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Abstract: This paper studies optimal fiscal and monetary policy in a stochastic economy with imperfectly competitive product markets and a discretionary government. We find that, in the flexible price economy, optimal time-consistent policy implements the Friedman rule independently of the degree of imperfect competition. This result is in contrast to the Ramsey literature, where the Friedman rule emerges as the optimal policy only if markets are perfectly competitive. Second, once nominal rigidities are introduced, the Friedman rule ceases to be optimal, inflation rates are low and stable, and tax rates are relatively volatile. Finally, optimal time-consistent policy under sticky prices does not generate the near-random walk behavior of taxes and real debt that can be observed under optimal policy in the Ramsey problem. A common reason for these results is that the discretionary government, in an effort to asymptotically eliminate its time-consistency problem, accumulates a large net asset position such that it can finance its expenditures via the associated interest earnings.

Keywords: Optimal macroeconomic policy, time-consistency, business cycles, Markov-perfect equilibrium

JEL classification: E31, E32, E61, E63

1 Introduction

The cyclical properties as well as the long-run implications of optimal macroeconomic policies have been studied extensively under the assumption that the government has commitment power (the so-called Ramsey problem).[1] The literature that analyzes optimal policies in the absence of a commitment technology, however, is much less developed, and most authors have focused on the long-run properties of optimal policies in relatively simple economies. In particular, attention has largely been restricted to deterministic settings so that our understanding of the cyclical properties of optimal discretionary policy in the presence of macroeconomic shocks is still very limited. In the present paper, we try to address this gap in the literature by analyzing a dynamic stochastic general equilibrium model of an economy with imperfectly competitive product markets and a benevolent government that conducts both monetary and fiscal policy but lacks commitment power. We characterize the equilibrium both under flexible prices and under sticky prices.

The framework we employ for our analysis is the stochastic production economy developed by Schmitt-Grohé and Uribe (2004a, 2004b). In this economy, product markets are imperfectly competitive and productivity shocks and government spending shocks are the driving forces behind macroeconomic fluctuations. Following the classical public finance literature, we assume that the government is benevolent and uses monetary and fiscal policy instruments in order to finance an exogenously given stream of public expenditures. This is a non-trivial problem, because the government does not have access to non-distortionary instruments: income taxes reduce the households' incentives to supply labor, whereas positive nominal interest rates generate non-zero opportunity costs of holding money. Moreover, by allowing the government to accumulate debt or assets, it can shift these distortions over time. The government's net asset position can therefore play an important role in absorbing macroeconomic shocks.

We have chosen the model from Schmitt-Grohé and Uribe (2004a, 2004b) for our investigation, because it constitutes a relatively detailed, yet tractable framework, which has several features that are deemed to be important for business cycle analysis. First, the model incorporates *monopolistic competition* and, therefore, allows for positive profits in the economy. This provides scope for inflation as an indirect tax on monopoly rents. Furthermore, the economy features *nominal rigidities* which are introduced via quadratic price adjustment costs. Although this modelling approach implies that inflation causes resource costs, it avoids relative price distortions which facilitates the numerical computation of equilibria. Finally, *monetary non-neutrality* is generated via a transaction cost motive for holding money. This turns out to be attractive from a computational perspective because, unlike the use of cash-in-advance constraints, it does not give rise to occasionally binding constraints in a stochastic economy. The second motive behind our choice of the particular model described above is to facilitate the comparison of our results for discretionary policy with those derived by Schmitt-Grohé and Uribe (2004a, 2004b) for the Ramsey problem. We treat the latter as a benchmark against which we assess the qualitative and quantitative implications of our alternative assumption regarding the government's intertemporal commitment capacity.

Under our maintained assumption of no commitment, the optimal policy problem is best de-

[1] See, e.g., the recent papers by Siu (2004), Schmitt-Grohé and Uribe (2004a, 2004b), and Chugh (2007).

scribed as a dynamic game between successive governments (sequential policy making). Following the pre-dominant approach of modern macroeconomics, we rule out reputational mechanisms and restrict the analysis to Markov-perfect equilibria of this game. Hence, policy choices may depend on history only via a set of endogenous and exogenous payoff-relevant state variables. In our setting, it turns out that the real level of public liabilities is the only endogenous payoff-relevant state variable, whereas the levels of productivity and government expenditure are the exogenous ones. By definition of a Markov-perfect equilibrium, the corresponding policy rules are time-consistent, i.e., a sequential policy maker will never find it optimal to deviate from these rules as time goes by. In contrast, optimal policies chosen by a government in the Ramsey problem are generically dynamically inconsistent. This distinction between Ramsey and Markov-perfect policies is particularly important in economic environments featuring an endogenous state variable because, in the absence of commitment, the current policy maker seeks to manipulate the future state of the economy such as to influence future policy choices. As has been pointed out above, the economy under consideration is an example of such an environment.[2]

A first key insight of our analysis is that, for the model under consideration, the steady state corresponding to a Markov-perfect equilibrium is (at least locally) unique and asymptotically stable. This is not true for the corresponding equilibrium of the Ramsey problem, which features a continuum of neutrally stable steady states and where the initial conditions determine which of these steady states is approached in the long-run. The steady state that emerges as the Markov-perfect equilibrium outcome has the property that the commitment problem faced by the policy maker disappears. In particular, we find that the discretionary government accumulates a large net asset position in order to finance its outlays via the associated interest earnings. In this respect, our paper relates to the results in Aiyagari et al. (2002). However, whereas the mechanism stressed by Aiyagari et al. (2002) is the precautionary savings motive of a Ramsey planner in a world of incomplete markets, in our case it is the sequential planner's intertemporal commitment problem which provides incentives to manipulate the endogenous state variable.

Our central findings for the Markov-perfect equilibrium allocation can be described as follows. Under *flexible prices* and irrespective of the degree of monopolistic competition, the Friedman rule is optimal and income taxes are perfectly smooth. This is in sharp contrast to the results derived by Schmitt-Grohé and Uribe (2004a) for the corresponding Ramsey problem, according to which the Friedman rule emerges only under perfect competition. The intuition behind our finding is that, as stated above, the policy maker accumulates a large stock of assets which allows him to finance expenditures and to subsidize labor effort via interest income. As after-tax wages are equated to the marginal product of labor, output is at the competitive level and the distortions due to monopolistic competition are eliminated. As a consequence, the policy maker's incentive to set positive interest rates disappears. In addition, inflationary

[2] As has been emphasized by Klein et al. (2007) in a similar environment, the motive for manipulation is an indirect one in the following sense. Given the value of the future state variables, there is no disagreement with respect to future policies between the current and the future government. However, the current government has an incentive to alter the private sector's expectations about future policies. This is achieved via the effect of current policies on intertemporal decisions taken by private agents (such as the purchase of government debt). These decisions, in turn, have an influence on future policies and, hence, on current expectations.

revaluations of the government's assets serve as the only shock absorber such that optimal tax rates are perfectly smooth.

Under *sticky prices*, the Friedman rule is no longer optimal. Moreover, we find that the volatilities of optimal taxes and inflation under discretion are similar to those under commitment. Both of these findings can be explained by the trade-off that the government faces in economies with imperfect competition and sticky prices. On the one hand, inflation is a powerful instrument which makes the real returns to nominal government debt state-contingent and also serves as a tax on monopoly rents. On the other hand, deviations from zero inflation are costly due to the price adjustment costs faced by firms. For reasons familiar from the Ramsey literature (Schmitt-Grohé and Uribe, 2004b; Siu, 2004), this trade-off between volatile and stable inflation rates is largely resolved in favor of stable (and low) inflation.

Finally, we observe that the same mechanism that generates a locally unique and stable steady state in a Markov-perfect equilibrium is also responsible for another important difference between optimal policy under discretion and under commitment. This difference concerns the time-series properties of optimal tax rates and debt. More specifically, in the Markov-perfect equilibrium considered in the present paper, taxes and debt under sticky prices do not display the near-random walk behavior that characterizes these variables in the Ramsey problem. Thus, temporary shocks to technology or government expenditure do not have permanent effects on the Markov-perfect equilibrium allocation. The explanation for this result is that the sequential policy maker always manipulates the endogenous state variable in such a way that the economy returns to the locally unique and stable steady state in the long-run.

The present paper contributes to a growing literature in macroeconomics that explores the properties of optimal policies under discretion. For example, in a recent paper, Díaz-Giménez et al. (2008) study optimal time-consistent fiscal and monetary policies in a deterministic setting with flexible prices and perfectly competitive markets. They point out the difficulty of credibly implementing non-inflationary policies in economies with outstanding nominal public debt, as governments are tempted to inflate in order to reduce the real value of their liabilities. In a similar environment, Martin (2007) provides a positive theory of government debt. He shows that the long-run level of debt is determined such that the sequential policy maker does not face incentives to employ the inflation tax. In particular, the sign and the size of the steady state level of government debt is determined by the relative ease/difficulty for households to substitute away from goods subject to the inflation tax. Moreover, time-consistent policies have also been studied extensively in the context of optimal taxation and fiscal policy. Recent contributions include Klein and Ríos-Rull (2003), Klein et al. (2005), Klein et al. (2007), Ortigueira (2006), and Ortigueira and Pereira (2007). However, all these papers are predominantly concerned with the characterization and computation of Markov-perfect optimal policies in deterministic environments. To the best of our knowledge, the present paper is the first one that characterizes the cyclical properties of monetary and fiscal policies in a stochastic economy governed by an authority that lacks commitment power.

The rest of this paper is organized as follows. Section 2 presents the model that we use for our formal analysis. In Section 3 we define and characterize optimal monetary and fiscal policies, as described by a Markov-perfect equilibrium. Section 4 presents the key quantitative results about Markov-perfect equilibria and compares them to existing findings on Ramsey equilibria.

Finally, Section 5 summarizes our results. Technical details and derivations are relegated to the Supplementary Appendices at the very end of this document.

2 The model

For our subsequent analysis, we employ the model developed by Schmitt-Grohé and Uribe (2004a, 2004b). As we have argued before, this model is a well-understood framework suitable to address the type of questions we ask. Moreover, it constitutes an established benchmark for the Ramsey literature against which we can compare our own results.

The model is an infinite-horizon production economy that features imperfectly competitive product markets and sticky prices. A demand for money arises due to its role in facilitating transactions. The government's problem is to finance an exogenous stream of public expenditures by levying distortionary taxes, printing money, and issuing nominally risk-free bonds. Apart from the government (policy maker), who decides over both fiscal and monetary instruments, the economy is populated by a continuum of identical households who act both as consumers and as producers. The following two subsections provide further details.

2.1 The private sector

Household $i \in [0,1]$ enters period $t \in \{0,1,2,...\}$ holding M_{t-1}^i units of money and B_{t-1}^i units of one-period bonds. The bonds are issued by the government and pay a nominal amount of B_{t-1}^i when they mature at the beginning of period t. In addition, the household has two different sources of income. First, it supplies h_t^i units of labor to the producers in the economy, whereby it takes the nominal wage W_t as given. Labor income is taxed at the rate τ_t such that the household's net income from supplying labor amounts to $(1-\tau_t)W_t h_t^i$. The second source of income are profits from the production of intermediate goods. In particular, household i is the monopolistic producer of the differentiated intermediate good i which is used as an input for the production of the final good. As a producer, household i has access to the linear technology $y_t^i = a_t \tilde{h}_t^i$, where y_t^i is output of intermediate good i and a_t denotes the stochastic productivity. The latter evolves according to

$$\log a_{t+1} = (1-\rho_a)\log \bar{a} + \rho_a \log a_t + \varepsilon_{t+1}^a, \tag{1}$$

where $\bar{a}$ is the steady state technology level, ρ_a measures its autocorrelation, and $\varepsilon_{t+1}^a \sim N(0, \sigma_{\varepsilon^a}^2)$ denotes a technology innovation. Note that, while h_t^i is the household's labor supply, $\tilde{h}_t^i$ is the amount of labor it hires from a perfectly competitive market in order to produce intermediate good i. The final good is a Dixit-Stiglitz aggregate of all intermediate goods,

$$y_t = \left[\int_0^1 (y_t^i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where y_t denotes the quantity of the final good produced and θ measures the constant elasticity of substitution between any two intermediate inputs. For an equilibrium to exist we assume that $\theta > 1$.

Let $\tilde{P}_t^i$ denote the price of intermediate good i charged by its monopolistic producer, and let P_t denote the price of the final good. The demand for intermediate good i then depends on aggregate demand and its relative price according to

$$d_t^i = y_t \left(\frac{\tilde{P}_t^i}{P_t} \right)^{-\theta}.$$

When setting its price, household i takes P_t and y_t as given. As in Schmitt-Grohé and Uribe (2004b), we allow for quadratic price adjustment costs à la Rotemberg (1982). In real terms, producer i faces the costs

$$\mathrm{PAC}_t^i = \frac{\kappa}{2} \left(\frac{\tilde{P}_t^i}{\tilde{P}_{t-1}^i} - 1 \right)^2.$$

The parameter κ measures the importance of price adjustment costs. In particular, if $\kappa = 0$, prices are flexible. Taking into account the price adjustment costs, the household's profits from producing good i are therefore given by

$$\Pi_t^i = \left[\tilde{P}_t^i y_t \left(\frac{\tilde{P}_t^i}{P_t} \right)^{-\theta} - W_t \tilde{h}_t^i - \frac{\kappa}{2} \left(\frac{\tilde{P}_t^i}{\tilde{P}_{t-1}^i} - 1 \right)^2 P_t \right]. \tag{2}$$

The household's income is used to finance consumption expenditures and savings. In particular, household i consumes c_t^i units of the final good in period t. When acquiring consumption goods, the household has to pay a proportional transaction cost $s(v_t^i)$ that depends on the household's consumption-based money velocity, $v_t^i = P_t c_t^i / M_t^i$. Thus, there is a transaction role for money.

The transaction cost function s is assumed to take non-negative values and to be twice continuously differentiable. Moreover, there exists a level of velocity $\underline{v} > 0$ (the satiation level of money) such that $s(\underline{v}) = s'(\underline{v}) = 0$ and $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$. Finally, it is assumed that $2s'(v) + vs''(v) > 0$ holds for all $v \geq \underline{v}$. As discussed by Schmitt-Grohé and Uribe (2004b), these assumptions imply that the Friedman rule need not be associated with an infinite demand for money, that the transaction cost as well as the distortion it introduces vanish when the nominal interest rate is equal to zero, and that, in equilibrium, the money velocity is always greater than or equal to the satiation level. Furthermore, the demand for money is decreasing in the nominal interest rate.

The household saves either in the form of money or government debt. Its nominal budget constraint is thus given by

$$M_{t-1}^i + B_{t-1}^i + (1 - \tau_t) W_t h_t^i + \Pi_t^i \geq P_t c_t^i [1 + s(v_t^i)] + M_t^i + B_t^i / (1 + R_t),$$

where R_t is the net nominal interest rate on bonds issued in period t. In addition to the budget constraint, the household is subject to a constraint that prevents it from engaging in Ponzi schemes.

Subject to these constraints, the household seeks to maximize its expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, h_t^i), \tag{3}$$

where u denotes the household's instantaneous utility function and $\beta \in (0,1)$ denotes a time discount factor. E_0 is the mathematical expectation operator conditional upon information available in period 0. We assume that the function u satisfies standard properties.

2.2 The government

The government controls monetary and fiscal policies. It faces expenditures g_t, which are exogenous, stochastic, and unproductive. In particular, g_t evolves according to

$$\log g_{t+1} = (1-\rho_g)\log \bar{g} + \rho_g \log g_t + \varepsilon^g_{t+1}, \tag{4}$$

where $\bar{g}$ denotes the steady state government expenditures, ρ_g is the autocorrelation coefficient, and $\varepsilon^g_{t+1} \sim N(0, \sigma^2_{\varepsilon g})$. To finance its expenditures, the government collects labor income taxes at rate τ_t, issues government bonds B_t, and receives seignorage income. Bonds issued in period t are nominally risk-free and promise to pay B_t units of currency when they mature in period $t+1$. Monetary policy manages the supply of money M_t and sets nominal interest rates R_t. The consolidated government budget constraint in nominal terms is thus given by

$$\tau_t W_t h_t + (M_t - M_{t-1}) + B_t/(1+R_t) \geq P_t g_t + B_{t-1}.$$

The policy instruments τ_t, B_t, R_t, and M_t are chosen in such a way that this constraint holds and the lifetime utility of the representative household given in (3) is maximized.

3 Equilibrium

Since we are interested in characterizing optimal policy by a government without commitment power, we assume that the government actually consists of an infinite sequence of separate policy makers, one for each period. The policy maker who is in charge in period t will be referred to as the period-t government. This government seeks to maximize social welfare from period t onwards, whereby it takes the behavior of its later incarnations as given. In other words, we consider a Nash equilibrium in the game among all period-t governments, where t ranges from 0 to $+\infty$.

We restrict our analysis to Markov-perfect equilibria, in which the policy choices in period t do not depend on the entire history up to period t but only on payoff-relevant state variables. In other words, we assume that reputational mechanisms are not at work. Letting $b_{t-1} = B_{t-1}/P_{t-1}$ and $m_{t-1} = M_{t-1}/P_{t-1}$ denote the real values of government debt and money, respectively, the amount of real government liabilities is given by $\ell_{t-1} = m_{t-1} + b_{t-1}$. The variable ℓ_{t-1} turns out to be the only endogenous aggregate state variable relevant for decisions taken in period t. In addition, there will be two exogenous state variables, namely, productivity a_t and government expenditure g_t.

In order to define a Markov-perfect equilibrium, we follow the three-step approach outlined in Klein et al. (2005). First, we define the private sector equilibrium under the assumption that a single but arbitrarily given policy rule determines government behavior from the present into the infinite future. In the second step, we define optimal policy for the period-t government, when the behavior of all future governments is determined by a single but arbitrarily given rule. Finally, we define a Markov-perfect equilibrium (optimal time-consistent policy rule) as a policy rule that is optimal for the period-t government provided that the period-t government expects all future governments to also choose this rule (policy fixed point).

3.1 Private sector equilibrium for an arbitrary policy rule

For notational convenience, let us denote the endogenous aggregate state variable that is relevant for period-t decisions by s_t, that is, $s_t = \ell_{t-1}$. Analogously, let us collect the two exogenous state variables in the vector $z_t = (a_t, g_t)$. For the time being, we assume that both the current and all future governments use the (arbitrary) policy rule Ω to map the aggregate state variables into their instrument variables. Since the vector of instruments implemented in period t is given by (τ_t, b_t, R_t, m_t), we thus assume that $(\tau_{t'}, b_{t'}, R_{t'}, m_{t'}) = \Omega(s_{t'}, z_{t'})$ holds for all $t' \geq t$. In the present subsection, we determine symmetric private sector equilibria induced by this policy rule. Note that we implicitly assume the policy rule to be anonymous. Accordingly, Ω does not condition on the distribution of individual states s_t^i to be defined next.

The individual state vector s_t^i of household i includes its beginning-of-period real wealth, $\ell_{t-1}^i = B_{t-1}^i/P_{t-1} + M_{t-1}^i/P_{t-1}$, and the relative price $p_{t-1}^i = \tilde{P}_{t-1}^i/P_{t-1}$ the household has charged as a producer of intermediate good i in the previous period. Formally, $s_t^i = (\ell_{t-1}^i, p_{t-1}^i)$. The household takes the real wage rate $w_t = W_t/P_t$, the inflation rate $\pi_t = P_t/P_{t-1}$, and the output quantity of the final good y_t as given. In particular, it perceives these variables to be functions of the aggregate state of the economy, given the policies Ω. Formally, $(w_t, \pi_t, y_t) = \mathcal{F}(s_t, z_t; \Omega)$.

We can now formulate the household's problem recursively as follows:

$$V(s_t^i, s_t, z_t; \Omega) = \max_{m_t^i, b_t^i, c_t^i, h_t^i, \tilde{h}_t^i, p_t^i} \left\{ u(c_t^i, h_t^i) + \beta E_t V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega) \right\} \tag{5}$$

subject to

$$c_t^i \left[1 + s\left(\frac{c_t^i}{m_t^i} \right) \right] + m_t^i + \frac{b_t^i}{1+R_t} \leq \frac{\ell_{t-1}^i}{\pi_t} + (1-\tau_t) w_t h_t^i + (p_t^i)^{1-\theta} y_t - w_t \tilde{h}_t^i - \frac{\kappa}{2} \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1 \right)^2, \tag{6}$$

$$\left(p_t^i\right)^{-\theta} y_t \leq a_t \tilde{h}_t^i, \tag{7}$$

$$\ell_t^i = m_t^i + b_t^i, \tag{8}$$

$$\begin{aligned}
\log a_{t+1} &= (1-\rho_a) \log \bar{a} + \rho_a \log(a_t) + \varepsilon_{t+1}^a, \\
\log g_{t+1} &= (1-\rho_g) \log \bar{g} + \rho_g \log(g_t) + \varepsilon_{t+1}^g, \\
w_t &= \mathcal{F}_w(s_t, z_t; \Omega), \\
\pi_t &= \mathcal{F}_\pi(s_t, z_t; \Omega), \\
y_t &= \mathcal{F}_y(s_t, z_t; \Omega), \\
\tau_t &= \Omega_\tau(s_t, z_t), \\
b_t &= \Omega_b(s_t, z_t),
\end{aligned}$$

$$R_t = \Omega_R(s_t, z_t),$$
$$m_t = \Omega_m(s_t, z_t).$$

Let λ_t^i denote the multiplier on household i's flow budget constraint (6). The solution to the household's problem is given by decision rules which, given the policy rule Ω, map the individual and aggregate state variables into decisions for m_t^i, b_t^i, c_t^i, h_t^i, $\tilde{h}_t^i$, p_t^i, and the multiplier λ_t^i, respectively. Formally:

$$\begin{aligned}
c_t^i &= f_c(s_t^i, s_t, z_t; \Omega),\\
h_t^i &= f_h(s_t^i, s_t, z_t; \Omega),\\
\tilde{h}_t^i &= f_{\tilde{h}}(s_t^i, s_t, z_t; \Omega),\\
m_t^i &= f_m(s_t^i, s_t, z_t; \Omega),\\
b_t^i &= f_b(s_t^i, s_t, z_t; \Omega),\\
p_t^i &= f_p(s_t^i, s_t, z_t; \Omega),\\
\lambda_t^i &= f_\lambda(s_t^i, s_t, z_t; \Omega).
\end{aligned}$$

Let us collect these decision rules in a vector-valued function f. The elements of f are characterized by the constraints (6), (7), and (8) holding with equality and the following first-order conditions:[3]

$$\begin{aligned}
0 &= \beta E_t \frac{\lambda_{t+1}^i}{\pi_{t+1}} - \lambda_t^i[1 - (v_t^i)^2 s'(v_t^i)],\\
0 &= \beta E_t \frac{\lambda_{t+1}^i}{\pi_{t+1}} - \frac{\lambda_t^i}{1+R_t},\\
0 &= \frac{\partial u(c_t^i, h_t^i)}{\partial c_t^i} - \lambda_t^i[1 + s(v_t^i) + v_t s'(v_t^i)],\\
0 &= \frac{\partial u(c_t^i, h_t^i)}{\partial h_t^i} + \lambda_t^i(1-\tau_t)w_t,\\
0 &= -\lambda_t^i w_t + mc_t \lambda_t^i a_t,\\
0 &= \lambda_t^i(1-\theta)(p_t^i)^{-\theta} y_t - \lambda_t \kappa \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1\right) \frac{\pi_t}{p_{t-1}^i} + mc_t \lambda_t^i \theta (p_t^i)^{-\theta-1} y_t\\
&\quad + \beta\kappa E_t \lambda_{t+1}^i \left(\frac{p_{t+1}^i \pi_{t+1}}{p_t^i} - 1\right) \frac{p_{t+1}^i \pi_{t+1}}{(p_t^i)^2}.
\end{aligned}$$

Finally, note that the equilibrium decision rules must also be consistent with the respective no-Ponzi and transversality conditions.

Having outlined the household's optimization problem for given policies Ω, we proceed to define the private sector equilibrium (again for given policies Ω). As hinted above, we restrict attention to symmetric equilibria in which initial bond and money holdings are identical and all households make identical choices. In particular, since there is a continuum of measure one

[3]Supplementary Appendix B provides a formal derivation of these conditions. Note that, for notational convenience, we present the first-order conditions in terms of allocations instead of the decision rules f.

of households, this implies that individual variables correspond to economy-wide aggregates, i.e., households are representative.

For the purpose of the formal definition of equilibrium, we introduce some more notation. First, let us introduce the vector-valued function F with the following components:

$$\begin{aligned} F_c(s_t, z_t; \Omega) &= f_c((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_h(s_t, z_t; \Omega) &= f_h((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_{\tilde{h}}(s_t, z_t; \Omega) &= f_{\tilde{h}}((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_m(s_t, z_t; \Omega) &= f_m((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_b(s_t, z_t; \Omega) &= f_b((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_p(s_t, z_t; \Omega) &= f_p((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_y(s_t, z_t; \Omega) &= a_t f_h((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega), \\ F_\lambda(s_t, z_t; \Omega) &= f_\lambda((m_{t-1}, b_{t-1}, 1)', s_t, z_t; \Omega). \end{aligned}$$

Note that the vector-valued function F defined in this way describes economy-wide aggregates of the respective variables, presuming that households are identical. Finally, let us introduce the functions $F_w(s_t, z_t; \Omega)$, $F_\pi(s_t, z_t; \Omega)$, and $F_y(s_t, z_t; \Omega)$, which denote the actual wage and inflation rates as well as the level of aggregate demand that are consistent with optimal household behavior in the symmetric equilibrium.

We are now ready to define a private sector equilibrium for a given policy rule Ω.

Definition 1 *A recursive competitive equilibrium given policy rule Ω consists of a household value function $V(\cdot, \cdot, \cdot; \Omega)$, household decision rules $f(\cdot, \cdot, \cdot; \Omega)$, and a perceived function $\mathcal{F}(\cdot, \cdot; \Omega)$ such that:*

(i) Households optimize, i.e., given states (s_t^i, s_t, z_t), policies Ω, and the perceived function $\mathcal{F}(\cdot, \cdot; \Omega)$, the value function $V(\cdot, \cdot, \cdot; \Omega)$ and the decision rules $f(\cdot, \cdot, \cdot; \Omega)$ solve the household's problem.

(ii) Households are representative, i.e., $\ell_{t-1}^i = \ell_{t-1}$ and $p_t^i = 1$ holds for all $i \in [0, 1]$ and all $t \in \{0, 1, 2, ...\}$.

(iii) Households are rational, i.e., the perceived equations for the real wage, the inflation rate, and aggregate output coincide with their respective actual equations:

$$\begin{aligned} \mathcal{F}_w(s_t, z_t; \Omega) &= F_w(s_t, z_t; \Omega), \\ \mathcal{F}_\pi(s_t, z_t; \Omega) &= F_\pi(s_t, z_t; \Omega), \\ \mathcal{F}_y(s_t, z_t; \Omega) &= F_y(s_t, z_t; \Omega). \end{aligned}$$

(iv) Markets clear:

$$\begin{aligned} F_h(s_t, z_t; \Omega) &= F_{\tilde{h}}(s_t, z_t; \Omega), \\ F_m(s_t, z_t; \Omega) &= \Omega_m(s_t, z_t), \\ F_b(s_t, z_t; \Omega) &= \Omega_b(s_t, z_t), \\ F_y(s_t, z_t; \Omega) &= F_c(s_t, z_t; \Omega) \left[1 + s\left(\frac{F_c(s_t, z_t; \Omega)}{F_m(s_t, z_t; \Omega)}\right)\right] + g_t + \frac{\kappa}{2}\left(F_\pi(s_t, z_t; \Omega) - 1\right)^2. \end{aligned}$$

(v) The policy rule Ω is feasible, i.e., it satisfies

$$\Omega_\tau(s_t, z_t)F_w(s_t, z_t; \Omega)F_h(s_t, z_t; \Omega) + \Omega_m(s_t, z_t) + \frac{\Omega_b(s_t, z_t)}{1 + \Omega_R(s_t, z_t)} = g_t + \frac{\ell_{t-1}}{F_\pi(s_t, z_t; \Omega)}$$

Having defined the private sector equilibrium for given policies Ω, we finally proceed to characterize the policy maker's value function (given Ω). Specifically, the policy maker's value function is implicitly given by

$$W(s_t, z_t; \Omega) = u(F_c(s_t, z_t; \Omega), F_h(s_t, z_t; \Omega)) + \beta E_t W(\mathcal{S}(s_t, z_t; \Omega), z_{t+1}; \Omega),$$

where $\mathcal{S}(s_t, z_t; \Omega) = F_m(s_t, z_t; \Omega) + F_b(s_t, z_t; \Omega)$.

3.2 Optimal current policy rule for given future policy rule

In the previous subsection, we have outlined the household's problem when both current and future policies are characterized by Ω. Furthermore, we have characterized the (symmetric) private sector equilibrium decision rules associated with this problem. In this subsection, we consider the household's problem, assuming that it faces a current policy rule which potentially differs from the rule employed in the future. More specifically, the household presumes that current policies are described by $\hat{\Omega}$ and that, from the next period onwards, policies are determined by the rule Ω. Again, we first outline the household's problem and the private sector equilibrium. Then, we characterize $\hat{\Omega}$, the optimal current macroeconomic policy rule from the perspective of a policy maker who faces the future policy rule Ω. We follow the notational convention in Klein et al. (2005) and mark by hats all those functions that are affected by the current policy.

The problem of a household facing a current policy rule $\hat{\Omega}$ followed by a future policy rule Ω can be formulated in a recursive way as follows.

$$\hat{V}(s_t^i, s_t, z_t; \hat{\Omega}, \Omega) = \max_{m_t^i, b_t^i, c_t^i, h_t^i, \tilde{h}_t^i, p_t^i} \left\{ u(c_t^i, h_t^i) + E_t \beta V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega) \right\} \tag{9}$$

subject to

$$c_t^i \left[1 + s\left(\frac{c_t^i}{m_t^i} \right) \right] + m_t^i + \frac{b_t^i}{1 + R_t} \leq \frac{\ell_{t-1}^i}{\pi_t} + (1 - \tau_t) w_t h_t^i + (p_t^i)^{1-\theta} y_t - w_t \tilde{h}_t^i - \frac{\kappa}{2} \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1 \right)^2, \tag{10}$$

$$\left(p_t^i\right)^{-\theta} y_t \leq a_t \tilde{h}_t^i, \tag{11}$$

$$\ell_t^i = m_t^i + b_t^i, \tag{12}$$

$$\log a_{t+1} = (1 - \rho_a) \log \bar{a} + \rho_a \log(a_t) + \varepsilon_{t+1}^a,$$

$$\log g_{t+1} = (1 - \rho_g) \log \bar{g} + \rho_g \log(g_t) + \varepsilon_{t+1}^g,$$

$$w_t = \hat{\mathcal{F}}_w(s_t, z_t; \hat{\Omega}, \Omega),$$

$$\pi_t = \hat{\mathcal{F}}_\pi(s_t, z_t; \hat{\Omega}, \Omega),$$

$$y_t = \hat{\mathcal{F}}_y(s_t, z_t; \hat{\Omega}, \Omega),$$

$$
\begin{aligned}
\tau_t &= \hat{\Omega}_\tau(s_t, z_t), \\
b_t &= \hat{\Omega}_b(s_t, z_t), \\
R_t &= \hat{\Omega}_R(s_t, z_t), \\
m_t &= \hat{\Omega}_m(s_t, z_t),
\end{aligned}
$$

where $\hat{\mathcal{F}}(\cdot, \cdot; \hat{\Omega}, \Omega)$ describes the perceived evolution of wages, inflation, and aggregate output conditional on policies being $\hat{\Omega}$ followed by Ω. Note also that the function V appearing on the right-hand side of (9) is the solution of the problem of maximizing (5) subject to the relevant constraints.

The solution to problem (9) is given by two sets of decision rules, one for the current period, $\hat{f}(\cdot, \cdot, \cdot; \hat{\Omega}, \Omega)$, and one for the future, $f(\cdot, \cdot, \cdot; \Omega)$. The future decision rules are those that we have derived in the previous section. The current rules, on the other hand, are characterized by the constraints (10), (11), and (12) holding with equality together with the following first-order conditions:

$$
\begin{aligned}
0 &= \beta E_t \frac{f_\lambda(s^i_{t+1}, s_{t+1}, z_{t+1}; \Omega)}{\mathcal{F}_\pi(s_{t+1}, z_{t+1}; \hat{\Omega}, \Omega)} - \frac{\lambda^i_t}{1 + R_t}, \\
0 &= \frac{\partial u(c^i_t, h^i_t)}{\partial c^i_t} - \lambda^i_t [1 + s(v^i_t) + v_t s'(v^i_t)], \\
0 &= \frac{\partial u(c^i_t, h^i_t)}{\partial h^i_t} + \lambda^i_t (1 - \tau_t) w_t, \\
0 &= -\lambda^i_t w_t + mc_t \lambda^i_t a_t, \\
0 &= \lambda^i_t (1 - \theta)(p^i_t)^{-\theta} y_t - \lambda^i_t \kappa \left(\frac{p^i_t \pi_t}{p^i_{t-1}} - 1 \right) \frac{\pi_t}{p^i_{t-1}} + \frac{w_t}{a_t} \lambda^i_t \theta (p^i_t)^{-1} a_t \tilde{h}^i_t + \beta \kappa E_t f_\lambda(s^i_{t+1}, s_{t+1}, z_{t+1}; \Omega) \\
&\times \left(\frac{f_p(s^i_{t+1}, s_{t+1}, z_{t+1}; \Omega) \mathcal{F}_\pi(s_{t+1}, z_{t+1}; \hat{\Omega}, \Omega)}{p^i_t} - 1 \right) \frac{f_p(s^i_{t+1}, s_{t+1}, z_{t+1}; \Omega) \mathcal{F}_\pi(s_{t+1}, z_{t+1}; \hat{\Omega}, \Omega)}{(p^i_t)^2}.
\end{aligned}
$$

Note that, when deciding about current variables, the household anticipates that future policy decisions are governed by the rules Ω so that the household's own decisions from period $t+1$ onwards are generated by the future decision rules $f(\cdot, \cdot, \cdot; \Omega)$. Again, in the symmetric private sector equilibrium, households optimize and are representative such that private sector behavior can be characterized by aggregate decision rules $\hat{F}(\cdot, \cdot; \hat{\Omega}, \Omega)$ for the current period t and $F(\cdot, \cdot; \Omega)$ from period $t+1$ onwards.

Finally, let us characterize the policy maker's problem of choosing optimal current policies when facing a continuation policy rule Ω. The current policy maker's maximization problem is

$$
\hat{W}(s_t, z_t; \Omega) = \max_{\tau_t, b_t, R_t, m_t} \left\{ u\left(\hat{F}_c(s_t, z_t; \hat{\Omega}, \Omega), \hat{F}_h(s_t, z_t; \hat{\Omega}, \Omega) \right) + \beta E_t W(\hat{\mathcal{S}}(s_t, z_t; \hat{\Omega}, \Omega), z_{t+1}; \Omega) \right\}
$$

subject to

$$
\tau_t \hat{F}_w(s_t, z_t; \hat{\Omega}, \Omega) \hat{F}_h(s_t, z_t; \hat{\Omega}, \Omega) + m_t + \frac{b_t}{1 + R_t} \geq g_t + \frac{\ell_{t-1}}{\hat{F}_\pi(s_t, z_t; \hat{\Omega}, \Omega)},
$$

where $\hat{\mathcal{S}}(s_t, z_t; \hat{\Omega}, \Omega) = \hat{F}_m(s_t, z_t; \hat{\Omega}, \Omega) + \hat{F}_b(s_t, z_t; \hat{\Omega}, \Omega)$ and where the function W satisfies (9). The solution to the problem is given by the policy rule $\hat{\Omega}(s_t, z_t; \Omega)$.

3.3 Policy fixed point

The results derived in the previous two subsections allow us to define the time-consistent policy rules (Markov-perfect equilibrium) in the following way.

Definition 2 *The policy rule Ω^* defines an optimal time-consistent policy if it is the current policy maker's optimal policy when future policies are expected to be determined by Ω^*. Formally, $\Omega^*(s_t, z_t) = \hat{\Omega}(s_t, z_t; \Omega^*)$.*

4 Quantitative results

This section presents the quantitative results of our analysis. We start by briefly discussing the numerical strategy for solving the model as well as the calibration. After that, we investigate the long-run properties and business cycle characteristics of optimal time-consistent taxes and inflation in a flexible price environment, and we compare them to the corresponding properties in the Ramsey problem. We find both noteworthy similarities and differences. Finally, we discuss the same issues for the economy with sticky prices.

4.1 Computation

In order to solve for the Markov-perfect equilibrium policies, we could in principle follow the theoretical approach laid out in Sections 3.1-3.3. In particular, we could apply a policy iteration algorithm to determine the functional fixed point Ω^*. This is computationally demanding, however, since, at each iteration step, we would have to solve the household's optimization problem to obtain private sector decision rules and the corresponding value functions. To avoid the computational burden associated with such a procedure, we choose to solve for the equilibrium following a primal approach. That is, we presume that the policy maker chooses both the policy instruments and the resulting allocation subject to a set of feasibility and implementability constraints. Feasibility requires that the policy maker's budget constraint is satisfied for each state and each period, whereas implementability requires that the allocation satisfies the household's optimality conditions. A formal description of the policy maker's problem in primal form is provided in Supplementary Appendix D.

To compute the Markov-perfect equilibrium numerically, we proceed as follows. We first compute the (approximate) steady state of the model based on an iterative first-order accurate perturbation method. Having identified the steady state, we define the relevant state space as a symmetric interval around this point. We then employ this information in a Galerkin projection method to compute third-order approximations to the equilibrium decision rules. Finally, as a numerical accuracy check, we verify that the Euler equation errors associated with simulated time series are sufficiently small. A detailed description of the numerical algorithm is available in Supplementary Appendix E.

4.2 Calibration

In our numerical analysis, we use the utility function

$$u(c,h) = \frac{c^{1-\vartheta}-1}{1-\vartheta} + \delta_h \frac{(1-h)^{1-\eta}-1}{1-\eta}, \tag{13}$$

where δ_h is the relative weight of leisure in the utility function, and where ϑ and η determine the elasticities of intertemporal substitution in consumption and leisure, respectively. As for the transaction cost function we assume that

$$s(v_t) = Av_t + B/v_t - 2\sqrt{AB}, \tag{14}$$

where A and B are positive parameters. The corresponding money demand function turns out to be

$$m_t = A^{1/2} c_t \left[B + R_t/(1+R_t)\right]^{-1/2},$$

which implies that money demand is increasing in consumption and decreasing in the interest rate. Also, note that money demand is well-defined even if $R_t = 0$.

To facilitate the comparison of our results to those in Schmitt-Grohé and Uribe (2004b), we adopt their parameterization as the benchmark calibration for our analysis. This requires in particular that the parameters ϑ and η are both equal to 1 such that the utility function is logarithmic in both arguments. Also for the remaining parameters of the model, we take exactly the values employed by Schmitt-Grohé and Uribe (2004b) as summarized in Table 1. Due to space constraints, we do not provide any justification for this parameterization or for the choice of the transaction cost function (14), but refer the reader to Schmitt-Grohé and Uribe (2004b), where these issues are discussed. Note, however, that the benchmark calibration features monopolistic markups of 20% and a degree of price stickiness consistent with a model of staggered price setting where individual firms change their prices on average every nine months.

Table 1: Benchmark calibration

β	ϑ	η	δ_h	θ	κ	ρ_a	$\bar{a}$	σ_a	ρ_g	$\bar{g}$	σ_g	A	B
0.96	1	1	2.9	6	17.5/4	0.82	1	0.0229	0.9	0.04	0.0302	0.0111	0.07524

4.3 Results for the flexible price economy

Recall that the literature on optimal policies under commitment has delivered two key insights for flexible price economies; see Schmitt-Grohé and Uribe (2004a, 2004b). First, if markets are perfectly competitive, then optimal policy implements the Friedman rule of setting the net nominal interest rate equal to zero. The planner finds it optimal to use surprise inflation to make the returns to nominal debt state-contingent, which allows for relatively smooth taxes over the business cycle. Consequently, inflation is on average negative, displays little serial correlation, and a relatively high degree of volatility. Secondly, the Friedman rule ceases to be optimal if markets are not perfectly competitive because positive nominal interest rates serve as an indirect tax on monopoly profits. Since monopoly rights are inelastic factors in production, the planner chooses to implement non-zero interest rates to allow for a less distortionary sequence of labor taxes. To summarize, the relevant literature on the Ramsey problem suggest that the degree of market power that exists in product markets has a decisive influence on the optimality of the Friedman rule. In this subsection we analyze whether these findings carry over to the case of a sequential policy maker.

In order to study a flexible price environment we eliminate the price adjustment costs, that is, we set $\kappa = 0$. In what follows, we first present some analytical results on the steady state that obtains in the non-stochastic version of the flexible price economy governed by a sequential policy maker. We call this steady state the Markovian steady state. After that, we present numerical results on the dynamic properties of optimal discretionary policies.[4]

Proposition 1 *If $\kappa = 0$, there exists a non-stochastic Markovian steady state characterized by*

$$
\begin{aligned}
a &= \bar{a}, \\
g &= \bar{g}, \\
R &= 0, \\
\tau &= 1 - \frac{\theta}{\theta - 1}, \\
\pi &= \beta, \\
w &= \frac{\theta - 1}{\theta} a, \\
v &= \sqrt{\frac{B}{A}} = \underline{v}, \\
\delta_h a^{\eta-1} c^{\vartheta} &= (a - c - g)^{\eta}, \\
m &= c/v, \\
h &= (c + g)/a, \\
y &= ah, \\
\ell &= \left(1 - \frac{1}{\beta}\right)^{-1} [g - \tau w h], \\
b &= \ell - m, \\
\lambda &= c^{-\vartheta}.
\end{aligned}
$$

[4]We have to emphasize the following caveat regarding the Markovian steady state. It is well known that uniqueness of Markov-perfect equilibria can hardly ever be established for reasonably complex models. We conjecture that this is true also for our model. Consequently, since we do not formally prove uniqueness of the Markov-perfect equilibrium, we cannot formally establish global uniqueness of the implied steady state either. Our further analysis proceeds subject to this caveat.

For $\vartheta = \eta = 1$, steady state consumption and labor effort can be derived explicitly as

$$c = \frac{a-g}{1+\delta_h}, \qquad h = \frac{a+\delta_h g}{a(1+\delta_h)}.$$

PROOF: The proof is available in Supplementary Appendix F. □

Proposition 1 illustrates that, in the long-run, the sequential policy maker accumulates a sufficiently large net asset position vis-à-vis the private sector to finance both government spending and a subsidy to labor effort via the real return on bonds. This means that the sequential planner manipulates the endogenous state variable such as to eliminate the distortions in the economy. Hence, in the long-run, the aggregate resource constraint is the only restriction that binds the planner, which is evidenced by the fact that all multipliers on the planner's implementability constraints except for the one that corresponds to the resource constraint are zero.[5]

This is achieved via sequential policy choices, which facilitate the steady state net nominal interest rate to be set equal to zero – the Friedman rule. Hence, the consumption-based velocity of money is at its satiation level and the wedge between the marginal utilities of consumption and wealth present due to transaction costs is removed ($s(v) = s'(v) = 0$). Furthermore, as long as $\theta < \infty$, the policy maker implements a negative labor tax rate such as to counteract the inefficiently low labor supply due to the imperfectly competitive market structure. That is, labor effort is subsidized to a point where the after tax real wage is equated to the marginal product of labor. Consequently, steady state output is at the competitive level and the consumption allocation does not depend on the degree of monopoly distortions θ.

In order to find out whether the optimal time-consistent policy implements the Friedman rule also out of the steady state, we simulate the model. Table 2 displays the cyclical properties of key macroeconomic variables, computed as the averages of $J = 500$ simulations of length $T = 100$. Results for taxes τ, inflation π, and the nominal interest rate R are reported in percentage points, whereas the remaining variables are reported in levels. For both perfectly and imperfectly competitive environments, our simulations confirm that sequentially optimal monetary policy indeed follows the Friedman rule. Actually, irrespective of the degree of monopolistic competition, nominal interest rates are equal to zero not only on average, but in all states over the business cycle. This is in contrast to the findings for optimal Ramsey policies reported in Schmitt-Grohé and Uribe (2004a), where average nominal interest rates are shown to increase along with the (gross) monopolistic markup $\theta/(\theta-1)$.[6]

This difference is to be understood as follows. In the imperfectly competitive economy, the policy maker does not have an instrument to directly tax the monopoly rents accruing to producers. Hence, as an indirect way to tax profit income, the Ramsey planner resorts to the inflation tax which distorts nominal interest rates away from zero.[7] However, this result

[5]See Supplementary Appendices D and F.

[6]Table 2 presents results only for the case of perfect competition ($\theta = \infty$) and the case of a 20% markup ($\theta = 6$). Further simulations for alternative markups yield results which are very similar to the latter case.

[7]Inflation acts as an indirect tax on profits because consumers must hold money in order to transform profits into consumption.

Table 2: Dynamic properties of the Markov-perfect equilibrium allocation under flexible prices

Variable	Mean	Std. dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,a)
Flexible prices and perfect competition ($\kappa = 0$ and $\theta = \infty$)						
τ	0	0	-	-	-	-
π	-3.8103	2.1357	-0.0808	-0.2951	-0.1705	-0.2681
R	0	0	-	-	-	-
y	0.2866	0.0096	0.7766	1.0000	0.1816	0.9814
h	0.2861	0.0021	0.8346	-0.3436	0.8467	-0.5097
c	0.2466	0.0095	0.7743	0.9663	-0.0636	0.9977
b	-1.0991	0.0389	0.7834	-0.9855	-0.3374	-0.9349
m	0.0947	0.0036	0.7743	0.9663	-0.0636	0.9977
ℓ	-1.0043	0.0356	0.7857	-0.9773	-0.3754	-0.9191
Flexible prices and imperfect competition ($\kappa = 0$ and $\theta = 6$)						
τ	-20	0	-	-	-	-
π	-3.8149	2.2118	-0.0896	-0.3196	-0.0825	-0.3097
R	0	0	-	-	-	-
y	0.2866	0.0096	0.7766	1.0000	0.1816	0.9814
h	0.2861	0.0021	0.8346	-0.3436	0.8467	-0.5097
c	0.2466	0.0095	0.7743	0.9663	-0.0636	0.9977
b	-2.2939	0.0823	0.7758	-0.9996	-0.1553	-0.9864
m	0.0947	0.0036	0.7743	0.9663	-0.0636	0.9977
ℓ	-2.1992	0.0788	0.7761	-0.9998	-0.1653	-0.9846

The parameters are: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

needs to be qualified in that a Ramsey planner operating at a steady state with a large net asset position against the private sector would implement a different policy. Specifically, if the interest earnings on assets are sufficient to finance public spending and the subsidy needed to correct the productive inefficiency due to monopolistic competition, then the policy maker implements zero nominal interest rates no matter whether there is commitment (Ramsey planner) or not (sequential planner).[8] For the Ramsey planner, though, there is no mechanism which determines the steady state debt-to-GDP ratio independently of the initial conditions. Hence, as Schmitt-Grohé and Uribe (2004a) correctly point out, their simulation results are conditional on the initial level of government liabilities. Conversely, starting from any initial asset position, the sequential planner accumulates assets until a non-distorted allocation can be implemented.

Besides this key difference between the nominal interest rates implemented by generic Ramsey versus Markov-perfect policies, we can report a number of further characteristics of optimal discretionary policies under flexible prices. As hinted above, average tax rates are non-positive and vary inversely with the gross markup $\theta/(\theta-1)$. More importantly, their standard deviation of zero indicates that not only nominal interest rates, but also the tax rates on labor are

[8]Table 6 in Appendix A provides results for the dynamic properties of the Ramsey allocation evaluated at the Markovian steady state. Note that our numerical algorithm to compute Ramsey allocations is virtually identical to the algorithm employed by Schmitt-Grohé and Uribe (2004b) and draws largely on computer code developed by these authors.

perfectly smooth. This implies that inflation serves as the main shock absorber, which works as a state-contingent lump-sum tax on households' financial wealth.[9] Indeed, while the volatility of inflation around the Markovian steady state is significantly lower than the one implied by the Ramsey allocation in Schmitt-Grohé and Uribe (2004a, 2004b), with a standard deviation of 2.21% it is still by far the most volatile variable in our economy. The serial correlation of inflation rates is close to zero which, in turn, suggests that the inflation tax is mainly used in response to unanticipated changes in the aggregate state of the economy.

4.4 Results for the sticky price economy

We now turn to the discussion of optimal time-consistent monetary and fiscal policies under sticky prices. In an environment of sticky prices and nominal non-state contingent debt, the policy maker faces a trade-off in choosing the optimal path of inflation. On the one hand, unexpected inflation serves as a non-distortionary tax on nominal wealth and helps to minimize tax distortions over the business cycle. On the other hand, deviations from zero in the rate of inflation are costly due to the price adjustment costs faced by firms. In this context, Schmitt-Grohé and Uribe (2004b) and Siu (2004) obtain two key results. First, they establish that, in the presence of nominal rigidities, the above trade-off is largely resolved in favor of price stability. Accordingly, the Ramsey planner chooses inflation rates that are close to zero at all dates and all states. Second, they argue that sticky prices introduce a near-random walk behavior of taxes and public debt. In what follows, we analyze whether these two results carry over to the sequential policy maker.

4.4.1 Optimal volatility of taxes and inflation

In analogy to what happens in the Ramsey problem, the Friedman rule ceases to be optimal also for the sequential policy maker once we allow for sticky prices. This holds despite the fact that the Markovian steady state involves a large net asset position (the debt-to-GDP ratio is at -8.02), which should help to minimize the need for distortionary taxes. However, when facing the trade-off underlying the choice of optimal inflation rates, the sequential planner still overwhelmingly opts for price stability. Table 3 illustrates this finding and further corroborates the view that the cyclical properties of optimal policies are remarkably robust to variations in the policy maker's intertemporal commitment capability, provided that the Ramsey policies are evaluated at the Markovian steady state.[10] This suggests that, in the neighborhood of this steady state, the lack of intertemporal commitment is not a binding restriction for a sequential planner. Indeed, optimal inflation rates are on average close to zero and display very little variation, irrespective of whether the policy maker has access to commitment or not. More precisely, annual inflation rates implemented by a sequential planner fall in a tight band between roughly -0.5% and 0.1%. On the other hand, with a standard deviation of 2.79% the labor tax rate is relatively volatile, ranging from approximately -12% to -28%. The intuition behind these results is the same as for the Ramsey policy maker. When price changes are costly,

[9] Compare Chari et al. (1991).

[10] Again, see Table 6 in Appendix A.

the policy maker chooses a low and stable inflation rate, which comes at the cost of relatively volatile labor income taxes.

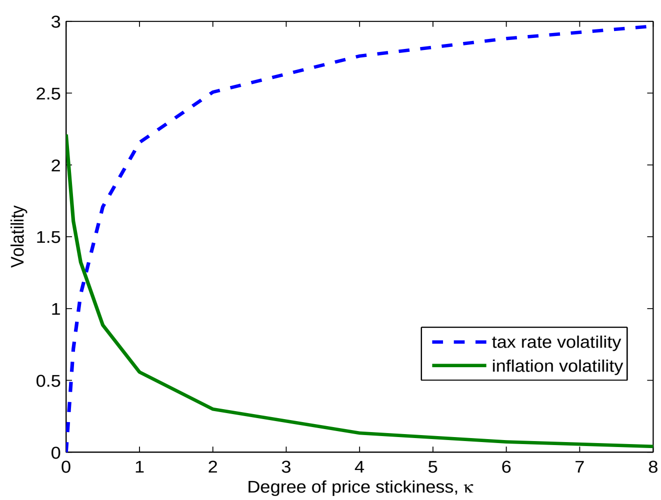
Table 3: Dynamic properties of the Markov-perfect equilibrium allocation under sticky prices

Variable	Mean	Std. dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,a)
τ	-20.0408	2.7859	0.4582	0.5046	0.1145	0.8270
π	-0.2051	0.1177	0.4419	-0.4742	-0.1317	-0.8042
R	3.7780	0.5108	0.4572	-0.2794	0.2525	0.1057
y	0.2844	0.0063	0.9349	1.0000	0.1838	0.8733
h	0.2841	0.0057	0.5214	-0.4928	0.2079	-0.8508
c	0.2441	0.0063	0.9190	0.9208	-0.1886	0.8687
b	-2.2810	0.0482	0.9241	-0.9965	-0.2139	-0.8977
m	0.0770	0.0030	0.7962	0.7515	-0.2587	0.5051
ℓ	-2.2039	0.0461	0.9161	-0.9917	-0.2410	-0.9049

The parameters are: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\theta = 6$, $\kappa = 17.5/4$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

To further illustrate the sequential policy maker's trade-off between inflation and tax volatility in the presence of price adjustment costs, Figure 1 displays the volatility of both policy instruments as a function of the parameter κ. As in Schmitt-Grohé and Uribe (2004b), we find that optimal policy stabilizes inflation already for relatively low levels of price stickiness. Furthermore, as κ grows, the optimal volatility of inflation approaches zero, whereas the volatility of taxes is increasing in κ. In light of these results, we conclude that the volatilities of taxes and inflation implemented by a sequential policy maker under sticky prices are virtually identical to the respective volatilities implemented by a Ramsey planner.

Figure 1: Effects of price stickiness on inflation and tax volatility



A final observation relates to the serial correlation properties of inflation. In contrast to our findings for the flexible price environment, where inflation displayed only little persistence at a serial correlation of -0.09, with sticky prices the serial correlation of inflation is considerable at 0.44. The intuition for this finding is that the government's large net asset position at the Markovian steady state facilitates the implementation of monetary and fiscal policies which allow a high degree of consumption smoothing. Indeed, the consumption allocation in the Markov-perfect equilibrium is characterized by little volatility and high persistence. As a consequence, also the real interest rate displays substantial persistence. In conjunction with price rigidity, which implies that optimal nominal interest rates are not too volatile, this calls for persistent inflation via the Fisher relationship.[11]

4.4.2 Dynamic properties of taxes and public debt

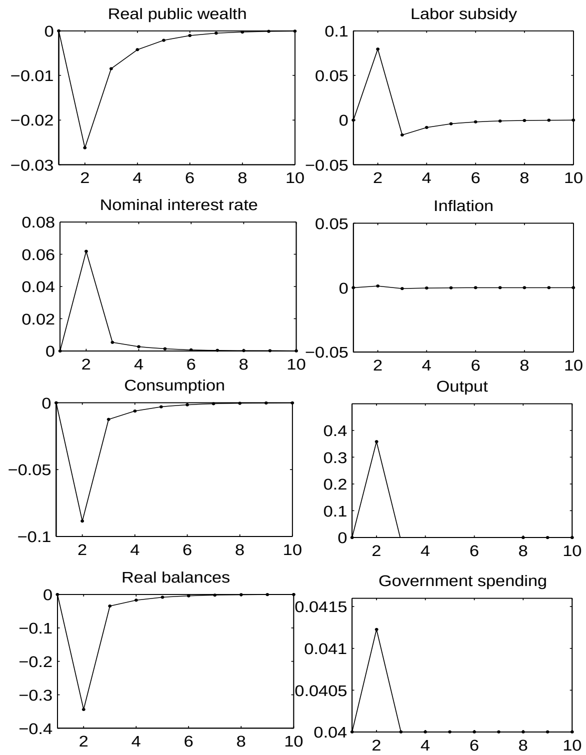
Price volatility is a way of introducing state-contingency in the real returns on nominal government debt. Under flexible prices the government can use changes in the price level as a non-distortionary tax or transfer on private asset positions. So, nominal non-state-contingent debt becomes state-contingent in real terms. Indeed, Chari et al. (1991) demonstrate that the Ramsey allocation in a flexible price economy with nominally non-state-contingent debt behaves exactly like that of an economy with real state-contingent debt. Seen from this perspective, introducing price adjustment costs works as if the government was limited in its ability to issue real state-contingent debt. This is reflected in the finding of the Ramsey literature that the presence of price adjustment costs implies a pronounced drop in the optimal volatility of inflation. Our results above indicate that this property carries over to the case of optimal time-consistent policies.

In the following, we therefore investigate for the sticky price economy whether taxes and public debt implemented by the sequential policy maker also display the near-random walk behavior that characterizes optimal Ramsey policies. To address this question, we analyze the economy's reaction to an uncorrelated government spending shock. More precisely, we study the implications of a one-standard deviation shock to government spending on public debt/wealth, labor taxes/subsidies, nominal interest rates, inflation, consumption, output, and real money holdings.

The impulse responses drawn in Figure 2 reveal the following pattern. The existence of price adjustment costs implies that the planner is reluctant to vary the inflation rate in response to the unanticipated expenditure shock. However, the labor subsidy is increased, thus boosting labor supply and output in order to absorb the increased public expenditure with a moderate reduction in private consumption. To finance these policies, the government must reduce its asset position. Hence, real public wealth decreases. Most importantly, however, the effects on taxes and public wealth have some persistence, but are far from the near-random walk behavior reported in Schmitt-Grohé and Uribe (2004b) and Siu (2004). In other words, the stochastic properties of the Markovian allocation fall in between the polar cases where taxes and public debt either inherit the stochastic process of the underlying exogenous shock or display a unit-root. The former case results under complete markets, i.e., in an economy with real state-contingent debt or, equivalently, an economy with nominal non-state-contingent debt

[11]See Chugh (2007) for a discussion of this effect in an economy with capital and/or habit persistence.

Figure 2: Impulse response to an uncorrelated government spending shock.



Note: The size of the innovation in government spending is one standard deviation. Public wealth, consumption, output, and real balances are measured in percent deviations from their pre-shock levels. The labor subsidy, the nominal interest rate, and inflation are measured in percentage points. Government spending is measured in levels.

and flexible prices (Lucas and Stokey, 1983). The latter case corresponds to an economy with non-state-contingent debt in real terms or, equivalently, a sticky price economy with nominally non-state-contingent debt (Aiyagari et al., 2002).

The reason behind the limited persistence appears to be the interplay between two forces. On the one hand, the stability of optimal inflation under sticky prices implies that there is only a minimal state-contingency of nominal debt in real terms. This generates the persistence in the response of real debt levels to the purely transitory exogenous spending shock. On the other hand, the model with a sequential policy maker features a model-inherent mechanism that ensures convergence to the locally unique and asymptotically stable steady state. With a Ramsey policy maker, the model lacks such a mechanism. Quite to the contrary, the latter admits a continuum of steady states, each of which is associated with different values of the initial state variables (s_{-1}, z_{-1}). Hence, in response to a spending shock the Ramsey economy simply moves from one steady state to another, thus generating a unit root in taxes and real debt. In the Markov-perfect equilibrium of the economy without commitment, however, monetary and fiscal policies are dynamically adjusted such as to bring these variables to their pre-shock (steady state) values over time. Paired with the sticky price effect, which prevents an immediate adjustment via variations in the rate of inflation, the convergence to the Markovian steady state generates the dynamics observed in Figure 2.

4.4.3 The effects of market power

Our benchmark calibration underlying Table 3 features a demand elasticity θ consistent with price markups over marginal costs of 20%. In order to assess the sensitivity of optimal time-consistent policies to changes in the degree of market power, we now let the parameter θ vary such as to induce markups of 10% and 30%, respectively. Inspection of Tables 3 and 4 demonstrates that average labor subsidies increase from 9.7% over 20.0% to 30.2% as monopoly markups increase. As we have already hinted above, this reflects the government's objective to counteract the inefficiently low labor supply present due to monopolistic competition. While average taxes are sensitive to changes in the competitive environment, inflation and nominal interest rates remain almost unaffected. In particular, optimal inflation rates are negative and close to zero, thus striking a balance between the distortions of positive nominal interest rates and the resource cost of price adjustments.[12] This policy is feasible because the sequential planner accumulates a larger net asset position against the private sector if monopoly distortions call for higher labor subsidies. Indeed, when monopolistic markups increase from 10% over 20% to 30%, the debt-to-GDP ratio in the Markovian steady state changes from -5.99 over -8.02 to -9.72. Higher interest earnings on the government's net assets then finance the increased subsidy without the need to resort to additional revenues from the inflation tax. This mechanism again highlights the importance of keeping track of the variation in the endogenous state variables when discussing the implications of changes in the economic environment on optimal monetary and fiscal policies.

4.4.4 Variations in elasticities of substitution

Finally, we investigate whether the preference parameters ϑ and η affect the dynamic properties of Markov-perfect equilibrium policies. Table 5 summarizes simulated time series statistics for several different values of ϑ^{-1}, the intertemporal elasticity of substitution in consumption, and η^{-1}, the Frisch elasticity of labor supply. A first observation is that changes in these elasticities have only relatively small effects on the average levels of tax and inflation rates. In contrast, they have a strong influence on the steady state generated by these policies. The intuition for this is as follows. A high ϑ implies a low consumption elasticity. This means that the cost of reduced consumption as a consequence of distortionary policies is relatively high such that the policy maker accumulates more assets in order to minimize these distortions. Similarly, a high η implies a low elasticity of labor supply. Hence, the distortions in terms of lost output due to the taxation of the returns from labor (via the wage tax or the inflation tax) are relatively low, and there is less need to accumulate a large net asset position.[13] These considerations are reflected in the Markov-perfect equilibrium allocations summarized in Table 5. As predicted, we find that, in absolute terms, the government's real net asset position increases with ϑ and

[12] In addition, we find that the volatility of taxes and interest rates is increasing in the markup, whereas the volatility of inflation does hardly change.

[13] See Martin (2007) for a further discussion of these aspects of Markov-perfect equilibria in the context of a flexible price economy with nominal government debt.

Table 4: Effects of market power on the Markov-perfect equilibrium allocation

Variable	Mean	Std. dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,a)
Low markup ($\theta = 11$)						
τ	-9.7271	2.4353	0.4982	0.5999	0.1632	0.8378
π	-0.2296	0.1119	0.4818	-0.5665	-0.1876	-0.8110
R	3.7484	0.4110	0.6203	-0.4102	0.3535	-0.1416
y	0.2839	0.0064	0.9202	1.0000	0.1559	0.9116
h	0.2836	0.0052	0.5593	-0.5936	0.1961	-0.8693
c	0.2436	0.0065	0.9024	0.9238	-0.2097	0.8995
b	-1.6997	0.0349	0.9326	-0.9895	-0.2783	-0.8628
m	0.0770	0.0030	0.8790	0.8118	-0.2989	0.6737
ℓ	-1.6227	0.0327	0.9271	-0.9821	-0.3254	-0.8598
High markup ($\theta = 13/3$)						
τ	-30.2323	3.0268	0.4381	0.4476	0.0915	0.8202
π	-0.1925	0.1203	0.4214	-0.4178	-0.1058	-0.7986
R	3.7932	0.5820	0.3919	-0.2431	0.2015	0.2013
y	0.2846	0.0063	0.9370	1.0000	0.1986	0.8487
h	0.2843	0.0060	0.5014	-0.4338	0.2132	-0.8408
c	0.2443	0.0063	0.9228	0.9199	-0.1759	0.8479
b	-2.7662	0.0595	0.9189	-0.9891	-0.1784	-0.9142
m	0.0771	0.0031	0.7279	0.7194	-0.2286	0.4060
ℓ	-2.6891	0.0575	0.9102	-0.9833	-0.1972	-0.9234

The parameters are: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\kappa = 17.5/4$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

decreases with η. Note, however, that the relevant implications are reversed when expressed in terms of debt-to-GDP ratios. The reason for this is that the changed elasticities also impinge on labor supply and output at the steady state.

5 Concluding remarks

In environments with imperfect competition and nominal debt, fiscal and monetary policy is typically subject to the well-known time-consistency problem. The Ramsey approach to optimal policy circumvents this problem by assuming that policy makers have access to a commitment technology. In this framework, several interesting properties of optimal fiscal and monetary policy have been established. For example, it has been shown that the optimal policy in environments with nominal rigidities implements stable prices.

In this paper, we have investigated optimal fiscal and monetary policies presuming that there is no commitment technology available to the policy maker. We have analyzed time-consistent optimal policies (in the sense of Markov-perfect equilibria) in environments of both flexible and sticky prices. Our first central finding is that, if prices are flexible, the Friedman rule is optimal irrespective of the degree of monopolistic competition. The intuition behind this result

Table 5: Effects of elasticities on the Markov-perfect equilibrium allocation

Variable	Mean	Std. dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,a)
$\vartheta = 1.5$						
τ	-19.0705	3.1559	0.4992	0.5726	0.0850	0.8452
π	-0.3085	0.1294	0.4800	-0.5338	-0.1041	-0.8168
R	3.6671	0.4426	0.5954	-0.3728	0.2198	-0.0431
y	0.3898	0.0069	0.9272	1.0000	0.1831	0.8981
h	0.3894	0.0087	0.6205	-0.6951	0.1477	-0.9388
c	0.3494	0.0068	0.9213	0.9323	-0.1604	0.8980
b	-2.6617	0.0611	0.9335	-0.9981	-0.1388	-0.8921
m	0.1108	0.0036	0.8609	0.7651	-0.2197	0.5533
ℓ	-2.5509	0.0582	0.9278	-0.9995	-0.1596	-0.9016
$\vartheta = 0.5$						
τ	-21.0050	2.4625	0.3709	0.3164	0.1691	0.7725
π	-0.0613	0.0712	0.3580	-0.2948	-0.1832	-0.7531
R	3.9308	0.7960	0.2813	-0.2016	0.2873	0.2762
y	0.1296	0.0045	0.9208	1.0000	0.2815	0.7750
h	0.1295	0.0030	0.4317	0.2456	0.4295	-0.4080
c	0.0895	0.0044	0.8709	0.8442	-0.2456	0.7822
b	-1.6336	0.0358	0.9058	-0.9537	-0.3604	-0.8788
m	0.0281	0.0019	0.6613	0.7034	-0.3122	0.4276
ℓ	-1.6055	0.0348	0.8953	-0.9393	-0.3878	-0.8782
$\eta = 1.5$						
τ	-20.2859	3.4119	0.5197	0.2750	0.0997	0.7470
π	-0.1704	0.1257	0.5143	-0.2276	-0.1109	-0.7112
R	3.7396	0.6053	0.5835	-0.4135	0.2225	0.0788
y	0.2585	0.0061	0.9291	1.0000	0.1834	0.8143
h	0.2583	0.0057	0.5660	-0.3000	0.2025	-0.7934
c	0.2183	0.0061	0.9135	0.9160	-0.2028	0.8059
b	-2.1649	0.0490	0.9190	-0.9979	-0.1950	-0.8379
m	0.0691	0.0033	0.8302	0.7719	-0.2327	0.4204
ℓ	-2.0958	0.0465	0.9129	-0.9944	-0.2218	-0.8517
$\eta = 0.5$						
τ	-20.0754	2.4805	0.4621	0.4993	0.1194	0.8284
π	-0.2424	0.1279	0.4431	-0.4628	-0.1379	-0.8014
R	3.7403	0.5010	0.4545	-0.2839	0.2423	0.1170
y	0.3214	0.0071	0.9360	1.0000	0.1745	0.8705
h	0.3210	0.0065	0.5198	-0.4890	0.1953	-0.8516
c	0.2810	0.0071	0.9215	0.9369	-0.1573	0.8694
b	-2.4440	0.0514	0.9256	-0.9956	-0.2226	-0.8928
m	0.0888	0.0034	0.7925	0.7647	-0.2335	0.4976
ℓ	-2.3551	0.0490	0.9171	-0.9897	-0.2500	-0.9010

Unless indicated otherwise, the parameters are: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\theta = 6$, $\kappa = 17.5/4$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

is that, over time, the sequential government accumulates enough assets to do away with the distortions introduced by monopolistic competition. In particular, the government subsidizes labor effort to equate real wages to the marginal product of labor.

Under sticky prices, we find that the optimal volatilities of taxes and inflation are similar to the corresponding volatilities in the Ramsey equilibrium. Independently of the government's net asset position, the existence of resource costs due to non-zero and volatile inflation induces optimal policy to focus on price stability. The cyclical properties of optimal policies are robust to changes in the degree of market power as well as to changes in the household's preferences.

One important difference between the solution to the Ramsey problem and the Markov perfect equilibrium allocation under sticky prices, however, concerns the dynamic behavior of taxes and real debt. In the Ramsey equilibrium, taxes and debt follow a near-random walk in response to an expenditure shock. In the Markov-perfect equilibrium, the dynamics of these two variables strike a balance between the persistence resulting from the low variability in the price level on the one hand, and the smooth convergence to the unique steady state on the other hand.

The key mechanism behind our results is the endogenous selection of a locally unique and asymptotically stable steady state in the Markov-perfect equilibrium. The sequential policy maker dynamically manipulates the economy's endogenous state variable such as to affect the incentive constraints faced by its future incarnations. This implies that the steady state is such that the time-consistency problem of the government becomes vacuous. In our particular context, the government accumulates sufficient claims against the private sector to cover its exogenous spending requirements without the need to resort to distortionary taxation. Indeed, the steady state of the Markov-perfect equilibrium allocation involves a significant positive net asset position for the government. This counterfactual property of the Markov-perfect equilibrium allocation may be viewed as a shortcoming of the present analysis. An interesting next step in this research programme would therefore be to introduce politico-economic frictions that typically lead to excessive fiscal deficits. In this way, we could perhaps move towards a positive theory of fiscal and monetary policy under the assumption of no commitment.

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Appendix

A Dynamic properties of the Ramsey allocation at the Markovian steady state

Table 6 below displays the dynamic properties of the Ramsey allocation evaluated at the steady state obtained in the Markov-perfect equilibrium.

Table 6: Dynamic properties of the Ramsey allocation around the Markovian steady state

Variable	Mean	Std. dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,a)
Flexible prices ($\kappa = 0$)						
τ	-20	0	-	-	-	-
π	-3.8426	2.1877	-0.0921	-0.3087	-0.0235	-0.3126
R	0	0	-	-	-	-
y	0.2866	0.0095	0.7672	1.0000	0.0959	0.9953
h	0.2861	0.0014	0.4578	-0.7049	0.6305	-0.7696
c	0.2464	0.0095	0.7738	0.9913	-0.0318	0.9992
b	-2.2889	0.0811	0.7737	-0.9909	0.0345	-0.9991
m	0.0946	0.0036	0.7738	0.9913	-0.0318	0.9992
ℓ	-2.1943	0.0771	0.7734	-0.9909	0.0347	-0.9991
Sticky prices ($\kappa = 17.5/4$)						
τ	-19.9392	2.0997	0.5335	0.5322	0.1100	0.7552
π	-0.2884	0.0785	0.3735	-0.5207	-0.1235	-0.6712
R	3.6873	0.4645	0.7028	-0.5952	0.1775	-0.4980
y	0.2842	0.0074	0.8936	1.0000	0.1818	0.9297
h	0.2839	0.0045	0.5770	-0.5326	0.3030	-0.8001
c	0.2439	0.0073	0.8883	0.9424	-0.1358	0.9366
b	-2.2675	0.0810	0.9818	-0.5507	-0.0859	-0.3659
m	0.0772	0.0035	0.9030	0.8773	-0.1658	0.8310
ℓ	-2.1904	0.0795	0.9825	-0.5179	-0.0958	-0.3339

Parameters: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\theta = 6$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

Supplementary Appendices

B Derivation of the household's first-order conditions

This Supplementary Appendix derives the household's optimality conditions stated in subsection 3.1. Note first that the household's problem is formally given by

$$V(s_t^i, s_t, z_t; \Omega) = \max_{m_t^i, b_t^i, c_t^i, h_t^i, \tilde{h}_t^i, p_t^i} \left\{ u(c_t^i, h_t^i) + \beta E_t V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega) \right\}$$
$$+\lambda_t^i \left[\frac{\ell_{t-1}}{\pi_t} + (1-\tau_t) w_t h_t^i + (p_t^i)^{1-\theta} y_t - w_t \tilde{h}_t^i - \frac{\kappa}{2} \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1 \right)^2 - c_t^i [1 + s(v_t^i)] - m_t^i - \frac{b_t^i}{1+R_t} \right]$$
$$+mc_t \lambda_t^i \left[a_t \tilde{h}_t^i - \left(p_t^i\right)^{-\theta} y_t \right],$$

where λ_t^i and $mc_t \lambda_t^i$ denote the Lagrangian multipliers on the household's budget and production constraints, respectively. Differentiating the above expression with respect to the decision variables yields the following set of first-order conditions:

$$0 = \beta E_t \frac{\partial V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega)}{\partial m_t^i} - \lambda_t^i [1 - (v_t^i)^2 s'(v_t^i)],$$
$$0 = \beta E_t \frac{\partial V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega)}{\partial b_t^i} - \lambda_t^i \frac{1}{1+R_t},$$
$$0 = \frac{\partial u(c_t^i, h_t^i)}{\partial c_t^i} - \lambda_t^i [1 + s(v_t^i) + v_t s'(v_t^i)],$$
$$0 = \frac{\partial u(c_t^i, h_t^i)}{\partial h_t^i} + \lambda_t^i (1-\tau_t) w_t,$$
$$0 = -\lambda_t^i w_t + mc_t \lambda_t^i a_t,$$
$$0 = \beta E_t \frac{\partial V(s_{t+1}^i, s_{t+1}, z_{t+1}; \Omega)}{\partial p_t^i} + \lambda_t^i (1-\theta)(p_t^i)^{-\theta} y_t - \lambda_t \kappa \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1 \right) \frac{\pi_t}{p_{t-1}^i} + mc_t \lambda_t^i \theta (p_t^i)^{-\theta-1} y_t.$$

Furthermore, from the envelope theorem we get

$$\frac{\partial V(s_t^i, s_t, z_t; \Omega)}{\partial m_{t-1}^i} = \frac{\partial V(s_t^i, s_t, z_t; \Omega)}{\partial b_{t-1}^i} = \frac{\partial V(s_t^i, s_t, z_t; \Omega)}{\partial \ell_{t-1}^i} = \frac{\lambda_t^i}{\pi_t},$$
$$\frac{\partial V(s_t^i, s_t, z_t; \Omega)}{\partial p_{t-1}^i} = -\lambda_t^i \kappa \left(\frac{p_t^i \pi_t}{p_{t-1}^i} - 1 \right) \left(-\frac{p_t^i \pi_t}{(p_{t-1}^i)^2} \right).$$

Combining envelope and first-order conditions, we obtain the system of equations stated in subsection 3.1.

C Private sector equilibrium conditions

In the following we list the private sector equilibrium conditions. We require that households are representative. This implies $p_t^i = 1$ for all t and all i, and we can drop the index i in our further analysis. Let the utility function be given by (13) and the transaction cost function by (14). Recall that, since policies are required to be feasible, we can replace the household's budget constraint by an aggregate resource constraint and the policy feasibility constraint. The conditions associated with the household's problem are thus given by:

$$
\begin{aligned}
0 &= c_t^{-\vartheta} - \lambda_t[1 + 2Av_t - 2\sqrt{AB}], \\
0 &= -\delta_h(1-h_t)^{-\eta} + \lambda_t(1-\tau_t)w_t, \\
0 &= Av_t^2 - B - \left(1 - \frac{1}{1+R_t}\right), \\
0 &= \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t}{1+R_t}, \\
0 &= \lambda_t \pi_t(\pi_t - 1) - \beta E_t \lambda_{t+1}(\pi_{t+1}-1)\pi_{t+1} - \frac{\lambda_t \theta a_t h_t}{\kappa}\left(\frac{w_t}{a_t} + \frac{1-\theta}{\theta}\right), \\
0 &= a_t h_t - c_t[1 + Av_t + B\frac{1}{v_t} - 2\sqrt{AB}] - g_t - \frac{\kappa}{2}(\pi_t - 1)^2, \\
0 &= \tau_t w_t h_t + m_t + b_t/(1+R_t) - \frac{\ell_{t-1}}{\pi_t} - g_t, \\
0 &= v_t - \frac{c_t}{m_t}, \\
0 &= \ell_t - m_t - b_t.
\end{aligned}
$$

D The sequential planner's problem in primal form

Let $x_{t+1} = (\ell_t, a_{t+1}, g_{t+1})$. Using a primal approach and the results from Supplementary Appendix C, the maximization problem of a benevolent single government without commitment power then reads as follows:

$$
\begin{aligned}
&\tilde{V}(\ell_{t-1}, a_t, g_t) \\
= \quad & \max_{c_t, h_t, \ell_t, m_t, b_t, \pi_t, w_t, \tau_t, R_t, \lambda_t, v_t} \left\{ \frac{c_t^{1-\vartheta} - 1}{1-\vartheta} + \delta_h \frac{(1-h_t)^{1-\eta} - 1}{1-\eta} + \beta E_t \tilde{V}(\ell_t, a_{t+1}, g_{t+1}) \right\} \\
& + \chi_t^1 \left[c_t^{-\vartheta} - \lambda_t \left(1 + 2Av_t - 2\sqrt{AB} \right) \right] \\
& + \chi_t^2 \left[-\delta_h (1-h_t)^{-\eta} + \lambda_t (1-\tau_t) w_t \right] \\
& + \chi_t^3 \left[Av_t^2 - B - \frac{R_t}{1+R_t} \right] \\
& + \chi_t^4 \left[\beta E_t \frac{\lambda(x_{t+1})}{\pi(x_{t+1})} - \lambda_t \frac{1}{1+R_t} \right] \\
& + \chi_t^5 \left[\kappa \lambda_t \pi_t (\pi_t - 1) - \kappa \beta E_t \lambda(x_{t+1}) [\pi(x_{t+1}) - 1] \pi(x_{t+1}) - \lambda_t \theta a_t h_t \left(\frac{w_t}{a_t} + \frac{1-\theta}{\theta} \right) \right] \\
& + \chi_t^6 \left[a_t h_t - c_t \left(1 + Av_t + B\frac{1}{v_t} - 2\sqrt{AB} \right) - g_t - \frac{\kappa}{2} (\pi_t - 1)^2 \right] \\
& + \chi_t^7 \left(\tau_t w_t h_t + m_t + b_t/(1+R_t) - \frac{\ell_{t-1}}{\pi_t} - g_t \right) \\
& + \chi_t^8 \left(v_t - \frac{c_t}{m_t} \right) \\
& + \chi_t^9 (\ell_t - m_t - b_t),
\end{aligned}
$$

where χ_t^1-χ_t^9 denote the multipliers on the respective implementability constraints. The first-order conditions resulting from this optimization problem are given by

$$
\begin{aligned}
0 &= c_t^{-\vartheta} - \vartheta \chi_t^1 c_t^{-\vartheta-1} - \chi_t^6 \left(1 + Av_t + B\frac{1}{v_t} - 2\sqrt{AB} \right) - \chi_t^8 \frac{1}{m_t}, \\
0 &= -\delta_h (1-h_t)^{-\eta} - \chi_t^2 \eta \delta_h (1-h_t)^{-\eta-1} - \chi_t^5 \lambda_t \theta a_t \left(\frac{w_t}{a_t} + \frac{1-\theta}{\theta} \right) + \chi_t^6 a_t + \chi_t^7 \tau_t w_t, \\
0 &= \beta E_t \frac{\partial \tilde{V}(\ell_t, a_{t+1}, g_{t+1})}{\partial \ell_t} + \chi_t^4 \beta \frac{\partial E_t \frac{\lambda(x_{t+1})}{\pi(x_{t+1})}}{\partial \ell_t} - \chi_t^5 \kappa \beta \frac{\partial E_t \lambda(x_{t+1})(\pi(x_{t+1}) - 1)\pi(x_{t+1})}{\partial \ell_t} + \chi_t^9, \\
0 &= \chi_t^7 + \chi_t^8 \frac{c_t}{m_t^2} - \chi_t^9, \\
0 &= \frac{\chi_t^7}{i_t} - \chi_t^9, \\
0 &= \chi_t^5 \kappa \lambda_t (2\pi_t - 1) - \chi_t^6 \kappa (\pi_t - 1) + \chi_t^7 \frac{\ell_{t-1}}{\pi_t^2},
\end{aligned}
$$

$$
\begin{aligned}
0 &= \chi_t^2\lambda_t(1-\tau_t) - \chi_t^5\lambda_t\theta h_t + \chi_t^7\tau_t h_t,\\
0 &= -\chi_t^2\lambda_t w_t + \chi_t^7 w_t h_t,\\
0 &= -\chi_t^3\frac{1}{(1+R_t)^2} + \chi_t^4\lambda_t\frac{1}{(1+R_t)^2} - \chi_t^7 b_t\frac{1}{(1+R_t)^2},\\
0 &= -\chi_t^1\left(1+2Av_t-2\sqrt{AB}\right) + \chi_t^2(1-\tau_t)w_t - \chi_t^4\frac{1}{1+R_t}\\
&\quad + \chi_t^5\left[\kappa\pi_t(\pi_t-1) - \theta a_t h_t\left(\frac{w_t}{a_t}+\frac{1-\theta}{\theta}\right)\right],\\
0 &= -2A\chi_t^1\lambda_t + 2A\chi_t^3 v_t - \chi_t^6 c_t\left(A-\frac{B}{v_t^2}\right) + \chi_t^8.
\end{aligned}
$$

Finally, the envelope condition is

$$
\frac{\partial\tilde{V}(\ell_{t-1},a_t,g_t)}{\partial\ell_{t-1}} = -\frac{\chi_t^7}{\pi_t}.
$$

Using the envelope condition allows us, after some algebraic manipulations, to rewrite the first-order conditions from above in the following way. This system of equations fully characterize the policy maker's optimal decision rules.

$$
\begin{aligned}
0 &= c_t^{-\vartheta} - \vartheta\chi_t^1 c_t^{-\vartheta-1} - \chi_t^6\left(1+Av_t+B\frac{1}{v_t}-2\sqrt{AB}\right) - \chi_t^8\frac{1}{m_t},\\
0 &= -\delta_h(1-h_t)^{-\eta} - \chi_t^2\eta\delta_h(1-h_t)^{-\eta-1} - \chi_t^5\lambda_t\theta a_t\left(\frac{w_t}{a_t}+\frac{1-\theta}{\theta}\right) + \chi_t^6 a_t + \chi_t^7\tau_t w_t,\\
0 &= -\beta E_t\frac{\chi^7(x_{t+1})}{\pi(x_{t+1})} + \chi_t^4\beta\frac{\partial E_t\frac{\lambda(x_{t+1})}{\pi(x_{t+1})}}{\partial\ell_t} - \chi_t^5\kappa\beta\frac{\partial E_t\lambda(x_{t+1})(\pi(x_{t+1})-1)\pi(x_{t+1})}{\partial\ell_t} + \frac{\chi_t^7}{1+R_t},\\
0 &= \chi_t^7\frac{R_t}{1+R_t} + \chi_t^8\frac{c_t}{m_t^2},\\
0 &= \chi_t^5\kappa\lambda_t(2\pi_t-1) - \chi_t^6\kappa(\pi_t-1) + \chi_t^7\frac{\ell_{t-1}}{\pi_t^2},\\
0 &= \chi_t^7 - \chi_t^5\lambda_t\theta,\\
0 &= -\chi_t^2\lambda_t + \chi_t^7 h_t,\\
0 &= -\chi_t^3 + \chi_t^4\lambda_t - \chi_t^7 b_t,\\
0 &= -\chi_t^1\left(1+2Av_t-2\sqrt{AB}\right) + \chi_t^2(1-\tau_t)w_t - \chi_t^4\frac{1}{1+R_t}\\
&\quad + \chi_t^5\left[\kappa\pi_t(\pi_t-1) - \theta a_t h_t\left(\frac{w_t}{a_t}+\frac{1-\theta}{\theta}\right)\right],\\
0 &= -2A\chi_t^1\lambda_t + 2A\chi_t^3 v_t - \chi_t^6 c_t\left(A-\frac{B}{v_t^2}\right) + \chi_t^8,\\
0 &= c_t^{-\vartheta} - \lambda_t\left(1+2Av_t-2\sqrt{AB}\right),\\
0 &= -\delta_h(1-h_t)^{-\eta} + \lambda_t(1-\tau_t)w_t,
\end{aligned}
$$

$$
\begin{aligned}
0 &= Av_t^2 - B - \frac{R_t}{1+R_t}, \\
0 &= \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t}{1+R_t}, \\
0 &= \lambda_t \pi_t (\pi_t - 1) - \beta E_t \lambda_{t+1} (\pi_{t+1} - 1) \pi_{t+1} - \frac{\lambda_t \theta a_t h_t}{\kappa} \left(\frac{w_t}{a_t} + \frac{1-\theta}{\theta} \right), \\
0 &= a_t h_t - c_t \left(1 + Av_t + B \frac{1}{v_t} - 2\sqrt{AB} \right) - g_t - \frac{\kappa}{2} (\pi_t - 1)^2, \\
0 &= \tau_t w_t h_t + m_t + b_t/(1+R_t) - \frac{\ell_{t-1}}{\pi_t} - g_t, \\
0 &= v_t - \frac{c_t}{m_t}, \\
0 &= \ell_t - m_t - b_t.
\end{aligned}
$$

E Numerical Strategy

The numerical approach we employ to solve for Markov-perfect equilibrium policies proceeds in two steps. In the first step, the non-stochastic steady state of the model is (approximately) computed based on a first-order accurate perturbation method. This allows us to determine the relevant state space, which facilitates the implementation of global solution methods. Finally, we use Galerkin projection methods to compute a third-order accurate solution of the policy maker's problem.[14]

Before going into details, let us first introduce some more notation. In particular, let $\varepsilon^{\lambda} = \frac{\partial \lambda(x^*)}{\partial l}$ and $\varepsilon^{\pi} = \frac{\partial \pi(x*)}{\partial l}$ denote the derivatives of the respective decision rules evaluated at the steady state.

To compute the (approximate) steady state, we employ an iterative procedure based on linearization methods. We proceed as follows:[15]

1.1 We guess initial values for the derivatives at the steady state. In particular, we set these (initial) derivatives to $\varepsilon_0^{\lambda} = 0$ and $\varepsilon_0^{\lambda} = 0$.

1.2 Given ε_0^{λ} and ε_0^{λ}, we solve the non-linear system of equations stated in Supplementary Appendix F for the implied steady state x^*.

1.3 Given the steady state x^*, we compute a first-order perturbation solution. We use the decision rules for λ and π to obtain new values for the derivatives, which we denote by ε_1^{λ} and ε_1^{π}.

1.4 We compute $\varsigma = |\varepsilon_1^{\lambda} - \varepsilon_0^{\lambda}| + |\varepsilon_1^{\pi} - \varepsilon_0^{\pi}|$. If ς is sufficiently small (i.e., smaller than 10^{-6}), we stop. Else we set $\varepsilon_0^{\lambda} = \varepsilon_1^{\lambda}$, $\varepsilon_0^{\pi} = \varepsilon_1^{\pi}$ and return to Step 1.2.

Having obtained the (approximate) steady state, we solve the model globally using projection methods. In particular, we follow the monomial Galerkin approach illustrated in Pichler (2007). For models with several state variables, this method is roughly equivalent to the traditional projection approach pioneered by Judd (1992) in terms of accuracy, but achieves convergence faster. Again, we proceed in several steps.

2.1 We decide upon the set of policies that are approximated with parametric forms. In the present application, we find it convenient to choose the decision rules for i, τ, λ, π, and the multiplier χ^7, respectively.

2.2 We postulate flexible functional forms. To be precise, we approximate each decision rule with a linear combination of Chebyshev polynomials, which are defined recursively as

$$T_0(y) = 1, \quad T_1(y) = y, \quad T_j(y) = 2yT_{j-1}(y) - T_{j-2}(y), \quad j = 2, 3, ...$$

[14] Our MATLAB computer code is available upon request.

[15] We do not formally establish convergence of this iterative procedure. However, in all our numerical examples, the algorithm did converge without posing any challanges.

We use a complete basis of polynomials up to order $p = 3$, such that our approximations take the following form:

$$\begin{aligned}
\hat{i}(\ell, a, g; \nu^i) &= \sum_{j_1=0}^{p} \sum_{j_2=0}^{p-j_2} \sum_{j_3=0}^{p-j_1-j_2} \nu^i_{j_1,j_2,j_3} \mathcal{T}_{j_1,j_2,j_3}(x), \\
\hat{\tau}(\ell, a, g; \nu^\tau) &= \sum_{j_1=0}^{p} \sum_{j_2=0}^{p-j_2} \sum_{j_3=0}^{p-j_1-j_2} \nu^\tau_{j_1,j_2,j_3} \mathcal{T}_{j_1,j_2,j_3}(x), \\
\hat{\lambda}(\ell, a, g; \nu^\lambda) &= \sum_{j_1=0}^{p} \sum_{j_2=0}^{p-j_2} \sum_{j_3=0}^{p-j_1-j_2} \nu^\lambda_{j_1,j_2,j_3} \mathcal{T}_{j_1,j_2,j_3}(x), \\
\hat{\pi}(\ell, a, g; \nu^\pi) &= \sum_{j_1=0}^{p} \sum_{j_2=0}^{p-j_2} \sum_{j_3=0}^{p-j_1-j_2} \nu^\pi_{j_1,j_2,j_3} \mathcal{T}_{j_1,j_2,j_3}(x), \\
\hat{\chi}^7(\ell, a, g; \nu^\chi) &= \sum_{j_1=0}^{p} \sum_{j_2=0}^{p-j_2} \sum_{j_3=0}^{p-j_1-j_2} \nu^\chi_{j_1,j_2,j_3} \mathcal{T}_{j_1,j_2,j_3}(x).
\end{aligned}$$

Here, ν^k collects the coefficients associated with decision rule k, and $\mathcal{T}_{j_1,j_2,j_3}(x)$ is the multivariate Chebyshev polynomial, i.e.,

$$\mathcal{T}_{j_1,j_2,j_3}(x) = T_{j_1}(\xi_\ell(\ell))T_{j_2}(\xi_a(a))T_{j_3}(\xi_g(g)).$$

In the above expression, ξ_{x_j} maps the state space for variable x_j into $[-1, 1]$, i.e., $\xi_{x_j} = 2(x_j - \underline{x}_j)/(\bar{x}_j - \underline{x}_j) - 1$. The application of this linear transformation requires to postulate some upper and lower bounds on each state variable. To facilitate convergence of our algorithm to a solution, these bounds must be chosen such that they cover the steady state. If this steady state vector was unknown, we would have to select the bounds following a trial-and-error strategy.[16] In our setting, however, the approximate steady state is known from the iterative procedure 1.1-1.4. Consequently, we can simply set $\underline{x}_j$ and $\bar{x}_j$ such that our relevant state space is a symmetric interval around the steady state. The precise size of the interval is chosen such that the economy stays within the bounds during the simulations.

2.3 We choose the projection conditions. Since we employ a Galerkin method, these conditions are given by

$$\int_{[-1,1]^3} R(x; \nu)\mathcal{T}_{j_1,j_2,j_3}(x) = 0, \quad j_1, j_2, j_3 \in \{0, 1, 2, 3\},\ j_1 + j_2 + j_3 \leq p,$$

where R summarizes the equilibrium conditions and $\nu = (\nu^i, ..., \nu^\chi)$. We use monomial rules as suggested in Pichler (2007) to evaluate the above integrals. This gives us a system of nonlinear equations which can be solved for the unknown coefficients ν.

The quality of numerical solutions can be assessed using standard measures of numerical accuracy. In particular, we investigate the Euler equation errors as suggested by Judd (1992).

[16] See, e.g., Ortigueira (2006).

The average and maximum error measures resulting from our simulations are summarized in Table 7. We find that our numerical solutions are sufficiently accurate, with average errors never exceeding 0.03% and the maximum errors below 0.5% for all model variants considered.

Table 7: Accuracy measures

Case	Average error	Maximum error
Benchmark	-4.0	-3.4
$\kappa = 0$, $\theta = \infty$	-4.9	-4.2
$\kappa = 0$	-5.1	-4.5
$\theta = 11$	-4.1	-3.4
$\theta = 13/3$	-4.3	-3.9
$\vartheta = 1.5$	-4.1	-3.6
$\vartheta = 0.5$	-4.1	-3.1
$\eta = 1.5$	-3.5	-2.3
$\eta = 0.5$	-4.3	-3.9

Note: Error measures are in base 10 logarithms; unless indicated otherwise, the parameters are: $\beta = 1/1.04$, $\vartheta = 1$, $\eta = 1$, $\delta_h = 2.9$, $\kappa = 17.5/4$, $\theta = 6$, $\rho_a = 0.82$, $\bar{a} = 1$, $\sigma_a = 0.0229$, $\rho_g = 0.9$, $\bar{g} = 0.04$, $\sigma_g = 0.0302$, $A = 0.0111$, $B = 0.07524$, $J = 500$, $T = 100$.

F Proof of Proposition 1

Proof: Imposing steady state on the system of optimality conditions listed at the end of Supplementary Appendix D and the two exogenous laws of motion (1) and (4) yields the following system of non-linear equations.

$$
\begin{aligned}
0 &= c^{-\vartheta} - \vartheta\chi^1 c^{-\vartheta-1} - \chi^6 \left(1 + Av + B\frac{1}{v} - 2\sqrt{AB}\right) - \chi^8 \frac{1}{m}, \\
0 &= -\delta_h(1-h)^{-\eta} - \chi^2\eta\delta_h(1-h)^{-\eta-1} - \chi^5\lambda\theta a\left(\frac{w}{a} + \frac{1-\theta}{\theta}\right) + \chi^6 a + \chi^7\tau w, \\
0 &= -\beta\frac{\chi^7}{\pi} + \chi^4\beta\frac{\partial[\lambda(x)/\pi(x)]}{\partial\ell} - \chi^5\kappa\beta\frac{\partial\{\lambda(x)[\pi(x)-1]\pi(x)\}}{\partial\ell} + \frac{\chi^7}{1+R}, \\
0 &= \chi^7\frac{R}{1+R} + \chi^8\frac{c}{m^2}, \\
0 &= \chi^5\kappa\lambda(2\pi-1) - \chi^6\kappa(\pi-1) + \chi^7\frac{\ell}{\pi^2}, \\
0 &= \chi^7 - \chi^5\lambda\theta, \\
0 &= -\chi^2\lambda + \chi^7 h, \\
0 &= -\chi^3 + \chi^4\lambda - \chi^7 b, \\
0 &= -\chi^1\left(1 + 2Av - 2\sqrt{AB}\right) + \chi^2(1-\tau)w - \frac{\chi^4}{1+R} + \chi^5\left[\kappa\pi(\pi-1) - \theta ah\left(\frac{w}{a} + \frac{1-\theta}{\theta}\right)\right], \\
0 &= -2A\chi^1\lambda + 2A\chi^3 v - \chi^6 c[A - \frac{B}{v^2}] + \chi^8, \\
0 &= c^{-\vartheta} - \lambda\left(1 + 2Av - 2\sqrt{AB}\right), \\
0 &= -\delta_h(1-h)^{-\eta} + \lambda(1-\tau)w, \\
0 &= Av^2 - B - \frac{R}{1+R}, \\
0 &= \beta\frac{\lambda}{\pi} - \frac{\lambda}{1+R}, \\
0 &= \kappa\lambda\pi(\pi-1) - \kappa\beta\lambda(\pi-1)\pi - \lambda\theta ah\left(\frac{w}{a} + \frac{1-\theta}{\theta}\right), \\
0 &= ah - c\left(1 + Av + B\frac{1}{v} - 2\sqrt{AB}\right) - g - \frac{\kappa}{2}(\pi-1)^2, \\
0 &= \tau wh + m + b/(1+R) - \frac{\ell}{\pi} - g, \\
0 &= v - \frac{c}{m}, \\
0 &= \ell - m - b, \\
0 &= (1-\rho_a)\log\bar{a} + \rho_a\log a - \log a, \\
0 &= (1-\rho_g)\log\bar{g} + \rho_g\log g - \log g.
\end{aligned}
$$

Presuming that $\kappa = 0$, it is easy to show that the allocation given in Proposition 1 satisfies this system of equilibrium conditions. □