COMPETITION AND EFFICIENCY IN MARKETS WITH QUALITY UNCERTAINTY

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ABSTRACT. This paper addresses the following question: Does competition enhance efficiency in markets with quality uncertainty? Using the mechanism design methodology, we characterize the *maximal* achievable level of efficiency in such markets, and then use this characterization to analyze how maximal efficiency varies with the degree of market competition. We show that the relationship between them is in general a non-trivial function of the main market parameters. In particular we show: (i) for some set of parameter values maximal efficiency is strictly increasing in the degree of market competition (although it never attains the first-best), but only until competition is sufficiently intense; thereafter, maximal efficiency is strictly decreasing in the degree of competition; (ii) for some set of parameter values maximal efficiency is strictly decreasing in the degree of market competition, attaining the first-best when there is no competition; and (iii) for some set of parameter values maximal efficiency is strictly increasing in the degree of market competition, attains the first-best once competition is sufficiently intense, and then remains at the first-best thereafter.
JEL Classification Numbers: C7, D4, D61, D82.

"... most cars traded will be the "lemons," and good cars may not be traded at all. The "bad" cars tend to drive out the good (in much the same way that bad money drives out the good)." GEORGE A. AKERLOF, *The Market for "Lemons": Quality Uncertainty and the Market Mechanism, 1970.*

1. INTRODUCTION

1.1. **Motivation and Overview.** It's conventional wisdom that in general competition is a good thing. The more the better. By fostering appropriate individual incentives, competition can help promote aggregate (or social) welfare. Economics textbooks are replete with models in which aggregate welfare increases with the degree of competition. One classic example of this key insight is provided by Cournot's model of oligopolistic competition: in this model, the difference between the Nash equilibrium market price and the

Date: March 7, 2005.

Key words and phrases. Lemons Markets, Mechanism Design, Maximal Efficiency, Competition.

We thank V. Bhaskar, Tilman Börgers, Bhaskar Dutta, Matt Jackson and various seminar participants for their helpful comments and suggestions.

constant marginal cost of production is strictly decreasing (and aggregate welfare is strictly increasing) in the number of competing firms.

Can competition have a similar beneficial impact in markets with asymmetric information? While it's well established that markets with asymmetric information tend in general to be inefficient (except perhaps in the restrictive, limiting scenario when they contain an arbitrarily large number of traders), much less is known about how the *degree* of inefficiency varies with the *degree* of competition in such markets. The main objective of this paper is to answer this question for markets with quality uncertainty, which, following Akerlof (1970), are termed "lemons" markets.

It goes without saying that a better understanding of the relationship between competition and efficiency in lemons markets is important not only from a theoretical perspective but also from a practical (market-design and policy) perspective. Such understanding would provide insight into the role played by market competition on how well lemons markets function and perform.

As is well-known, a distinguishing characteristic of the lemons market is that when contemplating the possibility of bilateral trade, one of the traders has relatively more information about something (eg., quality) that affects both traders' payoffs from trade. Such markets are ubiquitous, and have been intensely studied over the past three decades, both by economic theorists and by applied economists in the context of specific markets such as credit and labour markets. In his seminal paper, George Akerlof was the first to argue that lemons markets will typically be Pareto-inefficient: sellers owning high quality objects may fail to trade, although there are buyers who would wish to trade with them. The basic intuition for this fundamental observation stems from the incentives of the sellers owning low quality objects: each such seller has an incentive to pretend to own a high quality object in order to command a high price for her, actually, low quality object.

Our central objective is to analyze how the degree of market efficiency varies with the degree of market competition. We define the latter by the exogenously given number of sellers; the higher its value the greater is the degree of market competition. Notice that this is naturally defined independently of any specific trading rules (or game form), and the resultant market game that such rules would induce (when combined with trader preferences, beliefs and information structure). Market efficiency, on the other hand, is a concept associated with market outcomes, but interest (and hence our focus) centers on equilibrium market outcomes, which, in contrast, do depend on specific trading rules and/or the equilibrium concept. With any given equilibrium market outcome is associated an expected surplus which is generated from that outcome. Following Rustichini, Satterthwaite and Williams (1994), we define the efficiency of an arbitrary equilibrium outcome as the ratio of its associated expected surplus to the first-best expected surplus, where the latter is generated from the first-best outcome.[1]

We shall not address the issue under consideration — namely, the relationship between the degree of competition and the degree of efficiency — in the context of any specific set of trading rules. This is because doing so would leave open the possibility that with a different set of trading rules it might be the case that a higher degree of efficiency is attained for any given degree of competition. Instead, we characterize the *maximal* achievable level of efficiency that is attained for any given degree of competition. As such our approach involves considering all outcomes that can be achieved as a Bayesian Nash equilibrium of some game (induced by some set of trading rules). In order to conduct such a normative exercise, we use the mechanism design methodology, in which details of the trading rules are irrelevant. Of course, this exercise is made possible by appealing to the *Revelation Principle*, which allows us to confine attention to direct mechanisms in which agents truthfully report their private information.

We show that the relationship between maximal efficiency and competition in lemons market is a non-trivial function of the main market parameters. Our main results, some of which are briefly summarized in the subsection below, provide insight into the nature of that relationship. As a by-product of our main results, we show that in general (and particularly for arguably the most interesting set of parameter values) lemons markets will *not* attain the first-best outcome in the limit as the number of sellers becomes arbitrarily large. This specific result is perhaps not that unexpected, although it should be contrasted with the positive (limiting) result that has been established for markets with other kinds of asymmetric information (such as with private values); the related literature is discussed in section 7. As noted above, our main results speak to the issue of how maximal efficiency varies with the degree of competition. We show that this relationship depends in particular on two factors, namely, the likelihood of any seller owning a lemon rather than a peach, and whether the gains from trade are higher from trading a lemon or from trading a peach.[2] For example, in the (arguably most interesting) case when the likelihood of any seller owning a lemon is sufficiently high and the gains from trade are higher from trading a peach, the relationship between maximal efficiency and competition is non-trivial: For relatively small lemons markets, this relationship is either monotonic (strictly increasing or strictly decreasing) or non-monotonic, depending on exact parameter values; and for relatively large lemons markets, maximal efficiency is strictly decreasing in the degree of competition. One

[1]It may however be noted that our main results are robust to alternative definitions of efficiency that capture (or measure) the "difference" between equilibrium (or realized) expected surplus and first-best expected surplus.

[2]A "lemon" denotes a low quality object, while a "peach" denotes a high quality object.

(policy) implication of these results is that the "optimal" degree of competition — one that maximizes maximal market efficiency — is uniquely defined and is bounded away from being "too large".

1.2. **Summary of Main Results.** Our benchmark model is the standard one introduced by Akerlof (1970) with M buyers, N sellers and two grades ('high' and 'low') of the good. The only source of asymmetric information is the fact that the quality of the good owned by a seller is the private information of that seller. For this setting, we characterize the mechanism which maximizes expected surplus subject to satisfying appropriate incentive compatibility and individual rationality constraints, and being budget balanced. It turns out that the "optimal" mechanism when $M < N$ is distinctively different to that when $M \geq N$, as is the relationship between maximal efficiency and market competition.

1.2.1. *The Case of $M < N$.* We examine the behaviour of the optimal mechanism as the number of sellers increases (given a fixed number of buyers). It is shown that if maximizing the expected surplus requires transferring high quality objects to the buyers, then the first-best cannot be achieved, even in the limit as the number of sellers becomes arbitrarily large. Indeed, in this case, the optimal mechanism becomes *less* efficient relative to the first-best when the number of sellers is increased beyond a certain critical number. Until that critical point, however, the relationship between maximal efficiency and the number of sellers is non-trivial: it can be monotonic (strictly increasing or strictly decreasing), or non-monotonic.

In contrast, when maximizing the expected surplus requires that low quality objects be transferred to the buyers, then the first-best outcome can be obtained in this limit. Specifically, we show that there exists a number of sellers such that when the number of sellers is greater than that number, then the expected surplus from the optimal mechanism coincides with the first-best.

1.2.2. *The Case of $M \geq N$.* The analysis of this case turns out to be significantly simpler in both conceptual and technical terms. Furthermore, the results are very different as well. We show that that if the buyers are "soft" — which roughly speaking means that they believe each and every seller possesses a high quality object with a sufficiently large probability — then the maximal efficient outcome coincides with the first-best outcome for any $M \geq N$. If, on the other hand, the reverse is the case and the buyers are "tough," then for any $M \geq N$, the ratio of the maximal achievable level of expected surplus to the first-best expected surplus is a constant, strictly less than one and independent of M and N. Thus, when the number of buyers in the market is greater than or equal to the number of sellers, increasing the number of sellers (while maintaining the number of buyers at least equal to the number of sellers) does not affect maximal market efficiency.

1.2.3. *The model with buyer heterogeneity.* The negative results outlined above raise the question as to whether there is any way of overcoming them. We next show that if one extends the benchmark model to allow for private information on the buyers side, then it is possible to obtain asymptotic efficiency under some restrictions on the parameters. One might think that introducing private information on the buyers side will add to the inefficiency; this, though, is not necessarily correct because the private information on the buyers side can be used to relax the low type seller's incentive constraint as we discuss below.[3]

Let v_H and v_L be a buyer's valuation for the two types of the object, and c_H and c_L the corresponding seller's reservation prices. Then, efficiency dictates that —subject to availability— the high quality object be transferred to the buyer if $v_H - c_H > v_L - c_L$ and the low quality good otherwise. If v_H and v_L are commonly known, then as we show later, the low type seller has an incentive to misrepresent her type. Suppose now that there are *two* types of the buyer, one for whom the net surplus from the high quality good is more and the other for whom the net surplus from the low quality good is higher. Then, the socially optimal decision — viz., which type of good to transfer — depends on the type of the buyer which is unknown to the seller. In this extended scenario, it is no longer clear that the low type seller wants to pretend to be a high type: indeed, if the probability that a buyer is a "low" type is sufficiently high, then she would *not* want to do so. This raises the possibility that the inefficiency resulting from the seller's private information can be corrected by allowing for private information on the buyer's side.

Of course, introducing two types of the buyer makes the mechanism design problem more complex because we have to deal with additional individual rationality and incentive compatibility constraints. We have not been able to characterize the solution to the resulting mechanism design problem completely. However, we have been able to determine restrictions on parameters which ensure that when the number of sellers is large enough, then we can find an asymptotically efficient mechanism. Our results in this regard suggest that while asymptotic efficiency in a market for lemons settings is not a generic phenomenon, there are still significant cases where it is possible.

1.3. **Organization of the Paper.** The remainder of the paper is organized as follows. Section 2 lays down our market environment. Section 3 formulates the mechanism design problem for the case with a single buyer and an arbitrary number of sellers. Section 4 solves this problem, and characterizes the maximal achievable level of expected surplus. Section 5 derives our main results concerning the relationship between market competition and maximal achievable level of efficiency. Section 6 considers the extension to the case in which the buyer can be one of two types. Section 7 discusses

[3]This incentive constraint is the main reason for the negative results. We discuss this in detail later.

our results in the context of the related literature. Section 8 concludes. Appendix A contains our technical proofs, omitted from the main text.

Appendix B extends our framework, analysis and results to the many-buyers case. The main reasons for studying the single buyer case in the main text, while relegating the many-buyers case to the Appendix are two-fold. First, the analysis and the main results for the single buyer case carry over to the many-buyers case when the number of sellers is strictly greater than the number of buyers. In the reverse (less interesting and less important) case, the analysis is however different but pretty straightforward. Second, one obtains a relatively deeper understanding of the core aspects of the analysis and of main forces at work, and one develops sharper intuition for the main results especially concerning the impact of competition on market efficiency.

2. The Market Environment

We consider a market with finite numbers of sellers and buyers in which each seller has private information about the quality of the object (or commodity) that she owns. While a buyer may ascertain the quality of an object after acquiring it, the terms of trade cannot be made contingent on quality since that is non-verifiable by third parties (such as the courts).

Each of the N ($N \geq 1$) sellers owns one unit of an indivisible object whose quality is her private information. An object can be either low quality (L) or high quality (H). The qualities of the objects owned by any two sellers are uncorrelated. Indeed, the quality q_i of seller i's object (where $i = 1, 2, \ldots, N$) is considered an independent random draw from a binomial distribution, with the probability that $q_i = L$ being α and the probability that $q_i = H$ being $1 - \alpha$, where $0 < \alpha < 1$. Thus, there exist two "types" of sellers: a low-type seller (whose object is of low quality) and a high-type seller (whose object is of high quality).[4]

There are M ($M \geq 1$) buyers in the market, all of whom are identical. Each buyer is interested in acquiring one and only one unit of the object. We denote by v_L and v_H, respectively, the reservation values placed by a buyer on a low and a high quality object. Furthermore, c_L and c_H, respectively, denote the reservation values placed by a seller on a low and a high quality object. If a buyer acquires an object of quality q ($q = L, H$) at price p ($p \geq 0$) then his net payoff is $v_q - p$; and if a seller owning an object of quality q sells it at price p then her net payoff equals $p - c_q$. If an agent does not trade, then his net payoff is zero. All agents are risk-neutral and maximize expected utility. Throughout our analysis we maintain the following assumption on the valuations:

Assumption 1. $c_L \leq v_L < c_H < v_H$.

[4]In section 8 we discuss the extent to which our results and insights would be robust to alternative distributions of the sellers' types, including distributions that allow for a continuum of types and/or some degree of correlation.

The first inequality states that there may (but need not) exist gains from trade between a buyer and a low-type seller; the second inequality restricts attention to the interesting and non-trivial case; and the third inequality states that there exists gains from trade between a buyer and a high-type seller. For notational convenience, we denote by s_L and s_H the surpluses generated from trading the object of low quality and high quality, respectively; that is, $s_q = v_q - c_q$ $(q = L, H)$. Without loss of generality but in order to ease exposition, we assume that $s_H \neq s_L$.

This completes the description of our market environment. The core parameters include the number of sellers N, the number of buyers M, the traders' preferences as defined by the reservation valuations v_L, v_H, c_L and c_H, and the beliefs about quality, captured by α.

As discussed in section 1.1 above, our objective is to use the mechanism design methodology to characterize the maximal achievable level of efficiency in the market described above. We conduct this exercise by appealing to the Revelation Principle, which allows us to confine attention to direct, incentive-compatible mechanisms.

3. The single buyer case

Before proceeding to solve the mechanism design problem for this special case, we note that the first-best outcome and the associated first-best expected surplus are defined as follows. If $s_H > s_L$ then in the first-best outcome, the buyer trades with a high-type seller unless all sellers are of low type; symmetrically, if $s_H < s_L$ then the buyer trades with a low-type seller unless all sellers are of high type. Thus, the first-best expected surplus is

$$G \equiv \alpha^N s_L + (1-\alpha)^N s_H + [1 - \alpha^N - (1-\alpha)^N] \max\{s_L, s_H\}, \tag{1}$$

which is strictly increasing in N, and converges to $\max\{s_L, s_H\}$ as N tends to infinity.

3.1. Incentive and Participation Constraints.

A direct mechanism requires each seller to announce her type, and selects an outcome conditional on the announcements of all sellers.[5] However, without loss of generality but in order to simplify the analysis, we restrict attention to symmetric mechanisms; that is, mechanisms where the outcome does not depend on the names of the sellers. A symmetric, direct mechanism can be represented as $\{t_k^H, t_k^L, p_k^H, p_k^L\}_{k=0}^N$, where p_k^q is the probability with which a q-type seller sells her object and t_k^q is the expected transfer payment to her, conditional on there being k reported high-type sellers and $N-k$ reported low-type sellers ($q = L, H$ and $k = 0, 1, 2, 3, \ldots, N$).

[5] Formally, let $\hat{q} = (\hat{q}_1, \ldots, \hat{q}_N)$ (where $q_i \in \{H, L\}$ for any $i = 1, \ldots, N$) be the announcements of the N sellers, and let the tuple $(p_i(\hat{q}), t_i(\hat{q}))$ represent the outcome to seller i, where $p_i(\hat{q})$ is the probability with which she sells her object and $t_i(\hat{q})$ is the expected transfer payment to her.

Following the Revelation Principle, the mechanism needs to satisfy the incentive-compatibility constraints (i.e., no seller must have an incentive to misrepresent her type). We will also require the mechanism to satisfy the participation or individual-rationality constraints (i.e., the net gains to each seller and the buyer must be non-negative), which capture a fundamental feature of markets, namely, that trade is voluntary. We now proceed to define these constraints for an arbitrary symmetric, direct mechanism. In doing so, we assume that the mechanism satisfies budget balance, an assumption that is characteristic of markets in general; there are no subsidies by "third" parties (such as the state).

Suppose that an arbitrary high-type seller declares that she is a q-type seller ($q = L, H$), while all the other $N-1$ sellers behave truthfully. If $q = H$ (i.e., she also behaves truthfully), then her expected probability of trade and the expected transfer payment to her are respectively

$$\hat{p}^H = \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} p^H_{k+1}, \quad \text{and} \tag{2}$$

$$\hat{t}^H = \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} t^H_{k+1}. \tag{3}$$

But if $q = L$ (i.e., she misrepresents her type) then her expected probability of trade and the expected transfer payment to her are respectively

$$\hat{p}^L = \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} p^L_{k}, \quad \text{and} \tag{4}$$

$$\hat{t}^L = \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} t^L_{k}. \tag{5}$$

Now suppose that an arbitrary low-type seller declares that she is a q-type seller ($q = L, H$), while all the other $N-1$ sellers behave truthfully. If $q = L$ (i.e., she also behaves truthfully), then her expected probability of trade and the expected transfer payment to her are respectively $\hat{p}^L$ and $\hat{t}^L$. But if $q = H$ (i.e., she misrepresents her type) then her expected probability of trade and the expected transfer payment to her are respectively $\hat{p}^H$ and $\hat{t}^H$.

An arbitrary symmetric, direct mechanism is incentive-compatible (IC) if and only if each seller of either type does not have an incentive to misrepresent her type (when all other sellers behave truthfully), which is the case if and only if the following two inequalities hold:[6]

$$\hat{t}^H - \hat{p}^H c_H \geq \hat{t}^L - \hat{p}^L c_H, \quad \text{and} \tag{6}$$

$$\hat{t}^L - \hat{p}^L c_L \geq \hat{t}^H - \hat{p}^H c_L. \tag{7}$$

[6]The first inequality is an arbitrary high-type seller's IC condition while the second is an arbitrary low-type seller's IC condition.

Individual Rationality (IR) for an arbitrary high-type seller and an arbitrary low-type seller are obvious. They are

$$\hat{t}^H - \hat{p}^H c_H \geq 0, \quad \text{and} \tag{8}$$

$$\hat{t}^L - \hat{p}^L c_L \geq 0. \tag{9}$$

Individual Rationality for the buyer follows the same principle: the expected payoff from the mechanism must not be less than zero, which is[7]

$$(1-\alpha)\left[\hat{p}^H v_H - \hat{t}^H\right] + \alpha\left[\hat{p}^L v_L - \hat{t}^L\right] \geq 0. \tag{10}$$

The left-hand side of inequality (10) is the expected net payoff to the buyer from trading with an arbitrary seller.

3.2. **The Mechanism Design Problem.** Before stating the mechanism design problem, we derive three admissibility conditions and state the objective function. The symmetry of the mechanism entails that $p_k^H \leq 1/k$ (for $k > 0$) and $p_k^L \leq 1/(N-k)$ (for $k < N$). Using (2) and (4), these conditions imply that the mechanism must satisfy the following two conditions:

$$\hat{p}^H \leq \frac{1-\alpha^N}{N(1-\alpha)} \quad \text{and} \tag{11}$$

$$\hat{p}^L \leq \frac{1-(1-\alpha)^N}{N\alpha}. \tag{12}$$

Since the total probability with which trade occurs must be less than or equal to one, the mechanism must also satisfy

$$N[(1-\alpha)\hat{p}^H + \alpha\hat{p}^L] \leq 1. \tag{13}$$

In order to characterize the maximal achievable level of efficiency, we need to characterize the maximal achievable level of expected surplus attained across all (IC, IR, budget balance and admissible) symmetric, direct mechanisms. The expected surplus generated (or realized) from an arbitrary symmetric, direct mechanism is

$$\sum_{k=0}^{N} \binom{N}{k} (1-\alpha)^k \alpha^{N-k} [k\hat{p}^H s_H + (N-k)\hat{p}^L s_L],$$

which equals $N[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L]$. Thus, the mechanism design problem is to choose a symmetric, direct mechanism amongst those that satisfy the two IC constraints, three IR constraints and three admissibility constraints which generates the maximal expected surplus. Formally, this problem is:

$$E \equiv \max_{\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L} \quad N\Big[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\Big] \tag{14}$$

[7] The buyer's expected net payoff from an arbitrary symmetric, direct mechanism (which satisfies budget balance) is $\sum_{k=0}^{N}\binom{N}{k}(1-\alpha)^k\alpha^{N-k}[kp_k^H v_H + (N-k)p_k^L v_L - kt_k^H - (N-k)t_k^L]$, which is non-negative if and only if inequality (10) holds.

subject to (6)–(13).

Notice that this maximization problem involves four unknowns: $\hat{p}^q$ and $\hat{t}^q$ $(q = L, H)$, where the former is the expected probability with which an arbitrary q-type seller sells her object and the latter the expected transfer payment to her. The maximal achievable level of expected surplus is E; and the maximal achievable level of efficiency is E/G, where G is the first-best expected surplus (and is formally defined in (1)). Our main goal is to analyze the relationship between E/G and N.

4. Maximal Achievable Expected Surplus

4.1. **A Reduced-Form Problem.** We solve (14) by defining and solving a reduced-form problem that involves the following change of variables:

$$\tilde{p}^H = N(1-\alpha)\hat{p}^H \quad \text{and} \quad \tilde{p}^L = N\alpha\hat{p}^L,$$

where $\tilde{p}^H$ and $\tilde{p}^L$ are respectively the total expected probabilities with which trade occurs between the buyer and sellers owning high and low quality objects. Given this change of variables the three admissibility constraints (11), (12) and (13) respectively become

$$\tilde{p}^H \leq 1 - \alpha^N, \tag{15}$$

$$\tilde{p}^L \leq 1 - (1-\alpha)^N \quad \text{and} \tag{16}$$

$$\tilde{p}^L + \tilde{p}^H \leq 1, \tag{17}$$

and the expected surplus (the maximand in (14)) becomes $\tilde{p}^H s_H + \tilde{p}^L s_L$. What about the two IC and three IR conditions? We first note that the two IC conditions (namely, (6) and (7)) are satisfied *only if* the following inequality holds:[8]

$$\hat{p}^H \leq \hat{p}^L,$$

which, using the change of variables defined above, becomes:

$$\tilde{p}^H \leq \left[\frac{1-\alpha}{\alpha}\right]\tilde{p}^L. \tag{18}$$

Before proceeding further, we pause for a moment to take special note of the observation just made that incentive-compatibility implies that for any N, the expected probability with which a high-type seller trades must be no greater than the expected probability with which a low-type seller trades. This observation — which generalizes to the many-buyers case studied in Appendix B and to various other extensions of our market environment discussed in sections 6-8 — is one of the main critical factors driving our results concerning the relationship between competition and efficiency. Indeed, this is a robust feature of lemons markets, and is a crucial element that differentiates such markets from markets with other kinds of asymmetric information.

[8] This follows by rewriting (6) and (7) as $(\hat{p}^H - \hat{p}^L)c_H \leq \hat{t}^H - \hat{t}^L \leq (\hat{p}^H - \hat{p}^L)c_L$, and then applying the assumption that $c_H > c_L$.

Returning to the problem at hand, it is straightforward to show that at a solution to the mechanism design problem (14), the low-type seller's IC condition binds as does the high-type seller's IR condition:

Lemma 1. *At a solution to the mechanism design problem (14), the low-type seller's IC constraint, (7), binds as does the high-type seller's IR constraint, (8).*

Lemma 1 implies that (at a solution to (14)) $\hat{t}^H = \hat{p}^H c_H$ and $\hat{t}^L - \hat{p}^L c_L = \hat{t}^H - \hat{p}^H c_L$. Substituting for $\hat{t}^H$ in the second equality gives $\hat{t}^L = \hat{p}^L c_L + \hat{p}^H(c_H - c_L)$. Note that since $c_H > c_L$, the low-type seller's IR condition is automatically satisfied with these transfers. Furthermore, substituting for $\hat{t}^H$ and $\hat{t}^L$ in the individual rationality constraint of the buyer (10), it follows that it becomes (after using the change of variables defined above, and some simplification):

$$\tilde{p}^H\big[s_H - \alpha(v_H - c_L)\big] + (1-\alpha)\tilde{p}^L s_L \geq 0. \tag{19}$$

Now define the following *reduced-form* problem:

$$E^* \equiv \max_{\tilde{p}^H, \tilde{p}^L} \quad \tilde{p}^H s_H + \tilde{p}^L s_L \tag{20}$$

subject to (15)–(19).

The following lemma establishes the connection between the two maximization problems, (14) and (20):

Lemma 2. *Using the change of variables defined above and the expected transfer payments implied by Lemma 1, any solution of (20) defines a solution of (14) and vice-versa. Moreover, $E = E^*$.*

It follows from Lemma 2 that a solution to the mechanism design problem (14) can be obtained from a solution of the reduced-form problem (20), and we shall exploit this result in our analysis which characterizes the maximal achievable expected surplus, to which we now turn.

4.2. **The Solution.** The proposition below states the maximal achievable level of expected surplus. Immediately after its statement, we sketch the main elements of the argument in its support, relegating the detailed computations to Appendix A (where the solution that underpins the maximal expected surplus stated in the proposition is also derived).

Proposition 1 (Maximal Expected Surplus)**.** *Define $\alpha^* = s_H/(v_H - v_L)$, which* (*given Assumption 1*) *lies in the open interval* $(0,1)$.
(a) [The "Soft" Buyer Case]. *If $\alpha \leq \alpha^*$, then*

$$E = \begin{cases} (1-\alpha)s_H + \alpha s_L & \text{if } s_H > s_L, \\ (1-\alpha)^N s_H + [1-(1-\alpha)^N] s_L & \text{if } s_H < s_L. \end{cases}$$

(b) [The "Tough" Buyer Case]. *Assume that $\alpha > \alpha^*$. Define*

$$s_L \geq \frac{(1-\alpha)^{N-1}}{1-(1-\alpha)^N}\Bigg[\alpha(v_H - c_L) - s_H\Bigg]. \tag{21}$$

[Note: There exists an N^*, where $N^* \geq 2$, such that (21) holds if and only if $N \geq N^*$].[9]

(i) [Large Markets]. If $N \geq N^*$ (i.e., (21) holds), then

$$E = \begin{cases} \dfrac{\alpha(c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)(s_H - s_L)} & \text{if } s_H > s_L, \\ (1-\alpha)^N s_H + [1-(1-\alpha)^N]s_L & \text{if } s_H < s_L. \end{cases}$$

(ii) [Small Markets]. If $1 \leq N < N^*$ (i.e., (21) does not hold), then

$$E = \frac{\alpha[1-(1-\alpha)^N](c_H - c_L)s_L}{\alpha(v_H - c_L) - s_H}.$$

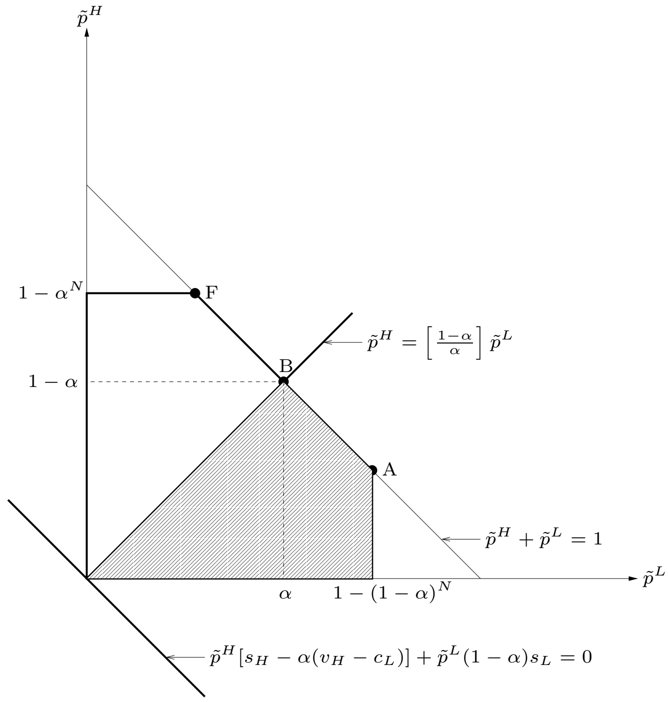

FIGURE 1. *Soft Buyer*: The feasible set when $\alpha \leq \alpha^*$

[9]This follows since (given the supposition that $\alpha > \alpha^*$) the RHS of (21) is strictly decreasing in N, converges to zero in the limit as $N \to \infty$ and is strictly greater than s_L when $N = 1$.

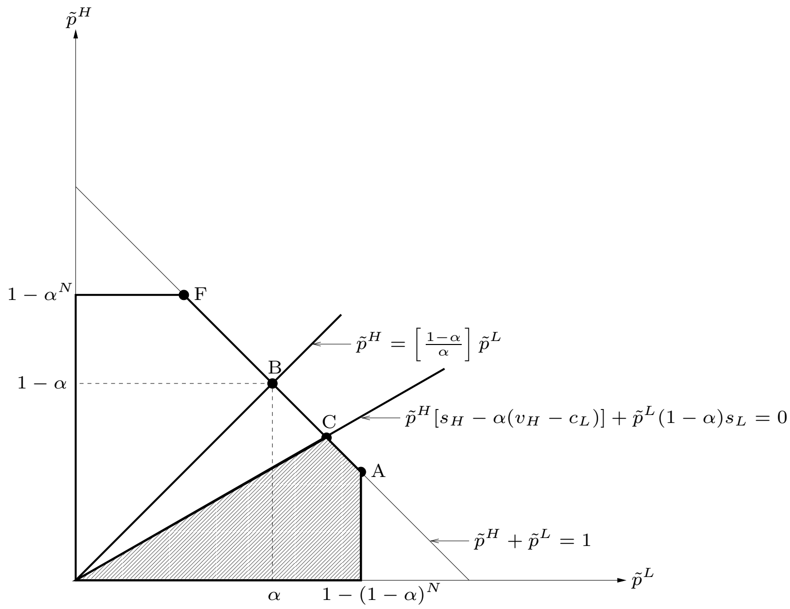

FIGURE 2. *Tough Buyer and Large Market*: The feasible set when $\alpha > \alpha^*$, and inequality (21) holds (i.e., $N \geq N^*$).

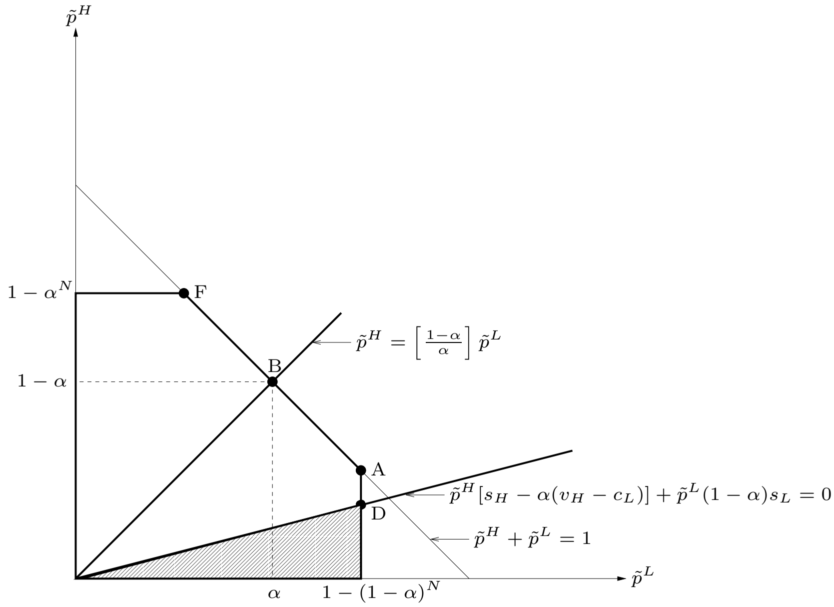

FIGURE 3. *Tough Buyer and Small Market*: The feasible set when $\alpha > \alpha^*$, and inequality (21) does not hold (i.e., $1 \leq N < N^*$).

Proof. [Sketch of the main argument; details are provided in Appendix A.] We begin by noting that for any $\alpha \in (0, 1)$, the set of all pairs $(\tilde{p}^L, \tilde{p}^H)$ that satisfy (15)-(18) comprises the shaded region in Figure 1. In the absence of (19), the solution of (20) lies either at point A or at point B depending on whether s_H is strictly less or strictly greater than s_L. Proposition 1(a) follows immediately since if $\alpha \leq \alpha^*$ then all points in the shaded region shown in Figure 1 satisfy (19), as is illustrated in Figure 1.

Now suppose that $\alpha > \alpha^*$. In this case not all points in the shaded region shown in Figure 1 satisfy (19). Figures 2 and 3 show how (19) affects the feasible set depending on whether or not point A remains a feasible point. If point A does remain a feasible point (i.e., $N \geq N^*$), then the shaded region shown in Figure 2 comprises the feasible set; in this case the solution of (20) lies either at point A or at point C depending on whether s_H is strictly less or strictly greater than s_L. If, on the other hand, point A does not remain a feasible point (i.e., $1 \leq N < N^*$), then the shaded region shown in Figure 3 comprises the feasible set; in this case the solution of (20) lies at point D. □

Proposition 1 characterizes the maximal expected surplus that can be generated in any Bayesian Nash equilibrium (BNE henceforth) of any market game which is derived from grafting some set of trading rules onto the market environment under consideration. How does this maximal achievable level of expected surplus compare to the first-best expected surplus? This can be answered quite easily with the aid of Figures 1–3.

It is straightforward to see that the first-best outcome lies at point F (shown in Figure 1) when $s_H > s_L$, but at point A when $s_H < s_L$.[10] It thus follows that when $s_H > s_L$, the first-best outcome can never be attained in any BNE of any market game except in the special case when there is a single seller *and* the buyer is soft (in this special case, points F and B in Figure 1 coincide); and when $s_H < s_L$, the first-best outcome can be attained in some BNE of some market game provided that it is *not* the case that the buyer is tough *and* $1 \leq N < N^*$ (in this case Figure 3 applies, and points A and D are different). These results are compactly stated in the following corollary:

Corollary 1 (Comparison with First-Best)**.** *(a) Assume $s_H > s_L$. If $N = 1$ and $\alpha \leq \alpha^*$ then $E = G$; otherwise $E < G$. (b) Assume $s_H < s_L$. If $\alpha > \alpha^*$ and $1 \leq N < N^*$ then $E < G$; otherwise $E = G$.*

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[10]This is because when $s_H > s_L$, in the first-best outcome trade occurs with a high-type seller unless all sellers are of low type; and hence the first-best outcome has $\tilde{p}^H = 1 - \alpha^N$ and $\tilde{p}^L = \alpha^N$. Symmetrically, when $s_H < s_L$, in the first-best outcome trade occurs with a low-type seller unless all sellers are of high type; and hence the first-best outcome has $\tilde{p}^H = (1 - \alpha)^N$ and $\tilde{p}^L = 1 - (1 - \alpha)^N$.

So, in arguably the most plausible set of circumstances (when $s_H > s_L$) and the most interesting case (when $\alpha > \alpha^*$), the first-best outcome can never be attained in any BNE of any market game.

We now briefly comment on the implications of these results as the number of sellers increases without bound. We have established that in this limiting scenario the lemons market attains the first-best outcome if $s_L > s_H$ but not if $s_H > s_L$. Thus, when first-best optimality requires transferring the high quality object (which would seem to be the relatively more plausible case), inefficiency in the lemons market is inescapable even as competition amongst the sellers to trade with the *single* buyer intensifies without bound. It may be noted that, in contrast, in markets with other kinds of asymmetric information the first-best is typically attainable in equilibrium in the limit as the number of traders increases without bound; for example, Rustichini, Satterthwaite and Williams (1994) establish such a limiting result in markets with private values. In Section 7 we discuss the fundamental differences between lemons markets and markets with private values which are responsible for such differing (limiting) market outcomes.

5. Competition and Efficiency

Having derived the maximal achievable level of expected surplus, we now address the main concern of this paper, namely, how maximal efficiency, E/G, varies with the degree of competition, as defined by the number N of sellers. In particular, whether, as perhaps seems intuitive, the gap between what can be achieved in equilibrium and the first-best outcome is decreasing in the number of sellers. We derive the relationship between maximal efficiency and competition by first considering the soft buyer case and then the tough buyer case.

5.1. **The Soft Buyer Case.** Suppose that $\alpha \leq \alpha^*$. Corollary 1(b) implies that if $s_H < s_L$ then $E/G = 1$ for all N. What if $s_H > s_L$? Corollary 1(a) implies that $E/G = 1$ when $N = 1$. Proposition 1(a) established that E is constant for all $N \geq 1$. At the same time, G (defined in (1)) is strictly increasing in N. Hence, if $s_H > s_L$ then E/G is strictly decreasing in N, and converges to $[(1-\alpha)s_H + \alpha s_L]/s_H$ in the limit as $N \to \infty$. We illustrate these results in Figure 4, and summarize them in the following proposition:

Proposition 2 (Competition–Efficiency with Soft Buyer)**.** *Assume that $\alpha \leq \alpha^*$. If $s_H < s_L$ then $E/G = 1$ for all $N \geq 1$. But if $s_H > s_L$ then E/G is strictly decreasing in N, with $E/G = 1$ when $N = 1$, and with $E/G \to [(1-\alpha)s_H + \alpha s_L]/s_H$ as $N \to \infty$.*

So, when the buyer is soft and $s_H > s_L$, competition does not enhance market efficiency. On the contrary, it affects it adversely. The reasons for this result are as follows: While first-best expected surplus is strictly increasing in the degree of market competition, the maximal achievable expected

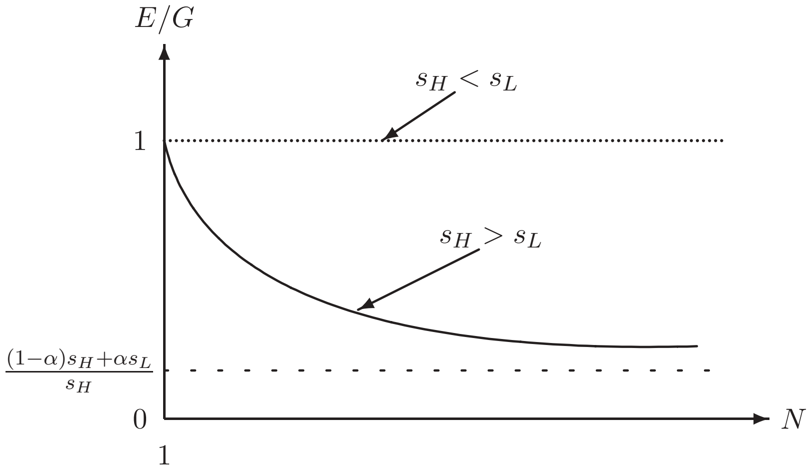

FIGURE 4. The competition-efficiency relationship for lemons markets with "soft" buyer.

surplus is not affected at all by it. Some intuition for this conclusion can be gained from the following observations. In the first-best outcome (depicted as point F in Figure 1) the total probability with which trade occurs with the high-type sellers is $1 - \alpha^N$, which is strictly increasing in N and converges to one in the limit as N diverges to infinity. In contrast, in the maximally efficient outcome (depicted as point B in Figure 1) the total probability with which trade occurs with the high-type sellers is $1 - \alpha$, which is independent of N; and this is partly because of the key implication of incentive-compatibility, namely, that for *any* N the probability with which a high-type seller trades must be no greater than the probability with which a low-type seller trades.

Even in the case where α is sufficiently small but strictly positive (i.e., the case where the probability that an arbitrary seller owns a high quality object is sufficiently large but not one), incentive constraints are unaffected by the degree of market competition. It may be noted that the latter is an important element of what distinguishes lemons markets from, say, markets with private values in which, in contrast, the ease with which incentive constraints are satisfied is increasing with the number of traders.

One implication of Proposition 2 is that maximal market efficiency is maximized at $N = 1$. This observation implies that in the lemons market with a single buyer and in which there is a high change that any seller would own a high quality object should optimally be a bilateral monopoly market (i.e., with a single seller as well). In Appendix B we study the many-buyers case and the results established there imply that in the lemons market with any given finite number of buyers (and in which there is a high chance that any seller would own a high quality object) should also optimally contain the same number of sellers as there are buyers; having more sellers than that affects market efficiency adversely for the same reasons as it does in the single-buyer case (under the assumption that $s_H > s_L$). A general message, then, that appears to emerge from our results is that lemons markets in which high quality objects are more likely to be owned by sellers should optimally (i.e., to maximize maximal market efficiency) contain identical numbers of sellers and buyers.

5.2. **The Tough Buyer Case.** Suppose now that $\alpha > \alpha^*$. Notice, first, from Proposition 1(b) that if $s_L = 0$ (in which case $N^* = \infty$), then $E = 0$; and hence $E/G = 0$ for all $N \geq 1$. This arises because if $s_L = 0$ then in order to satisfy the buyer's individual rationality constraint there should be no trade between the buyer and the high-type sellers, no matter how large are the gains from trade and no matter how intense market competition is. It should be emphasized that this conclusion is obtained under the supposition that $\alpha > \alpha^*$ (i.e., the probability of an arbitrary seller owning a low quality object is sufficiently large).

Now let's focus attention on the case when $s_L > 0$. First consider values of $N \geq N^*$ (the case of large markets); in this case Proposition 1(b)(i) is applicable. If $s_L > s_H$ then $E = G$. But if $s_H > s_L$ then (since E is independent of N while G is strictly increasing in N) it immediately follows that E/G is strictly decreasing in N; and moreover $E/G < 1$ at $N = N^*$.

Now consider values of $N < N^*$ (the case of small markets); here Proposition 1(b)(ii) is applicable. Over such values of N, it is easy to verify that E is strictly increasing in N. But how does the ratio E/G vary with N? If $s_L > s_H$, then it's easy to verify that E/G is strictly increasing. But if $s_H > s_L$, then it turns out that E/G is a non-trivial function of N: it can be monotonic (either strictly increasing or strictly decreasing), or even non-monotonic — the precise characterization of the relationship between E/G and N is contained in Appendix A but its key features are stated in the following proposition, which also summarizes our results for the tough buyer case (and which are illustrated in Figure 5):

Proposition 3 (Competition–Efficiency with Tough Buyer)**.** *Assume that* $\alpha > \alpha^*$.
(a) If $s_L = 0$*, then* $E/G = 0$ *for all* N.
(b) If $s_H < s_L$*, then* E/G *is strictly increasing in* N *over values of* $N \in [1, N^*)$*, and then* $E/G = 1$ *for all* $N \geq N^*$.
(c) If $s_H > s_L > 0$*, then* E/G *is strictly decreasing in* N *over values of* $N \geq N^*$*, with* $E/G < 1$ *at* $N = N^*$*. For values of* $N < N^*$*,* E/G *— which is strictly less than one — is a non-trivial function of* N*; depending*

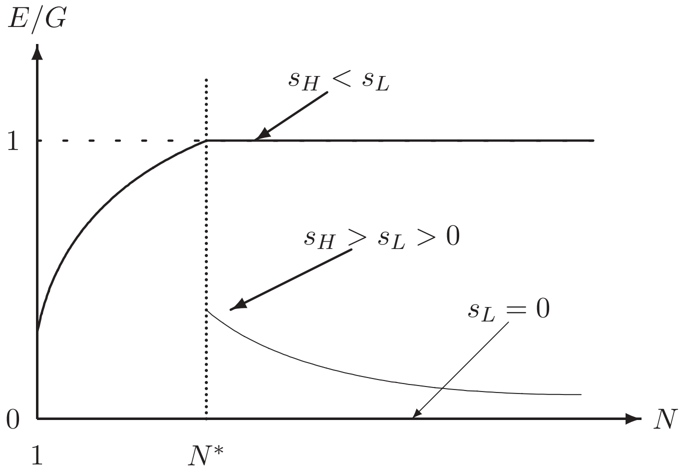

FIGURE 5. The competition-efficiency relationship for lemons markets with “tough” buyer. The relationship between E/G and N on the interval $[1, N^*)$ when $s_H > s_L > 0$ is not illustrated here since it's non-trivial, as is summarized in Proposition 3.

on parameter values, it may be strictly increasing or strictly decreasing or non-monotonic. Its key features are as follows (see Appendix A for full characterization):

- *At $N = 1$, the value of E/G equals the value of E/G at $N = 1$ when $s_H < s_L$ (which is strictly less than one).*
- *If $\alpha < 1/2$ then, depending on parameter values, E/G can either be strictly increasing, or strictly decreasing, or non-monotonic (strictly decreasing at first, achieving a minimum, and then strictly increasing).*
- *If $\alpha = 1/2$ then E/G is strictly increasing.*
- *If $\alpha > 1/2$ then, depending on parameter values, E/G can either be strictly increasing, or strictly decreasing, or non-monotonic (strictly increasing at first, achieving a maximum, and then strictly decreasing).*

So, when $s_L > s_H$ the relationship between maximal market efficiency and competition is straightforward and accords with a priori intuition: Competition enhances market efficiency and once competition is sufficiently intense, the first-best is achieved, and stays so forever after. But when $s_H > s_L$ things are much more complex when competition is not sufficiently intense, and moreover, the relationship does not accord with a priori intuition. The intuition for the relationship for large markets ($N \geq N^*$) — namely, that competition affects maximal market efficiency adversely — is similar to that provided for the soft buyer case (cf. Proposition 2). However, the intuition for the relationship for small markets is complex as it depends on the interplay of both incentives and individual rationality. Further insight about this is provided in sections 6 and 7 where we report the results of numerical simulations.

We now briefly discuss the implications of our results in this tough buyer case for the optimal degree of market competition (i.e., the degree of competition which maximizes maximal market efficiency). When $s_L > s_H$, then the lemons market with a single buyer and where it's the case that any seller would own a lemon (low quality object) with a high probability, it's optimal to have a sufficiently large number of sellers ($N \geq N^*$ where N^* is strictly greater than one). What if, more plausibly, $s_H > s_L$? In that case, the optimal number of sellers in this lemons market is between 1 and N^*. It can be N^*, or one, or some number in between, depending on parameters values. It definitely is not more than N^*.

6. The Case of Two Types of Buyers

6.1. **The Extended Framework.** We now extend the framework studied above by allowing for the buyer to be one of two "types," where his type is his private information. For reasons that will shortly become clear, we call the two possible buyer types as "high" and "low," and respectively denote them by the symbols H and L. Let v_{wq} denote the buyer's valuation of a q ($q = H, L$) quality object when he is a w-type ($w = H, L$), where $v_{wH} > v_{wL}$ for each $w = H, L$ (i.e., each type of buyer prefers the high quality good to the low quality good). Furthermore, let s_{wq} denote the social surplus generated from trading an object of quality q to a w-type buyer; formally, $s_{wq} \equiv v_{wq} - c_q$. We assume that the social surplus is maximized by transferring the high quality good when the buyer is of "high" type, but when buyer is of "low" type it's maximized by transferring the low quality good.[11] That is:

Assumption 2. *$s_{HH} > s_{HL}$ and $s_{LL} > s_{LH}$.*

As mentioned above, the buyer's type is his private information. From the sellers' perspective (and hence this defines their commonly known beliefs about the buyer's type), the probability that the buyer is low-type is β and probability that he is high-type is $1-\beta$, where $\beta \in (0, 1)$. Finally, we assume that for each $w = H, L$, $c_L \leq v_{wL} < c_H < v_{wH}$ (which parallels Assumption 1).

A mechanism asks all agents to report their types and implements a corresponding outcome. As before, we invoke the Revelation Principle to focus on

[11] This would be implied, for example, by a single-crossing type assumption in which the amount of the increase in the buyer's valuation from an increase in quality is monotonic in the buyer's type.

direct revelation mechanisms which are incentive compatible. Furthermore, we require the mechanism to satisfy both budget balance and individual rationality. A (symmetric) mechanism is a vector $(\hat{p}_H^H, \hat{p}_L^H, \hat{p}_H^L, \hat{p}_L^L, \hat{t}_H^H, \hat{t}_L^H, \hat{t}_H^L, \hat{t}_L^L)$, where $\hat{p}_w^q$ and $\hat{t}_w^q$ are respectively the expected probability with which an arbitrary q-type seller's sells her good and the expected transfer to her when the buyer reports that he is of w-type.

Incentive compatibility requires that no agent has an incentive to misrepresent her type. We thus have, for the two types of sellers:

$$(1-\beta)[\hat{t}_H^H-\hat{p}_H^H c_H]+\beta[\hat{t}_L^H-\hat{p}_L^H c_H] \geq (1-\beta)[\hat{t}_H^L-\hat{p}_H^L c_H]+\beta[\hat{t}_L^L-\hat{p}_L^L c_H], \tag{22}$$

$$(1-\beta)[\hat{t}_H^L-\hat{p}_H^L c_L]+\beta[\hat{t}_L^L-\hat{p}_L^L c_L] \geq (1-\beta)[\hat{t}_H^H-\hat{p}_H^H c_L]+\beta[\hat{t}_L^H-\hat{p}_L^H c_L]. \tag{23}$$

And for the two types of buyers:

$$(1-\alpha)[\hat{p}_H^H v_{HH}-\hat{t}_H^H]+\alpha[\hat{p}_H^L v_{HL}-\hat{t}_H^L] \geq (1-\alpha)[\hat{p}_L^H v_{HH}-\hat{t}_L^H]+\alpha[\hat{p}_L^L v_{HL}-\hat{t}_L^L], \tag{24}$$

$$(1-\alpha)[\hat{p}_L^H v_{LH}-\hat{t}_L^H]+\alpha[\hat{p}_L^L v_{LL}-\hat{t}_L^L] \geq (1-\alpha)[\hat{p}_H^H v_{LH}-\hat{t}_H^H]+\alpha[\hat{p}_H^L v_{LL}-\hat{t}_H^L]. \tag{25}$$

The individual rationality constraints require that the expected payoff to any agent be non-negative. We thus have, for the two types of sellers:

$$(1-\beta)[\hat{t}_H^H-\hat{p}_H^H c_H]+\beta[\hat{t}_L^H-\hat{p}_L^H c_H] \geq 0, \tag{26}$$

$$(1-\beta)[\hat{t}_H^L-\hat{p}_H^L c_L]+\beta[\hat{t}_L^L-\hat{p}_L^L c_L] \geq 0. \tag{27}$$

And for the two types of the buyer:

$$(1-\alpha)[\hat{p}_H^H v_{HH}-\hat{t}_H^H]+\alpha[\hat{p}_H^L v_{HL}-\hat{t}_H^L] \geq 0, \tag{28}$$

$$(1-\alpha)[\hat{p}_L^H v_{LH}-\hat{t}_L^H]+\alpha[\hat{p}_L^L v_{LL}-\hat{t}_L^L] \geq 0. \tag{29}$$

The admissibility constraints for this problem follow straightforwardly from the observation that the planner can offer a different gamble to each seller depending on the announcement of the buyer. We thus have:

$$\hat{p}_w^H \leq \frac{1-\alpha^N}{N(1-\alpha)}, \quad (w=H,L) \tag{30}$$

$$\hat{p}_w^L \leq \frac{1-(1-\alpha)^N}{N\alpha}, \quad (w=H,L) \tag{31}$$

$$N[(1-\alpha)\hat{p}_w^H+\alpha\hat{p}_w^L] \leq 1 \quad (w=H,L). \tag{32}$$

The expected surplus generated by an arbitrary direct, symmetric mechanism is

$$N\left[(1-\beta)\left[(1-\alpha)\hat{p}_H^H s_{HH}+\alpha\hat{p}_H^L s_{HL}\right]+\beta\left[(1-\alpha)\hat{p}_L^H s_{LH}+\alpha\hat{p}_L^L s_{LL}\right]\right]. \tag{33}$$

The mechanism design problem is to maximize (33) subject to (22)–(32). Before proceeding to analyze this problem, we note that the first-best expected surplus is

$$(34)\quad (1-\beta)\left[(1-\alpha^N)s_{HH}+\alpha^N s_{HL}\right]+\beta\left[(1-\alpha)^N s_{LH}+[1-(1-\alpha)^N]s_{LL}\right],$$

which is easily interpreted, as follows. When the buyer's type is high, we want to transfer the high quality good from one of the sellers to the buyer except when all sellers are of low type. Similarly, when the buyer's type is low, we want to transfer the low quality good from one of the sellers to the buyer except when all sellers are of high type.

6.2. **Asymptotic Efficiency.** We have not been able to characterize the solution to the mechanism design problem for an arbitrary set of parameter values. To address an issue that we are particularly interested in — namely, the impact of competition on market efficiency — we instead focus attention on markets with a large number of sellers, and for that case we adopt a different route to studying this mechanism design problem.

Observe that when the number of sellers is large, the probability of a "mismatch" between the sellers and the buyer (i.e., the buyer is a low type and all sellers are high type and vice versa) is small. In the limit as N becomes arbitrarily large, we can thus ignore these two cases. Our analysis to follow derives and analyzes the restrictions on the parameters such that a mechanism implementing the first-best outcome (in all cases except the two involving a mismatch) satisfies (22)–(32). Hence, our results below concern conditions under which the lemons market in the context of our extended framework is efficient in the limit as competition intensifies without bound.

By comparing (33) and (34), it follows that (for any parameter values) the values of $\hat{p}_w^q$ $(q, w = H, L)$ given by $(\bar{p}_H^H, \bar{p}_H^L, \bar{p}_L^H, \bar{p}_L^L)$, where these are defined below, generate the first-best expected surplus:

$$\bar{p}_H^H = \frac{1-\alpha^N}{N(1-\alpha)}, \bar{p}_H^L = \frac{\alpha^{N-1}}{N}, \bar{p}_L^H = \frac{(1-\alpha)^{N-1}}{N}, \bar{p}_L^L = \frac{1-(1-\alpha)^N}{N\alpha}.$$

Now define a mechanism, which for notational convenience we denote by $\tilde{\mu}$, in which

$$(35)\qquad \hat{p}_L^H = \hat{p}_H^L = \hat{t}_L^H = \hat{t}_H^L = 0, \hat{p}_H^H = \bar{p}_H^H, \hat{p}_L^L = \bar{p}_L^L.$$

Clearly, this mechanism fails to generate the first-best outcome *only* when the buyer's type is low and all sellers are of high type and *vice versa*. Therefore, this mechanism would be asymptotically efficient. However, for any arbitrary parameters values it will typically *not* satisfy (22)–(32). In the proposition below, we establish restrictions on the values of the parameters under which this mechanism would (in the limit as N becomes arbitrarily large) satisfy the IC, IR and admissibility conditions. In order to simplify the involved computations, but with some loss of generality, we adopt the following additional assumptions about the players' valuations:

Assumption 3. *(a) $v_{HH} > v_{LH}$ and $v_{LL} > v_{HL}$; (b) $v_{HL} = c_L$; and (c) $c_H = 1$ and $c_L = 0$.*

Assumption 3(a) states that the high-type buyer values the high quality good more than does the low-type buyer; and symmetrically, the low-type buyer values the low quality good more than does the high-type buyer. Assumption 3(b) states that there are no gains from trade between a high-type buyer and a low-type seller (i.e., $s_{HL} = 0$).[12] Assumption 3(c) is no more than a normalization and is made only for convenience; the proposition below holds even if we drop this assumption.

Before proceeding with the statement of the proposition, we note that Assumption 3, when combined with the assumption made above, one that parallels Assumption 1, implies that:

$$v_{HH} > v_{LH} > c_H = 1 > v_{LL} > v_{HL} = c_L = 0.$$

We are now ready to state our main result for our extended framework:

Proposition 4. *The mechanism $\tilde{\mu}$ defined above satisfies (22)–(32) in the limit as the number of sellers becomes arbitrarily large if*

$$\frac{\beta(1-\alpha)}{\alpha(1-\beta)} > \frac{v_{LH}}{v_{LL}}. \tag{36}$$

It may be noted that the RHS of (36) is strictly greater than one, and hence this implies that (36) holds only if $\beta > \alpha$. This necessary condition reflects the point that the likelihood of the buyer being of low-type has to be sufficiently large in order to provide the low-type seller with incentives to reveal his type.

6.3. **Numerical Analysis.** As mentioned before, we have not been able to characterize the solution to the mechanism design problem analytically. However, we have been able to gain some insights through numerical analysis which suggest significant differences between the case when the buyer's type is commonly known and when it is not known publicly. We illustrate these differences by looking at the numerical solution for the following parameter values: $v_{HH} = 1.45, v_{HL} = 0, c_H = 1, c_L = 0, v_{LH} = 1.35, v_{LL} = 0.45, \alpha = 0.5$. For this set of parameter values, Proposition 4 implies that the approximating mechanism $\tilde{\mu}$ satisfies incentive compatibility and individual rationality as the number of sellers becomes large if $\beta \geq 0.75$.

[12] This parallels the case in which $s_L = 0$ in our earlier analysis with a single type of buyer. There we showed, for example, that when the buyer is tough and $s_L = 0$ then no trade ever occurs, and the best possible market equilibrium outcome is far away from the first-best outcome, no matter how intense competition is (see Figure 4). Indeed, this observation may suggest that Assumption 3(b) may similarly make it impossible to achieve the first-best outcome in the limit as N becomes arbitrarily large. But we show in Proposition 4 that even with Assumption 3(b), in this extended framework the first-best outcome is implementable in this limit *provided* that the probability with which the buyer is low-type is sufficiently high.

The first observation from our simulation results is that the condition identified in Proposition 4 constitutes only a sufficient condition for convergence to full efficiency. Figure 6 illustrates the relationship between market

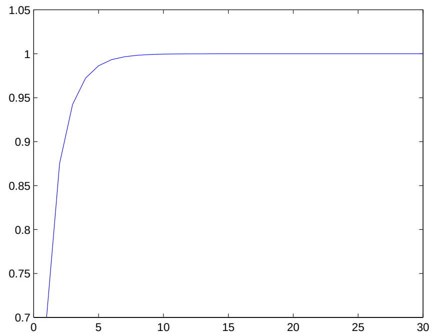

FIGURE 6. Market efficiency v/s Number of sellers for $\beta = 0.55$

efficiency and the number of sellers for $\beta = 0.55$. Here, the market converges to efficiency when $N \geq 13$. Incidentally, $\beta = 0.55$ appears to be the smallest value for which convergence to full efficiency obtains; furthermore, if we choose $\beta \geq 0.75$, then convergence to full efficiency obtains for $N = 2$.

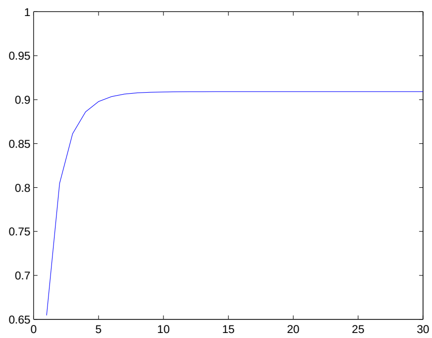

FIGURE 7. Market efficiency v/s Number of sellers for $\beta = 0.5$

The second observation, illustrated in Figure 7, is that increasing the number of sellers can increase market efficiency even if there is no convergence to full efficiency. This is in contrast to what we saw in the case where the buyer's type was common knowledge: there, except for one case, increasing the number of sellers had either a negative impact or no impact whatsoever on market efficiency.

The third observation, illustrated in Figure 8, is that the convergence towards full or partial efficiency may not be monotonic. We have not been

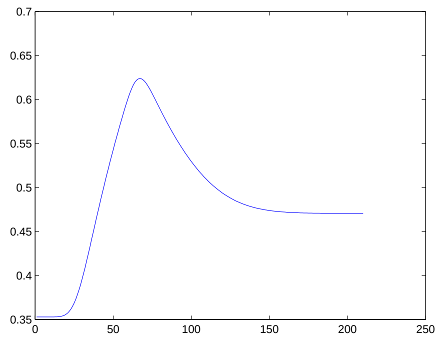

FIGURE 8. Market efficiency v/s Number of sellers for $\beta = 0.4$

able to explain this non-monotone behaviour; obviously, further work is needed to understand what is going on here.

7. Discussion and Related Literature

In the context of markets with asymmetric information, the relationship between competition and market efficiency has been examined for the *private values* case, which concerns trade in a homogenous object.[13] It is instructive to compare our results with those obtained there.

7.1. **Asymptotic efficiency.** Satterthwaite and Shneyerov (2003) note that asymptotic efficiency has been established under very general conditions for the private values case. Indeed, in some papers—for example, Rustichini, Satterthwaite and Williams (1994) and Tatur (2004)—the concern has gone beyond establishing asymptotic efficiency to looking at the rate of convergence to efficiency. In contrast, in our benchmark model, we obtain asymptotic efficiency only in the case when the incentive constraint $\hat{p}^H \leq \hat{p}^L$ does not bind. The failure to obtain asymptotic efficiency is related to the fact that the incentive constraint $\hat{p}^H \leq \hat{p}^L$ is independent of the number of sellers; it therefore follows that when this constraint binds, inefficiency will always be present. In contrast, Gresik and Satterthwaite (1989) establish that in the private values case, the incentive constraint of a seller (buyer) weakens as the number of sellers (buyers) increases.

One can understand the difference in the two situations as follows. In the private values case, inefficiency results solely as a result of some profitable trades not taking place.[14] In the market for lemons, there is an additional

[13] See, for example, the papers of Gresik and Satterthwaite (1989), Satterthwaite and Williams (1989), Williams (1991), Rustichini, Satterthwaite and Williams (1994), and Tatur (2004).

[14] There is, of course, the possibility of a "wrong" kind of trade wherein a seller with a higher reservation price sells his object in preference to a seller with a lower reservation price. A similar possibility exists on the buyers side too. Note though that such a trade

source of inefficiency—viz., trade involving the "wrong" type of object. More precisely, it being inefficient to transfer a low quality object when it is optimal to transfer the high quality object. Competition amongst sellers helps in overcoming the first type of inefficiency but not the second. It is worth noting that we obtain asymptotic efficiency only when the second source of inefficiency is not present.

7.2. **Non-monotonicity.** One important aspect of our results, both in the benchmark and the extended models, is that efficiency may be a non-monotonic function of the number of sellers. It is not easy to provide economic intuition for this result. In the benchmark model, non-monotonicity arises when the probability that each seller owns a lemon is high enough so that the high quality object cannot be transferred with probability one even when all traders report that they have high quality objects.[15] When one adds additional sellers, the achievable expected surplus increases because the probability that all traders have high quality objects decreases. However, the first-best level of expected surplus also increases, and so it is not clear what happens to maximal efficiency (i.e., the ratio of the maximal achievable level of surplus to the first-best surplus). As we showed in Proposition 3, the behaviour of this ratio depends on whether α is bigger, equal to or less than 0.5. Note that this non-monotonicity result persists in the extended model as well, though the reasons for this behaviour in that model are less clear. There is no equivalent of this non-monotonicity result in the private values case.

7.3. **Impact of market size.** Our analysis for the most part involves holding the number of buyers constant while changing the number of sellers. In contrast, the literature in the private values case has considered the more traditional replication scenario, where the number of buyers and sellers are both varied while keeping the buyer-seller ratio constant. We have been able to derive only limited results with regard to the impact on efficiency of changing the market size for our benchmark model. If we start from a scenario where $N \leq M$ (the number of buyers is more than the number of sellers), then our analysis shows that replicating this economy has no impact on market efficiency. On the other hand, if we start from a scenario where $M < N$ and $s_H > s_L$ then our analysis shows that replicating this setup will not lead to asymptotic efficiency. It appears difficult saying anything more because the feasible set (in the $(\tilde{p}^H, \tilde{p}^L)$ space) changes.[16] It is thus difficult determining the behavior of the efficiency ratio. Note though, that

can occur only if a seller (buyer) understates (overstates) her valuation. Such a strategy is never in the trader's own interest.

[15]This is the case illustrated in Figure 3.

[16]As an example, consider Figure 12 which concerns the case $M < N$ and $\alpha \leq \alpha^*$. If we increase both M and N while keeping M/N constant, then the following happens: (i) the line $\tilde{p}^H + \tilde{p}^L = M$ shifts outward, (ii) the lines $\tilde{p}^H = N(1-\alpha)b^H$ and $\tilde{p}^L = N\alpha b^L$ shift outward.

in the case $M < N$ we showed that asymptotic efficiency could be obtained only when $s_L > s_H$ and N increases while holding M constant. Intuitively, therefore, we would not expect an increase in the market size to enhance efficiency. This is what our simulation results suggest, some of which we report below to illustrate the different possibilities regarding the behavior of the efficiency ratio. These simulations have been done starting with an initial setting of $M = 1, N = 2$ and $c_H = 2, c_L = 1$.

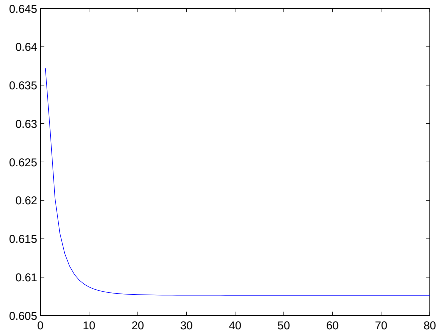

FIGURE 9. Market efficiency v/s Market size when $s_H > s_L$ illustrating decreasing efficiency with increasing market size

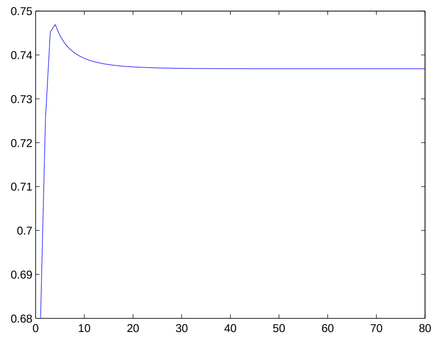

FIGURE 10. Market efficiency v/s Market size when $s_H > s_L$ illustrating non-monotonic behaviour of the efficiency ratio

Figures 9 and 10 both concern the case when $s_H > s_L$. Both illustrate that in this case, increasing the market size actually makes the market less efficient. Figure 10 suggests that there may be an initial phase when the efficiency of the market increases, which reinforces our earlier point about non-monotonicity.[17]

[17]The parameters used in Figure 9 are $\alpha = 0.35, v_H = 2.2.5, v_L = 1.15$ and in Figure 10, $\alpha = 0.75, v_H = 2.25, v_L = 1.1$.

The last simulation considers the case of $M < N$ and $s_L > s_H$. Note that if the first-best is achievable for the given (M, N), then clearly it will be achievable in the replicated economy as well. If the first-best is not achievable in the initial setting, then it may not be obtainable in the replicated economy as shown in Figure 11.[18]

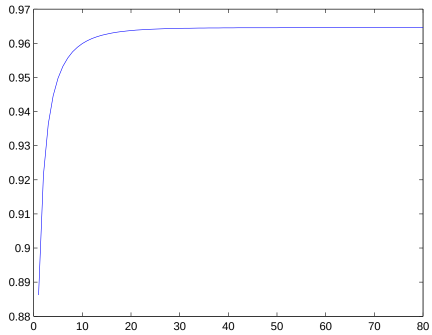

FIGURE 11. Market efficiency v/s Market size when $s_L > s_H$

7.4. **Relationship to the literature on the market for lemons.** The literature on the market for lemons is too large to be summarized here; we confine ourselves to discussing those papers which have a direct bearing on our paper. The use of the mechanism design methodology in our benchmark model can be regarded as a direct follow-up to the work of Samuelson (1984) who studied the bilateral lemons problem; our extension consists of analyzing the general case of finite but arbitrary number of sellers and buyers.[19]

The inefficiency pointed to by Akerlof (1970) has prompted economists to examine ways by which this inefficiency can be overcome. For instance, Klein and Leffler (1981) and Tirole (1996) have suggested that repeated interactions may overcome the adverse selection problem; on another dimension, the works of Hendel and Lizzeri (1999), Janssen and Roy (2002) and Hendel, Lizzeri and Siniscalchi (2004) suggest that particular features of the durable goods market, when taken into account, can overcome partially or even fully the inefficiency associated with the lemons market.[20] Our extended model is in the same spirit as these papers; however, there are two differences. Firstly, the other papers mostly use models with a continuum of agents; their models cannot therefore directly address the question of interest to us which is the impact of competition on market efficiency. In

[18]Note that for this case, we showed that if the number of buyers is kept constant, then efficiency is achieved for sufficiently large N. The parameters used are $v_L = 1.15, v_H = 2.05$ and $\alpha = 0.35$.

[19]See also Evans (1989), Vincent (1989), and Manelli and Vincent (1995).

[20]Note that the motivating example used by Akerlof in his seminal paper was the 'market for second hand cars' which suggests a durable good.

this context, note that in our extended model, we need private information on the buyers' side as well as a large number of sellers to obtain efficiency. Secondly, in contrast to the other papers which use the dynamic element, our extended model is still a static one and as such, closer to the basic Akerlof model. Our results thus show that it is possible to obtain asymptotic efficiency even in markets for non-durable goods and without using repeated game effects.

8. Concluding Remarks

We end by discussing the sensitivity of our results to the particular specification of our model. One issue concerns our use of a finite type space: we assume that both sellers and buyer can be of one of two types. We think that variants of our results can be proved for the continuum type case, but this will require different techniques to that used here; we intend pursuing this route in future work.

A second issue concerns our assumption that sellers types are independent. One might ask, if by allowing for correlation, one can obtain positive results as in Crémer and McLean (1988). They showed that if agents' types are correlated in a Bayesian mechanism, then one can construct a two-stage 'augmented mechanism' which implements the same outcome as the original mechanism, but where all agents' informational rents are driven down to zero. This fact can be used to implement efficient outcomes in some circumstances *provided* the efficient outcome is implementable without requiring budget balance: essentially, one can construct an 'augmented mechanism' which recovers from the agents the implicit subsidy needed to implement the efficient outcome. Thus, the key to seeing whether correlation amongst sellers' types can help in our context is to see whether we can implement the efficient outcome if we do not require individual rationality and/or budget balance. Note, however, that incentive compatibility by itself requires that $\hat{p}^H \leq \hat{p}^L$ and this condition implies inefficiency whenever $s_H > s_L$. Therefore, the first-best outcome is unimplementable even if we are willing to give up budget balance and individual rationality and this shows that correlation amongst sellers' types in unlikely to change the nature of our results.

Appendix A: Omitted Proofs

Proof of Lemma 1. The lemma can be established using standard techniques. Here is an outline of the argument:

(I) *Incentive Compatibility implies that* $\hat{p}^H \leq \hat{p}^L$: We can write (6) and (7) as

$$(\hat{p}^H - \hat{p}^L)c_H \leq \hat{t}^H - \hat{t}^L \leq (\hat{p}^H - \hat{p}^L)c_L.$$

The result follows since $c_H > c_L$.

(II) *For each type of seller, either her incentive compatibility constraint binds or her individual rationality constraints binds*: Suppose, to the contrary, that neither (6) nor (8) bind for the high-type seller. Then, one can increase the expected surplus by increasing $\hat{p}^H$ and lowering $\hat{t}^H$ without violating incentive compatibility or individual rationality. The same argument applies with respect to the low-type seller.

(III) *It cannot be the case that both incentive compatibility constraints bind*: Suppose, to the contrary, that this is the case. Then, neither individual rationality constraint can bind. We can thus increase the expected surplus by increasing $\hat{p}^H$ and $\hat{p}^L$ both by $\epsilon > 0$ and decreasing $\hat{t}^H$ and $\hat{t}^L$ both by $\delta > 0$. Note that these changes do not affect the incentive compatibility constraints. If ϵ and δ are small enough, then the individual rationality constraints are unaffected.

(IV) *It cannot be the case that the high-type seller's incentive compatibility constraint and the low-type seller's individual rationality constraint bind*: Suppose, to the contrary, that this is the case. Then, we have $\hat{t}^H = \hat{t}^L + (\hat{p}^H - \hat{p}^L)c_H = \hat{p}^L c_L + (\hat{p}^H - \hat{p}^L)c_H$. Therefore, $\hat{t}^H - \hat{p}^H c_H = \hat{p}^L c_L + (\hat{p}^H - \hat{p}^L)c_H - \hat{p}^H c_H = \hat{p}^L(c_L - c_H)$. Since $c_L < c_H$, this implies that we must have $\hat{p}^L = 0$, which in turn implies that $\hat{p}^H = 0$, and via the incentive compatibility constraints, that $\hat{t}^H = \hat{t}^L$. This implies that both incentive compatibility constraints bind, a contradiction.

Hence, at a solution to (14), inequalities (7) and (8) will be binding constraints.

Proof of Lemma 2. Suppose that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ solves the mechanism design problem (14). Then, $\tilde{p}^H = N(1-\alpha)\hat{p}^H$ and $\tilde{p}^L = N\alpha\hat{p}^L$ satisfy (15)–(19), and so $E \leq E^*$. Now suppose that $(\tilde{p}^H, \tilde{p}^L)$ solves the reduced-form mechanism design problem (20). Define $\hat{p}^H = \tilde{p}^H/N(1-\alpha)$, $\hat{p}^L = \tilde{p}^L/N\alpha$, $\hat{t}^H = \hat{p}^H c_H$, and $\hat{t}^L = \hat{p}^L c_L + \hat{p}^H(c_H - c_L)$. It is straightforward to verify that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ satisfies (6)–(13), and hence $E^* \leq E$. Therefore $E = E^*$.

Proof of Proposition 1. In the proof of this proposition stated in the text (immediately after the statement of the proposition) we sketched the main elements of the argument. Here we provide a fuller argument, with attention being focused on the main omitted computations. We conveniently break our argument into two main cases, depending on whether Z is negative or positive, where

$$Z \equiv s_H - \alpha(v_H - c_L)$$

is the coefficient of $\tilde{p}^H$ in (19). First consider the case when $Z \geq 0$ (i.e., $\alpha < s_H/(v_H - c_L)$).[21] In this case (19) can be rewritten as

$$\tilde{p}^H \geq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L,$$

and hence (since $Z \geq 0$) the feasible set of the maximization problem (20) is the shaded region in Figure 1. It thus follows that in this case the unique solution of (20) is at point B if $s_H > s_L$ and at point A if $s_H < s_L$, i.e.,

$$(\tilde{p}^L, \tilde{p}^H) = \begin{cases} (\alpha, 1-\alpha) & \text{if } s_H > s_L, \\ (1-(1-\alpha)^N, (1-\alpha)^N) & \text{if } s_H < s_L. \end{cases}$$

[21]It may be noted that $s_H/(v_H - c_L) < \alpha^*$, and hence this case refers to part (a) of the proposition.

Now consider the case when $Z < 0$ (i.e., $\alpha > s_H/(v_H - c_L)$). In this case (19) can be rewritten as

$$\tilde{p}^H \leq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L.$$

Notice that in this case the line

$$\tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L \tag{A.1}$$

is positively sloped; whereas in the previous case when $Z \geq 0$, the line (A.1) was non-positively sloped. There are three subcases to consider here, depending on the relative position of the line (A.1).

If the slope of the line (A.1) is greater than or equal to $(1-\alpha)/\alpha$ — which is the case if and only if $\alpha \leq \alpha^*$ — then (A.1) lies above the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$, and hence the feasible set of the maximization problem (20) in this case [when $\alpha \in (s_H/(v_H - c_L), \alpha^*)$] continues to be the shaded region in Figure 1. It thus follows that in this case the unique solution of (20) is the same as for the case above when $\alpha < s_H/(v_H - c_L)$.

Now suppose that $\alpha > \alpha^*$ — which means that the line (A.1) lies below the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$. This is shown in Figures 2 and 3, depending on whether it intersects the line $\tilde{p}^H + \tilde{p}^L = 1$ to the left of (or at) point A or to the right of point A. After some simplification, it can be shown that the former is the case if and only if inequality (21) holds; and that the latter is the case if and only if (21) does not hold — notice that Proposition 1(b)(i) concerns the former case while Proposition 1(b)(ii) the latter.

When (21) holds, the unique solution of (20) lies, as shown in Figure 2, at point C if $s_H > s_L$ and at point A if $s_H < s_L$, i.e.,

$$(\tilde{p}^L, \tilde{p}^H) = \begin{cases} \left[\dfrac{-Z}{(1-\alpha)s_L - Z}, \dfrac{(1-\alpha)s_L}{(1-\alpha)s_L - Z}\right] & \text{if } s_H > s_L, \\ (1-(1-\alpha)^N, (1-\alpha)^N) & \text{if } s_H < s_L. \end{cases}$$

When (21) does not hold, then the unique solution of (20) lies, as shown in Figure 3, at point D, i.e.,

$$\tilde{p}^L = 1 - (1-\alpha)^N \quad \text{and} \quad \tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right][1-(1-\alpha)^N].$$

Proof of Proposition 3. Here we provide a full characterization of the competition-efficiency relationship when $\alpha > \alpha^*$, $s_H > s_L > 0$ and $N \in [1, N^*)$. Maximal efficiency is E/G where, for this set of parameter values, E is defined in Proposition 1(b)(ii) and it follows from (1) that $G = \alpha^N s_L + (1-\alpha^N)s_H$.

It's straightforward to show that since $dE/dN > 0$ and $dG/dN > 0$,

$$\frac{d(E/G)}{dN} \gtreqqless 0 \quad \text{iff} \quad \frac{G}{dG/dN} \gtreqqless \frac{E}{dE/dN}.$$

Hence, after substituting for E, dE/dN, G and dG/dN, simplifying and re-arranging terms, we obtain that

$$\frac{d(E/G)}{dN} \gtreqqless 0 \qquad \text{iff}$$

$$\text{(A.2)} \qquad \left[\frac{\alpha}{1-\alpha}\right]^N \lesseqqgtr \left[1 - \frac{\ln(1-\alpha)}{\ln\alpha}\right]\alpha^N + \left[\frac{\ln(1-\alpha)}{\ln\alpha}\right]\left[\frac{s_H}{s_H - s_L}\right].$$

We now divide the rest of argument into three cases, depending on whether α equals, or is less than, or is greater than $1/2$.

Case 1: $\alpha = 1/2$. In this case, (A.2) reduces to

$$1 \lesseqqgtr \frac{s_H}{s_H - s_L}.$$

Hence it follows that E/G is strictly increasing in N over the interval $[1, N^*)$.

Case 2: $\alpha > 1/2$. In this case, the RHS of (A.2) is strictly decreasing while the LHS is strictly increasing (in N). This gives rise to two subcases:

(i) *At $N = 1$, the LHS of (A.2) is strictly greater than the RHS of (A.2).* In this subcase, the LHS is strictly greater than the RHS for all N, and hence it follows that E/G is strictly decreasing in N over the interval $[1, N^*)$.

(ii) *At $N = 1$, the LHS of (A.2) is less than or equal to the RHS of (A.2).* In this subcase, there exists an $\tilde{N} \geq 1$ such that LHS $\lesseqqgtr$ RHS if and only if $N \lesseqqgtr \tilde{N}$. This implies that if $\tilde{N} \geq N^*$ then E/G is strictly increasing in N over the interval $[1, N^*)$. But if $\tilde{N} < N^*$ then E/G is non-monotonic in N over the interval $[1, N^*)$: it's strictly increasing over $[1, \tilde{N})$, achieves local maximum at $\tilde{N}$ and is strictly decreasing over $(\tilde{N}, N^*)$.

Case 3: $\alpha < 1/2$. In this case, the RHS of (A.2) is strictly increasing while the LHS is strictly decreasing (in N). This gives rise to two subcases:

(i) *At $N = 1$, the RHS of (A.2) is strictly greater than the LHS of (A.2).* In this subcase, the RHS is strictly greater than the LHS for all N, and hence it follows that E/G is strictly increasing in N over the interval $[1, N^*)$.

(ii) *At $N = 1$, the RHS of (A.2) is less than or equal to the LHS of (A.2).* In this subcase, there exists an $\hat{N} \geq 1$ such that RHS $\lesseqqgtr$ LHS if and only if $N \lesseqqgtr \hat{N}$. This implies that if $\hat{N} \geq N^*$ then E/G is strictly decreasing in N over the interval $[1, N^*)$. But if $\hat{N} < N^*$ then E/G is non-monotonic in N over the interval $[1, N^*)$: it's strictly decreasing over $[1, \hat{N})$, achieves local minimum at $\hat{N}$ and is strictly increasing over $[\hat{N}, N^*)$.

Proof of Proposition 4. The mechanism $\tilde{\mu}$ leaves unspecified the transfers $\hat{t}^H_H$ and $\hat{t}^L_L$. So, let us define $\hat{t}^H_H = \bar{p}^H_H c_H + \delta$ for some $\delta \geq 0$ and $\hat{t}^L_L = \bar{p}^L_L v_{LL} - \gamma$ for some $\gamma \geq 0$. Given these transfers and the mechanism $\tilde{\mu}$, we now have a completely specified (symmetric, direct) mechanism, which of course varies as δ and/or γ vary. Our analysis below will show that if the parameters satisfy (36) then (in the limit as N tends to infinity) there exists a $\delta \geq 0$ and a $\gamma \geq 0$ such that this mechanism satisfies the four IC conditions and four IR conditions. This would then establish the proposition.

First, we note that (by definition) this mechanism satisfies the IR condition of the high-type seller (namely, (26)) and the IR condition of the low-type buyer (namely, (29)). Furthermore, it's easy to verify that the high-type seller's IC condition (22) is also satisfied since (by assumption) $c_H > v_{LL}$.

Now making use of Assumption 3, it's easy to verify that if the mechanism satisfies the low-type seller's IR condition (27) and the high-type buyer's IR condition (28), then it would also satisfy the high-type buyer's IC condition (24). Hence, given the observations made so far, we now need to explore whether or not there exists $\delta \geq 0$ and $\gamma \geq 0$ that satisfy the low-type seller's and the low-type buyer's IC conditions ((23) and (25)), *and* the low-type seller's and high-type buyer's IR conditions ((27) and (28)). After substituting for the mechanism $\tilde{\mu}$ with the transfers specified above (and making use Assumption 3), conditions (23), (25), (27) and (28) respectively become:

$$z_1 - \left[\frac{1-\beta}{\beta}\right]\delta \geq \gamma \tag{A.3}$$

$$z_2 - \left[\frac{1-\alpha}{\alpha}\right]\delta \leq \gamma \tag{A.4}$$

$$\bar{p}_L^L v_{LL} \geq \gamma \tag{A.5}$$

$$\bar{p}_H^H (v_{HH} - 1) \geq \delta, \quad \text{where} \tag{A.6}$$

$$z_1 = \bar{p}_L^L v_{LL} - \left[\frac{1-\beta}{\beta}\right]\bar{p}_H^H \quad \text{and} \quad z_2 = \left[\frac{1-\alpha}{\alpha}\right]\bar{p}_H^H (v_{LH} - 1).$$

We now divide the argument according to whether $z_2 > z_1$ or $z_1 > z_2$.

First consider the case in which $z_2 > z_1$. It's straightforward to verify that there exists $\delta \geq 0$ and $\gamma \geq 0$ satisfying (A.3)–(A.6) if and only if

$$\frac{\beta z_1}{1-\beta} > \frac{\alpha z_2}{1-\alpha}.$$

Note that since $z_2 > 0$, the latter condition implies that $z_1 > 0$. It may also be noted that our claim follows since (given the assumption that $v_{HH} > v_{LH}$) it is always the case that

$$\frac{\alpha z_2}{1-\alpha} < \bar{p}_H^H (v_{HH} - 1).$$

Now, after substituting for the values of z_1 and z_2 — and in the process substituting for the values of $\bar{p}_H^H$ and $\bar{p}_L^L$ in z_1 and z_2 — it's thus straightforward to show that the existence of a $\delta \geq 0$ and $\gamma \geq 0$ is assured in the limit as N tends to infinity if and only if the parameters satisfy the following condition:

$$\frac{v_{LH}}{v_{LL}} < \frac{\beta(1-\alpha)}{\alpha(1-\beta)} < \frac{1}{s_{LL} - s_{LH}}. \tag{A.7}$$

This defines a non-empty set of parameter values.

We now turn attention to the case in which $z_1 > z_2$. It's straightforward to verify that there exists $\delta \geq 0$ and $\gamma \geq 0$ satisfying (A.3)–(A.6) if and only if

$$\textit{Either} \quad (i) \quad \frac{\beta z_1}{1-\beta} > \frac{\alpha z_2}{1-\alpha},$$

$$\textit{Or} \quad (ii) \quad \frac{\beta z_1}{1-\beta} < \frac{\alpha z_2}{1-\alpha} \quad \text{and} \quad \bar{p}_L^L v_{LL} \geq \underline{\gamma}, \quad \text{where}$$

$$\underline{\gamma} = \frac{(1-\beta)(1-\alpha)}{\beta - \alpha}\left[\frac{\beta z_1}{1-\beta} - \frac{\alpha z_2}{1-\alpha}\right].$$

Note that the first inequality in part (ii) implies (since $z_1 > z_2$) that $\alpha > \beta$, which, in turn, means that $\underline{\gamma} > 0$. It however turns out that in the limit as N tends to infinity, the second inequality in part (ii) holds (given that here $\alpha > \beta$) if and only if $v_{LH} < v_{LL}$, which is inconsistent with our maintained assumption. Hence, in this limit the conditions in part (ii) (in conjunction of course with the hypothesis of this case that $z_1 > z_2$) cannot all be satisfied.

This means that in the limit as N tends to infinity, we obtain (after substituting for the values of z_1 and z_2; and in the process substituting for the values of $\bar{p}_H^H$ and $\bar{p}_L^L$ in z_1 and z_2) that the existence of a $\delta \geq 0$ and $\gamma \geq 0$ is assured if and only if the parameters satisfy the following condition (which relates to part (i) in conjunction of course with the hypothesis of this case that $z_1 > z_2$):

$$\text{(A.8)} \qquad \frac{\beta(1-\alpha)}{\alpha(1-\beta)} > \frac{1}{s_{LL} - s_{LH}}.$$

This defines a non-empty set of parameter values. The Proposition follows immediately by combining (A.7) and (A.8).

Appendix B: Generalization to the Many-Buyers Case

In this appendix we extend our analysis to the case in which the market has an arbitrary number M of buyers. It turns out that the analysis for the case when $M < N$ is essentially identical to the analysis conducted in the text of the single buyer case; and the main results obtained above for the single buyer case carry over to this case. The analysis and results for the case when $M \geq N$ are however different.

The only difference between the mechanism design problem with $M = 1$, as stated in (14), and the problem with an arbitrary number M of buyers concern the three admissibility conditions. For any M, the symmetry of the mechanism now entails that $p_k^H \leq \min\{1, M/k\}$ (for $k > 0$) and $p_k^L \leq \min\{1, M/(N-k)\}$ (for $k < N$). Using (2) and (4), these conditions imply that the mechanism must satisfy the following two conditions:

$$\text{(B.1)} \qquad \hat{p}^H \leq \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} \min\left\{1, \frac{M}{k+1}\right\} \qquad \text{and}$$

$$\text{(B.2)} \qquad \hat{p}^L \leq \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} \min\left\{1, \frac{M}{N-k}\right\}$$

Furthermore, since the expected number of objects transferred to the buyers must be less than or equal to $\min\{M, N\}$, the mechanism must also satisfy

$$\text{(B.3)} \qquad N[(1-\alpha)\hat{p}^H + \alpha\hat{p}^L] \leq \min\{M, N\}.$$

We can now formally state the mechanism design problem for an arbitrary M and arbitrary N; it is as follows:

$$\text{(B.4)} \qquad E \equiv \max_{\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L} \quad N\Big[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\Big]$$

subject to (6)–(10) and (B.1)–(B.3).

The Case of M < N. We solve the mechanism design problem (B.4) in this case in exactly the same manner as we did above for the case when $M = 1$. The only difference is that now the three admissibility constraints (B.1), (B.2) and (B.3) respectively become (using the same change of variables)

$$\tilde{p}^H \leq N(1-\alpha)b^H, \tag{B.5}$$

$$\tilde{p}^L \leq N\alpha b^L \quad \text{and} \tag{B.6}$$

$$\tilde{p}^L + \tilde{p}^H \leq M, \tag{B.7}$$

where b^H and b^L respectively denote the right-hand sides of (B.1) and (B.2). Hence, the mechanism design problem (B.4) for the case when $M < N$ can be solved by instead solving the following reduced-form problem:

$$E \equiv \max_{\tilde{p}^H, \tilde{p}^L} \quad \tilde{p}^H s_H + \tilde{p}^L s_L \tag{B.8}$$

$$\text{subject to (18)–(19) and (B.5)–(B.7)} .$$

With the aid of Figures 12–14 — which parallel Figures 1–3 — it's relatively easy to characterize the solution to (B.8) by using exactly the same arguments to those used in establishing Proposition 1.[22] As such it's easy to obtain a proposition that characterizes the maximal achievable level of expected surplus, a proposition that would parallel (and look very similar to) Proposition 1. Since nothing significantly new is obtained over and above the arguments, results and insights derived and discussed in the single buyer case, we instead proceed to discuss the robustness (or otherwise) of the results in Propositions 2 and 3 (illustrated in Figures 4 and 5 respectively) concerning the "competition-efficiency" relationship when there are an arbitrary number of buyers, but whose number M is strictly less than N.

In discussing the impact of the degree of competition on the maximal achievable level of efficiency, we keep M fixed at some arbitrary level and allow N to vary over the set $\{M+1, M+2, \ldots\}$. First we consider the case when the buyers are soft (and hence we address the robustness or otherwise of the results in Proposition 2). We first note that when $s_L > s_H$ then point A in Figure 12 depicts both the first-best outcome and the outcome associated with the maximal achievable expected surplus. On the other hand, when $s_H > s_L$ then point F depicts the first-best outcome while point B the outcome associated with the maximal achievable expected surplus. Since point B is unaffected by N while point F moves upwards along the $\tilde{p}^H + \tilde{p}^L = M$ line, where the former means that the maximal achievable expected surplus is independent of N while the latter means that first-best expected surplus is increasing in N, we obtain that maximal efficiency is decreasing in N (for a fixed $M < N$). In conclusion, the competition-efficiency relationship when $M < N$ and the buyers are soft is as illustrated in Figure 4.

Now consider the case when the buyers are tough (and hence we address the robustness or otherwise of the results in Proposition 3). If $s_L = 0$ then $E = 0$ for exactly the same reasons as in the single buyer case. Now suppose the $s_L > 0$. Just like in the single buyer case it can be shown that point C lies above point

[22]It should be noted that both b^H and b^L are bounded from below by M/N. Furthermore, b^H and b^L are respectively bounded from above by $M(1-\alpha^N)/N(1-\alpha)$ and $M[1-(1-\alpha)^N]/N\alpha$.

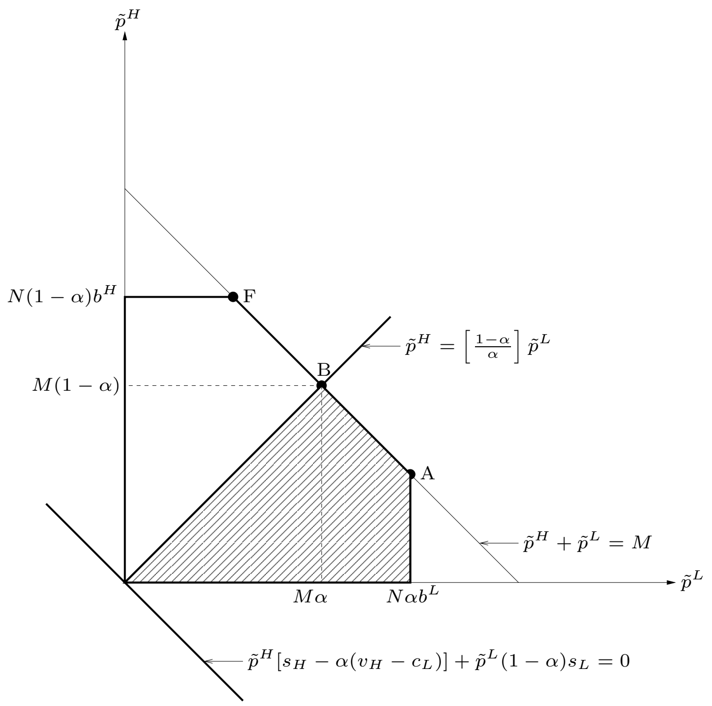

FIGURE 12. The feasible set when $M < N$ and $\alpha \leq \alpha^*$.

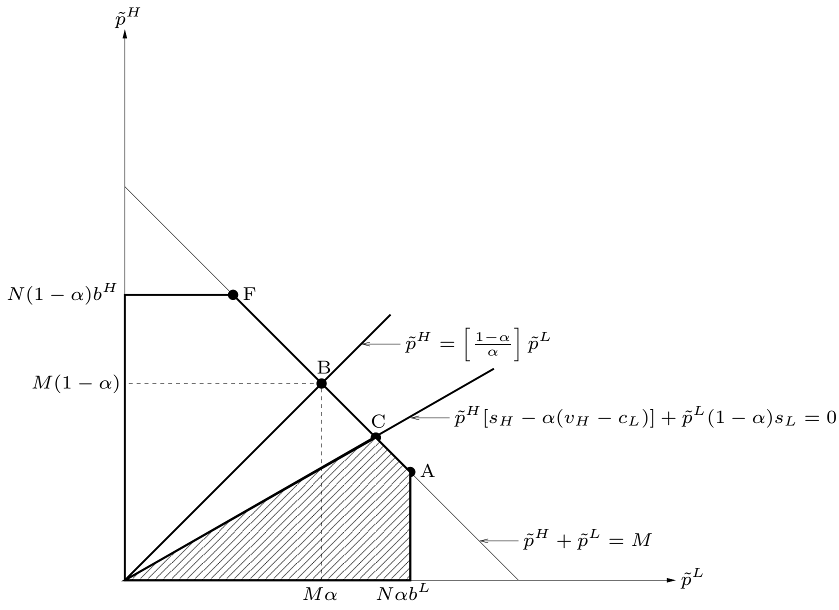

FIGURE 13. The feasible set when $M < N$, $\alpha > \alpha^*$ and point C lies above point A.

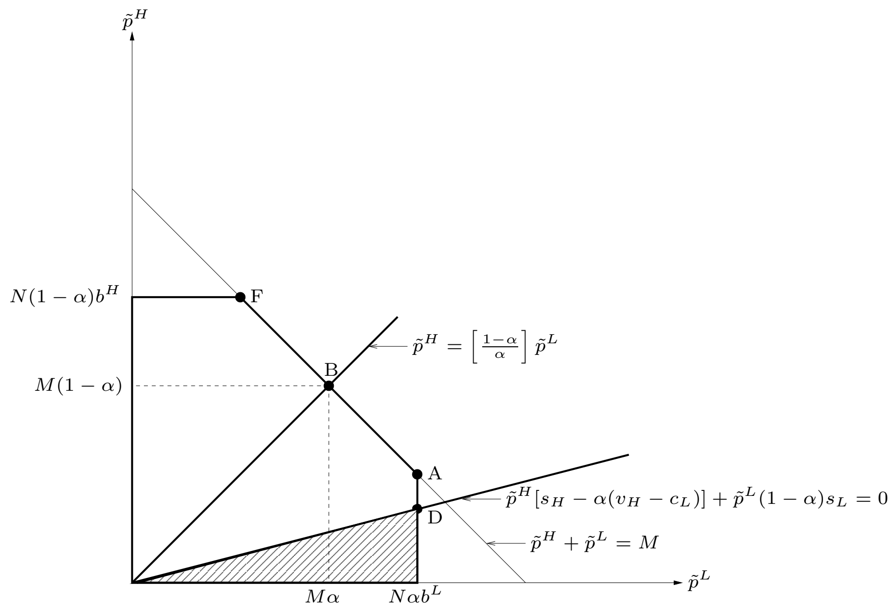

FIGURE 14. The feasible set when $M < N$, $\alpha > \alpha^*$ and point C, which is not shown, lies below point A.

A (and hence Figure 13 applies) when $N \geq N^*$, where $N^* \geq 2$; and that C lies below A (and hence Figure 14 applies) when $N < N^*$. The rest of the argument is similar to the argument supporting Proposition 3 and hence the competition-efficiency relationship when $M < N$ and the buyers are tough is as illustrated in Figure 4.

The Case of M ≥ N. Since $M \geq N$, it follows that $b^H = b^L = 1$ (i.e., it is possible for all sellers to sell their objects since the total number of buyers exceeds the number of sellers). Hence, the admissibility conditions (B.1) and (B.2) respectively become

$$\hat{p}^H \leq 1 \quad \text{and} \tag{B.9}$$

$$\hat{p}^L \leq 1. \tag{B.10}$$

Since $M \geq N$ the admissibility condition (B.3) becomes:

$$(1-\alpha)\hat{p}^H + \alpha\hat{p}^L \leq 1 \tag{B.11}$$

Finally, since the analysis in section 3.1 applies, it follows that the relevant IC and IR conditions are [after undoing the change of variables]:

$$\hat{p}^H \leq \hat{p}^L \quad \text{and} \tag{B.12}$$

$$\hat{p}^H\left[s_H - \alpha(v_H - c_L)\right] + \alpha\hat{p}^L s_L \geq 0. \tag{B.13}$$

This means that the mechanism design problem (B.4) becomes:

$$E \equiv \max_{\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L} \quad N\left[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\right] \tag{B.4}$$

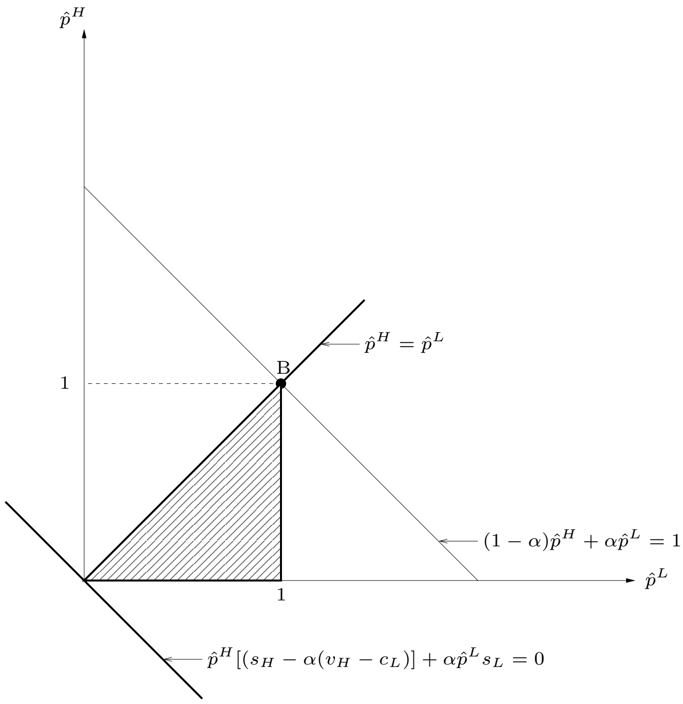

FIGURE 15. The feasible set when $M \geq N$ and $\alpha \leq \alpha^*$.

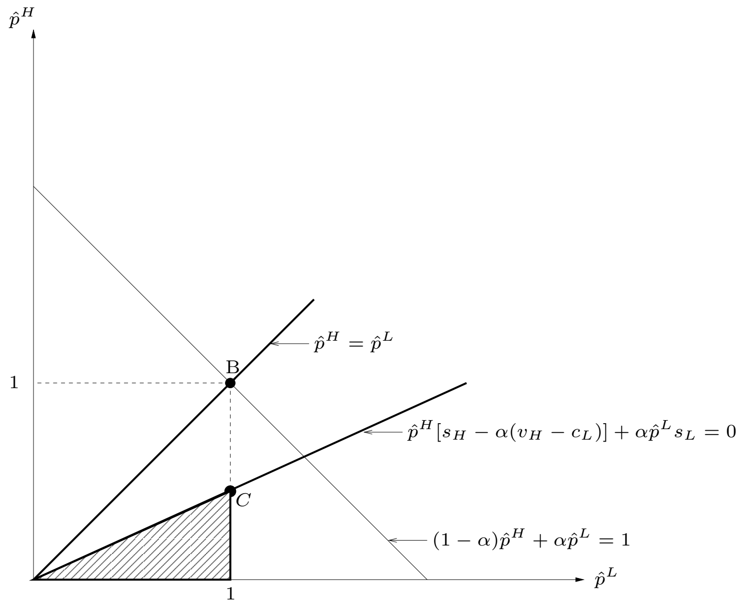

FIGURE 16. The feasible set when $M \geq N$ and $\alpha > \alpha^*$

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subject to (B.9)–(B.13).

Figures 15 and 16 respectively illustrate the feasible sets for the soft buyer and tough buyer cases. In both figures, the efficient point is defined uniquely by the intersection of the three admissibility constraints; this is point B in both diagrams. Figure 15 shows that when $\alpha \leq \alpha^*$, the first-best is achievable even with one seller (and at least one buyer). Increasing the number of sellers while maintaining the number of buyers at least equal to the number of sellers preserves efficiency. When $\alpha > \alpha^*$, on the other hand, Figure 16 shows that the first-best is no longer achievable. Increasing the number of sellers (while maintaining the number of buyers at least equal to the number of sellers) has no effect on the ability to achieve efficiency.

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