Hybrid R&D[*]

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Abstract

We develop a model of R&D competition and collaboration in which individual firms carry out independent in-house research and also undertake joint research projects with other firms. We examine the impact of collaboration on in-house research and explore the circumstances under which a hybrid organization of R&D which combines the two is optimal for firms and society.

We find that investments in independent research and in joint research are complementary: an increase in the number of joint projects also increases in-house research. Firm profits are highest under a hybrid organization if the number of firms is small (less than 5) while they are highest with pure in-house research if the number of firms is large (5 or more). However, social welfare is maximized under a hybrid organization of R&D in all cases.

Our analysis also yields new results on the role of cooperative R&D. We find that non-cooperative decision making by firms leads to larger R&D investments and higher social welfare than fully cooperative decision making. However, a hybrid form of decision making where there is bilateral cooperation in joint projects and non-cooperative decision making in in-house research yields the highest level of welfare in concentrated industries.

1 Introduction

Technological innovations introduced in the private sector have traditionally been the result of in-house R&D undertaken by individual firms. In the last three decades this pattern has altered: there has been a significant increase in the number of R&D collaboration agreements between firms. This number rose sharply in the 1990's and has remained high in recent years (Hagedoorn, 2002). As a result, nowadays a typical firm carries out independent in-house R&D and in addition is also engaged in several joint research projects with distinct partners.[1] This hybrid form of organization of research raises a number of interesting questions such as: What is the impact of bilateral collaborations on independent research? How do firms allocate resources between their different projects? What are the relative merits of in-house vs joint projects? What are the implications of different modes of cooperative decision making for investments, profits and welfare in hybrid organizations? What are the circumstances under which such hybrid forms of organization are optimal for firms and society?

The existing models in the literature have viewed independent in-house research and collaborative joint research as mutually exclusive.[2] This formulation is inadequate for addressing the above questions. In the present paper, we develop a simple model of R&D competition and collaboration which accommodates hybrid forms of organization of R&D. In our model, individual firms carry out independent in-house research but can also enter into joint research projects with other firms. A firm's investments in the independent project and in its joint projects taken along with the efforts of its collaborators determine its operating costs. Given these costs, firms operate in the market by setting quantities.

We are interested in three basic forms of R&D organization: *pure in-house research*, *exclusive partnerships* and *networks*. A network of collaboration in which all the firms have the same number of joint projects is said to be symmetric, and this number of projects is referred to as the degree of collaborative activity, k. In a world with n firms, k ranges from from 0 to $n-1$; $k = 0$ yields pure in-house research, $k = 1$ defines exclusive partnerships, and

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[1]In the pharmaceutical sector, leading firms such as Bayer, Bristol-Myers, and Glaxo-Smith-Kline now explicitly state that their research strategy relies upon a combination of in-house research and collaborations with other firms to deliver innovation. For example, GSK has on-going collaborations with Bristol-Myers, Merck and Hoffmann-La Roche, while Bayer currently collaborates with Millenium and CuraGen and ComGenex. Similarly, in the aeronautic defense sector firms regularly collaborate to complement their core competencies. For instance, Boeing has been engaged in joint research with Alenia Spazio and EADS in the area of missile defense, while EADS (Eurocopter) collaborates with Onera on developing the technology for high-speed helicopter flight. In the engine manufacturing sector, Snecma holds patents on production processes of high temperature resistant composites with Rolls-Royce, Volvo-Aero and Sochata. Finally, research collaborations are very popular among firms in the IT sector. For instance, CSC conducts joint research with Oracle, IBM and SAS.

[2]See e.g., d'Aspremont and Jacquemin (1988), Brander and Spencer (1983), Kamien, Muller and Zang (1992), Katz (1986), Leahy and Neary (1997), and Suzumura (1992).

$k > 1$ generates network forms of organization in R&D. The latter two are special cases of hybrid forms of organization. Our analysis focuses on the relationship between the degree of collaborative activity and the variables of interest such as R&D efforts in independent in-house research and in joint projects, firm profits and social welfare.

We first show that for any degree of collaborative activity there exists a symmetric equilibrium in research investments. For any positive level of collaboration, the equilibrium is always interior, that is, firms allocate positive economic resources to independent research as well as to joint research with other firms. We find that a firm's investment in the independent project as well as in each of the joint projects increases with the degree of collaborative activity. This suggests that independent research and collaborative research are complementary.[3]

We next study the issue of relative importance of the two forms of R&D. We find that the ratio of investment in independent project to investment in a single joint project is constant across different degree networks. This implies that the share of independent R&D in total research budget falls as the number of joint projects a firm undertakes increases.

We then examine the relation between forms of R&D organization and firm profits and social welfare. We find that firm profits are increasing in the number of joint projects if the number of firms is small (less than 5) while the relationship is reversed if the number of firms is large (5 or more). In other words, in concentrated markets dense networks are desirable while in markets with many firms pure in-house research is optimal for the firms.

We conclude our analysis of equilibrium outcomes by noting that social welfare is increasing in the degree of collaborative activity and hence the complete network, in which every pair of firms is collaborating in a joint project, is socially optimal.

Given these differences in the aggregate performance of different forms of R&D organization, we ask if one of these forms is more likely to arise as compared to others? Our analysis reveals that incentives to form collaborations are quite significant. We show that purely in-house research cannot be sustained in equilibrium since any pair of firms would gain by establishing a joint collaborative project. Moreover, the *complete* network is a strategically stable network. In some examples we find that the complete network is in fact the unique strategically stable form of R&D organization. This result highlights a strategic motive behind the recent burst of collaborations in R&D networking (Hagedoorn, 2002). Taken in combination with our earlier results on firm profits under different forms of organization, this result also suggests that individual firm incentives may be in conflict with what is collectively

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[3]This finding is consistent with existing empirical work which argues that the different innovation activities of a firm are complementary; see e.g. Arora and Gambardella (1990), Cassiman and Veugelers (2002) and Cockburn and Henderson (1998).

desirable for firms in some settings.

We now turn to the issue of spillovers and incentives for R&D. Several authors have argued that if individual R&D has positive spillovers on other firms then non-cooperative decision making will lead to underinvestment in R&D (see footnote 2 above). This has led them to propose cooperative R&D as a way of internalizing these spillovers. Our analysis sheds new light on this classical problem. We analyze two possible scenarios of cooperative decision making by firms. In the first cooperative regime, we examine the benchmark case of *full cooperation* where firms decide on investments in in-house research and in joint projects collectively to maximize the joint profits of all firms. In this case we find that there is less R&D as compared to non-cooperative decision making and social welfare is lower as well. This finding shows that the dampening effects of bilateral spillovers are modest as compared to the investment enhancing effects of competition in our model of bilateral collaboration in research. We now turn to the second mode of cooperative decision making: *bilateral cooperation*. In this mode of cooperation, every pair of firms involved in a collaborative research project chooses the aggregate level of R&D investment of the common project so as to maximize the joint profit of the two research partners, while decisions on in-house research activities are kept private.[4] In this case we find that there is more R&D and profits are lower as compared to non-cooperative decision making (as well as the fully cooperative case). We now discuss the welfare implications of bilateral cooperation. We observe that this type of hybrid R&D and bilateral cooperative decision making by firms has the potential of bringing together the gains from R&D collaboration and cooperative decision making and the gains from R&D competition. We indeed find that in concentrated industries welfare is highest under bilateral cooperation; moreover, bilateral cooperation yields higher welfare than full cooperation in all cases.

Our paper contributes to the study of R&D in oligopolistic industries. The main contribution of our paper is a model of R&D which accommodates both independent in-house research and collaborative research. To the best of our knowledge this is the first attempt to integrate these two standard forms of R&D within a common framework. The analysis of our model addresses classical questions such as the role of cooperative vs non-cooperative decision making in R&D and the desirability of increasing collaboration among firms more generally. We now relate our paper to two strands of the existing literature more specifically. The *first* strand is the extensive body of work on collaborative R&D in oligopolistic markets (see e.g., d'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Katz (1986) and

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[4]Several authors have argued that firms are moving towards a organizational form in which divisions of different firms have very close ties, and sometimes these ties may be closer than the ties they have with other divisions within their own firms (see e.g., Delapierre and Mytelka (1998) and Podolny and Page (1998)). The above mode of decision making offers a natural model for this form of organization.

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Suzumura (1992)). In the terminology of Kamien, Muller and Zang (1992) our basic model with non-cooperative firms is a Research Joint Venture competition type of model, where firms forming a collaboration commit to completely sharing the R&D results arising from research efforts which are unilaterally decided. A standard result in this literature is that non-cooperative firms, in an attempt to control business-stealing effects, invest too little in R&D relative to what is optimal for them collectively (see e.g., Kamien, Muller and Zang (1992)). By contrast, we find that non-cooperative decision making leads firms to over-invest in R&D. The main reason for this difference is the nature of spillovers. In Kamien, Muller and Zang (1992) spillovers of individual efforts accrue to all firms in the market, while in our model spillovers are controlled and only accrue to the collaborator in the specific project in question. In contexts where spillovers are bilateral – due to technological or contractual reasons – the findings in our model provide a useful benchmark.[5]

Our paper also contributes to the literature on strategic models of collaboration in oligopoly. Traditionally, these issues have been addressed using models of coalition formation (for a survey of this work see Bloch (1997)). In recent years several authors have also attempted to develop strategic models of networks see e.g., Bala and Goyal (2000), Dutta, van den Nouweland and Tijs (1995), Jackson and Wolinsky (1996) and Kranton and Minehart (2001). Specifically, our model builds on two papers Goyal and Joshi (2003) and Goyal and Moraga-González (2001). There are two novel elements in the present paper: the possibility of in-house research as well as joint research and the local nature of spillovers. These differences lead to results which are quite different from the results obtained earlier. For instance, Goyal and Moraga-González (2001) find that R&D investments are decreasing in the level of collaborative activity while profits and welfare initially increase but eventually decline. By contrast, in the present paper we observe that investment in independent projects as well as in each of the joint projects is increasing while the profits are declining and social welfare is increasing in the number of joint projects of a representative firm. This difference in relationship between level of collaborative activity and equilibrium outcomes points to the significance of different forms of spillovers.

The rest of the paper is organized as follows. The model is presented in section 2. In section 3 we present our main results. We first take up the case of symmetric networks of firm collaboration and then asymmetric networks. In section 4 we discuss the role of spillovers across projects and the role of market competition. Section 5 concludes.

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[5]The influence of spillover control on the number of collaborations of a firm is examined empirically in Cassiman and Veugelers (2002a).

2   The model

We consider a three-stage game Γ. In the first stage firms form pair-wise collaboration projects. In the second stage, each firm unilaterally allocates resources to its private R&D project and to every joint R&D project it is engaged in. These decisions determine the effective costs of production of every firm. In stage three, firms engage in Cournot competition. The details of the game and the notation follow.

Joint projects and collaboration networks: Let $N = \{1, \ldots, n\}$ be the set of identical firms. We shall assume that $n \geq 2$. The underlying R&D network formed in the first stage is denoted by g. Whenever $ij \in g$, this means that there exists a *competitive RJV* between firms i and j. Let $N_i(g) = \{i \in N | ij \in g\}$ be the set of firms with which firm i has a collaboration link in network g. Also let $n_i(g) = |N_i(g)|$, be the cardinality of the set of partner firms for firm i in network g. One part of our analysis will focus on networks in which every firm has the same number of collaborations, i.e., $n_i(g) = n_j(g) = k$. In this case, we shall refer to k as the degree of collaborative activity. Clearly, $k \in \{0, 1, 2, ..., n-1\}$.

If g is a network obtained in the first stage of the game, denote by $\Gamma(g)$ a subgame of Γ. In $\Gamma(g)$ firms choose their research expenditures anticipating their profits in the ensuing Cournot game correctly. Given the network g, every firm chooses the amount of money $x_{ij} \in \mathbb{R}_+$ it is going to spend on the joint project $ij \in g$. At the same time, every firm chooses the level of the R&D expenditure in its own project $x_{ii} \in \mathbb{R}_+$. Let $x_i = ((x_{ij})_{j \in N_i(g)}, x_{ii})$ be a strategy of firm i in the subgame $\Gamma(g)$. Denote by $x = (x_i)_{i \in N}$ a strategy profile of this subgame.

Cost-reducing R&D: We assume that the returns to R&D investments are given by the function $f(X) = (1/\gamma)\sqrt{X}$, where $\gamma \in (0, +\infty)$ is a parameter related to the cost of research investment, and X is effective R&D investment, that is, the amount of money that a firm alone would have had to invest in the project to achieve the same unit cost reduction. A cost reduction function is assumed to be associated with every research project. Specifically, we put

$$f_{ii}(x) = \frac{1}{\gamma}\sqrt{x_{ii}} \tag{1}$$

for a firm i's own project, and

$$f_{ij}(x) = \frac{1}{\gamma}\sqrt{x_{ij} + x_{ji} + \beta\sqrt{x_{ij}x_{ji}}} \tag{2}$$

for a joint project $ij \in g$. The exogenous parameter $\beta \in [0, +\infty)$ reflects the complementarity of firms' research assets. Finally, the unit cost of the firm i is given as follows

$$c_i(g, x) = \bar{c} - f_{ii}(x) - \sum_{j \in N_i(g)} f_{ij}(x). \tag{3}$$

We would like to elaborate on this cost-reduction formulation. The first aspect is the additive structure: we think of the product as consisting of several components. Investment in a research project – independent or joint – reduces the cost of producing a particular component. The additive structure reflects the assumption that there are no direct or technological spillovers across projects. For example, investments in lowering packaging costs have no impact on the cost of producing specific components of the product. We explore the implications of cross project spillovers in section 4.

We note that the partial derivative $(c_i)'_{x_{ij}}(g, x)$ equals minus infinity whenever $x_{ij} = 0$, $\beta > 0$ for any $i \in N$, $j \in N_i(g) \cup \{i\}$. Similarly, the partial derivative with respect to own project investments is infinity at $x_{ii} = 0$. These observations imply that marginal gains from both in-house and joint research are very high at very low levels of investments, which materializes in that a firm's maximization problem in R&D variables does not have a corner solution.

The market stage: Given the costs $c_i(g, x)$, firms operate in the market by choosing quantities $(q_i(g, x))_{i \in N}$. The inverse demand function is given by:

$$p = A - \sum_{i \in N} q_i(g, x).$$

Then, the equilibrium quantity of firm i in a homogeneous-product oligopoly is

$$q_i(g, x) = \frac{A - nc_i(g, x) + \sum_{j \neq i} c_j(g, x)}{n + 1}, \tag{4}$$

and the profits of Cournot competitors are given by

$$\pi_i(g, x) = (q_i(g, x))^2 - \sum_{j \in N_i(g)} x_{ij} - x_{ii}. \tag{5}$$

We shall compare the individual incentives with socially desirable outcomes. For this comparison we shall use the following measure of social welfare.

$$W(g, x) = \sum_{i \in N} \pi_i(g, x) + \frac{1}{2} Q(g, x)^2, \tag{6}$$

where $Q(g, x) = \sum_{i \in N} q_i(g, x)$.

3 Analysis

In this section we present two sets of results. The first set of results pertains to the performance of different forms of organization in R&D. In this part, we assume that firms make decisions on R&D non-cooperatively. The second set of results compares non-cooperative with cooperative firm decision making.

Given the network g and other firms' R&D investments, firm i maximizes $\pi_i(g, x)$ in x_i subject to the constraints $x_{ii} \geq 0$, $x_{ij} \geq 0$, $j \in N_i(g)$:

$$
\begin{array}{ll}
\max & \pi_i(g, x) \\
\text{s.t.} & x_{ii} \geq 0, \\
& x_{ij} \geq 0, \quad j \in N_i(g).
\end{array}
\tag{7}
$$

The first-order conditions for the interior solution of this problem are

$$
\frac{\partial \pi_i}{\partial x_{ii}}(x) = \frac{q_i(g, x)n}{\gamma^2(n+1)f_{ii}(x)} - 1 = 0,
\tag{8}
$$

and

$$
\frac{\partial \pi_i}{\partial x_{ij}}(x) = \frac{q_i(g, x)(1 + \frac{\beta}{2}\sqrt{x_{ji}/x_{ij}})(n-1)}{\gamma^2(n+1)f_{ij}(x)} - 1 = 0, \quad j \in N_i(g).
\tag{9}
$$

A close look at the first order conditions reveals the strategic nature of R&D variables. From the viewpoint of a firm i, all R&D investments by rival firms in own projects as well as in joint projects are strategic substitutes except the R&D investments of firms linked to i in joint projects with firm i, which are strategic complements. Moreover, it can be seen that the marginal gains to invest in both own and joint research are increasing in complementarity parameter β and the degree of collaborative activity k. This suggests that dense networks are more attractive than in-house research and partnerships from the point of view of an individual firm.

For a network g, a profile of R&D investments $x \in \mathbb{R}_+^{(k+1)n}$ is a Nash equilibrium of the subgame $\Gamma(g)$ if and only if every x_i is a solution of the maximization problem (7) given the R&D investments of other firms $x_{-i} = (x_j)_{j \neq i}$. An equilibrium x is called *symmetric* if $x_{ij} = x_{lm}$ whenever $i \neq j$ and $l \neq m$, and $x_{ii} = x_{jj}$ for every pair of firms $i, j \in N$.

In a symmetric equilibrium, the first-order conditions can be rewritten as:

$$
\sqrt{x_{ii}} = \frac{\gamma n(A - \bar{c}) + nk\sqrt{2 + \beta}\sqrt{x_{ij}}}{\gamma(n+1)^2 - n},
\tag{10}
$$

$$\sqrt{x_{ij}} = \frac{(A - \bar{c} + \frac{1}{\gamma}\sqrt{x_{ii}})(n-1)}{\frac{2\gamma(n+1)}{\sqrt{2+\beta}} - \frac{k}{\gamma}(n-1)\sqrt{2+\beta}} \tag{11}$$

3.1 Symmetric Networks

We study R&D intensity, firm profits and social welfare under different forms of organization in R&D. In this part we focus on symmetric networks; pure in-house research obtains if $k = 0$, exclusive partnerships arise if $k = 1$, while symmetric networks of increasing collaboration arise as k increases from 2 till $n-1$.

We begin with a preliminary result on existence and characterization of symmetric equilibrium in investment levels.

Proposition 3.1 *Suppose that $g = g^k$ is a symmetric network of degree k. There exists an interior symmetric Nash equilibrium $x^* \in I\!\!R_{++}^{(k+1)n}$ of the subgame $\Gamma(g^k)$. The equilibrium investment levels are given as follows:*

$$x_{ii}^* = \left(\frac{(A - \bar{c})n\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(1+\beta/2)} \right)^2, \tag{12}$$

$$x_{ij}^* = \frac{1}{2+\beta} \left(\frac{(A - \bar{c})(n-1)(1+\beta/2)\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(1+\beta/2)} \right)^2. \tag{13}$$

Proof: See the appendix.

The proof proceeds as follows. We note that, if a symmetric equilibrium exists, it is given by the solution to the system of equations (10) and (11). The right-hand-sides of (10) and (11) increase monotonically with $\sqrt{x_{ij}}$ and $\sqrt{x_{ii}}$, respectively. Thus, there is at most a single solution to this system of equations. We note that the first order conditions are necessary for an interior optimum but not sufficient since the profit functions are not necessarily quasi-concave in own strategy. The final step in the proof exploits specific features of the payoff function to deduce that the solution worked out on the basis of the first order conditions above indeed constitutes a Nash equilibrium.

Upon substituting these equilibrium investment levels in (4), (5) and (6), we get the following expressions for equilibrium quantities, profits and social welfare, respectively.

$$q_i(g^k) = \frac{2(A - \bar{c})\gamma^2(n+1)}{2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)} \tag{14}$$

$$\pi_i(g^k) = \frac{(A - \bar{c})^2\gamma^2[4(\gamma^2(n+1)^2 - n^2) - k(n-1)^2(2+\beta)]}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))^2} \tag{15}$$

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$$W(g^k) = \frac{(A - \bar{c})^2 \gamma^2 [2n\gamma^2(n+2)(n+1)^2 - kn(n-1)^2(2+\beta) - 4n^3]}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))^2} \tag{16}$$

We now examine the sensitiveness of equilibrium with respect to the degree of collaborative activity k. Inspection of (12) and (13) reveals that, for a given complementarity parameter β, equilibrium research expenditures in own project as well as in joint projects are increasing in the degree of collaborative activity. The intuition is as follows. As the degree of collaboration rises, it becomes cheaper for a firm to lower costs of production; this effect tends to raise the incentives to invest in in-house as well as in joint research. Greater investment by collaborators tend to increase further the incentives to put money in research; by contrast, investments by firms that have no joint projects with a given firm lower its marginal returns from carrying out R&D. Our result shows that the former effects dominate the latter effects. However, it is interesting to note that as the degree of collaborative activity k increases, the proportion of R&D expenditures dedicated to joint projects increases; thus, firms become more keen to put money in the network of collaboration projects. This can be seen in the following equation:

$$\frac{kx_{ij}^*}{x_{ii}^* + kx_{ij}^*} = \frac{1}{\frac{4n^2}{k(n-1)^2(2+\beta)} + 1}. \tag{17}$$

Our next result summarizes these observations.

Proposition 3.2 *An increase in the degree of collaborative activity k leads to: (i) an increase in firm investment in own project as well as in joint projects, (ii) an increase in the proportion of investment in joint projects relative to investment in own project.*

This result shows that in-house research and joint research are complementary in the sense that an increase in the number of joint projects raises investments in each joint project and in in-house research as well. It also points to an increase in the relative importance of overall investments in joint projects as compared to in-house research as firms have more joint projects. This latter finding is consistent with empirical observation, see e.g., Delapierre and Mytelka (1998).

Since investment in both own and joint projects rises, it follows that cost reduction increases with k. Inspection of (14) reveals that individual firm quantity — and by implication aggregate quantity and consumer surplus — increases with the degree of collaboration k. We now examine how firm profits change with k. Using (15), we obtain:

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$$\frac{\partial \pi_i(g^k)}{\partial k} = \frac{-(A-\bar{c})^2(2+\beta)(n-1)\gamma^2(2\gamma^2(n+1)^2(n-5) + k(n-1)^2(2+\beta) + 2n(3n+1))}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))^3}$$

Inspection of this equation reveals that if $n < 5$ then (for sufficiently large γ) firm profits are increasing in k while if $n \geq 5$ then the profits of each firm are decreasing in the degree of collaborative activity. This result suggests that firms prefer dense networks of collaboration in concentrated industries and pure in-house R&D in non-concentrated industries.

We finally examine the welfare implications of an increase in k. Using (16), we compute:

$$\frac{\partial W(g^k)}{\partial k} = \frac{(A-\bar{c})^2(2+\beta)n(n-1)\gamma^2(2\gamma^2(n+1)^2(n+5) - k(n-1)^2(2+\beta) - 2n(3n+1))}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))^3} \quad (18)$$

This derivation tells us that welfare is increasing in the complementarity parameter β provided γ is sufficiently large. Thus the effect of an increase in consumers surplus (due to increase in quantity supplied) dominates the negative effect of a fall in firm profits as the level of collaborative activity increases. As a result, we conclude that networks of collaboration are more attractive than partnerships or in-house research from the social point of view.

The following result summarizes these findings:

Proposition 3.3 *If $n < 5$ then profits increase in the degree of collaborative activity and the complete network maximizes firm profits while if $n \geq 5$ then profits decrease in the degree of collaborative activity and pure in-house research maximizes firm profits. However, social welfare increases in the degree of collaborative activity and the complete network maximizes social welfare for all n.*

We now comment briefly on the role of the complementarity parameter β. Here we find that the qualitative effects of changing β are similar to those of changing the degree of collaboration. This is easily seen by inspection of expressions (8), (9), (14) and (17) and by noting that

$$\frac{\partial \pi_i(g^k)}{\partial \beta} = \frac{k}{(2+\beta)}\frac{\partial \pi_i(g^k)}{\partial k}$$

$$\frac{\partial W(g^k)}{\partial \beta} = \frac{k}{(2+\beta)}\frac{\partial W(g^k)}{\partial k}$$

Thus the effects of an increase in β on efforts, quantities, profits and welfare are analogous to those reported in Proposition 3.2. In particular, we note that an increase in the complementarity parameter β leads to an increase in firm investment in own project as well as in joint projects. This is somewhat surprising at first glance since an increase in β actually makes an independent project relatively less attractive. This result thus illustrates the complementary nature of investments in independent and joint projects.

The above results provide a clear picture of the relationship between network density and incentives for conducting R&D. Individual investments both in own projects as well as in joint projects are increasing in the level of collaborative activity. Social welfare is increasing in degree of collaboration as well. However, the behaviour of individual profits depends on the number of firms in the market.

These findings raise two related questions that we address in what follows: The *first* question is about the mode of decision making by firms: does cooperative decision making in R&D lead to higher effort, profits and social welfare as compared to non-cooperative decision making? The second question pertains to the relative stability of different networks.

We start with the first question and analyze two possible scenarios of cooperative decision making by firms. In the first cooperative regime, we examine the benchmark case of *full cooperation* where firms decide on investments in in-house research and in joint projects collectively to maximize the joint profits of all firms. Given a symmetric network g^k, R&D investments that maximize industry profits solve the problem:

$$\max \sum_{i=1}^{N} \pi_i(x; g^k)$$
$$\text{s.t. } x_{ii} \geq 0, \ i \in N$$
$$x_{ij} \geq 0, \ i \in N, \ j \in N_i(g^k)$$

Straightforward computations (presented in the appendix), which exploit the symmetry of the problem, yield the following optimal research investments.

$$x_{ii}^{FC} = \left(\frac{\gamma(A - \bar{c})}{\gamma^2(n+1)^2 - k(2+\beta) - 1} \right)^2, \tag{19}$$

$$x_{ij}^{FC} = \left(\frac{\gamma(A - \bar{c})\sqrt{2+\beta}}{\gamma^2(n+1)^2 - k(2+\beta) - 1} \right)^2. \tag{20}$$

In the second scenario of cooperative decision making, we examine *bilateral cooperation*. Under this mode of cooperation, every pair of firms involved in a collaborative research

project chooses the aggregate level of R&D investment of the common project so as to maximize the joint profit of the two research partners, while decisions on in-house research activities are kept private. The investments in a joint project ij solve then the following problem

$$\max \quad \pi_i(x; g^k) + \pi_j(x; g^k)$$
$$\text{s.t. } x_{ij} \geq 0, \ x_{ji} \geq 0$$

given the investments of firms i and j in independent research and in all other joint projects. Firm i investment in in-house research is decided unilaterally and thus solves the problem

$$\max \quad \pi_i(x; g^k)$$
$$\text{s.t. } x_{ii} \geq 0$$

given firm i investments in all joint projects. The symmetric solution of these problems is as follows:

$$x_{ii}^{BC} = \left(\frac{(A - \bar{c})n\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(2+\beta)} \right)^2, \tag{21}$$

$$x_{ij}^{BC} = (2 + \beta) \left(\frac{(A - \bar{c})(n-1)\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(2+\beta)} \right)^2. \tag{22}$$

How do these investment levels compare with the equilibrium investment levels obtained under non-cooperative behaviour? The following result provides a clear response.

Proposition 3.4 *Bilateral cooperation yields the highest level of R&D investments, while full cooperation yields the lowest:*

$$x_{ii}^{BC} > x_{ii}^* > x_{ii}^{FC}, \quad k > 0, \quad i \in N,^{[6]} \tag{23}$$

$$x_{ij}^{BC} > x_{ij}^* > x_{ij}^{FC}, \quad k > 0, \quad ij \in g. \tag{24}$$

Further, the proportion of investment in joint R&D satisfies the following:

$$\frac{kx_{ij}^{FC}}{x_{ii}^{FC} + kx_{ij}^{FC}} > \frac{kx_{ij}^{BC}}{x_{ii}^{BC} + kx_{ij}^{BC}} > \frac{kx_{ij}^*}{x_{ii}^* + kx_{ij}^*}, \quad i \in N. \tag{25}$$

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[6]We note that $x_{ii}^* > x_{ii}^{FC}$, $i \in N$, for $k = 0$ also.

Furthermore, if $n \geq 3$ and γ is sufficiently high, then firms' profits satisfy

$$\pi_i(x^{FC}, g^k) > \pi_i(x^*, g^k) > \pi_i(x^{BC}, g^k), \quad k > 0, \quad i \in N. \tag{26}$$

Proof: See the appendix.

We now elaborate on some aspects of this result. The first point to note is that non-cooperative R&D investments are lower than under bilateral cooperation. We have noted above that a firm investment in a joint project brings about detrimental business-stealing effects for this firm because the collaborator is conferred a cost advantage. Under bilateral cooperation, this externality is internalized and thus firms investments in joint R&D projects are larger. The second point to note is that under non-cooperative decision making investments in joint projects are excessive as compared to what is good for the firms collectively. There are two forces at work here: on the one hand spillovers are internalized under full cooperation and this tends to increase R&D investments relative to non-cooperative firms. On the other hand, there is the joint-profit externality which tends to lower R&D investments under full cooperation. The above result shows that the latter effect dominates. This result is in contrast to the results of Kamien, Muller and Zang (1992) who show that R&D investments and hence cost reduction is lower under non-cooperative behaviour. This difference in result is important for policy purposes and we explain the reasons for it. In our model of bilateral collaborative projects the investment put in a joint research project is shared only with a single collaborator. This implies that business-stealing effects are moderate; in the model of Kamien, Muller and Zang (1992), by contrast, a firm's investment confers a cost advantage to *all* the firms in the collaboration structure. This global spillover dampens incentives for spending in joint R&D greatly under non-cooperative decision making.

The second point is about independent R&D. In the pure in-house research case ($k = 0$), we find that independent R&D under non-cooperative decision making is excessive as compared to what firms would like collectively. This result is in line with the finding of Leahy and Neary (1997), who also obtain an excessive individual incentives result for low global spillovers. The complementary nature of investments in independent and joint R&D reinforces this result for any level of collaboration.

The third point is about the relative proportion invested in independent research. We find that for all degrees of the collaborative network, this proportion is lowest under non-cooperative behaviour, intermediate under bilateral cooperation and highest under full cooperation. This feature of the result reflects the presence of positive spillovers in joint research which are not fully internalized at an intermediate or no level of cooperation.

Finally, the profits result deserves a comment. The first part of the result is to be expected since fully cooperating firms can always replicate non-cooperative decisions. The second part

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of the result is interesting and somewhat surprising as it shows that bilateral cooperation is worse than no cooperation for the firms. The intuition behind this result is that under bilateral cooperation, firms internalize spillovers in joint projects and this leads them to invest more in all projects as compared to the non-cooperative case. This aggravates the overinvestment problem which exists in the non-cooperative case (as pointed out above).

The next question that arises is whether non-cooperative firm behavior yields higher or lower welfare as compared to cooperative behaviour. The answer is given by the following result:

Proposition 3.5 *Suppose $3 \leq n \leq 6$. Then, for any degree of collaboration k and γ sufficiently large*

$$W(x^{BC}, g^k) > W(x^*, g^k) > W(x^{FC}, g^k).$$

If $n \geq 7$ then

$$W(x^*, g^k) > W(x^{BC}, g^k) > W(x^{FC}, g^k)$$

for any k and for γ sufficiently large.

Proof: See the appendix.

We note that welfare under full cooperation is always lower than under non-cooperative behavior. This result, which is in contrast to the finding in Kamien, Muller and Zang (1992), highlights the detrimental effects of fully coordinated behavior to restrict R&D investments.

To understand the welfare implications of bilateral cooperation is useful to examine the benchmark case of welfare maximization. Given the network g, R&D investments that maximize social welfare solve the following problem:

$$\max W(x; g)$$
$$\text{s.t. } x_{ii} \geq 0, \ i \in N$$
$$x_{ij} \geq 0, \ i \in N, \ j \in N_i(g)$$

Solving this problem yields the following socially optimal levels of research investments:

$$\widehat{x}_{ii} = \left(\frac{(A - \bar{c})\gamma(n + 2)}{2\gamma^2(n + 1)^2 - (n + 2)(1 + k(2 + \beta))} \right)^2 \tag{27}$$

$$\widehat{x}_{ij} = \left(\frac{(A - \bar{c})\gamma(n + 2)\sqrt{2 + \beta}}{2\gamma^2(n + 1)^2 - (n + 2)(1 + k(2 + \beta))} \right)^2 \tag{28}$$

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Comparing the equilibrium levels of R&D with the socially optimal ones we observe that the private incentives to invest in independent project are excessive while the incentives to invest in joint projects are insufficient from the social viewpoint (for a proof see the appendix). Since bilateral cooperation yields greater investments in all projects, this means that bilateral cooperation strengthens the excessive in-house R&D result while weakens the insufficient joint R&D result. In concentrated industries, the second effect has a dominating influence and bilateral cooperation yields greater welfare than non-cooperative behavior. By contrast, in non-concentrated industries bilateral cooperation yields too much duplication of R&D results and welfare is thus lower than under non-cooperative decision making.

We now move to the second question and examine the strategic stability of different forms of organization in R&D. To proceed with this, we set $\beta = 2$, which ensures that the equilibrium in efforts and quantities is unique for every symmetric network.[7] For a given network g, the equilibrium R&D investments are denoted by $x(g)$. Let firm i's profit in network g be given by

$$\pi_i(g) = (q_i(g, x(g)))^2 - \sum_{j \in N_i(g)} x_{ij}(g) - x_{ii}(g).$$

We say that a network g is *pair-wise stable* if (i) $\pi_i(g) \geq \pi_i(g-ij)$ and $\pi_j(g) \geq \pi_j(g-ij)$, for all $ij \in g$ and (ii) if $\pi_i(g+ij) > \pi_i(g)$ then $\pi_j(g+ij) < \pi_j(g)$ for all $ij \notin g$. We now present a result which shows that firms' have significant incentives to form collaborative projects.

Proposition 3.6 *The empty network is not pair-wise stable while the complete network is pair-wise stable.*

This result shows that pure in-house research cannot arise in our model as an equilibrium outcome, while a network in which every pair of firms collaborates is stable. The fact that the complete network is stable suggests that firms

may have excessive incentives to form collaborations in unconcentrated industries.

3.2   Asymmetric networks

It is frequently argued that firms can acquire market power by forming alliances. Moreover, a dominant position in a network can enable a firm to 'exploit' its partners by having them make greater investments in joint projects. An examination of these arguments requires an analysis of networks in which firms have a different number of connections, i.e., asymmetric

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[7]We present a proof of existence and uniqueness of equilibrium in the appendix.

networks. We have been unable to develop a general set of results for asymmetric networks; in what follows we restrict attention to an example with four firms.

It is possible to calculate equilibrium values of R&D investments $x(g)$, profits $\pi(g)$ and welfare $W(g)$, for every network g.[8] We begin by noting a simple link monotonicity property: the firm with more joint projects puts in higher R&D efforts in the independent project as well as in each of the joint projects. Moreover, the firms with more joint projects obtain higher profits as compared to the firms with fewer projects. This observation implies that partners in a joint research project will put unequal efforts into their common project if they have a different number of joint projects, with the more linked partner putting in a higher level of effort. In this sense poorly connected firms are at an advantage in a bilateral relationship. However, this inference should be seen in the context of the fact that the larger investments of the more connected firms are suitably compensated and such a firm also makes larger profits.

Our next observation concerns firms' incentives to establish joint research projects: we find that establishing a link always increases the profits of the two participating firms while such a link decreases the profits of the rest of the firms. If we take the definition of stability outlined above, this result implies that the complete network is the *unique* strategically stable network.

This finding leads us to explore the properties of the different networks of collaboration from the point of view of the collective interest. We obtain a simple monotonicity property between the total number of collaboration projects in a network and aggregate performance: If $\sum_{i \in N} |N_i(g)| > \sum_{i \in N} |N_i(g')|$ then $W(g) > W(g')$ and $\Pi(g) > \Pi(g')$. Our final observation is on how aggregate performance depends on the distribution of joint projects in a network of collaboration. To explore this issue, we fix the number of joint projects in a network and study how aggregate firms' profits and social welfare change when we move from less to more asymmetric networks. We find that the higher the degree of asymmetry in a network of collaboration, the better are the welfare and joint profit properties of the network. In fact, the lowest values of W and Π are attained under the symmetric network. Among the three possible 3-link architectures, a triangle has a slightly higher welfare than a star, and a chain has the lowest welfare of all three.

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[8]We have computed these equilibrium values using the software Mathematica 4.1. All the computations are available from the authors upon request.

4 Discussion

4.1 The role of spillovers across projects

In the basic model we have assumed that a firm i's investment in a joint project with firm j has no direct effect on the productivity of other investments of firm i, whether in own project or in projects with other firms. Thus there are no direct knowledge spillovers across projects. In this section we will examine the role of this assumption.

Perhaps the simplest way to do so is by looking at the polar opposite case where spillovers across projects are perfect. To implement this idea, we take firm i's strategy to be a one-dimensional variable, x_i. We can then posit that, given a network g, the cost reduction for firm i is as follows:

$$c_i(\{x_i(g)\}_{i \in N}) = \bar{c} - \frac{1}{\gamma}\sqrt{x_i} - \sum_{k \in N_i(g)} \frac{1}{\gamma}\sqrt{x_k}. \tag{29}$$

This formulation assumes that the outcome of a firm's own research is shared fully with all the collaborators. We shall use the superscript ps to refer to this case of perfect spillovers across projects. As before, given a network g, and a profile of investments $x = \{x_1, x_2, ..., x_n\}$, the Cournot profits are:

$$\pi_i^{ps}(g) = q_i^2 - x_i(g), \text{ with } q_i^{ps} = \frac{a - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1}.$$

We now compute the equilibrium level of investments for a symmetric network of degree k.

$$x_i^{ps} = \left[\frac{(A - \bar{c})(n - k)}{\gamma(n+1)^2 - \frac{(k+1)}{\gamma}(n - k)} \right]^2. \tag{30}$$

The corresponding equilibrium profits and social welfare are as follows:

$$\pi_i^{ps}(g^k) = \frac{(A - \bar{c})^2 \gamma^2 [\gamma^2(n+1)^2 - (n-k)^2]}{[\gamma^2(n+1)^2 - (n-k)(k+1)]^2}, \tag{31}$$

$$W^{ps}(g^k) = \frac{n(A - \bar{c})^2 \gamma^2 [\gamma^2(n+2)(n+1)^2 - 2(n-k)^2]}{2[\gamma^2(n+1)^2 - (n-k)(k+1)]^2}. \tag{32}$$

We note that this model of collaboration activity in networks with perfect spillovers is essentially the same model (apart from some reparameterization) as that studied in Goyal

and Moraga-González (2001). We can therefore use their analysis to derive the following relationship between equilibrium R&D investments and the degree of collaborative activity in a network:

Proposition 4.1 *In the case of perfect spillovers across projects, as the degree of collaborative activity k increases equilibrium R&D investments decrease while profits and social welfare initially increase but eventually fall.*

A comparison between Propositions 3.3 and 4.1 reveals that firm R&D investments, profits and social welfare display very different relations in the two spillover settings. For instance, with no spillovers across projects, per project investment is increasing in the level of collaborative activity, while the reverse is true in the case of perfect spillovers. On the other hand, with no spillovers across projects profits fall as collaboration increases, while under perfect spillovers profits initially rise and eventually fall. Finally, social welfare increases in the absence of spillovers while initially increases and eventually declines under perfect spillovers.

These remarks lead us to ask whether firms prefer one or the other regime of spillovers? The following proposition responds to this question:

Proposition 4.2 *Suppose that $n \geq 5$. For a given degree of the collaboration network, individual firm profits are higher in the perfect spillovers regime as compared to the zero spillovers regime*[9].

Proof: See the appendix.

This result tells us that firms prefer a regime with perfect spillovers. Why is this the case? We note that, for a given level of R&D investment, an increase in spillovers is clearly beneficial for a firm since this firm potentially benefits from other firms investments. This effect tends to increase profits, ceteris paribus. On the other hand, perfect spillovers across projects induce greater business-stealing effects and this lowers the marginal returns from investments in R&D; this effect tends to reduce R&D efforts and in turn profits. However, we note that in the present context firms make excessive investments in R&D (see Proposition 3.4) and thus a pressure toward lower investments can actually increase profits. These considerations underlie the conclusion of the above result: the overall effect of moving from zero sillovers to perfect spillovers is that profits of firms increase.

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[9]If $n < 5$, individual firm profits can be higher in the zero spillovers regime provided β is sufficiently large.

We finally discuss the role of spillovers across projects from the social welfare point of view. A comparison of equations (16) and (32) shows that $W(g^{k=0}) = W^{ps}(g^{k=0})$. Furthermore, Proposition 3.3 and Proposition 4.1 imply the following relationship between the social welfare levels attained under the two spillover regimes.

Proposition 4.3 *In the empty network, social welfare under perfect spillovers is equal to the social welfare under zero spillovers. Moreover, if γ is sufficiently large, then there exists a cut-off level of collaboration k^*, such that for all $1 \leq k \leq k^*$, social welfare under perfect spillovers is larger than the social welfare under zero spillovers. The reverse is true for $k > k^*$.*

The intuition behind this result is as follows. First note that perfect spillovers across projects lead to less duplication of R&D activities and that this is desirable from the point of view of the society. We note however that business-stealing effects become more acute when the degree of collaboration increases, which dampens incentives for R&D spending. It turns out that for low levels of collaboration social gains from less duplication of R&D offset the social losses from lower investments in R&D. Eventually business-stealing effects are of such magnitude that welfare is higher under the no spillovers case.

4.2 The role of market competition

In the basic model it is assumed that firms compete in the market by choosing quantities. This is a very specific form of market competition and it is worth examining the impact of alternative forms of competition on the incentives to do R&D in collaboration networks. This motivates the following model in which firms are local monopolies in their own product markets.

Let $P = A - Q_i$ now be the inverse market demand for firm $i \in N$. As before firms first choose partners, then R&D investments and finally their quantity. For a given degree of collaboration k the equilibrium efforts, quantities, profits and social welfare are given as follows:

$$x_{ii}^{lm} = \frac{(A - \bar{c})^2 \gamma^2}{(4\gamma^2 - 1 - k(\beta/2 + 1))^2}; \quad x_{ij}^{lm} = \frac{(2 + \beta)(A - \bar{c})^2 \gamma^2}{(8\gamma^2 - 2 - k(\beta + 2))^2}; \tag{33}$$

$$q_i^{lm}(g^k) = \frac{2(A - \bar{c})\gamma^2}{4\gamma^2 - 1 - k(\beta/2 + 1)}; \quad \pi_i^{lm}(g^k) = \frac{(4\gamma^2 - 1 - k(1/2 + \beta/4))(A - \bar{c})\gamma}{4\gamma^2 - 1 - k(1 + \beta/2)}; \tag{34}$$

$$W^{lm}(g^k) = \frac{(6\gamma^2 - 1 - k(1/2 + \beta/4))(A - \bar{c})\gamma n}{4\gamma^2 - 1 - k(1 + \beta/2)};$$ (35)

The following result on the relationship between the degree of collaborative activity and equilibrium variables can be easily computed from the above equations.

Proposition 4.4 *Suppose firms are local monopolists. Then an increase in the degree of collaborative activity increases efforts in own projects as well as joint projects, increases firm profits and also increases social welfare.*

A comparison of Proposition 3.2 and Proposition 4.4 suggests that the effects of competition are felt most in the behaviour of firm profits. As the degree of collaborative activity increases, firm profits are decreasing under Cournot competition while they are increasing if firms are local monopolies. The other equilibrium variables behave similarly: firm investments – both on own project as well as joint project – are increasing, quantities are increasing and social welfare is increasing with respect to the degree of collaboration in both settings. The difference with respect to firm profits arises due to the competitive effects in the Cournot model. From Proposition 3.4 we know that firms overinvest in R&D relative to the industry profit maximizing levels. An increase in collaboration makes investments cheaper and this has the perverse effect of making this overinvestment even greater and this leads to a fall in profits.

5   Concluding remarks

In R&D intensive industries, firms increasingly combine independent in-house research and bilateral collaborations with other firms to further their objectives. In this paper we have developed a model which accommodates these hybrid forms of R&D organization. The focus has been on the relationship between different forms of R&D activity and on understanding the circumstances under which a hybrid organization of R&D which combines the two is optimal for firms and society.

Our analysis has shown that independent in-house research and joint research are complementary activities for a firm. The performance of a hybrid form of organization in R&D depends on the level of concentration in the market. If the number of firms is small then hybrid forms of R&D organization are best for firms otherwise pure in-house research yields higher profits. However, social welfare is maximized under a hybrid organization of R&D in all cases.

We have also explored alternative modes of decision making among firms. We have found that non-cooperative firms make larger investments in R&D as compared to industry-profit maximizing firms and this has the consequence that social welfare is higher with non-cooperative firms. This result is in contrast to standard results in the existing literature and points to the importance of controls on spillovers. Perhaps a more natural cooperative regime in our setting is that of bilateral cooperation, that is, a mode of decision making by which the two partners involved in a collaboration project decide jointly the R&D investment to be put in the joint project, while keeping decision on in-house R&D private. We have found that bilateral cooperation yields greater welfare than non-cooperative behavior in concentrated industries, and greater than the benchmark case of full cooperation in all cases.

6 Appendix

Proof of Proposition 3.1: The first-order conditions imply that at interior solution x^* the following equality is necessarily satisfied:

$$\frac{\partial q_i}{\partial x_{ii}}(g^k, x^*) = \frac{\partial q_i}{\partial x_{ij}}(g^k, x^*), \quad j \in N_i(g^k). \tag{36}$$

Substituting expressions $(1) - (2)$ into (3), and (3) into the formula (36) and invoking symmetry gives

$$\frac{n}{\sqrt{x_{ii}^*}} = \frac{(n-1)\sqrt{2+\beta}}{2\sqrt{x_{ij}^*}},$$

or

$$\frac{x_{ij}^*}{x_{ii}^*} = \frac{(2+\beta)(n-1)^2}{4n^2}. \tag{37}$$

Solving the system of the first-order conditions, one obtains the levels of R&D expenditures at the interior symmetric solution:

$$x_{ii}^* = \left(\frac{(A-\bar{c})n\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(1+\beta/2)}\right)^2, \tag{38}$$

$$x_{ij}^* = \frac{1}{2+\beta}\left(\frac{(A-\bar{c})(n-1)(1+\beta/2)\gamma}{\gamma^2(n+1)^2 - n - k(n-1)(1+\beta/2)}\right)^2. \tag{39}$$

Additionally, one can observe that if γ is large enough ($\gamma > \sqrt{2n(2+\beta)+4}$), then the Hessian matrix of the system at x^* has the diagonal domination property, which implies that x_i^* is a local maximizer for every i. However, it does not imply that x_i^* maximizes the

profit function globally, since the latter is not necessarily quasi-concave in the firm's own strategy.

Suppose that strategies of all firms other than i are given by the expressions $(12) - (13)$, that is $x_{tt} = x_{ii}^*$ for every $t \in N$, $t \neq i$, and $x_{st} = x_{ij}^*$ for every $st \in g^k$ such that $s \neq i$. We are going to prove that playing $x_i^* = (x_{ii}^*, (x_{ij}^*)_{j \in N_i(g^k)})$ is the only optimal strategy of i.

Since $\lim_{\|x_i\| \to +\infty} \pi_i(x_i, x_{-i}^*) = -\infty$, the profit function $\pi_i(x_i, x_{-i}^*)$ is negative outside some compact set $K \subset \mathbb{R}_+^{k+1}$. Moreover, it never reaches its maximum on the boundary of ith strategy set $\mathbb{R}_+^{k+1}$. Therefore, $\pi_i(x_i, x_{-i}^*)$ has at least one interior maximum $y_i = (y_{ii}, (y_{ij})_{j \in N_i(g^k)}) >> 0$. Conditions $(8) - (9)$ imply that $y_{ij} = y_{im}$ for every $j, m \in N_i(g^k)$. Let

$$\delta = \frac{\sqrt{y_{ij}}}{\sqrt{x_{ij}^*}}.$$

Then it follows from the equality

$$\frac{\partial \pi_i}{\partial x_{ii}}(g^k, y_i, x_{-i}^*) = \frac{\partial \pi_i}{\partial x_{ij}}(g^k, y_i, x_{-i}^*)$$

that

$$\sqrt{y_{ii}} = \frac{2n\delta\sqrt{1 + \beta\delta + \delta^2}}{(n-1)(2\delta + \beta)}\sqrt{x_{ij}^*}. \tag{40}$$

The first order condition (8) transforms into

$$\frac{n}{\gamma(n+1)^2}\frac{1}{\sqrt{y_{ii}}}\left(A - \bar{c} + \frac{n}{\gamma}\sqrt{y_{ii}} + \frac{k(n-1)}{\gamma}\sqrt{x_{ij}^*}\sqrt{1 + \delta^2 + \delta\beta} - \right.$$
$$\left. - \frac{n-1}{\gamma}\sqrt{x_{ii}^*} - \frac{k(n-2)}{\gamma}\sqrt{x_{ij}^*}\sqrt{2 + \beta}\right) - 1 = 0.$$

We will show that if γ is large enough, then this equation has only one solution $\delta = 1$ and, therefore, $y_{ij} \neq x_{ij}^*$ is impossible. Taking into account (37) and (40) one obtains:

$$\frac{\gamma(n+1)^2}{n} = \frac{(n-1)(2\delta + \beta)(A - \bar{c})}{2n\delta\sqrt{1 + \beta\delta + \delta^2}}\frac{1}{\sqrt{x_{ij}^*}} + \frac{n}{\gamma} + \frac{k(n-1)^2(2\delta + \beta)}{2\gamma n\delta} -$$
$$- \frac{(n-1)(2\delta + \beta)}{\gamma\delta\sqrt{2 + \beta}\sqrt{1 + \beta\delta + \delta^2}} - \frac{k(n-1)(n-2)(2\delta + \beta)\sqrt{2 + \beta}}{2n\gamma\delta\sqrt{1 + \beta\delta + \delta^2}}.$$

After the substitution of x_{ij}^* this expression transforms to

$$\left(\frac{\gamma(n+1)^2}{n} - \frac{n}{\gamma}\right)\left(\frac{2\delta+\beta}{\delta\sqrt{2+\beta}\sqrt{1+\delta^2+\beta\delta}} - 1\right) - \frac{k(n-1)^2}{2n\gamma}\frac{2\delta+\beta}{\delta}\left(\frac{\sqrt{2+\beta}}{\sqrt{1+\delta^2+\beta\delta}} - 1\right) = 0.$$

This equation can be rewritten as

$$\left(\frac{\gamma(n+1)^2}{n(2+\beta)} - \frac{n}{\gamma(2+\beta)} - \frac{k(n-1)^2}{2n\gamma}\right)\left(\frac{(2\delta+\beta)\sqrt{2+\beta}}{\delta\sqrt{1+\delta^2+\beta\delta}} - (2+\beta)\right) + \qquad (41)$$

$$\frac{k(n-1)^2}{2n\gamma}\left(\frac{2\delta+\beta}{\delta} - (2+\beta)\right) = 0. \qquad (42)$$

First, note that $\delta = 1$ solves this equation. Second, note that the first factors of the first and the second summation terms are positive (if γ is large enough) and independent of δ while the second factors are strictly decreasing functions of δ. Thus the L.H.S is strictly decreasing with respect to δ. It then follows that $\delta = 1$ is the unique solution and thus $y_{ij} = x_{ij}^*$. $\square$

Proof of Proposition 3.4: The first order conditions of the maximization problem for industry profits are as follows:

$$\frac{\partial \Pi(\cdot)}{\partial x_{ii}} = \sum_{j=1}^{N} \frac{\partial \pi_j(\cdot)}{\partial x_{ii}} = 0, \ i \in N$$

$$\frac{\partial \Pi(\cdot)}{\partial x_{ij}} = \sum_{m=1}^{N} \frac{\partial \pi_m(\cdot)}{\partial x_{ij}} = 0, \ i \in N, \ j \in N_i(g)$$

In an interior and symmetric solution x^{FC} the first order conditions simplify to:

$$\frac{q_i(\widetilde{x}, g)}{\gamma(n+1)\sqrt{x_{ii}^{FC}}} - 1 = 0 \qquad (43)$$

$$\frac{q_i(\widetilde{x}, g)\sqrt{2+\beta}}{\gamma(n+1)\sqrt{x_{ij}^{FC}}} - 1 = 0 \qquad (44)$$

Invoking symmetry and solving for the joint-profit maximizing R&D investments yields (19) and (20).

We turn next to a comparison between equilibrium investments and the industry-profit maximizing levels. First, we take investments in own projects. Using (12) and (19) we can establish the comparison

$$\frac{\sqrt{x_{ii}^*} - \sqrt{x_{ii}^{FC}}}{\gamma(A - \bar{c})} = \frac{2n}{2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)} - \frac{1}{\gamma^2(n+1)^2 - k(2+\beta) - 1}$$

$$= \frac{(n+1)\left[2\gamma^2(n^2 - 1) - k(2+\beta)\right]}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))(\gamma^2(n+1)^2 - k(2+\beta) - 1)}$$

$$\underset{\text{large } \gamma}{>} 0.$$

Similarly, for joint projects using (13) and (20) we get

$$\frac{\sqrt{x_{ij}^*} - \sqrt{x_{ij}^{FC}}}{\gamma(A - \bar{c})\sqrt{2+\beta}} = \frac{n-1}{2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)} - \frac{1}{\gamma^2(n+1)^2 - k(2+\beta) - 1}$$

$$= \frac{\gamma^2(n+1)^2(n-3) + n + 1}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))(\gamma^2(n+1)^2 - k(2+\beta) - 1)}$$

$$\underset{\text{large } \gamma}{>} 0.$$

The first order conditions corresponding to the problem of bilateral cooperation are as follows:

$$\frac{\partial \pi_i(\cdot)}{\partial x_{ii}} = 0, \ i \in N,$$

$$\frac{\partial \pi_i(\cdot) + \partial \pi_j(\cdot)}{\partial x_{ij}} = 0, \ ij \in g^k.$$

In an interior and symmetric solution x^{BC} these equations can be rewritten as

$$\sqrt{x_{ii}^{BC}} = \frac{\gamma n(A - \bar{c}) + nk\sqrt{2+\beta}\sqrt{x_{ij}^{BC}}}{\gamma(n+1)^2 - n}, \quad i \in N, \tag{45}$$

$$\sqrt{x_{ij}^{BC}} = \frac{(A - \bar{c} + \frac{1}{\gamma}\sqrt{x_{ii}^{BC}})(n-1)}{\frac{\gamma(n+1)}{\sqrt{2+\beta}} - \frac{k}{\gamma}(n-1)\sqrt{2+\beta}}, \quad ij \in g. \tag{46}$$

Solving this system of equations gives us expressions (21) and (22).

Using (12), (13), (21), and (22) we obtain

$$\frac{\sqrt{x_{ii}^{BC}} - \sqrt{x_{ii}^*}}{(A - \bar{c})n\gamma} =$$

$$= \frac{(2 + \beta)(n - 1)k}{(2n + k(-1 + n)(2 + \beta) - 2(1 + n)^2\gamma^2)(n + k(-1 + n)(2 + \beta) - (1 + n)^2\gamma^2)} \underset{\text{large } \gamma}{>} 0,$$

and

$$\frac{\sqrt{x_{ij}^{BC}} - \sqrt{x_{ij}^*}}{(A - \bar{c})(n - 1)\gamma\sqrt{2 + \beta}} =$$

$$= \frac{\gamma^2(n + 1)^2 - n}{(2n + k(-1 + n)(2 + \beta) - 2(1 + n)^2\gamma^2)(n + k(-1 + n)(2 + \beta) - (1 + n)^2\gamma^2)} \underset{\text{large } \gamma}{>} 0.$$

We can use these investment levels to compute:

$$\frac{kx_{ij}^{FC}}{x_{ii}^{FC} + kx_{ij}^{FC}} = \frac{1}{\frac{1}{k(2+\beta)} + 1} \tag{47}$$

and

$$\frac{kx_{ij}^{BC}}{x_{ii}^{BC} + kx_{ij}^{BC}} = \frac{1}{\frac{n^2}{k(n-1)^2(2+\beta)} + 1}. \tag{48}$$

Relation (25) follows immediately.

Finally, to show (26) it is sufficient to prove that a firm's profit in the case of bilateral cooperation

$$\pi_i(x^{BC}, g^k) = \frac{(A - \bar{c})^2\gamma^2(-k(-1 + n)^2(2 + \beta) + (n(-1 + \gamma) + \gamma)(n + \gamma + n\gamma))}{(n + k(-1 + n)(2 + \beta) - (1 + n)^2\gamma^2)^2}$$

is lower than the equilibrium profit (15). Notice that

$$\lim_{\gamma \to \infty} \gamma^2(\pi_i(x^*, g^k) - \pi_i(x^{BC}, g^k)) = \frac{k(A - \bar{c})^2(n - 1)(3n - 7)(2 + \beta)}{4(n + 1)^4},$$

which implies that the equilibrium profit is indeed higher if $n \geq 3$ and γ is large enough. $\square$

Proof of Proposition 3.5: The equilibrium welfare is given by (35). The welfare level attained under bilateral cooperation is

$$W(x^{BC}, g^k) = \frac{(A - \bar{c})^2 \gamma^2 (-2n^3 - 2k(-1+n)^2 n(2+\beta) + n(1+n)^2(2+n)\gamma^2)}{2(n + k(-1+n)(2+\beta) - (1+n)^2\gamma^2)^2},$$

and the welfare level under full cooperation is

$$W(x^{FC}, g^k) = \frac{(A - \bar{c})^2 n \gamma^2 (-2 - 2k(2+\beta) + (1+n)^2(2+n)\gamma^2)}{2(1 + k(2+\beta) - (1+n)^2\gamma^2)^2}.$$

First, notice that

$$\lim_{\gamma \to \infty} \gamma^2 (W(x^*, g^k) - W(x^{BC}, g^k)) = \frac{(A - \bar{c})^2 k(n-7)(n-1)n(\beta+2)}{4(n+1)^4}.$$

Therefore, for a sufficiently large γ, welfare under non-cooperative decision making is higher than under bilateral cooperation if $n > 7$ and lower if $n < 7$. If $n = 7$ then

$$\lim_{\gamma \to \infty} \gamma^4 (W(x^*, g^k) - W(x^{BC}, g^k)) = \frac{21(A - \bar{c})^2 k(\beta+2)(70 + 9k(\beta+2))}{524288},$$

so the equilibrium welfare is higher than that under bilateral cooperation. Furthermore,

$$\lim_{\gamma \to \infty} \gamma^2 (W(x^*, g^k) - W(x^{FC}, g^k)) = \frac{(A - \bar{c})^2 n(4(n-1) + k(n^2 - 9)(\beta+2))}{4(n+1)^4},$$

which means that welfare under non-cooperative behaviour is higher than under full cooperation if $n \geq 3$ and γ is large enough.

Finally,

$$\lim_{\gamma \to \infty} \gamma^2 (W(x^{BC}, g^k) - W(x^{FC}, g^k)) = \frac{(A - \bar{c})^2 n(n-1 + 2k(n-2)(\beta+2))}{(n+1)^4},$$

which shows that the welfare attained under bilateral cooperation is always greater than that under full cooperation. $\square$

Welfare maximizing R&D investments: The first order conditions of the social welfare maximization problem are:

$$\frac{\partial W(\cdot)}{\partial x_{ii}} = Q(g) \left[\sum_{j=1}^{N} \frac{\partial q_j(\cdot)}{\partial x_{ii}} \right] + \sum_{j=1}^{N} \frac{\partial \pi_j(\cdot)}{\partial x_{ii}} = 0, \ i \in N$$

$$\frac{\partial W(\cdot)}{\partial x_{ij}} = Q(g) \left[\sum_{j=1}^{N} \frac{\partial q_j(\cdot)}{\partial x_{ij}} \right] + \sum_{j=1}^{N} \frac{\partial \pi_j(\cdot)}{\partial x_{ij}} = 0, \ i \in N, \ j \in N_i(g)$$

In an interior and symmetric solution $\widehat{x}$ the first order conditions simplify to:

$$\frac{q_i(\widehat{x}, g)(n+2)}{2\gamma(n+1)\sqrt{\widehat{x}_{ii}}} - 1 = 0 \tag{49}$$

$$\frac{q_i(\widehat{x}, g)(n+2)\sqrt{2+\beta}}{2\gamma(n+1)\sqrt{\widehat{x}_{ij}}} - 1 = 0 \tag{50}$$

Applying symmetry and solving for welfare maximizing R&D investments yields (27) and (28).

We turn next to a comparison of the equilibrium values with the social welfare maximizing values. We start with the case of own projects. Using (12) and (27) we establish the comparison

$$\frac{\sqrt{x_{ii}^*} - \sqrt{\widehat{x}_{ii}}}{(A - \overline{c})\gamma} = \frac{2n}{2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)} - \frac{n+2}{2\gamma^2(n+1)^2 - (n+2)(1+k(2+\beta))}$$

$$= \frac{(n+1)(2\gamma^2(n+1)(n-2) - k(n+2)(2+\beta))}{(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta))\, 2\gamma^2(n+1)^2 - (n+2)(1+k(2+\beta))}$$

$$\underset{\text{large } \gamma}{>} 0.$$

Therefore, from the point of view of the social planner, firms invest too much in own projects.

Similarly, using (13) and (28), we can compare equilibrium investments in joint projects with the socially optimal levels.

$$\frac{\sqrt{\widehat{x}_{ij}} - \sqrt{x_{ij}^*}}{(A - \overline{c})\gamma\sqrt{2+\beta}} = \frac{n+2}{2\gamma^2(n+1)^2 - (n+2)(1+k(2+\beta))} - \frac{n-1}{2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)}$$

$$= \frac{(n+1)\left(6\gamma^2(n+1) - (n+2)\right)}{(2\gamma^2(n+1)^2 - (n+2)(1+k(2+\beta)))\left(2\gamma^2(n+1)^2 - 2n - k(n-1)(2+\beta)\right)}$$

$$\underset{\text{large } \gamma}{>} 0$$

Therefore, from the point of view of the social planner, firms invest insufficiently in joint R&D projects.

Investment in own project is excessive while investment in joint projects is insufficient relative to the social optimum. These observations taken together imply imply that the proportion of resources devoted to the own project is excessive. Finally, we note that for a given network g^k (except for $k = 0$), the comparison between $x_{ii}^* + kx_{ij}^*$ and $\widehat{x}_{ii} + k\widehat{x}_{ij}$ does not yield clear-cut results. $\square$

Proof of existence and uniqueness of equilibrium if $\beta = 2$: In this case, $f_{ij}(x) = \frac{1}{\gamma}(\sqrt{x_{ij}} + \sqrt{x_{ji}})$, and the first order conditions (8) and (9) transform into

$$\frac{\partial \pi_i}{\partial x_{ii}}(x) = \frac{q_i(g, x)n}{\gamma(n+1)\sqrt{x_{ii}}} - 1 = 0, \quad i \in N, \tag{51}$$

and

$$\frac{\partial \pi_i}{\partial x_{ij}}(x) = \frac{q_i(g, x)(n-1)}{\gamma(n+1)\sqrt{x_{ij}}} - 1 = 0, \quad ij \in g. \tag{52}$$

After the substitution of variables $z_{ii} = \sqrt{x_{ii}}$, $z_{ij} = \sqrt{x_{ij}}$, the system $(51) - (52)$ becomes linear. If γ is sufficiently large, the matrix of this system has the diagonal domination property, which guarantees the existence and uniqueness of the equilibrium. $\square$

Proof of Proposition 3.6 We need to compare profits of firm i in the empty network g^e:

$$\pi_i(g^e) = \frac{4\gamma^2(\gamma^2(n+1)^2 - n^2)}{(2(n+1)^2\gamma^2 - 2n)^2},$$

with profits in the network $g^e + ij$ for some $j \in N$, $j \neq i$:

$$\pi_i(g^e + ij) = \frac{(-1 + 2n - 2n^2 + (1+n)^2\gamma^2)(n\gamma - (1+n)\gamma^3)^2}{(n(-2 + 3n) - (2 + n^2 + 3n^3)\gamma^2 + (1+n)^3\gamma^4)^2}.$$

It is possible to show that $\lim_{\gamma \to +\infty} \gamma^2(\pi_i(g^e + ij) - \pi_i(g^e)) = \frac{3(n-1)^2}{(n+1)^4} > 0$. Moreover, it is easy to find out that $\pi_i(g^e + ij) > \pi_i(g^e)$ for all γ that are sufficiently large to guarantee non-negativity of subgame solutions.

We now prove the stability of the complete network. The profits under the complete network g^c are:

$$\pi_i(g^c) = \frac{\gamma^2(-4(n-1)^3 + 4(-n^2 + (1+n)^2\gamma^2))}{(-4(-1+n)^2 - 2n + 2(1+n)^2\gamma^2)^2}.$$

If two firms i and j cancel their collaboration project, they obtain a profit:

$$\pi_i(g^c - ij) = \frac{\gamma^2(2 + (-2+n)n - (1+n)\gamma^2)^2(2 - 5n + 3n^2 - n^3 + (1+n)^2\gamma^2)}{(12 - 28n + 26n^2 - 11n^3 + 2n^4 - (2 + 3n + n^4)\gamma^2 + (1+n)^3\gamma^4)^2}.$$

It is easy to see that $\lim_{\gamma \to +\infty} \gamma^2(\pi_i(g^c) - \pi_i(g^c - ij)) = \frac{3(n-1)^2}{(n+1)^4} > 0$, which shows that g^c is stable for all sufficiently large γ. Consider a polynomial $\mathcal{P}(n, \gamma) = A(n, \gamma)B'(n, \gamma) - B(n, \gamma)A'(n, \gamma)$, where $A(n, \gamma)$ and $A'(n, \gamma)$ are, respectively, the numerator and the denominator of the expression for $\pi_i(g^c)$ and $B(n, \gamma)$, $B'(n, \gamma)$ are those of the expression for $\pi_i(g^c - ij)$. Consider a function

$$F(n) = \max\{\gamma \mid \mathcal{P}(n, \gamma) = 0\}.$$

It is possible to show that the value of $x_{ii}(g^c - ij)$ is negative for any n whenever $\gamma = F(n)$. This implies that the the complete network g^c is stable for any relevant values of parameters n and γ. $\square$

Proof of Proposition 4.2: We first note that in the perfect spillovers model, the profit level in the empty network is:

$$\pi_i^{ps}(g^{k=0}) = \frac{(A - \bar{c})^2 \gamma^2 [\gamma^2(n+1)^2 - n^2]}{[\gamma^2(n+1)^2 - n]^2}. \tag{53}$$

Next note that the profits under perfect spillovers in the complete network are:

$$\pi_i^{ps}(g^{k=n-1}) = \frac{(A - \bar{c})^2 \gamma^2 [\gamma^2(n+1)^2 - 1]}{[\gamma^2(n+1)^2 - n]^2}. \tag{54}$$

It can be checked that $\pi_i^{ps}(g^{k=0}) < \pi_i^{ps}(g^{k=n-1})$. From Proposition 4.1 we know that profits are initially increasing and eventually decreasing. These observations imply that $\pi_i^{ps}(g^{k=0}) < \pi_i^{ps}(g^k)$, for any $k \geq 1$.

We now turn to the profits in the zero spillovers model. Here we find that:

$$\pi_i(g^{k=0}) = \frac{(A - \bar{c})^2 \gamma^2 [\gamma^2(n+1)^2 - n^2]}{(\gamma^2(n+1)^2 - n)^2} \tag{55}$$

From above equations it follows that $\pi_i(g^{k=0}) = \pi_i^{ps}(g^{k=0})$. We know from Proposition 3.3 that individual profits are declining in k. Putting together the above observations on $\pi_i(g^k)$ and $\pi_i^{ps}(g^k)$ then gives us the desired result. $\square$

Proof of Proposition 4.3: The equality $W(g^0) = W^{ps}(g^0)$ can be checked directly. Moreover, one can verify that $W(g^{n-1}) > W^{ps}(g^{n-1})$ and $(W(g^k) - W^{ps}(g^k))''_k > 0$ for all γ sufficiently large. The latter condition implies that the difference $W(g^k) - W^{ps}(g^k)$ is strictly convex in k. There are two possible cases. First, assume that $W(g^k) - W^{ps}(g^k)$ is decreasing in the neighborhood of 0. One can find $k' \in (0, n-1)$ such that $W(g^{k'}) = W^{ps}(g^{k'})$. Such k' is unique by the convexity condition. It is sufficient to take the integer part of k' as a cut-off level of collaboration, $k^* = [k']$. The second case, $W(g^k) - W^{ps}(g^k)$ is increasing in the proximity of 0, is analogous. By continuity and convexity, $k^* = 0$. $\square$

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