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A Search Model with a Quasi-Network

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Abstract

In a standard search model the expected duration of unemployment is independent of the duration of previous employment, as well as of the current length of the unemployment spell. This paper offers a network mechanism to generate these correlations. Here, employed workers invest in social contacts with other employed workers, which will help them find jobs in the event of unemployment. These social contacts "depreciate" because they can also become unemployed and unemployed contacts are assumed to be useless. In this model the longer you have been working, the more contacts you are likely to have, and the more contacts you have the shorter your expected unemployment duration will be. The model is a simple and tractable way of introducing network ideas in one of the workhorses of labour and macroeconomics. The model also suggests that networks are less productive during periods of high unemployment, mainly because high unemployment destroys part of the network. In addition, the model provides guidance for indirect inference of network effects from the data.

JEL Classification: D83, D85, J21, J64.

Keywords: Unemployment, Contacts, Duration.

1 Introduction

In a texbook version of the Mortensen and Pissarides (1994) search model the expected duration of unemployment is independent of the duration of previous employment, as well as of the current length of the unemployment spell. These are clearly counterfactual implications, and, as an example, models of loss of skill during unemployment such as Pissarides (1992) and Larsen (2001) sucessfully generate duration dependence in unemployment.[1] This paper provides an alternative mechanism inspired by the growing literature on networks and their implications for labour markets.

Here, as a natural by-product of working activity, employed workers invest in social contacts with other employed workers. In turn these contacts will help them find jobs in the event of unemployment. These social contacts "depreciate" because they can also become unemployed and unemployed contacts are assumed to be useless. In this model the longer you have been working, the more contacts you are likely to have, and the more contacts you have the shorter your expected unemployment duration will be.[2]

The assumption used here that unemployed workers do not constitute useful contacts is largely a simplifying assumption. It does stand in contrast with some of the literature on networks in labour markets, where the goal is to model the flow of information about available jobs.[3] In these models everyone is important because if an unemployed worker is offered two jobs he will pass one of the jobs onto another unemployed acquaintance. In the present paper there is an implicit assumption that these information flow propagation mechanisms are of second order in the dynamics of unemployment.[4] If what matters is who you know, or here how many people you know, there is a presumption that it is best if these people are working.

A direct outcome of the model is that the network is less productive during periods of high unemployment, since unemployment duration increases and the network gets partially destroyed. This result seems contrary to, but is not necessarily inconsistent with, empirical findings that the fraction of people finding jobs through friends increases during times of high unemployment.

The idea that the number of people one knows matters has some indirect empirical support. Weatherall (2008) finds that displaced workers that exit establishments with a small number of workers have a higher probability of becoming long term unemployed. Also, Addison and Portugal (1989) show for US data that the length of tenure prior to unemployment has a positive impact on post displacement wages.

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[1] Also, models of endogenous search effort where a longer unemployment duration signals a lower quality imply the payoff of search and search intensity declines with duration.

[2] An additional implication is that the time series process for unemployment is no longer a first order autoregressive process, but rather a potentially infinite order one.

[3] See for example Calvó-Armengol and Jackson (2007) and Galeotti and Merlino (2008).

[4] The literature on networks is extremely suggestive but the fact that network use and density are correlated with unemployment does not *prove* that the network is *causing* unemployment behaviour. For example, it is very difficult to disentangle the network implications developed here from those of human capital models.

Simplifying assumptions aside, the mechanism and model presented here offers a simple marriage between the search and the network literatures that we believe is interesting and useful in its own right.

This paper explores first a deterministic model to highlight the properties of the resulting equilibrium which arise from the network structure. This version of the model allows for an examination of cross sectional properties of an artificial dataset, where we obtain the negative correlation between employment and unemployment durations. It also allows us to study which properties the new additions to the standard search model should have if we want to fit the data. Following that, a model with aggregate shocks is developed and explored quantitatively. This version of the model allows us to evaluate time series properties of the model, such as serial correlations, volatilites and response to shocks.

The difficulties of the standard model to match simultaneously the quantitative behaviour of vacancies, unemployment, productivity and wages are well documented in Shimer (2005). While it is not the primary goal of this paper to adress those shortcomings, we also explore how the model offered here performs on this score.

Finally, the model is also useful as it suggests one should look into the detailed relationship between employment and unemployment durations as a way to identify network effects.

2   Deterministic Model

2.1   Firms

There is a unit mass of workers. When a vacancy is created, the probability a vacancy meets an unemployed worker is denoted by α^f. In Montgomery (1991) the network helps in reducing the effects of adverse selection. In the present paper all workers are identical from the perspective of the firm so that this mechanism is absent. Danish survey data from Filges (2008) also contains information suggestive of the importance of personal contacts in the hiring process for firms: around one third of the latest hires across the firms surveyed were achieved through personal contacts. Here, this mechanism is summarized inside the matching function because in the standard search model each match is itself a firm, and so there are no current in-house workers to rely on.[5]

The value of posting a vacancy is

$$V = -k + \beta \alpha^f J + \beta(1 - \alpha^f)V$$

where k is the flow cost of keeping a vacancy open.

Free entry will drive V to zero so that $k = \beta \alpha^f J$, where J is the value of a filled vacancy. Here β is the discount factor (over one week). A filled job

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[5] Two exception to this modelling framework are Cooper, Haltiwanger, and Willis (2007), and Ortigueira and Faccini (2008), where firms have many workers.

produces an ouput y, which is divided between the firm and the worker.[6] The value of the wage, w, is above the unemployment output b the worker gets, which can be labelled home production. All matches have an exogenous break up rate of $1 - \lambda$.[7] The value of a filled vacancy is

$$J = y - w + \beta\lambda J + \beta(1-\lambda)V = \frac{y-w}{1-\beta\lambda}$$

which implies we must have

$$\alpha^f = \frac{k\left(1-\beta\lambda\right)}{\beta(y-w)}$$

which is a constant, implying (as we will see later) that vacancies v adjust to keep the probability of success constant. We can also see that if the cost of vacancies rises relative to the wage then the equilibrium probability α^f must rise, which happens by decreasing vacancies relative to the number of unemployed.

2.2 Some probability algebra

The algebra that follows rests on the Pascal Triangle relationship. Employed workers create relationships or contacts in the course of their working activity. They can make at most one extra contact each period and the model is such that there will be a voluntary upper bound $\bar{n}$ on the number of contacts any worker is willing to have. This assumption that it takes time to reach the upper bound on contacts is essencial for the model to be able to generate the correct correlation between employment and unemployment durations.

All workers face the same exogenous probability of losing their jobs, $(1 - \lambda)$, and we assume here that only employed workers count as contacts.[8] Therefore workers lose contacts because their contacts can become unemployed.[9] Given n employed contacts - and if there is no investment in an additional contact today - we have the following probability distribution over the set $[0, 1, 2, ..., n]$ contacts tomorrow:

$$f(n-k) = \left(\begin{array}{c} n \\ k \end{array}\right)(\lambda)^{n-k}\left(1-\lambda\right)^k = \frac{n!}{k!(n-k)!}\left(\lambda\right)^{n-k}\left(1-\lambda\right)^k$$

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[6] The appendix discusses extensions which include allowing wages to depend exogenously on the number of contacts the worker has (as a measure of his outside option), and also wages determined by Nash bargaining over the surplus of the match.

[7] Conceivably, well connected employees may have lower job destruction rates, but we do not exmine this issue here.

[8] This is justified by the fact that the number of connections of any agent is finite, but the population is an everywhere dense unit mass.

[9] We assume that an individual knows only how many contacts he has, but never knows how many contacts his contacts have. From an individual perspective each of his contacts is an identical random variable. Of course, each of his contacts is in fact a realization of a random variable. An individual knows his own employment history, but does not know anyone else´s employment history (and therefore cannot infer the number of contacts anyone else may have). In fact, these issues are assumed away.

with $f(n) = \lambda^n$, and $f(0) = (1-\lambda)^n$.

In case an additional contact is made today, the same type of distribution applies but over the set $[0, ..., n+1]$. The Pascal Triangle will give rise to two important matrices in this problem. The matrix M which governs transitions of **unemployed** agents, of which we show the examples with supports $[0,1]$ and $[0,1,2]$:

$$M = \begin{bmatrix} & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & (1-\lambda) & \lambda \end{bmatrix}, \quad M = \begin{bmatrix} & 0 & 1 & 2 \\ \hline 0 & 1 & 0 & 0 \\ 1 & (1-\lambda) & \lambda & 0 \\ 2 & (1-\lambda)^2 & 2\lambda(1-\lambda) & \lambda^2 \end{bmatrix}$$

and the matrix $\hat{M}$ which governs the transitions of **employed** agents, of which we show the examples here also with supports $[0,1]$ and $[0,1,2]$, corresponding to the upper bounds $\bar{n} = 1$ and $\bar{n} = 2$:

$$\hat{M} = \begin{bmatrix} & 0 & 1 \\ \hline 0 & (1-\lambda) & \lambda \\ 1 & (1-\lambda) & \lambda \end{bmatrix}, \quad \hat{M} = \begin{bmatrix} & 0 & 1 & 2 \\ \hline 0 & (1-\lambda) & \lambda & 0 \\ 1 & (1-\lambda)^2 & 2\lambda(1-\lambda) & \lambda^2 \\ 2 & (1-\lambda)^2 & 2\lambda(1-\lambda) & \lambda^2 \end{bmatrix}$$

2.3 Workers

Managing contacts is costly which implies there will be a finite upper bound on the number of connections any worker accumulates. These connections or contacts are useful because they will help the worker find a job in the event of unemployment. The number of connections a worker has is his individual state variable or "network type". When the unemployed worker gets a job his "network type" vanishes. He then starts reconstructing his network from zero contacts. This is a strong assumption but it greatly simplifies the analysis.

There are two distributions that matter in this model. One is the distribution of the unemployed population over contacts, S_t^u, and the second one is the distribution of the employed population over contacts, S_t^e. The level of unemployment, $0 < \bar{u}_t < 1$, is also a state variable. All of these are summarized in the state vector S.

For simplicity we assume where convenient that $\bar{n} = 1$, in which case the state vector S, effectively has three numbers, $S = (\bar{u}, u_0, e_0)$, where u_0 is the fraction of unemployed workers with zero contacts, and e_0 is the fraction of employed workers with zero contacts.

<center>Value of Unemployment</center>

The value of unemployment of a worker with zero contacts given state S satisfies

$$U_0(S) = b + \beta \alpha_0^w(S) W_0(S') + \beta(1 - \alpha_0^w(S)) U_0(S')$$

<center>5</center>

and similarly, the value of unemployment of a worker with one contact obeys:

$$U_1(S) = b + \beta \alpha_1^w(S) W_0(S') + \beta(1 - \alpha_1^w(S))[(1 - \lambda)U_0(S') + \lambda U_1(S')]$$

Here, $\alpha_n^w(S)$ is the probability that an unemployed worker with n employed connections will find a vacancy this period, given the current state vector.

It is important to note here that, despite the fact that for each agent the transition of his own contacts is stochastic, the transition of the aggregate state vector is deterministic. That is why there are no expectational operators over S' in the equations above.

We can write this in matrix form as:

$$U(S) = b + \beta \alpha(S) W_0(S') + \beta T(S) U(S')$$

where $T(S) = Diag(1 - \alpha(S)) \times M$, and for the example with $\bar{n} = 1$ this yields:

$$
\begin{aligned}
\alpha(S) &= \left[\begin{array}{c} \alpha_0^w(S) \\ \alpha_1^w(S) \end{array} \right], U(S) = \left[\begin{array}{c} U_0(S) \\ U_1(S) \end{array} \right], \\
T(S) &= \left[\begin{array}{cc} 1 - \alpha_0^w(S) & 0 \\ 0 & 1 - \alpha_1^w(S) \end{array} \right] \times \left[\begin{array}{cc} 1 & 0 \\ (1 - \lambda) & \lambda \end{array} \right]
\end{aligned}
$$

Value of Employment

The value of being employed having already built n connections is then:

$$W_n(S) = \max \left\{ \begin{array}{c} w - g(n) + \beta(1 - \lambda)EU_j(S') + \beta\lambda EW_j(S') \\ w - c - g(n) + \beta(1 - \lambda)\tilde{E}U_j(S') + \beta\lambda\tilde{E}W_j(S') \end{array} \right.$$

There are two expectation signs in the above equation, because they are taken with respect to different probability distributions over different sets of contacts, with $\tilde{E}$ being taken over a support with one extra contact. Each period the worker has the choice of adding one connection to his private network at a cost $c > 0$. There is also a convex cost $g(n)$ of maintaining connections, which ensures the existence of a finite upper bound on the number of connections any individual creates, $\bar{n}$. This also ensures there are no "stars" in the network because no worker is willing to manage more than $\bar{n}$ contacts. If this cost $g(n)$ is not incurred the connections are lost. Note also the timing convention that $g(n)$ is associated with the beginning of period state variable.

An employed worker stops making contacts when

$$
\begin{aligned}
c &> \beta\lambda \left[\tilde{E}W_j(S') - EW_j(S') \right] \\
&\quad + \beta(1 - \lambda) \left[\tilde{E}U_j(S') - EU_j(S') \right]
\end{aligned}
$$

so that there is a decision rule whereby the worker stops investing at a given number of connections and then invests again when some of these connections are lost (because they have become unemployed).

In the example where $\bar{n} = 1$, we have[10]

$$c > \beta\lambda\left[(1-\lambda)^2 W_0 + 2\lambda(1-\lambda)W_1 + \lambda^2 W_2 - (1-\lambda)W_0 - \lambda W_1\right]$$
$$+\beta(1-\lambda)\left[(1-\lambda)^2 U_0 + 2\lambda(1-\lambda)U_1 + \lambda^2 U_2 - (1-\lambda)U_0 - \lambda U_1\right]$$

<div align="center">Example</div>

In our example with a maximum of one connection this is simpler and can be written with the pair of equations (where the maximization is already resolved in both of them):

$$W_0(S) = w - c - g(0) + \beta(1-\lambda)EU_j(S') + \beta\lambda EW_j(S')$$
$$W_1(S) = w - g(1) + \beta(1-\lambda)EU_j(S') + \beta\lambda EW_j(S')$$

We can write this in matrix form as:

$$W(S) = wg + \beta(1-\lambda)\hat{M}U(S') + \beta\lambda\hat{M}W(S')$$

where

$$wg = \left[\begin{array}{c} w - c - g(0) \\ w - g(1) \end{array}\right], \quad \hat{M} = \left[\begin{array}{cc} (1-\lambda) & \lambda \\ (1-\lambda) & \lambda \end{array}\right]$$

We are assuming that the maximum number of connections the agent is willing to acquire is one, so that one contact is always desirable while $g(2)$ is such that two contacts are never desirable. In general we have $g(n+1) > g(n) \geq 0$.

There is a conjecture here that a random wage would not affect the upper limit on the number of contacts for each worker, which implies that this upper limit is unique and the same for all workers. The reason this is so is that contacts only affect the probability of finding a job, and once agents get unemployed they would all have the same expected wage once they find a job. In addition, utility is linear in income and the job destruction probability does not depend on the number of contacts.[11]

A more substantive issue regarding wages is that here they are exogenously set, as opposed to the standard practice of using Nash bargaining to do so. This is a tractability device. However, Nash bargaining is not the only wage determination procedure, and following Cooper, Haltiwanger, and Willis (2007) we set output and wages to depend only on the aggregate state of the economy which, in this first version of the model without aggregate shocks, is constant.[12]

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[10]Note that W_1 for $\bar{n} = 1$ differs from W_1 for $\bar{n} = 2$. The same is true for U_1. Still, we can design a $g(n)$ function such that we can get any $\bar{n}$ we want, which greatly simplifies the solution of the model.

[11]We can therefore simplify by having a constant wage. However if the lower bound of the hypothetical wage distribution is less than b this might not be the case.

[12]See also references in their paper. In their model the wage bill is firm specific and varies with idiosyncratic shocks because hours vary within the firm, but the outside option of the worker depends only on the aggregate state.

2.4 Mechanics of employment and unemployment

Two types of attrition are at work in this model: attrition on the number of contacts, and attrition on the transition to and out of unemployment. To characterize the mechanics for the unemployed population we need to detail two transitions: the transition from unemployment to unemployment, and the transition from employment to unemployment. Here we use the example where the maximum number of contacts is $\bar{n} = 1$.

<div align="center">Mechanics of Unemployment</div>

Let $\begin{bmatrix} \bar{u}_0^t & \bar{u}_1^t \end{bmatrix}$ be a row vector containing the *number* of unemployed at time t by number of contacts,

$$\begin{bmatrix} \bar{u}_0^t & \bar{u}_1^t \end{bmatrix} \equiv \bar{u}_t \begin{bmatrix} u_0^t & u_1^t \end{bmatrix} \equiv \bar{u}_t \begin{bmatrix} u_0^t & 1 - u_0^t \end{bmatrix} \equiv \bar{u}_t \left(S_t^u \right)'$$

Here $\bar{u}_t$ is the unemployment rate (a scalar), $(\bar{u}_t S_t^u)$ is a *population* vector, and S_t^u is a *density* vector with elements summing to one. Now let $\begin{bmatrix} \tilde{u}_0^{t+1} & \tilde{u}_1^{t+1} \end{bmatrix}$ be the vector that contains *the subset* of these workers that remain unemployed the following period. The transition between unemployment and unemployment is then given by:

$$\begin{bmatrix} \tilde{u}_0^{t+1} & \tilde{u}_1^{t+1} \end{bmatrix} = \begin{bmatrix} \bar{u}_0^t & \bar{u}_1^t \end{bmatrix} \times \begin{bmatrix} 1 - \alpha_0^w(S_t) & 0 \\ 0 & 1 - \alpha_1^w(S_t) \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ (1-\lambda) & \lambda \end{bmatrix}$$

The transition of total unemployment is an operator:

$$\left\{ \bar{u}_t \left(S_t^u \right)' \equiv \begin{bmatrix} \bar{u}_0^t & \bar{u}_1^t \end{bmatrix} \right\} \implies \begin{bmatrix} \bar{u}_0^{t+1} & \bar{u}_1^{t+1} \end{bmatrix}$$

To get this operator we must add the incoming cohort to our previous algebra.

The incoming cohort is the transition from employment into unemployment and is given by

$$(1-\lambda)(1-\bar{u}_t)\left(S_t^e \right)' \hat{M} = (1-\lambda)\left[\bar{e}_t \left(S_t^e \right)' \right] \hat{M}$$

where $\bar{e}_t$ is the employment rate (a scalar), $(\bar{e}_t S_t^e)$ is a *population* vector, and S_t^e is a *density* vector with elements summing to one. We have then:

$$\left(Z_{t+1}^u \right)' = \bar{u}_t \left(S_t^u \right)' \times T(S_t) + (1-\lambda)(1-\bar{u}_t)\left(S_t^e \right)' \hat{M}$$

which in our example:

$$
\begin{aligned}
\left(Z_{t+1}^u \right)' &\equiv \bar{u}_{t+1} \begin{bmatrix} u_0^{t+1} & 1 - u_0^{t+1} \end{bmatrix} \\
&= \bar{u}_t \begin{bmatrix} u_0^t & 1 - u_0^t \end{bmatrix} \times T(S_t) \\
&\quad + (1-\lambda)(1-\bar{u}_t)\begin{bmatrix} e_0^t & 1 - e_0^t \end{bmatrix} \times \hat{M}
\end{aligned}
$$

and then

$$
\begin{aligned}
\bar{u}_{t+1} &= \left(Z_{t+1}^u \right)' \times [1] \\
S_{t+1}^u &= Z_{t+1}^u / \bar{u}_{t+1}
\end{aligned}
$$

which gives both the unemployment level/rate and its distribution.

Mechanics of Employment

This is the last piece of the problem. We need it because we want to know the *distribution* of employment (the *level* we already know since we know unemployment from the previous equation). It involves the transition from unemployment into employment:

$$\bar{u}_t \left[\alpha \left(S_t^u \right)' S_t^u \right] \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and the transition from employment to employment:

$$\lambda(1 - \bar{u}_t) \left[\left(S_t^e \right)' \hat{M} \right]$$

to get:

$$\left(Z_{t+1}^e \right)' = \lambda(1 - \bar{u}_t) \left[\left(S_t^e \right)' \hat{M} \right] + \bar{u}_t \left[\alpha \left(S_t^u \right)' S_t^u \right] \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where

$$\left(Z_{t+1}^e \right)' \equiv \bar{e}_{t+1} \begin{bmatrix} e_0^{t+1} & 1 - e_0^{t+1} \end{bmatrix} = (1 - \bar{u}_{t+1}) \begin{bmatrix} e_0^{t+1} & 1 - e_0^{t+1} \end{bmatrix}$$

and in detail

$$\begin{aligned} \left(Z_{t+1}^e \right)' &= \lambda(1 - \bar{u}_t) \begin{bmatrix} e_0^t & 1 - e_0^t \end{bmatrix} \hat{M} \\ &+ \bar{u}_t \begin{bmatrix} u_0^t & 1 - u_0^t \end{bmatrix} \begin{bmatrix} \alpha_0^w (S_t) \\ \alpha_1^w (S_t) \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

where $\alpha^w (S_t)'$ is a row vector. The distribution of employment by contacts next period is then:

$$S_{t+1}^e = Z_{t+1}^e / \left(\left(Z_{t+1}^e \right)' [1] \right)$$

2.5 Equilibrium

These last two sets of equations are enough to determine the equilibrium given the current state variables. But to get the long run equilibrium we need three equations. So far we have:

$$\begin{aligned} \bar{u} \begin{bmatrix} u_0 & 1 - u_0 \end{bmatrix} &= \bar{u} \begin{bmatrix} u_0 & 1 - u_0 \end{bmatrix} \times \begin{bmatrix} 1 - \alpha_0 & 0 \\ 0 & 1 - \alpha_1 \end{bmatrix} \times M \\ &+ (1 - \bar{u})(1 - \lambda) \begin{bmatrix} e_0 & 1 - e_0 \end{bmatrix} \times \hat{M} \end{aligned}$$

$$\begin{aligned} (1 - \bar{u}) \begin{bmatrix} e_0 & 1 - e_0 \end{bmatrix} &= \lambda(1 - \bar{u}) \begin{bmatrix} e_0 & 1 - e_0 \end{bmatrix} \times \hat{M} \\ &+ \bar{u} \begin{bmatrix} \alpha_0 & \alpha_1 \end{bmatrix} \begin{bmatrix} u_0 \\ 1 - u_0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

but to close the model we need two more steps. First we find the equilibrium distribution for employed agents, S^e.

Finding S^e.

Given an upper bound $\bar{n}$ on the number of connections we can derive the expected distribution of connections at the point of job destruction for a given agent, when he starts working. Starting from zero in a new job the worker makes a connection in the first period and continues making a new connection each period until he reaches $\bar{n}$ connections. Along the way he can lose connections, and of course his own job.

If the worker is unemployed after one period of work, an event which occurs with probability $(1 - \lambda)$, he can have either zero or one contacts with respective probabilities $((1 - \lambda), \lambda)$. If he becomes unemployed after two periods, which happens with probability $\lambda(1 - \lambda)$, he can have entered the second period with one contact (probability λ), and then he can have $[0, 1, 2]$ contacts at the end of period two with probability vector $\left((1 - \lambda)^2, 2\lambda(1 - \lambda), \lambda^2\right)$. But he can also have entered the second period with only zero contacts - an event with probability $(1 - \lambda)$ - and then he can have at most one contact as above.

If we assume that the upper bound on the number of contacts is 2, we have the following graphic representation of the probability paths:
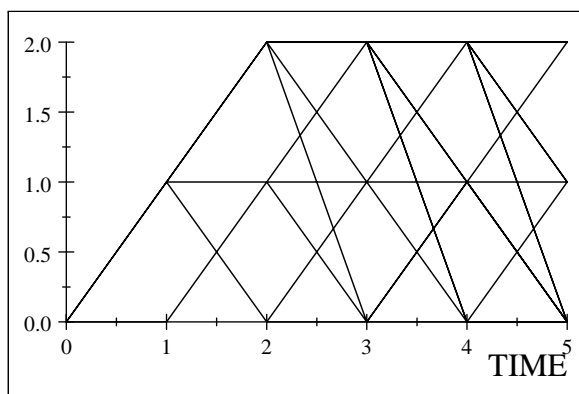
and the probability associated with each final node is the sum of all paths that leads to that node.

We can write this algorithm in matrix form. The appendix details this algebra for the cases where $\bar{n} = 1$, and $\bar{n} = 2$. All the algebra is from the perspective of time zero, when the worker starts his new job. For the case where $\bar{n} = 1$ the density over contacts is

$$D^0 = (1 - \lambda) \hat{M}_0 \left(I - \lambda \hat{M}_1\right)^{-1} \equiv \left[\begin{array}{cc} e_0 & 1 - e_0 \end{array}\right] = S^e$$

where $\hat{M}_0 = \left[\begin{array}{cc} (1 - \lambda) & \lambda \end{array}\right]$ and $\hat{M}_1 = \hat{M}$. In steady state equilibrium this is the density over contacts of the employed population, S^e, as well as the equilibrium distribution over contacts of the cohort entering unemployment (since the probability of losing a job does not depend on the number of contacts).

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The final step involves solving a non linear expression. We need it because the matrix T depends on α, and α depends on vacancies, and vacancies depend on the distribution of unemployed workers over contacts. We therefore need an extra loop to make sure the unemployment distribution inside T is consistent with the distribution on the left hand side of the above expression. We do not solve this problem here but numerically - as long as care is taken in the selection of the matching function - there is no problem converging on this operator.

2.6 The matching function

The simple Cobb-Douglas matching function is not enough in this model. We want the matching function to have several standard properties. The total number of matches must be increasing in both the number of vacancies and the number of unemployed workers. And the probability that an unemployed worker finds a vacancy must be increasing in the number of vacancies and decreasing in the number of unemployed workers. Additionally, this probability must be increasing in the number of contacts a worker has. This extra feature of our problem provides us with the possibility of matching time patterns of job finding rates in the data which the standard model does not, and therefore may help us better understand the matching process.

Equilibrium consistency of the matching probabilities then implies for the total number of matches:

$$m = \sum_{i=0}^{\bar{n}} \alpha_i^w u_i \bar{u}$$

where

$$1 = \sum_{i=0}^{\bar{n}} u_i$$

and for $\alpha_i^w = g(i, \bar{u}, v)$ we must have *at least* that $g_i > 0$, $g_{\bar{u}} < 0$, and $g_v > 0$. Note that we are not imposing that these probabilities depend explicitly on the distribution of workers over contacts. We now check whether a close relative of the standard Cobb-Douglas will satisfy our requirements. Consider the function

$$\alpha_i^w = \gamma \left(\frac{v}{\bar{u}}\right)^\theta \phi(i)$$

with $0 < \theta < 1$, and $\phi_i > 0$. This function satisfies all our derivatives above. Furthermore it also satisfies the condition that the total number of matches is increasing in both vacancies and unemployment:

$$m = \gamma \left(\frac{v}{\bar{u}}\right)^\theta \bar{u} \sum_{i=0}^{\bar{n}} \phi(i) u_i = \gamma (v)^\theta (\bar{u})^{1-\theta} \sum_{i=0}^{\bar{n}} \phi(i) u_i$$

The final steps in the logic of this construction come from the side of the firm. We know that α^f is a constant:

$$\alpha^f = \frac{k\left(1 - \beta\lambda\right)}{\beta(y - w)}$$

and because for the firm it does not matter which type of worker they meet, this probability can be written simply as $\alpha^f = m/v$.

We then have

$$\frac{m}{v} = \gamma\left(\frac{\bar{u}}{v}\right)^{1-\theta} \sum_{i=0}^{\bar{n}} \phi\left(i\right) u_i = \frac{k\left(1 - \beta\lambda\right)}{\beta(y - w)}$$

which then implies

$$v = \bar{u}\left(\frac{1}{\gamma}\frac{k\left(1 - \beta\lambda\right)}{\beta(y - w)}\left[\sum_{i=0}^{\bar{n}} \phi\left(i\right) u_i\right]^{-1}\right)^{-\frac{1}{1-\theta}}$$

that vacancies depend on the *distribution* of types. This also provides an additional mechanism affecting the dynamics and volatility of vacancies.

Given that vacancies depend on the distribution of contacts, worker probabilities also do:[13]

$$\alpha_i^w = g\left(i, \bar{u}, v\right) = g\left(i, \bar{u}, v\left(S^u\right)\right)$$

This construction allows us to now specify the function $\phi\left(i\right)$ independently of any other considerations. Its main property is that it should be increasing in the number of contacts, but we should also define whether this function should be concave or convex. The following function:

$$\phi\left(i\right) = \frac{\log(\tau_0 + \tau_1 i)}{N}$$

where N is an arbitrary number chosen for calibration purposes, is positive and strictly concave in (i). The choice of (N, τ_0, τ_1) ensures the match probability is positive even at zero contacts and grows with the number of contacts, and determines the shape of ϕ over the relevant range. With the appropriate calibration we can choose any shape we want, either concave, linear or convex.

2.7 Unemployment Duration Algebra

Consider a worker who enters unemployment with zero contacts.

His probability of reemployment is constant every period at α_0^w. Therefore he finds a job after one period with probability α_0^w. Finds a job after two periods

---

[13]The level of unemployment at the start of the period summarizes all the useful information about the distribution of contacts in the *employed* population S_t^e.

with probability $\alpha_0^w(1-\alpha_0^w)$. After three periods with probability $\alpha_0^w(1-\alpha_0^w)^2$, etc. We have expected duration of unemployment given by

$$
\begin{aligned}
d_0 &= 1\alpha_0^w + 2\alpha_0^w(1-\alpha_0^w) + 3\alpha_0^w(1-\alpha_0^w)^2 + ... \\
&= \alpha_0^w\left[1 + 2(1-\alpha_0^w) + 3(1-\alpha_0^w)^2 + ...\right] \\
&= 1/\alpha_0^w
\end{aligned}
$$

This algebra, however, depends on the state vector:

$$
\begin{aligned}
d_0(S_t) &= 1\alpha_0^w(S_t) + 2\alpha_0^w(S_{t+1})(1-\alpha_0^w(S_t)) \\
&\quad + 3\alpha_0^w(S_{t+2})(1-\alpha_0^w(S_{t+1}))(1-\alpha_0^w(S_t)) + ...
\end{aligned}
$$

so that the value of this sum is not trivial to compute (even though the law of large numbers ensures the transition of the state vector is deterministic).

We can write the expected duration algebra for all contacts in matrix form as:

$$
D(S_t) = \alpha^w(S_t) + 2\alpha^w(S_{t+1})T(S_t) + 3\alpha^w(S_{t+2})T(S_{t+1})T(S_t) + ...
$$

and in steady state equilibrium we can write the expected duration algebra in matrix form as:

$$
D = \left\{I + 2T + 3T^2 + ...\right\}\alpha^w = [I-T]^{-1}[I-T]^{-1}\alpha^w
$$

where α^w is a column vector.

3   Numerical Simulations

The experiments conducted in this paper take the shortcut of determining $\bar{n}$ by assuming the $g(n)$ function to be such that the optimal $\bar{n}$ does not depend on the state variables in the problem. This circumvents the need to solve the optimization problem at each step and reduces the evolution of the economy to the mechanics presented above. There is a loss of the endogenous transmission mechanism which would come from changing $\bar{n}$ as the state of the world changes, but this is compensated by an enormous gain in numerical implementability.

Calibration

Here we follow Shimer (2005), Mortensen and Nagypal (2007), Cooper, Haltiwanger and Willis (2007), and Zhang (2008), adapting where appropriate for our weekly frequency. CHW use an annual interest rate of 4%, while Shimer uses a value around 5%. We use 5%. CHW report that a value of θ, the weight of firms in the matching function, of 0.64. Shimer uses a value of 0.28 for this parameter. We use the mid point of these two values, 0.46, which is within the plausible range proposed by Petrongolo and Pissarides (2001).

CHW report a monthly job finding rate equal to 0.61, while Shimer reports a value of 0.45, and Zhang reports a value of 0.309 for canadian data. Disregarding contacts this implies on a weekly frequency:

$$\alpha^w + (1 - \alpha^w) \alpha^w + (1 - \alpha^w)^2 \alpha^w + (1 - \alpha^w)^3 \alpha^w = 0.61$$

which in turn implies $\alpha^w = 0.21$ or for Shimer's number $\alpha^w = 0.14$, while for Zhang's number it is $\alpha^w = 0.08825$.[14] We note here that over four weeks we expect few contacts to lose their jobs, so that we can use this number to benchmark $\alpha_{\bar{n}}^w$.

$$\alpha_{\bar{n}}^w = \gamma \left(\frac{v}{\bar{u}} \right)^\theta \phi(\bar{n})$$

and so we calibrate the ϕ function such that we obtain $\alpha_7^w \approx 0.15$.[15]

Shimer uses a quarterly separation rate of 10% which delivers an expected employment duration equal to 30 months or 130 weeks. This then implies a value of $(1 - \lambda) = 1/130$, and we use this value. Zhang finds that canadian jobs have an expected duration of 146 weeks. Given a value of the monthly separation rate in Canada of 0.03 we can compute a gross value of our weekly λ simply by doing $(1 - \lambda) = 0.03/4.35 = 1/145$.[16] In the case of Shimer the separation rate used is 0.10 at the quarterly frequency, and a gross computation yields $(1 - \lambda) = 0.1/13 = 1/130$. The numbers match and the value of $1/130$ is our benchmark.

---

[14] Lynch (1989) reports a value of 0.30 for the first week of unemployment. This number falls fast with duration though, suggesting the ϕ function is quite possibly convex rather than concave. Alternatively, this quick drop reflects mismeasurement by including in the unemployment pool job to job transitions (which require a week or two to clean up a desk).

[15] We achieve this by setting $(\tau_0 = 1.2, \tau_1 = 2, N = 18)$.

[16] This is not the only way to compute the weekly separation rate out of quarterly numbers. The reason is that we do not know how the following event is accounted for in the data: a firm and a worker separate 2 weeks into the quarter and 4 weeks later, well before the quarter is over, both the firm and the worker have found new matches.

Given that the values of unemployment in Canada and the US reported by Zhang and by Shimer are respectively 0.0778 and 0.0567, the difference must come from the job finding rates we construct above.

CHW report that the average value of labor market tightness, the ratio v/u, is around 0.46. However, Shimer argues that this ratio is essentially meaningless and he calibrates it to equal 1, which in turn implies that in equilibrium $\alpha^w = \alpha^f$. We therefore use the vacancy cost k to target the vacancy filling rate directly to be $\alpha^f \approx 0.14$. This implies a value of $k = 16$. The tightness ratio is then implied by the rest.[17] CHW estimate $\gamma = 1.0072$, and we set this parameter to 1. We note here that in Shimer (2005), setting $\gamma \geq 1$, and at the same time normalizing the tightness ratio to be one, implies the ratio of matches over vacancies or over unemployment is greater than one. Irrespective of the frequency with wich one looks at the data, this implies awkwardly that more matches are formed than vacancies or the number of unemployed workers. In the present paper, this is not the case because the matching function is different and the quantity

$$\sum_{i=0}^{\bar{n}} \phi(i) u_i$$

adds up to a small number, bringing down the number of matches formed.

The following table summarizes the calibration of the deterministic model:

Table 1: Deterministic Model Parameters

$\bar{n}$	$1-\lambda$	γ	θ	β	$y-w$	k	τ_0	τ_1	N
9	$\frac{1}{130}$	1	0.46	$(0.95)^{1/52}$	1	$75*0.213$	1.2	2	18

Note that what matters in our construction - and following the discussion of Hagedorn and Manovski (2007) and Mortensen and Nagypal (2007) potentially makes the surplus of the match small - is the ratio of k to $y - w$, and not the value of $y - w$ itself. The value of α^f is 0.1387 in this calibration example.

Simulations

The following simulation outcomes average thirteen runs of a panel with four thousand individuals and four hundred periods. From each panel only about 200 individuals (5%) are unemployed in the last period of the panel. For these 200 individuals we measure the length of their current unemployment spell, the length of the employment spell that preceded it, and the number of contacts they have today. Then we compute three correlations. All this is done for the different values of $\bar{n}$ shown.[18]

Row 1 in table one shows the correlation between the length of the current unemployment spell and the length of the employment spell that immediately

---

[17]Shimer sets the ratio $k/(y-w)$ to 0.213. Here since $y - w = 1$ this would be the value of k. But these values do not work in our model. He also sets home production at $b = 0.4$.

[18]The model is run for a number of periods first until the unemployment rate and the contact distributions converge. It is from then on that the simulations begin, and all individuals begin the simulations with maximum contacts $\bar{n}$. After 400 periods the simulation stops and we pick the cross sectional status of the panel at this period.

Table 2: Correlations in artificial data

$\bar{n}$	0	1	...	7	8	9	26
ρ_1	-0.5396	-0.3562		-0.0791	-0.0716	-0.0925	-0.1109
ρ_2	-	0.5904		0.6680	0.5947	0.6242	0.7205
ρ_3	-	-0.1465		-0.1570	-0.1628	-0.1634	-0.1879
$\bar{u}$	0.8759	0.6329		0.0496	0.0459	0.0431	0.0272
n	45427	32112		2627	2430	2278	1439
0	917	479		96.0	92.5	89.7	72.0
1		222		22.9	21.8	20.9	15.7
2				12.2	11.7	11.3	8.7
3				9.4	9.0	8.7	6.9
4				8.2	7.8	7.6	6.0
5				7.4	7.1	6.9	5.5
6				6.9	6.7	6.5	5.1
7				6.5	6.3	6.1	4.9
8					6.0	5.8	4.7
9						5.6	4.5
26						...	3.3

preceeded it. **Row 2** shows the correlation between the length of the current unemployment spell and expected unemployment duration, and **Row 3** computes the correlatiuon between the length of the employment spell and current expected unemployment duration. **Row 4** shows the observed average unemployment rate for each $\bar{n}$, and the following row the total sum of unemployed agents over the thirteen panels. The following rows show expected duration by contacts in weeks.

Table 2 shows, for the case of $\bar{n} = 9$, the functions ϕ and α^w, and the histogram of the distribution of unemployed agents by contacts:

Table 3: Contacts in the matching function when max(n)=9

	0	1	2	3	4	5	6	7	8	9
ϕ	0.010	0.065	0.092	0.110	0.123	0.134	0.143	0.151	0.158	0.164
α^w	0.011	0.071	0.101	0.121	0.136	0.148	0.158	0.167	0.174	0.181
HU	32	40	28	28	25	28	48	130	459	1460

The histogram of contacts on the bottom row is slightly U-shaped because of the steep drop in the ϕ function close to zero contacts. It is interesting that the histogram of contacts for $\bar{n} = 1$, is reversed at $[24579, 7533]$. Because the technology ϕ is not changing with $\bar{n}$, a small value for the maximum number of contacts implies that nobody has a large probability of reemployment ($\alpha^w = [0.0021, 0.0133]$) which implies most workers will be unemployed long enough to lose their single contact.

One characteristic that stands out from this calibration is that, for the $\bar{n} = 9$ case, only about 1.5% of the unemployed population (the ones with zero con-

tacts) is really long term unemployed. If we add the workers with one contact (which have 21 weeks of expected unemployment duration) this proportion comes up to 3.5% which is closer to the number found in Weatherall (2008) on Danish data. In any case, the shape of the duration histogram is clearly a pattern which this model hopes to match.

3.1 Convex versus concave matching technologies

Here we pick the experiment with $\bar{n} = 9$ and compare the distributional implications of a convex function ϕ, keeping the unemployment rate at roughly the same level. The convex ϕ function has three parameters $(\tau_0, \tau_1, \tau_2) = (1.15, 0.6, 0.015)$, and is constructed as follows. First define $\phi_j = (\tau_0)^j - \tau_1$. Then define $\phi_j = -\tau_2 + \phi_j / \left(\sum_{j=0}^{nbar} \phi_j \right)$. The correlations, expected duration by contacts, and the job finding rates and contact distribution of unemployed workers are given in tables 4 and 5:

Table 4: Correlations and expected duration (weeks)

$\bar{n} = 9$	Concave	Convex	n	Concave	Convex
ρ_1	-0.0925	-0.2194	0	90	72
ρ_2	0.6242	0.7495	1	21	47
ρ_3	-0.1634	-0.3471	2	11	32
$\bar{u}$	0.0431	0.0465	3	8.7	23
n	2278	2420	4	7.6	17
			5	6.9	13
			6	6.5	10
			7	6.1	7.9
			8	5.8	6.4
			9	5.6	5.3

Table 5: Contacts with a convex matching function

$\bar{n} = 9$	0	1	2	3	4	5	6	7	8	9
ϕ	0.013	0.023	0.035	0.049	0.065	0.084	0.105	0.129	0.157	0.189
α^w	0.014	0.025	0.038	0.053	0.070	0.089	0.112	0.138	0.167	0.202
HU	74	138	74	56	46	50	56	135	446	1345

We can see that while the signs of the different correlations are naturally the same, both the magnitude of the correlations and the shape of the distributions change with the shape of the ϕ function, which is good news for identification. At this stage there is a variety of moments that characterize the dynamic behaviour of the model which we cannot measure without adding aggregate shocks. We do this in the next section.

4 Aggregate Shocks

The aggregate state affects productivity, y, and the destruction rate, λ. We add to this the cost of opening a vacancy, k, only for linear algebra reasons. For simplicity of exposition we work here with two aggregate states, indexed x_1 and x_2. The transition between them is governed by the Markov matrix:

$$\Pi = \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}$$

4.1 Firms

The value of posting a vacancy is now

$$V\left(x_i\right) = -k_i + \beta\alpha^f\left(x_i\right)E_i\left(J\right) + \beta(1-\alpha^f\left(x_i\right))E_i\left(V\right)$$

where the probability a vacancy meets an unemployed worker is denoted by $\alpha^f\left(x_i\right)$. Free implies $V = 0$ whatever the state so that

$$k_i = \beta\alpha^f\left(x_i\right)E_i\left(J\right)$$

A filled job produces an ouput $y_i \equiv y\left(x_i\right)$, which is divided between the firm and the worker, with $w_i \equiv w\left(x_i\right) > b$. Matches have an exogenous break up rate of $1 - \lambda_i \equiv 1 - \lambda\left(x_i\right)$. The value of a filled vacancy is

$$J\left(x_i\right) = y_i - w_i + \beta\lambda_i E_i\left(J\right) + \beta(1-\lambda_i)E_i(V)$$

and in matrix form

$$
\begin{aligned}
J &= \begin{bmatrix} J\left(x_1\right) \\ J\left(x_2\right) \end{bmatrix} = \begin{bmatrix} y_1 - w_1 \\ y_2 - w_2 \end{bmatrix} + \beta \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \Pi \begin{bmatrix} J\left(x_1\right) \\ J\left(x_2\right) \end{bmatrix} \\
J &= \left[I - \beta Diag\left(\lambda\right)\Pi\right]^{-1} \begin{bmatrix} y_1 - w_1 \\ y_2 - w_2 \end{bmatrix}
\end{aligned}
$$

which implies we must have a vector of constants with dimension given by the number of aggregate states:

$$
\begin{aligned}
\frac{1}{\alpha^f\left(x_i\right)} &= \frac{\beta}{k_i}E_i\left(J\right) \\
1/\alpha^f &= \begin{bmatrix} 1/\alpha^f\left(x_1\right) \\ 1/\alpha^f\left(x_2\right) \end{bmatrix} = \beta \begin{bmatrix} 1/k_1 & 0 \\ 0 & 1/k_2 \end{bmatrix} \Pi J
\end{aligned}
$$

4.2 Probability matrices.

We still have the transition matrices M and $\hat{M}$ as before, and one difference is that they are indexed by the aggregate state since the destruction rate varies with the aggregate shock, $\lambda\left(x_i\right)$. This is, however not the only difference. In our example of two states we will assume that it is possible for the maximum

18

number of contacts to differ in state zero and state one. In this case the two transition matrices change in a subtle way. We give here an example where $\bar{n}(x_2) = 2$, and $\bar{n}(x_1) = 1$. We have then for unemployed agents in state i:

$$M(x_i) = \begin{bmatrix} 1 & 0 & 0 \\ (1-\lambda_i) & \lambda_i & 0 \\ (1-\lambda_i)^2 & 2\lambda_i(1-\lambda) & \lambda_i^2 \end{bmatrix}$$

with $i = 1, 2$, while for **employed** agents we have:

$$\hat{M}(x_2) = \begin{bmatrix} (1-\lambda_2) & \lambda_2 & 0 \\ (1-\lambda_2)^2 & 2\lambda_2(1-\lambda_2) & \lambda_2^2 \\ (1-\lambda_2)^2 & 2\lambda_2(1-\lambda_2) & \lambda_2^2 \end{bmatrix}$$

$$\hat{M}(x_1) = \begin{bmatrix} (1-\lambda_1) & \lambda_1 & 0 \\ (1-\lambda_1) & \lambda_1 & 0 \\ (1-\lambda_1)^2 & 2\lambda_1(1-\lambda_1) & \lambda_1^2 \end{bmatrix}$$

which marks a qualitative difference from the deterministic model.

We emphasize here that the exercise with aggregate shocks will keep $\bar{n}$ constant across the aggregate states. This will allow us to see the impact of the mere inclusion of this mechanism in the standard model, without the contributions of *changes* in its intensity.

Some reflections are nevertheless in order. The aggregate state will always be ranked by its impact on total match productivity, y. But it is unclear what the variation of $\bar{n}$ should be across aggregate states. Furthermore with several parameters likely to change with the aggregate state this is hard to predict.

4.3 Mechanics

The transition from unemployment to unemployment when the current aggregate state is x_t is now:

$$[\tilde{u}_{t+1}] = \bar{u}_t[u_t] \times T(x_t, S_t)$$
$$T(x_t, S_t) = Diag(1 - \alpha^w(x_t, S_t)) \times M(\lambda_t)$$

The incoming cohort (employment into unemployment) is:

$$(1 - \lambda_t)(1 - \bar{u}_t)[e_t]\hat{M}(\lambda_t)$$

The transition of employment into employment is:

$$\left(Z_{t+1}^e\right)' = \lambda_t(1 - \bar{u}_t)[e_t]\hat{M}(\lambda_t)$$

and the transition of unemployment into employment yields the scalar:

$$\alpha(x_t, S_t)\bar{u}_t[u_t]$$

We will proceed with the simplifying assumption that the maximum number of contacts never changes, $\bar{n}(x_t) = \bar{n}$.

4.4　The matching function

With a constant $\bar{n}$ the algebra here is trivial again. From the firm side we know that α^f is a constant which depends on the aggregate state:

$$\begin{bmatrix} 1/\alpha^f (x_1) \\ 1/\alpha^f (x_2) \end{bmatrix} = \beta \begin{bmatrix} 1/k_1 & 0 \\ 0 & 1/k_2 \end{bmatrix} \Pi J$$

and where $\alpha^f (x_t) = m (x_t) / v_t$. All we need is

$$\frac{m_t}{v_t} = \gamma \left(\frac{\bar{u}_t}{v_t} \right)^{1-\theta} \sum_{i=0}^{\bar{n}} \phi (i) u_{i,t} = \alpha^f (x_t)$$

which then implies

$$v_t = \bar{u}_t \left(\frac{\gamma}{\alpha^f (x_t)} \sum_{i=0}^{\bar{n}} \phi (i) u_{i,t} \right)^{\frac{1}{1-\theta}}$$

We see that it is only through the expectation of J inside $\alpha^f (x_t)$ that the parameters of the matrix Π enter vacancies and affect dynamics, in the world where $\bar{n}$ is fixed. Other than that the matrix Π manifests itself through the realized path of the aggregate shock.

Discussion

We can also explore having (N, τ_0, τ_1) change with the aggregate state. This requires discussion. Is it the case that in good times the contribution of a contact for finding a job is smaller than in bad times (perhaps because its is easier to find a job without contacts in good times)? Or is the contribution of a contact for finding a job higher in good times? One reason this matters is because the distribution of contacts can either smooth or magnify the effect of the aggregate shock. We have no a-priori reason to think one way or another, and perhaps the data will be able to tell us something about this. At this stage we do not pursue these ideas, and the ϕ function is not affected by aggregate shocks.

4.5　Unemployment Duration Algebra

Consider a worker who enters unemployment with zero contacts. His probability of reemployment α_0^w now can change every period depending on the aggregate state. We can show that for zero contacts, given initial aggregate state x_t, expected duration is given by

$$\begin{aligned} d_0 (x_t) & = \alpha_0^w (x_t) + 2(1 - \alpha_0^w (x_t)) \Pi_t [\alpha_0^w] \\ & \quad + 3(1 - \alpha_0^w (x_t)) \Pi_t [Diag (1 - \alpha_0^w) \Pi] [\alpha_0^w] \\ & \quad + 4(1 - \alpha_0^w (x_t)) \Pi_t [Diag (1 - \alpha_0^w) \Pi]^2 [\alpha_0^w] + ... \\ & = \alpha_0^w (x_t) + (1 - \alpha_0^w (x_t)) \Pi_t \left[(I - Z)^{-1} \left(1 + (I - Z)^{-1} \right) \right] [\alpha_0^w] \end{aligned}$$

However, this is not enough. In practice the computation of expected unemployment duration is much harder because as aggregate shocks hit, the distribution of workers over contacts changes. The $\alpha_n^w(x)$ functions are themselves changing over time because the distributions are changing.

To overcome this difficulty, in the stochastic model we measure actual durations. Once we pick a cross section of workers at a given period, we measure its characteristics both backward and forward looking. For all workers at this moment we measure their unemployment spell to date, the length of the previous employment spell, and the length of the subsequent duration of unemployment from this period onwards. Rather than measuring expected duration of unemployment we measure observed subsequent duration.

5   Numerical Simulations

While the dynamics arising from the proper solution to the model are certainly more interesting, we believe it is useful to use a constant $\bar{n}$ to examine the time series behaviour of this economy. We preserve the calibration used previously, but now we need to target new moments. The following table summarizes moments for the US and Canadian economies taken from Zhang (2008).[19]

Table 6: Actual data. USA and Canada

		u	v	v/u	α^w	δ	y
CA	σ	0.162	0.237	0.367	0.105	0.096	0.021
US	σ	0.190	0.202	0.382	0.118	0.075	0.020
CA	ρ_1	0.956	0.956	0.959	0.791	0.795	0.876
US	ρ_1	0.936	0.940	0.941	0.908	0.733	0.878
CA	μ	0.078			0.309	0.030	
US	μ	0.057			0.452	0.023	

And also the following correlations

Table 7: Quarterly data. USA and Canada

	$\rho(v,u)$	$\rho(v/u, \alpha^w)$	$\rho(\delta, y)$
CA	-0.689	0.753	-0.396
US	-0.894	0.948	-0.524

The correlation between the destruction rate, $1 - \lambda \equiv \delta$, and productivity dictates the construction of the shocks. We form the two shocks as different linear combinations of two identical and independent Markov processes (they have the same transition matrix):

$$
\begin{aligned}
y &= x_1 + x_2 \\
\delta &= x_1 - \psi x_2
\end{aligned}
$$

where the support of δ is constructed around the value $1/130$ and the support of y is constructed around 1.[20] With the parameter ψ set at 3.1, the correlation between the two variables is around -0.36. Both x_1 and x_2 are generated from a discretization of an AR1 process on a five point support using Tauchen's method.[21] The serial correlation parameter is 0.98. The standard deviation is

---

[19] The top four rows have moments for quarterly frequency data, while the bottom two rows have moments for monthly frequency data. The data are measured always as $log(x_t) - log(\bar{x})$, where $\bar{x}$ is the Hodrick-Prescot trend. CHW estimate the serial correlation of market tightness to be 0.93 using monthly data. The average monthly separation rate for the US is computed simply as 0.1 (the quarterly rate in Shimer (2005)) divided by 4.3482, which is the average number of weeks in a month - a year has 365.25 days on average.

[20] In fact here it is the support of $y - w$, since the two are indistinguishable.

[21] We suspect that the Ornstein-Uhlenbeck process used by Shimmer (2005) may be useful to tackle some correlations arising from time aggregation.

set to match the standard deviation of productivity, although we fall just a little short in the present calibration.

Tables 8 and 9 show standard deviations, first order serial correlations, and averages, of the stochastic model with the concave ϕ function used before and for $\bar{n} = 104$. Everything is identical to the previous numerical experiments except where adjustments must be made for the stochastic nature of the problem. The initial run of the model is made using a constant realization of the aggregate state at its highest value. After convergence the stochastic path of aggregate shocks is used for the following 600 periods, and we make use of 520 weeks which are ten years of (artificial) data.[22]

Table 8: Moments in artificial data with a maximum of 104 contacts

	u	v	v/u	α^w	δ	y	$\hat{\alpha}^w$	$\hat{\delta}^m$
σ_w	0.105	0.049	0.064	0.041	0.087	0.0173		
σ_m	0.106	0.050	0.064	0.042	0.099	0.0173	0.155	0.0063
ρ_{1w}	0.999	0.962	0.982	0.990	0.973	0.970		
ρ_{1m}	0.982	0.869	0.941	0.970	0.957	0.888	0.594	-0.018
μ_w	0.058	0.058	1.01	0.127	0.0078	1.001		
μ_m	0.058	0.058	1.01	0.127	0.0078	1.001	0.401	0.032

We have data here for weekly and monthly frequencies. The monthly frequency data is sampled every fourth week, - in the first 6 columns - except in the last two columns where the relevant measure must be constructed.

The job finding rate at lower frequencies, $\hat{\alpha}^w$, is defined as a moving average and thus has weekly values. For the monthly frequency, we take the unemployed population at the start of each week and measure their finding rate over the *following* four weeks:

$$
\begin{aligned}
\alpha_t^{w,m} &= \alpha_t^w + \alpha_{t+1}^w \left(1 - \alpha_t^w\right) + \alpha_{t+2}^w \left(1 - \alpha_{t+1}^w\right) \left(1 - \alpha_t^w\right) \\
&\quad + \alpha_{t+3}^w \left(1 - \alpha_{t+2}^w\right) \left(1 - \alpha_{t+1}^w\right) \left(1 - \alpha_t^w\right)
\end{aligned}
$$

where each weekly α_t^w is the weighed average

$$
\alpha_t^w = \sum_{j=0}^{n} \alpha_t^w (j) \, u_j
$$

Now, for example, the serial correlation of this variable is then measured by first sampling it every fourth week and then running an AR1 regression on this sampled time series. The other measures are also taken on the sampled time series.

The job destruction rate at lower frequencies, $\hat{\delta}^m$, is constructed in a similar way. For the monthly frequency we have the forward looking measure:

$$
\hat{\delta}^m = \delta_t + \delta_{t+1}(1-\delta_t) + \delta_{t+2}\left(1 - \delta_{t+1}\right)\left(1 - \delta_t\right) + \delta_{t+3}\left(1 - \delta_{t+2}\right)\left(1 - \delta_{t+1}\right)\left(1 - \delta_t\right)
$$

---

[22]One issue is whether the cross sectional moments are affected by the particular aggregate state at which the cross section is selected.

where δ_t is simply the destruction rate in week t. It is worth noting how both $\alpha_t^{w,m}$ and $\hat{\delta}^m$ perform.

And finally, a set of correlations where we can see that vacancies and un-

Table 9: Moments with a maximum of 104 contacts

	$\rho(v,u)$	$\rho(v/u,\alpha^w)$	$\rho(v/u,\hat{\alpha}^w)$	$\rho(\delta,y)$	$\rho\left(\hat{\delta},y\right)$
w	0.904	0.951		-0.361	
m	0.903	0.950	0.069	-0.361	-0.004

employment are strongly positively correlated (we have not adressed this issue) and at the weekly frequency the other correlations are close to the data. unfortunately, when we aggregate to monthly frequencies, these correlations are lost for the more relevant measures.

5.0.1 Network productivity

In an interesting paper, Galeotti and Merlino (2008) use the measure: "the proportion of newly employed workers that found a job through a friend or acquaintance that worked in the same place as the new employee". This measure is positively correlated (0.44) with the unemployment rate.[23]

This measure of network productivity is only part of the picture, since it can be positively correlated with the unemployment rate and still the network be creating fewer jobs. The network can be absolutely less productive during times of high unemployment. In fact, we can consider a world where everyone has one cousin, and therefore the "network" is constant and exogenous, and where all jobs are first filled by asking current employees about their cousins. When the unemployment rate is high, few new jobs are being created and more "cousins" are likely to be unemployed. For a firm with an open vacancy, it is more likely that one of its employees will have an unemployed cousin and so at times of high unemployment, more jobs will be filled by cousins, and less by a random draw from the market. In this world there is no notion of "network productivity", and yet the correlation of the empirical measure with the unemployment rate may be positive.[24]

Here, in the present paper, by construction close to 100% of jobs are found with some unspecified use of contacts (inside the ϕ function) - the absolute exception being the finding rate for workers with zero contacts. We therefore need to construct a different measure of "network productivity". We use two notions, which are related to the number of contacts of the set of unemployed workers, and its subset of those who find jobs on a given week.

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[23]These authors also state that "between 30% and 50% of jobs are filled through social exchange of information.", and this number matches well with the numbers obtained from the Danish survey.

[24]This is in fact not obvious because in equilibrium firms would tend to employ workers in pairs (of cousins), and so it is not clear that more cousin-pairs would be broken in times of high unemployment.

The first measure of network productivity is: the difference between, on one hand, the average number of contacts of the cohort of unemployed workers, and, on the other hand, the average number of contacts of the subset of the unemployed that finds a job in the period (job finders):

$$NP_1 = \frac{\text{Total Contacts of Job Finders}}{\text{Number of Job Finders}} - \frac{\text{Total Contacts of all Unemployed}}{\text{Number of Unemployed}}$$

Regarding this measure, it is not clear whether the average number of contacts of the unemployed population should rise or fall as the unemployment rate increases. This is because as unemployment increases more workers with lots of contacts enter the unemployment pool, but at the same time unemployment duration might increase. In fact, it does: taking in each period the cohort of workers that enters unemployment, their average onward realized duration of unemployment is positively correlated - with a coefficient of 0.72 - with the unemployment rate verified at the time they lost their job.

We know a priori less about what happens to the average number of contacts of job finders. This depends on the shape of the job finding rate function which depends on the shape of the distribution of contacts and of the ϕ function. The behaviour of this gap measure will tell us something about how the labour market works in this model.

The second measure of network productivity is the fraction of job finders with less than $\frac{\bar{n}}{4}$ contacts.

$$NP_2 = \frac{\text{Number of Job Finders with less than } \frac{\bar{n}}{4} \text{ contacts}}{\text{Number of Job Finders}}$$

It is useful to emphasize that it is hard to draw a normative inference from just looking at this measure. Is it a good or a bad thing if the measure NP_2 rises? In fact, in the data, is it a good or a bad thing if a measure of "network productivity" is positively correlated with unemployment? In light of this, what the measures used here tell us is how to understand the way shocks interact with the model of the labour market we have constructed.

The following table shows the weekly frequency correlation between these two network productivity measures and the unemployment rate in an artificial panel with 58500 workers. We use the concave ϕ function described above.

Table 10: NP and unemployment

	NP_1	NP_2	U
NP_1	1	0.22	-0.10
NP_2		1	-0.00
U			1

In this simulation average unemployment is about 5.77.% or 3375 workers, and the average number of job finders each week is 434 or about 12.86% of the

unemployed. Of these job finders, an average of 87 workers has less that $\bar{n}/4$ contacts (or roughly 20% of job finders).[25]

We see that these "network productivity" measures are negatively correlated with the unemployment rate. The tail measure (NP2) has a zero correlation with unemployment. Our "gap" measure is negatively correlated. When unemployment increases, the average number of contacts of the unemployed rises more (see below) than the average number of contacts of job finders. The question is how can this be interpreted as a productivity measure? Well, job finders are less driven by contacts than by the macroeconomy. This is a model where the network is important, but the network productivity seems to be lower during periods of high unemployment because high unemployment destroys part of the network. This makes sense: as workers lose their jobs and spend more time unemployed, their contacts vanish faster and matter less.

It is useful to decompose the correlation of NP_1, in its components:

Table 11: NP1 Decomposed

	$ACJF_1$	ACU_1	U
$ACJF_1$	1	0.98	0.16
ACU_1		1	0.17
U			1

We see that it is the average number of contacts of the unemployed that is responsible for the bigger share of the correlation. This is also the part of the model where we have an explicit mechanism. The response of the average number of contacts of job finders depends more on the ϕ function for which we have not much direct empirical guidance.[26]

Note that the average number of contacts of job finders is positively correlated with the unemployment rate, and in fact we can think of this as a similar measure to that of Galeotti and Merlino, since as job finders have more contacts they are more likely to find jobs where ther cousins are working (if the place where their cousins work has many employees and if their cousins don't necessarily have all the information about the job openings- because they work in a different division perhaps).

5.0.2   Duration

This model delivers clear implications for the pattern of employment and unemployment durations. Specifically, it predicts that workers separating from jobs with longer durations, will have on average shorter unemployment durations. It also generates exponential decay in unemployment duration. These are patterns we can look for in the data.

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[25] In a simulation with $\bar{n} = 52$, the unemployment rate is 6.3% or 3700 workers, the average number of job finders is 431 or 11.6% of unemployed, and the number of job finders with less than $\bar{n}/4$ contacts is 43 or 10% of job finders.

[26] This is one of the empirical challenges: to use indirect inference, or find moments using the model that may help us identify this function in the data.

We now construct a duration matrix in the same manner as what we also obtain from the data.[27] From our artificial data we select a given period, and for that period we select first all employed workers. Then we follow these employed workers for the next 13 weeks and extract all those that start an unemployment spell in this period. Then we measure the length of their subsequent unemployment spell, as well as the length of their previous employment spell.

The element (i,j) of this matrix has the number of unemployed workers that find a job after (j) weeks of unemployment, following a previous employment spell of (i) weeks. Dividing by the total number of elements in the matrix this is a joint density value $f(i,j)$. There are several aspects of this density matrix which are of interest. The first obvious one is the correlation coefficient between durations:

$$\rho_d^{week} = \frac{\sum \sum (x_i - \bar{x})(y_j - \bar{y}) f(x_i, y_j)}{\left[\sum (x_i - \bar{x})^2 g(x_i)\right]^{1/2} \left[\sum (y_j - \bar{y})^2 g(y_j)\right]^{1/2}} = -0.1893$$

and here, the empirical counterpart of this measure is positive at 0.121.[28]

But other less obvious characteristics have to do with where the matrix is populated, and how the shape of this density matches what we obtain from the data. A first indicator is the pattern of exponential decay in unemployment duration. We have data on a cohort of 18473 danish workers that start their unemployment spell in first quarter of 2002. The unconditional density of unemployment duration - here shown only for the first 24 weeks which uses 4975 observations - is concave (left picture) and log-linear as we can see in the right hand side picture:
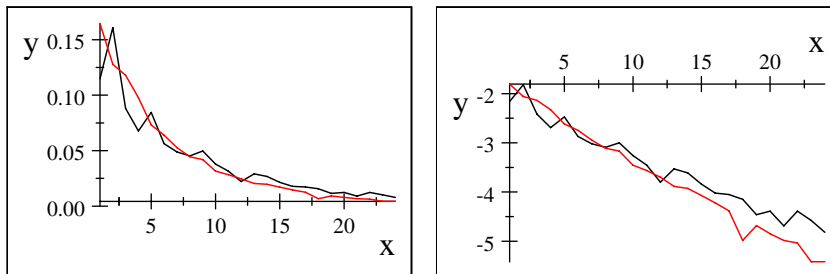
Interestingly, in red (thinner line) we have the corresponding values for the artificial data.

6   Conclusion

In this paper we construct a version of the Mortensen-Pissarides search model where the duration of unemployment is not independent of the duration of

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[27] In the data we select all Danish workers that start an unemployment spell in the first quarter of 2002.

[28] In a simulation with $\bar{n} = 52$, this value is -0.147.

employment. Here, in the course of their working activity, employed workers invest in social contacts with other employed workers. These professional contacts will help them find jobs in the event of unemployment. These social contacts are a type of capital, similar to experience or skill. In the same spirit they also"depreciate" during unemployment spells. Here this happens because the contacts a worker has acquired can also become unemployed and unemployed contacts are assumed to be useless - an extreme version of less useful. In this model the longer you have been working, the more contacts you are likely to have, and the more contacts you have the shorter your expected unemployment duration will be.

Interestingly, on a first unconditional examination of a cohort of unemployed workers in Danmark, this pattern seems to be the reverse: the longer you have been working previously, the longer unemployment duration is. One task in progress is to examine the Danish data to see if we can separate out the contributions of age, gender and education from possible network characteristics of the different individuals. We expect the initial empirical result to be reversed.

We simulate a stochastic version of the model to generate duration moments as well as "network productivity" moments. This is a model where the network is important, but the network productivity seems to be lower during periods of high unemployment. This makes sense: as workers lose their jobs and spend more time unemployed, their contacts vanish faster and matter less. This seems contrary to some empirical evidence, but given that the model presented in this paper is quite straightforward, the mechanisms it generates are useful to understand possible network effects in the data.

Much identification work remains to be done. The model generates exponential decay in unemployment duration, and this is exactly what we see in the data. This is no more than very superficial evidence of network mechanisms and can be replicated with other theories. However, the exact shape - slope and intersept - of the duration density may provide further identifying variation. Also, the entire shape of the joint density of unemployment versus employment duration is also potentially revealing.

Finally, the model can be extended to tackle other possible implications of network mechanisms in the labour market.[29] For example, we see in the data that workers displaced (due to firm closure) from smaller firms have a higher chance of becoming long term unemployed. This is over and above individual and sector characteristics. This suggests market structures where average firm size is bigger may be more efficient in terms of unemployment dynamics.

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[29] An interesting extension of this model is to consider the vacancy supply model of Cooper, Haltiwanger and Willis (2007) to see how the richer dynamics of vacancies induced by this mechanism interact with the network structure.

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7   Appendix: The probability distribution for an employed agent.

Given an upper bound $\bar{n}$ on the number of connections we can derive the expected distribution of connections at the point of job destruction for a given agent, when he starts working. Starting from zero in a new job the worker makes a connection in the first period and continues making a new connection each period until he reaches $\bar{n}$ connections. But the worker can also lose its job along the way, as can his connections.

If the worker is unemployed after one period of work, an event which occurs with probability $(1 - \lambda)$, the worker can have either zero or one contacts with respective probabilities $((1 - \lambda), \lambda)$. If he becomes unemployed after two periods, which happens with probability $\lambda(1 - \lambda)$, he can have entered the second period with one contact (probability λ), and then he can have $[0, 1, 2]$ contacts at the end of period two with probability vector $\left((1 - \lambda)^2, 2\lambda(1 - \lambda), \lambda^2\right)$. But he can also have entered the second period with only zero contacts - an event with probability $(1 - \lambda)$ - and then he can have at most one contact as above.

We can write this algorithm in matrix form. Here we use an example where $\bar{n} = 2$. All the algebra is from the perspective of time zero, when the worker starts his new job. The first period transition is between zero contacts and the set of $(0, 1)$ possible contacts, which defines a (1×3) transition vector over the set $[0, 1, 2]$, given by $\hat{M}_0 = \left[\begin{array}{ccc} (1 - \lambda) & \lambda & 0 \end{array} \right]$.

The second period transition is between the set of $(0, 1)$ possible contacts and the set of $(0, 1, 2)$ possible contacts, which defines a transition matrix (with one extra rows of zeros):

$$\hat{M}_1 = \left[\begin{array}{ccc} (1 - \lambda) & \lambda & 0 \\ (1 - \lambda)^2 & 2\lambda(1 - \lambda) & \lambda^2 \\ 0 & 0 & 0 \end{array} \right]$$

Since we are assuming that $\bar{n} = 2$, the third period transition is between the set of $(0, 1, 2)$ possible contacts and the set of $(0, 1, 2)$ possible contacts, which defines a (3×3) transition matrix:

$$\hat{M}_2 = \left[\begin{array}{ccc} (1 - \lambda) & \lambda & 0 \\ (1 - \lambda)^2 & 2\lambda(1 - \lambda) & \lambda^2 \\ (1 - \lambda)^2 & 2\lambda(1 - \lambda) & \lambda^2 \end{array} \right]$$

This generates the following sequence:

$$(1 - \lambda)\hat{M}_0, \quad \lambda(1 - \lambda)\hat{M}_0\hat{M}_1, \quad \lambda^2(1 - \lambda)\hat{M}_0\hat{M}_1\hat{M}_2,$$
$$\lambda^3(1 - \lambda)\hat{M}_0\hat{M}_1(\hat{M}_2)^2, \quad \lambda^4(1 - \lambda)\hat{M}_0\hat{M}_1(\hat{M}_2)^3, ...$$

and we can show that the distribution of connections contains a matrix geometric series which converges to $\left(I - \lambda\hat{M}_2\right)^{-1}$, if all the eigenvalues of $\hat{T} = \lambda\hat{M}_2$ are

less than 1 in absolute value. The probability of losing the job after k periods of employment is $\lambda^{k-1}(1-\lambda)$. Taking this into account, after some algebra the density over contacts becomes

$$S^e = (1-\lambda)\,\hat{M}_0\left[I + \lambda\hat{M}_1\left(I - \lambda\hat{M}_2\right)^{-1}\right]$$

and this is a vector with three elements which sum to one.

$$S^e = \left[\begin{array}{ccc} e_0 & e_1 & 1-e_0-e_1 \end{array}\right]$$

where e_0 is the fraction of the employed population with zero contacts.

We repeat the algebra quickly for the example where $\bar{n}=1$. All the algebra is from the perspective of time zero, when the worker starts his new job.

The first period transition is between zero contacts and the set of $(0,1)$ possible contacts, which defines the transition vector over the set $[0,1]$, $\hat{M}_0 = \left[\begin{array}{cc} (1-\lambda) & \lambda \end{array}\right]$. The second period transition (and the last step) is between the set of $(0,1)$ possible contacts and the set of $(0,1)$ possible contacts, which defines a a (2×2) transition matrix:

$$\hat{M}_1 = \left[\begin{array}{cc} (1-\lambda) & \lambda \\ (1-\lambda) & \lambda \end{array}\right]$$

After some algebra the density over contacts becomes

$$\begin{aligned} D^0 &= (1-\lambda)\,\hat{M}_0\left(I - \lambda\hat{M}_1\right)^{-1} \\ &= \left[\begin{array}{cc} e_0 & 1-e_0 \end{array}\right] = \left[\begin{array}{cc} 1-\lambda & \lambda \end{array}\right] \end{aligned}$$

and in steady state equilibrium this is the density over contacts of the employed population, S^e, as well as the equilibrium distribution over contacts of the cohort entering unemployment (since the probability of losing a job does not depend on the number of contacts). The last equality is specific to the $\bar{n}=1$ example and is easy to prove. The correspondence of D^0 to the Pascal Triangle expression is not generalizable.