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Serial and Parallel Krylov Methods for Implicit Finite Difference Schemes Arising in Multivariate Option Pricing

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Abstract:
This paper investigates computational and implementation issues for the valuation of options on three underlying assets, focusing on the use of the finite difference methods. We demonstrate that implicit methods, which have good convergence and stability prooperties, can now be implemented efficiently due to the recent development of techniques that allow the efficient solution of large and sparse linear systems. In the trivariate option valuation problem, we use nonstationary iterative methods (also called Krylov methods) for the solution of the large and sparse linear systems arising while using implicit methods. Krylov methods are investigated both in serial and in parallel implementations. Computational results show that the parallel implementation is particularly efficient if a fine grid space is needed.

Executive Summary:
In recent years the demand for numerical computations in financial applications has greatly increased. Several fields of application have benefited from the combination

of efficient algorithms and fast computers. In particular, the valuation of derivative securities has been pushed forward by the use of intensive computational procedures.

This paper focuses on computational and implementation issues of finite difference methods for the valuation of multivariate contigent claims. Examples of problems resulting in multivariate partial differential equations in finance include the pricing of foreign currency debts, compensation plans, risk-sharing contracts, multifactor interest rate models to mention a few.

It is generally accepted that the dimensionality of the problem is a nontrivial issue. Up to a dimension of three, methods like the finite differences or the finite elements can still be used. With a greater number of state variables, Monte Carlo is thought to be the only way out.

For bivariate problems, finite difference methods, both explicit and implicit, have been successfuly implemented. In the trivariate case the dimensionality of the problem increases and it is generally accepted that implicit methods are greatly desirable as much smaller grid sizes need to be used in order to obtain acceptable precision in reasonable computation times.

Based on the computational results performed in two different computing environments, we conclude that implicit finite difference methods can be efficiently us ed for the valuation of options on three underlying assets, allowing to take advantage of their good stability and convergence features. The avail- ability of efficient methods for the solution of large and sparse linear systems, namely nonstationary iterative methods, makes the use of implicit finite difference methods possible.

In our experiments, the size of the system that we solve in parallel is approximatively three million. However, we think that, with faster processors and more memory, we can go even further, allowing for a finer space grid (of the order of 200 in each of the three directions).

In the serial case, the maximum grid size that we can solve in a standard PC in Matlab environment is of about 70 in each direction.

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