partial x}n^1(x,m|c^1)^{-1} = -\frac{\partial}{\partial x}\mathbf{F}(x,m|c^1,n^1)\bigg/\frac{\partial}{\partial(1/n^1)}\mathbf{F}(x,m|c^1,n^1)$$
$$= \left[\frac{\partial E^1(x,m|c^1)}{\partial x}(1-\frac{1}{n^1}) + \frac{\partial dD(x,m|c^1)}{\partial x}\right]\bigg/E^1(x,m|c^1)$$

where c^1 is again treated as a parameter, and

$$\frac{\partial}{\partial x}E^1(x,m|c^1) = [\beta\lambda x + \frac{\gamma}{1-\tau}\varphi_l^1(x,m)^{\frac{1}{1-\tau}}(\beta\lambda l^1 - \delta c^1)]\frac{1}{x},$$
$$\frac{\partial}{\partial x}dD(x,m|c^1) = \frac{\gamma}{1-\tau}\alpha\varphi_l^1(x,m)^{\frac{1}{1-\tau}}\frac{\delta}{\phi}(\frac{c^1-c^0}{x}).$$

Finally since $p^1(b^1(m;B),m|c^1,n^1) = B$,

$$\frac{\partial}{\partial x}p^1(b^1(m;B),m|c^1(x,m),n^1(x,m|c^1)) = (n^1B)\frac{\partial n^1(x,m|c^1)^{-1}}{\partial x}$$

In turn, the l.h.s of (73) equals

$$\frac{\partial}{\partial x}v^0(x,m;B) = \pi\lambda + \frac{\partial}{\partial x}\varphi_b^0(x,m;B)[B - (\pi\lambda b^0(m;B) - \delta c^0)]$$
$$+ \frac{\partial}{\partial x}\varphi_u^0(x,m;B)[v^1(u(m),m;B) - (\pi\lambda u(m) - \delta c^0)]$$

with

$$\frac{\partial}{\partial x}\varphi_b^0(x,m;B)|_{x=u(m)} = \frac{(\eta-\gamma)\mathbf{x}^{\gamma+\eta}}{\mathbf{x}^\eta - \mathbf{x}^\gamma}u(m)^{-1},$$
$$\frac{\partial}{\partial x}\varphi_u^0(x,m;B)|_{x=u(m)} = \frac{\gamma\mathbf{x}^\gamma - \eta\mathbf{x}^\eta}{\mathbf{x}^\eta - \mathbf{x}^\gamma}u(m)^{-1},$$

where $\mathbf{x} \equiv b^0(m;B)/u(m)$. ∎

Proposition 4: The Optimal Financing Comparative Statics

Denote the owners' basis by $B = p^0(m,m)$ with reset trigger $b^0(m;B) = m$, and let the firm's operating income at the investment date be $x = u(m)$. The firm's optimal financial policy can be characterized as a policy function $c^1(x,m|B)$ which represents the total promised debt coupon flow after investment. The number of shares outstanding after investment, $n^1(x,m|B)$, follows from the budget constraint (65).

The claim to be verified is

$$\frac{\partial}{\partial B}c^1(x,m|B) \geq 0. \tag{74}$$

By application of the Implicit Function Theorem (IFT),

$$\frac{\partial}{\partial B}c^1(x,m|B) = -\left[\frac{\partial^2 v^1(x,m;B|c^1,b^1)}{\partial c^1 \partial b^1}\frac{\partial b^1(m;B)}{\partial B} + \frac{\partial^2 v^1(x,m;B|c^1)}{\partial c^1 \partial B}\right] \bigg/ \frac{\partial^2 v^1(x,m;B|c^1)}{(\partial c^1)^2}. \tag{75}$$

In order for (74) to be satisfied, the numerator in (75) must be positive, since the second-order optimality condition for $c^1(x,m|B)$ implies that the denominator in (75) is negative. I define the numerator in (75) (the term enclosed in brackets) by $\mathbf{F}(x,m;B|c^1)$. That is,

$$
\begin{aligned}
\mathbf{F}(x,m;B|c^1) &\equiv \varphi_b^1(x,m;B|c^1) \\
&\times \Bigg\{ \frac{\gamma}{b^1(m;B)}\left[(1-\tau)\frac{\partial p^1(b^1(m;B),m|c^1)}{\partial c^1} - \mathbf{G}(x,m;B|c^1)\right]\frac{\partial b^1(m;B)}{\partial B} \\
&\qquad\qquad + (1-\tau)\mathbf{H}(x,m;B|c^1) - \lambda\frac{\partial(1/n^1)}{\partial c^1}\frac{\partial b^1(m;B)}{\partial B} \Bigg\}
\end{aligned} \tag{76}
$$

where

$$
\begin{aligned}
\mathbf{G}(x,m;B|c^1) &\equiv (\lambda b^1(m;B) - \delta c^1)\frac{\partial(1/n^1)}{\partial c^1} - \frac{\delta}{n^1}, \\
\mathbf{H}(x,m;B|c^1) &\equiv \frac{\partial^2 p^1(b^1(m;B),m|c^1)}{\partial c^1 \partial b^1}\frac{\partial b^1(m;B)}{\partial B} + \frac{\partial^2 p^1(b^1(m;B),m|c^1,B)}{\partial c^1 \partial B} \\
&= \frac{1}{1-\tau}\left\{\lambda\frac{\partial(1/n^1)}{\partial c^1} - \frac{\gamma}{b^1(m;B)}\left[\frac{\partial p^1(b^1(m;B),m|c^1)}{\partial c^1} - \mathbf{G}(x,m;B|c^1)\right]\right\}\frac{\partial b^1(m;B)}{\partial B}.
\end{aligned}
$$

Application of the IFT gives

$$\frac{\partial b^1(m;B)}{\partial B} = \left[\frac{\partial p^1(b^1(m;B),m)}{\partial b^1}\right]^{-1} \geq 0,$$

which is positive since $\frac{\partial}{\partial x}p^1(x,m) \geq 0$ for all $x \geq m$. After substitution, (76) equals

$$\mathbf{F}(x,m;B|c^1) = (-\tau)\,\frac{\gamma}{b^1(m;B)}\varphi_b^1(x,m;B|c^1)\frac{\partial p^1(b^1(m;B),m|c^1)}{\partial c^1} \bigg/ \frac{\partial p^1(b^1(m;B),m)}{\partial b^1}.$$

Hence, the claim (74) is verified if and only if at the optimal financing mix,

$$\frac{\partial p^1(b^1(m;B),m|c^1)}{\partial c^1}\bigg|_{c^1 = c^1(x,m|B)} \leq 0,$$

The first-order condition (70) implies that

$$
\begin{aligned}
\frac{\partial(c^1/n^1)}{\partial c^1}\bigg|_{c^1=c^1(x,m|B)} &= \mathbf{I}(x,m;B|c^1)\{[u(m) - \varphi_b^1(u(m),m;B)b^1(m;B)] \\
&\qquad + \varphi_b^1(u(m),m;B)(1-\tau)\beta[b^1(m;B) - \varphi_l^1(b^1(m;B),m)^{\frac{1}{1-\tau}}l^1]\}
\end{aligned}
$$

where

$$\mathbf{I}(x,m;B|c^1) \equiv \lambda\frac{\partial(1/n^1)}{\partial c^1} \bigg/ \{1 - \varphi_b^1(u(m),m;B) + \varphi_b^1(u(m),m;B)(1-\tau)[1 - \varphi_l^1(b^1(m;B),m)^{\frac{1}{1-\tau}}]\}.$$

Therefore, using (71, 69) at $c^1 = c^1(x, m|B)$,

$$
\begin{aligned}
\frac{\partial p^1(b^1(m; B), m|c^1)}{\partial c^1} &= \beta\lambda[b^1(m; B) - \varphi_l^1(b^1(m; B), m)^{\frac{1}{1-\tau}}l^1]\frac{\partial(1/n^1)}{\partial c^1} \\
&\quad -\delta\frac{\partial(c^1/n^1)}{\partial c^1}[1 - \varphi_l^1(b^1(m; B), m)^{\frac{1}{1-\tau}}] \\
&= \mathbf{I}(x, m; B|c^1)\{\beta[b^1(m; B) - \varphi_l^1(b^1(m; B), m)^{\frac{1}{1-\tau}}l^1][1 - \varphi_b^1(u(m), m; B)] \\
&\quad -[u(m) - \varphi_b^1(u(m), m; B)b^1(m; B)][1 - \varphi_l^1(b^1(m; B), m)^{\frac{1}{1-\tau}}]\}
\end{aligned}
\tag{77}
$$

The term in (77) enclosed in curly brackets has to be negative. The latter condition amounts to an inequality of the form

$$
\beta\frac{1 - \mathbf{z}^{1+\frac{\gamma}{1-\tau}}}{1 - \mathbf{z}^{\frac{\gamma}{1-\tau}}} \leq \frac{1 - \mathbf{x}^{1+\gamma}}{\mathbf{x} - \mathbf{x}^{1+\gamma}} \qquad \text{for all } \mathbf{z}, \mathbf{x} \in (0, 1)
\tag{78}
$$

where $\mathbf{z} \equiv l^1/b^1(m; B)$ and $\mathbf{x} \equiv b^1(m; B)/u(m)$. By L'Hospital's rule, the l.h.s of (78) is monotonically increasing in $\mathbf{z}$ on $(0, 1)$ and bounded above by $1 + \frac{1}{\gamma}$, whereas the r.h.s of (78) is monotonically decreasing in $\mathbf{x}$ and bounded below by $1 + \frac{1}{\gamma}$. $\blacksquare$

D   Tables and Figures

Table 1
The Base Parametrization.

The table reports the values in the base parametrization of the various model parameters. They reflect the recent U.S. tax code and otherwise represent consensus values of parameterizations found in the literature.

Parameter	Value	Description
r	5%	After-Tax Risk-Free Rate
μ	2%	Drift of Operating Income Process under $\mathbb{Q}$
σ	20%	Volatility of Operating Income Process
ω	50%	Direct Bankruptcy Cost
τ	25%	Capital Gains Tax Rate
τ^p	30%	Ordinary Income Tax Rate on Interest
τ^d	30%	Ordinary Income Tax Rate on Dividends
τ^c	30%	Effective Corporate Income Tax Rate
$1-\phi$	30%	Double-Taxation of Corporate Income
I	100	Capital Expenditure (Normalized)

Table 2
The Financial Policy.

The table reports comparative statics for the *ex-post* optimal capital structure. The base parametrization is from Table 1, $c^0 = 0$ and $\pi = 100\%$.

| | Basis-Price Ratio | | | | |
	0%	50%	100%	Δ	$\tau = 0\%$
	Target Leverage (%)				
Base	60.7	67.5	69.9	9.2%	66.4
$\tau = 15\%$	63.6	67.1	68.4	4.8%	66.4
$\tau = 35\%$	56.3	68.0	71.5	15.2%	66.4
$\mu = 1\%$	59.0	65.8	68.0	9.0%	64.6
$\mu = 3\%$	62.4	69.3	70.5	8.2%	68.3
$\sigma = 15\%$	66.3	72.7	75.3	9.0%	72.0
$\sigma = 25\%$	56.6	63.5	65.5	8.9%	62.2
$r = 4\%$	59.3	66.3	68.5	9.2%	65.1
$r = 6\%$	61.8	68.6	70.9	9.1%	67.5
$\phi = 75\%$	53.5	62.3	65.3	11.8%	61.4
$\phi = 65\%$	66.5	71.9	73.7	7.2%	70.7
$\omega = 25\%$	69.3	76.8	79.2	9.9%	77.1
$\omega = 100\%$	51.3	57.0	59.2	7.9%	54.1
	Target Credit Spread (Basis Points)				
Base	72	103	117	45	159
$\tau = 15\%$	106	125	134	28	159
$\tau = 35\%$	41	81	100	59	159
$\mu = 1\%$	90	125	139	49	188
$\mu = 3\%$	57	86	92	35	135
$\sigma = 15\%$	37	58	71	34	96
$\sigma = 25\%$	118	159	173	55	235
$r = 4\%$	67	94	106	39	144
$r = 6\%$	76	112	127	51	173
$\phi = 75\%$	53	85	98	46	135
$\phi = 65\%$	91	122	135	44	183
$\omega = 25\%$	83	122	138	55	190
$\omega = 100\%$	64	89	102	38	136

Table 3
The Investment Policy.

The table reports comparative statics for the optimal investment threshold. The base parametrization is from Table 1, $c^0 = 0$ and $\pi = 100\%$.

| | Operating History $(m - l^0)/\overline{m}$ | | | | | |
	0%	33%	67%	100%	Δ	$\tau = 0\%$
Investment Threshold $u(m)/I$ (%)						
Base	128.4	92.5	53.2	46.2	178%	94.4
$\tau = 15\%$	109.1	86.1	65.0	60.0	82%	94.4
$\tau = 35\%$	166.3	117.5	45.7	36.1	361%	94.4
$\mu = 1\%$	146.2	103.9	67.7	60.9	140%	107.8
$\mu = 3\%$	113.2	86.8	44.5	34.7	226%	83.2
$\sigma = 15\%$	89.7	75.1	46.5	35.9	150%	66.4
$\sigma = 25\%$	174.8	106.8	63.9	58.0	201%	129.9
$r = 4\%$	112.6	75.2	39.9	34.8	223%	82.9
$r = 6\%$	143.1	108.2	65.3	56.3	154%	105.2
$\phi = 75\%$	196.8	142.6	68.5	54.7	260%	122.4
$\phi = 65\%$	92.7	68.7	45.0	40.2	131%	75.9
$\omega = 25\%$	108.2	76.5	44.0	38.4	182%	75.9
$\omega = 100\%$	157.4	115.5	66.6	57.5	174%	123.4
Threshold Equity Value $E^0(u(m), m)/I$						
Base	10.8	7.3	3.7	3.0	260%	6.1
$\tau = 15\%$	8.2	6.2	4.4	3.9	109%	6.1
$\tau = 35\%$	16.2	10.8	3.3	2.3	592%	6.1
$\mu = 1\%$	9.7	6.4	3.7	3.2	205%	5.4
$\mu = 3\%$	13.7	10.1	4.6	3.3	313%	7.7
$\sigma = 15\%$	6.5	5.2	2.8	1.9	238%	3.7
$\sigma = 25\%$	16.3	9.3	5.0	4.3	276%	9.2
$r = 4\%$	14.8	9.3	4.3	3.6	314%	8.3
$r = 6\%$	8.8	6.3	3.3	2.7	231%	5.0
$\phi = 75\%$	20.1	13.9	5.7	4.2	377%	9.3
$\phi = 65\%$	6.5	4.5	2.6	2.2	192%	4.2
$\omega = 25\%$	7.4	4.7	2.2	1.8	311%	3.5
$\omega = 100\%$	16.0	11.3	5.9	4.9	225%	10.5

Table 4
The Effect of Project Size.

The table reports comparative statics for the investment threshold $u(m)$ and the target leverage ratio as a function of the size of the investment project relative to existing operations. Each line represents a different project size $(1 - \pi)$. The base parametrization is from Table 1. The capital gains tax rate equals 25%.

Project Size	Operating History $(m - l^0)/\overline{m}$				
$(1 - \pi)$	0%	50%	100%	Δ	$\tau = 0\%$
		Investment Threshold $u(m)/I$ (%)			
0%	128.4	55.6	46.2	178%	94.4
10%	67.3	40.5	35.5	90%	60.5
20%	45.5	31.9	28.9	58%	44.4
30%	34.3	26.3	24.4	41%	35.0
40%	27.5	22.4	21.1	31%	28.9
50%	23.0	19.5	18.6	24%	24.7
60%	19.8	17.3	16.7	19%	21.5
70%	17.3	15.5	15.1	15%	19.0
80%	15.4	14.1	13.8	12%	17.1
90%	13.9	12.9	12.7	9%	15.5
100%	12.6	11.9	11.8	7%	14.2
		Target Leverage (%)			
0%	60.7	67.5	69.9	9.2%	66.4
10%	61.1	67.6	69.9	8.8%	66.4
20%	61.4	67.7	69.9	8.4%	66.4
30%	61.8	67.8	69.9	8.1%	66.4
40%	62.2	67.9	69.9	7.7%	66.4
50%	62.5	67.9	69.9	7.3%	66.4
60%	62.9	68.0	69.9	7.0%	66.4
70%	63.2	68.1	69.9	6.6%	66.4
80%	63.6	68.2	69.9	6.3%	66.4
90%	63.9	68.2	69.9	6.0%	66.4
100%	64.2	68.3	69.9	5.6%	66.4

Table 5

The Value of Path-Dependency in the Firm's Policy.

The table reports comparative statics for the increase in firm value from switching to the path-dependent firm policy—evaluated at the market-value maximizing investment trigger. Each line represents a different parametrization. The base parametrization is from Table 1. The capital gains tax rate equals 25%, and $\pi = 100\%$.

	Operating History $(m - l^0)/\overline{m}$			
	0%	33%	67%	100%
	Δ Firm Value (%)			
Base	5.6	3.8	2.0	0.0
$\mu = 1\%$	5.1	3.2	1.6	0.0
$\mu = 3\%$	6.4	4.6	2.7	0.0
$\sigma = 15\%$	6.9	5.3	3.4	0.0
$\sigma = 25\%$	4.5	2.7	1.2	0.0
$r = 4\%$	5.4	3.5	1.8	0.0
$r = 6\%$	5.8	4.0	2.2	0.0
$\phi = 75\%$	6.2	4.3	2.4	0.0
$\phi = 65\%$	5.1	3.3	1.7	0.0
$\omega = 25\%$	5.9	3.9	2.1	0.0
$\omega = 100\%$	3.9	2.7	1.5	0.0

Table 6
The Simulated Data.

The table reports descriptive statistics for the simulated data set. The first column for each variable contains the sample average, and the second column the standard deviation. The number of observations is $100,000$. The base parametrization is summarized in Table 1. In the base parametrization, the definition of book value is from (35), and $\pi = 24\%$, $c^0 = 6.0$, $x_0 = l^0 + .692 \times (\overline{m} - l^0)$.

(a) Descriptive Statistics

IPO+t	Leverage L_t		Market-to-Book MB_t^{efwa}		Market-to-Book MB_t		Asset Growth $\Delta B_t/B_{t-1}$	
0	39.18	(0)	2.33	(0)	2.33	(0)	–	–
1	47.06	(14.14)	2.30	(0.05)	2.20	(0.44)	19.64	(46.63)
2	54.97	(16.77)	2.28	(0.07)	2.05	(0.52)	20.35	(47.32)
3	59.69	(17.21)	2.27	(0.07)	1.97	(0.59)	12.58	(38.49)
4	62.45	(17.21)	2.26	(0.07)	1.94	(0.68)	8.52	(32.23)
5	64.07	(17.19)	2.25	(0.07)	1.94	(0.76)	6.29	(27.93)
6	65.01	(17.25)	2.25	(0.07)	1.96	(0.84)	4.95	(24.91)
7	65.46	(17.29)	2.25	(0.07)	2.00	(0.92)	4.12	(22.81)
8	65.64	(17.40)	2.24	(0.07)	2.05	(1.01)	3.65	(21.51)
9	65.51	(17.60)	2.24	(0.07)	2.10	(1.11)	3.00	(19.55)
10	65.21	(17.83)	2.24	(0.07)	2.17	(1.20)	2.70	(18.56)

(b) Event Likelihood (%)

IPO+t	Inert	Bankrupt before Invested	Invested	Invested, Not Bankrupt	Bankrupt after Invested
0	100	0	0	0	0
1	84.9	0	15.1	15.1	0
2	69.3	0	30.7	30.6	0
3	59.6	0.1	40.3	39.9	0.4
4	52.8	0.4	46.7	45.4	1.3
5	47.5	1.1	51.4	48.8	2.6
6	43.0	2.1	54.9	50.8	4.1
7	39.1	3.1	57.8	51.9	5.9
8	35.5	4.3	60.3	52.5	7.7
9	32.3	5.5	62.2	52.6	9.6
10	29.5	6.6	63.9	52.6	11.3

Table 7
The Capital Structure Regressions.

The table reports coefficient estimates from capital structure regressions using simulated data. I run the following cross-sectional regression specification in IPO time

$$L_t = a_0 + a_1 MB_{t-1}^{efwa} + a_2 MB_{t-1} + f(X_t, \kappa_u) + \varepsilon_t.$$

In the specification (BW1) the function $f(\cdot)$ is additive in $\ln(X_t)$ and X_t to control for contemporaneous variation in firm size and operating performance. In the second set of regressions (BW2) I include investment fixed effects $\mathbf{1}_{\{\kappa_u \leq t\}}$, $\mathbf{1}_{\{\kappa_u \leq t-1\}}$. In the third set of regressions (BW3) I run the first specification on the subsample of firms that have invested by time $t-1$. Since in the model there is at most one external financing event after the IPO, I construct the external-financing weighted average market-to-book ratio as $MB_{t-1}^{efwa} = w_{t-1} MB_0 + (1 - w_{t-1}) MB_{\kappa_u}$. The weight w_{t-1} equals 1 if the firm has not invested by date $t-1$ and $w_{t-1} = p_0^0/(p_0^0 + I)$ otherwise. The base parametrization is summarized in Table 1. In the base parametrization, the definition of book value is from (35), and $\pi = 24\%$, $c^0 = 6.0$, $x_0 = l^0 + .692 \times (\overline{m} - l^0)$.

IPO+t	(BW1) MB_{t-1}^{efwa}	MB_{t-1}	(BW2) MB_{t-1}^{efwa}	MB_{t-1}	(BW3) MB_{t-1}^{efwa}	MB_{t-1}
			Book Leverage			
2	-186.4	8.6	-32.7	-0.1	-42.6	0.0
4	-242.1	2.4	-20.0	-0.2	-38.6	0.0
6	-261.5	0.8	-24.8	-0.3	-35.4	0.0
8	-266.9	0.2	-30.4	-0.3	-34.4	0.0
10	-274.3	-0.1	-34.4	-0.2	-34.2	0.0
			Market Leverage			
2	-215.4	9.9	-52.2	0.0	-52.2	0.0
4	-281.3	3.0	-46.6	0.0	-46.6	0.0
6	-306.0	1.2	-44.5	0.0	-44.5	0.0
8	-314.0	0.5	-43.4	0.0	-43.4	0.0
10	-324.4	0.2	-42.8	0.0	-42.8	0.0

Table 8
**Persistence in the Capital Structure Regressions.**

The table reports coefficient estimates from capital structure regressions with varying lag length using simulated data. I run the following cross-sectional regression specification for $t = IPO + 10$:

$$L_t = a_0 + a_1 MB_{t-\iota}^{efwa} + a_2 MB_{t-\iota} + f(X_t, \kappa_u) + \varepsilon_t.$$

In the specification (BW1) the function $f(\cdot)$ is additive in $\ln(X_t)$ and X_t to control for contemporaneous variation in firm size and operating performance. In the second set of regressions (BW2) I include investment fixed effects $\mathbf{1}_{\{\kappa_u \leq t\}}$, $\mathbf{1}_{\{\kappa_u \leq t-\iota\}}$. In the third set of regressions (BW3) I run the first specification on the subsample of firms that have invested by time $t - \iota$. Since in the model there is at most one external financing event after the IPO, I construct the external-financing weighted average market-to-book ratio as $MB_{t-\iota}^{efwa} = w_{t-\iota} MB_0 + (1 - w_{t-\iota}) MB_{\kappa_u}$. The weight $w_{t-\iota}$ equals 1 if the firm has not invested by date $t - \iota$ and $w_{t-\iota} = p_0^0/(p_0^0 + I)$ otherwise. The base parametrization is summarized in Table 1. In the base parametrization, the definition of book value is from (35), and $\pi = 24\%$, $c^0 = 6.0$, $x_0 = l^0 + .692 \times (\overline{m} - l^0)$.

	(BW1)		(BW2)		(BW3)	
Lag ι	$MB_{t-\iota}^{efwa}$	$MB_{t-\iota}$	$MB_{t-\iota}^{efwa}$	$MB_{t-\iota}$	$MB_{t-\iota}^{efwa}$	$MB_{t-\iota}$
			Book Leverage			
1	-274.3	-0.1	-34.4	-0.2	-34.2	0.0
3	-198.0	1.0	-37.0	-0.1	-36.1	0.0
5	-138.9	2.4	-38.0	-0.1	-37.4	0.0
7	-92.4	4.7	-37.1	0.0	-38.1	0.0
9	-63.0	6.8	-38.9	0.2	-42.7	0.0
			Market Leverage			
1	-324.4	0.2	-42.8	0.0	-42.8	0.0
3	-233.9	1.3	-43.3	0.0	-43.4	0.0
5	-164.0	2.9	-44.1	0.0	-44.4	0.0
7	-109.1	5.6	-45.6	0.1	-46.5	0.0
9	-74.5	8.0	-52.0	0.3	-52.1	0.0

Figure 2. Bankruptcy Region ($\mathcal{L}$), Continuation Region ($\mathcal{C}$), and Investment Region ($\mathcal{U}$).
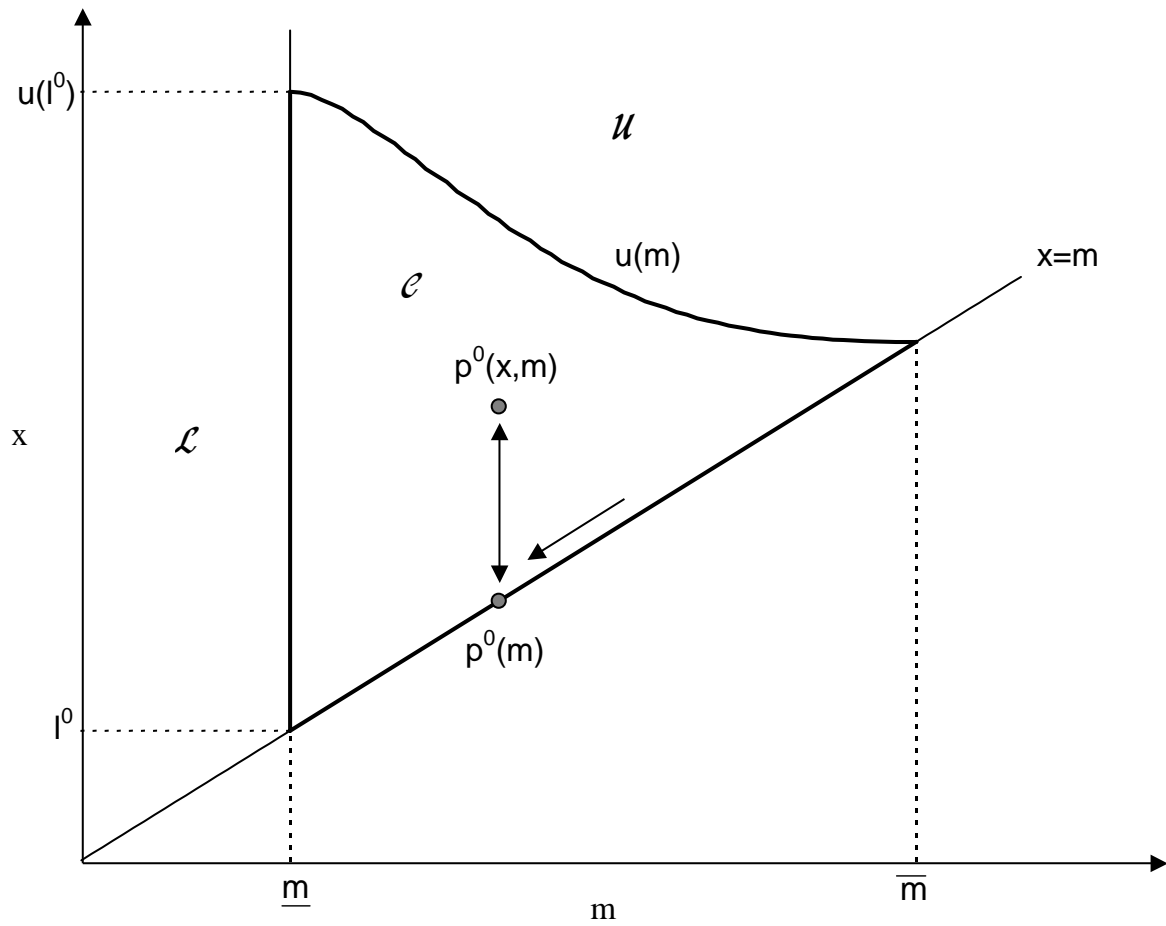
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Martin Hoesli is acting Head of the Research Paper Series. Please email any comments or queries to the following address: Martin.Hoesli@hec.unige.ch.

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The University of Geneva

The University of Geneva, originally known as the Academy of Geneva, was founded in 1559 by Jean Calvin and Theodore de Beze. In 1873, The Academy of Geneva became the University of Geneva with the creation of a medical school. The Faculty of Economic and Social Sciences was created in 1915. The university is now composed of seven faculties of science; medicine; arts; law; economic and social sciences; psychology; education, and theology. It also includes a school of translation and interpretation; an institute of architecture; seven interdisciplinary centers and six associated institutes.

More than 13'000 students, the majority being foreigners, are enrolled in the various programs from the licence to high-level doctorates. A staff of more than 2'500 persons (professors, lecturers and assistants) is dedicated to the transmission and advancement of scientific knowledge through teaching as well as fundamental and applied research. The University of Geneva has been able to preserve the ancient European tradition of an academic community located in the heart of the city. This favors not only interaction between students, but also their integration in the population and in their participation of the particularly rich artistic and cultural life. ***http://www.unige.ch***

The University of Lausanne

Founded as an academy in 1537, the University of Lausanne (UNIL) is a modern institution of higher education and advanced research. Together with the neighboring Federal Polytechnic Institute of Lausanne, it comprises vast facilities and extends its influence beyond the city and the canton into regional, national, and international spheres.

Lausanne is a comprehensive university composed of seven Schools and Faculties: religious studies; law; arts; social and political sciences; business; science and medicine. With its 9'000 students, it is a medium-sized institution able to foster contact between students and professors as well as to encourage interdisciplinary work. The five humanities faculties and the science faculty are situated on the shores of Lake Leman in the Dorigny plains, a magnificent area of forest and fields that may have inspired the landscape depicted in Brueghel the Elder's masterpiece, the Harvesters. The institutes and various centers of the School of Medicine are grouped around the hospitals in the center of Lausanne. The Institute of Biochemistry is located in Epalinges, in the northern hills overlooking the city. ***http://www.unil.ch***

The Graduate Institute of International Studies

The Graduate Institute of International Studies is a teaching and research institution devoted to the study of international relations at the graduate level. It was founded in 1927 by Professor William Rappard to contribute through scholarships to the experience of international co-operation which the establishment of the League of Nations in Geneva represented at that time. The Institute is a self-governing foundation closely connected with, but independent of, the University of Geneva.

The Institute attempts to be both international and pluridisciplinary. The subjects in its curriculum, the composition of its teaching staff and the diversity of origin of its student body, confer upon it its international character. Professors teaching at the Institute come from all regions of the world, and the approximately 650 students arrive from some 60 different countries. Its international character is further emphasized by the use of both English and French as working languages. Its pluralistic approach - which draws upon the methods of economics, history, law, and political science - reflects its aim to provide a broad approach and in-depth understanding of international relations in general. ***http://heiwww.unige.ch***

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