

AME

F



# Times-to-Default: Life Cycle, Global and Industry Cycle Impacts

F. Couderc<sup>\*</sup>, O.Renault<sup>†</sup>

First Version: 10/02/2003

This Versio  $n^{\ddagger} : 02/09/2005$ 

### ABSTRACT

This paper studies times-to-default of individual firms across risk classes. Using Standard & Poor's ratings database we investigate common drivers of default probabilities and address two shortcomings of many papers in the credit literature. First, we identify relevant determinants of default intensities using business cycle and credit market proxies in addition to financial markets indicators, and reveal the time-span of their impacts. We show that misspecifications of financial based factor models are largely corrected by non financial information. Second, we show that past economic conditions are of prime importance in explaining probability changes: current shocks and long term trends jointly determine default probabilities. Finally, we exhibit industry contagion indicators which might be helpful to capture leading and persistency patterns of the default cycle.

JEL Classification: C14, C41, G20, G33.

Keywords: censored durations, proportional hazard, business cycle, credit cycle, default determinants, default prediction.

<sup>&</sup>lt;sup>\*</sup> FAME & University of Geneva.

<sup>&</sup>lt;sup>†</sup> Associate Fellow, FERC, Warwick Business School, Coventry CV4 7AL, UK.

<sup>&</sup>lt;sup>‡</sup> The authors gratefully acknowledges data support by Standard & Poor's. In addition the first author thanks financial support from the Swiss National Science Foundation through the National Center of Competence: Financial Valuation and Risk Management (NCCR FINRISK) and from the Geneva Research Collaboration Foundation (GRC). We thank Arnaud de Servigny, Olivier Scaillet, René Stulz, Laurent Barras, anonymous participants to the 2004 GRETA Credit conference and members of Standard & Poor's European academic panel for useful comments. The views expressed therein are those of the authors but not necessarily those of Standard & Poor's or any other institution. All remaining errors are ours.

# Introduction

The academic literature on credit risk has historically been more focused on the pricing of credit risk rather than on risk management issues. As pricing relies on the risk neutral measure researchers concentrate on modelling the dynamics of credit quantities (in particular default probabilities and spreads) using stock and bond markets factors (e.g. Duffee (1999) or Driessen (2002)). The approach has theoretical grounds both from structural models (firmvalue based) and reduced-form models (intensity based). The former are closely linked to financial markets as they model default as the first time at which firm assets value fall below liabilities. The intensity based methodology is more convenient for pricing and provides simple extensions to multiple assets (e.g. see Duffie and Singleton (2003)), but ignores what the default mechanisms are. Recent works have tried to bridge the gap between the two approaches by keeping the flexibility of intensity models but introducing default determinants (e.g. Duffie & Wang (2004)).

However, some short cuts have been made in the understanding of the default process. More precisely structural models insights have not been fully exploited. A first series of caveats lie in the source of information used to predict credit riskiness. Changes in default probabilities have been reduced as consequences of financial markets movements. This might explain the reported weak explanatory performance of proposed common factors on credit spread variations. Meanwhile analyses demonstrate that common factors should account for the largest part of observed deformations (e.g. see Collin-Dufresne, Goldstein & Martin (2001)). Yet business conditions, or more generally the business cycle, have strong implications on firm assets values. This part of the global economy cannot be left aside without empirical justifications. In the same way, observed patterns of the default cycle have not been encompassed. It is smoother than the financial markets changes and more persistent after economic crisis. Such particular features have to be endogenously extracted from credit markets and from the rating process. A second important source of misspecification resides in the use of contemporaneous explanatory variables. Past information should arguably convey strong explanatory power on the behaviour of realized default probabilities. This is motivated by the fact that in a large majority defaults do not arise suddenly but are rather the conclusion of a long lasting process. Some structural models already take advantage of this observation requiring the liabilities barrier to be crossed by assets value several times or during a defined amount of time before they signal the default (e.g. François & Morellec (2004) or Moraux (2004)).

Obviously such issues are of prime importance for risk management. It calls for knowledge and ideally forecasts of realized default probabilities, which translates into a deep knowledge of their determinants. Remark however that the comprehension of risk-neutral credit risk is not a substitute for historical default probabilities. On the one hand, to filter out market assessment of default probabilities from prices, we need specific assumptions on liquidity or/and recovery risk. On the other hand, the adequate change of measure required to recover default behaviour under the historical measure is still unknown<sup>1</sup> and the practical implementation appears to be a matter of adhoc adjustments. Yet historical default probabilities constitute critical inputs of popular commercial credit risk portfolio models. Even for pricing purposes, information

<sup>&</sup>lt;sup>1</sup>Note that Jarrow Lando & Yu (2005) provided a theoretical framework in that direction.

arising from historical probabilities are relevant for defaultable asset pricing models such as the Jarrow Lando & Turnbull (1997) specification as well as for hedging as shown by Bielecki, Jeanblanc & Rutkowski (2004). Moreover as shown by Fledelius, Lando & Nielsen (2004) and Couderc (2004), the historical measure allows for a finer analysis of the default drivers. Indeed calendar time effects can easily be separated out from duration or "life cycle" effects on the default probabilities deformations.

Using the intensity-based technology to estimate historical default probabilities on Standard & Poor's ratings database, this paper largely extends previous analyses by putting more structure on the intensity model. It provides new insights into the determinants of intensities. More precisely we perform a detailed analysis of explanatory variables through parametric and semi-parametric factor models. Parametric models let us identify marginal impacts of the credit markets, the business cycle and the financial market. The semi-parametric models, accommodating possible "life cycle" effects (i.e. changes in default probabilities arising from the "age" of a company or from the time it has spent in a given risk class)<sup>2</sup> are particularly convenient diagnostic tools. They allow us to check misspecifications of multifactor financial market based models, and to assess the benefits of considering other identified predictors.

The contributions of the paper are fourfold. First, we provide extensive empirical analyses of the behaviour of default probabilities conditioning on financial markets, business cycle and credit market indicators. We explore each driver of default and its horizons, showing that all these economic components contribute to the default likelihood over the following three to five years. Second, we test the appropriateness of factor models relying solely on financial information. We show that it leads to underevaluate default peaks and to overshoot probabilities during stable and low-default periods. But including business and endogenous information largely reduces these errors and greatly increases changes in probabilities explained by common factors. Third, we exhibit the critical importance of past information because it both captures economic trends and lead-lag effects between the economy and the default cycle. Past covariates partly take into account the lower speed and higher pesistence of the default cycle. Finally, exploring other potential sources of endogenous information from industries, we prove that some industries are forerunner of the global business cycle whereas others just suffer from its consequences. This is achieved using autoregressive models that are more traditionally applied to high frequency trade data. Thus industrial default cycles appear as good candidates to characterize and forecast more accurately the behaviour of default probabilities.

The paper is organised as follows. In the first Section we briefly present the ratings data and discuss potential default drivers included in our analyses. Section 2 studies sensitivities of the default cycle with respect to financial markets, business and credit indicators through conditional single factor models of intensities. It provides a more thorough analysis of the time span of covariates' impacts and tests possible "life cycle" effects. A semi-parametric setting is proposed in Section 3 in order to evaluate strengths and weaknesses of factor models according

 $<sup>^{2}</sup>$ In retail credit both age and calendar time effects are routinely used, but the literature on corporate credit often ignores those effects without empirical support. Furthermore emark that duration frameworks and more particularly Cox-type semi-parametric specifications such as that proposed in this study have already been used in finance. For instance Lunde, Timmermann & Blake (1999) applied the methodology to study the performance of mutual funds.

to the information they use. We then proceed with the assessment of market driven multifactor models and show that business and credit components are essential additional determinants of default. In addition we demonstrate that economic trends and past information play a crucial role in explaining default. In Section 4 we discuss exploratory issues. In particular thanks to parsimonious ACD models we show strong differences between industrial default cycles which should prove useful to further explain default riskiness. Our results deliver guidelines for future research, and should be meaningful for the broad class of models that are used by practitioners as well as for the specification of reduced form pricing models. They ultimately offer analyses of predictive variables of future default probabilities while keeping in mind the history of companies in the rating process.

# I. Potential Determinants of Default

### A. Ratings and Duration Data

Ratings allow to classify firms into "homogenous" classes of default risk, the default being the ultimate grade which can be attributed. Our ratings data was extracted from Standard & Poor's Credit Pro 6.6 database. This database contains S&P's rating histories for 10439 companies over the period January 1981 to December 2003. Overall 33044 rating migrations are recorded in CreditPro as well as 1386 defaults and default rate ranges from 3% to 29% across industries. Remark that credit reviews concluding to no changes in rating are not reported in the database. Such data could indicate whether agencies revise their credit risk assessment more frequently during specific part of the business cycle or not. This could induce a bias in our findings as it would lead to sharper decrease/increase in estimated default intensities during peaks and throughs. However S&P argues that ratings are reviewed on a regular basis, and more frequently if substantial new information arises. Furthermore a rating is only modified when the likelihood of default changes significantly and if this change is not purely transitory. Therefore this should not constitute a restriction in our case. The Credit Pro database has already been used and extensively described by Bangia & al. (2002) over the period January 1981 to December 1998.

Rating events require careful treatment as three sources of censoring are present in the database. Left truncation arises from the fact that 1371 issuers had already received a rating before they were included in the database (i.e. before January 1981). We do not have information about the attribution date of their first rating and therefore for robustness checks we run all estimations both on the full sample and on the reduced sample excluding left-censored data (the reduced sample contains 9068 companies and 25993 rating migrations). Obviously, a first type of right censoring is also an inherent feature of any ratings database as most companies survive after the end of the recordings. Another specific type of right censoring requires specific consideration. Some companies leave the ratings process and fall into the not-rated (NR) category. Several reasons may explain this fact: the rated company may be acquired by another firm or may simply decide no longer to be rated by S&P. The database has the nice feature to identify firms that migrated to NR and subsequently defaulted. Therefore the NR

class is not a complete loss of information: although there is no longer any indication of credit quality, a NR firm is a non defaulter.

Within our sample, firms are classified by industrial groups and each of them has been refined by subindustry criteria. Practically, we have at our disposal 13 industries or 526 distinct subindustries distributed among 93 countries. But 6897 firms or 66% are US ones. Moreover, S&P attributes 25 distinct ratings plus the NR one, but we aggregate the data coming from a grade and its plus/minus modifiers because of minimal population requirements. Besides, all grades below B- have been put in the CCC class. Let us notice that such a database allows to consider two types of durations, implying two different approaches to the behaviour of default probabilities. On the one hand we can look at times-to-default from entry in a risk class up to the last available observation. This perspective constitutes the primary goal of ratings : agencies use ratings to rank firms in cross-section with respect to their expected default probability. Thus splitting the sample of firms on the basis of ratings should lead to homogenous classes of risk. On the other hand, we can examine times-to-default conditional on staying in a given risk class up to the default time. By doing so, the emphasis is put on rating migrations giving stonger relevance to rating changes. Actually the difference lies in the way default probabilities are assumed to evolve. In the first case firms can change continuously their default probability. In the second case the default probability should remain constant within a rating class and jump when the rating is revised. The latter corresponds to the standard Markov chain assumption.

#### **B.** Default Drivers

In this section we present potential determinants of default intensities that will be used to calibrate log-linear models. Several authors calibrate mainstream models on financial variables : interest rates for reduced form models and equity information for structural models. For instance Duffee (1999), Driessen (2002) or Collin-Dufresne & al. (2001) examine impacts of selected financial covariates on credit spreads. The business cycle has also been factored in other papers (e.g. see Koopman & Lucas (2004)) whose primary focus is on the cyclicality of credit and macroeconomic variables. To our knowledge there exists no systematic study of the determinants of default including both financial and non financial variables. In addition all studies concentrate on contemporaneous market variables, and ignore lag effects. Default is typically reduced to a short term process. Keeping in mind these two points we investigate potential drivers from various sectors of the economy, i.e. from financial markets, from the business cycle and from the default cycle itself. We also distinguish specific credit cycle information from financial markets. Given that our rating and default sample is primarily American, we use US explanatory variables. Many of these variables are redundant and will be eliminated at the estimation stage in multi-factor analysis. Our data was extracted from the Federal Reserve of St. Louis website and Bloomberg.

#### **Financial Markets Information**

- Return on S&P500: Short and mid term economic performance should be positively correlated with S&P500's returns and we expect a negative impact on default intensities.
   Furthermore, an increase in equity prices tends to decrease firm leverage and therefore also push down default probabilities.
- Volatility of S&P500 returns: In a traditional Merton (1974)-type model, the two drivers of default probability are leverage and the volatility of firms' assets. The volatility of equity returns is often used as a proxy for the latter and we expect it to have a positive impact on default intensities. We use the realized annualized volatility computed over the last 60 trading days<sup>3</sup>.
- 10 year treasury yield: Higher interest rate levels imply higher cost of borrowing. Hence, this
  variable could impact positively on default probabilities. However interest rates tend to
  be lower in contraction periods and higher in expansions. Thus the ultimate impact on
  intensities is uncertain and may depend on issuer quality.
- Slope of term structure (10 year rate minus 1 year rate): Steep term structures of interest rates are usually associated with strong growth prospects and we expect this variable to impact negatively on mid- to long-term intensities.

# **Business Cycle**

We believe that it is crucial to extract information from the business cycle. If stocks were available for all firms and markets were fully efficient, financial markets and the business cycle might be redundant. As it is not the case, we include standard proxies of business health.

- Real GDP growth: As a signal of current macro-economic conditions this variable should be negatively correlated with short term probabilities.
- Industrial production growth: This is an alternative growth measure which should have a similar impact as that of GDP growth. Advantage lies in the more frequent update of these series.
- Personal Income Growth: Same expected impact as the previous two variables.
- CPI growth: Inflation is again a general indicator of economic conditions. We expect to observe a negative correlation with short term default probabilities, as high inflation has often been associated with growth.

<sup>3</sup>Using the historical volatility or an implied volatility from the VIX or the VXO could be an issue. We argue that implied volatility essentially contains information on the next transitory shock in financial market, but as our results reveal, trends are much more important in determining future changes in default probabilities. Furthermore, no implied volatility time series is available over our entire observation period starting 1981. Starting from 1987, unreported results from the VXO percentage changes prove that implied volatility has no explanatory power on default intensities.

### **Credit Market Information**

Beside general economic variables and financial information, more specific credit factors should prove valuable in explaining default intensities.

- Spread of long term BBB bonds over treasuries: Spreads should reflect the default probability as well as expected recoveries and a liquidity premium. It should therefore be positively correlated with default intensities.
- Spread of long term BBB bonds over AAA bonds: This variable factors in the risk aversion of investors and may be a measure of their risk forecast. It filters out mixed effects contained into the BBB spread. Furthermore, an increase in the relative spread may reflect an increase in firms' asset volatilities (see Prigent, Renault & Scaillet (2001)). We therefore expect default intensities to increase with this variable.
- Net issues of Treasury securities: This indicator should positively impact short term probabilities of default as higher deficit and borrowing is an indicator of economic difficulties (it is at least negatively correlated with the business cycle). Furthermore, high public sector borrowing may crowd out private borrowers and lead to increased financial difficulties for firms. However, if borrowing is used for investments, an increase in Treasury issuance may be linked to stronger growth in the long term and decreasing probabilities of default.
- Money lending (M2-M1) and bank credit growth: These factors measure credit liquidity and should be associated with default intensities. However it is well known that the information content of this indicator and more particularly of M2 has changed a lot on our period. In particular a series of adjustments have been done by the Federal Reserve. As a consequence this indicator cannot be conclusive on the short run, but its implications on the long run (more than one year) have turned out to be pretty stable. We thus expect clearer impacts when using the lag operator.

Our dataset includes both forward looking and current information. In particular, stock market components, CPI and personal income growth deliver snapshots of current global business conditions whereas interest rate-based measures also contain expectations of future economic conditions. Default is not a fully exogenous process but is often the result of renegotiations between the firm and its creditors. Good economic prospects should induce investors to renegotiate contracts rather than trigger liquidation. This should be reflected in the significance of some "forward looking" explanatory variables.

Before turning to regressions of intensities on the above variables, we run a principal component analysis (PCA) on this set of economic indicators to determine how many factors were necessary to explain most of the variations in intensities. Using the eigenvalue criteria, we found that five significant factors explain a cumulated percentage of 71% of the total variations captured by this information set. PCA analysis on non-parametric estimates of default intensities based on the Gamma kernel (rather than on the raw data) also suggests that four to five factors are relevant and account for 73% to 94% of the changes in intensities<sup>4</sup>. The relatively low explanatory power obtained with 5 factors in the PCA analysis indicates that it is unlikely that we will be able to explain more than 75% of variations in default probabilities using the set of variables presented above but we can expect to reach a higher figure than the 25% reported in empirical applications as Collin-Dufresne & al. (2001) for the credit cycle.

#### Inner Dynamics of the Default Cycle

A striking feature of the default cycle might not be captured by the above variables. After the last two recessions strong persistencies in default rates have been observed. The number of defaults remained high even during economic recoveries. The default cycle seems to exhibit its own dynamics. Thus we believe that the set of predictive variables should be expanded with default endogenous variables. Kavvathas (2000) used as explanatory variables the weighted log upgrade-downgrade ratio and the weighted average rating of new issuers. He actually only took into account the first PCA factor of these variables, but other variables may also be relevant. The average rating of financial institutions may be of primary interest in describing the short term trend of the global economy (in terms of credit crunch for instance). This trend can also be captured by the ratio of downgrades over all non-stayer transitions. As representative of the default cycle trend we choose to include the following rating-based variables:

- IG and NIG<sup>5</sup> upgrade rates : both variables should include information on economic health.
- IG and NIG downgrade rates : downgrades should be higher in bad conditions.

One may also want to add firm specific factors such as leverage, cash flows or size which constitute the main determinants of bankruptcy as pointed out by Lennox (1999). In particular, historically the size of the firms seems to induce very different behaviours. Moreover, these factors may be introduced at aggregate levels, for instance to provide a measure of the solvency of new issuers. Recently, Duffie & Wang (2004) used the earnings ratio and firm size as specific factors, jointly with a measure of the distance-to-default. However ratings should constitute stable and good proxies for firm-specific components and a fair alternative as specific variables are not always available. From an accounting perspective, default cannot realistically be initiated by small changes in earnings, leverage or any balance sheet information but rather by negative trends or by unexpected large changes in cash flows. Any negative trend should have been incorporated in issuer ratings. Furthermore as pointed out by Collin-Dufresne & al. (2001) who used leverage, idiosyncratic factors do not represent the dominant factor in credit risk changes and seem to exhibit lower explanatory power than common components.

#### [INSERT TABLE I HERE]

<sup>4</sup>The number of factors and cumulated explanatory power depend on the risk class considered, i.e. whether one uses the entire sample, or only non investment grades, BB, B or CCC firms. The non-parametric PCA inputs are estimated increments in intensity from the Gamma kernel intensity estimator (see Section III.A).

<sup>&</sup>lt;sup>5</sup>The investment grades class (IG) gathers the AAA, AA, A and BBB classes whereas BB, B and CCC classes are collected in the non investment grades (NIG) class.

Table I presents basic statistics on the set of retained factors. Obviously some of the above variables such as the real GDP growth and the industrial production growth are highly correlated, which would deteriorate statistical significance on the full set of variables. However our main purpose consists of identifying the relevant factors and their relationship with default probabilities. Therefore, we will concentrate on univariate and parsimonious multivariate analysis.

# II. Predictors and Indicators of the Default Cycle

The analysis of the impacts and explanatory powers of the potential determinants of default discussed above requires to define a basic framework. In finance a simple and traditional practice to study the effects of structural factors on stock returns consists of performing regressions of returns on the explanatory variables. Investigations of spreads follow the same approach. Instantaneous default probabilities (or intensities) are not directly observable, and we therefore propose to use an analogous technique on conditional distributions in order to examine sensitivities of default intensities on our information set. As intensities have to be positive, we define "regressions" of log-intensities<sup>6</sup>. These models are known as log-linear duration models. We briefly recall the basics of these models and then turn to estimations. We start with time-dependent covariates which embed the impact of successive shocks of the economic environment on intensities. We differentiate impacts of current and past conditions. Then we consider time-independent covariates which aim at capturing potential effects of initial conditions (e.g. the state of the economy at a firm's entry in the risk class). We refer to this last phenomenon as life cycle effects or time profiles since a factor is likely to modify the risk of default of firms over their whole life in a given risk class.

# A. Factor Models of Intensities

For a firm i, let  $D_i$  denote the uncensored duration up to default and  $C_i$  the censored duration.  $U_i = \min(C_i, D_i)$  is the time at which the firm leaves the class either because of censoring  $(C_i)$  or default  $(D_i)$ . The  $U_i$  are the true observations, jointly with indicators of censoring. We also let  $\mathbf{Z}$  denote a vector of explanatory variables. We consider intensities as exponential affine functions of factors which remain constant between two observations of the factors. Hence, conditional on the realization of the covariates, durations are piecewise exponential :

$$\lambda^{i}\left(u,t_{i}\right) = \exp\left(\gamma + \beta' \mathbf{Z}\left(t_{i}, u + t_{i}\right)\right) \qquad \forall i.$$

$$(1)$$

where  $\mathbf{Z}$  can include a mixture of time-dependent and time-independent covariates. The exponentiality assumption could be relaxed by replacing the constant  $\exp(\gamma)$  by another formulation. For instance one could impose a conditional Weibull hazard where the covariates may

<sup>&</sup>lt;sup>6</sup>Let us recall that continuous intensities fully characterize continuous distributions. Therefore, we cannot talk about regressions because intensities are not estimated in a first step and then log-regressed on independent variables. Actually factor models of intensities specify conditional distributions and are estimated in a single stage.

be time-dependent (through  $u + t_i$ ) and/or time-independent (through  $t_i$ ).

Our chosen parametric framework allows to use efficient and tractable estimation techniques for  $\hat{\beta}$ , namely maximum likelihood<sup>7</sup>. The standard estimation procedure works in the following way. Assuming that structural variables dynamics are independent, the likelihood is separable into two terms, one related to the dynamics of covariates and the other one dealing with conditional durations. Therefore if we are not interested in factors dynamics, we can ignore this part and focus purely on durations. For a given firm *i*, the likelihood *l* of observed duration  $u_i$  can be written conditionally on factors realizations at firm's "death or exit" but the whole construction of the risk classes information set has to be known :

$$l(u_i) = l_1\left(u_i \middle| \mathcal{F}_{t_i+u_i}^Z\right) \times l_2\left(\mathcal{F}_{t_i+u_i}^Z\right)$$

where  $l_1$  is the univariate likelihood of conditional durations and  $l_2$  the likelihood associated with the dynamics of covariates. From that point, letting  $L_1$  and  $L_2$  denote the multivariate counterparts of  $l_1$  and  $l_2$ , the multivariate likelihood function for a sample of n firms observed up to time  $t = \max_i \{t_i + u_i\}$  is defined by

$$L\left(u_{1},..,u_{n}\right) = L_{1}\left(u_{1},..,u_{n} \mid \mathcal{F}_{t}^{Z}\right) \times L_{2}\left(\mathcal{F}_{t}^{Z}\right)$$

with

$$L_1(u_1, ..., u_n | \mathcal{F}_t^Z) = \prod_{i=1}^n \exp\left(-\int_0^{u_i} \lambda^i(s, t_i) \, ds\right) \left(\mathbb{I}_{(d_i > c_i)} + \lambda^i(u_i, t_i) \, \mathbb{I}_{(d_i \le c_i)}\right)$$
(2)

where  $c_1, ..., c_n$  (resp.  $d_1, ..., d_n$ ) are realizations of censoring variables  $C_1, ..., C_n$  (resp. default durations  $D_1, ..., D_n$ ).

The estimation of this model is therefore quite straightforward and both censored and default durations contribute to the likelihood. The main task is the selection of appropriate explanatory variables. Empirical results on this specification are provided in next sections.

# B. Economic Shocks over Time and their Persistency

In this section, we determine whether intensities are sensitive to each factor identified previously. We also explore the necessity to lag factors to extract more information. Surprisingly, the issue of lagged information has been ignored in most papers, although Koopman & Lucas (2004) and Kwark (2002) have reported lagged effects between the market and the credit cycle. We analyse the explanatory power of each factor performing maximum likelihood estimations. For each covariate we run distinct lagged estimations to examine the persistency of its effects. In all cases, we look at 95% and 99% confidence tests, and likelihood ratio. The alternative model of the likelihood ratio test corresponds to unconditional exponentiality, i.e. the case of constant intensity. We also break our dataset into several samples, namely investment grade (IG), non investment grade (NIG), AA, A, BBB, BB, B and CCC samples. For each of these

<sup>&</sup>lt;sup>7</sup>See Duffie & Wang (2004) for a clear presentation of maximum likelihood in this case.

risk classes we look both at durations to first exits from the risk class and durations to last days of observation. Further robustness checks are performed by leaving out left truncated firms and, focusing on the US subsample.

Tables II, III and IV present results on the IG and NIG<sup>8</sup> samples respectively over financial, business and credit indicators. Table V reports estimates of sensitivities with respect to upgrade and downgrade rates over rating classes. We have found that the sample used makes little differences to the results. For example, considering durations up to the first exits keeps sensitivities almost unchanged and only lowers the significance of parameters. Focusing on the US subsample does not modify estimates more than 10% on average, and does not alter signs. Such robustness could be expected as risk classes are quite stable and the whole sample is made of 66% US firms. From a general perspective all variables are significant. In addition, these findings do not change across ratings. Nevertheless sensitivities have to be expounded with care. For a same level, with an average intercept around -13.1 for IG and -8.8 for NIG, the explanatory power is indeed weaker for IG. It is worth mentioning that due to the exponential specification, only signs, significance levels and likelihood ratios can provide insights, whereas possible differences in sensitivity levels across distinct samples cannot be interpreted. We observe that lagged covariates are significant at all stages too.

# [INSERT TABLE II HERE]

#### **Recent Economic Changes**

Looking carefully at signs for lags up to two years, we observe that the probability of default covaries with the expected signs. Financial markets impact default probabilities as predicted by structural models. Increases in the market index decrease the probability of default while increases in volatility push up probabilities. Increases in interest rates are good news because they reflect anticipation of growth. On the contrary decreases in short term yield increase default probabilities as low rates are strongly correlated with recessions. A steep contemporaneous or recent (less than one year) slope of the term structure of riskless rates tends to be associated with higher intensities of default while past steep slopes (over one year lags) tend to decrease intensities. The only exception to this short term/long term split is for the CCC class, for which a steep slope is always associated with lower intensities, irrespective of the lags. This can be explained by changes in the dominant effect according to the firm's structure. First, low short term interest rates can indicate a slowing down of economic activity and it increases competition in the corporate bond market. Second, increases in long term interest rates are often interpreted as expectation of higher growth. Future growth may be dominant effect for junk issuers, as these companies are highly levered and require strong business conditions to move up the rating ladder.

# [INSERT TABLE III HERE]

<sup>8</sup>Further results are available on request.

The business cycle appears to have large effects on the default cycle. Of course business expansion provides good news for default whereas credit crunch amplifies defaults from the money lending variable. However if average levels of intensities (not reported) from the models using the S&P500 returns and the real GDP growth are comparable, the GDP involves much larger impacts: from contemporaneous variables, an increase by one percent in the real GDP growth levels down intensities by 17.5% for IG (resp. 20.3% for NIG) whereas the same increase in the S&P500 return decreases intensities by 2.7% (resp. 2.1%). The higher stability of the business cycle indicators is certainly partly responsible for such differences, since the default cycle is far less volatile than financial markets. Credit markets also display significant explanatory power, the BBB spread being a key indicator. Net treasury issues and money lending variables have minor impacts on intensities. We argue that this is due to their weak short term informational content. Furthermore, looking at likelihood ratios (LR), we observe that the default cycle and the credit cycle are not necessarily synchronous. Estimated sensitivities exhibit the best LR using forward credit covariates (IG spread, BBB yield). Similarly some factors like the personal income growth are not appropriate as they seem to lag the default cycle rather than lead it. Causality analysis in that direction would be particularly relevant for future research. Finally aggregate default indicators appear as major explanatory components. They express the persistency of the default cycle both in declines and recoveries. Studies of rating migrations (e.g. Nickell, Perraudin & Varotto (2000)) have shown that the NIG downgrade ratio is highly correlated with increases in the number of defaults. Further estimations show that including such an endogenous factor is highly relevant (see below).

#### [INSERT TABLE IV HERE]

# [INSERT TABLE V HERE]

#### Behaviour w.r.t Past Conditions

Older information provides further insights on the explanatory power and the time-span of economic shocks over the default cycle. Sensitivities to covariates are either constant, increasing or decreasing as lags increase. They evidence the high degree of persistency of economic shocks on the firms likelihood of default. It bears major implications from a modelling standpoint even for reduced form models because Markovian processes are unlikely to provide such features. Besides we find that some factors impact differently on default probabilities in the long run. The S&P500, the term structure slope, the real GDP growth and net treasury issues appear to be leading indicators of future peaks of defaults, thus showing that the default cycle lags the economy. The interesting point lies in the signs of these covariates which come as warnings: expansion peaks of the financial market or of the business cycle seem to announce increases in the number of defaults three years later. This has to be taken with care as it could only represent the singularities of the global economy over the past 25 years and not necessary apply to the future. For example early repayments and small levels of issues by US Treasury signaled the peak of the US cycle which was later followed by a major default crisis. Hence negative net treasury issues increase future default probability at a three year horizon in our sample. Interestingly notice that these lagged effects should also be significant because the default process is time consuming from its origination as reported by Altman (1989). From that perspective, the default cycle has to remain high after economic recoveries, generating explanatory power for lagged covariates and persistency for economic shocks. Therefore we argue that lagged factors when used as supplementary information could at least help in capturing business and market trends, which constitute the essential information on future default probability<sup>9</sup>. Section III.D examines this issue. Finally we point out that, as expected, the money lending indicator has a much larger influence on the long run and could be used in that way.

#### C. Factors as Determinants of Time profiles

Results on the time-span of economic shocks suggest that economic conditions could determine firms' default time profiles within rating classes. Moreover economic variables are intuitive candidates for the explanation of the shape of intensities over distinct vintages. For example Bassett & Zakrajsek (2003) have reported singularities for loans granted during recessions. The average quality of new loans is lower than usual but comes back to standards after the crisis. This should translate into higher intensities over the first months for such vintages. In order to test this hypothesis, we rely on the time-independent covariates setup.

Considering intensities up to the last day of observation from entry in the rating process, estimates  $\hat{\beta}$  do not show any evidence at any conventional confidence level that some covariates have significant impacts over the whole intensity curve whatever the rating class we consider. We can reasonably believe that if a company faces credit difficulties when it enters into a specific risk class, either these difficulties should be absorbed or at least diluted after a while, or the company should default. In the latter case, the company should be quickly downgraded. Similarly, strong business growth in a sector may vanish quickly as that sector is likely to become more competitive, and also because any worthwhile project has limited duration. Hence as the duration increases, economic shocks on its distribution should prevail over possible differences in initial exposures in default risk.

As a consequence, instead of looking for impacts on full time profiles, we focus on short and mid term horizons within rating classes. Practically, we run maximum likelihood estimations on subsamples imposing additional right and left truncation on durations. We considered durations, up to 1 year, 3 years, 5 years, between 1 and 3 years, and finally between 3 and 5 years. Such cut-outs are suggested from Table II. As before we looked at factors' impact individually (results not reported here). All empirical results confirmed the findings of Tables II and V in the following way. If economic conditions do not affect the whole default time profile, the duration of a company within a rating class at such horizons is influenced by conditions at entry in the class.

 $<sup>^{9}</sup>$ Indeed, for instance, from time series cycle analysis between the GDP and bankruptcy rates, Koopman & Lucas (2004) observe differences in magnitude and lengths. The default cycle being much smoother and persistent than the financial market or the business cycles, transitory shocks should not represent the most relevant information.

Using time-independent covariates, we find that factor signs are the same as those we obtained previously in the case of time-dependent covariates with lags from 0 to 2 years. This implies that the significance of the impacts of the real GDP, the S&P 500 returns, net treasury issues and the slope of the term structure, for lags longer than 2 years are weaker than those obtained for shorter lags. Shorter lags are intuitively those that provide the dominant effects on default intensities.

As a consequence, observed decreases and increases in intensities in the very short run could easily be handled through conditioning on information at the firm's entry in the risk class. This would be especially relevant in short term risk management. It captures cyclical quality of new issues as well as possible conservatism in rating analysts' classifications without relying on adhoc adjustments. Moreover it implies that, at the time of a rating change, short term risk predictions (less than 2 years) can be proxied through factors at that date and do not necessary require the prediction of factors. In addition our findings reveal the following behaviour in default distributions: as time from entry elapses, default risks of all firms in a given class globally converge toward the same level which is in turn influenced by shocks and trends in the economy.

# III. Factors Efficiency and Default Decomposition

We now turn to the assessment of these factor models and, more generally, of the efficiency of the information set in explaining and predicting observed patterns of the default cycle. Following such goals Kavvathas (2000) and Aunon-Nerin & Burkhard (2003) have shown that economic variables do not explain a huge part of transition probability changes. Closer to our study, Collin-Dufresne & al. (2001) also report 25% to 30% of explanatory power by macroeconomic factors on spread changes. Their residuals are highly cross correlated and there is only one significant underlying factor which cannot be explained by their state variables. Such a factor will be examined later in Section IV.A. The explanatory variables used in Collin-Dufresne & al. included interest rates indicators, changes in volatility, expected recovery rates and leverage. One noticeable result is that, contrary to structural model predictions, systematic factors seem to be more important than firm-specific ones in explaining spread changes. Yet the authors argue that the failure of their state variables to capture a large amount of the systematic part of spreads should be due to the strong impact of local demand/supply shocks. We now bring complementary answers to these issues.

In the following we propose a semi-parametric framework which allows us to extract variations in default intensities which are not explained by a given factor model. By doing so, we are able to study misspecifications of factor models and to compare the pertinence of the various covariates. In particular we show that explained variations in intensities could be widely underestimated because of inappropriate choices in the information set. For instance leaving aside information provided by the business cycle can be damaging for the performance of the model. Hence looking carefully at models relying on financial market information, we show how traditional models can be enhanced while remaining parsimonious.

#### A. A Semi-Parametric Framework for Intensities

In this section we develop a semi-parametric framework to model default intensities and test the impact of the covariates presented above. We start from a fully non-parametric estimator of default intensities based on the Gamma kernel. We then add a parametric component using Cox proportional hazard methodology (Cox (1972) (1975)), a well known tool in biostatistics, both with time-independent and time-dependent covariate specifications. The baseline hazard is estimated using the GRHE estimator<sup>10</sup> (Gamma Ramlau-Hansen Estimator) as the solution of the maximum likelihood objective function, while the parametric part is estimated by partial likelihood. A complete survey of related models, estimation techniques and asymptotics can be found in Andersen & Gill (1982) or Andersen & al. (1997).

#### The GRHE estimator

The GRHE estimator has been introduced by Couderc (2004). It is based on a convenient Gamma kernel smoother of the cumulative hazard rate belonging to the popular Nelson-Aalen class<sup>11</sup>. As duration increases, the number of firms under observation tends to zero. Standard smooth estimators with finite and symmetrical support need large bandwidths (smoothing parameters) to provide estimations in the long run. Therefore they suffer from oversmoothing, which reinforces the boundary bias inherent to these kernels (e.g. see Bouermani & Scaillet (2001) for the properties of asymmetric kernels on density function estimations). Couderc [2004] has shown that the GRHE is free of boundary bias<sup>12</sup> and is able to capture changes in intensities in the short run (which may cover up to 5 years) as well as subsequent deformations, whereas Fledelius & al. (2004) obtained flat intensities using the standard Epanechnikov kernel. Empirical applications hereafter prove that not using an unbiased estimator would lead to inaccurate assessments of intensity variations as well as of factor models.

The necessary conditions ensuring the consistency of the estimator are assumed to be met. Accordingly, all firms in a given risk class are supposed to be homogenous and conditionally independent. Censoring mechanisms which may prevent from observing firms up to their default time are random and independent from the default process. For a given firm, these mechanisms are reported through the process  $Y^{i}(u)$ . The most important building block of the GRHE lies in the following assumption :

Assumption III.1 The intensity of individual firms satisfies the Multiplicative Intensity Model:

$$\lambda^{i}\left(u\right) = \alpha\left(u\right)Y^{i}\left(u\right) \tag{3}$$

where  $\alpha(u)$  is deterministic and called the hazard rate whereas  $Y^{i}(u)$  is a predictable and observable process.

<sup>&</sup>lt;sup>10</sup>The use of a Gamma kernel estimator is crucial to capture variations in intensities of default and deficiencies of models. The definition is given below.

<sup>&</sup>lt;sup>11</sup>See Andersen & al. (1997) (1982) for details on the Nelson-Aalen estimator and its properties.

 $<sup>^{12}</sup>$ This feature has already been widely documented in the case of density function estimation with semi-finite support. For instance see Chen (2000).

Remark that the difference between the intensity and the hazard rate resides in their observability. The hazard rate is the relevant quantity. The estimator is specified as :

**Definition III.1** The gamma kernel estimator  $\hat{\alpha}(u)$  of the hazard rate (Gamma Ramlau-Hansen Estimator, GRHE) is defined by

$$\widehat{\alpha}\left(u\right) = \int_{0}^{\infty} \frac{1}{Y\left(s\right)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma\left(\frac{u}{b}+1\right)} dN_s.$$

$$\tag{4}$$

where  $dN_s$  counts the number of default at time s and Y (u) is computed as the number of firms for which the last time of observation is greater than u. Y (u) is usually described as the risk set and handles censoring. b is a smoothing parameter, the so-called bandwidth. The intuition behind this non-parametric estimator is as follows: the probability of default over the next infinitesimal time step is estimated as a weighted average of past, current and subsequent instantaneous default rates. The weights are determined by the choice of the kernel, by the bandwidth as well as by the durations between default events. Default rates are computed as the ratio of the number of firms defaulting at the same time over the number of firms in the sample which survived up to that time.

The restrictions imposed on the process Y(u) are sufficiently weak to permit more complex specifications of this process. In what follows we rely on the multiplicative intensity model, to enrich the firm-specific part of the intensity specification by introducing covariates. As a consequence, one should think about this non-parametric estimator as a useful diagnostic tool to model the unexplained baseline hazard.

## Assessment of Factor Models

Let  $\mathbf{Z}$  denote a vector of covariates. We assess the performance of factor models in explaining intensities by relaxing the conditional exponentiality assumption. Therefore the parametric part reflects implications of covariates, whereas the baseline hazard or conditional distribution of the errors is estimated through a slight modification of the GRHE.

**Assumption III.2** The intensities conditional on structural factors are proportional to a class baseline intensity  $\lambda^{\circ}(u)$  representing the common intensity shape:

$$\lambda^{i}(u,t_{i}) = \lambda^{\circ}(u) \exp\left(\beta' \mathbf{Z}(t_{i},u+t_{i})\right) \quad \forall i$$

$$= \alpha^{\circ}(u) Y^{i}(u) \exp\left(\beta' \mathbf{Z}(t_{i},u+t_{i})\right) \quad \forall i$$
(5)

where  $\mathbf{Z}(t_i, u + t_i)$  is the set of structural variables taken at the date of entry  $t_i$  of the firm *i* in the class or at calendar time  $u + t_i$ , and  $\boldsymbol{\beta}$  is the vector of sensitivities associated with a given risk class.

In this framework, provided that structural variables dynamics are not explosive, the Gamma kernel estimator of the baseline hazard  $\alpha^{\circ}(u)$  becomes:

**Corollary 1** Under assumptions III.1 and III.2, a semi-parametric estimator of the baseline hazard function is given by

$$\widehat{\widehat{\alpha}}^{\circ}(u) = \int_{0}^{\infty} \frac{1}{\widehat{\widehat{Y}}(s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma\left(\frac{t}{b}+1\right)} dN_s$$
(6)

$$\widehat{\widehat{Y}}(s) = \sum_{i} Y^{i}(s) \exp\left(\widehat{\widehat{\beta}}' \mathbf{Z}(t_{i}, u + t_{i})\right)$$
(7)

A convenient feature of this model is that an estimate  $\hat{\beta}$  of the sensitivities  $\beta$  can be derived separately from the baseline intensity through Cox partial likelihood:

$$\widehat{\widehat{L}} = \prod_{k=1}^{n} \frac{Y^{k}(u_{k}) \exp\left(\boldsymbol{\beta}^{\prime} \mathbf{Z}\left(t_{k}, u_{k} + t_{k}\right)\right)}{\sum_{i} Y^{i}(u_{k}) \exp\left(\boldsymbol{\beta}^{\prime} \mathbf{Z}\left(t_{i}, u_{k} + t_{i}\right)\right)}$$
(8)

where  $u_k$  is the observed duration of firm k and  $t_k$  is its date of entry in the class. This powerful two-stage estimation technique does not affect the non-parametric estimation of the baseline intensity as the speed of convergence of the partial likelihood estimator is of order  $\frac{1}{\sqrt{n}}$  and therefore higher than that of the kernel estimator. In particular, confidence intervals on the baseline estimator are not affected by the estimation of  $\beta$ . It then corresponds to the parametric factor model case from equation (1) where the constraint  $\lambda^{\circ}(u) = \exp(\gamma)$  has been imposed. Corollary 1 is a direct consequences of Andersen & al. (1997) and Couderc (2004),  $\hat{\alpha}^{\circ}$  being the maximum likelihood gamma kernel estimator of the baseline intensity.

#### **B.** Contemporaneous Financial Markets Factors Failure

The semi-parametric framework presented above allows us to test the relative performance of the various factors and its evolution through time. This is achieved by comparing fully non-parametric specifications and semi-parametric models. In particular the time-dependent covariate framework allows to value the quality of factor models reflecting insights of the two standard modelling setting, namely structural and reduced form models.

For each semi-parametric model we can associate a factor model counterpart. Remark that even if the estimation process is not the same, sensitivities to covariates should be equivalent<sup>13</sup> in both methodologies. We checked this last point and found that all factors keep the same signs at the same horizons when switching from a parametric factor model to a semi-parametric specification. Only small variations in magnitude can be observed as expected. On the IG and NIG samples significant coefficients do not change. On rating subsamples, some coefficients

<sup>&</sup>lt;sup>13</sup>The difference between the estimation approaches of  $\hat{\beta}$  and  $\hat{\beta}$  only lies in the slower rate of convergence of the semi-parametric specification to its asymptotic distribution with respect to the parametric model.

become insignificant. However the main issue consists of the capacity of stock and treasury bond markets shocks to capture real intensities of default. If financial markets factors provide an appropriate representation of default intensities, then the estimated baseline hazard  $\hat{\alpha}^{\circ}(u)$ should be close to a constant function. Indeed previous analyses and results from Fledelius & al (2004) and Couderc (2004) indicate that after initial informational adjustments (at entry in a new class, up to 2 to 4 years), the credit riskiness of firms from a homogenous risk class should be similar whatever their duration in the class. In other words, on the long run the baseline hazard is constant and shifts in this baseline results from changes in the economy through time. Tables VI and VII present estimates of sensitivities  $\hat{\beta}$  for different specifications on the basis of financial predictors over rating classes.

#### [INSERT TABLE VI HERE]

Table VI stages three multifactor models fitted on contemporaneous stock market and interest rate information over various rating classes. For robustness checks, we included a dummy indicating non-US firms. Sensitivities to this non-US indicator were not significant. Likelihood ratios select the joint model as the best one, whereas interest rates alone provide the poorest fits. These results confirm findings of Driessen (2002) or Janosi & al. (2002) on credit spreads. From unreported estimates of  $\widehat{\alpha}^{\circ}(u)$ , interest rates appear to be unhelpful explanatory covariates of default accross rating classes. On the contrary stock market information brings significant explanatory power. In a Merton-like intensity model with additional stochastic liabilities, it could be interpreted as evidence of the level and higher variability of assets being the main determinants of the default probability changes. Figures 1(a) and 1(c) focus on IG and NIG classes displaying baseline hazard rates for non-parametric ( $\hat{\alpha}(u)$ ), semi-parametric  $(\widehat{\alpha}^{\circ}(u))$  and parametric specifications (dashed lines). In the latter, the baseline hazard is always constant. The baseline hazard is leveled down by 29% for the IG category thanks to stock indicators. However it is very clear from the graph that a constant intensity either unconditional or conditional on stock information does not represent the data accurately, in particular in the NIG class. Indeed deviations from the constant (blue lines) remain significant, implying that the S&P500 returns and volatility do not succeed in capturing shocks of the economy which affect the default riskiness. As expected short term default probabilities are poorly predicted and for all classes but the CCC class, and completely overstated (the dashed line is higher than  $\hat{\alpha}^{\circ}(u)$  up to 2 to 4 years). Deviations are not significantly reduced in the NIG class, and default probabilities are overestimated on a larger part of the debt life. In an attempt to capture effects of the 2001 default peak, the model grants too much weight to covariations between the stock market and defaults<sup>14</sup>. Notice that given our sample window, the 2001 recession is responsible to a large extent for the first hump of  $\hat{\alpha}(u)$  (among NIG observed durations which range between 1 to 5 years approximately one fourth faced the 1991 recession at these horizons while one half faced the 2001 recession).

 $<sup>^{14}</sup>$ We check this overfitting problem due to the 2001 recession by estimating the "Stock Market" model on the subsample constituted by firms entered in the process after the 03/01/1991. This basically cuts durations higher than 10 years, and thus the second hump on intensity graphs. On this reduced sample the model delivers higher sensitivities of -2.82 on the S&P500 return and of 3.14 on its volatility for IG, -1.99\*\* and 0.88\*\* respectively for NIG. At the same time it also exhibits a higher overestimation of instantaneous probabilities from the 6 to 10 remaining years after the peak.

Interestingly, the market volatility exhibits lower significance than the market return in Table VI. Its relative impact on default is minor with respect to other factors. Corresponding coefficients are insignificant in fifty percent of the cases. Such a finding is highly challenging for structural models as the volatility determines the dynamics of equities and as a consequence default probabilities. However the main impact of volatility may not immediate but may arise at a longer horizon: default is usually a progressive and lenghty process. These results prove that transitory market shocks are not drivers of the default cycle. We still have to determine if these shocks are meaningful on the long run. Yet remark that the BBB class seems to be much more affected by market volatility. As BBB corporate bonds are much shocked by demand and supply effects, this result agrees with the hypothesis of Collin-Dufresne & al. (2001). For instance, some fund managers systematically rule out non investment grade corporate bonds from their portfolios: at the time of a downgrade from BBB, numerous funds close their positions. The market volatility should be a good proxy for that kind of market segmentation behaviour and consequently has to be a key indicator for the BBB class.

Stock and interest rate-based factor models have now become a market standard to model default probabilities. However their empirical performance has been challenged in many occasions. We have just shown that this failure is still true under the historical probability measure, meaning that such a failure should not be due to control problems or mixing of risk effects present under the pricing measure. More importantly by decomposing their performance through time, we have shown that they lead to overestimating default riskiness over the bottom part of the default cycle but still missing levels of default probabilities during top parts. Yet, some researchers as Yu (2002) believe that factor models are not doomed to fail. Our previous outcomes suggest that this incapacity to explain changes in default risk exposure may lie first in the reduced information set used, and then in the fact that contemporaneous persistent shocks modify the default cycle several months after they happen. We now test these hypotheses.

#### C. Joint Performance of Default Indicators

Univariate analyses have established a strong impact of the business cycle on default intensities. To our knowledge, such a relationship, although highly intuitive, has never been taken into account in factor models. We may explain this phenomenon by the fact that market factor are expected to integrate business cycle information. The market supplies notwithstanding for a noisy signal of the business cycle. As we already stressed, these are not substitute. Thus we study whether the business cycle and financial markets are complementary predictors of the default cycle, and to what extent business indicators reduce the unexplained part in the intensities variations. We select the most successful factors from our univariate analysis according to their likelihood ratios. Table VII presents the outputs.

# [INSERT TABLE VII HERE]

The model including the six factors outperforms models using only stock market or interest rate information, according to LR tests. It manifests that all mentioned components (financial, business and credit) of the economy bring their own contribution to the behaviour of default probabilities. All covariates are statistically significant at standard confidence levels. Most of the selected factors are not highly correlated, except the GDP and the S&P500 return which exhibit a correlation of 36%. This explains the fading of weights on stock market in this 6factors model as the GDP offers a higher univariate marginal impact. LR tests suggest that all covariates bring additional information and should be kept. They all enter the model with the predicted signs. Figures 1(b) and 1(d) show that the baseline hazard is strongly shifted downwards by 62% and that distortions have been reduced, compared to Figures 1(a) and 1(c). This evidence that the new factors are key determinants of the default regime changes. The default process cannot be reduced to a simple outcome of a decrease in firms' asset value. Notice that most of the additional explanatory power comes from real GDP growth and NIG downgrade rates, indicating a heavy impact of business conditions, as well as a persistence in erosion of firms' quality.

#### [INSERT FIGURE 1 HERE]

We can observe that the humps still exists which implies that our contemporaneous determinants do not capture all joint movements of the default cycle through time. The presence of humps suggests that the default cycle is longer than the business cycle and that some persistency has to be incorporated. Furthermore, we observe that the tendency to overestimate default probabilities on low grades is due to incapacity of traditional factors to explain the huge number of defaults observed in the latest default crisis. As correlations of default seemed to be particularly high during this period, we may believe that this failure could be reduced by modelling snowball effects, for instance through industry contagion as we will see later.

# **D.** Trends and Persistency of Shocks

So far, all selected variables have been contemporaneous, but potential lead-lag effects from financial markets or the business cycle on the default cycle should also be considered. Looking back at Table II, in a univariate setting, we can observe that lagged variables may have more explanatory power than current economic conditions, and some variables lead the default cycle by an average of three years. Besides, lagged information could capture parts of persistency patterns. Considering both contemporaneous and lagged factors indeed pick up economic trends. Therefore, we adapt the "stock market" model which is the most akin to structural credit risk models, and add lagged volatility and stock market return. Estimated parameters  $\hat{\beta}$  are provided in Table VIII.

#### [INSERT TABLE VIII HERE]

Results show that lagged information is relevant in addition to contemporaneous one. While current volatility is not statistically significant for some classes, lagged volatility always is, except for the AA class. Moreover for all classes but the BBB a one percent change in past volatility has an impact three times higher than the same change in contemporaneous volatility. This is consistent with our earlier findings. In other words current demand and supply shocks are dominated by past market movements. It shows that the default cycle lags the economy. However both current and lagged volatilities are associated with higher default rates, which implies that trends induce higher intensity shifts. The results on lagged returns still imply that high equity returns tend to be associated with higher default probabilities three years later. As emphasized earlier, by combining current and lagged returns we capture the long term trend in financial markets. Figure 3 shows that it has a stronger influence on the behaviour of the default cycle. We can propose explanations for this finding. First, it may reflect some cyclicality in equity returns. We have found that high current equity returns tend to be associated with low current default rates. If there is cyclicality in equity returns, with a peak-to-trough time of approximately 3 years, it is plausible that high returns will be associated with high default rates 3 years on. Nonetheless we have found no evidence of such a cyclicality. An alternative explanation would be that in good times (when the equity market is performing well) companies can afford to raise large amounts of debt while preserving acceptable levels of leverage. Several years later, this level of debt may become unsustainable for some firms, thereby raising the default rate. Table VIII does not contradict such a hypothesis: if the market keeps the same upward trend during three years, the market appreciation induces a decrease in intensities for Investment Grades whereas those of Non Investment Grade corporates are pushed up.

> [INSERT FIGURE 2 HERE] [INSERT FIGURE 3 HERE]

We find that including lags substantially improves the models from LR tests. Figures 2(a) and 2(b) diagnose significant decreases in baseline hazards (about 64% for IG) as well as in deviations from the constant, compared to Figures 1(a) and 1(c). The improvement is even more substantial than that achieved by additional business and credit indicators for the NIG class. We can observe a global levelling down of the errors. Once again the latter result may be strongly dependent on our sample window but Altman (1989) among others already suggested that lagged dependencies would be useful because of the lag between the time at which a firm starts experiencing difficulties and its default time.

# **IV.** Exploratory Issues

# A. Industries as Alternative Source of Information

In the previous section, we observed that a significant part of variations in NIG intensities was still unexplained by covariates. We pointed out the huge number of defaults that occurred during and after the 2001 recession and the difficulty for factor models to fit this peak. We have ignored contagion effects so far. Reduced form models relying on structural factors have often been criticised for failing to replicate empirically the observed default correlation. Jarrow & Yu (2001), Yu (2002) and Gagliardini & Gouriéroux (2003) documented the fact that when a firm defaults in a portfolio, other firms' intensities may jump and generate substantial default correlation. Contagion models (e.g. Davis (1999), Davis & Lo (2001a) (2001b), Schoenbucher & Schubert (2001)) are able to replicate some of these effects but they are often difficult to calibrate. Kyiotaki & Moore (1997) have shown through a theoretical equilibrium model that the business cycle may only be a contagion vehicle. "Disease" starts from local changes in the credit cycle (roughly among an industry) and leads to global shocks in defaults. Therefore, in such a context it will be impossible to design a consistent model using only calendar time dependent factors, and leaving aside pure default information and industrial factors. Koopman & Lucas (2004) studied this cyclicality using the well-established machinery of VAR models and including in cycles. Using GDP, bankruptcy rates and credit spreads as respective proxies for the business cycle, global credit cycle and pure default cycle, they show that co-movements between economic conditions and defaults may arise in the long term. However, as in previous studies, no general pattern can be extracted from the data but their findings support the idea that parts of the credit and default cycles contain their own dynamics. In this section we further document this phenomenon which may be instrumental in explaining the high level of defaults observed in the last recession.

In a two-state hidden Markov chain model, Crowder, Davis & Giampieri (2003) showed that adverse economic conditions do not affect all industries in the same way. There is evidence of sector-specific crisis, such as that affecting the energy sector in the mid-eighties or the telecom crisis of the early 2000s. Therefore, the number of defaults occurring in a given industry and in a given time step may represent a good indicator of the health of this industry and be useful in predicting default probabilities over the next time step. If the business cycle is really a contagion vehicle, such factors may efficiently enlarge the information set we previously used.

In order to test this, we rely on a class of autoregressive models that have been introduced to study the durations between trades in microstructure econometrics. These models are called Autoregressive Conditional Duration (ACD) models. More specially we focus on a log-ACD specification (see Bauwens & Giot (2000) and Engle & Russel (1998)). The intuition is the following. If one observes short times between defaults in a given sector, it probably means that the sector faces a crisis and therefore that one can expect the next default to occur shortly. In terms of intensities, it implies that the intensity of a firm in a given sector should be inversely related with past durations between two defaults in that sector. ACD models allow to take those effects into account by assuming that the expected duration until the next default is a function of past durations. Hence, defaults will tend to cluster. We control for sector size because, even if probabilities of default remain constant over time, variations in the sector size will artificially create clusters in the sector intensity of default. Obviously large sectors will face a higher number of defaults than smaller ones for equivalent default probabilities.

For a given risk class c, we consider an aggregate counting process  $N_t^c$ . We introduce additional left-censoring for each firm i defining  $S_i = \max\{\tau_{j+1}; \tau_j < t_i\}$  where  $\tau_k$  denotes the  $k^{th}$  jump time of the process  $N_t^c$ . This censoring scheme is designed to take into account only firms which were already in the class c before the last observed default time in this class. Thus the process  $N_t^c$  can be written as  $N_t^c = \sum_{i \in c} \mathbb{I}_{(t \ge S_i; D_i + t_i \le t; D_i \le U_i)}$ , and  $\Delta N_{\tau_i}^c = \sum_{i \in c} \mathbb{I}_{(\tau_i \ge S_i; D_i + t_i = \tau_i; D_i \le U_i)}$ .<sup>15</sup> We now specify the intensity of the process  $N_t^c$ .

**Assumption IV.1** Durations between two jumps of the counting process  $N_t^c$  follow a log-ACD(1,1) model :

$$\tau_k - \tau_{k-1} = \psi_c(k) \epsilon(k) \tag{9}$$

$$\log(\psi_{c}(k)) = w_{c} + a_{c}\log(\tau_{k-1} - \tau_{k-2}) + b_{c}\log(\psi_{c}(k-1))$$

$$= w_{c} + a_{c}\log(\epsilon(k-1)) + (a_{c} + b_{c})\log(\psi_{c}(k-1))$$
(10)

where  $\epsilon(k)$  are independent unit exponential variables<sup>16</sup>.

 $\psi_c(k)$  is known right after the  $(k-1)^{th}$  default and represents the expected duration up to the  $k^{th}$  default given the population under observation at time  $\tau_{k-1}$ . In other words, conditionally on the past, durations between defaults are exponentially distributed and we assume that both right truncation and left censoring are uninformative. We now extract the relevant information for firms in the simplest way<sup>17</sup>:

**Assumption IV.2** The intensity of default within the risk class c affects all firms in the same way, and intensity  $\lambda^i$  of firm i is given by

$$\lambda^{i}\left(u,t_{i}\right) = \lambda^{\circ}\left(u+t_{i}\right)$$

Therefore, the intensity associated with the log-ACD model is given by

$$\lambda_{c}(k) = \frac{1}{\psi_{c}(k)} = \sum_{i \in c \text{ at } \tau_{k-1}} \lambda^{\circ}(k)$$

If X(k) is the number of firms under observation for the  $k^{th}$  duration, it simply states that the common intensity  $\lambda^{\circ}$  of firms which belong to the class c is given by  $\lambda^{\circ}(k) = \frac{1}{\psi_c(k) X(k)}$ . This last statement says that durations between consecutive default are also inversely proportional to the number firms in the risk class.

#### [INSERT TABLE IX HERE]

Table 6 provides the estimated parameters for the above model on 11 broad industry categories defined by Standard & Poor's. We find high levels of persistency for most industries.

<sup>&</sup>lt;sup>15</sup>Remark that by considering durations between default times, we do not focus on the complete natural filtration generated by all firms as we do not take entry dates into account.

<sup>&</sup>lt;sup>16</sup>In order not to introduce bias,  $\tau_0$  will be the date at which the first default has been observed in the risk class.

<sup>&</sup>lt;sup>17</sup>Assumption IV.2 could be enriched by conditioning on business and financial covariates.

Implied intensities provide the most interesting results. Figures 4, 5 and 6 show that the macroeconomic cycle does not have the same impact on default intensities in all industries, as found by Crowder, Davis & Giampieri (2003) in a simpler framework. For instance Figure 6(a) shows that the telecommunication sector was not affected by the 1990-1991 downturn but was the most hit by the 2001 recession. 1986-1987 appears to have been a crisis period for the energy sector, while other sectors were little affected.

> [INSERT FIGURE 4 HERE] [INSERT FIGURE 5 HERE] [INSERT FIGURE 6 HERE]

Several other phenomena can be identified in these pictures. First, the persistency of the default cycle can be observed: inter-default intensities remain high several years after the peak of a recession. Second, we can see that some sectors appear to be forerunners of economic downturns, while others seem to follow recessions. As a consequence, information relating to sectors whose default cycle leads the economic cycle could prove valuable for credit risk management. The default rate in these industries may be a good variable to forecast the aggregate default rate in the economy. This is left for further research. Remark that we ran estimations on BB, B and CCC rating classes too. Obviously the levels of implied intensities were found to be increasing with decreasing rating quality but variations and log-ACD(1,1) coefficients do not display different patterns. Therefore default rates in various rating classes cannot be used to forecast default rates in other classes. However remark that it gives support to the fact that migrations should be mainly driven by only one underlying factor.

# **B.** Modelling Default Probabilities

We showed that large errors can be made when evaluating default probabilities over the first months in a given risk class. Our results suggest that a way to correct for this phenomenon would be to include a variable reflecting economic conditions at the firm's entry in the class. This term would be specified such that its effect would vanish with time, i.e. initial conditions would progressively become irrelevant. However Figure 3 displays other insights in that direction, putting forward assumptions on times-to-default distributions over rating classes as a major issue.

The practical implications of such assumptions are critical in the valuation of complex derivatives such as Collateralized Debt Obligations (CDOs) or nth-to-default. Madan & al. (2004) investigate empirical distributions of the life of such derivatives under the pricing measure. They find evidences toward increasing intensity shapes but they cannot identify whether the phenomenon is due to the time profile of default probabilities or to the market assessment of credit riskiness (i.e. risk premia). Our study allows to confirm that intensities of times-to-default are not constant. It strongly support the use of Weibull distributions as common baseline instead of the exponential distribution. The Weibull distribution produces monotonically increasing or decreasing intensities exactly as we obtained. These results echo findings of

Madan & al. and prove that, conditional on the realization of the factors, the Weibull independence assumption should be preferred to the usual exponential independence of firms. Among all rating classes but the B and CCC we find that default intensities globally increase with durations (Figure 3 present estimations on the BBB to CCC classes). Junk issuers exhibit a globally decreasing intensity implying that, as time elapses and conditional on non defaulting, their financial standing should improve. Junk issuers can be seen as "do or die" firms. They will either default quickly or, given their high level of leverage and firm risk, they may be very successful in the longer term. Therefore, conditional on surviving the first few years, their default probability should fall substantially over the long term. Most startups would fall in this category but they are not captured in our sample as very few of them are rated. Non-junk issuers exhibit an increasing hazard rate, reflecting increasing uncertainty in the longer term. Madan & al. explain such a result by an over-exposure to innovation for established firms. Increasing exposures to managerial inefficiency and agency conflicts can also induce increasing likelihood of default for large companies. Economic conditions enter then the default problem producing shocks along with the intensity trajectory.

Finally remark that, coming back to reduced form models, our findings indicate that the choice of intensity's dynamics has to be made with care. Single factor models such as CIR processes, are unlikely to be good performers in the long run. If a two factor model with time varying long term trend might be more appropriate, we overall stress that the importance of past information once again casts doubt on Markovian specifications.

# Summary

In this paper we study times-to-default in the Standard & Poor's rated universe. We rely on a simple framework that enables us to analyse the behaviour of default probabilities with respect to changes in stock and bond markets indicators under the historical measure. The setting decomposes explanatory errors through time. More importantly, we investigate other sources of information which should alter the default cycle and which have been left aside by the credit literature, namely the business cycle and endogenous proxies from credit markets and the default cycle. We explore further the sensitivity of probabilities to past economic conditions. Surprisingly, this question has been bypassed by researchers whereas short run cocyclicality is doubtful (e.g. Koopman & Lucas (2004)).

Our first empirical results confirm the weak explanatory power of contemporaneous financial market factors. They overall show that changes in intensities cannot be attributed solely to financial variables such as equity returns, their volatility or interest rates but the business cycle and the specific behaviour of credit markets are key determinants of future default probabilities. As a consequence, the significant explanatory variables found in our study can be used to improve traditional credit risk models. In particular, a set of carefully selected factors from each information source can substantially enhance the efficiency of factor models in capturing movements in default probabilities. They demonstrate that common factors should account for a larger part of probability changes than reported by studies on corporate spreads. Additional research quantifying these effects on spreads and repercussions in standard risk management models would be highly valuable. Non financial indicators indeed are able to partially correct the tendency of financial based factor models to overstate real default probabilities in expansions and stable periods, and to undershoot default peaks during and following recessions.

Our results also show that past information has special benefits. Economic trends and large past shocks appears as main drivers of default probabilities. Both structural and reduced form models usually only feature short term shocks, whereas long term business trends are ignored. However, our results evidence that default is triggered by their joint impact, indicating that efficient models should incorporate them. Intuitively, corporate defaults may be induced by large changes in local or global economic conditions but also by successive declines in a company's performance. The legal process may also delay the default event which in turn might not be explained anymore by contemporaneous financial or business indicators. We consequently argue that past information constitutes a crucial component of adequate modelling, implying that Markovian specification of intensities cannot provide pertinent pictures of their evolution.

Finally, we highlight issues for future research. In an analysis of durations between consecutive defaults within industrial classes we show that strong differences prevail between sectors. Some industries lead the global default cycle while others maintain high levels of defaults during economic recoveries. This suggests the existence of bidirectional contagion between the default cycle and the business cycle. Loosely speaking, this phenomenon makes the default cycle slower and more persistent than economic factors can predict. Therefore, to succeed in capturing default probability variations, we suggest that more predictive proxies could be endogenously extracted from the default process itself, and for instance from industrial classes.

# References

- Altman E.I., 1989, Measuring Corporate Bond Mortality and Performance, Journal of Finance 44, 909-922
- Andersen P., Borgan Ø., Gill R. and N. Keiding, 1997, Statistical Models Based on Counting Processes, Springer, 2<sup>nd</sup> Edition
- Andersen P. and R. Gill, 1982, Cox's Regression Model for Counting Processes : A Large Sample Study, The Annals of Statistics 10, n°4, 1100-1120
- Aunon-Nerin D. and J. Burkhard, April 2003, Conditional Credit Migration Matrices for a Bank Loan Portfolio, Working Paper
- Bandopadhyaya A., 1994, An Estimation of the Hazard Rate of Firms Under Chapter 11 Protection, Review of Economics and Statistics 76, 346-350
- Bangia A., Diebold F.X., Kronimus A., Schagen C. and T. Schuermann, 2002, Ratings migration and the business cycle, with application to credit portfolio stress testing, Journal of Banking and Finance 26, 445-474
- Bassett W.F. and E. Zakrajsek, December 2003, Recent Developments in Business Lending, Federal Reserve Bulletin
- Bauwens L. and P. Giot, 2000, The logarithmic ACD model: an application to the bid-ask quote process of three NYSE stocks, Annales d'Economie et Statistique 60, 117-149
- Bielecki T., Jeanblanc M. and M. Rutkowski, 2004, Replication of Defaultable Claims within the Reduced-Form Framework, Forthcoming Paris-Princeton Series, Springer
- Bielecki T., Jeanblanc M. and M. Rutkowski, 2004, Mean-Variance Hedging of Defaultable Claims, Forthcoming Paris-Princeton Series, Springer
- Bouezmarni T. and O. Scaillet, 2001, Consistency of asymmetric kernel density estimators and smoothed histograms with application to income data, Forthcoming Econometric Theory
- Chen S.X., 2000, Probability Density Functions Estimation Using Gamma kernels, Ann. Inst. Statis. Math. 52, 471-480
- Collin-Dufresne P., Goldstein R.S. and J. Helwege, August 2003, Is Credit Event Risk Priced ? Modeling Contagion via the Updating of Beliefs, Working Paper, University of Berkeley
- Collin-Dufresne P., Goldstein R.S. and S.J. Martin, December 2001, The Determinants of Credit Spread Changes, The Journal of Finance 66, n°6, 2177-2207
- Couderc F., 2005, Understanding Defaut Risk Through Nonparametric Intensity Estimation, Working Paper, Forthcoming FAME Research Series
- Cox D.R., 1972, Regression Models and Life Tables, Royal Statistical Society 2, 187-220
- Cox D.R., 1975, Partial Likelihood, Biometrika 62, 269-276
- Crowder M., Davis M. and G. Giampieri, 2003, A Hidden Markov Model of Default Interaction, Working Paper, Department of Mathematics, Imperial College, London
- Davis M., 1999, Contagion Modelling of Collateralized Bond Obligations, Working Paper, Department of Mathematics, Imperial College, London

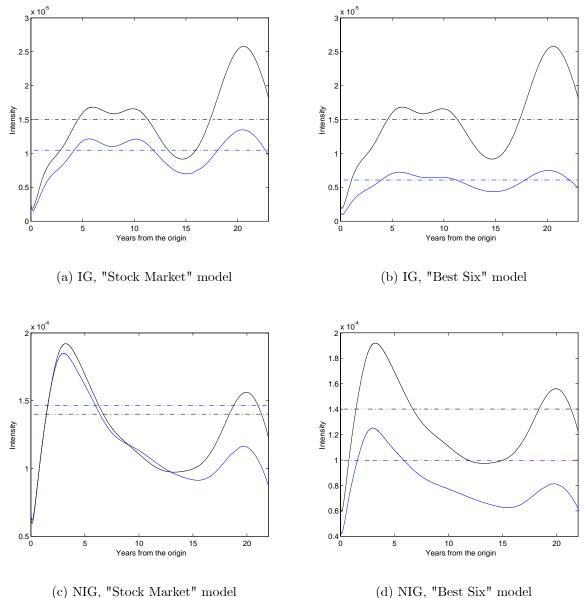
Davis M. and V. Lo, 2001, Infectious Defaults, Quantitative Finance 1, 382-387

- Davis M. and V. Lo, 2001, Modelling Default Correlation in Bond Portfolios, in Mastering Risk Volume 2: Applications, ed. Carol Alexander, Financial Times Prentice Hall, 141-151
- Driessen J., 2002, Is default event risk priced in corporate bonds?, Forthcoming Review of Financial Studies
- Duffee G.R., 1999, Estimating the price of default risk, Review of Financial Studies 12, 197-226
- Duffie D. and K.J. Singleton, 2003, Credit Risk Pricing, Measurement, and Management, Princeton Series in Finance
- Duffie D. and K. Wang, June 2004, Multi-Period Corporate Failure Prediction With Stochastic Covariates, Working Paper, Graduate School of Business, Stanford University
- Engle R.F. and J.R. Russel, September 1998, Autoregressive Conditional Duration : a New Model for Irregularly Spaced Transaction Data, Econometrica 66, n°5, 1127-1162
- Fledelius P., Lando D. and J.P. Nielsen, 2004, Non-parametric analysis of rating transition and default data, Forthcoming Journal of Investment Management
- François P. and E. Morellec, April 2004, Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures, Journal of Business 77, n°2, 387-412
- Gagliardini P. and C. Gouriéroux, Mai 2003, Spread Term Structure and Default Correlation, HEC Montreal, Cahiers du CREF n°03-02
- Janosi T., Jarrow R. and Y. Yildirim, 2002, Estimating expected losses and liquidity discounts implicit in debt prices, Journal of Risk 5, n°1
- Jarrow R.A., Lando D. and S.M. Turnbull, 1997, A Markov Model for the Term Structure of Credit Risk Spreads, Review of Financial Studies 10, n°2, 481-523
- Jarrow R.A., Lando D. and F. Yu, January 2005, Default Risk and Diversification: Theory and Empirical Implications, Mathematical Finance 15, n°1, 1-26
- Jarrow R.A. and F. Yu, 2001, Counterparty Risk and the Pricing of Defaultable Securities, Journal of Finance 56, 1756-1799
- Kavvathas D., May 2000, Estimating Credit Rating Transition Probabilities for Corporate Bonds, Working Paper, University of Chicago
- Kiyotaki N. and J. Moore, 1997, Credit Cycles, Journal of Political Economy 105, n°2, 211-248
- Koopman S.J. and A. Lucas, 2004, Business and Default Cycles for Credit Risk, Forthcoming Journal of Applied Econometrics
- Kwark N.-S., 2002, Default risks, interest rate spreads, and business cycles: Explaining the interest rate spread as a leading indicator, Journal of Economic Dynamics and Control 26, n°2, 271-302
- Leland H.E., September 2002, Predictions of Expected Default Frequencies in Structural Models of Debt, Working Paper, Haas School of Business, University of California
- Lennox C., 1999, Identifying Failing Companies: A Re-Evaluation of the Logit, Probit and DA Approaches, Journal of Economics and Business 51, 347-364

- Lunde A., Timmermann A. and D. Blake, 1999, The hazards of mutual fund underperformance: A Cox regression analysis, Journal of Empirical Finance 6, 121-152
- Madan D.B., Konikov M. and M. Marinescu, September 2004, Credit and Basket Default Swaps, Working Paper, Robert H. Smith School of Business and Bloomberg LP
- Merton R., 1974, On the Pricing of Corporate Debt : the Risk Structure of Interest Rates, Journal of Finance 29, 449-470
- Moraux F., 2004, Valuing Corporate Liabilities when the Default Threshold is not an Absorbing Barrier, Working Paper, University of Rennes
- Nickell P., Perraudin W. and S. Varotto, 2000, Stability of ratings transitions, Journal of Banking and Finance 24, 203-227
- Prigent J-L., Renault O. and O. Scaillet, 2001, An Empirical Investigation into Credit Spread Indices, Journal of Risk 3, 27-55
- Ramlau-Hansen H., 1983, Smoothing Counting Process Intensities by Means of Kernel Functions, The Annals of Statistics 11, n°2, 453-466
- Schönbucher P.J. and D. Schubert, Copula-Dependent Default Risk in Intensity Models, Working Paper, University of Bonn
- Yu F., November 2002, Correlated Defaults in Reduced-Form Models, Working Paper, University of California at Irvine

# Figure 1 **Baseline Hazards of Multifactor Models**

Estimated non-parametric baseline hazard rates  $\alpha^{\circ}(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i)$ ) =  $\alpha^{\circ}(u)$ ) and blue lines show semi-parametric specifications  $(\alpha(u, t_i) = \alpha^{\circ}(u) \exp(\beta' \mathbf{Z}(u + t_i)))$ . Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts  $(\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i)))$ . The "Stock Market" model uses the contemporaneous return and volatility on the S&P500. The "Best Six" model includes in addition the US Term Structure Slope, the real GDP growth, the BBB spread and the NIG downgrade rate.

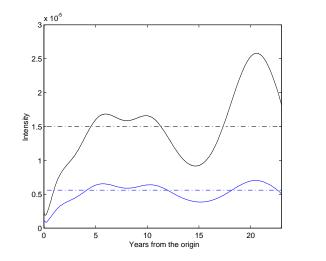


(d) NIG, "Best Six" model

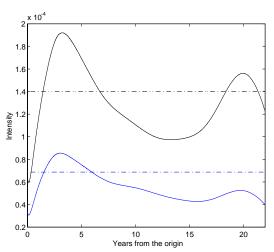
# Figure 2

## IG and NIG Baseline Hazards of MultiFactor Models with Past Information

Estimated non-parametric baseline hazard rates  $\alpha^{\circ}(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i) = \alpha^{\circ}(u)$ ) and blue lines show semi-parametric specifications ( $\alpha(u, t_i) = \alpha^{\circ}(u) \exp(\beta' \mathbf{Z}(u + t_i))$ ). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts ( $\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i))$ ). The improved "Stock Market" model uses the contemporaneous return and volatility on the S&P500 as well as their three year lags.



(a) IG, improved "Stock Market" model

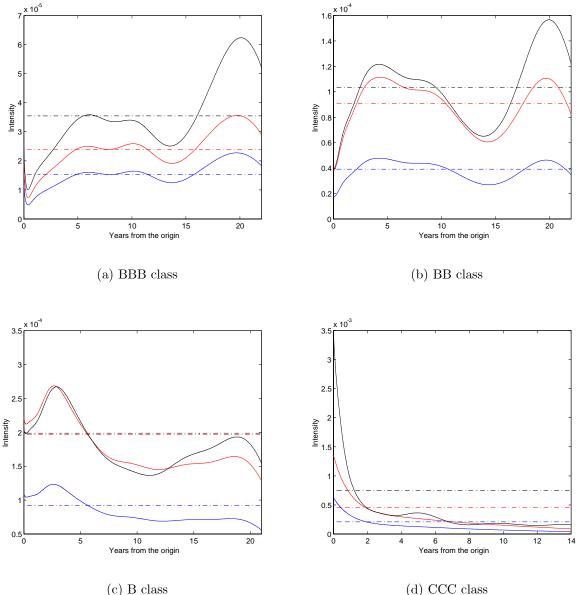


(b) NIG, improved "Stock Market" model

# Figure 3

### Ratings Baseline Hazards of MultiFactor Models with Past Information

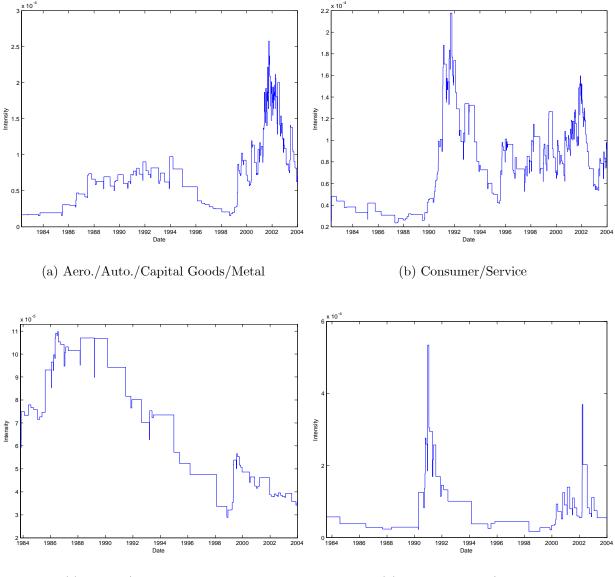
Estimated non-parametric baseline hazard rates  $\alpha^{\circ}(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i)$ ) =  $\alpha^{\circ}(u)$ ), blue and red lines show semi-parametric specifications  $(\alpha(u, t_i) = \alpha^{\circ}(u) \exp(\beta' \mathbf{Z}(u + t_i)))$ . Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts  $(\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i)))$ . The "Stock Market" model (red) uses the contemporaneous return and volatility on the S&P500. The improved "Stock Market" model (blue) adds the three year lags.



(d) CCC class

Figure 4 Intra-Industry Implied Hazard Rates

Predicted intensity of default  $\lambda^{\circ}$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.

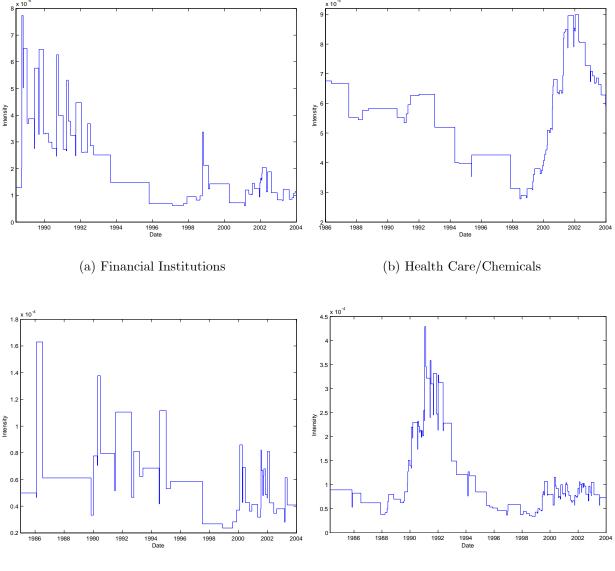


(c) Energy/Natural Ressources

(d) Forest Product/Building

Figure 5 Intra-Industry Implied Hazard Rates

Predicted intensity of default  $\lambda^{\circ}$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.

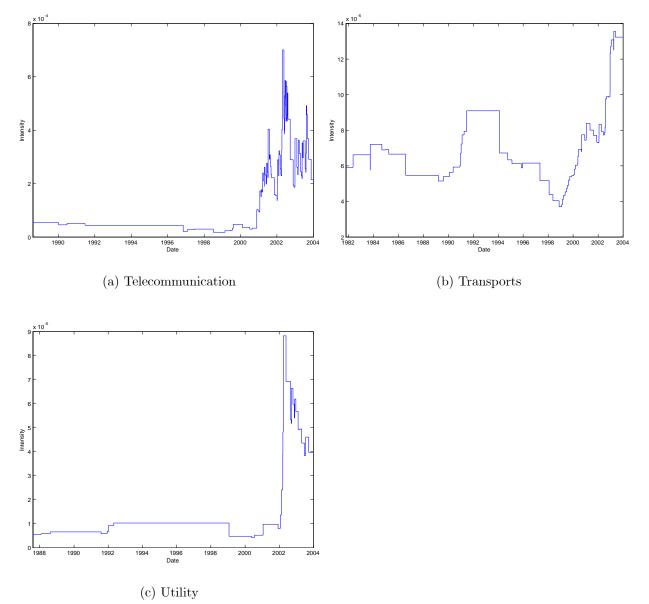


(c) High Tech./Office Eq.

(d) Leisure/Time/Media

Figure 6 Intra-Industry Implied Hazard Rates

Predicted intensity of default  $\lambda^{\circ}$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.



### Table I

#### **Statistics on Covariates**

Basic statistics on retained factors. Figures are given on an annual basis. All variables but upgrade and downgrade rates are US indicators.

	Mean	Min	Max	Volatility	3 Year Autocorrelation
S&P500 Return	0.093	-0.324	0.439	0.158	-0.12
S&P500 Vol.	0.154	0.063	0.628	0.073	0.08
Treas. 10 yr. Yield	0.079	0.033	0.153	0.028	-0.23
Term Struc. Slope	0.013	-0.021	0.033	0.011	-0.08
Real GDP Growth	0.030	-0.028	0.081	0.019	0.17
Des. Ind. Prod. Growth	0.002	-0.018	0.020	0.006	0.22
CPI Growth	0.035	0.011	0.118	0.019	0.60
Pers. Inc. Growth	0.060	0.015	0.134	0.023	-0.32
BBB Yield	0.100	0.062	0.172	0.027	0.62
BBB Spread	0.022	0.013	0.038	0.006	0.09
BBB-AAA Spread	0.011	0.006	0.027	0.004	0.44
Treas. Net Issues	0.144	-0.391	0.748	0.163	-0.12
Money Lending Growth	0.062	-0.039	0.128	0.039	0.16
IG Upgrade Rate	0.005	0	0.020	0.003	0.09
NIG Upgrade Rate	0.010	0	0.109	0.009	0.06
IG Downgrade Rate	0.011	0	0.031	0.006	0.04
NIG Downgrade Rate	0.017	0	0.131	0.013	-0.05

#### Table II

#### Sensitivities w.r.t Financial Markets Information

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

_										
-	Investment Grades									
_	Lag	S&P500 Return	S&P500 Vol.	Treas Yield	Term Str.Slope					
	-2M	-2.19**	2.92**	-15.87**	42.31**					
_	-1M	-2.25**	$3.36^{**}$	-15.98**	40.69**					
	0M	-2.72**	3.80**	-15.33**	38.44**					
-	1M	-2.85**	3.99**	-15.35**	33.11**					
-	2M	-2.77**	$3.65^{**}$	-14.57**	29.59**					
	3M	-2.61**	3.50**	-13.90	28.29**					
	6M	-2.40**	3.11**	-11.84**	18.84**					
_	1Y	-1.96**	2.59**	-11.07**	-3.56					
-	2Y	49	$3.85^{**}$	-9.08**	-28.18**					
-	3Y	1.33**	$3.85^{**}$	-11.75**	-20.84**					
_	5Y	1.12**	3.54**	-10.79**	-7.55					
=										
=		No	on Investment G	Grades						
_	Lag	S&P500 Return	S&P500 Vol.	Treas Yield	Term Str.Slope					
	-2M	-1.99**	1.78**	-10.88**	24.83**					
	-1M	-1.89**	1.81**	-10.91**	22.54**					
-	0M	-1.96**	2.10**	-10.41**	19.66**					
_										

 $2.39^{**}$ 

2.47\*\*

2.47\*\*

 $2.51^{**}$ 

 $1.99^{**}$ 

2.84\*\*

3.60\*\*

2.49\*\*

-10.44\*\*

-10.45\*\*

-10.03\*\*

-8.57\*\*

-7.05\*\*

-6.53\*\*

 $-12.21^{**}$ -7.64\*\*  $16.93^{**}$ 

 $13.\overline{53^{**}}$ 

10.98\*\*

.45

-19.00\*\*

-29.72\*\*

 $-2\overline{1.08^{**}}$ 

-12.30\*\*

-2.05\*\*

-2.02\*\*

-2.01\*\*

-1.76\*\*

-1.02\*\*

.16

 $1.66^{**}$ 

1.28\*\*

1M

2M

3M

6M

1Y

2Y

3Y

5Y

# Table III Sensitivities w.r.t. Business Cycle Information

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

		Investment Grades	S	
Lag	Real GDP Growth	Ind. Prod Growth	CPI Growth	Pers. Inc Growth
-2M	-17.70**	-21.54	-16.36**	-25.96**
-1M	-17.47**	-48.37**	-15.85**	-27.13**
0M	-19.23**	-38.40**	-15.08**	-26.74**
1M	-20.81**	-46.55**	-12.77*	-25.14**
2M	-19.97**	-20.68	-11.57*	-23.49**
3M	-21.91**	-57.27**	-9.85**	-22.16**
6M	-19.84**	-46.64**	-4.47	-18.09**
1Y	-12.62**	-47.77**	-2.76	-9.72**
2Y	3.29	-24.67*	-3.91	3.13**
3Y	15.32**	5.99	-16.07**	40
5Y	17.42**	23.95	-22.23**	-4.42
		Non Investment Gra	des	
Lag	Real GDP Growth	Ind. Prod Growth	CPI Growth	Pers. Inc Growth
-2M	-21.27**	-35.17**	-9.09**	-20.58**
-1M	-21.44**	-39.38**	-7.87**	-19.82**
0M	-22.66**	-36.20**	-5.87**	-18.78**
1M	-23.01**	-50.57**	-4.47*	-17.26**
2M	-22.32**	-37.65**	-3.56**	-15.11**
3M	-22.58**	-54.06**	-2.02	-13.16**
$\frac{3M}{6M}$	-22.58** -17.57**	-54.06** -53.61**	-2.02 1.83	-13.16** -7.93**
			-	
6M	-17.57**	-53.61**	1.83	-7.93**
6M 1Y	-17.57** -6.78**	-53.61** -35.55**	1.83 .53	-7.93** 45

# Table IV Sensitivities w.r.t. Credit Markets Information

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

		Invest	tment Grades	3	
Lag	BBB Yield	BBB Spread	IG Spread	Treas. Issues	Money Lending
-2M	-10.28**	73.55**	$50.50^{**}$	1.70**	2.05
-1M	-10.23**	75.08**	50.38**	$1.55^{**}$	2.54
0M	-9.65**	74.35**	44.63**	.99**	3.39
1M	-9.39**	76.89**	41.38**	.90*	$4.45^{*}$
2M	-9.05**	72.86**	$34.49^{**}$	.98**	4.78**
3M	-8.38*	72.70**	35.73**	$1.09^{**}$	5.32**
6M	-6.95**	65.61**	25.75	.24	6.33**
1Y	-7.51*	49.11**	-7.02	75*	9.34**
2Y	-6.24*	44.19**	-19.64	-2.43**	9.02**
3Y	-8.60	41.57**	-13.48	-2.41**	12.09**
5Y	-9.84**	-4.38	-52.73**	-3.39**	6.13**
		Non Inv	restment Grad	des	
Lag	BBB Yield	BBB Spread	IG Spread	Treas. Issues	Money Lending
-2M	-5.62**	62.97**	53.08**	.59**	$4.56^{**}$
-1M	-5.76**	62.07**	$51.84^{**}$	.46**	4.88**
0M	-5.54**	59.09**	$45.95^{**}$	.14	$5.41^{**}$
$1\mathrm{M}$	-5.45**	$60.12^{**}$	$39.87^{**}$	02	6.00**
2M	-5.42**	60.30**	$31.96^{**}$	22	6.34**
3M	-5.02**	59.68**	$28.85^{**}$	26*	6.56**
6M	-4.24**	51.92**	$7.58^{*}$	71**	7.10**
1Y	-4.94**	28.91**	-13.84**	-1.61**	7.55**
2Y	-5.42**	18.04**	-23.09**	-2.11**	7.17**
3Y	-9.70**	30.16**	-15.08**	-1.76**	11.01**
<u> </u>	-7.91**	30.10	10.00	1.10	4.71**

#### Table V

#### Sensitivities to Aggregate Default Indicators

Estimations of log-linear intensities  $\lambda^{i}(u, t_{i})$  with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^{i}(u, t_{i}) = \exp(\gamma + \beta' Z(u + t_{i}))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Default Factors $\backslash$ Ratings	AA	А	BBB	BB	В	CCC
IG Upgrade Rate	-6.339	-16.691	-38.853	-42.941**	-68.568**	-70.051**
NIG Upgrade Rate	-5.054	-7.651	-22.777	-28.416	-33.657**	-41.229**
IG Downgrade Rate	38.517	53.027**	64.619**	59.399**	50.417**	$34.746^{**}$
NIG Downgrade Rate	22.426	24.379**	23.822**	$25.408^{**}$	$26.206^{**}$	23.990**

#### Table VI

#### **Contemporaneous Financial Multifactor Models**

Estimations of log-linear intensities  $\lambda^{i}(u, t_{i})$  with time-varying covariates over rating classes for durations up to the first exits and all countries. The table displays sensitivities  $\beta$  from multivariate specifications  $\lambda^{i}(u, t_{i}) = \exp(\gamma + \beta' \mathbf{Z} (u + t_{i}))$  where the default arrival is assumed to be piecewise exponential conditional on realizations of covariates. We focus on financial market information as used by several studies. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Model	Stock	Interest	Both	Stock	Interest	Both
	Market	Rates		Market	Rates	
Factors $\setminus$ Class		AA			А	
S&P500 Return	$-3.169^{**}$		-3.033*	-2.027**		-1.674*
S&P500 Vol.	0.029		0.097	0.349		0.350
Treas. Yield		-10.548	-5.704		-9.951**	-6.675*
Term. Str. Slope		19.72	12.008		$27.326^{**}$	19.683
		BBB			BB	
S&P500 Return	-2.112**		-1.441**	-1.920**		-1.436**
S&P500 Vol.	$2.132^{**}$		$2.552^{**}$	0.756**		$1.15^{*}$
Treas. Yield		-4.467	-1.899		-3.055*	-1.137
Term. Str. Slope		$33.254^{**}$	$25.661^{**}$		$25.799^{**}$	$17.109^{**}$
		В			CCC	
S&P500 Return	$-1.965^{**}$		$-1.687^{**}$	-1.273**		-1.272**
S&P500 Vol.	0.329		$0.166^{**}$	0.972*		0.351
Treas. Yield		-8.364**	-4.535**		$-14.611^{**}$	$-10.252^{**}$
Term. Str. Slope		16.932**	4.913**		-12.851*	-10.981**

#### Table VII

#### Parsimonious Multivariate Proportional Hazard Models

Estimations of semi-parametric models of default intensities with time-varying covariates over IG and NIG classes for durations up to the first exits and all countries. The table displays sensitivities  $\hat{\beta}$  from multivariate specifications  $\lambda^{i}(u, t_{i}) = \lambda^{\circ}(u) \exp(\beta' \mathbf{Z}(u + t_{i}))$ . \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Model	Stock	Interest	Both	Best	Stock	Interest	Both	Best
	Market	Rates		Six	Market	Rates		Six
Factors $\setminus$ Class		IG				NIG		
S&P500 Return	-2.26**		$-1.39^{**}$	-0.62*	-1.87**		$-1.62^{**}$	-0.23*
S&P500 Vol.	2.35**		$2.53^{*}$	2.99**	0.50*		$0.33^{*}$	$0.66^{**}$
Treas. Yield		$-10.67^{**}$	-4.37			-7.97**	-4.21	
Term. Str. Slope		32.22**	24.87**	$26.91^{**}$		$15.52^{**}$	$4.09^{*}$	4.18**
GDP				$-10.15^{*}$				-14.02**
BBB Spread				8.06				$12.16^{**}$
NIG Down. Rate				$14.92^{**}$				$15.86^{**}$

#### Table VIII

#### Improved Multivariate Proportional Hazard Models

Estimations of semi-parametric models of default intensities with time-varying covariates over rating classes for durations up to the first exits and all countries. The table displays sensitivities  $\hat{\beta}$  from multivariate specifications  $\lambda^{i}(u, t_{i}) = \lambda^{\circ}(u) \exp(\beta' \mathbf{Z}(u + t_{i}))$ . \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Factors \ Class	AA	А	BBB	BB	В	CCC	IG	NIG
S&P500 Return								
Contemporaneous	-2.74*	$-1.23^{*}$	$-1.64^{**}$	-1.08**	-1.25**	-0.653**	$-1.62^{*}$	-1.15**
Three Year Lagged	1.39	0.94	0.45	$1.56^{**}$	$1.66^{**}$	$2.49^{**}$	0.83	$1.65^{**}$
S&P500 Volatility								
Contemporaneous	-0.84	0.25	$2.28^{**}$	$0.62^{*}$	$0.65^{*}$	$0.77^{*}$	$2.52^{**}$	0.31
Three Year Lagged	1.94	3.54**	$2.12^{**}$	$3.65^{**}$	$3.11^{**}$	2.41**	$2.76^{**}$	3.13**

### Table IX Intra-Industry Default Behaviour

Log-ACD(1,1) estimates on inter-default durations within various industry categories. Ljung-Box Q-test and Arch-test on residuals including successively 1, 5, 10 and 20 lags (figures correspond to 20 lag) were found to be insignificant. \* (resp. \*\*) denotes significance at the 95% (resp. 99%) confidence level. Other sectors, namely Insurance and Real Estate, were too sparse to run estimations.

	a.			1	
Risk Class	Size	$w_c$	$a_c$	$b_c$	
Aerospace					
Automotive	232	0.1327	$0.1356^{*}$	0.8434**	
Capital Goods	292	0.1327	0.1350	0.0434	
Metal					
Consumer	200	0.1500*	0 1077**	0.0007**	
Service Sector	296	$0.1560^{*}$	$0.1077^{**}$	0.8627**	
Energy and	00	0 7501	0.0404	0 7020**	
Natural Resources	88	0.7581	0.0494	0.7938**	
Financial	80	9 1905*	0.9409**	0.2964	
Institutions	89	$2.1295^{*}$	0.2483**	0.2304	
Forest Products	68	$0.9152^{*}$	0.3383**	0.5042**	
and Building	08	0.9152	0.3383	0.0042	
Health Care	01	0.1450	0.0400	0.0004**	
and Chemicals	81	0.1459	0.0423	0.9284**	
High Tech	54	3.7032*	0.2779**	0.1516**	
and Office Eq.	-04	5.7052	0.2779**	$0.1516^{**}$	
Leisure Time	169	0 4079	0.1401*	0 7745**	
and Media	162	0.4072	0.1421*	$0.7745^{**}$	
Telecommunications	141	$0.2056^{*}$	$0.1894^{**}$	0.7730**	
Transports	79	0.0654	0.0422	0.9449**	
Utility	57	0.1671	0.1203**	$0.8532^{**}$	

# **The FAME Research Paper Series**

The International Center for Financial Asset Management and Engineering (FAME) is a private foundation created in 1996 on the initiative of 21 leading partners of the finance and technology community, together with three Universities of the Lake Geneva Region (Switzerland). FAME is about **Research**, **Doctoral Training**, and **Executive Education** with "interfacing" activities such as the FAME lectures, the Research Day/Annual Meeting, and the Research Paper Series.

The **FAME Research Paper Series** includes three types of contributions: First, it reports on the research carried out at FAME by students and research fellows; second, it includes research work contributed by Swiss academics and practitioners interested in a wider dissemination of their ideas, in practitioners' circles in particular; finally, prominent international contributions of particular interest to our constituency are included on a regular basis. Papers with strong practical implications are preceded by an Executive Summary, explaining in non-technical terms the question asked, discussing its relevance and outlining the answer provided.

Martin Hoesli is acting Head of the Research Paper Series. Please email any comments or queries to the following address: Martin.Hoesli@hec.unige.ch.

The following is a list of the 10 most recent FAME Research Papers. For a complete list, please visit our website at www.fame.ch under the heading 'Faculty and Research, Research Paper Series, Complete List'.

- **N°141 Understanding Default Risk Through Nonparametric Intensity Estimation** Fabien COUDREC, University of Geneva and FAME
- N°140 Robust Mean-Variance Portfolio Selection Cédric PERRET-GENTIL, Union Bancaire Privée, Maria-Pia VICTORIA-FESER, HEC, University of Geneva
- N°139 Trading Volume in Dynamically Efficient Markets Tony BERRADA, HEC Montréal, CIRANO and CREF, Julien HUGONNIER, University of Lausanne, CIRANO and FAME, Marcel RINDISBACHER, Rotman School of Management, University of Toronto and CIRANO.
- N°138 Growth Options in General Equilibrium: Some Asset Pricing Implications Julien HUGONNIER, University of Lausanne and FAME, Erwan MORELLEC, University of Lausanne, FAME and CEPR, Suresh SUNDARESAN, Graduate School of Business, Columbia University.
- N°137 On the Demand for Budget Constrained Insurance Richard WATT, Universidad Autónoma de Madrid, Henri LOUBERGÉ, University of Geneva and FAME
- N°136 Direct Prference for Wealth in Aggregate Household Portfolios Pascal ST-AMOUR, HEC, University of Lausanne, FAME, CIRANO and CIRPEE
- N°135 Indirect Robust Estimation of the Short-Term Interest Rate Process Veronika CZELLAR, Dept. of Econometrics, University of Geneva, G. Andrew KAROLYI, Fisher College of Business, Ohio State University, Elvezio RONCHETTI, Dept. of Econometrics, University of Geneva.
- N°134 Do Major Financial Crises Provide Information on Sovereign Risk to the Rest of the World? A Look at Credit Default Swap Markets Didier COSSIN, IMD International and FAME, Gero JUNG, Graduate Institute of International Studies.
  - Dider Cobbitt, hub incritational and 17142, Gero Jorto, Graduate institute of incritational studie.
- N°133 Are European Corporate Bond and Default Swap Markets Segmented? Didier COSSIN, IMD International, Hongze LU, IMD International, HEC, University of Lausanne.
- N°132 Conditional Asset Allocation Under Non-Normality: How Costly is the Mean-Variance Criterion? Eric JONDEAU, HEC Lausanne and FAME, Michael ROCKINGER, HEC Lausanne and FAME.



HEI



## **International Center FAME - Partner Institutions**

#### **The University of Geneva**

The University of Geneva, originally known as the Academy of Geneva, was founded in 1559 by Jean Calvin and Theodore de Beze. In 1873, The Academy of Geneva became the University of Geneva with the creation of a medical school. The Faculty of Economic and Social Sciences was created in 1915. The university is now composed of seven faculties of science; medicine; arts; law; economic and social sciences; psychology; education, and theology. It also includes a school of translation and interpretation; an institute of architecture; seven interdisciplinary centers and six associated institutes.

More than 13'000 students, the majority being foreigners, are enrolled in the various programs from the licence to high-level doctorates. A staff of more than 2'500 persons (professors, lecturers and assistants) is dedicated to the transmission and advancement of scientific knowledge through teaching as well as fundamental and applied research. The University of Geneva has been able to preserve the ancient European tradition of an academic community located in the heart of the city. This favors not only interaction between students, but also their integration in the population and in their participation of the particularly rich artistic and cultural life. *http://www.unige.ch* 

#### The University of Lausanne

Founded as an academy in 1537, the University of Lausanne (UNIL) is a modern institution of higher education and advanced research. Together with the neighboring Federal Polytechnic Institute of Lausanne, it comprises vast facilities and extends its influence beyond the city and the canton into regional, national, and international spheres.

Lausanne is a comprehensive university composed of seven Schools and Faculties: religious studies; law; arts; social and political sciences; business; science and medicine. With its 9'000 students, it is a mediumsized institution able to foster contact between students and professors as well as to encourage interdisciplinary work. The five humanities faculties and the science faculty are situated on the shores of Lake Leman in the Dorigny plains, a magnificent area of forest and fields that may have inspired the landscape depicted in Brueghel the Elder's masterpiece, the Harvesters. The institutes and various centers of the School of Medicine are grouped around the hospitals in the center of Lausanne. The Institute of Biochemistry is located in Epalinges, in the northern hills overlooking the city. *http://www.unil.ch* 

#### **The Graduate Institute of International Studies**

The Graduate Institute of International Studies is a teaching and research institution devoted to the study of international relations at the graduate level. It was founded in 1927 by Professor William Rappard to contribute through scholarships to the experience of international co-operation which the establishment of the League of Nations in Geneva represented at that time. The Institute is a self-governing foundation closely connected with, but independent of, the University of Geneva.

The Institute attempts to be both international and pluridisciplinary. The subjects in its curriculum, the composition of its teaching staff and the diversity of origin of its student body, confer upon it its international character. Professors teaching at the Institute come from all regions of the world, and the approximately 650 students arrive from some 60 different countries. Its international character is further emphasized by the use of both English and French as working languages. Its pluralistic approach - which draws upon the methods of economics, history, law, and political science - reflects its aim to provide a broad approach and in-depth understanding of international relations in general. *http://heiwww.unige.ch* 

UNIVERSITÉ DE GENÈVE







40, Bd. du Pont d'Arve PO Box, 1211 Geneva 4 Switzerland Tel (++4122) 312 09 61 Fax (++4122) 312 10 26 http: //www.fame.ch E-mail: admin@fame.ch



