Financial Structure and Market Equilibrium in a Vertically Differentiated Industry

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Abstract

This paper examines the effects of uncertainty and the choice of financial structure in a vertically differentiated duopoly. In the market model consumers are located along a continuum of taste parameters and prefer unanimously higher to lower qualities when quality prices are set at average variable cost. In such a model only two firms can survive with a positive market share. We introduce uncertainty in demand by varying the range of the consumer taste parameter and consider a simultaneous game of sequential choices of quality, financial structure and product price, with varying order of decision-making and revelation of information. We consider both restricted and free entry. It is shown that financial structure affects market equilibrium, which is also heavily dependent on the order of choice of structure and quality, as well as on whether uncertainty exists in the lower or the upper limit of the taste parameter. In all cases leverage increases the lower quality and in most cases it also increases the lower quality price. There are also welfare implications, with the use of leverage when it is optimal improving both total and consumer surplus.

Keywords: Vertical differentiation; uncertainty; financial structure; leverage; quality; sequential choice.

JEL codes: C70 ; G32; L11; L13; L15.

I. Introduction

The fact that financial structure affects investment, pricing and output decisions in oligopolistic industries has been known for several years, at least since Brander and Lewis' 1986 article. These effects have, however, attracted relatively little attention, as noted in the 1991 survey by Harris and Raviv. A few more studies, especially empirical ones, have been added in the decade since that survey[1], but overall the topic seems to be relatively neglected in both the economics and financial literature. Existing studies have mostly dealt with homogenous product oligopolies, and have examined the effect of leverage on pricing and output, as well as on barriers to entry and the feasibility of entry deterrence[2]. A few recent works have also examined leverage effects in industries with differentiated products, although the nature of differentiation has been left unspecified and the demand functions have been taken as given in modeling the firms' interaction[3]

This paper examines the effect of leverage and financial structure on the investment, pricing and output decisions of firms in an imperfectly competitive industry in which product is differentiated by *quality*. We consider vertically differentiated products, in which there is consumer unanimity in ranking the various products available in the market. In such markets it is well known that firms compete along both price and quality dimensions, by segmenting the market along the characteristics that differentiate consumers from each other.

We examine such markets under uncertainty about consumer preferences for quality and consider the effect of financial structure on market equilibrium. It turns out that financial structure has a major impact on market equilibrium, affecting both product price and product quality under both restricted and free entry. We also show that the optimal financial structure is dependent on both the nature of uncertainty as well as the sequence of decision-making in the choices of structure and product quality. Depending on the case, the optimal structures may contain both equity and debt or be all-debt or all-equity. Further, we show that the use of debt when it is optimal has positive effects on aggregate welfare and on consumer surplus in many important cases.

Markets with vertically differentiated products were introduced by Gabsewicz and Thisse (1980) and extended by Shaked and Sutton (1982, 1983). The dominant characteristic of such differentiated products is that their market structure is demand-driven and depends on the width of the income (or taste) distribution of consumers. Markets contain products with a finite set of quality-price combinations and a set of consumers with utility functions dependent on both price and quality, with the consumers buying only one unit of product each (or none at all). In such markets it can be shown that, subject to relatively mild constraints, only a finite number of firms, each one producing a single quality, can survive, even in the face of unrestricted entry. This property, termed the *finiteness property* by Shaked and Sutton (1983), determines "natural" market structures that can be shown to depend on the aforementioned width of the consumer taste or income distribution.

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[1] See, for instance, Phillips (1994), Chevalier (1995), Chevalier and Scharfstein (1996) and Showalter (1995).
[2] See Brander and Lewis (1986), Maksimovic (1988), and Poitevin (1989).
[3] See Maksimovic and Titman (1991), and Dasgupta and Titman (1998).

Much attention has been devoted in these markets to the topics of firm entry and entry deterrence under varying assumptions about the structure of the entry game. Thus, while it is not in general possible to deter entry in a market whose natural structure is not a monopoly and where firms are restricted to one quality each, it can be shown that entry deterrence can be both feasible and profitable if this last assumption is relaxed[4]. Similarly, the industry equilibrium depends crucially on whether entry is simultaneous or sequential, with the lower-quality entrant able to observe the incumbent's decisions, as well as on whether quality can be altered costlessly once it has been chosen.

By contrast, neither uncertainty nor financial structure has been examined so far in such vertically-differentiated industries. Yet these characteristics are undoubtedly relevant real life features of these industries. For instance, if consumers are differentiated by income there may be *a priori* randomness in consumer income, depending on general economic conditions, which makes the width of the income distribution also random. Similarly, in cases where consumers are differentiated by tastes there may also be randomness on the width of the distribution of the taste (or willigness to pay) parameter, which depends on other products available in the market, as well as on consumer income.

In this paper we examine duopoly models of vertically differentiated products under various assumptions concerning the entry game and the financial structures of the participating firms. It turns out that the effect of financial structure on the duopoly market equilibrium is crucially dependent on the type of uncertainty as well as on the sequence of information revelation with respect to the sequence of decision-making by the firm. In markets where price and output must be chosen under uncertainty financial structure affects the product market equilibrium, with the use of debt raising both qualities and, in most cases, prices as well. This result is robust with respect to various assumptions about the type of uncertainty and the sequence of firm decision-making, even though the nature of market equilibrium is sensitive to these assumptions.

We also examine the existence of an optimal financial structure. This structure depends on the nature of uncertainty, the parameters of the problem and the sequence of decision-making. We identify conditions under which an optimal degree of leverage exists in equilibrium in several important cases. This optimal leverage is independent of taxation or bankruptcy costs, unlike the conventional Modigliani-Miller (1958, 1963). This result also has several consequences with respect to the feasibility of entry deterrence.

The use of debt in industries with products differentiated by quality presents interest also because such industries have special features that distinguish them sharply from conventional homogenous product oligopolies. Firms are, by nature, asymmetric and compete in two dimensions of the product, price and quality, with the latter chosen upon entry and the former chosen subsequently given the quality choice. Further, the limited capacity of the market to accommodate new entry allows incumbent firms to relax price competition by occupying opposite ends of the quality spectrum.

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[4] See Constantatos and Perrakis (1997).

The asymmetric nature of the incumbent firms also has a crucial impact on the way uncertainty affects the choice of capital structure. In the duopoly structure that emerges uncertainty on the width of the taste parameter distribution, the equivalent of demand uncertainty in homogenous products industries, can affect only one of the two competing firms if it affects the lower or the higher parameter limit. In the former case debt is always profitable for the firm that is affected by uncertainty, while in the latter case the optimal structure can be all-equity. This difference stems from the fact that, given optimal price choices, competition on quality levels has a differential impact on the two firms: *ceteris paribus* an increase of the higher quality level raises the profit of the lower quality firm, while an increase in the lower quality level *lowers* the profit of the higher quality firm.

In the next section we present the general model and summarize the results under certainty. We consider markets in which consumers are differentiated with respect to the taste parameter distribution. Uncertainty is introduced in the following section, while subsequent sections examine financial structure under various uncertainty specifications. To keep the models simple, most of our results are examined in structures in which a natural duopoly exists under all uncertainty situations. We also abstract from the impact of uncertainty on entry by assuming initially that no entry takes place. Entry deterrence is discussed as an extension in another section.

II. The General Model

We consider a market with a universe of consumers, each one buying one unit of the product, and characterized by the following consumer utility function, which is commonly used in vertically differentiated markets:

$$U = u_i t - p_i , \quad i=1,2, \tag{1}$$

where u_i is a product quality index, p_i its corresponding price, and t is a consumer taste parameter that indicates the preference for quality of a particular consumer, the marginal utility *with respect to quality*. The parameter t is drawn from a given set of consumers uniformly distributed in $t \in [a,b]$. For simplicity, assume the density of the income distribution equals one. Equation (1) implies that at equal prices consumers unanimously prefer the product with the higher u.

Firms in this market produce one quality each. A natural duopoly exists in such a market[5] if the average variable cost does not rise "too much" with quality and if the width of the income distribution is such that unanimity is preserved when prices are set equal to average variable costs. This happens if the width of the interval in which consumer tastes are located is within the bounds $2a < b < 4a$. In such a case it can be shown that the market can support exactly two firms with positive market shares, and that the market is fully covered, with each consumer buying one unit of the product. For $b < 2a$ the market can support only one firm (a natural monopoly), while for $b > 4a$ the market can support a third firm. In our duopoly firm 1 is the high quality and firm 2 the low quality firm, implying that $u_1 > u_2$.

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[5] See Shaked and Sutton (1982, 1983).

In a natural duopoly any third firm entering the market can gain market share only by displacing an existing lower quality firm. Consumers with a high (low) value of the taste parameter t buy from firm 1 (firm 2). Let also t_d indicate the taste parameter of the marginal consumer, the one who is indifferent between buying quality 1 or quality 2. It is easy to see that:

$$t_d = (p_1 - p_2)/(u_1 - u_2) . \qquad (2)$$

Hence, the market shares of firms 1 and 2 are $b-t_d$ and $t_d - a$ respectively.

For the production side of our model we assume that the variable cost of production is the same for both firms and, without loss of generality, it is taken to be equal to 0, in which case the prices are interpreted as the excess over unit cost. Higher qualities, however, are allowed to have a higher fixed cost. The cost of quality u will be generally assumed to be an increasing and convex function $F(u)$, with $F' \geq 0$, $F'' \geq 0$. Alternatively, F can be assumed constant, like a patent or a license fee; both assumptions are relevant and both will be used in this paper. This cost is assumed irrecoverable (sunk) once entry has taken place. Let $F_i \equiv F(u_i)$, i=1,2.

We assume that the entry game is simultaneous, and that firm 1 is predetermined to choose the higher quality. It will also be assumed initially that no further entry will be allowed into the sector, an assumption that will be relaxed later on. Hence, the game sequence is entry and choice of quality, followed by production decisions taken simultaneously by choosing prices independently (a Bertrand-type of game). The game terminates after one period.

Under certainty the equilibrium of this game starts with the choice of the two prices under the Bertrand assumption given the quality levels. The solution is well-known and only the results will be stated below. The game starts by maximizing the net revenues R_i, i=1,2, given by:

$$R_1 = p_1(b-t_d) - F_1 = p_1(b-(p_1 - p_2)/(u_1 - u_2)) - F_1, \quad R_2 = p_2((p_1 - p_2)/(u_1 - u_2) - a) - F_2. \qquad (3)$$

Maximizing in p_1 and p_2, we find that the optimal prices p_i^*, i=1,2 are given by[6]:

$$p_1^* = (2b-a)(u_1 - u_2)/3, \quad p_2^* = (b-2a)(u_1 - u_2)/3. \qquad (4)$$

Substituting (2) and (4) into (3) we get the optimal revenues R_i^*, i=1,2:

$$R_1^* = [(2b-a)/3]^2(u_1 - u_2) - F_1, \quad R_2^* = [(b-2a)/3]^2(u_1 - u_2) - F_2. \qquad (5)$$

The optimal qualities are chosen simultaneously and independently (a Cournot-type assumption), implying that the optimal u_1^* is given from the equation $dR_1^*/du_1=F'$ if $F'>0$; otherwise, if F is independent of quality, u_1^* is equal to the maximum technologically feasible quality level $\bar{u}$. As for u_2^*, it is chosen by decreasing u_2 to the

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[6] This maximization assumes that at the existing quality levels we have $u_2a \geq (b-2a)(u_1 - u_2)/3$. This technical assumption effectively insures that there is full market coverage at the existing market equilibrium. See Tirole (1988), p. 296. Such an assumption will also be adopted when uncertainty is introduced further on in this section.

minimum level consistent with the full market coverage (see note 6). This is the level at which the consumer with taste parameter a is indifferent between purchasing or not, which occurs when $u_2^* = p_2^*/a$. Substituting and solving, we find the optimal qualities:

$$u_1^* = F'^{-1}([(2b-a)/3]^2), \quad u_2^* = [u_1^*(b-2a)/3]/[a + (b-2a)/3]. \qquad (6)$$

Before closing this section on the firm under certainty, we note that the two choice variables, prices and qualities, have different strategic effects in interfirm rivalry. Indeed, it can be easily seen from (3) that the marginal revenues $\partial R_i/\partial p_i$ are increasing functions of the other firm's strategic variable p_j, for both $i,j= 1, 2, i \neq j$; prices are, therefore, *strategic complements* in the terminology of Bulow *et al* (1985). For qualities, on the other hand, the marginal revenues $\partial R_i/\partial u_i$ are decreasing functions (hence, strategic substitutes) of the opponent's quality u_j in (3), while the strategic effects of quality disappear if the sequential nature of quality choice are taken into account in (5); in the latter case the marginal revenues are independent of the opponent's quality. This fact plays a role in the optimal financial structure when uncertainty and debt are introduced in subsequent sections[7].

In the following sections we introduce uncertainty in the width [a, b] of the buyers' taste parameter distribution, which corresponds in our formulation to uncertainty over the size of product demand facing the two firms.

III. Uncertainty in the All-Equity Firm

We introduce uncertainty by assuming that the width of the set within which the taste parameter lies varies randomly at the time the entry decisions are taken. There are many ways of representing such randomness. In what follows we shall initially assume that the upper limit parameter b remains unchanged, while the lower limit a varies continuously within a given interval [a , ā], and let **a** denote the expectation of a, whose distribution function is G(.). Alternatively, we could have chosen a fixed a and a randomly varying b. This model, which is discussed as an extension in subsequent sections, *does not* yield the same results concerning financial structure as when a is taken as random. In real life both a and b may vary randomly, but when it is a that varies the entire impact of uncertainty falls on the lower quality firm 2. In all cases it is assumed that firms are risk-neutral, maximizing expected profits.

Of particular interest is the revelation of uncertainty with respect to the sequence of decision-making. If uncertainty is revealed before prices are chosen then (2), (4) and (5) still hold, and uncertainty affects only the optimal choice of qualities, chosen by maximizing the expectations of the expressions in (5). The optimal qualities are then given by (6), with the expectation of the expression in the argument for u_1^*, and with the replacement of a by the lower limit a for u_2^*.

The only interesting element that the introduction of uncertainty brings to the analysis is the fact that both firms' profits are increasing functions of the variance of the random parameter a, as it can be easily seen by taking the expectations in (5). Otherwise, uncertainty will have no effect on market equilibrium. When there is debt in the firm's

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[7] The differential impact on the optimal capital structure of strategic substitutes and complements in homogenous firms appears very clearly in Showalter (1995).

capital structure the revelation of the value of a will also reveal whether default will occur before prices are chosen, implying that the debt holders will write into the debt contract provisions for taking control of the firm in such cases. Hence, financial structure will have no effect on market equilibrium.

We assume, therefore, that the true value of a is revealed after prices have been chosen. Since consumer tastes are unobservable *ex ante*, such an assumption has also the advantage of being closer to reality. We also assume that the variations in a are such that the market remains always a natural duopoly, with $2\underline{a} < b < 4\underline{a}$ for all a. This limits the allowable width $[\underline{a}, \bar{a}]$, since we must have $2\bar{a} < b$ and $4\underline{a} > b$, implying that we must have $\bar{a} < 2\underline{a}$. This assumption is innocuous with respect to the interesting results, but it does simplify the analysis by avoiding the changes in functional forms which would be necessary if the market were to become a monopoly or remain uncovered for some values of a.

For all-equity firms the choices of the optimal prices p_i, i=1,2, is the same as if we were to replace a by **a** in (3), implying that (4) also holds with **a** instead of a. Now, however, while the profit R_1^* is still given by (5) with the substitution of **a** for a, the profit R_2^* is *ex ante* random and given by:

$$R_2^* = [(b-2\mathbf{a})/3](u_1 - u_2)[(b+\mathbf{a})/3 - a] - F_2. \tag{7}$$

Taking the expectation in (7) it can be easily seen that for an all-equity firm uncertainty still yields the same result as certainty, with (5) holding for both firms with **a** replacing a. The optimal quality u_1^* is chosen by the same expression (6) as in the certainty case, while u_2^* is also chosen by (6), by setting it so that full market coverage will exist for all values of a, with $\underline{a}$ replacing a in (6) and with u_2^* equal to $p_2^*/\underline{a}$.

The situation changes, however, when we examine the introduction of debt in the financial structure of firm 2. A key element in this case is the time point at which financial structure is chosen, relative to the choices of price and quality. We distinguish the following cases:

i)   The financial structure is chosen *after* the choice of prices and qualities.
ii)   The financial structure is chosen before prices but after quality choices.
iii)   The financial structure is chosen before both qualities and prices.

Case (i) is equivalent to the conventional Modigliani-Miller (MM) solution, according to which financial structure is irrelevant in the absence of corporate income taxes. Indeed, we can easily see that the relevant objective functions to maximize in order to find the optimal prices and qualities are the values of the two firms, which are found by taking the expectations in (3) for given prices and qualities whether there is debt or not. Hence, the product market equilibrium is identical to that of the all-equity case and is, therefore, independent of financial structure. This case is also not very realistic on economic grounds, since the choice of product prices normally takes place after the firm has been established and financial structure has been chosen.

The decision-making sequences in cases (ii) and (iii) are shown in Figure 1. In these cases the MM result definitely does *not* hold, as it will be seen when we examine them in detail in the next section.
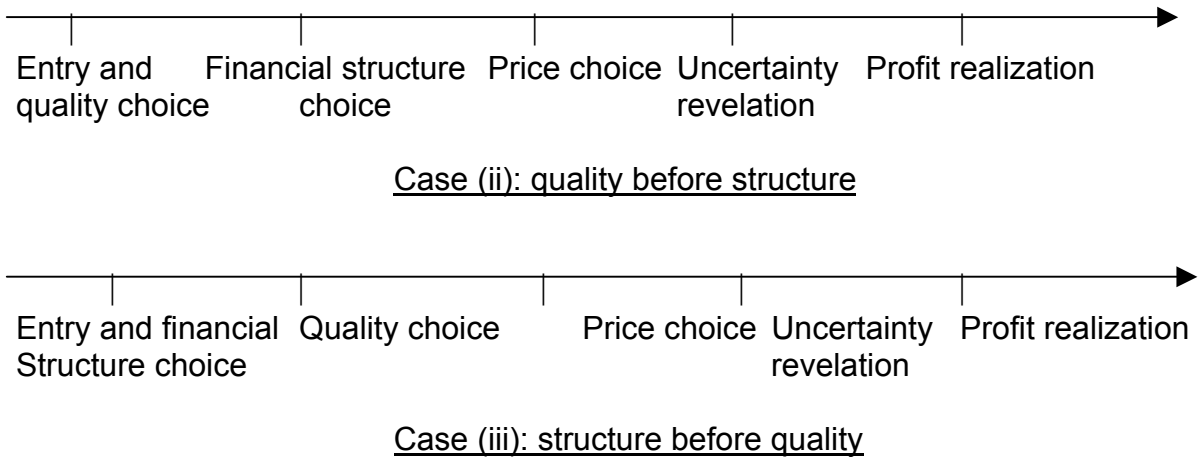
Entry and
quality choice

Financial structure
choice

Price choice

Uncertainty
revelation

Profit realization

Case (ii): quality before structure

Entry and financial
Structure choice

Quality choice

Price choice

Uncertainty
revelation

Profit realization

Case (iii): structure before quality

Figure 1

IV. The Effect of Leverage on Product Market Equilibrium

We assume that firm 2 is partially financed by debt, whose promised repayment amount is equal to D. For a sufficiently large D the firm will go into default at the end of the production period, with the bondholders acquiring control. In both cases shown in Figure 1 it is initially assumed that D is exogenously fixed when prices are chosen. As for firm 1, its financial structure is not relevant since it is not affected directly by uncertainty, even though its choices of price and quality are clearly dependent on those of firm 2. We also assume that D is common knowledge at the times the qualities and prices are chosen.

From (3) it is clear that the firm 2 revenue R_2 is a decreasing function of the parameter a. Let a_1 denote the value of a at which the firm is just able to repay its debt obligation. The firm is solvent for $a \in [\underline{a}, a_1]$, and in default for higher values of a. The value a_1 is given by:

$$p_2(t_d - a_1) - F_2 = p_2[(p_1 - p_2)/(u_1 - u_2) - a_1] = D, \tag{8}$$

where t_d is given by (2).

To find the optimal prices we now maximize the firm 2 equity value E_2 with respect to p_2 for a given p_1. Since E_2 is equal to the expected profit under solvency minus the promised payment, we find by subtracting (8) from the value of R_2 given by (3) that,

$$E_2 = \int_{\underline{a}}^{a_1} p_2 (a_1 - a) \, dG(a), \tag{9}$$

whose maximization with respect to p_2 taking into account (2) and (7) yields:

$$\int_{\underline{a}}^{a_1} [(p_1 - 2p_2)/(u_1 - u_2) - a]dG(a) = 0 . \tag{10}$$

This can be rewritten under the following form:

$$[(p_1 - 2p_2)/(u_1 - u_2)] = E[a \mid a \leq a_1] \equiv \mathbf{a_1}, \quad p_2 = p_1/2 - \mathbf{a_1}(u_1 - u_2)/2, \tag{11}$$

where $\mathbf{a_1}$ denotes the truncated expectation of a given that it is less than a. On the other hand, the maximization of R_1 with respect to p_1 yields:

$$p_1 = b(u_1 - u_2)/2 + p_2/2 . \tag{12}$$

Solving now (11) and (12), we find that the optimal prices $p_{1D}{}^*$ and $p_{2D}{}^*$ under leverage are given with the same expressions as (4), with $\mathbf{a_1}$ replacing a.

Consider now the unlevered solution for firm 2, which is similar to the certainty case with **a** replacing a in (4). In the presence of debt this solution is infeasible if at the lowest level of random firm 2 profit, which occurs when a = $\bar{a}$, we have:

$$p_2{}^*[(p_1{}^* - p_2{}^*)/(u_1{}^* - u_2{}^*) - \bar{a}] - F(u_2{}^*) < D.$$

Assuming that in such a case a feasible choice of price and quality for firm 2 exists given the firm 1 choices (p_1, u_1), there is a value $a_1 \in (\underline{a}, \bar{a})$ of a such that relation (8) holds at the optimal choices (p_2, u_2) of firm 2. Since, as argued above, the optimal prices $p_{1D}{}^*$ and $p_{2D}{}^*$ are given by the same expressions as (4), with $\mathbf{a_1}$ replacing a, the optimal revenue $R_{1D}{}^*$ of firm 1 is now given by :

$$R_{1D}{}^* = [(2b - \mathbf{a_1})/3]^2(u_1 - u_2) - F_1. \tag{13}$$

Similarly, replacing p_2 by $p_{2D}{}^*$ into (9) we get the firm 2 equity:

$$E_2 = G(a_1) (b - 2\mathbf{a_1})(u_1 - u_2)(a_1 - \mathbf{a_1})/3. \tag{14}$$

The value of firm 1 is also given by (13), while the value of firm 2 is equal to the equity plus the debt. The latter is equal to D for $a \in [\underline{a}, a_1]$ and to the entire revenue $p_{2D}{}^*(t_d - a) - F_2$ for $a \in (a_1, \bar{a}]$. Replacing E_2 from (14), D from (8), and taking the expectation, we get:

$$V_{2D} = (b - 2\mathbf{a_1})(u_1 - u_2)[(b + \mathbf{a_1})/3 - a]/3 - F_2. \tag{15}$$

A key issue at this point is whether quality choice comes before or after the choice of capital structure, cases (ii) and (iii) in Figure 1 of the previous section. In case (ii) one should choose first the level of D by maximizing V_{2D} given u_1 and u_2. We may then prove the following result.

Proposition 1: If there is uncertainty in the lower limit of the consumer taste parameter and if the lower quality firm 2 chooses its financial structure after the

choices of the quality levels u_1 and u_2 then there exists a unique leveraged optimal capital structure for the firm. This optimal capital structure is defined by the debt level that sets,

$$6\mathbf{a} - b - 4\mathbf{a_1} = 0, \tag{16}$$

provided this relation yields a value $a_1 \in [\underline{a}, \ \overline{a}]$; at that structure the firm will have positive debt and equity levels. If, on the other hand, there is no value of $a_1 \in [\underline{a}, \ \overline{a}]$ satisfying (16) the optimal capital structure is 100% debt financing[8].

<u>Proof</u>: Differentiating (15) with respect to D for given u_1 and u_2,, we note that $\partial V_{2D}/\partial D$ is proportional to the quantity $(\partial \mathbf{a_1}/\partial D)(6\mathbf{a} - b - 4\mathbf{a_1})$. Since $2a < b < 4a$ by the natural duopoly assumption, the sign of the factor $6\mathbf{a} - b - 4\mathbf{a_1}$ is ambiguous. Rewriting (8):

$$(b-2\mathbf{a_1})(u_1 - u_2)[(b + \mathbf{a_1})/3 - a_1] /3 - F_2 = D, \tag{17}$$

we can easily see that for given u_1 and u_2 the left-hand-side expression decreases in a_1, implying that $\text{sign}(\partial a_1 / \partial D) < 0$. Hence,

$$\text{sign} (\partial V_{2D} / \partial D) = (-) \, \text{sign}(\partial \mathbf{a_1} / \partial D) = (-) \, \text{sign}(\partial a_1 / \partial D) < (>) \, 0,$$

if $6\mathbf{a} - b - 4\mathbf{a_1}$ is greater than (less than) zero. For the all-equity firm $D = 0$ and $\mathbf{a_1} = \mathbf{a}$, implying that $6\mathbf{a} - b - 4\mathbf{a_1} = 2\mathbf{a} - b < 0$ and $\text{sign} (\partial V_{2D} / \partial D) > 0$. On the other hand, for the all-debt firm we have $\mathbf{a_1} = \underline{a}$ and $6\mathbf{a} - b - 4\mathbf{a_1}$ can be > 0 or <0, since the sign of $6\mathbf{a} - b - 4\underline{a}$ is ambiguous. If $6\mathbf{a} - b - 4\underline{a} >0$ then $\text{sign} (\partial V_{2D} / \partial D) < 0$ and there exists an optimal structure with positive amounts of debt and equity, given by $6\mathbf{a} - b - 4\mathbf{a_1} = 0$. If $6\mathbf{a} - b - 4\underline{a} \leq 0$ then the optimal structure is all-debt, QED.

  Given Proposition 1, we can now describe completely the effects of leverage in this case (ii), where structure is chosen after the quality levels have been set. This is done in the next result.

<u>Proposition 2</u>: Under the conditions of Proposition 1 and at the optimal capital structure defined in Proposition 1 the following industry equilibrium exists:

a) The optimal quality levels $u_{1D}{}^*$ and $u_{2D}{}^*$ are higher[9] under leverage than the all-equity levels $u_1{}^*$ and $u_2{}^*$;
b) The corresponding optimal price level $p_{2D}{}^*$ is similarly higher than the all-equity level $p_2{}^*$; while the relation between $p_{1D}{}^*$ and $p_1{}^*$ is ambiguous;
c) The *relative* quality level $u_{1D}{}^*/ u_{2D}{}^*$ of the optimal quality choices of firms 1 and 2 under
     leverage is *less* than that same ratio $u_1{}^*/ u_2{}^*$ in the absence of leverage.

<u>Proof</u>: Define $X \equiv (b-2\mathbf{a})/3$, $X_1 \equiv [(b-2\mathbf{a_1})/3]$, $Y \equiv (2b-\mathbf{a})/3$, $Y_1 \equiv (2b-\mathbf{a_1})/3$, where $\mathbf{a_1} = \underline{a}$ whenever the optimal structure is 100% debt. It is clear that $X_1 > X$ and $Y_1 > Y$. Maximizing (5) and (13) with respect to u_1 for a given u_2 yields $Y^2 = \partial F/\partial u_1$ and $Y_1{}^2 = \partial F/\partial u_1$ respectively, from which $u_{1D}{}^* > u_1{}^*$ by the convexity of F. For the choice

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[8] The notion of an all-debt capital structure is to be considered an abstraction, since it is not compatible with the subsequent price-setting decision-making for firm 2, which was assumed to maximize the value of the equity.
[9] If the cost of quality is independent of the quality level we obviously have $u_{1D}{}^* = u_1{}^*$.

of u_2 we maximize (5) and (15), in both of which the optimal u_2 turns out to be the lowest possible consistent with full market coverage, namely $u_2^* = p_2^*/\underline{a}$ and $u_{2D}^* = p_{2D}^*/\underline{a}$. Hence, we have, from (4):

$$u_2^* = X(u_1^* - u_2^*)/\underline{a}, \quad u_{2D}^* = X_1(u_{1D}^* - u_{2D}^*)/\underline{a}. \tag{18}$$

From (18) it is clear that $(u_1^* - u_2^*)/u_2^* > (u_{1D}^* - u_{2D}^*)/u_{2D}^*$, which corresponds to $(u_1^*/u_2^*) > (u_{1D}^*/u_{2D}^*)$, thus proving part (c). This last relation, together with the already proven $u_{1D}^* > u_1^*$, shows that $u_{2D}^* > u_2^*$ as well, thus completing part (a). Part (b) follows immediately from $u_2^* = p_2^*/\underline{a}$ and $u_{2D}^* = p_{2D}^*/\underline{a}$, QED.

Next we examine case (iii), where qualities are chosen given the capital structure. In such a case u_1 and u_2 in (15) are functions of the debt level D and must be chosen by maximizing R_{1D}^* from (13) and the equity E_2 from (14), taking also (16) into account with a given D[10]. The solution is, in general, parameter-dependent. We can, however, obtain results in an important special case. This is when the cost of quality F is *fixed* and independent of the quality level, like a patent or a license. This case is particularly important in the entry-deterrence literature[11], since a fixed cost of quality implies economies of scale in quality. With this simplifying assumption we can show the following.

Proposition 3: If there is uncertainty in the lower limit of the consumer taste parameter and if firm 2 chooses its capital structure before it chooses its quality the optimal quality u_{2D}^* will be equal to $p_{2D}^*/\underline{a}$, equal to the minimum level consistent with full market coverage for all a if the fixed cost F is independent of quality level.

Proof: Differentiating (14) with respect to u_2, we find that the derivative is proportional to the quantity,
$G(a_1)(b-2\mathbf{a_1})(u_1 - u_2)(\partial a_1/\partial u_2) - (b-2\mathbf{a_1})(a_1 - \mathbf{a_1}) - 2(\partial \mathbf{a_1}/\partial u_2)(u_1 - u_2)(a_1 - \mathbf{a_1})$, whose sign is negative if $(\partial \mathbf{a_1}/\partial u_2) < 0$ and the sign of the quantity $-(b-2\mathbf{a_1}) - 2(\partial \mathbf{a_1}/\partial u_2)(u_1 - u_2)$ is similarly negative. To find the sign of this latter quantity we differentiate (16) with respect to u_2. The differentiation yields,
$$[(b + \mathbf{a_1})/3 - \mathbf{a}][-(b-2\mathbf{a_1}) - 2(\partial \mathbf{a_1}/\partial u_2)(u_1 - u_2)] + $$
$$+(b-2\mathbf{a_1})(u_1 - u_2)(\partial[(b + \mathbf{a_1})/3 - a_1]/\partial a_1)(\partial a_1/\partial u_2) = 0,$$
from which it follows immediately that $\partial a_1/\partial u_2 < 0$. Further, the last term of the above equation is positive, implying that the other terms must be negative, which in turn implies that $-(b-2\mathbf{a_1}) - 2(\partial \mathbf{a_1}/\partial u_2)(u_1 - u_2) < 0$. Hence, u_2 must be reduced as much as feasible, implying that $u_{2D}^* = p_{2D}^*/\underline{a}$, QED.

From Proposition 3 we can then derive the following.

Proposition 4: Under the conditions of Proposition 3 assume the debt level D is given and corresponds to a positive default probability for firm 2 for at least some of the values of the random parameter a. Then leverage has the following effects:

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[10] We assume that D is common knowledge, observable by firm 1 at the time u_1 is chosen.
[11] See, for instance, Hung and Schmidt (1988), and Constantatos and Perrakis (1997, 1999).

a) The quality level of firm 1 becomes equal to the maximum technologically feasible level $\bar{u}$ for both levered and unlevered firm 2 cases;
b) Leverage decreases *relative* product differentiation, in the sense that the ratio $u_{1D}{}^*/ u_{2D}{}^*$ of the optimal quality choices of firms 1 and 2 under leverage is *less* than that same ratio $u_1{}^*/ u_2{}^*$ in the absence of leverage, implying in turn that the quality level of firm 2 increases under leverage, with $u_{2D}{}^* > u_2{}^*$;
c) Leverage increases the price level of firm 2, while its effect on the firm 1 price is ambiguous.

Proof: To prove part (b) first, since D is given and F is independent of quality the optimal choice of u_2 is given by $u_{2D}{}^* = p_{2D}{}^*/\underline{a}$, where $p_{2D}{}^* = (b-2\mathbf{a_1})(u_1 - u_2)/3$ as given by (4). Similarly, $u_2{}^* = p_2{}^*/\underline{a} = (b-2\mathbf{a})(u_1 - u_2)/3$. Hence, (18) holds here as well, and $(u_1{}^* - u_2{}^*)/ u_2{}^* > (u_{1D}{}^* - u_{2D}{}^*)/ u_{2D}{}^*$, which corresponds to $u_1{}^*/ u_2{}^* > u_{1D}{}^*/ u_{2D}{}^*$. For part (a), we note first from (5) that $R_1{}^*$ increases linearly with u_1, implying that $u_1{}^* = u$. Maximizing now (13) with respect to u_1, we note that $\partial R_{1D}{}^*/\partial u_1 = Y_1{}^2 - 2Y_1(\partial \mathbf{a_1}/\partial u_1)(u_1 - u_2)/3$. To show that this quantity is positive it suffices to show that $Y_1 > 2(\partial \mathbf{a_1}/\partial u_1)(u_1 - u_2)/3$. From (16), by differentiating with respect to u_1, it can easily be seen that this last relation holds, implying that $\partial R_{1D}{}^*/\partial u_1 > 0$ always and that $u_{1D}{}^* = \bar{u} = u_1{}^*$, thus proving part (a). From parts (a) and (b) it also follows that $u_{2D}{}^* > u_2{}^*$ and, hence, $p_{2D}{}^* > p_2{}^*$, thus proving part (c), QED.

The last step is now to determine whether there is an optimal level of debt D, for which we maximize (15) taking also into account (16) and the optimally determined values $u_{1D}{}^*$ and $u_{2D}{}^*$. This allows us to derive the market equilibrium for this case.

Proposition 5: Under the conditions of Propositions 3 and 4 and if the cost of quality is independent of the quality level then the optimal capital structure of firm 2 contains a positive amount of debt. It may contain positive amounts of both debt and equity or it may consist of 100% debt depending on parameter values.

Proof: From (18) and Propositions 3 and 4 we have the following relation:

$$u_{1D}{}^* - u_{2D}{}^* = \bar{u}X_1 / (\underline{a} + X_1) . \qquad (19)$$

Replacing (19) into (15), differentiating with respect to a_1 and taking into account from (16) that $\partial a_1/\partial D < 0$, we get:

$$\text{sign}(\partial V_{2D}/\partial D) = [\text{sign}(\partial a_1/\partial D)][\text{sign}[[(b + \mathbf{a_1})/3 - \mathbf{a}](-4(\underline{a} + X_1) + 2X_1) + X_1(\underline{a} + X_1)]] =$$

$$= - \text{sign}[[(b + \mathbf{a_1})/3 - \mathbf{a}](-4(\underline{a} + X_1) + 2X_1) + X_1(\underline{a} + X_1)]. \qquad (20)$$

For D = 0 we have $\mathbf{a_1} = \mathbf{a}$ and $(b + \mathbf{a_1})/3 - \mathbf{a} = X_1 = X$, and the expression in (20) becomes clearly positive, implying that the optimal amount of debt is positive. For an all-debt structure, we have $\mathbf{a_1} = \underline{a}$, and the sign of the last expression in (20) can be >0 or < 0, corresponding respectively to a structure with both debt and equity and to an all-debt structure, QED.

Although the optimal capital structure in Proposition 5 is also parameter-dependent as in Proposition 1, the two cases yield a different optimal structure, implying that the order of decision-making in choosing structure or quality first is an important

determinant of market equilibrium. This appears clearly in the following numerical example: the parameter a is distributed uniformly in the interval $[\underline{a}, \bar{a}]$, $\bar{a} = 1.8\underline{a}$, $b = 3.8\underline{a}$, implying that $a = 1.4\underline{a}$. Then under the conditions of Proposition 1 the optimal structure is at the value at which $a_1 = (6a – b)/4 = 1.15\underline{a}$, implying that the structure is part equity and part debt. On the other hand, the condition (19) for $a_1 = \underline{a}$ has $X_1 = 0.6\underline{a}$ and $(b+\underline{a})/3 = 1.6\underline{a}$, which yield:

$$[(b + a_1)/3 – a](-4(\underline{a} + X_1) + 2X_1) + X_1(\underline{a} + X_1)] = -.08\underline{a}^2.$$

This means that under the conditions of Proposition 5 $\partial V_{2D}/\partial D > 0$ at $a_1 = \underline{a}$, and the optimal structure is 100% debt in this case.

Nonetheless, both these cases of uncertainty in the taste parameter's lower limit yield qualitatively similar results. In both cases the use of debt has definite advantages for the lower quality firm. Although debt reduces the range of values over which equity holders realize positive returns, it also increases the size of the equity returns whenever the latter occur. The positive effect is sufficiently strong to favor the use of debt under all circumstances, even though the strategic advantages conferred by debt are clearly dependent on the order of decision-making.

An important consideration in examining the effects of leverage is the impact of leverage on the total value of the two firms. The following result will also be used in assessing the impact of leverage under free entry conditions.

Proposition 6: If the cost of quality is independent of quality level then under the conditions of both Propositions 1 and 5 leverage increases the value of the lower quality firm and decreases the value of the higher quality firm.

Proof: If the cost of quality F is independent of quality level then we have $u_1^* = u_{1D}^* = \bar{u}$ and $u_{2D}^* = X_1(u_{1D}^* - u_{2D}^*)/a = X_1(\bar{u} - u_{2D}^*)/\underline{a}$ from (18). Solving for u_{2D}^* and for $\bar{u} – u_{2D}^*$, we get $u_{2D}^* = \bar{u}X_1/(\underline{a} + X_1)$, $\bar{u} – u_{2D}^* = \bar{u}\underline{a}/(\underline{a} + X_1)$. Replacing this last expression into (13) we find that the two firm values R_{1D}^* and V_{2D}^* are respectively proportional to $Y_1^2/(\underline{a} + X_1)$ and $X_1[(b+a_1)/3 – a]/(\underline{a} + X_1)$. Differentiating the first one with respect to a_1 we find a positive sign for the derivative, implying that R_{1D}^* is larger when $a_1 = a$ (no debt for firm 2). Differentiating similarly V_{2D}^* we find that the sign of the derivative is equal to,

$\text{sign}\{(X_1 + \underline{a})[-2[(b+a_1)/3 – a]/3 + X_1/3] + 2[(b+a_1)/3 – a]X_1/3\} =$
$=\text{sign}\{-2[(b+a_1)/3 – a]\underline{a} + X_1^2 + X_1\underline{a}\}.$

At $a_1 = a$ this sign is negative, implying that taking on debt increases the firm 2 value, QED.

When the cost of quality depends on the quality level one or both statements of Proposition 6 may still hold depending on the slope of the function F(u). For instance, if the slope F' increases sharply around u_1^* then u_{1D}^* is "close" to u_1^* and the value of firm 1 still decreases with leverage; the firm 2 value may, however, decline with leverage since the higher quality u_{2D}^* increases the cost of quality. The leverage effects are reversed when F' increases slowly around u_1^*.

We close this section by examining the welfare consequences of the use of debt in the firms' capital structure. Propositions 1 to 5 have some important implications for the welfare analysis of the duopoly, expressed by the combined consumer and producer surplus. In the vertically differentiated market modeled in this paper

aggregate welfare given the qualities produced remains more or less constant whenever prices change[12], with such changes representing transfers from producers to consumers or vice versa. For this reason it is very simple to prove the following result.

Proposition 7: In the presence of uncertainty in the lower limit of the consumer taste parameter range and under the conditions of Propositions 1 and 5 aggregate welfare gross of quality costs[13] *increases* for all realizations of the random lower limit if the lower quality firm 2 is allowed to use debt in its capital structure. Consumer surplus, on the other hand, increases unequivocally for all those purchasing the lower quality but the effect of firm 2 debt on consumer surplus for the customers of the higher quality firm is ambiguous.

Proof: Aggregate welfare or total surplus in this model for a consumer with taste parameter equal to t is equal to $u_i t$, while consumer surplus is equal to $u_i t - p_i$, i = 1,2. Hence, aggregate welfare W when the lower limit on the taste parameter is equal to a is given by:

$$W = \int_{(b+a_1)/3}^{b} u_1^* t \, dt + \int_{a}^{(b+a_1)/3} u_2^* t \, dt. \qquad (21)$$

From Propositions 2 and 4 we know that both quality levels (u_1^*, u_2^*) increase under leverage, while the integration limit is $(b+\mathbf{a})/3$ and $(b+\mathbf{a_1})/3$ respectively without and with leverage, with the former exceeding the latter. Hence, W increases under leverage for all values of the lower limit a of the consumer taste parameter. On the other hand, aggregate consumer surplus is found by replacing the integrands in (21) by $(u_i^* t - p_i^*)$ for i = 1 and 2 in the first and second integrand respectively. From Propositions 2, 3 and 4 we know that in all cases $u_{2D}^* = p_{2D}^*/\underline{a}$, and similarly that $p_{2D}^* > p_2^*$. Hence, consumer welfare is $p_{2D}^*(t/\underline{a} - 1)$ under leverage and $p_2^*(t/\underline{a} - 1)$ without leverage for every consumer who purchases low quality with and without leverage, while for those consumers in the interval $[(b+\mathbf{a_1})/3, (b+\mathbf{a})/3]$ who switch to the high quality firm under leverage consumer surplus similarly increases, since otherwise they would have stayed with the lower quality. On the other hand, there are cases in which the customers of the high quality firm see their consumer surplus reduced as a result of the adoption of leverage by the lower quality firm. Indeed, assume that leverage increases p_{1D}^* above p_1^* under the conditions of Proposition 3, in which the optimal quality level u_1^* remains equal to the highest technologically feasible level $\underline{u}$ for both levered and unlevered firm 2 cases. In such a case every firm 1 customer will have lower consumer surplus under firm 2 leverage than under its absence, QED.

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[12] The only change may come from the migration of consumers from one to the other quality because the marginal consumer t_d changes when prices change.
[13] The statement in the proposition also holds net of quality costs if the cost of quality is independent of quality level.

V. The Effect of Uncertainty in the Upper Limit of the Taste Parameter

Consider now uncertainty in the upper limit b of the taste parameter, which is assumed to vary in an interval $[\underline{b}, \bar{b}]$, with mean **b**. The assumption of a natural duopoly for all b implies that $\underline{b} > 2a$ and $\bar{b} < 4a$, implying $\bar{b} < 2\underline{b}$. As with the case of the previous section, relations (3)-(6) hold true for the all-equity firm, with the mean value **b** replacing b everywhere.

Although the analysis of the optimal financial structure in this case is very similar to that of the previous section, the results are qualitatively different insofar as the all-equity case appears as an optimal choice of capital structure whenever structure is chosen before qualities, while a positive amount of debt is optimal when qualities are chosen before structure. In other words, a change in the sequence of decision-making is sufficient to nullify the strategic advantages of debt, even though prices are strategic complements and qualities have similar strategic properties as when there is uncertainty in the lower limit of consumer tastes.

Let $b_1 \in (\underline{b}, \bar{b}]$ denote the value of the consumer taste parameter for which the firm 1 revenue is just enough to cover the debt payments D, with default (solvency) taking place for b < (>) b_1. The following relation is the counterpart of (8) in defining b_1:

$$p_1(b_1 - t_d) - F_1 = p_1[b_1 - (p_1 - p_2)/(u_1 - u_2)] - F_1 = D. \tag{22}$$

Here the firm 2 revenue R_2 is given by (3) and is non-random given prices and qualities. The value of the equity E_1 is 0 for $b \leq b_1$, while for $b > b_1$ it is found by subtracting (22) from (3). Taking expectations and maximizing with respect to p_1 and p_2 we get the following expressions, which form counterparts of (13)-(15) and (17):

$$R_{2D}{}^* = [(\mathbf{b_1}-2a)/3]^2(u_1 - u_2) - F_2, \tag{23}$$

$$E_1 = (2\mathbf{b_1}-a)(u_1 - u_2)[1 - G(b_1)](\mathbf{b_1} - b_1)/3, \tag{24}$$

$$V_{1D} = (2\mathbf{b_1}-a)(u_1 - u_2)[\mathbf{b} - (\mathbf{b_1} + a)/3]/3 - F_1, \tag{25}$$

$$(2\mathbf{b_1}-a)(u_1 - u_2)[b_1 - (\mathbf{b_1} + a)/3]/3 - F_1 = D, \tag{26}$$

where the quantity $\mathbf{b_1}$ is the conditional expectation of b given that it is greater than b_1, or $\mathbf{b_1} \equiv E[b \mid b \geq b_1]$. We shall adopt the assumption that the distribution of b is such that this conditional expectation increases more slowly than b_1 when b_1 increases, or that $\partial \mathbf{b_1}/\partial b_1 \leq 1$. This technical assumption is innocuous in most cases, but it does simplify the proofs of some of the results. In the benchmark uniform distribution case this derivative is equal to ½.

Relation (4) now holds for the optimal prices $p_{1D}{}^*$ and $p_{2D}{}^*$, with $\mathbf{b_1}$ replacing b; a similar replacement in (5) yields the optimal firm 2 revenue $R_2{}^*$. We distinguish two cases, depending on whether qualities or financial structure is chosen first. If qualities are chosen first (case (ii) of Figure 1) we maximize (25) with respect to D for given u_1 and u_2, taking into account (26). We then have the following result.

Proposition 8: If there is uncertainty in the upper limit of the consumer taste parameter and if the higher quality firm chooses its qualities before it chooses its financial structure then the optimal financial structure would always contain a positive amount of debt. The optimal financial structure is part equity and part debt if a $b_1 \in [\underline{b}, \bar{b}]$ exists satisfying the relation $2[\mathbf{b} - 2\mathbf{b_1}/3] - a/3 = 0$; otherwise the optimal structure is 100% debt.

Proof: From (26) it is easy to see that the assumption that $\partial \mathbf{b_1}/\partial b_1 \leq 1$ guarantees that $\partial b_1/\partial D > 0$. Hence, sign($\partial V_{1D}/\partial D$) = sign($\partial V_{1D}/\partial b_1$) = sign($\partial V_{1D}/\partial \mathbf{b_1}$), and differentiating (25) we note that this latter sign is also that of the expression,

$$2[\mathbf{b} - (\mathbf{b_1} \, \partial F/\partial u_1 + a)/3] - (2\mathbf{b_1}-a)/3 = 2[\mathbf{b} - 2\mathbf{b_1}/3] - a/3.$$

For the all-equity firm $b_1 = \underline{b}$ and the sign of the expression is positive, while the expression decreases as b_1 increases, implying that an optimal structure exists if $2[\mathbf{b} - 2\mathbf{b_1}/3] - a/3 = 0$ at some $b_1 < \bar{b}$, QED.

For the benchmark case of a uniformly distributed b we have $\mathbf{b} = (\underline{b} + \bar{b})/2$ and $\mathbf{b_1} = (b_1 + \bar{b})/2$. For the all-debt firm we have $b_1 = \bar{b} = \bar{\mathbf{b_1}}$, and the restriction $\bar{b} < 2\underline{b}$ implies that the expression $2[\mathbf{b} - 2\bar{b}/3] - a/3$ is positive and an all-debt structure is optimal.

The next result is similar to Proposition 2 of the previous section.

Proposition 9: Under the conditions and at the optimal capital structure of Proposition 8 the following industry equilibrium exists:

   a) The optimal quality levels $u_{1D}*$ and $u_{2D}*$ are higher[14] under leverage than the all-equity levels u_1* and u_2*;
   b) The corresponding optimal price levels $p_{1D}*$ and $p_{2D}*$ are similarly higher than the all-equity levels p_2* and p_1*;
   c) The *relative* quality level $u_{1D}*/u_{2D}*$ of the optimal quality choices of firms 1 and 2 under leverage is lower than that same ratio u_1*/u_2* in the absence of leverage.

Proof: The proof will only be sketched since it follows along similar lines to that of Proposition 2. u_1* is found from the solution of the equation $[(2\mathbf{b} - a)/3]^2 = \partial F/\partial u_1$, while from (25) $u_{1D}*$ solves the equation $(2\mathbf{b_1}-a)[\mathbf{b} - (\mathbf{b_1} + a)/3]/3 = \partial F/\partial u_1$. From Proposition 7 the left-hand-side of the second equation can be shown to exceed that of the first one thus proving that $u_{1D}*> u_1*$. Since $u_{2D}*= p_{2D}*/a$ and $u_2* = p_2*/a$, replacing the optimal prices in (4) with $\mathbf{b_1}$ and $\mathbf{b}$ instead of b and solving for the optimal firm 2 quality in the levered and all-equity cases, we can show that $u_{2D}*> u_2*$, thus proving part (a) of the Proposition. Parts (b) and (c) also follow immediately from part (a) after substitution of the optimal qualities in (4).

Although the result in Proposition 9 appears broadly similar to that of Proposition 2, there are some important differences in the structure of the solution that have welfare consequences. Under uncertainty in the lower limit a as in Proposition 2 leverage reduces quality differentiation and expands the range of consumers buying the higher quality for all values of a, thus improving aggregate welfare gross of quality costs as

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[14] When the cost of quality is independent of the quality level we have $u_1* = u_{1D}*$.

shown in Proposition 7. When there is uncertainty in the upper limit b, on the other hand, the limits of integration in the aggregate welfare expression (21) become $(\mathbf{b} + a)/3$ and $(\mathbf{b_1} + a)/3$ for the all-equity and the levered firms respectively, implying that the range of consumers buying the lower quality expands under leverage. Proposition 7 may not, therefore, necessarily hold for this case, since some consumers switch to the lower quality under leverage; this effect will be demonstrated further on in this section. As for the consumer surplus, it increases for all those who used to purchase the lower quality when the firm was unlevered, but not necessarily for those who purchased the upper quality.

Next we consider the case in which financial structure is chosen before qualities. As in the previous section, we obtain unequivocal results by assuming that $F' = 0$, or that the cost of quality is independent of the quality level. We can then prove the following.

Proposition10: If there is uncertainty in the upper limit of the consumer taste parameter, if the higher quality firm chooses its financial structure before it chooses its quality level, and if the cost of quality is independent of quality level then the optimal financial structure for firm 1 is all-equity.

Proof: We maximize (24) with respect to u_1, with b_1 given by (26) for a fixed D. First we differentiate (26) with respect to u_1 and we observe that for a given D and a fixed F and for $\partial \mathbf{b_1}/\partial b_1 \le 1$ we must have $\partial b_1/\partial u_1 < 0$. Next we differentiate (24) with respect to u_1 and we find:

$$\text{sign}(\partial E_1/\partial u_1) = \text{sign}\{(2\mathbf{b_1} - a)(\mathbf{b_1} - b_1) + (u_1 - u_2)[2(\mathbf{b_1} - b_1)\partial \mathbf{b_1}/\partial u_1 - (2\mathbf{b_1} - a)\partial b_1/\partial u_1]\}. \quad (27)$$

The term in brackets in this last expression decreases if we replace $\partial \mathbf{b_1}/\partial u_1$ by $\partial b_1/\partial u_1$, yet the sign turns out to be positive, implying that it is optimal to increase u_1 till the maximum limit $\overline{u}$. On the other hand, (23) still implies that $u_{2D}{}^* = p_{2D}{}^*/a = (2\mathbf{b_1} - a)(u_1 - u_2)/3$. Replacing and solving, we find that:

$$u_{1D}{}^* - u_{2D}{}^* = 3a\ \overline{u}/(a + \mathbf{b_1}), \quad u_{2D}{}^* = \overline{u}(\mathbf{b_1} - 2a)/(\mathbf{b_1} + a). \quad (28)$$

Replacing into (25) and (26) we observe immediately that $\partial b_1/\partial D > 0$. It suffices, therefore, to maximize V_{1D} with respect to $\mathbf{b_1}$, since $\text{sign}(\partial b_1/\partial D) = \text{sign}(\partial \mathbf{b_1}/\partial D)$. From (25) we get:

$$\partial V_{1D}/\partial D \sim 2[b_1 - (\mathbf{b_1} + a)/3]/(\mathbf{b_1} + a) - (2\mathbf{b_1} - a)[b_1 - (\mathbf{b_1} + a)/3]/(\mathbf{b_1} + a)^2 - (2\mathbf{b_1} - a)/(3((\mathbf{b_1} + a)). \quad (29)$$

For $D = 0$ we have $b_1 = \underline{b}$ and $\mathbf{b_1} = \mathbf{b}$, and replacing into (29) we find that $\partial V_{1D}/\partial D > 0$ only if $\mathbf{b} < 2a$, which obviously does not hold. Hence the optimal financial structure is all-equity in this case, QED.

The dependence of industry equilibrium on the sequence of decision-making and the optimal leverage that it entails appears clearly in a comparison of the results of Propositions 9 and 10 for the case where the cost of quality is independent of quality level. In such a case $u_{1D}{}^* = u_1{}^* = \overline{u}$, and Proposition 10 yields a value,
$$u_2{}^* = (\mathbf{b} - 2a)\ \overline{u}/(\mathbf{b} + a),$$
while under Proposition 9 we have $u_{2D}{}^* = (\mathbf{b_1} - 2a)\ \overline{u}/(\mathbf{b_1} + a) > u_2{}^*$, and a similar relation holds for $p_{2D}{}^*$ and $p_2{}^*$. Aggregate welfare is given by (21) with limit of

integration (**b** + a)/3 or (**b₁** + a)/3 under Propositions 10 and 9 respectively. Replacing u_1^* and u_2^* for the all-equity Proposition 10 and the levered Proposition 9 cases we find that aggregate welfare is proportional to the expression:

$$[\mathbf{b}^2 - ((\mathbf{b}+a)/3)^2] + (\mathbf{b} - 2a)/(\mathbf{b} + a)[((\mathbf{b}+a)/3)^2 - a^2]. \qquad (30)$$

The derivative of (30) with respect to **b** is negative, implying that when **b** is replaced by its larger value **b₁** as in Proposition 9 aggregate welfare decreases. On the other hand, consumer surplus for any firm 2 consumer with taste parameter t is equal to $(\mathbf{b} - 2a) \, \bar{u} \, (t-a)/(\mathbf{b} + a)$, which increases when **b** is replaced by **b₁** as in Proposition 9, while the surplus for any firm 1 consumer with parameter t is $\bar{u} \, [t - ((2\mathbf{b}-a)/(\mathbf{b} + a))]$, which decreases[15] when **b** is replaced by **b₁**. Hence, the order of decision-making in choosing structure or quality first brings important changes to both total and consumer surplus.

Proposition 10 should also be contrasted to Proposition 5 of the previous section, in which the order of decision-making is the same but uncertainty exists in the lower, rather than in the upper limit of the consumer taste parameter. In the former case leverage is profitable and, as Proposition 4 shows, it results in a decrease in differentiation and an increase in aggregate welfare. In the latter case leverage is unprofitable. The difference stems from the fundamental asymmetry of the strategic interactions between the two firms and from the fact that uncertainty affects only one of the two firms whenever it exists in only one of the two limits of the consumer taste range. In the presence of leverage the strategic interactions with respect to quality choice are represented by the signs of the derivatives $\partial[\partial R_{jD}/\partial u_j]/\partial u_i$ and $\partial[\partial E_i/\partial u_i]/\partial u_j$ for i = 2 and j = 1 (i = 1 and j = 2) when there is uncertainty in the lower (upper) limit of the taste parameter, where R_{jD} and E_i are given by (13) and (14) (by (23) and (24)) respectively. The derivations must also take into account the dependence on leverage through (17) (through (26). In general the signs of these derivatives are parameter-dependent, implying that the nature of the interactions is hard to ascertain.

VI. Generalizations and Extensions: The Role of Free Entry

Uncertainty and leverage also affect the market equilibrium that emerges when free entry of competitors is allowed. In such a case the vulnerable firm is the lower quality firm 2, since the natural duopoly assumption guarantees that only the two highest qualities can obtain a positive market share. If quality choice is irreversible, and if the cost of quality is sunk once quality choice is made, the low quality firm 2 must locate at a position that would preclude any subsequent entry of a higher quality firm able to realize positive profits. This generally implies that firm 2 must locate sufficiently close to firm 1 to reduce its own profits to zero[16].

We model free entry in this natural duopoly market by assuming that there are a large number n of potential entrants producing substitute goods.. These firms play the following (n+2)-stage game: in the first n stages each firm, in a predetermined order

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[15] The consumer surplus also decreases for those consumers who switch from firm 1 to firm 2 when **b₁** replaces **b** in the lower limit of integration of the first integral in (21).
[16] See Hung and Schmitt (1992) and Constantatos and Perrakis (1997, 1999).

given by its index, makes an irreversible entry decision, which can be either a choice of quality or a choice of structure. In the next stage the second irreversible decision (structure or quality) is made, and the possibility of non-entry is represented by the choice of a quality level u=0 at the stage at which quality is chosen. In the last stage those firms with u>0 decide whether to produce or to withdraw their product from the market; the withdrawal of those firms whose market share is going to be zero is ensured by introducing a "small" marketing cost ε that firms must pay before starting production.

In such an entry game the subgame perfect equilibrium of n quality-price pairs ends up with only the two top qualities having positive market shares since the market is a natural duopoly by our assumptions about the range of consumer taste parameters. Under general conditions about the cost of quality function F(u) the outcome of the entry game is quite complicated because of the possibility of "leapfrogging" by an entrant who may enter the market with a quality level higher than u_1. Firm 1 must, therefore, take this into account in its own quality choice, as it must also take into account the firm 2 quality choice which wants to avoid being displaced by a higher quality entrant[17]. Since our purpose is to investigate the effects of financial structure on industry equilibrium under free entry, we shall abstract from these complicating features of the entry process by assuming that the quality cost F is independent of quality level, implying that firm 1 will always locate its quality at the technological upper limit $\bar{u}$ as in Propositions 4 and 10.

With this assumption free entry only affects the lower quality firm 2, which must now locate sufficiently close to $\bar{u}$ in order to make any further entry unprofitable. Since leverage, when it is profitable, affects the value of firm 2, it would also have an impact on the firm 2 entry-deterring quality choice. Our main interest in this section is to ascertain whether the important results of the previous sections on the impact of leverage, increased quality level and reduction of differentiation, are still preserved in the presence of free entry.

The simplest case is when there is uncertainty in the lower limit a of the consumer taste parameter range and qualities are chosen before financial structure. In such a case Proposition 1 shows that there exists an optimal degree of leverage that determines a critical value a_1 of the taste parameter, independent of quality levels, which determines the value V_{2D} of firm 2 under leverage in (15). Since V_{2D} is a monotone decreasing function of the quality level u_2, this level must be increased sufficiently under free entry so that V_{2D} would become zero. Indeed, suppose that u_2 were chosen at a lower level, at which V_{2D} was positive. Then an entrant firm would be able to locate at a marginally higher level of u_2, in which by choosing the same optimal leverage given by Proposition 1 it would manage to realize a nonnegative market value, while the incumbent firm 2 market share would decline to zero.

Let the superscript e denote the values of the superscripted variables under free entry and the ensuing firm 2 entry-deterring policy. In the absence of leverage the quality u_2^e is chosen by firm 2 by setting the revenue $R_2^e = (b - 2a)^2(\bar{u} - u_2^e)/3 - F$ equal to zero. When there is leverage it is the value $V_{2D}^e = (b-2a_1)(u_1 - u_{2D}^e)[(b + a_1)/3 - a]/3 - F$ that is set equal to zero. In both cases entry deterrence increases quality level u_2 and decreases both prices p_1 and p_2 as

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[17] For a full analysis of the entry process see Constantatos and Perrakis (1999).

given by (4) with **a** or $\mathbf{a_1}$ replacing a. Of more interest, however, is the comparison between the entry-deterring industry equilibrium with and without leverage. From the proof of Proposition 1 it is clear that for $u_2^e = u_{2D}^e$ we have $R_2^e < V_{2D}^e$, implying that for entry deterrence we should have $\underline{u}_{2D}^e > u_2^e$. On the other hand the comparison between the prices $p_2^e = (b - 2\mathbf{a})(\overline{u} - u_2^e)/3$ and $p_{2D}^e = (b - 2\mathbf{a_1})(\overline{u} - u_{2D}^e)/3$ shows that $p_{2D}^e > p_2^e$; similarly, it can be shown that $p_{1D}^e > p_2^e$ as well. Hence, the results of Proposition 2 are robust in the presence of entry.

The same conclusions also hold in the case of free entry when there is uncertainty in the lower taste limit a but financial structure is chosen before quality, as in Propositions 3-5. Indeed, our analysis of this case in section IV took also place under the assumption that the cost of quality is independent of quality level, implying that the same entry-deterring levels u_2^e and u_{2D}^e as in the previous case are also chosen here, even though the optimal financial structure and the value $\mathbf{a_1}$ may be different. Last, we note that the results of Proposition 7 are also unaffected by the possibility of free entry: total surplus obviously increases under leverage; a simple calculation shows that consumer surplus increases for every firm 2 consumer under leverage when the entry-deterring quality level is used instead of u_2^* in (21); consumer surplus decreases for every firm 1 consumer, given that the optimal quality choice of firm 1 is by assumption the same with and without leverage, while the price increases under leverage. In other words the threat of entry does not affect the conclusions as to the effect of leverage on industry equilibrium when there is uncertainty in the lower taste parameter limit.

By contrast the market equilibrium under leverage and threat of entry has a different outcome from the no entry case when there is uncertainty in the upper limit b of the taste parameter and qualities are chosen before structure. In this case the quality level u_{2D}^e is found by setting $R_{2D}^* = 0$ in (23) for $u_1 = \overline{u}$, $F_2 = F$ and $\mathbf{b_1}$ defined as in Proposition 8, while for u_2^e it is $\mathbf{b}$ that must be used instead of $\mathbf{b_1}$. Since $\mathbf{b_1} > \mathbf{b}$, it is immediately obvious that u_{2D}^e must be greater than u_2^e in order to set the firm 2 profit equal to zero. Here, however, $p_{2D}^e = 3F/(\mathbf{b_1} - 2a)$ and $p_2^e = 3F/(\mathbf{b} - 2a)$, and it is clear than $p_{2D}^e < p_2^e$, unlike the case where a was uncertain; a similar calculation also shows that $p_{1D}^e < p_1^e$. In this case, therefore, consumer surplus is unequivocally higher under leverage for every consumer. Similarly, total surplus can be calculated from (21) by setting $(\mathbf{b_1} + a)/3$ as the limit of integration and $u_{2D}^e = F/[(\mathbf{b_1} - 2a)/3]^2$; the calculation shows that total surplus increases as well when $\mathbf{b_1}$ is replaced by $\mathbf{b}$. Hence, leverage is unequivocally welfare-enhancing under free entry when there is uncertainty in the upper limit of the taste parameter b.

VII. Conclusions

We have examined the choice of financial structure in industries in which products are differentiated by quality and there is unanimity of consumer preferences when prices are set equal to average variable costs. In such industries the fundamental asymmetry of the two firms that can survive in equilibrium with positive market shares also produces different results depending on whether uncertainty is introduced in the

lower or the upper limit of the consumer taste distribution. In both cases, however, there are instances where the use of leverage is optimal and has an important impact on the industry equilibrium.

Our main findings are that equilibrium is also crucially dependent on the order of decision-making with respect to the quality or the financial structure choice. In all cases, however, leverage increases the level of the lower quality, and in most cases it also increases the price of the lower quality as well. This has important welfare implications, generally raising both total and consumer surplus. These results are robust with respect to assumptions about restricted or free entry and the adoption of entry deterrence by the lower quality firm.

In our analysis we have chosen the parameters in order to maintain the natural duopoly structure, and we have examined separately uncertainty in the lower or the upper limit of the consumer taste distribution. None of the conclusions is expected to be affected under restricted entry if the market becomes a monopoly or can accommodate a third firm under some values of the random parameters. The market would remain a natural duopoly as long as the expected profit of the lower quality firm is positive, and provided the expected profit of a third firm is negative. Only the functional forms would change for some values of the random parameter, since the lower quality firm may have zero sales or leave the market uncovered. Similarly, the existence of simultaneous uncertainty in both the upper and the lower limit of consumer taste distribution would complicate the analysis but leave the basic insights unchanged, since the impact of leverage has similar effects in both cases.

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