Games Played in a Contracting Environment*

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Abstract

We analyze situations where a player must contract with the monopoly supplier of an essential input in order to play an action in a strategic form game. Supplier monopoly power does not distort the equilibrium distribution over player actions under private contracting, but may dramatically affect the equilibrium actions under public contracting. When a player randomizes between actions, suppliers for the different actions behave as though they are producing perfect substitutes when contracts are private; when contracts are public, it is as though they are producing perfect complements.

Keywords: multi-party contracting, mixed strategy equilibria, marginal contribution, perfect substitutes, perfect complements.

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1 Introduction

This paper belongs to the intersection of monopoly price theory and game theory. Monopoly or oligopoly price theory traditionally assumes that buyers are not engaged in any strategic interaction. Given the prices quoted by competing sellers, a buyer solves a simple *decision* problem, choosing the product that maximizes her utility. When preferences and costs are common knowledge, the prediction of the theory is also straightforward. First, monopoly power *does not distort decisions*, as compared to the competitive benchmark – if the buyer seeks to purchase one of several competing goods, price competition between the sellers will ensure that she chooses the option or product that maximizes social surplus (the difference between her valuation and the cost of production). Second, the seller of this product earns profits that equal his *marginal contribution* to social welfare, that is the difference the between social surplus from the consumption of this product, and that arising from the consumption of the next best option.

Buyers are often engaged in strategic interaction, and the utilities of the different options are not fixed, but depend upon the actions of other agents. This is especially true in the market for intermediate goods, where the buyer is often a firm that is engaged in strategic interaction with other firms. For example, for a firm that is considering buying advertising services, the payoff to advertising may also depend upon whether or not a competing firm also advertises. This paper investigates how the two main insights above – no distortion, and profits equal marginal contribution — generalize when the agent is involved in a game.

The paper may also be seen as addressing a lacuna in economic applications of game theory, where it is standard to treat a game in isolation from the wider economic environment in which it is played. In most economic contexts a player has to transact with another party in order to take some action in the game. For instance, a firm that invests in order to deter entry will normally have to purchase one or more investment goods (from a *supplier* of investment goods) in order to make this investment. Treating the game in isolation from the wider economic environment is well justified when this environment is perfectly competitive, for in this case, the prices that must be paid by a player for any inputs required to take an action may be treated as exogenous. Thus payoffs associated with any action profile are therefore fixed. However, if supplier of an input required for an action possesses market power, then he will typically have some leeway in setting the prices. The consequences are two fold. First, the terms of contract dictated by suppliers will affect players' payoffs, and will therefore potentially affect the outcome of the game. Second, the action profile played will have payoff consequences for suppliers, who will in turn seek to influence these actions.

In this paper we analyze situations where players have to contract with suppliers in order to take actions in a strategic form game. We model this as an extensive form game where suppliers quote prices, and then players choose actions simultaneously – we call this a game played in a contracting environment. Our focus is on the *competition* between suppliers who provide inputs for distinct actions. To this end we assume that any supplier is involved with at most one action, for a single player.[1] Contracts can either be public (i.e. observed by all the players in the game) or private (observed only by contracting parties). In the case of private contracts, we provide a sharp characterization of equilibrium outcomes. Our main finding is that supplier market power does not alter the equilibrium distribution over player actions, as compared to a situation where market power is absent and inputs are supplied competitively. Furthermore, any supplier earns his marginal contribution to a player's payoff. We therefore find that with private contracts, the basic insights from non-strategic case, where the buyer solves a single person decision problem, generalize to the strategic context.

Our results in the case of public contracts are very different, since supplier competition has subtle and complex effects. Take for example the case where the underlying game (i.e. when prices equal cost) is the prisoners' dilemma, with a unique pure strategy equilibrium. We find that supplier monopoly power may result in very different outcomes, where the players randomize across different actions. These phenomena arise since a supplier to a player may be able to influence his *opponent's* behavior by the choice of price. In games where players play mixed strategies, competing suppliers for the same player may act partially as though they are producing complementary goods, and in part as though they are producing substitutes. This contrasts sharply with mixed equilibria in the private contracting case, where competition between suppliers is intense.

This paper may be seen as a contribution to the literature on multi-party contracting, which includes the literature on common agency (Bernheim and

[1] In contrast, the literature on strategic delegation assumes that a single supplier (the principal) has monopoly power over the entire action set of the player (the agent).

Whinston, [2]), and vertical contracting between a single principal and many agents (Hart and Tirole, [8], McAfee and Schwartz, [12], Segal [14]). Its connections are closest to the case where there are many agents as well as principals, as in Prat and Rustichini, [13], or Jackson and Wilkie [9], where players in the game may make side payments to other players). However, while these papers allow general forms of contracting between principals and agents, and focus on the question of whether contracting ensures efficiency, the "principals" in our context have rather more limited power, since they may only demand transfers in the event that the player chooses to take the action over which they have monopoly power. Furthermore, in our context the actions taken by the "agents" have no direct payoff consequences for principals, which is not the case in common agency models. As such, our focus is on how equilibrium outcomes in the presence of competing monopolists differ from equilibrium outcomes under perfect competition. It is also related to the literature on strategic delegation (Fershtman and Judd [4], Sklivas [17], Vickers [18], Fershtman et. al. [5]), which emphasizes that delegation with public contracts allowed a principal to secure favorable outcomes. Katz [10], Fershtman and Kalai [6] and Kockesen and Ok [11] examine the implications of private contracting in this context. In the strategic delegation context, an agent requires to contract with the principal to take *any* action in the game, the principal effectively has monopoly power over all the actions. In contrast, we have focus on the situation where any supplier has monopoly power over only a single action, and thus supplier competition plays an important role.

The remainder of this paper is organized as follows. Section 2 sets out the basic model of a game played in a contracting environment. Section 3 analyzes private contracting, while section 4 focuses on public contracts. Section 5 discusses the robustness of our results and possible extensions, and the final section concludes.

2 The Model

A game played in a contracting environment is defined as follows. We will use the term player for someone who plays the game in question, and the term supplier to denote someone with whom a player may need to contract with in order to be able to adopt some strategy in the game. Let $I = \{1, 2, ..., n\}$ be the set of players and let each player i have a finite action set A_i, whose generic element will also be denoted by a_i^j or a_i. Let $A = \times_{i \in I} A_i$ be the set of action profiles, and let $g_i : A \to R$ be the *gross payoff* of player i. These gross payoffs at the profile $a = (a_i)_{i \in I}$ will in general differ from the usual (net) payoffs of a player since she may have to contract with a supplier in order to be able to play the action a_i. Let $\bar{A}_i \subset A_i$ be the set of actions for which the player needs a supplier. We shall assume that for any player i and any action $a_i^j \in \bar{A}_i$, there exists exactly one supplier, $\lambda(a_{ij})$. We may identify this supplier with the action, and with a slight abuse of terminology, call him supplier a_i^j or supplier ij. $\Lambda_i = \{\lambda(a_{ij})\}_{j \in \bar{A}_i}$ denotes the set of suppliers for player i. Let p_i^j denote the price which is charged by supplier $\lambda(a_{ij})$ for enabling the action a_i^j, and let $p_i = (p_i^j)_{j \in \bar{A}_i}$. If $a_i^j \notin \bar{A}_i$ we set the price of this action, p_i^j, to zero. The net payoff at the profile $a = (a_i^j, a_{-i})$ where a_{-i} is the vector of actions of players $h \neq i$, is given by

$$u_i(a, p_i) = g_i(a_i^j, a_{-i}) - p_i^j. \tag{1}$$

If player i plays action j and $a_i^j \in \bar{A}_i$, the payoff to supplier ij is given by $p_i^j - c_i^j$, where c_i^j is the cost of supplying this action. If the player does not play action a_i^j, the payoff to the supplier is zero. Let us normalize prices and gross payoffs by measuring them net of cost, so that a zero price corresponds to pricing at cost. Henceforth, the gross payoff $g_i(a_i, a_{-i})$ will denote the payoff when player pays the cost of action a_i. We extend, in the usual way, the gross payoff function g_i to mixed action profiles: $g_i(\alpha_i, \alpha_{-i})$ is the payoff to $\alpha_i \in \Delta(A_i)$ when $\alpha_{-i} \in \times_{h \neq i} \Delta(A_h)$ is the vector of (mixed) actions played by the other players.

In comparison with common agency models [2] or games played through agents (Prat and Rustichini [13]), our formulation differs in two respects. First, the actions taken by the players have no payoff consequences for suppliers whereas principals directly care about the actions taken by agents in common agency models. This implies that the suppliers in our setting have no reason to make transfers to the agents. Their monopoly power allows them to *demand* transfers, but this power is only relevant in the event that a player takes the monopolized action. In contrast, in common agency models, principals are assumed to have no direct monopoly power − the agent can always take any action without making any payment to any principal. One assumption in our formulation deserves comment. The payment to supplier $\lambda(a_i^j)$ is contingent only on whether the action a_i^j has been played or not. In particular, it cannot be made contingent upon the entire action profile that is played. This reflects the assumption that in the relation between player i

and supplier $\lambda(a_i^j)$, it is verifiable whether action a_i^j has been enabled by the latter or not – in particular, some physical good may need to be delivered by $\lambda(a_i^j)$ in order to enable a_i^j. On the other hand, the action profile that is played is assumed not to be verifiable, preventing transfers from being conditioned on the profile.

A point of reference, before we proceed to analysis of the extensive form games, is the normal form game $G =< I, A, g >$, i.e. where all actions are supplied at cost, and players net payoffs equal their gross payoffs. The payoffs we write down, in the various examples, will be of the game G. Let E^G denote the set of Nash equilibria of G. Let $\alpha = (a_i^j, a_{-i}) \in E^G$, where player i plays the pure action a_i^j. Define $\delta_i(\alpha) = g_i(\alpha) - \max_{k\neq j} g_i(a_i^k, a_{-i}^j)$ as player i's deviation loss from his pure action at α. If player i plays a mixed action at $\alpha, \delta_i(\alpha) = 0$. If a supplier is needed in order to take action a_i^j, $\delta_i(\alpha)$ may be thought of as the *marginal contribution* made by (active) supplier ij to player i's gross payoff at the profile α. These marginal contributions will play an important role in our characterization results.

We have two possible extensive forms to analyze: private contracts and public contracts. The contracting game with *private contracts*, Γ^{pvt}, is as follows:

1. Each supplier in $\Lambda = \cup_{i\in I}\Lambda_i$ quotes a price for each input that he supplies.

2. Each player i observes the price vector p_i (but not the prices quoted to other players), and players simultaneously choose actions $a_i \in A_i$.

3. Players receive the net payoffs as defined above, and suppliers receive their payoffs.

If we replace (2) above so that every player observes $(p_i)_{i\in I}$, the prices quoted to all, we have a game with *public contracts,* which we can call Γ^{pub}.

In either game, a (pure) strategy for a supplier is a price, i.e. a real number. In the game with private contracts, a pure strategy for player i is a function $s_i : R^{m_i} \rightarrow A_i$, where m_i is the cardinality of Λ_i.In the game with public contracts, a pure strategy is a map $s_i : \times_j R^{m_j} \rightarrow A_i$. Both these games are an instance of a *continuum extensive form game,* in the terminology of Simon and Stinchcombe [15].We will restrict attention to perfect Bayesian equilibria of these extensive form games. In the case of public contracts, these are the same as subgame perfect equilibria. In addition, we want to rule out "unreasonable equilibria", where inactive suppliers (i.e. those who do not make a sale) choose strictly negative prices. Such equilibria are sometimes called *cautious,* and can be ruled out by considerations of trembling hand perfection.[2] So henceforth, by "equilibrium" we mean a cautious perfect Bayesian equilibrium where all supplier prices are non-negative.

3 Private Contracts

We make explicit our assumptions on the relation between players and suppliers. First, we assume that each supplier is a monopolist in the market for the action that he enables. This assumption is easily relaxed. Our other assumptions are as follows.

Assumption **A1.** (No Complementary Inputs): For any player i and any action a_i^j, no more than one supplier is required.

Assumption **A2**: Each player has at least two actions

If a player was to have only one action, the supplier enabling that action would have completely inelastic demand.

Assumption **A3.** A supplier supplies at most one player, i.e. Λ_i and Λ_j are disjoint if $i \neq j$.

This assumption plays an essential role in our analysis of private contracts, since it ensures that the beliefs of player i regarding the actions chosen by other players do not vary with the prices that player i is quoted by her supplier.

Assumption **A4.** A supplier supplies at most one action of any player, i.e. if $j \neq k, \lambda(a_i^j) \neq \lambda(a_i^k)$.

This assumption is less essential for our results and is made mainly for expositional convenience.

Restricting attention to deterministic price choices by suppliers,[3] a strategy profile in Γ^{pvt} consists of a pair $(\hat{p}, \sigma)$, where $\hat{p}$ is the vector of prices chosen by all suppliers, and for each player $i, \sigma_i : R^{m_i} \to \Delta(A_i)$. The (mixed) action profile which is played under this strategy profile is $\sigma(\hat{p}) = (\sigma_i(\hat{p}_i))_{i \in I}$, and is called the *action outcome* of this profile. Clearly, the set of action

[2]More precisely, we can discretize the price space, and consider trembling hand perfect equilibria of the discretized game. We may restrict attention to equilibria of the continuum game which are limit points of a sequence of trembling hand equilibria of discrete games as the grid of prices becomes increasingly finer. It is easy to see that any equilibrium with negative prices will not be a limit of such trembling hand perfect equilibria.

[3]This assumption does not appear to be essential for our results in the case of private contracts, but simplifies notation and exposition. However, in the case of public contracts, supplier randomization enlarges strategic possibilities significantly by enabling players to correlate their actions.

outcomes of Γ^{pvt} coincide with the set of mixed action profiles in G, i.e. $\sigma(\hat{p}) \in \times_i \Delta(A_i)$. Let $\Omega^{\Gamma^{pvt}}$ denote the set of equilibrium action outcomes of Γ^{pvt}. Let $\sigma_{-i}(\hat{p}) \in \times_{h \neq i} \Delta(A_h)$ denote the beliefs of player i regarding the actions taken by the other players. By assumption A3, the beliefs of player i regarding the actions of other players do not vary if the prices that he faces, p_i, change. Also, for player i, only the prices he faces (p_i) and the actions of other players are payoff relevant. The prices paid by other players are payoff irrelevant.

The following theorem is the main result of this section.

Theorem 1 $\Omega^{\Gamma^{pvt}} = E^G$*, the equilibrium action outcomes of* Γ^{pvt} *and the equilibria of* G *coincide. Let* $(\hat{p}, \sigma)$ *be an equilibrium of* Γ^{pvt} *with action outcome* α*. If* α_i *assigns probability one to action* a_i^j*, then* $\hat{p}_{ij} = \delta_i(\alpha)$*. If* α_i *assigns positive probability to more than one action, then* $\hat{p}_{ij} = 0$ *for every active supplier* ij*.*

Proof. See Appendix. ■

If we allowed for equilibria with negative prices, the only difference that is that when i plays a pure action, $\hat{p}_{ij}$ could be any number in the interval $[0, \delta_i(\alpha)]$.

Although the monopoly power of suppliers allow them to charge non-negative prices, thereby causing net payoffs to differ from gross payoffs, the theorem shows that this does not alter the equilibrium distribution over action profiles in Γ^{pvt}. Furthermore, when the profile α is played at an equilibrium of Γ^{pvt}, supplier ij gets a payoff which equals his marginal contribution to player i's gross payoff at α. A supplier's marginal contribution is generically positive when a pure action is played, but equals zero for any mixed action.

	L	H
L	$1,1$	$0,0$
H	$0,0$	$2,2$

Fig. 1: A Coordination Game

The example in Fig. 1, a coordination game, will illustrate our results. Let us consider an equilibrium where (L, L) is played. The equilibrium prices faced by a player, say the row player, must satisfy $p(L) = 1$ and $p(H) = 0$.Thus the prices (which equal profits) of the supplier $1L$ to the row player equals 1. The net payoff of the row player is therefore zero. Note that prices are such that H weakly dominates L. Nevertheless, this equilibrium is a limit of perfect equilibria of the discretized version of the game — with discrete prices, equilibrium prices will be such that each player has a strict incentive to play L.[4] If we consider any equilibrium where the players randomize between their two actions, then all prices must equal zero, and hence the each player must play his first action with probability $\frac{2}{3}$. The net payoffs of the players equals $\frac{2}{3}$, and hence the players benefit by "not-coordinating" and playing the mixed equilibrium of G, since this improves their position vis-a-vis the supplier.

We have established that if player i randomizes between any two actions, say j and k, in an equilibrium of Γ^{pvt}, then the prices paid to these suppliers is zero. Suppliers of different actions act as though there are producing goods which are perfect substitutes. We call this condition a *generalized indifference principle for mixed strategies* with private contracts, since the suppliers as well as the player are indifferent between his choice of actions. On the other hand, when player i plays action a_i^j in a pure strategy equilibrium $(\hat{p}, \sigma)$, supplier ij will in general make positive profits. The profits that ij makes equal $\delta_i(a^j)$, which will generically be strictly positive. When we consider more complicated extensive form games, this distinction between the incentive constraints faced by a supplier at a pure action profile and the constraints faced at a mixed action profile has subtle effects. In particular, it implies that supplier incentives may be qualitatively different at player action profiles that are arbitrarily close together. The implications of this are explored in a companion paper (Bhaskar [3]). This paper shows that shows that imperfect observability generates a failure of lower-hemicontinuity of equilibrium outcomes in extensive games that are played in a contracting environment.

[4]Simon and Stinchcombe [15] develop a theory of equilibrium refinement for infinite normal form games, and argue that a minimal condition for reasonableness in such games is *limit admissibility*, i.e. that equilibrium strategies should be limits of admissible (undominated) strategies. In our contracting extensive form game, limit admissibility in conjunction with sequentiality precludes existence. Although the coordination game example possesses a limit admissible equilibrium, where the mixed strategy equilibrium is played, this will not be the case in general, as can be seen from the example of the prisoner's dilemma (see Fig. 2, subsection 4.1), where $\ell > g_1$ and $\ell > g_2$.

4 Public Contracts

We now consider public contracts. Recall that with public contracts, a (pure) strategy for a supplier is a price, i.e. a real number. A mixed strategy for a player is a map $s_i : \times_j R^{m_j} \to \Delta(A_i)$. We focus on subgame perfect equilibrium. As before, we may require additionally that prices are non-negative even for inactive suppliers. Let Γ^{pub} denote the game with public contracts, and let $\Omega^{\Gamma^{pub}}$ denote the set of equilibrium action outcomes.

Our first concern is to ensure existence of equilibrium. Assumption A2 is no longer sufficient to ensure existence of equilibrium. We therefore replace it with the following:

Assumption **A5.** For every player i there exists an action a_i^0 such that no input is required to play this action.

This assumption ensures that the minimum payoff that any player in I can receive is bounded and given by $\min_{a_{-i}} g_i(a_i^0, a_{-i})$. [5]Therefore the maximum price that any supplier ij can charge and possibly make a sale is no greater than $\max_{a_{-i}} g_i(a_i^j, a_{-i}) - \min_{a_{-i}} g_i(a_i^0, a_{-i})$. Hence the strategy set for any supplier is in effect compact, since higher prices cannot be optimal. Let us now assume that there exists a public signal which is uniformly distributed on $[0, 1]$. We assume that suppliers cannot observe the realization of this random variable while choosing prices, but that players observe this before choosing their actions. It is easily verified that players' payoffs as well as supplier payoffs are continuous in the strategies. By the results of Simon and Zame [16] and Harris, Reny and Robson [7], there exists a subgame perfect equilibrium of this game. [6]

With public contracts, equilibrium outcomes are surprisingly complex, even in simple games such as the prisoners' dilemma. Our focus is on the question, under what conditions is an equilibrium of G an equilibrium action outcome of Γ^{pub}. When G has multiple equilibria, these conditions are easy to satisfy. The question is more interesting when G has a unique equilibrium. We therefore consider, in turn, games G with a unique pure strategy equilibrium, then those with a unique mixed equilibrium, and finally games

[5]We are unable to establish boundedness in the absence of A5 – the mixed strategy example in section 4.2 may help clarify why this is the case, although in fact A5 is not required for existence in this specific example.

[6]In the examples we consider in this paper, we do not have invoke a public randomization device in order to ensure existence.

with multiple equilibria.

4.1 Games with a unique pure strategy equilibrium

Let us consider first as an example, the prisoners' dilemma game below, where for each player, playing D requires a contracting with a supplier, but playing C does not. This simple game, when played in a public contracting context, displays a surprising complexity of equilibrium behavior.

	C	D
C	$1, 1$	$-\ell, 1+g_2$
D	$1+g_1, -\ell$	$0, 0$

Fig. 2: Prisoners' Dilemma: $g_1, g_2, \ell > 0$

Recall that theorem 1 ensures that in Γ^{pvt}, the game with private contracting, (D, D) must be played and each active supplier earns ℓ, his marginal contribution at (D, D). We now show that if $g_1 > \ell$ and $g_2 < \ell$, then there does not exist any equilibrium of Γ^{pub} where (D, D) is played. To see this, let suppose that (D, D) is played with probability one in an equilibrium. We must have $p_i \leq \ell$ for $i \in \{1, 2\}$, since otherwise playing D is not optimal. We now show that $p_2 = \ell$ if (D, D) is played. If $p_2 < \ell$, supplier 2 can increase his price, while still keeping it less than ℓ. In the induced subgame, player 1 finds it strictly optimal to play D if player 2 plays C, and is indifferent between C and D if player 2 plays D. Thus 1 can only play C if two plays D with probability one. Therefore, if two plays D with probability less than one, one must play D with probability one, in which case playing D is optimal for two. Thus in any equilibrium of induced subgame, two continues to play D with probability one, making supplier 2's price increase profitable.

We conclude therefore that any candidate equilibrium where (D, D) is played, we must have $p_2 = \ell$. However, if $p_2 = \ell$, supplier 1 has a profitable deviation. If he chooses $p_1 \in (\ell, g_1)$, then (D, D) is no longer an equilibrium. Indeed, in induced subgame, action D is weakly dominated for player 2, and in any equilibrium, player 1 must play D with probability one, while player 2 plays C with sufficiently high probability in order to make this optimal (since $\ell > p_1 > g_1$). Thus supplier 1 has a profitable price increase, since he can increase his price above ℓ without reducing the probability of a sale. We have therefore demonstrated that there does not exist an equilibrium where

(D, D) is played with probability one. Indeed, since no other action profile is a Nash equilibrium of G, there does not exist an equilibrium where any pure action profile is played.

We now show that there exists an equilibrium where sellers randomize between prices, inducing a correlated distribution over action profiles. First, notice that if seller 2 chooses the price ℓ, then seller 1 can choose a higher price, at g_1 or arbitrarily close to g_1, and in the resulting subgame, seller 2 makes a sale with probability arbitrarily close to zero. Seller 2's best response is therefore to choose a price of g_1, which ensures a sale for sure. On the other hand, if seller 2 chooses g_1, seller 1's cannot make a sale by choosing a price higher than ℓ; his best response is to choose ℓ, which ensures a sale with probability one. Each seller can achieve a payoff arbitrarily close to $\min\{\ell, g_i\}$ by choosing a price below this number, and therefore in any equilibrium, D must be played with positive probability by each player. We now show that there exists an equilibrium in mixed strategies, where both each seller randomizes between two prices, ℓ and g_i. The table in Fig. 3 specifies equilibrium actions by the two players for each price profile. Let π_i denote the probability that supplier i chooses ℓ. If $\pi_2 = \frac{\ell}{g_1}$ and $\pi_1 = \frac{g_2}{\ell}$, then each two supplier is indifferent between the two prices.[7]

	g_2	ℓ
ℓ	D, D	D, D
g_1	C, D	D, C

Fig. 3: Actions as a function of supplier prices

Although the prisoners' dilemma game G has a unique equilibrium (D, D), we find that this is not an equilibrium action outcome of Γ^{pub}. Indeed, equilibrium action outcome is a correlated distribution over the set of action pro-

[7]It is easy to verify that supplier prices other than g_i and ℓ are not optimal at this equilibirum. Since no player will play D if $p_i > \max\{g_i, \ell\}$, and will play D for sure if $p_i < \min\{g_i, \ell\}$, prices outside the interval are sub-optimal (notice is that if $p_i = \min\{g_i, \ell\}$, then D is played for sure in the equilibrium, regardless of the realization of p_j). Suppose that supplier 1 chooses a price $p_1 \in (\ell, g_1)$. If supplier 2 chooses g_2, then it is easily verified that D is weakly dominant for player 2, and that D cannot be played by player 1 in any equilibrium of this subgame. Thus p_1 and ℓ yield the same payoff of zero in this event. On the other hand, if supplier 2 chooses ℓ, then D must be played with probability one in this subgame, and therefore the price of ℓ yields more profits than any lower price in this event. Similar reasoning verifies that intermediate prices between g_2 and ℓ are dominated for supplier 2 as well.

files, since under this distribution all profiles other than (C, D) have strictly positive probability.

We now set out the conditions that must be satisfied for an unique pure equilibrium of G to be played in the game Γ^{pub}. Let a^* be a pure strategy Nash equilibrium of G. We say that the marginal contribution of supplier a_i^* is *maximal* at a^* if

$$a_{-i}^* \in \arg\max_{a_{-i} \in A_{-i}} \left\{ g_i(a_i^*, a_{-i}) - \max_{a_i \neq a_i^*} g_i(a_i, a_{-i}) \right\}.$$

Similarly, the marginal contribution of supplier a_i^* is *minimal* at a^* if

$$a_{-i}^* \in \arg\min_{a_{-i} \in A_{-i}} \left\{ g_i(a_i^*, a_{-i}) - \max_{a_i \neq a_i^*} g_i(a_i, a_{-i}) \right\}.$$

In our prisoners' dilemma example, the marginal contribution of supplier 1 is maximal at (D, D) if $\ell \geq g_1$, and it is minimal if $\ell \leq g_1$. Note that these definitions also apply when the player does not require to contract with any supplier in order to take action a_i^*. The following theorems provides sufficient conditions under which a pure strategy equilibrium of G is played in Γ^{pub}.

Theorem 2 *Let a^* be a pure strategy equilibrium of G, such that either i) for every player i, the marginal contribution of supplier $\lambda(a_i^*)$ is maximal at a^*, or ii) for every player i, the marginal contribution of supplier $\lambda(a_i^*)$ is minimal at a^*. Then there exists an equilibrium of Γ^{pub} with outcome a^*, where each active supplier earns his marginal contribution $\delta_i(a^*)$.*

Proof. See Appendix. ■

In the prisoners' dilemma, (D, D) is an equilibrium action outcome of Γ^{pub} when either $\ell \geq \max\{g_1, g_2\}$ or $\ell \leq \max\{g_1, g_2\}$. In the former case, the marginal contribution of each active supplier is minimal at (D, D), and the unique subgame perfect equilibrium has each active supplier earning his marginal contribution, ℓ. In the latter case, the marginal contribution of each active supplier is maximal at (D, D); consequently, there exist equilibria where active suppliers earn their marginal contribution ℓ, but there also exist equilibria where they earn payoffs in the interval $[g_i, \ell]$. [8]

[8] For any $\bar{p}_i \in [g_i, \ell\,]$, there exists an equilibrium where supplier i chooses $\bar{p}_i$ and (D, D) is played with probability one on the equilibrium path. A player chooses D as long as $p_1 \leq \bar{p}_1$ and $p_2 \leq \bar{p}_2$. If either supplier deviates by choosing a higher price, *both* players play (C, C).

The sufficient conditions required for the theorem are rather strong, since they require either that the marginal contributions of active suppliers are *maximal for all* suppliers, or that they are *all minimal*. Our prisoners' dilemma example shows why rather strong assumptions are required if a pure strategy equilibrium of G is to continue to be an equilibrium outcome of Γ^{pub}. Even if a profile a^* is a pure strategy equilibrium of G, there may be a supplier for action a_i^* who may be able to secure a higher payoff than his marginal contribution $\delta_i(a^*)$ by inducing some other player j to play a different action.

4.2 Games with a unique mixed equilibrium

Let us now consider the properties of equilibria of $\Gamma^{pub}$where a player randomizes between different actions, a_i^j and a_i^k. In the private contracts case, we saw that this lead to intense competition between the suppliers, ij and ik. This is very much as though these suppliers were supplying perfectly substitutable goods. In the case of public contracts, we show that things are dramatically different – it is as though the two suppliers are supplying *complementary goods.* To see this, let us consider the game in Fig. 4, matching pennies game where the row player has an outside option, OUT. We assume that $0 < b < a/2$ so that the game G has a unique mixed equilibrium where the player 1 (the row player) plays T and B each with probability one-half, while player 2 chooses both her actions with equal probability. In the corresponding game with public contracts, player 2 does not have to contract with anyone to play either L or R, so that net payoffs equal her gross payoffs. Player faces a single supplier in the case of action T and also in the case of action B, but can play OUT without contracting with anyone.

	L	R
T	$a, 0$	$0, 1$
B	$0, 1$	$a, 0$
OUT	$b, 0$	$b, 0$

Fig. 4: Matching Pennies with an Outside Option: $0 < 2b < a$.

We will discuss two different parameter configurations. First, we assume that $b \geq 0$, so that the outside option bites ($b \geq 0$). In this case, it is as though the two suppliers, of T and B, are producing a complementary inputs for single good – the randomization that mixes these two actions equally. Let

x, y denote the prices to be paid for playing T and B respectively. Let us exclude OUT from consideration for the moment. Now given this restricted subgame, given y, supplier T can make a sale for sure by choosing $x = y - a - \varepsilon$, and can therefore get a payoff arbitrarily close to $y - a$. Alternatively, T can also choose a price in the interval $(y - a, y + a)$, in which case the (restricted) subgame has a completely mixed equilibrium where T and B are played with equal probability and the column player plays L with probability $\pi = \frac{1}{2} + \frac{x-y}{2a}$. The expected payoff to row in this equilibrium equals $\frac{a-(x+y)}{2}$. This must be greater than b or else OUT will be better for row. Now, the probability with which T is played equals one-half, independent of the value of x or y, so long as $|x - y| \leq a$. This is so because this probability is such that column is kept indifferent between his two actions, and hence it does not depend upon the prices row pays. Hence supplier T seeks to choose x to maximize $\frac{x}{2}$ subject to the constraint $\frac{a-(x+y)}{2} \geq b$, and supplier B seeks to choose y to maximize $\frac{y}{2}$, also subject to the same constraint. Hence the equilibria are non-negative values of x, y such that

$$x + y = a - 2b > 0. \tag{2}$$

Thus we have a continuum of equilibria, where x and y satisfy the above equation, where x and y are subject to the further constraint that the payoff of each supplier is greater in this equilibrium than from making a sale for sure. Notice that for supplier T, the highest price for y in any such candidate equilibrium is $y = a - 2b$, at which supplier T makes a profit of zero at $x = 0$ by sharing the market. By undercutting, he can earn $y - a$, i.e. $-2b$, and therefore as long as $b \geq 0$, each element of this continuum is an equilibrium. To conclude, we have demonstrated that as long as $b \geq 0$, we have a continuum of equilibria $x, y \in [0, a - 2b]$, where the probability with which L is played varies between 0 and 1. This is very similar to the situation where there are two monopoly producers of perfect complementary goods – equilibrium prices must sum to the value to the consumer of the composite good. This example also illustrates that action profiles which are not an equilibrium of G may be sustained as equilibrium outcomes of Γ^{pub}. Let $x = y = a - 2b$. In this case, the resulting subgame has an equilibrium where row plays OUT and column chooses any mixed action. Each supplier chooses a high price since the price of the other supplier is so high that only a zero price can ensure a sale. This is an example of a inefficiency due to excessively high pricing by producers of complementary goods, in the mixed strategy context.

Let us now consider the second case, where the value to the outside option, b, is sufficiently negative that it becomes irrelevant. For expositional simplicity, let us assume that it no longer exists, i.e. $b = -\infty$. In this case, it is as though the goods offered by the two suppliers are partly complements but also partly substitutes. We show first that this game does not have an equilibrium where supplier prices are deterministic. To see this, note that supplier T can ensure himself a payoff arbitrarily close to $a/2$, by choosing a price equal to $a - \varepsilon$, since in this event play of T is not dominated, and T must be played with probability at least one-half. So in any deterministic price equilibrium, T and B must be played with probability one-half. But in this case, any ε-optimal best response for T is $x = y + a - \varepsilon$, and similarly $y = x + a - \varepsilon$, so that each supplier wants to price higher than his rival. So there cannot be a deterministic price equilibrium.

Let us now consider mixed price equilibria. Consider a symmetric mixed equilibrium where the price chosen by each supplier, x has support $[\underline{x}, \bar{x}]$, and continuous distribution F. The payoff to a supplier from choosing x is given by

$$U(x, F) = x[1 - F(x + a)] + \frac{x}{2}[F(x + a) - F(x - a)]. \tag{3}$$

$$\frac{\partial U}{\partial x} = 2 - [F(x+a) + F(x-a)] - x[F'(x+a) + F'(x-a)] = 0, x \in [\underline{x}, \bar{x}]. \tag{4}$$

The solution to this differential equation is given by

$$F(x + a) + F(x - a) = 2 + \frac{k}{x}, x \in [\underline{x}, \bar{x}]. \tag{5}$$

We show first that $\underline{x}$ and $\bar{x}$ must satisfy some inequality constraints. Let y denote my opponent's price. Since $y \geq 0$, the best response to it is either $x = y + a$ (if $y \leq 3a$) or $y - a$ (if $y \geq 3a$). Hence any rationalizable strategy must satisfy $x \geq a$. Thus $\underline{x} \geq a$.

Turning now to $\bar{x}$, we know that the payoff from $\bar{x}$ equals $\frac{\bar{x}}{2}[1 - F\ (\bar{x} - a)]$. But the payoff from $\bar{x} - 2a$ is no less than $(\bar{x} - 2a)[1 - F\ (\bar{x} - a)]$, so we must have $\frac{\bar{x}}{2} \geq \bar{x} - 2a$, or $\bar{x} \leq 4a$. Finally we must have $\bar{x} - \underline{x} \geq 2a$, since otherwise at the midpoint of the support, the payoff will be strictly increasing in x, and thus this cannot be optimal. Note that we have shown that $\bar{x} - \underline{x} \in [2a, 3a]$.

We now construct an equilibrium based on equation (5). Let $\underline{x} > a$, and let $\bar{x} = \underline{x} + 2a$. For $x \in [\underline{x}, \underline{x} + a]$, (5) implies that

$$F(x) = 1 - \frac{\underline{x} + a}{x + a}, x \in [\underline{x}, \underline{x} + a]. \tag{6}$$

For $[\underline{x} + a, \bar{x}]$, (5) implies that

$$F(x) = 2 - \frac{\underline{x} + a}{x - a}, x \in [\underline{x} + a, \bar{x}]. \tag{7}$$

Since (7) and (6) must yield the same value for $F(\underline{x} + a)$, we deduce that

$$1 - \frac{\underline{x} + a}{\underline{x} + 2a} = 2 - \frac{\underline{x} + a}{\underline{x}}. \tag{8}$$

This implies that

$$\underline{x} = a\sqrt{2}. \tag{9}$$

The payoff of the supplier in the equilibrium is given by

$$\frac{\underline{x} + a}{2} = \frac{a(1 + \sqrt{2})}{2}. \tag{10}$$

.

This completes the construction of the equilibrium. That is, we have an equilibrium with support $[a\sqrt{2}, a(\sqrt{2} + 2)]$, with the associated distribution given by equations (6) and (7). Indeed, we have also demonstrated that any equilibrium where $\bar{x} - \underline{x} = 2a$ coincides with our construction.

Note that the action outcome of this equilibrium is not the same as the mixed equilibrium of G. Indeed, it corresponds to a correlated distribution over action profiles in G. To see this, note that there is positive probability that the difference in prices between supplier T and supplier B is greater than a, in which case (B, L) is played. Similarly, the probability that this difference is less than $-a$ is also the same, in which case (T, R) is played. Finally, in the case that the difference lies between $-a$ and a, a mixed action profile is played, where the T and B are always played with probability one-half, while the probability of L depends upon the two prices.

We now show that this equilibrium is essentially unique within this class. Suppose that $\bar{x} = \underline{x} + 2a + \varepsilon$ where $\varepsilon \in (0, a]$ (we have already shown that $\bar{x} - \underline{x} \leq 3a$). Consider $y \in (x + a, x + a + \varepsilon)$. We will show that equation (5) implies two different values for $F(y)$, and therefore a contradiction. Consider first the indifference condition at $y - a$. By equation (5) we must have

$$F(y) = 2 + \frac{k}{y-a}. \tag{11}$$

On the other hand, the indifference condition at $y + a$ implies

$$F(y) = 1 + \frac{k}{y+a}. \tag{12}$$

Thus we must have

$$\frac{k}{y+a} = 1 + \frac{k}{y-a}, \forall y \in (x+a, x+a+\varepsilon). \tag{13}$$

This implies that $y^2 - a^2 + 2ak = 0$ for an interval of values y, which is clearly impossible.

To summarize, we have considered the case where G is a simple matching pennies game with an outside option, that has a unique mixed equilibrium. We find that equilibria in Γ^{pub} show interesting and complex behavior. When the outside option bites, the suppliers of the two actions act as though they are producing complementary goods that combine to form a single composite good, that must compete with the outside option. This gives rise to multiplicity of pricing equilibria and consequently, a continuum of equilibrium distributions over action profiles. In the case where the outside option does not bite, we see that suppliers act partially as though they are producing complementary goods, but also partially as producers of substitute goods. This arises since one supplier's price may be so high that it becomes profitable for the other to undercut this price sufficiently to take the entire market, i.e. make a sale with probability one. This mix between substitutes and complements precludes existence of a deterministic price equilibrium. We show that a mixed equilibrium exists where the distribution over prices is continuous.

Finally, let us consider the interaction between suppliers of different players in the context of this example. Assume now that there is a single supplier for T for the row player, and a single supplier for L for the column player, the other actions not requiring any contracting. Assume also that $b = -\infty$, i.e. the action OUT is not available for the row player. Let p be the price chosen by the supplier for T and q be the price for L. As long as $p < a$ and $q < 1$, the resulting subgame has a unique mixed equilibrium, where row plays T with probability $\frac{1-q}{2}$, and column plays L with probability $\frac{a+p}{2a}$. That is, the probability with which T is played does not depend upon the price of that action p, as long as it is less than a. Similarly, the supplier of B also faces completely inelastic demand as long as $q \leq 1$. We must therefore have $p = a, q = 1$, so that the resulting subgame is given by Fig. 5.

	L	R
T	$0, -1$	$-a, 1$
B	$0, 0$	$a, 0$
OUT	$b, 0$	$b, 0$

Fig. 5

To complete the description of subgame perfect equilibrium, (B, L) is played at this subgame. Notice that seller T does not make a sale; however, if he reduces price below a, then (B, R) is played, making any price reduction unprofitable. Thus we see that a pure profile (B, L) is played although this is not an equilibrium of the original game.

To summarize: we have considered a classic matching pennies type game and shown that supplier competition can yield complex results under public contracts. Specifically, suppliers of different actions taken by the same player act as though they are producing complements; this can be combined with elements of Bertrand style competition if one supplier's price is very high.

4.3 Games with multiple equilibria

If the game G has multiple equilibria, it is rather easy to ensure that any such equilibrium is played in a subgame perfect equilibrium of Γ^{pub}. For example, in the coordination game of Fig. 1, one can support the play of (L, L) at any arbitrary price $p_i(L) \leq 1$ (for $i = 1, 2$) by pricing strategies where $p(H) < 2$ (for $i = 1, 2$), where players choose (L, L) as long as neither L supplier has deviated, but switch to playing (H, H) in the event of a deviation. Similarly, the mixed equilibrium can be played if equilibrium prices are $p_i(L) = p_i(H) = k$,(for $i = 1, 2$) where k is some constant. If say one of the L supplier deviates, the players respond by choosing (H, H). The same logic allows one to support a wide range of prices, and therefore mixed action profiles that are not equilibria of the normal form game G. We therefore have the following result:

Theorem 3 *Let a^j and α be distinct equilibria of G, where a^j is pure and α is such that for every i, a_i^j is not in the support of α_i. There exists an equilibrium $(\hat{p}, \sigma)$ of Γ^{pub}, with $\sigma(\hat{p}) = a^j$, with $\hat{p}_i^j \in [0, \delta_i(a^j)]$ for every active supplier.*

Proof. See Appendix. ■

5 Robustness and Extensions

We now consider how our results would be modified if our assumptions were changed. Since our strongest characterization results are for the case of private contracting, we focus our discussion mainly on this case.

5.1 Competition among suppliers

We have assumed throughout each supplier has monopoly power over the action that he is needed for. Let us assume that there is more than one source from which the player may obtain the input required for an action, a_i^j. Let Λ_{ij} be the set of suppliers for input ij, each of whom (indexed ij_k) produces a product which is perfectly substitutable with any other. Let ij_1 be the least cost supplier. Define the gross payoff function so that the $g_i(a_i^j, a_{-i})$ is the utility to player i minus the cost to ij_1 of supplying action a_i^j and measure prices of the suppliers in Λ_{ij} net of the cost to ij_1. Cautious equilibria are now such that no supplier prices below cost. It is clear that basic point of theorem 1 is unaltered in this case, i.e. the equilibrium action outcomes Γ^{pvt} continue to coincide with equilibria of G. In the case of pure strategy equilibria, the prices paid to supplier ij_1 must now satisfy an additional constraint, of being lower than the cost of the next best supplier in Λ_{ij}. In the case of mixed strategy equilibria of G, nothing is altered at all, and the generalized indifference principle continues to apply.

5.2 Complementary Inputs

Assumption A1, that there are no complementary inputs necessary for taking an action, is essential for our main result. In its absence, coordination failures can enlarge the set of equilibria, so that action profiles which are not equilibria of G can be sustained as equilibrium outcomes of Γ^{pvt}. To see this consider a one player game, where it is efficient for the player to consume a composite good, which yields utility one, whereas the outside option of the player yields zero. Assume that two inputs are required in order to make up the composite good, that there is a monopoly supplier for each of these inputs. Now if each supplier chooses a price of 1 (or greater), it is strictly optimal to take the outside option. No supplier can benefit by reducing price, since he must choose a price of zero (or lower) to ensure a sale.

The generalized indifference principle for mixed equilibria must also be modified in this case. Let us consider the coordination game in Fig. 1, and let us assume that for both players, no inputs are required for action L and that two inputs are required playing H. Let the supplier of the first input to row choose a price equal to one, while the supplier of the second input randomizes, choosing a price of zero with probability $\frac{1}{2}$, and a price of one with probability $\frac{1}{2}$. Each player chooses to play H if and only if the total price is less than equal to one. It can be verified that this is an equilibrium, with action outcome different from the mixed equilibrium of G. The supplier of the first input makes positive profits at this equilibrium while the supplier of the second input makes zero profits. The generalized indifference principle in this context implies that at least one of the producers of the complementary goods is indifferent between making a sale or not.

5.3 Sequential Move Games

Our analysis has focused entirely on the case where the underlying game G is a simultaneous move game, where we find that private contracting and public contracting yield very different results. Consider a two player sequential move game where one player (the leader) moves first, and the follower moves after observing the leader's action. To embed this in a contracting setting, assume that suppliers of the leader choose prices first, while the suppliers of the follower choose prices after observing the leader's chosen action. The distinction between private and public contracting becomes irrelevant in this context – since the prices chosen by the leader's suppliers are payoff irrelevant to the follower it makes little difference whether they are observed or not by the follower or her suppliers. More generally, one might conjecture that when the underlying game is one of perfect information with generic gross payoffs (no ties at any terminal nodes), private and public contracting yield the same outcomes.

We believe that the conclusion – that the form of contracting makes no difference — is too strong, even in this very special context. Following Bagwell [1], one may ask, are the equilibrium outcomes of perfect information games robust to small imperfections in observation? If one modifies the sequential move game in a contracting environment, so that the follower (and his supplier) have noisy observations of the leader's choice, and restricts attention to pure strategy equilibria, then it is as though we are analyzing a simultaneous move game. On the other hand, one may consider mixed equilibria of the game with noisy observation in a contracting environment, where the leader randomizes between different actions. In this case, the analysis of the present paper indicates that one is likely to obtain very different results depending upon whether one has private or public contracting. Specifically, with public contracting, it will be as though the suppliers of the different actions are producing complements, whereas with private contracting, it is as though they are producing perfect substitutes. [9]

6 Conclusion

We have analyzed games where players have to contract with other suppliers in order to take actions. As in the literature on principal-multi agent games, we find that it is important whether contracts are public (i.e. observed by all the players in the game) or private (observed only by contracting parties). We also find that there is a major difference between mixed equilibrium in the game, where players randomize between their actions, and pure equilibria. In particular, the incentive constraints which apply to mixed equilibria are dramatically different from incentive constraints which apply to pure equilibria. This can have important consequences, as we show in a companion paper (Bhaskar, [3]).

7 Appendix

Proof of Theorem 1:

We first prove the following lemma:

[9] Bhaskar [3] examines some of these issues in the private contracting case, and shows that the set of equilibrium payoffs with almost perfect observation can be disjoint from the set of equilibrium outcomes with perfect observation.

Lemma 4 *$(\hat{p}, \sigma)$ is an equilibrium of Γ^{pvt} if and only for every player i and any action a_i^j in the support of $\sigma_i(\hat{p}_i)$: (i) $g_i(a_i^j, \sigma_{-i}(\hat{p})) - p_i^j \geq g_i(a_i^k, a_{-i}^*) - p_i^k$ for any k. (ii) If $ij \in \Lambda_i$, there exists an action $a_i^k, k \neq j$ such that $g_i(a^*) - p_i^j = g_i(a_i^k, a_{-i}^*) - p_i^k$, with $p_i^k \leq 0$ (iii)$p_i^j \geq 0$.*

Proof. We prove the necessity of these conditions first. Condition (i) follows from individual optimization by player i. Condition (ii) follows from Bertrand competition between suppliers ij and ik. By assumption A3 player i's gross payoff, as a function of his own action, does not vary with the prices p_i^j or p_i^k. If the inequality in (i) was to be strict for every k, then supplier j could increase his price, so this must hold with equality for some k. Given this equality, this supplier (k) can ensure purchase by decreasing his price slightly, so such a price reduction must not be profitable, i.e. $p_i^k \leq 0$. Finally, condition (iii) ensures that supplier j does not make a loss. ■

Turning to the proof of the theorem, to prove only if, note that $\alpha = (\alpha_i)_{i \in I}$ is an equilibrium of the game G if for any player i and any action a_i^j which is assigned positive probability by α_i,

$$g_i(a_i^j, \alpha_{-i}) \geq g_i(a_i^k, \alpha_{-i}) \forall k. \tag{14}$$

In particular, if two actions a_i^j and a_i^k are both assigned positive probability under α_i, then:

$$g_i(a_i^j, \alpha_{-i}) = g_i(a_i^k, \alpha_{-i}). \tag{15}$$

We now show that $\sigma(\hat{p})$ satisfies the same conditions. Let a_i^j be an action which is played with positive probability at $\hat{p}_i$, and suppose that there exists some k such that $g_i(a_i^j, \sigma_{-i}(\hat{p})) < g_i(a_i^k, \sigma_{-i}(\hat{p}))$. Now p_i^kmust be strictly positive since otherwise by condition (iii) of theorem 1, $g_i(a_i^j, \sigma_{-i}(\hat{p})) + p_i^j < g_i(a_i^k, \sigma_{-i}(\hat{p})) + p_i^k$, and $\sigma_i(\hat{p})_i$ cannot be optimal for player i. Since supplier k sells with a probability strictly less than one, he can do better by choosing a price $p_i^\kappa = g_i(a_i^k, \alpha_{-i}) - g_i(a_i^j, \alpha_{-i}) + \hat{p}_i^j - \varepsilon$ (ε small and positive), then he can ensure that i chooses action k with probability one. We conclude that $g_i(a_i^j, \sigma_{-i}(\hat{p})) \geq g_i(a_i^k, \sigma_{-i}(\hat{p}))$. This implies also that if both a_i^j and a_i^k are assigned positive probability at $\hat{p}_i$, then $g_i(a_i^j, \sigma_{-i}(\hat{p})) = g_i(a_i^k, \sigma_{-i}(\hat{p}))$. Since this is true for every i, this proves (i).

To prove (ii), let α be an equilibrium of G, and let $\sigma_i(\hat{p}_i) = \alpha_i$. If α_i randomizes across two or more actions, let $\hat{p}_i^k = 0 \, \forall k$. If α_i assigns probability one to a single action, a_i^j, let $\hat{p}_i^k = 0 \, \forall k \neq j$, and let $\hat{p}_i^k = g_i(a_i^j, \alpha_{-i}) - \max_{k \neq j} g_i(a_i^k, \alpha_{-i})$. ■

Proof of Theorem 2:

For each i, let any active supplier $\lambda(a_i^*)$ choose his price equal to $\delta_i(a^*)$, and let every other supplier choose a price of zero. In the resulting subgame, it is clearly an equilibrium for each player to choose a_i^*. It remains to specify behavior after a deviation by any one seller, and to verify that such a deviation is not profitable.

Suppose that condition (i) holds. Let players play an arbitrary equilibrium in any other subgame that results after different prices. It is clear that no active supplier can do better by choosing a different price, since at any profile, the maximum that he can earn is his marginal contribution, which by assumption is smaller than $\delta_i(a^*)$

Suppose ii) holds. If active supplier $\lambda(a_i^*)$ deviates by choosing a higher price, then all players $j \neq i$ continue to choose action a_j^*, while player i chooses the action

$$a_i' \in \arg\max_{a_i \neq a_i^*} g_i(a_i, a_{-i}^*). \tag{16}$$

This response by player i makes a price increase for the supplier unprofitable. It remains to verify that the continuation play constitutes a Nash equilibrium in the subgame. For player i, choosing a_i' rather than a_i^* is clearly preferable since the price chosen by supplier $\lambda(a_i^*)$ is greater than $\delta_i(a^*)$. For every player j different from i, continuing to play a_j^* is a best response since the marginal contribution at any profile (a_j^*, a_{-j}) is greater than the price $\delta_j(a^*)$. ■

Proof of Theorem 3:

Let $\hat{p}_i$ be such that $\hat{p}_i^j \in [0, \delta_i(a^j)]$ and $\hat{p}_i^k = 0$ for $k \neq j$. If the chosen price vector is $\hat{p}$, the players play a^j; this is clearly optimal since $\hat{p}_i^j \leq \delta_i(a^j)$. If supplier ij deviates and chooses a higher price, the players play α, so that ij gets zero. Since α is an equilibrium of G, this is an equilibrium in this subgame since only the payoff to action a_i^j, which is not in the support of α_i, has been reduced. Clearly no other supplier ik can benefit by raising his price, if the players continue to play a^j after this deviation. ■

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