

**Systematic patterns before
and after large price changes:**

**Evidence from high frequency
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Systematic patterns before and after large price changes:

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ABSTRACT

This paper examines the intra-day behavior of asset prices shortly before and after large price changes. Whereas similar studies so far have been based on daily closing prices, I use three years of high frequency data of 120 stocks listed on the French stock exchange. Various systematic patterns, in addition to those often reported in the literature, emerge from this data. Evidence is found that prices do overreact and that a correction takes place after large price movements, especially those to the downside. The correction does not take place immediately after the large price change. Prior to this, some very significant and sometimes economically important patterns can be observed. When the bid-ask spread is taken into account, I still find some *ex-post* profitable trading strategies that are too small in magnitude to suggest market inefficiency.

Keywords: predictable patterns, large price changes, high frequency data.

JEL classification: G10.

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Executive Summary

The recent availability on a large scale of intra-day data has allowed researchers to investigate the behavior of asset prices at new time scales. Thousands of quotes and transaction prices a day and for each asset comes along with considerable noise but may also provide interesting information about the behavior of prices. But who cares about these intra-day price changes? Traders, of course, who earn their bread with small but repeated profits generated from trading. I believe, however, that long-term fund managers should also become increasingly interested in such types or analyses as increased competition reduces profit margins. Furthermore, a half or one percentage point higher annual performance may significantly change the fund's rank. A better understanding of asset prices' behavior may help to achieve such goals.

For instance, a large price decline (of, say, 10% or more) of an asset's price may be seen as a good buying opportunity. Previous studies, based on daily closing prices, have indeed shown that stocks experiencing such heavy losses on a single day generally recover slightly in the days after the "event". In practice, fund managers have other opportunities to trade than only at the close of the trading day. This is exactly what the purpose of the present study is. If prices "bounce back" after large declines, it can be expected to begin before the end of the trading day. Identifying the moment of the trend reversal within the day may yield a few extra percentage points in return, eventually leading to a better ranking of the manager's annual performance.

Numerous problems of empirical nature have to be addressed in order to lead such an investigation. For instance, a 10% decline in price from one daily

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closing price to the next is a clear observable signal. Using intra-day data, other definitions have to be chosen, as a price will generally not experience such a large loss from one second to the next.

In this study I investigate three years of intra-day data for the 120 most widely traded stocks on the French Bourse. The results confirm the “bouncing back” phenomenon already reported in the literature dealing with daily closing prices. Interestingly, I also clearly identify a period of time necessary before the rebound takes places. In other words, once the downtrend is considered over, potential buyers take their time before entering the market.

A similar, although slightly different, pattern can be observed after large price increases. The results do depend on the definitions that are used to identify large price changes and trend reversals. The purpose of this study is not to build a trading model but rather to show that historical intra-day data may be very informative about the reaction of asset prices after important “shocks”. I am convinced that such analyses may very well lead a portfolio manager to generate some additional performance to his overall portfolio return.

This paper follows another study (available from the author) using intra-day data in which I jointly modeled the duration (measured in seconds) between significant price changes and returns derived from these price changes. The duration is seen as an inverse measure of the instant volatility. I find that past durations have a significant power in predicting future returns. Of course, these returns are small in magnitude and given the relatively high transaction costs associated with such trades, even for a floor trader, I do not suggest that these strategies could be considered as a profitable trading system. Identifying the

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right moments of buying or selling a stock within the trading day may, however, generate some much needed additional portfolio return.

Many more practical oriented studies using intra-day data (also referred to as “high frequency” data) can be expected in the coming years. The availability of these data, combined with cheap computing power, will give more insight in how prices are formed and change over time. They will provide the tools needed by the next century’s portfolio managers to remain on the competitive edge.

I. Introduction

The behavior of assets' returns before and especially after large price changes has now been widely documented in the financial literature. The strong interest in the subject is due to the fact that most of these studies have reported a systematic, and therefore predictable, pattern in the returns after the "event". At first glance, such predictability is incompatible with the notion of market efficiency. De Bondt and Thaler (1987) were among the first to report that stocks experiencing a more than 10% increase (or decrease) in price on a single day do earn, on average, a lower (higher) return in the next one to five years than the market portfolio. In other words, the initial overreaction is followed by a movement in the opposite direction; the greater the initial movement, the greater the correction. Numerous subsequent studies have investigated the issue, using different methodologies and data frequencies. While Ball and Kothari (1989), Chan (1988), Chan and Chen (1991) or Zarowin (1989, 1990) investigate long term returns up to five years after the initial event, recent studies use daily closing prices to study systematic patterns shortly after the initial movement. These studies include the ones by Atkins and Dyl (1990), Bremer and Sweeney (1991), Bremer, Hiraki and Sweeney (1997), Brown, Harlow and Tinic (1988, 1993), Cox and Peterson (1994), Howe (1986), Lehmann (1990), Park (1995) and Renshaw (1984). The rebound phenomenon is usually observed, although the significance is often low, and usually disappears when the so called "data selection bias" is taken into account. This bias is introduced by the fact that closing prices on the day of a large price decrease (increase) are more likely to be close to the bid (ask) price, whereas there is no such obvious bias on the days following the large price change.

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This paper tries to make two contributions to the large existing literature. To begin with, three years of high frequency data are used, which allows capturing the details of the intra-day price movements. Studies based on closing prices inevitably leave out of the analysis all events where the rebound phenomenon took place within the trading day. Furthermore, I add to the international evidence on the subject by investigating all 120 stocks of the French SBF 120 index. To my knowledge, only the study by Bremer, Hiraki and Sweeney (1997) is based on non-US data. For a sample of liquid stocks traded on the Tokyo Stock Exchange they report results similar to those obtained so far from US data.

The French database contains all necessary information for a good analysis: all bid and ask quotes and trading prices are reported throughout the day, together with the volumes, with a very high degree of accuracy. However, important methodological issues have to be addressed before the analysis can be conducted within a high frequency framework, the most important one being the definition of a large price change. Whereas previous studies generally set a minimum one-day price change (ranging from 2.5% to 10%), the use of tick by tick data requires a more appropriate definition that takes into account the very specific meaning of “time” in this context¹. It would not be rational, for instance, to consider as “event” a change of 10% or more on two consecutive bid-ask quotes. Although this may be the case at the opening of the market compared to the

¹ For a general discussion about time in the context of high frequency data, see, for instance, Müller *et al.* (1990) or Goodhart and O’Hara (1997).

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close on the previous trading day, we are typically interested in many more situations, for example, where the assets' price moves by a "large amount" over a "short period" of time. These are some of the issues that will be addressed in the next sections.

The remainder of this paper is organized as follows. Section II presents the data and the methodology. The results are analyzed in Section III and Section IV provides some concluding remarks.

II. Data and Methodology

The data was provided by the SBF (Société de Bourse Française) and covers the full years from 1995 to 1997. Although the initial data set contains all bid and ask quotes each time one of the quotes changes, in addition to all transaction prices, an initial screening is performed by keeping only those quotes that immediately precede a transaction. In other words, bid and ask quotes that do not result in an actual trade are deleted. A possible bias due to illiquidity in the market is thus avoided. All prices are adjusted for dividend payouts, splits and other changes in capital.

The use of high frequency data necessitates some further filtering. Successive quotes are often very close to each other and would introduce noise in the model rather than provide exploitable information about price changes. In Engle and Russell (1997), Dollar/Deutschmark tick by tick quotes are filtered in the following way: a bid-ask quote is retained in the filtered database only if the average of the bid and ask prices is at least 5 pips (approximately 0.03% for the Dollar/Deutschmark) above or below the average of the previously retained bid-

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ask quote. The 5 pips threshold also corresponds to the most often observed bid-ask spread. By applying this filter, the database is reduced to 5% of its initial size. In the case of individual stocks, the market is much less liquid and the quoted bid-ask spread is generally much larger than 0.03%. In order to consider various possibilities, I construct data sets by retaining only those prices for which the price variation compared with the previous retained price is at least 0.25%, 0.50% and 1%. This minimum price variation will be referred to as the filter value applied to the initial raw data set. In order to account for a possible bias due to the sample selection, all prices are defined as the average of the bid and the ask quote. Cox and Peterson (1994), for instance, find that the bias introduced when daily transaction data are used instead of daily bid and ask quotes explains earlier documented price reversals. This bias can also be expected to be economically important at the level of high frequency data. Furthermore, all prices and price changes can be considered economically meaningful, as they correspond to actual transactions.

The existing literature considers various definitions of large price changes and abnormal returns following the event. Both may be calculated in absolute terms [as in Cox and Peterson (1994) for large price changes, for instance], but also relative to a benchmark. This benchmark may be an average historical return [see Brown and Warner (1980, 1985) and Atkins and Dyl (1990)] or the market model. When the market model is chosen, the abnormal returns are adjusted for risk, but rely on the strong implicit assumption that the market model is valid. Brown, Harlow and Tinic (1988) argue that securities' systematic risk can be expected to be significantly different before and after the event. In the literature various β estimates have been proposed in order to overcome, at least partially,

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this problem [for a review, see, for instance, Bremer, Hiraki and Sweeney (1997)]. Nevertheless, throughout this study, only the case where β is equal to one will be considered, that is, the market return is simply subtracted from the asset returns [as in Park (1995)]. The main reason is that the instability of the estimated systematic risk coefficients is likely to play a more important role in the case of high frequency data. The estimation of the market return itself is also an issue that has to be carefully addressed in this context. The SBF database contains the level of the SBF 120 index at a frequency of one price every 30 seconds. The 120 stocks for which price changes of a given magnitude only are retained, do not, in most cases, have time stamps that correspond exactly to those of the index. The market return that is subtracted from the asset returns is obtained by interpolating the index levels with the closest time stamps. Although this method is imperfect, I consider the frequency of the index data (30 seconds) high enough for the results to be meaningful.

The definition of a “large price change” varies in the literature. The one-day price change is usually at least 10%, either positive or negative, and results for other values of the filter are often provided as well. Throughout this study I will consider three levels of large price changes: 2.5%, 5% and 10%. Although the choice of both the filter value and the definition of a large price change is arbitrary in a sense, the filter can be seen as a measure of a trader’s time horizon. Short-term traders are likely to be interested in small price movements, whereas longer term traders look for larger price changes. The former will therefore consider smaller large price movements more important than the latter. The end of a large price movement (of a given minimum magnitude) is defined by the first observation of the opposite sign to the large price movement. The

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beginning of the movement is defined by the last observation also of the opposite sign. In order to remain in a framework of a sharp price increase or decrease during a short period of time, I also impose that the large price change should occur within 6 trading hours. Again, this is arbitrary but necessary in order not to consider the case where a stock price slowly drops over a couple of days².

Abnormal returns are measured here in framework that has become standard by now and which is well described, for instance, in Atkins and Dyl (1990). An equally weighted portfolio containing the 120 stocks is formed and the portfolios' average abnormal return for period h is defined as the sum of individual assets' abnormal returns for period h divided by the number of observations in the portfolio. The Cumulated Average Returns (CARs) are obtained by summing the individual periods' abnormal returns. All returns are calculated as the first difference of the logarithms of the price levels. Following Bremer and Sweeney (1991), Simple cross-sectional t -statistics are used to test for the significance of period's h CAR. A period corresponds to a price move of at least (and mostly not very different from) the value of the filter applied to the initial data set (0.25%, 0.5% or 1%) and is therefore independent from "calendar" time. Up to 20 periods before and after the large price change are analyzed. All events, pre-event and post-event periods are non-overlapping.

² I have performed a variety of runs using different maximum durations for the large price change. The results (available on request) are not much affected by changes in this maximum duration value.

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Nevertheless, in order to keep as many events as possible to analyze and to have a realistic picture of what would have happen *ex post*, I only require that all events with a given h number of periods are non overlapping. In other words, two successive large price changes with a small number of significant price changes in between may be accounted for as two events when $h=1$ or $h=2$ periods after the event are analyzed and only as one event when $h=20$ periods are analyzed. In the latter case, the second large price movement will influence the CARs up to period $h=20$. Thus the number of 1 period abnormal returns is larger than the number of 20 periods abnormal returns. I argue that this basis is more realistic than considering all periods to be non-overlapping. In the latter case, the CARs for $h=1$ period would depend on the maximum number of periods (20) investigated. In this sense, my approach is different from the one taken for instance by Bremer and Sweeney (1991) or Cox and Peterson (1994).

A number of other variables, in addition to the CARs, are interesting to analyze. To begin with, the average bid ask spread as a percentage of the average of the bid and ask prices is reported, as well as the closing price ratio (CPR). As in Park (1995), this latter ratio captures whether a transaction is buyer or seller initiated and is defined by the difference between the transaction price and the bid price, divided by the absolute spread (difference between the ask price and the bid price). I also report the percentage of the trades occurring at the bid price and those occurring at the ask price.

III. Results

This section presents the results obtained with the full sample of stocks belonging to the SBF 120 between 1995 and 1997 and is in four parts. I first

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give a descriptive analysis of the large price changes obtained with the different filters. The global characteristics of the *pre* and *post* event periods are then presented for various holding periods h . In a third part, a detailed analysis of the *post* event observations is given. Finally, I investigate whether the previously obtained results could have generated significant excess market trading returns once the bid-ask spread is taken into account.

A. An Analysis of Large Price Changes

Table 1 reports the main characteristics of the events. All events are taken as non-overlapping and do not depend on the number of periods before or after the event, which will be investigated later. Large price changes of 2.5%, 5% and 10% are considered for each of the three filters (0.25%, 0.5% and 1%) applied to the initial data. Large price decreases are shown in Panel A, whereas Panel B deals with large price increases. The magnitude of the average price change is almost always slightly below the value of the definition of the large price change. This is due to the definition used for identifying the end of the trend characterizing the large price change. The number of identified events is almost 9000 for the 2.5% price increases and as low as 55 for price increases of 10% at least. A large price change of 2.5%, either positive or negative, occurs on average once a week for each stock. The bid-ask spread observed at the end of the event is clearly higher for the 5% and 10% price changes than for the 2.5% ones. It is also higher for price decreases than for price increases. Both the CPR figures and the percentage of trades occurring at the bid price and at the ask price show that after a large price decrease (increase), a majority of trades occur at or close to the bid (ask) price. The only exception is the 10% or more price decline with the 1% filtered data. Although this is what one may intuitively

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expect after negative or positive events, it shows that the definition used here to identify the reversal after the trend characterizing the large price change does not correspond exactly to what is perceived by the market. The bid-ask asymmetry is more pronounced after large price increases, suggesting the presence of more buyers after a large price increase than sellers after a large price decrease. The duration is the effectively elapsed time covered by the large price change and ranges between 2:26 and 5:43 hours. Not surprisingly, data filtered with larger filters show longer durations. Large price changes are faster incorporated into the prices than smaller ones. The bid volumes are significantly higher than the ask volumes after large price decreases, indicating a selling pressure after large price decreases. No clear pattern is observed after large price increases. Trading volume is also almost always higher after large price decreases.

B. Some aspects of *pre* and *post* event periods

This section discusses the behavior of a few characteristics of the data around large price changes. The focus will be on a global analysis, while a more deep going one will be provided for *post* event periods in section C.

Figure 1 shows the CARs for $h=1,\dots,20$ periods before and after large price changes for the 1% filtered data³. After large (10% or more) decreases, the

³ The results for the other filters are very similar, although generally less pronounced, and therefore not reported here but available from the author.

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Table 1: Descriptive statistics of the samples obtained from three levels of large price changes applied to high frequency data filtered with three levels of minimum price changes

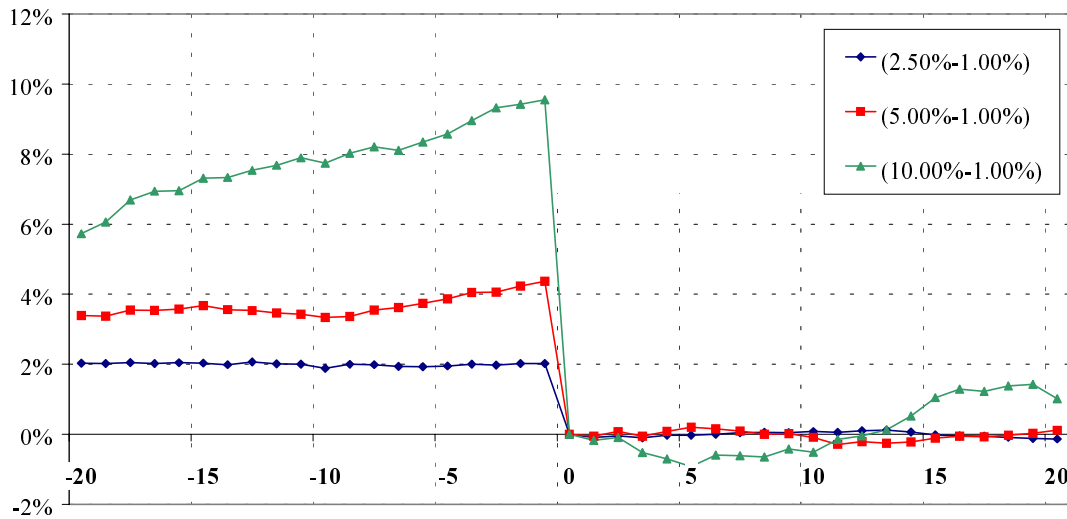
SigPC	BigPC	Mean	N	Spread	CPR	On Bid	On Ask	Duration (hours)	Bid Volume	Ask Volume	Trading Volume
<i>Panel A: After a Large Price Decrease</i>											
0.25%	2.5%	-2.15%	7 185	0.95%	39.2%	59.1%	38.6%	02:37	56 314	53 686	7 459
0.25%	5%	-4.54%	1 066	1.33%	41.2%	57.3%	40.1%	02:28	206 642	99 953	9 795
0.25%	10%	-9.47%	95	1.27%	46.8%	52.6%	47.4%	02:26	415 229	352 636	18 724
0.5%	2.5%	-2.14%	4 732	1.04%	42.2%	56.2%	41.3%	03:53	353 523	111 752	21 952
0.5%	5%	-4.59%	846	1.39%	44.8%	54.3%	43.0%	03:29	1 313 592	263 312	26 270
0.5%	10%	-9.60%	85	1.38%	41.6%	56.5%	40.0%	03:32	575 153	254 229	29 094
1%	2.5%	-2.27%	2 234	1.12%	47.5%	51.2%	46.7%	05:36	478 429	395 731	63 276
1%	5%	-4.73%	500	1.46%	46.5%	52.2%	46.0%	04:49	855 993	529 132	67 718
1%	10%	-9.74%	57	1.27%	52.4%	45.6%	50.9%	04:17	1 516 898	1 241 325	105 853
<i>Panel B: After a Large Price Increase</i>											
0.25%	2.5%	2.22%	8 979	0.84%	59.4%	39.5%	58.4%	02:31	76 680	33 962	7 395
0.25%	5%	4.57%	1 511	1.14%	60.4%	37.7%	59.8%	02:33	61 451	67 619	7 817
0.25%	10%	9.31%	101	1.16%	68.2%	31.7%	67.3%	02:28	257 132	200 125	12 867
0.5%	2.5%	2.29%	5 923	0.89%	56.5%	42.7%	55.1%	03:53	152 052	110 143	20 085
0.5%	5%	4.63%	1 192	1.12%	56.8%	42.6%	55.6%	03:36	340 219	112 176	17 514
0.5%	10%	9.40%	78	1.31%	71.0%	28.2%	69.2%	03:03	1 129 724	582 392	31 157
1%	2.5%	2.46%	2 799	0.94%	53.4%	45.7%	52.1%	05:43	713 396	1 074 904	63 011
1%	5%	4.63%	701	1.15%	53.5%	45.8%	50.8%	04:57	587 997	545 297	54 593
1%	10%	10.28%	55	1.22%	61.6%	38.2%	60.0%	03:45	528 790	1 631 083	71 188

The data was provided by the SBF (Société de Bourse Française) and covers three full years from 1995 to 1997. The first column gives the value of the filter that is applied to the raw high frequency data set: only those observations (an observation is a transaction price with the corresponding bid and ask quotes) are retained for which the price variation compared with the previously retained price is at least of the magnitude of the filter. A price is defined as the average of the bid and ask quote. The second column indicates the minimum magnitude of the “event”, that is, the large price change. The reported statistics pertain to all non-overlapping large price changes. In order to be retained, the large price change has to take place within 6 trading hours.

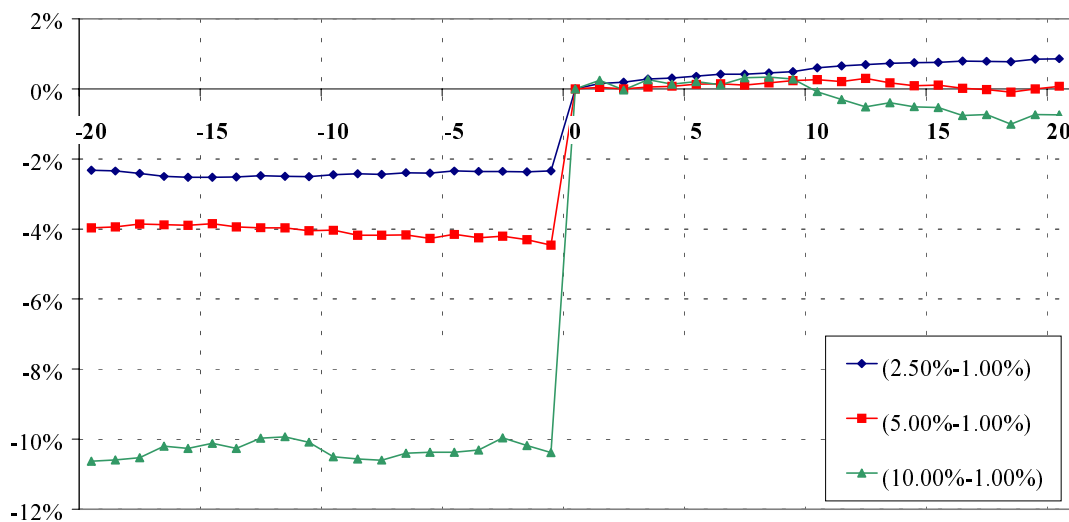
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Figure 1: Cumulated Average Returns around large price changes for the 1% filtered data

Panel A: Around large price decreases



Panel B: Around large price increases



The CARs for price changes of 2.5%, 5% and 10% are calculated from the end of period zero (the large price change). No evidence of knowledge by some market participants about the future event is to be seen from the graphs.

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prices show a little persistence in the downtrend over 5 periods before bouncing back. This is less pronounced for smaller price changes. Prior to the event, a strong positive trend is detected for the largest event, suggesting the large price decrease is (partially) the consequence of a recent significant price increase. In none of the cases do we observe a negative trend prior to the large price decrease, which would have to be interpreted as evidence that information about the coming price decline is known to some investors.

The use of intra-day, high frequency data allows to pay a closer attention to variables that can be expected to have fewer informative contents within a daily framework using closing prices. The average spread is one of these variables. Figure 2 reports the average spread (defined as a percentage of the average of the bid and ask price) around the large price change. The Figure only reports the results for the 0.25% filtered data, but results are similar for larger filters. The broad picture is the same both for large price decreases and increases: prior to small price changes the average spread slightly increases over the 20 periods; in all cases the spread strongly increases at the moment of the event; the decay that follows is progressive and slower after price decreases than price increases. Interestingly, in all cases the pre-event spread is a positive function of the magnitude of the large price change to come. A possible explanation is that larger price changes tend to occur more often with risky stocks, for which the spread is expected to be larger. On the other hand, 15 periods after the event the spread for the 10% moving stocks is almost identical to the one observed for 5% moving stocks. Twenty periods before the event the stocks that are going to experience the 10% move have a much larger spread (respectively, 0.97% prior to price declines and 0.93% prior to price increases) than the stocks that will

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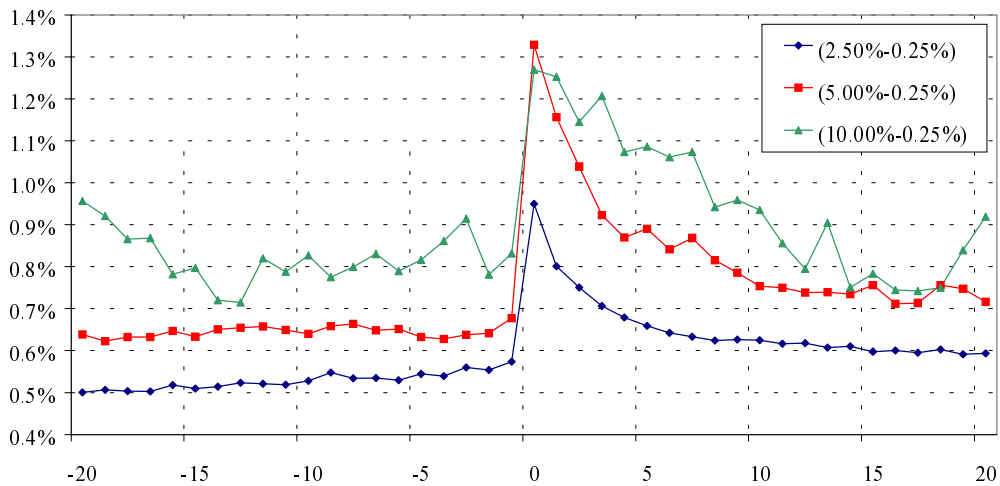
experience a 5% move (respectively 0.64% and 0.62%). This is either evidence that the perceived risk level of 10% moving stocks drops to the same level as the one perceived for stocks having moved by 5%, or that the market –or at least some participants– have an ability to anticipate whether the future price move will be of the 5% or 10% magnitude. Another observation that can be made is that in the period following the 10% increase the average spread still increases and remains at an exceptionally high level for two more periods. This suggests that for very large price movements, either the simple rule used to define the end of the upward trend does not correspond to how the market perceives the very short term continuation of the trend, or that the anticipated volatility remains extremely high for a few more periods.

I now turn to the “Closing Price Ratio” (CPR) indicator. This is the average difference between the transaction price and the bid price divided by the spread. A value above 50% indicates that more trades occur closer to the ask price than to the bid. Figure 3 reports the results for the 0.25% filtered data: the larger the large price change, the more important the volatility of the CPR indicator for the different periods. Just prior to the large price decrease, the CPR indicator is very high, indicating that most trades occur on the ask. This is due to the method chosen to set the beginning of the trend: a price movement in the opposite direction of the trend. A similar observation can be made just prior to large price increases.

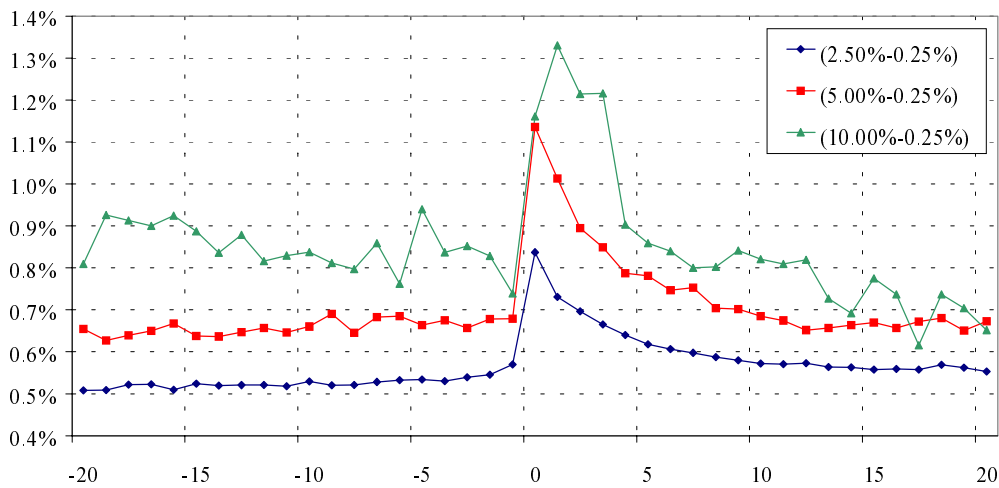
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Figure 2: Average spreads before and after large price changes for the 0.25% filtered data

Panel A: Around large price decreases



Panel B: Around large price increases

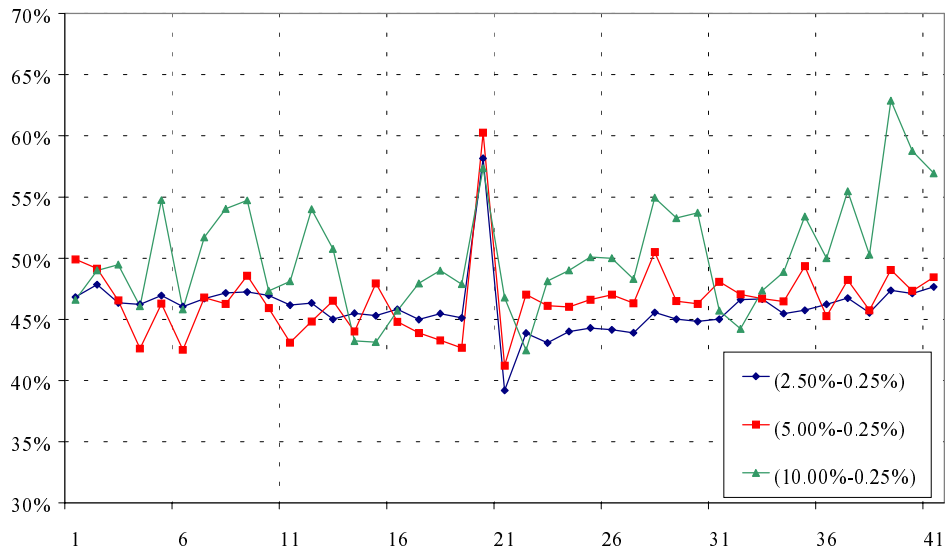


The average spread, as a percentage of the average of the bid and ask quotes, increases sharply with the large price change. The effect is more pronounced for large price decreases. The decay is progressive but faster after price increases. Large pre-event spreads are associated with large price changes.

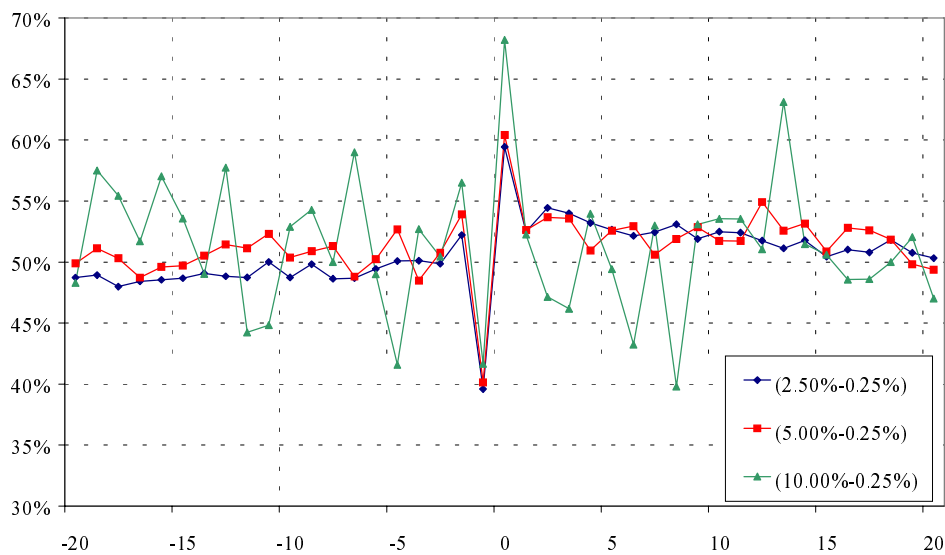
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Figure 3: CPR values around large price changes of at least 5%

Panel A: Around large price decreases



Panel B: Around large price increases

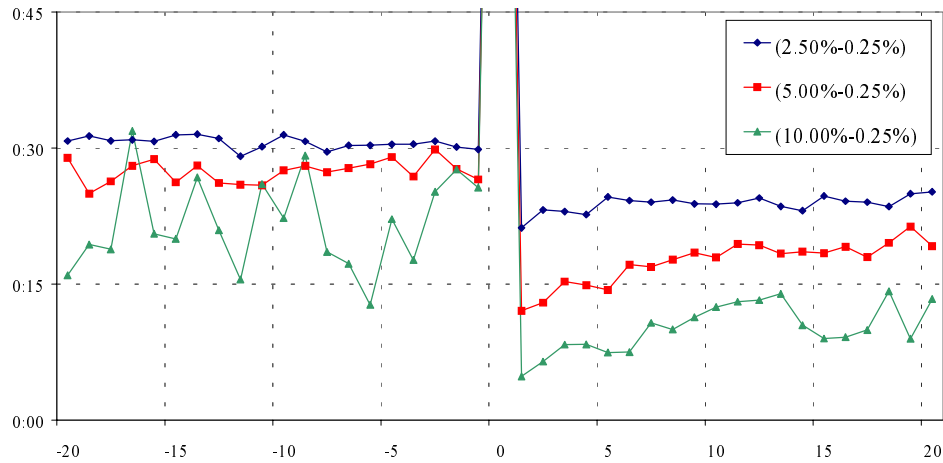


The CPR is the difference between the transaction price and the bid price divided by the spread value.

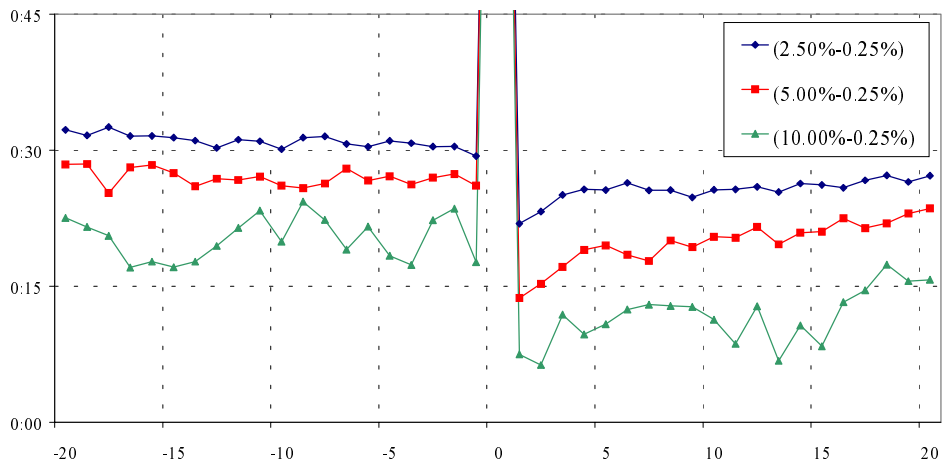
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Figure 4: Average duration (in minutes) around large price decreases for the 0.25% filtered data

Panel A: Around large price decreases



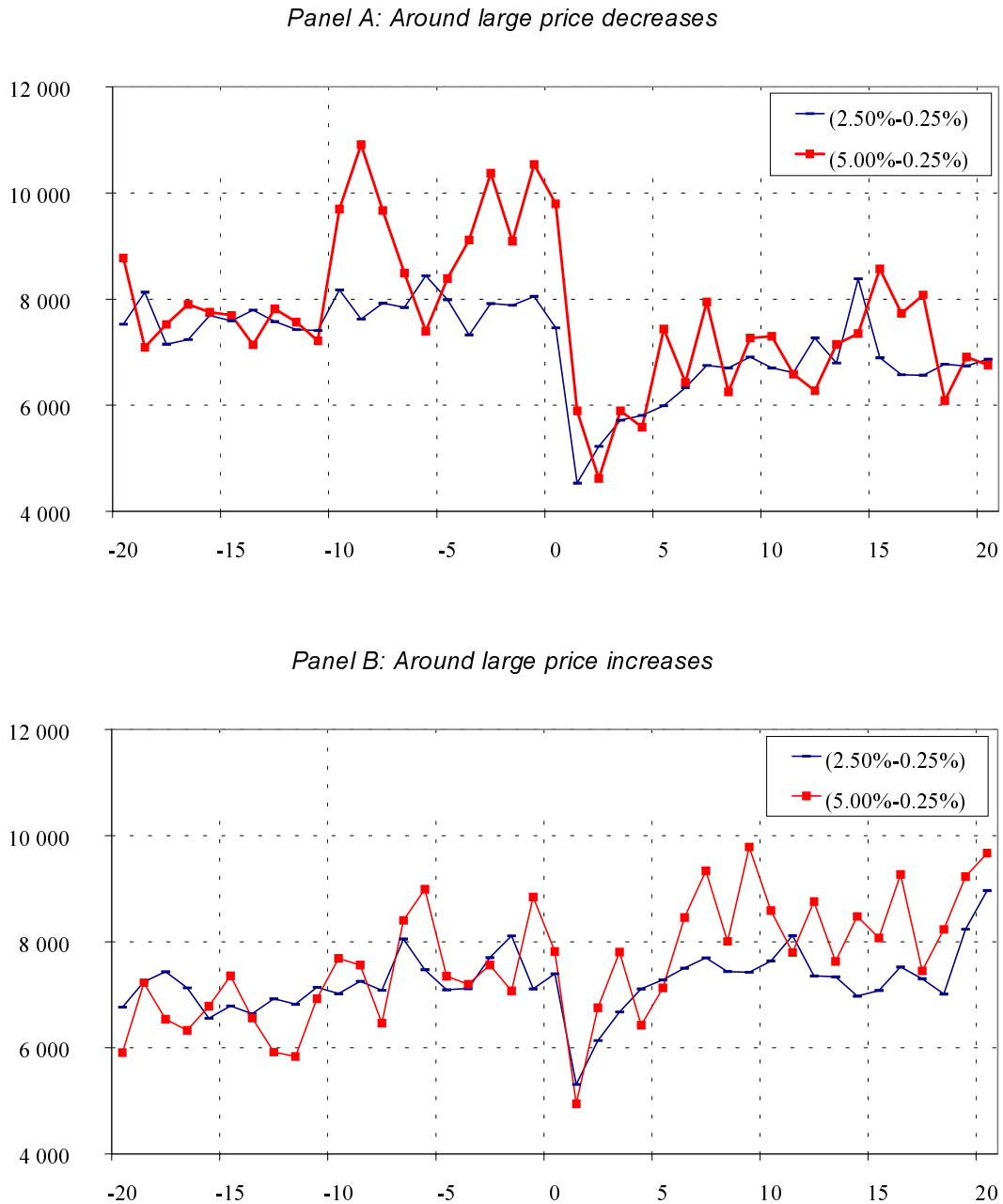
Panel B: Around large price increases



This table reports the average duration for each individual period h (not the cumulated durations). The much larger duration for the large price change has been omitted from the graph. Smaller durations (and therefore larger volatility) are observed prior to events of larger magnitude.

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Figure 5: Average traded volume around large price changes for the 0.25% filtered data



The traded volume sharply declines after larger price changes. The decline is more pronounced for larger price changes and also stronger for price decreases than for price increases.

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Figure 4 reports the duration for each individual period h . Smaller durations are observed prior to larger price changes, either of positive or negative sign. After the large price change the recovery of the duration between price changes is progressive, but the difference between 2.5%, 5% and 10% price changes remains important. Also, the durations are notably lower after the event than before.

The average traded volume is represented in Figure 5 for the 0.25% filtered data and price changes of 2.5% and 5%. Clearly, a large price change comes along with a sharp drop in trading volume. The drop is much stronger in the case of a price decrease. Large price changes can be the consequence of large market orders while the market is thin or the volume can drop because there is a large price movement. The latter seems to be the case because there is no sign that the volume is smaller prior to the large price change. The contrary is true: there is some evidence that the trading volume increases prior to the move, which could, again, indicate that some market participants trade in the market large volumes in anticipation of the price change to come. The trading volume returns quicker to its normal level after large price increases than large price decreases.

C. Post event results

In this section I analyze the post-event periods in details. While the pre-event periods are interesting to study, because they may indicate insiders' information held by some agents, the post-event periods throw light on matters such as market efficiency. In particular, a large price change should not be a trading signal after which a systematic, and potentially profitable, pattern is found in the

Table 2: Characteristics of Cumulated Average Returns after large price changes

Panel A. Raw data with the 0.25% Filter

Holding Period	Large Price Changes of 2.5% at least						Large Price Changes of 5% at least						Large Price Changes of 10% at least					
	Large Price Decreases			Large Price Increases			Large Price Decreases			Large Price Increases			Large Price Decreases			Large Price Increases		
	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N
1	-0.080% *	-10.79	7006	0.090% *	14.67	8749	-0.044%	-1.65	1060	0.074% *	3.90	1503	-0.090%	-1.14	95	0.059%	0.80	101
2	-0.069% *	-7.63	6828	0.087% *	11.35	8532	0.043%	1.29	1052	0.042% *	1.84	1497	0.021%	0.21	94	-0.005%	-0.05	101
3	-0.076% *	-7.02	6657	0.104% *	11.33	8303	0.100% *	2.65	1045	0.024%	0.93	1481	0.033%	0.21	94	0.079%	0.64	101
4	-0.078% *	-6.24	6494	0.112% *	10.39	8086	0.099% *	2.46	1038	0.017%	0.57	1472	-0.021%	-0.11	93	0.055%	0.38	101
5	-0.082% *	-5.82	6323	0.132% *	10.76	7875	0.122% *	2.76	1030	-0.006%	-0.19	1466	-0.112%	-0.49	93	0.033%	0.21	101
6	-0.076% *	-4.97	6168	0.142% *	10.62	7645	0.158% *	3.47	1023	0.001%	0.04	1457	-0.124%	-0.58	92	0.000%	0.00	100
7	-0.078% *	-4.61	6012	0.153% *	10.44	7440	0.180% *	3.73	1017	-0.001%	-0.02	1448	-0.055%	-0.26	91	-0.122%	-0.77	100
8	-0.077% *	-4.22	5839	0.171% *	10.63	7231	0.236% *	4.45	1010	0.010%	0.25	1439	-0.012%	-0.05	91	-0.032%	-0.13	100
9	-0.076% *	-3.93	5703	0.197% *	11.23	7028	0.251% *	4.42	1003	0.015%	0.35	1431	-0.117%	-0.53	90	-0.199%	-0.81	100
10	-0.083% *	-4.04	5565	0.202% *	10.89	6843	0.256% *	4.21	997	0.036%	0.81	1425	-0.081%	-0.38	90	-0.153%	-0.63	100
11	-0.093% *	-4.28	5436	0.220% *	11.25	6680	0.254% *	4.11	987	0.076%	1.62	1411	-0.169%	-0.67	88	-0.104%	-0.38	99
12	-0.090% *	-3.94	5315	0.219% *	10.56	6521	0.257% *	3.77	980	0.066%	1.35	1401	-0.416%	-1.24	88	-0.126%	-0.44	99
13	-0.094% *	-3.89	5186	0.226% *	10.48	6385	0.278% *	3.94	977	0.065%	1.29	1388	-0.305%	-0.96	88	-0.057%	-0.18	99
14	-0.096% *	-3.81	5072	0.236% *	10.42	6253	0.266% *	3.66	975	0.074%	1.40	1381	-0.319%	-1.01	88	-0.082%	-0.26	99
15	-0.104% *	-3.99	4963	0.265% *	11.10	6113	0.299% *	3.93	970	0.075%	1.38	1373	-0.298%	-0.89	88	-0.125%	-0.40	98
16	-0.103% *	-3.75	4865	0.274% *	10.97	5969	0.296% *	3.77	966	0.071%	1.25	1361	-0.359%	-1.06	88	-0.212%	-0.66	98
17	-0.093% *	-3.28	4761	0.292% *	11.15	5831	0.297% *	3.70	961	0.085%	1.44	1352	-0.442%	-1.24	88	-0.245%	-0.78	98
18	-0.085% *	-2.89	4658	0.297% *	10.89	5705	0.287% *	3.51	959	0.097%	1.59	1344	-0.501%	-1.38	88	-0.172%	-0.53	98
19	-0.078% *	-2.58	4560	0.304% *	10.71	5580	0.250% *	2.95	957	0.092%	1.46	1336	-0.494%	-1.35	88	-0.171%	-0.53	98
20	-0.084% *	-2.68	4477	0.316% *	10.63	5473	0.235% *	2.67	949	0.065%	1.01	1328	-0.534%	-1.43	88	-0.128%	-0.42	98

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Panel B. Raw data with the 0.5% Filter

Holding Period	Large Price Changes of 2.5% at least						Large Price Changes of 5% at least						Large Price Changes of 10% at least					
	Large Price Decreases			Large Price Increases			Large Price Decreases			Large Price Increases			Large Price Decrease			Large Price Increases		
	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N
1	-0.113% *	-8.80	4505	0.089% *	8.33	5641	-0.012%	-0.33	837	0.058% *	2.12	1178	-0.070%	-0.56	85	-0.006%	-0.07	78
2	-0.080% *	-4.78	4288	0.107% *	7.54	5374	0.091% *	2.14	826	0.033%	0.97	1163	0.024%	0.14	85	-0.281% *	-2.44	78
3	-0.098% *	-4.69	4084	0.104% *	5.72	5134	0.102% *	1.99	818	-0.005%	-0.11	1152	-0.052%	-0.30	84	-0.283% *	-1.87	78
4	-0.075% *	-3.03	3909	0.137% *	6.23	4919	0.120% *	2.09	806	-0.017%	-0.35	1141	-0.089%	-0.38	84	-0.175%	-0.92	78
5	-0.069% *	-2.49	3747	0.167% *	6.62	4689	0.172% *	2.58	793	-0.002%	-0.03	1123	-0.246%	-1.01	82	-0.173%	-0.58	78
6	-0.054%	-1.74	3612	0.183% *	6.48	4508	0.155% *	2.11	786	0.029%	0.48	1108	-0.063%	-0.24	82	-0.120%	-0.40	78
7	-0.062% *	-1.83	3492	0.192% *	6.19	4366	0.174% *	2.13	780	0.055%	0.84	1095	-0.156%	-0.56	81	-0.024%	-0.07	78
8	-0.055%	-1.50	3375	0.220% *	6.49	4225	0.141%	1.56	773	0.086%	1.22	1081	-0.308%	-0.98	81	-0.010%	-0.03	77
9	-0.062%	-1.57	3245	0.257% *	6.86	4063	0.114%	1.19	767	0.101%	1.36	1072	-0.456%	-1.19	81	0.196%	0.49	77
10	-0.030%	-0.71	3117	0.293% *	7.39	3926	0.108%	1.06	761	0.066%	0.86	1064	-0.529%	-1.39	81	0.201%	0.50	77
11	-0.021%	-0.46	3012	0.334% *	7.88	3800	0.094%	0.86	753	0.053%	0.63	1051	-0.683%	-1.66	81	0.209%	0.52	77
12	-0.002%	-0.05	2919	0.363% *	8.02	3656	0.101%	0.88	746	0.033%	0.37	1045	-0.853% *	-1.87	80	0.118%	0.29	76
13	0.009%	0.18	2832	0.380% *	7.89	3510	0.123%	1.03	737	0.049%	0.53	1038	-0.775%	-1.66	80	-0.010%	-0.02	76
14	0.005%	0.08	2744	0.384% *	7.55	3404	0.150%	1.24	732	0.052%	0.53	1028	-0.934% *	-2.02	80	-0.053%	-0.12	75
15	0.042%	0.73	2666	0.398% *	7.40	3298	0.156%	1.23	727	0.135%	1.30	1023	-0.819%	-1.75	80	-0.088%	-0.20	74
16	0.057%	0.96	2592	0.402% *	7.18	3203	0.159%	1.22	714	0.090%	0.87	1018	-0.684%	-1.48	80	-0.027%	-0.06	74
17	0.074%	1.18	2506	0.459% *	7.81	3101	0.215%	1.59	700	0.109%	1.03	1009	-0.405%	-0.87	80	0.053%	0.11	74
18	0.089%	1.36	2441	0.455% *	7.38	3016	0.214%	1.55	693	0.085%	0.78	1005	-0.382%	-0.79	80	0.141%	0.30	74
19	0.069%	1.02	2360	0.464% *	7.24	2920	0.191%	1.31	690	0.015%	0.14	998	-0.236%	-0.43	79	0.207%	0.43	74
20	0.072%	1.02	2306	0.489% *	7.30	2844	0.198%	1.34	689	0.014%	0.12	990	-0.031%	-0.06	79	0.403%	0.79	74

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Panel C. Raw data with the 1% Filter

Holding Period	Large Price Changes of 2.5% at least						Large Price Changes of 5% at least						Large Price Changes of 10% at least					
	Large Price Decreases			Large Price Increases			Large Price Decreases			Large Price Increases			Large Price Decrease			Large Price Increases		
	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N	Mean	t-stat	N
1	-0.087% *	-2.86	2107	0.157% *	5.96	2625	-0.057%	-0.88	488	0.046%	0.85	679	-0.176%	-0.97	57	0.249%	1.37	55
2	-0.046%	-1.11	1996	0.197% *	4.85	2492	0.077%	0.86	482	0.004%	0.05	668	-0.092%	-0.32	56	-0.031%	-0.13	55
3	-0.098% *	-1.87	1897	0.283% *	5.65	2372	-0.058%	-0.51	467	0.056%	0.59	656	-0.523%	-1.27	55	0.266%	0.81	54
4	-0.030%	-0.47	1806	0.309% *	5.32	2266	0.086%	0.64	456	0.075%	0.65	644	-0.703%	-1.42	54	0.138%	0.29	54
5	-0.032%	-0.44	1705	0.358% *	5.33	2165	0.204%	1.36	442	0.136%	1.04	638	-0.927%	-1.57	54	0.210%	0.40	53
6	-0.005%	-0.06	1623	0.419% *	5.52	2042	0.157%	0.91	435	0.147%	1.02	626	-0.591%	-0.97	54	0.117%	0.20	53
7	0.042%	0.45	1552	0.417% *	5.03	1940	0.088%	0.48	428	0.114%	0.70	619	-0.612%	-0.95	54	0.321%	0.48	53
8	0.055%	0.55	1485	0.457% *	5.02	1856	0.002%	0.01	423	0.175%	1.02	611	-0.652%	-0.94	54	0.337%	0.50	53
9	0.046%	0.42	1414	0.489% *	4.98	1783	0.015%	0.07	418	0.237%	1.32	602	-0.418%	-0.61	54	0.281%	0.39	53
10	0.077%	0.65	1341	0.603% *	5.74	1709	-0.087%	-0.40	415	0.263%	1.36	597	-0.512%	-0.74	54	-0.082%	-0.11	51
11	0.058%	0.46	1291	0.660% *	5.84	1644	-0.290%	-1.23	408	0.208%	1.03	591	-0.144%	-0.19	54	-0.303%	-0.42	51
12	0.104%	0.78	1231	0.695% *	5.79	1571	-0.212%	-0.87	407	0.301%	1.42	586	-0.045%	-0.05	54	-0.512%	-0.62	51
13	0.122%	0.86	1181	0.734% *	5.74	1493	-0.258%	-1.02	401	0.175%	0.80	580	0.124%	0.14	54	-0.396%	-0.49	51
14	0.062%	0.42	1125	0.747% *	5.49	1432	-0.216%	-0.83	394	0.091%	0.40	576	0.521%	0.57	54	-0.511%	-0.66	51
15	-0.018%	-0.12	1078	0.761% *	5.33	1376	-0.109%	-0.41	387	0.109%	0.47	570	1.048%	1.10	54	-0.535%	-0.67	51
16	-0.043%	-0.27	1035	0.794% *	5.28	1314	-0.056%	-0.20	382	0.018%	0.07	565	1.289%	1.29	54	-0.758%	-0.97	51
17	-0.057%	-0.33	994	0.789% *	4.98	1268	-0.069%	-0.24	377	-0.018%	-0.07	558	1.221%	1.17	54	-0.734%	-0.95	51
18	-0.087%	-0.48	959	0.774% *	4.73	1227	-0.025%	-0.09	371	-0.090%	-0.36	556	1.384%	1.29	54	-1.010%	-1.33	51
19	-0.118%	-0.61	918	0.850% *	5.01	1183	0.030%	0.10	367	0.001%	0.00	549	1.428%	1.22	52	-0.734%	-0.97	51
20	-0.132%	-0.65	882	0.860% *	4.87	1139	0.117%	0.37	366	0.077%	0.29	546	1.013%	0.83	52	-0.742%	-0.94	51

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data. Table 2 reports the characteristics of the Cumulated Average Returns after large price changes. Panels A, B and C deal respectively with the 0.25%, 0.5% and 1% filtered data. The results can be interpreted as the *ex post* performance of a trading strategy where a buy or sell signal is generated by the large price change. The trading strategy can be considered realistic, because the end of the large price change is clearly identified by a price change in the opposite direction.

The results in Table 2 show that after a large price change of 2.5% there is a strong persistence in the trend. The Cumulated Average Returns are highly significant immediately after the price change and for some periods, depending on the filter applied to the raw data set. The economic significance of these CARs, however, is very small (mostly below 0.1%) in the case of large price decreases. They are much larger (up to 0.8%) after large price increases. The persistence of positive trends is also much stronger: whatever the filter for the raw data, all 20 holding periods after the event are highly significant. In other words, price changes of 2.5% are the beginning of a longer trend in the same direction and the fact that the filtered data yields one observation in the opposite direction is not a valid indication for the end of the trend.

When large price changes of 5% are considered, the results and interpretation are very different. For large price decreases, the first period after the event always shows a persistence in the trend, however, this negative return is never significantly different from zero. The CAR at period 2 is in all cases positive. After this, smaller filters applied to the raw data clearly show a significant reversal. The larger the filter, the shorter the number of periods during which

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positive abnormal returns are observed: the price adjustment process has fully taken place after a few significant price movements. The economic significance is greatest for the 0.25% filtered data: a highly significant CAR of 0.3% is observed. For large positive price changes there is only a statistical significant but economically insignificant positive CAR at period 1 (for the 0.5% filtered data) and at periods 1 and 2 (for the 0.25% filtered data). The results for the 5% large price changes indicate that a significant price change in the opposite direction to the trend is on average a good indication that the large price change has taken place. Only after price declines will small filter values reveal that the market then needs some time before “bouncing back”.

The results for the 10% price changes are less significant due to the limited number of observations: between 50 and 100, depending on the filter applied. Nevertheless, the general trend is that after price declines of 10% or more the price continues to slide a little more for some time before bouncing back. After price declines of this magnitude, the market clearly needs more time to react than after price falls of 5%. The persistence in the downtrend in the first stage leads to negative CARs as large as almost minus one percent (period 5 with the 1% filter) before returning to a positive CAR of 1.4% (period 19 with the same filter). Although the one significant price change in opposite direction after the large price fall does indicate that the end of the trend is close, it would not be the best buy signal: waiting five price changes of 1% each, buying the stock, and keeping it for 14 periods would yield an average return of 2.3%! The smaller filters (0.25% and 0.5%) do not show the price reversal within the 20 periods considered here. The only revealed pattern is a strong persistence in the down trend. For price increases of 10% or more there is also a persistence to begin

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with, before a correction takes place. This is clearly seen with the 1% filter. At a higher frequency level, with the 0.5% filter, a significant negative CAR is revealed for periods 2 and 3. The magnitude of these average returns is -0.28% . The 1% filtered data also reports a negative CAR at period 2, although it is now insignificant and very small in magnitude due to the larger filter. Although the results for these 10% price increases should be interpreted with some caution because of the number of observations, the general pattern is a persistence in a first stage, and a bouncing back in a second stage. Economically, selling the stock at period 8 measured with the 1% filter and buying it back 10 periods later would have generated, *ex post*, a CAR of 1.34% . It should be noted that all these returns are in excess of the market return over the same period.

D. Possible trading strategies and market efficiency

Previous studies dealing with large price changes using daily closing prices have generally argued that the systematic patterns found after large price changes are significant, but too small in magnitude to allow systematic trading profits once transaction costs, even the smallest ones like the bid-ask spread, have been taken into account. The purpose of this section is to investigate whether our results would have allowed to generate abnormal profits. Besides, it is important to assess the implications of such findings in terms of market efficiency.

The trading strategy I investigate in this section is based on buying or short selling the stock (respectively a long or a short position) at period zero, that is, the moment the end of the large price movement is detected. The long or short decision is based on the sign of the CARs calculated from the averages of the bid-ask quotes: a negative CAR generates a sell signal, whereas a positive CAR

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generates a buy signal. CARs that are small in magnitude will generate negative trading profits, because the bid-ask spread is not covered, on average, by the price changes. Table 3 reports the only strategies that would have been profitable over time. All were generated using the 1% filter for the initial data. The first strategy consists in buying the stock after a large price increase of 2.5% and holding it for 14 to 20 periods. The average return ranges between 0.01% and 0.15% after deduction of the bid-ask spread. If the strategy is followed for each of the 120 stocks and assuming no overlap of signals among stocks (that is, that a buy signal for one stock is never generated while another stock is already held), the annualized returns range from 4.9% for a 14 period holding to 56.4% for a 20 period holding. The associate t-statistics are low. For the strategy consisting in buying the stock after a large price decrease of 10%, the average return per transaction is as high as 1.24% for 18 holding periods, with an annualized return of 22.2%. Again, the t-statistic of 0.91 is low.

Although some of the strategies considered here show profitable trading opportunities, the interpretation should be done with care. To begin with, the sample that estimates the CARs is the same as the one used to calculate the trading profits. At the beginning of 1995 it was unknown which strategy would generate profitable trading results. Furthermore, especially for the large price drops of 10%, the sample is small and there is no evidence of the stability over time of the results. From a practical point of view, even a floor trader at the French stock exchange faces other costs than only the bid-ask spread. These results cannot, therefore, be seen as evidence against market efficiency. Nevertheless, they are puzzling.

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Table 3: Statistics of *ex post* profitable trading strategies taking into account the bid-ask spread for the 120 stocks.

Holding Period	Strategy: BUY after a Large Price Increase of 2.5%				Strategy: BUY after a Large Price Decrease of 10%			
	Annual Return	Average Return	# Obs	t-stat	Annual Return	Average Return	# Obs	t-stat
14	4.9%	0.01%	1432	0.07				
15	9.5%	0.02%	1376	0.13	0.3%	0.02%	54	0.02
16	32.1%	0.07%	1314	0.45	5.2%	0.29%	54	0.28
17	23.7%	0.06%	1268	0.33	19.7%	1.10%	54	0.82
18	23.8%	0.06%	1227	0.33	22.2%	1.24%	54	0.91
19	54.1%	0.14%	1183	0.75	21.2%	1.22%	52	0.84
20	56.4%	0.15%	1139	0.78	15.5%	0.89%	52	0.61

The only trading strategies that remain profitable once the bid-ask spread is taken into account are obtained with the 1% filter applied to the raw data set and for large price increases of 2.5% and large price decreases of 10%. All figures are in excess of the market return. The annualized returns are the sums of all strategies for each of the 120 stocks in the sample.

IV. Conclusion

In this study I tried to contribute to the existing empirical literature about large price changes by examining the intra-day price movements around the event. The high frequency data is very rich in information, but also comes along with specific problems which have to be dealt with and for which there is not always a straight answer. A good example is the question what should be considered a “large price change”. Prices seldom go up or down by large amounts along a smooth continuous path. A large price increase will typically be characterized by

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numerous smaller price increases alternating with a few price drops of smaller magnitude. This requires filtering the data according to some consistent but arbitrary rule.

The various definitions that have been tried out in this study have revealed some systematic patterns which were more or less pronounced according to the value used in filtering the initial raw high frequency data set and to the minimum magnitude of a large price change. In particular, I found that the first reaction of the prices after a large price change is a persistence in the trend of the large price movement. Although a clear signal of a significant price change in the direction opposite to the trend is found, the prices continue to move in the direction of the large price movement for some time. A clear rebound takes place at a second stage, some periods after the initial movement of persistence.

The abnormal returns that were detected in some cases would have been large enough in magnitude to remain profitable even after transaction costs, in terms of the bid-ask spread, had been taken into account. Although this could be an indication of market inefficiency, there is no evidence of stability over time of the generated profits. Furthermore, one should have known at the beginning of the high frequency sample, in January 1995, which filter to use, which large price movements to track and the direction of the trades to be done. Following the wrong trading strategies would have generated substantial losses. I therefore consider the results consistent with most of those reported in the existing literature about large price changes.

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