Habit formation
and the transmission of financial crises

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Abstract

We study how external habit formation by investors affects the transmission of financial crises. Habit formation increases the effective risk premium on assets when there is a negative wealth shock and introduces non-linearities which can lead to multiple equilibria. We embed this investor's behavior in the Jeanne (1997) model which allows for a competitiveness effect and for contagion through changes in fundamentals. Habit formation, however, can lead to transmission of financial crises even in the absence of the competitiveness effect, and makes multiple equilibria more likely. The possible stabilization effects of capital controls and a Tobin tax on the international transmission of financial crises are also discussed.

Keywords: Financial crises; contagion; habit formation; international asset pricing; capital controls; Tobin tax.

JEL Classification: D91, F31, F32, G11, G15.

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1 Introduction

One of the main features of the financial crises of the nineties was their tendency to spread across countries. The Mexican devaluation of 1994 affected other Latin American countries (the Tequila effect); the currency crisis in Thailand of July 1997 spread across East Asia; several months after the Russian crisis of August 1998, the Brazilian crisis ended up with the floatation of Real.

In this paper we model investors' behavior as affected by (a slow moving) external habit based on past aggregate consumption (see Abel (1990) and Campbell and Cochrane (1999)) rather than an internal habit based on past own consumption (see Constantinides (1990) and Sundaresan (1989)). One of the key insights of this class of models is that as investors' wealth and consumption fall to the habit level, the effective curvature of the utility function increases making investors act as if they were more risk averse. This increases the risk premium that investors need in order to hold risky assets. Through this mechanism, financial linkages can act as an independent source of transmission of financial crises. In fact, given a financial crisis in one country, if the wealth of investors decreases sufficiently they will demand higher risk premia on assets of other countries even when the fundamentals of these countries are unchanged. This will increase the coverage ratio, make debt servicing more difficult and trigger a financial crisis. Note that the effect of habit formation is similar to the Goldstein and Pauzner (2004) assumption of decreasing risk aversion. However, in our model habits drive the attitude towards risk, microfounding the decreasing risk aversion effect.

It is important to remark that the traditional way of modeling investors' behavior, i.e. by CARA preferences, cannot account for these wealth effects. With such specification of the utility function following a financial crisis investors will increase the demand for assets whose returns are negatively correlated to their wealth (which is lower as a consequence of the crisis) - this, in turn, will reduce risk premia on other countries' risky assets (*ceteris paribus*) and serve to limit the spread of the crisis. However, this outcome is counterfactual since the evidence shows that risk premia, which can be measured by bond spreads, increase in the event of a financial crisis (see, among others, Eichengreen *et al.* (2001)).

We elaborate on this behavioral approach in the context of a model of speculative attacks with self-fulfilling expectations built on Jeanne (1997) and Masson (1999). The paper innovates in several ways. First, it provides an explicit microeconomic foundation based on habit formation of the investors' behavior triggering contagion. Asymmetric information is not necessary to explain contagion. Second, the portfolio choices deriving from the investor's optimizing behavior add a further channel of contagion to the monsoonal and spillover effects introduced by Masson (1999). Finally, the explicit modeling of the risk premium required by the investor introduces an additional source of nonlinearity in the multiple equilibria framework. In our model even without a trade competitiveness effect crises can still be transmitted by a feedback effect on investors' behavior.

One of the implications of this model is that it is the very nature of free international capital movements that introduces the risk of financial contagion. To see whether restrictions on such movements can reduce contagion we consider the effect of capital controls and Tobin taxes, i.e. taxes levied on foreign exchange transactions.

The problem of contagion, or transmission of financial crises, has been addressed by a growing body of literature in the last decade (recent surveys include Pericoli and Sbracia (2003) and Williamson (2004)). Most of the papers tend to split the explanations of contagion between those related to fundamentals and those related to the investors' behavior.

Theories of contagion based on fundamentals' deterioration mostly refer to trade and financial links as the main channels of transmission of shocks. In Gerlach and Smets (1995), for example, contagion occurs through a loss of competitiveness of country B as a consequence of the devaluation of country A's exchange rate, in a bilateral trade link framework. Glick and Rose (1999) find that trade patterns and competitiveness on third markets can indeed be the rationale for contagion and its regional character. On the other hand, Kaminsky and Reinhart (2000) show that the role of financial linkages among countries is crucial to understand the regional attribute of the Asian crisis of 1997 and the Latin American troubles of 1994-95. This may be due to the necessity of commercial banks to adjust their lending to their lower level of wealth following a crisis in one country in order to reduce their overall risk. Kaminsky *et al.* (2001) study US based mutual funds and find evidence of feedback behavior and mutual funds selling assets in emerging markets in the event of a crisis. Van Rijckeghem and Weder (2001) test for the relative strength of the trade linkages and macroeconomic factors as opposed to the spillovers from a common lender. They find that financial linkages generally have predictive power even when trade and macroeconomic factors are taken into account. Their subsequent paper, Van Rijckeghem and Weder (2003), specifies that it is the banks' exposure to a crisis country that has predictive power. The intuition is that as a financial crisis occurs in an origin country the banks with the largest exposures will face potential losses, and hence will need to restore capital asset ratios, meet margin calls, or readjust risk exposures. Financial channels may work also through simple portfolio management rules (Schinasi and Smith (2001)) and portfolio rebalancing effects may be magnified by asymmetric information (Kodres and Pritsker (2002)). In the Russian crisis of 1998 the turmoil was not restricted within a same regional or financial bloc, but rather spread across countries, like Brazil, with small direct economic connections to Russia. Baig and Goldfajn (2001) find empirical support to the hypothesis of contagion through financial links. Eichengreen *et al.* (2001) study the behavior of emerging market bonds in times of crisis and find evidence of "flight to quality". The literature, thus, suggests that common lenders effects and financial linkages are very important in understanding the transmission of financial crises.

"Pure contagion" occurs when the transmission of the crisis cannot be related to changes in fundamentals. In this case contagion turns out to be the outcome of investors' behavior, specifically a shift in market sentiments due to a change in risk aversion, changes in the interpretation given to existing information that may lead to a "wake up call", or changes in behavior due to other considerations. In the literature the three main channels for "pure contagion" are (i) A self-fulfilling bank run (Sachs *et al.* (1996), and Allen and Gale (1998)), which leads to the inefficient liquidation of assets. (ii) Information acquisition (Goldstein and Pauzner (2004)) and herding behavior (Chari and Kehoe (2004)). Goldstein and Pauzner (2004) use the idea of global games to show that investors receive correlated signals on which they update their beliefs. This model does not rely on multiple equilibria and a unique probability of crisis can be calculated. The model however relies crucially on decreasing risk aversion. They demonstrate that the strategic interaction of investors holding equities of two different countries can generate contagious withdrawal of money from one country (the relation of this paper to our model is discussed further below). (iii) Jumps between multiple equilibria which arise in the underlying macroeconomic model (Jeanne (1997), Masson (1999), and Jeanne and Masson (2000)). In such situations contagion can arise because of, or can be exacerbated by, self fulfilling beliefs of private agents: crises spread just because agents believe they are going to spread. For example, Masson (1999), building on Jeanne (1997), shows that a second generation model of speculative attacks can generate multiple equilibria as a consequence of the interaction between the government and the private sector, with contagion effects emerging as the expectations of devaluation of a competitor country's currency enter the model.

To sum up, the empirical literature suggests that both trade and financial linkages are important with perhaps the latter being the most important channel. The theoretical literature stresses that "pure contagion" is also likely to occur. Our paper builds on this. We model contagion as arising from investors' behavior in full information, frictionless markets through a feedback effect based on habit formation. This is embedded in a second generation model of currency crises such that both financial and trade linkages can be considered simultaneously.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses policy measures commonly regarded to as stabilizing devices. Section 4 concludes.

2 The model

This section derives the risk premium from an international asset pricing model with habit formation and then embeds it into a second generation model of currency crises *à la* Jeanne (1997). In modeling the investor's behavior we follow closely the specification of Campbell and Cochrane (1999). There are several reasons why treating the investor as habit forming can be justified in our context. Firstly, this specification has been used to explain the equity premium puzzle in closed economies. Thus, a similar specification should be used when understanding the asset prices in an open economy context. This is a behavioral view. Secondly, if we are to interpret investors as large financial intermediaries or large firms, then behavior similar to habit formation may be generated. There are two different reasons for this. The first is that if we interpret "consumption" as payouts to claimants (either depositors or shareholders) then financial intermediaries will not want the payouts to drop below some level. Cuts in the level of the payout is costly. The second is that since we can interpret the "utility function" as that of fund managers of financial intermediaries, if the fund managers' performance is evaluated relative to a benchmark which depends on past average performance, they will not want performance to fall behind the benchmark level so that a similar behavior will be generated. There is a third class of reasons why the habit formation can be rationalized. This has to do with portfolio insurance and risk management practices of large investors. Under portfolio insurance, investors do not want the value of the portfolio to fall below some level, i.e. they do not want the withdrawals from the portfolio (consumption in our model) to fall below some threshold. The portfolio insurance models have the similar property that as the prices of assets fall the allocation to them falls as well.[1] The close relationship between portfolio insurance and habit formation has already been noted in the literature (see, for example, Campbell and Viceira (2002)). Thus, there are three entirely different mechanisms through which the investors' behavior can be rationalized.[2]

There are N identical price-taking international investors and M countries. The time horizon is infinite. The representative international investor maximizes the period utility flow which depends on current consumption and is affected by external habit formation.

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s - X_s) \right\} \quad (1)$$

where E_t is the expectation operator conditional on information available at date t, $\beta \in \{0, 1\}$ is the constant subjective time-preference factor, $u(\cdot)$ is the period utility function which is assumed to be thrice-continuously differentiable, strictly increasing, and strictly concave, i.e. $u'(\cdot) > 0$, $u''(\cdot) < 0$, C_s is real consumption on period s, X_s is the habit level and depends on the history of aggregate consumption. The price of the consumption good is normalized to 1.

Each country may issue either dollar-denominated and local currency-denominated debt. $*$ denotes dollar-denominated variables. The period-by-period budget constraint is given by:

$$B_{s+1}^{*f} + \sum_{m=1}^{M} x_{s+1}^{*m} B_s^{*m} + \sum_{m=1}^{M} x_{s+1}^{m} \frac{B_s^m}{e_s^m} = (1 + r_s^{*f}) B_s^{*f} + \sum_{m=1}^{M} x_s^{*m} \left(I_s^{*m} + B_s^{*m} \right) + \sum_{m=1}^{M} x_s^{m} \left(\frac{I_s^m + B_s^m}{e_s^m} \right) - C_s \quad (2)$$

where B_s^{*f} is the real net risk-free bond purchase at time $s-1$, x_s^{*m} and x_s^m are respectively the fractional shares of country m's dollar- and local

[1] See Brennan and Schwartz (1988) and Grossman and Zhou (1996) for a general equilibrium model with portfolio insurance.

[2] See Goldstein and Pauzner (2004) for further discussion of how such considerations can induce decreasing risk aversion.

currency-denominated debt purchased by the agent in period $s-1$, B_s^{*m} and B_s^m denote respectively the date s real market value of country m's dollar- and local currency-denominated debt, r_s^{*f} is the net real interest rate on the risk-free bond B_s^{*f} between period $s-1$ and s, I_s^{*m} and I_s^m are the coupons paid on country m's securities at time s, and e_s^m is time s spot exchange rate (price of dollars in terms of country m's currency). Equation (2) expresses the link between period s's saving and period $s+1$ financial wealth. One can think of B_s^{*f} as the net purchase of a United States Treasury bill.

The rationale for pricing the emerging market debt within the framework of a consumption-based CAPM, usually applied to equities, is threefold. Firstly, arbitrage on interest rates will equate returns on bonds to returns on equities through price adjustments, according to the Fisher equation (De La Grandville (2001), ch. 2). Secondly, if we assume that markets are complete, then the Modigliani-Miller theorem holds and the value of the portfolio is invariant to the debt-equity mix. Thirdly, dealing with emerging markets bonds requires taking into account default and exchange rate risk thus hiding the sharp distinction between stocks and bonds (Obstfeld and Rogoff (1996), ch. 5).

Maximizing the utility function (1) subject to the constraints (2) with respect to x_{s+1}^{*m}, x_{s+1}^m, and B_{s+1}^{*f}, gives the following Euler equations:

$$u'(C_s - X_s)B_s^{*m} = \beta E_s \left\{ u'(C_{s+1} - X_{s+1}) \left(I_{s+1}^{*m} + B_{s+1}^{*m} \right) \right\} \tag{3}$$

$$u'(C_s - X_s)\frac{B_s^m}{e_s^m} = \beta E_s \left\{ u'(C_{s+1} - X_{s+1}) \left(\frac{I_{s+1}^m + B_{s+1}^m}{e_{s+1}^m} \right) \right\} \tag{4}$$

and

$$u'(C_s - X_s) = (1 + r_{s+1}^{*f})\beta E_s \left[u'(C_{s+1} - X_{s+1}) \right] \tag{5}$$

Define the *ex post* net real rates of return on country m's dollar- and local currency-denominated risky bond as:

$$r_{t+1}^{*m} \equiv \frac{I_{t+1}^{*m}}{B_t^{*m}} + \frac{B_{t+1}^{*m} - B_t^{*m}}{B_t^{*m}}; \text{ and } r_{t+1}^m \equiv \frac{I_{t+1}^m}{B_t^m} + \frac{B_{t+1}^m - B_t^m}{B_t^m}$$

Therefore, from (3), recalling that $E(XY) = Cov(X, Y) + E(X)E(Y)$, we obtain:

$$
\begin{aligned}
u'(C_s - X_s) &= \beta Cov\left\{u'(C_{s+1} - X_{s+1}), (1 + r^{*m}_{s+1})\right\} \\
&\quad + \beta E_s\left[u'(C_{s+1} - X_{s+1})\right] E_s(1 + r^{*m}_{s+1}) \qquad (6)
\end{aligned}
$$

Dividing both sides by $u'(C_s - X_s)$, using (5) to substitute out $\beta E_s u'(C_{s+1} - X_{s+1})/u'(C_s - X_s)$, and rearranging, we obtain, for $s = t$:

$$
E_t(1 + r^{*m}_{t+1}) - (1 + r^{*f}_{t+1}) = -(1 + r^{*f}_{t+1}) Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r^{*m}_{t+1}\right\} \quad (7)
$$

Equation (7) is the crucial expression of the consumption-based international CAPM with habit formation. It says that, given the assumptions on the period utility function, the risk premium on asset m depends positively on the covariance of the asset's return with the surplus consumption growth. If the covariance term is negative, the risk premium will be positive, meaning that the asset yields unexpectedly high returns in states of nature when the level of surplus consumption is unexpectedly high. Therefore, the asset does not provide a hedge against consumption fluctuations and the investor will require an excess return with respect to the risk-free bond's return to be persuaded to hold the asset.

Following the same steps, from (4) we obtain:

$$
E_t\left[\frac{e^m_t}{e^m_{t+1}}(1 + r^m_{t+1})\right] - (1 + r^{*f}_{t+1}) = -(1 + r^{*f}_{t+1}) Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r^{*m}_{t+1}\right\} \quad (8)
$$

where we assume that:

$$
Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r^{*m}_{s+1}\right\} = Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, \frac{e^m_t}{e^m_{t+1}}(1 + r^m_{t+1})\right\}
$$

so that the risk premium is invariant across assets, either dollar- or local currency denominated, issued by the same country.

Proposition 1 *The local currency-denominated asset must yield a premium over the riskless rate of return to compensate the investor's risk aversion and the exchange rate risk.*

Proof. Let π^m_{t+1} be the probability of a devaluation of country m's currency occurring at time $t + 1$, and $\Delta e = \ln\left(e^m_{t+1}/e^m_t\right)$ the proportional, time-invariant, extent of such devaluation, where Δ is the difference operator. Hence, equating the l.h.s. of (7) and (8) and taking logarithms, yields:

$$E_t(r_{t+1}^m) = E_t(r_{t+1}^{*m}) + \pi_{t+1}^m \Delta e$$

where $r_{t+1}^m \approx \ln\left(1 + r_{t+1}^m\right)$. Adding and subtracting r_{t+1}^{*f} on the r.h.s. of the equation above, gives:

$$E_t(r_{t+1}^m) = r_{t+1}^{*f} + \rho_{t+1}^m + \pi_{t+1}^m \Delta e \tag{9}$$

where $\rho_{t+1}^m \equiv E_t(r_{t+1}^{*m}) - r_{t+1}^{*f}$ is the risk premium given by equation (7). ■

We assume the Campbell and Cochrane (1999) specification for the utility function with external habit formation. Thus, the utility function (equation (1)) becomes:

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{(C_s - X_s)^{1-\gamma} - 1}{1-\gamma} \right\} \tag{10}$$

Define the surplus consumption ratio $S_t \equiv (C_t - X_t)/C_t$ and let $s_t = \ln S_t$, and $c_t = \ln C_t$. We assume that the logarithm of the surplus follows an AR(1) process:

$$s_t = (1-\omega)\overline{s} + \omega s_{t-1} + \lambda(s_{t-1})(c_t - c_{t-1} - \overline{c}) \tag{11}$$

where $\overline{c}$ is the systematic component of consumption growth, with the latter given by:

$$\Delta c_t = \overline{c} + v_t, \quad v_t \sim i.i.d.N(0, \sigma^2).$$

The specification of s_t implies that consumption is always greater than habit, and we can let consumption affect habit differently in different states, as implied by $\lambda(s_{t-1})$. In other words, we allow surplus-consumption ratio to react slowly to changes in consumption in order to keep the risk-free rate constant and to avoid C falling short of X (see Campbell and Cochrane (1999)). Furthermore, consumption affects surplus differently in different states, as implied by $\lambda(s_{t-1})$. Note that the results of the paper do not depend on the specification of the process governing the evolution of s_t. We adopt the specification of Campbell and Cochrane (1999) so that the evolution of the risk-free rate is consistent with their paper.

Proposition 2 *The risk premium is negatively related to the surplus consumption ratio:*

$$E_t(r_{t+1}^{*m}) - r_{t+1}^{*f} \approx (1 + r_{t+1}^{*f})\beta \frac{\gamma}{S_t} Cov \left\{ \frac{(C_{t+1} - X_{t+1})}{C_t}, r_{t+1}^{*m} \right\}$$

Proof. We take a second order Taylor expansion at the points $C_{t+1} - X_{t+1} = C_t - X_t$ and $r_{t+1}^{*m} = E_t\left(r_{t+1}^{*m}\right)$ of the function

$$G(C_{t+1} - X_{t+1}, r_{t+1}^{*m}) \equiv \frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)} \left[r_{t+1}^{*m} - E\left(r_{t+1}^{*m}\right)\right]$$

the expected value of which equals the covariance entering the risk premium in equation (7).[3]

$$G\left[C_t - X_t, E_t\left(r_{t+1}^{*m}\right)\right] = \frac{\beta u'(C_t - X_t)}{u'(C_t - X_t)} \left[E_t\left(r_{t+1}^{*m}\right) - E_t\left(r_{t+1}^{*m}\right)\right] = 0;$$

$$\frac{\partial G\left[C_t - X_t, E_t\left(r_{t+1}^{*m}\right)\right]}{\partial(C_{t+1} - X_{t+1})} = \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} \left[E_t\left(r_{t+1}^{*m}\right) - E_t\left(r_{t+1}^{*m}\right)\right] = 0;$$

$$\frac{\partial G\left[C_t - X_t, E_t\left(r_{t+1}^{*m}\right)\right]}{\partial r_{t+1}^{*m}} = \frac{\beta u'(C_t - X_t)}{u'(C_t - X_t)} = \beta;$$

$$\frac{\partial^2 G\left[C_t - X_t, E_t\left(r_{t+1}^{*m}\right)\right]}{\partial(C_{t+1} - X_{t+1})^2} = \frac{\beta u'''(C_t^* - X_t^*)}{u'(C_t - X_t)} \left[E_t\left(r_{t+1}^{*m}\right) - E_t\left(r_{t+1}^{*m}\right)\right] = 0;$$

$$\frac{\partial^2 G\left[C_t - X_t, E_t\left(r_{t+1}^{*m}\right)\right]}{\partial\left(r_{t+1}^{*m}\right)^2} = 0;$$

[3] Recall that the second-order approximation to $G(X,Y)$ near $X = \overline{X}$ and $Y = \overline{Y}$ is:

$$\begin{aligned} G(X,Y) \simeq\ & G(\overline{X},\overline{Y}) + G_X(\overline{X},\overline{Y})(X - \overline{X}) + G_Y(\overline{X},\overline{Y})(Y - \overline{Y}) \\ & + \frac{1}{2}G_{XX}(\overline{X},\overline{Y})(X - \overline{X})^2 + \frac{1}{2}G_{YY}(\overline{X},\overline{Y})(Y - \overline{Y})^2 \\ & + G_{XY}(\overline{X},\overline{Y})(X - \overline{X})(Y - \overline{Y}). \end{aligned}$$

$$\frac{\partial^2 G\left[C_t - X_t, E_t\left(r^{*m}_{t+1}\right)\right]}{\partial(C_{t+1} - X_{t+1})\partial r^{*m}_{t+1}} = \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)};$$

Therefore:

$$\begin{aligned} G(C_{t+1} - X_{t+1}, r^{*m}_{t+1}) &\approx \beta\left[r^{*m}_{t+1} - E_t\left(r^{*m}_{t+1}\right)\right] + \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} \cdot \\ &\quad \left[C_{t+1} - X_{t+1} - (C_t - X_t)\right]\left[r^{*m}_{t+1} - E_t\left(r^{*m}_{t+1}\right)\right] \end{aligned} \tag{12}$$

Taking conditional expectations of both sides of (12), yields:

$$\begin{aligned} E_t\left[G(C_{t+1} - X_{t+1}, r^{*m}_{t+1})\right] &= Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r^{*m}_{t+1}\right\} \\ &\approx \frac{\beta u''(C_t - X_t)}{u'(C_t - X_t)} \cdot \\ &\quad E_t\{[(C_{t+1} - X_{t+1}) - (C_t - X_t)] \cdot \\ &\quad \left[r^{*m}_{t+1} - E_t\left(r^{*m}_{t+1}\right)\right]\} \\ &= \beta\frac{C_t u''(C_t - X_t)}{u'(C_t - X_t)} \cdot \\ &\quad Cov\left\{\frac{(C_{t+1} - X_{t+1})}{C_t}, r^{*m}_{t+1}\right\}. \end{aligned}$$

Hence equation (7) becomes:

$$E_t(r^{*m}_{t+1}) - r^{*f}_{t+1} \approx (1 + r^{*f}_{t+1})\beta\frac{\gamma}{S_t}Cov\left\{\frac{(C_{t+1} - X_{t+1})}{C_t}, r^{*m}_{t+1}\right\} \tag{13}$$

where

$$\frac{\gamma}{S_t} = \frac{-C_t u''(C_t - X_t)}{u'(C_t - X_t)}$$

denotes the local curvature of the utility function with habits. ■

Thus, it is clear that risk aversion is negatively related to the surplus consumption ratio: a fall in consumption towards the habit level will increase the time-varying local curvature of utility function and, hence, the risk premium required on the country m's asset.

2.1 Linkages through portfolio choice

In this subsection we show how the probability of a currency crisis in one country influences the risk premium through a wealth effect. We extend to an internationally diversified portfolio framework the log-linear approximation to the budget constraint proposed by Campbell (1993). This allows us to relate unexpected changes in consumption to changes in expectations about future returns.

Consider that the representative international investor's dynamic budget constraint (equation (2)) can be alternatively written as:

$$W_t^* = \left(W_{t-1}^* - C_{t-1}\right)(1 + r_t^{*w}) \tag{14}$$

where W_t^* denotes total real wealth (denominated in dollars) and $(1 + r_t^{*w})$ is defined to be the gross real return on wealth invested from period $t-1$ to period t. Given international portfolio diversification, the *ex post* gross return can be decomposed as follows:

$$(1 + r_t^{*w}) = q_t^{*f}(1 + r_t^{*f}) + \sum_{m=1}^{M} q_t^{*m}(1 + r_t^{*m}) + \sum_{m=1}^{M} q_t^{m} \frac{e_s^m}{e_{s+1}^m}\left(1 + r_{t+1}^m\right) \tag{15}$$

where q_t^{*f} is the proportion of wealth invested in the risk-free bond and q_t^{*m} and q_t^m are, respectively, the proportions of wealth invested in country m's dollar- and local currency-denominated assets at time $t-1$, implying that $q_t^{*f} + \sum_{m=1}^{M} q_t^{*m} + \sum_{m=1}^{M} q_t^m = 1$.

Taking logarithms of expectations of both sides of (15) gives:

$$\begin{aligned} E_{t-1}\left(r_t^{*w}\right) \quad \approx \quad & \log\{q_t^{*f}(1 + r_t^{*f}) + \sum_{m=1}^{M} q_t^{*m} \exp\left[E_{t-1}\left(r_t^{*m}\right)\right] \\ & + \sum_{m=1}^{M} q_t^m \exp\left[E_{t-1}\left(r_t^m\right) - \pi_{t+1}^m \Delta e\right]\} \end{aligned} \tag{16}$$

Proposition 3 *An unexpected decrease in wealth through an unexpected fall in current consumption and a decrease in the surplus consumption ratio, leads to an increase in the risk premium.*

Proof. Dividing (14) by W_{t-1}^* and taking logarithms, we obtain:

$$\Delta w_t^* \approx r_t^{*w} + \log\left[1 - \exp(c_{t-1} - w_{t-1}^*)\right] \tag{17}$$

where $r^{*w}_{t+1} \approx \ln\left(1 + r^{*w}_{t+1}\right)$.

Taking a first-order Taylor expansion around the mean $(\overline{c} - \overline{w}^{*})$ of the second term on the right hand side of (17) we get the following approximation to the budget constraint (14):

$$\Delta w^{*}_{t} \approx r^{*w}_{t} + k + \left(1 - \frac{1}{\eta}\right)(c_{t-1} - w^{*}_{t-1}) \tag{18}$$

where $k = \log(1 - \exp(\overline{c} - \overline{w}^{*})) - \left(1 - \frac{1}{\eta}\right)(\overline{c} - \overline{w}^{*})$, $\left(1 - \frac{1}{\eta}\right) = -\frac{-\exp(\overline{c} - \overline{w}^{*})}{1 - \exp(\overline{c} - \overline{w}^{*})}$, and $\eta \equiv 1 - \exp(\overline{c} - \overline{w}^{*})$.

Next, consider the equality:

$$\Delta w^{*}_{t} = \Delta c_{t} + (c_{t-1} - w^{*}_{t-1}) - (c_{t} - w^{*}_{t}) \tag{19}$$

Equating the left hand sides of (18) and (19), solving forward the resulting difference equation in $c_{t-1} - w^{*}_{t-1}$, assuming that $\lim_{j \longrightarrow \infty} \eta^{j}\left(c_{t+j} - w^{*}_{t+j}\right) = 0$, and taking expectations at time $t-1$ we obtain:

$$c_{t-1} - w^{*}_{t-1} = E_{t-1} \sum_{j=1}^{\infty} \eta^{j} (r^{*w}_{t-1+j} - \Delta c_{t-1+j}) + \frac{\eta k}{1 - \eta} \tag{20}$$

Finally, substitute out equation (20) into (18) and (19) to obtain:

$$\begin{aligned} c_{t} - E_{t-1} c_{t} &= (E_{t} - E_{t-1}) \sum_{j=0}^{\infty} \eta^{j} r^{*w}_{t+j} \\ &\quad - (E_{t} - E_{t-1}) \sum_{j=1}^{\infty} \eta^{j} \Delta c_{t+j} \end{aligned} \tag{21}$$

Recalling now that, by assumption, $(E_{t} - E_{t-1})\Delta c_{t+j} = 0$, for $j = 1, ..., \infty$, equation (21) simplifies to:

$$c_{t} - E_{t-1} c_{t} = (E_{t} - E_{t-1}) \sum_{j=0}^{\infty} \eta^{j} r^{*w}_{t+j} \tag{22}$$

Paraphrasing Campbell (1993), equation (22) indicates that an unexpected decrease in consumption today must be determined by an unexpected reduction of return on wealth today, as shown by the first term in the sum on the right hand side of the equation, or by news that future returns will be lower, as shown by the remaining terms in the sum.

The decrease in current consumption implies a decrease in the surplus consumption ratio through equation (11), and an increase in the risk premium by Proposition 1. ■

Equation (22) provides the link between the risk premium and the probability of devaluation of country m's currency. An increase in the probability of devaluation of country m's currency at time $t+1$ will decrease the expected return on total wealth $E_t\left(r_{t+1}^{*w}\right)$ through equation (16) which in turn will determine an unexpected decrease in consumption, as implied by equation (22), through bad news about future returns;[4] finally, this process will increase the excess return required by the investor through equation (13). Notably, the increase in the risk premium will involve *all* assets since the recession state will affect the investor's attitude towards risk. In the habit formation framework, a relatively small change in expected future wealth can lead to a large change in the risk premium. One can see this from equation (22) and equation (13). A small change in wealth will lead to a corresponding change in consumption from equation (22), but this can cause a large change in the surplus consumption ratio and hence, an increase in the risk premium (equation (13)).

2.2 Habit Formation and models of currency crises

Building on a second generation model of currency crises in the spirit of Jeanne (1997), and Bratsiotis and Robinson (2004), we will specify now the process through which the risk premium affects the probability of devaluation showing how the investor's portfolio choice can in itself be a source of transmission of financial crises.[5] In what follows contagion is implicitly defined as an increase in the probability of a crisis in one country given a crisis is occurring in another country, which is equivalent to Definition 1 in the classification proposed by Pericoli and Sbracia (2003).

Consider an emerging economy whose government pegs the currency to the US dollar.[6] As long as the country's debt is partly denominated in local currency, the government faces a policy dilemma as it has an incentive to

[4] This effect can be better seen by expanding the sum on the right-hand side of equation (22): a reduction in $E_t\left(r_{t+1}^{*w}\right)$ constitutes bad news about future returns since reduces the term $\theta(E_t - E_{t-1})r_{t+1}^{*w}$ of the sum.

[5] While the risk premium is derived in an infinitely lived agents framework, the following analysis focuses on a given two time period. This is consistent with the entire class of models of currency crises.

[6] In order to reduce notation, in this subsection we suppress the superscript denoting the country.

devalue in order to reduce the cost of debt. It will take any decisions about either maintaining the peg or not by minimizing the following quadratic loss function:[7]

$$L_{t+1} = (u_{t+1})^2 + (\Delta d_{t+1})^2 + \delta\Gamma_{t+1} \tag{23}$$

where u_{t+1} is the unemployment rate, Δd_{t+1} is the growth in government real debt proportional to GDP, δ is a dummy variable which is equal to 1 if devaluation occurs and 0 if the peg is maintained, and Γ_{t+1} denotes the exogenous cost of devaluing.

We assume that the dynamics of unemployment is determined by an expectations-augmented Phillips curve, that a relative purchasing power parity (between the emerging market's currency and the US dollar) holds and that foreign prices are normalized to unity, so that $\Delta p_{t+1} = \Delta e$, where $\Delta p_{t+1} = \ln(P_{t+1}/P_t) \approx \Delta P_{t+1}/P_t$, and P_t is the domestic price level:

$$u_{t+1} = \theta u_t - \zeta(\Delta e - E_t\Delta e).$$

As there is a devaluation with probability π_{t+1}, we see that $E_t\Delta e = \pi_{t+1}\Delta e$. Thus, we have:

$$u_{t+1} = \theta u_t - \zeta(\Delta e - \pi_{t+1}\Delta e) \tag{24}$$

with $\zeta > 0$ and $0 < \theta < 1$. We make the further assumptions that the rate of growth of real GDP is zero and that $r_{t+1} = E_t(r_{t+1}) + \epsilon_{r,t+1}$and $r^*_{t+1} = E_t(r^*_{t+1}) + \epsilon_{r^*,t+1}$ where $\epsilon_i \sim i.i.d.(0, \sigma_{\epsilon_i})$, for $i = r, r^*$ are stochastic shocks.

To derive the deficit, we consider the budget constraint of the consolidated government (including the central bank) as a proportion of GDP:

$$\begin{aligned}\frac{\Delta\widehat{B}_{t+1} + e_t(\Delta\widehat{B}^*_{t+1} - \Delta\widehat{F}^*_{t+1})}{P_tY_t} &= \frac{(\widehat{G}_t - \widehat{T}_t)}{P_tY_t} + r_{t+1}\frac{\widehat{B}_t}{P_tY_t} \\ &\quad + r^*_{t+1}e_t\frac{(\widehat{B}^*_t - \widehat{F}^*_t)}{P_tY_t}\end{aligned} \tag{25}$$

where $\widehat{B}_t$ and $\widehat{B}^*_t$ are the country's currency-denominated and US dollar-denominated stocks of bonds respectively, $\widehat{F}_t$ is the amount of official foreign reserves, $\widehat{G}_t$ and $\widehat{T}_t$ are the government expenditure and taxes, P_tY_t is the country's nominal GDP, and $\wedge$ indicates nominal variables, i.e. $\widehat{X} = PX$.

[7] This loss function is similar to that in Sachs *et al.* (1996), among others. The present formulation follows Jeanne (1997) and Bratsiotis and Robinson (2004).

Denoting the country's proportional debt as $d_t = D_t/Y_t = (\widehat{B}_t + e_t(\widehat{B}_t^* - \widehat{F}_t^*))/P_tY_t$, then the change in d can be approximated by the total differential:

$$\Delta d_{t+1} = \frac{\Delta \widehat{B}_{t+1} + e_t(\Delta \widehat{B}_{t+1}^* - \Delta \widehat{F}_{t+1}^*)}{P_tY_t} - (\Delta p_{t+1})\, d_t - e_t(f_t^* - b_t^*)\Delta e \quad (26)$$

where $f_t^* = \widehat{F}_t^*/\left(P_tY_t\right)$ and $b_t^* = \widehat{B}_t^*/\left(P_tY_t\right)$. Substituting equation (25) into (26), recalling that purchasing power parity holds, and rearranging, we obtain:

$$\Delta d_{t+1} = \overline{g}_t + \rho_{t+1} d_t - (\Delta e - \pi_{t+1}\Delta e)\,(d_t + z_t) \quad (27)$$

where $\overline{g} = g_t - \tau_t + (r_{t+1}^{*f} + \epsilon_{r,t+1})d_t + (\epsilon_{r,t+1} - \epsilon_{r^*,t+1})\, z_t$, $z_t = e_t(f_t^* - b_t^*)$, $g_t = \widehat{G}_t/\left(P_tY_t\right)$, and $\tau_t = \widehat{T}_t/\left(P_tY_t\right)$. From equation (27) we see how an unexpected devaluation decreases the cost of government debt by reducing the debt service and increasing the returns on net foreign assets. It is also evident that a rise in the risk premium increases the government deficit by driving up the expected return on debt.

Proposition 4 *The probability of devaluation of a country's currency depends on the risk premium asked by the investor on the country's asset*

Proof. The net benefit of maintaining the peg is given by the difference between the loss functions corresponding to the devaluation and to the fixed peg hypothesis, respectively:

$$\begin{aligned}
\overline{V}_{t+1} &= L_{t+1}^d - L_{t+1}^f \\
&= \left[(u_{t+1})^d\right]^2 + \left[(\Delta d_{t+1})^d\right]^2 + \Gamma_{t+1} - \left[(u_{t+1})^f\right]^2 - \left[(\Delta d_{t+1})^f\right]^2 \\
&= \Gamma_{t+1} + (\Delta e)^2\left[\zeta^2 + (d_t + z_t)\right]^2 - 2\theta\zeta u_t \Delta e \\
&\quad -2\left(\overline{g}_t + \rho_{t+1} d_t\right)(d_t + z_t)\,\Delta e \\
&\quad -2\left\{(\Delta e)^2\left[\zeta^2 + (d_t + z_t)\right]^2\right\}\pi_{t+1} \\
&= V_{t+1} - \varphi\rho_{t+1} - \alpha\pi_{t+1} \qquad (28)
\end{aligned}$$

where

$$V_{t+1} = \Gamma_{t+1} + \frac{1}{2}\alpha - 2\Delta e\left[\theta\zeta u_t + \overline{g}_t\,(d_t + z_t)\right]$$

is the gross benefit of maintaining the peg,

$$\varphi = 2d_t\,(d_t + z_t)\,\Delta e$$

and

$$\alpha = 2\left\{(\Delta e)^2\left[\zeta^2 + (d_t + z_t)\right]^2\right\}$$

Equation (28) says that the net benefit of keeping the peg depends not only on the fundamentals, summarized by the term V_{t+1}, but also on the credibility of the government commitment to it, indicated by π_{t+1}, and on the risk aversion of the international investor, as captured by the risk premium term ρ_{t+1}. Given the macroeconomic conditions, a lower credibility, as implied by a higher π_{t+1}, or a higher risk aversion, as implied by a higher ρ_{t+1}, reduces the benefit of the peg. The decision rule of the policymaker is derived optimally, taking into account the probability of devaluation π_{t+1} formulated by the private international investor, that is:

$$\phi_{t+1} = E_t\left(V_{t+1}\right) \tag{29}$$

and:

$$V_{t+1} - \phi_{t+1} = \epsilon_{t+1} \sim i.i.d.N(0, \sigma_\epsilon) \tag{30}$$

The variable ϕ_{t+1} summarizes the exogenous fundamentals affecting the probability of devaluation at time $t+1$.

The international investor, in turn, formulates rational expectations about the probability of devaluation as depending on the net benefit of the peg becoming negative, i.e.:

$$\pi_{t+1} = \Pr[\overline{V}_{t+1} < 0]$$

Using equations $(28)-(30)$, the probability of devaluation may be rewritten as:

$$\pi_{t+1} = \Pr[\epsilon_{t+1} < \alpha\pi_{t+1} + \varphi\rho_{t+1} - \phi_{t+1}]$$

or:

$$\pi_{t+1} = F[\alpha\pi_{t+1} + \varphi\rho_{t+1} - \phi_{t+1}] \tag{31}$$

where $F(\cdot)$ is the cumulative distribution function of $f(\cdot)$, with the latter being the density function of ϵ_{t+1}. ■

Equation (31) implies that, even at this stage, only taking into account the portfolio management rules of a risk-averse investor, contagious currency crises with self-fulfilling expectations can arise; in fact, the devaluation probability of country m, for example, depends on itself and on the risk premium on country m's asset, the latter depending, in turn, on the probability of devaluation of country m's and of all other countries in the investors' portfolios, as discussed above.

Notice that if we were to introduce an investor with CRRA preferences into the Jeanne's framework then the risk premium would be a constant. What habit formation does is to make the risk premium vary with shocks to expected wealth and hence to current consumption.

To give an intuitive idea of how the above model works, consider a representative investor holding risky assets issued by two emerging-market countries as well as a risk-free bond issued by a developed country (USA is a reasonable assumption). In keeping with the portfolio choice problems, the investor will require a risk premium on the risky assets to hold them. The risk premium is inversely related to unexpected changes in consumption, which in turn depends on the devaluation probability of emerging-market currencies. Hence, an increase in the devaluation probability of country m's currency will decrease the consumption of the investor who will in turn ask for a higher risk premium on all the portfolio assets, including those of country n. This will raise the probability of devaluation of country n's currency through equation (31), which in turn will widen the risk premium on country m's asset, and so forth, thus feeding a self-fulfilling expectation process. Hence, the international transmission of financial crises relies on portfolio re-balancing driven by wealth effects, and is magnified by self-fulfilling expectations. Finally, the wealth effect acts differently on the probability of devaluation according to the amount of debt accumulated by the emerging country, as equation (28)-(31) show.

Such a model can provide a theoretical framework useful to explain some of the empirical evidence reported in the introduction. For example, a commercial bank holding claims issued by a number of countries in the same region (e.g. East Asia) may be hit by the devaluation of one of its debtors' currency and see its financial wealth reduced. This may lead the bank to reassess the risk premium required on the bonds issued by all other countries as a consequence of the increase in effective risk aversion. The spreading of the currency crisis to the whole region is thus triggered by an investors' sentiment shift. On the other hand, international investors holding a geographically diversified portfolio can be induced to a reallocation of their wealth as it is hit by a currency devaluation, consistently with the strong correlation of emerging markets sovereign spreads documented by Baig and Goldfajn (2001). The same model can help explain why other financial crises did not spread from the origin country, as it happened for Argentina in 2001-

2002[8]. In that case, the sharp decline in the correlation between emerging markets sovereign spreads and stock indexes can be the result of a previous reallocation of the international investors' portfolios away from Argentine assets triggered by frequent signals of instability over the months preceding December 2001, the conventional starting date of the Argentine crisis.

2.3 Market segmentation and regional contagion

Most of contagion episodes in the nineties had a regional character, involving countries belonging to the same economic and financial area, like Latin America, East Asia and the European Monetary System. A model of contagion, then, should be capable of accounting for such a feature. Our model, indeed, is able to explain why most of financial crises tend to spread to neighboring countries, at the same time as it can explain why other crises affect countries not related to the origin one. In the present model a change in the probability of devaluation affects the expected rate of return on the risky asset, which in turn affects the total rate of return on wealth invested through equation (16), but this effect will vary in accordance with the proportion of wealth invested in that specific asset, q_t^m. If markets are segmented, due to legal, regulatory or trade reasons, then the proportion of wealth invested in a certain country's asset will be substantial and this will increase the effect of a change in the probability of devaluation through a higher q_t^m. Moreover, the effect of an unexpected change in consumption through equation (13) will be larger the larger is the covariance of surplus-consumption with the rate of return on that country's asset. But, in turn, the latter is larger the larger is the market segmentation. In fact, the investor's consumption will be presumably more related to the rate of returns on a specific asset as the portfolio is not diversified efficiently. In other words, for the feedback mechanism here presented to be significant, it is important that investors have non-negligible holdings of assets in the relevant countries and they are not fully diversified. If investors are already on the efficient frontier then losses in one country will not lead them to re-optimize in a significant way. Shore and White (2002) show that with external habit formation investors may be incompletely internationally diversified. Their model focuses on the home equity bias. The intuition is that some investors may be constrained to hold domestic equity due to regulatory, agency or control reasons. If there is external habit formation, then other investors will mimic the domestic bias in their own portfolio leading to incomplete international diversification. This

[8] The lack of contagious effects from the Argentine crisis has been documented, among others, by the IMF (2002) and Boschi (2005).

idea can be extended to our model. If some investors hold assets in some countries due to trade, regulatory or ownership reasons, then other investors will also mimic their behavior leading to the incomplete diversification we need in our model. Goldstein and Pauzner (2004) give an alternative explanation for incomplete diversification as due to informational and operational advantages.

Notably, the above arguments contradicts the general wisdom according to which international investors may be concerned with the benefits of diversification: contagion - the argument goes - may prevent diversification from delivering its benefits exactly when they are needed most, that is during periods of crisis, as cross-country correlations of asset prices are significantly higher. Quite on the contrary, the present model suggests how important is the role of international portfolio diversification in reducing the probability of contagion.

Alternatively, the regional feature of contagion can be explained by heterogeneous investors. Li and Zhong (2005) model investors with country-specific external habit formation, which, in presence of weakly correlated cross-country business cycle synchronization, implies country-specific risk aversion and locally determined surplus consumption ratios. In such a framework, if the business cycle is synchronized at a regional level (see, among others, Loayza *et al.* (2001)), the comovement of the devaluation probabilities will happen at a regional level as well.

2.4 The range of fundamentals for multiple equilibria

Jeanne (1997) illustrates the conditions under which an equation similar to (31) may have multiple solutions. Such conditions require the fundamentals to lie within a certain interval $(\underline{\phi}, \overline{\phi})$. In this Subsection, following the Jeanne's line of argument, we show how the introduction of a risk premium in the Jeanne's model affects the conditions for multiple equilibria and the range of fundamentals.

Propositions 2 and 3 show that the risk premium is a function of the probability of devaluation, i.e.: $\rho_t = h(\pi_t)$. Hence, equation (31) can be rewritten as follows:

$$\pi = F[\alpha\pi + \varphi h(\pi) - \phi] \tag{32}$$

where time subscripts and country superscripts are omitted for simplicity. We assume that the density function $f(\cdot)$ is continuous, symmetric (i.e. $\forall\epsilon$, $f(-\epsilon) = f(\epsilon)$), and strictly increasing (decreasing) in $(-\infty, 0)$ $((0, \infty))$ respectively, that is it reaches its maximum at zero, where $\alpha\pi_0 + \varphi h(\pi_0) = \phi$.

The possible multiplicity of equilibria arises from the fact that both sides of equation (32) are increasing with π. Figure (1) shows the case of a unique equilibrium and figure (2) the case of multiple equilibria.[9] The 45° line plots the l.h.s. of equation (32), while the curve $C_\phi \equiv F[\alpha\pi + \varphi h(\pi) - \phi]$ plots the r.h.s. Equation (32) is satisfied at the intersections of the 45° line with C_ϕ. Following Jeanne (1997), if at its maximum the slope of C_ϕ is smaller than 1, i.e. $\frac{dC_\phi}{d\pi} < 1$, then it is such everywhere. Hence π is uniquely determined by ϕ and strictly decreasing with it. There may be multiple equilibria if $\frac{dC_\phi}{d\pi} > 1$ at its maximum. If this necessary condition holds, then there are two critical values of the fundamentals $\underline{\phi} < \overline{\phi}$ such that: if $\phi < \underline{\phi}$ or $\phi > \overline{\phi}$, the devaluation probability π is uniquely determined by (and strictly decreasing with) the fundamental ϕ. Therefore, the further condition for having multiple equilibria is that the fundamentals ϕ lie in the range $(\underline{\phi}, \overline{\phi})$. The critical values of fundamentals $\underline{\phi}$ and $\overline{\phi}$ are identified by the tangency points between the 45° line and C_ϕ (see figure (3)). Whether these tangency points lie to the right or to the left of the point of maximum value of the pdf $f(\cdot)$ cannot be determined analytically. As shown below, in fact, whether the value of π at which C_ϕ is tangent to the 45° line lies to the right or to the left of π_0 depends on the sign of $h''(\pi)$ which depends on the sign of $f'(\cdot)$ which in turn depends on whether π is on the right or on the left of π_0. Fortunately enough, it can be shown easily that the following results are independent of where the tangency points lie with respect to π_0. Therefore, we will make the further simplifying assumptions that $\overline{\pi} > \pi_0$, where $\overline{\pi}$ is the probability of devaluation corresponding to $\overline{\phi}$, and $\underline{\pi} < \pi_0$, with $\underline{\pi}$ defined correspondingly. This also allows an easier comparison to the derivation in Jeanne (1997).

We will now determine the upper bound $\overline{\phi}$ and lower bound $\underline{\phi}$ of the range of fundamentals within which multiple equilibria may arise.

Upper bound $\overline{\phi}$ The tangency conditions between the function $C_{\overline{\phi}} = F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}]$ and the 45° line are:

$$\overline{\pi} = F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}] \quad (33)$$

$$1 = f[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}] \cdot \left[\alpha + \varphi h'(\overline{\pi})\right] \quad (34)$$

From equation (34), recalling that by assumption $\overline{\pi} > \pi_0$, we have:

[9] We are grateful to Paul Masson for providing us with the Gauss codes of the graphs of his 1999 paper.

$$\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi} = f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right] \tag{35}$$

where $f^{-1}(\cdot) : [0, f(0)] \rightarrow \Re^{+}$ denotes the inverse function of $f(\cdot)$ that takes positive values.

Substituting out $\overline{\pi}$ and rearranging yields:

$$\begin{aligned} \overline{\phi} &= \alpha F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}] + \varphi h\left\{F[\alpha\overline{\pi} + \varphi h(\overline{\pi}) - \overline{\phi}]\right\} \\ &\quad - f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right] \end{aligned} \tag{36}$$

Finally, substituting out $\alpha\overline{\pi} + \varphi g(\overline{\pi}) - \overline{\phi}$ given by equation (35) into equation (36), we obtain:

$$\begin{aligned} \overline{\phi} &= \alpha F\left\{f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right]\right\} + \varphi h\left(F\left\{f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right]\right\}\right) \\ &\quad - f^{-1}\left[\frac{1}{\alpha + \varphi h'(\overline{\pi})}\right] \end{aligned} \tag{37}$$

Proposition 5 *If α and φ are large enough, the upper bound of the range of fundamentals for which there are multiple equilibria increases with the probability of devaluation.*

Proof. In order to prove this proposition we need to know how $h(\cdot)$ and $h'(\cdot)$ change with $\overline{\pi}$, i.e. we need to know the sign of $h'(\overline{\pi})$ and of $h''(\overline{\pi})$. We know from Proposition 3 that $h'(\overline{\pi}) > 0$, $\forall\overline{\pi}$: this is because an increase in the probability of devaluation decreases the expected rate of return on assets which in turn makes the risk premium go up through a wealth effect. From equations (33) and (35) we have:

$$\overline{\pi} = F\left(f^{-1}\left\{L\left[h'(\overline{\pi})\right]\right\}\right) \tag{38}$$

where:

$$L\left[h'(\overline{\pi})\right] = \frac{1}{\alpha + \varphi h'(\overline{\pi})} \tag{39}$$

We can sign $h''(\overline{\pi})$ applying the Implicit Function Theorem (IFT) to equation (38). Define:

$$G\left[\overline{\pi}, h'(\overline{\pi})\right] \equiv \overline{\pi} - F\left(f^{-1}\left\{L\left[h'(\overline{\pi})\right]\right\}\right)$$

Then:

$$\frac{dh'(\overline{\pi})}{d\overline{\pi}} = -\frac{\partial G/\partial \overline{\pi}}{\partial G/\partial h'(\overline{\pi})}$$

where:

$$\partial G/\partial \overline{\pi} = 1$$

and

$$\partial G/\partial h'(\overline{\pi}) = -\frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} < 0$$

since $\frac{\partial F}{\partial f^{-1}} > 0 \ \forall f^{-1}$ being F the cdf of a normal distribution; $\frac{\partial f^{-1}}{\partial L} < 0$ being f^{-1} the inverse function of a normal pdf that takes positive values and, by the Inverse Function Theorem, $\left(f^{-1}\right)' = 1/f'$; and, finally, being from equation (39):

$$\frac{\partial L}{\partial h'} = \frac{-\varphi}{\{[\alpha + \varphi h'(\overline{\pi})]\}^2} < 0, \ \forall h'$$

Then:

$$\frac{dh'(\overline{\pi})}{d\overline{\pi}} = -\frac{1}{-\frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'}} > 0$$

Applying the Chain rule to equation (37) we find how $\overline{\phi}$ changes with $\overline{\pi}$:

$$\begin{aligned}
\frac{\partial \overline{\phi}}{\partial \overline{\pi}} &= \alpha \cdot \frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} \\
&\quad + \varphi \cdot \frac{\partial h}{\partial F} \cdot \frac{\partial F}{\partial f^{-1}} \cdot \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} - \frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}} \\
&= \left[\underbrace{\frac{\partial F}{\partial f^{-1}} \left(\alpha + \varphi \frac{\partial h}{\partial F} \right) - 1}_{>0} \right] \cdot \underbrace{\frac{\partial f^{-1}}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \overline{\pi}}}_{>0} > 0 \\
&\quad \text{for } \alpha \text{ and } \varphi \text{ large enough}
\end{aligned}$$

■

Lower bound $\underline{\phi}$ Recall that we are assuming $\underline{\pi} < \pi_0$, although this is not essential to the derivation of what follows.

The tangency conditions between the function $C_{\underline{\phi}} = F[\alpha \underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}]$ and the 45° line are:

$$\begin{aligned} \underline{\pi} &= F[\alpha\underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}] \quad (40) \\ 1 &= f[\alpha\underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}] \cdot \left[\alpha + \varphi h'(\underline{\pi})\right] \quad (41) \end{aligned}$$

From equation (41) we have (recall that $f^{-1}(\cdot)$ is defined as the inverse function of $f(\cdot)$ that takes positive values, and thus we need to change the sign of $\alpha\underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}$, being this negative since $\underline{\pi} < \pi_0$):

$$-\left[\alpha\underline{\pi} + \varphi h(\underline{\pi}) - \underline{\phi}\right] = f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right] \quad (42)$$

Substituting out $\underline{\pi}$ and rearranging we obtain:

$$\begin{aligned} \underline{\phi} &= \alpha F\left\{-f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right]\right\} + \varphi h\left(F\left\{-f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right]\right\}\right) \\ &\quad + f^{-1}\left[\frac{1}{\alpha + \varphi h'(\underline{\pi})}\right] \quad (43) \end{aligned}$$

Proposition 6 *If α and φ are large enough, the lower bound of the range of fundamentals for which there are multiple equilibria increases with the probability of devaluation.*

Proof. In order to prove this proposition we need to know the sign of $h'(\underline{\pi})$ and of $h''(\underline{\pi})$. By Proposition 3 $h'(\underline{\pi}) > 0 \ \forall \underline{\pi}$. From equations (40) and (42) we have:

$$\underline{\pi} = F\left(-f^{-1}\left\{L\left[h'(\underline{\pi})\right]\right\}\right) \quad (44)$$

where:

$$L\left[h'(\underline{\pi})\right] = \frac{1}{\alpha + \varphi h'(\underline{\pi})} \quad (45)$$

Again, we can sign $h''(\underline{\pi})$ applying the IFT to equation (44). Define:

$$G\left[\underline{\pi}, h'(\underline{\pi})\right] \equiv \underline{\pi} - F\left(-f^{-1}\left\{L\left[h'(\underline{\pi})\right]\right\}\right)$$

Then:

$$\frac{dh'(\underline{\pi})}{d\underline{\pi}} = -\frac{\partial G/\partial\underline{\pi}}{\partial G/\partial h'(\underline{\pi})}$$

where:

$$\partial G/\partial\underline{\pi} = 1$$

and

$$\partial G/\partial h'(\underline{\pi}) = -\frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} > 0$$

since $\frac{\partial F}{\partial(-f^{-1})} > 0 \ \forall \left(-f^{-1}\right)$; $\frac{\partial(-f^{-1})}{\partial L} > 0$, and from equation (45):

$$\frac{\partial L}{\partial h'} = \frac{-\varphi}{\left\{\left[\alpha + \varphi h'(\underline{\pi})\right]\right\}^2} < 0, \ \forall h'$$

Then:

$$\frac{dh'(\underline{\pi})}{d\underline{\pi}} = -\frac{1}{-\frac{\partial F}{\partial(-f^{-1})} \cdot \frac{\partial(-f^{-1})}{\partial L} \cdot \frac{\partial L}{\partial h'}} < 0$$

From (43) we have:

$$\begin{aligned}
\frac{\partial \underline{\underline{\phi}}}{\partial \underline{\pi}} &= \alpha \cdot \frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} \\
&\quad + \varphi \cdot \frac{\partial h}{\partial F} \cdot \frac{\partial F}{\partial\left(-f^{-1}\right)} \cdot \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} - \frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}} \\
&= \left[\underbrace{\frac{\partial F}{\partial\left(-f^{-1}\right)}\left(\alpha + \varphi \frac{\partial h}{\partial F}\right) - 1}_{>0}\right] \cdot \underbrace{\frac{\partial\left(-f^{-1}\right)}{\partial L} \cdot \frac{\partial L}{\partial h'} \cdot \frac{\partial h'}{\partial \underline{\pi}}}_{>0} > 0 \\
&\quad \text{for } \alpha \text{ and } \varphi \text{ large enough}
\end{aligned}$$

■

Propositions 5 and 6 imply that an increase in the probability of devaluation shifts to the right the interval of fundamentals within which multiple equilibria may arise. This makes more difficult for the emerging country to avoid dropping in the multiple equilibria range by running very good fundamentals. Conversely, running bad fundamentals increases the chance that equation (31) holds with a unique equilibrium characterized by a high crisis probability. Notice that if α and φ are small enough, then $\frac{\partial \overline{\phi}}{\partial \overline{\pi}}, \frac{\partial \underline{\underline{\phi}}}{\partial \underline{\pi}} < 0$ and an increase in the probability of devaluation makes the region for multiple equilibria shift to the left. We see, then, that taking into account investors' risk aversion makes a sound fiscal policy more compelling to the policymaker.

2.5 Adding linkages through competitiveness effects

The literature reviewed in the introduction emphasizes that trade linkages, both bilateral and with third markets, are among the main channels of contagion. In this subsection a terms of trade effect is introduced in the model following Masson (1999). However, we extend Masson's contribution by modeling the policymaker as an optimizing agent.[10]

Suppose, without loss of generality, that the world consists of three countries, the US, the emerging market a, and the emerging market b. We assume that the emerging markets trade balance as a proportion to nominal GDP evolves as follows:

$$tb_t^i = \overline{tb}^i + \varsigma RER_t^i, \quad i = a, b \tag{46}$$

where $tb_t^i = \widehat{TB}_t^i / P_t^i Y_t^i$, $\widehat{TB}_t^i$ is the nominal value of the trade balance, $\overline{tb}^i$ is a constant, and RER is the (log) real effective exchange rate, which gives a weight w^i on country j and $(1 - w^i)$ on the US, with $i, j = a, b$, $i \neq j$, i.e.:

$$RER_t^i = (1 - w^i)\left[\ln(e_t^i) - (p_t^i - p_t^*)\right] + w^i\left[\ln(e_t^{ij}) - (p_t^i - p_t^j)\right]$$

where e_t^i and e_t^{ij} are the country i's nominal exchange rates *vis-a-vis* the US dollar and the country j's currency respectively. Since $\ln(e_t^i) - \ln(e_t^j) = \ln(e_t^{ij})$, then we have:

$$RER_t^i = \ln(e_t^i) - (p_t^i - p_t^*) - w^i\left[\ln(e_t^j) - (p_t^j - p_t^*)\right] \tag{47}$$

Equation (47) shows that a devaluation of country b's currency *vis-a-vis* the US dollar (that is, an increasing competitiveness of country b on the American market) decreases RER_t^a, that is the general competitiveness of country a. We can refer to this as to a competitiveness effect. Assuming for sake of simplicity that $\overline{tb}^i = 0$ for $i = a, b$, we can approximate the change in the trade balance tb_t for country a by the total differential:

$$\begin{aligned}\Delta tb_{t+1}^a &= \varsigma \Delta RER_{t+1}^a \\ &= \varsigma\left[\left(\pi_{t+1}^a - 1\right)\Delta e^a - w^a(\pi_{t+1}^b - 1)\Delta e^b\right]\end{aligned}$$

where, as in Subsection 2.2, we have assumed that US prices are normalized to 1, that relative PPP between each single emerging market's currency and

[10] See Berger and Wagner (2005) for a different way of extending the Masson's model.

the US dollar holds so that $\Delta p^i_{t+1} = \Delta e^i$ for $i, j = a, b$, and where π^b_{t+1} is the probability of devaluation of country b's currency at time $t+1$.

We assume that the social welfare is affected by the change in the trade balance, then the policymaker's welfare loss function under trade competition is given by:

$$L^{aT}_{t+1} = \left(u^a_{t+1}\right)^2 + \left(\Delta d^a_{t+1}\right)^2 - (\Delta tb^a_{t+1})^2 + \delta\Gamma^a_{t+1}$$

Proposition 7 *The probability of devaluation of one country depends on the probability of devaluations of its trading partners (π^b_t) through trade effects. It is increasing in π^b_t. Even if the trade competitiveness effect is absent, if there is a devaluation in country b, there can be a devaluation in country a through common lender effects.*

Proof. Given:

$$\begin{aligned}\left[(\Delta tb^a_{t+1})^d\right]^2 - \left[(\Delta tb^a_{t+1})^f\right]^2 &= \\ &\varsigma^2[(\Delta e^a)^2 - 2\,(\Delta e^a)^2\,\pi^a_{t+1} \\ &+2w^a\Delta e^a\Delta e^b\left(\pi^b_{t+1} - 1\right)]\end{aligned}$$

equation (28) becomes:

$$\begin{aligned}\overline{V}^{aT}_{t+1} &= L^{aTd}_{t+1} - L^{aTf}_{t+1} \\ &= \left[\left(u^a_{t+1}\right)^d\right]^2 + \left[\left(\Delta d^a_{t+1}\right)^d\right]^2 - \left[(\Delta tb^a_{t+1})^d\right]^2 + \Gamma^a_{t+1} \\ &\quad -\left[\left(u^a_{t+1}\right)^f\right]^2 - \left[\left(\Delta d^a_{t+1}\right)^f\right]^2 + \left[(\Delta tb^a_{t+1})^f\right]^2 \\ &= \Gamma^a_{t+1} + (\Delta e^a)^2\left\{\left[\zeta^2 + (d^a_t + z^a_t)\right]^2 - \varsigma^2\right\} \\ &\quad -2\Delta e^a\left[\theta\zeta u^a_{t+1} + \overline{g}^a_t\,(d^a_t + z^a_t) + \varsigma^2 w^a\Delta e^b\left(\pi^b_{t+1} - 1\right)\right] \\ &\quad -2\Delta e^a\rho^a_{t+1}d^a_t\,(d^a_t + z^a_t) \\ &\quad -2\,(\Delta e^a)^2\left\{\left[\zeta^2 + (d^a_t + z^a_t)\right]^2 - \varsigma^2\right\}\pi^a_{t+1} \\ &= V^{aT}_{t+1} - \varphi\rho^a_{t+1} - \alpha^T\pi^a_{t+1} \qquad (48)\end{aligned}$$

Where

$$V^{aT}_{t+1} = \Gamma^a_{t+1} + \frac{1}{2}\alpha^T - 2\Delta e^a\left[\theta\zeta u^a_{t+1} + \overline{g}^a_t\,(d^a_t + z^a_t) + \varsigma^2 w^a\Delta e^b\left(\pi^b_{t+1} - 1\right)\right]$$

and

$$\alpha^T = 2(\Delta e^a)^2 \left\{ \left[\zeta^2 + (d_t^a + z_t^a)\right]^2 - \varsigma^2 \right\}$$

Now the probability of devaluation in country a depends on the probability of devaluation in b, π_t^b, through a further channel, since $\pi_{t+1}^a = \Pr[\overline{V}_{t+1}^{aT} < 0]$ will increase with the probability of devaluation of country b's currency. ∎

This shows how contagion can occur because of trade competitiveness as well as wealth effects and time-varying risk aversion. The specification of the two countries' trade balance as dependent on the real exchange rate makes the devaluation expectations depend on trade competitiveness. This framework can give a complete picture of how contagion works. In fact, a portfolio effect, a competitiveness effect, or both can increase the probability of devaluation in one country as a consequence of the expected devaluation in another country. The link between the expectations of devaluation in the two countries is provided by the inclusion of the risk premium and by trade competitiveness on a third market. Finally, the crisis can also be triggered simultaneously in the two countries by a common global shock captured by the rate of interest of a risk-free bond issued by the industrialized country. The empirical evidence provided by, for example, Glick and Rose (1999), according to which contagion tends to occur as a consequence of trade links and competitiveness effects, is consistent with the theoretical model here presented.

3 Policy measures to avoid crises and contagion

Recently, a number of authors have invoked some sort of restrictions on capital mobility in order to stabilize the international financial system. For example, Krugman (1999) argues that countries which cannot adopt either currency unions or free floating exchange rates should limit capital flows. Stiglitz (1999) endorses the same view with special reference to developing countries. Eichengreen *et al.* (1995) argue that as real markets adjust sluggishly to shocks, a second best can be achieved by "throwing sand in the wheels" of international finance through a global foreign exchange transaction tax in the spirit of Tobin (1978). This would help prevent destabilizing speculation in international financial markets and make capital mobility compatible with modest autonomy in monetary and macroeconomic policy.

These views were broadly discussed in the aftermath of the frequent currency crises of the 1990s. In this Section we analyze the effects of such restrictions on the probability of transmission of crises in the context of our model. We distinguish between the effects of two different measures: we define the first as capital controls, which can be thought of as an administrative or direct intervention and amounts to outright prohibition or discretionary approval of certain cross-border capital movements; the second is a Tobin tax, i.e. a market-based or indirect control which aims at discouraging capital movements through cost-increasing measures.

3.1 Capital controls

As mentioned above, one way to stabilize international financial markets could be by means of administrative restrictions on capital flows. Montiel and Reinhart (1999) show that, unlike policies of sterilized intervention, capital controls exert an effect on the composition of inflows, rather than on their overall volume, in the direction of reducing the share of short-term and portfolio flows with respect to FDI. Furthermore, as documented by Edwards (1999), the introduction of such restrictions involves a number of issues regarding the sequencing of reforms in emerging markets, the effectiveness and timing of controls, the kind of flows to be restricted (whether inflows or outflows), as well as the maturity of capital flows to be discouraged.

The present setup cannot accommodate for such features of capital controls. Thus we resort to modeling this restriction introducing a no-short-selling constraint on either dollar- and local currency-denominated risky assets held by private investors:

$$x_s^{*m}, x_s^m \geq 0 \tag{49}$$

In order to simplify notation and make the argument clearer, we assume that the marginal effect on expected utility of relaxing the nonnegative constraints (49) is invariant across different currency-denominated assets issued by the same country, and that the market value of dollar denominated debt is equal to the market value of local currency denominated debt converted to dollars, i.e. $B_s^{*m} = B_s^m / e_s^m$. As shown below, these assumptions allow for the same effect of capital controls on country m's assets denominated in different currencies.

Proposition 8 *The introduction of a no-short-selling constraint on risky assets reduces the probability of currency devaluation.*

Proof. When the constraints (49) are added to the model, the Euler equations (3) and (4) change as follows:

$$u'(C_s - X_s)B_s^{*m} = \beta E_s\left\{u'(C_{s+1} - X_{s+1})\left(I_{s+1}^{*m} + B_{s+1}^{*m}\right)\right\} + v_{s+1}^m \quad (50)$$

$$u'(C_s - X_s)\frac{B_s^m}{e_s^m} = \beta E_s\left\{u'(C_{s+1} - X_{s+1})\left(\frac{I_{s+1}^m + B_{s+1}^m}{e_{s+1}^m}\right)\right\} + v_{s+1}^m \quad (51)$$

where v_{s+1}^m is the Lagrange multiplier associated with constraint (2) at time $s+1$, denoting the increase in expected lifetime utility that would result if the current constraint were relaxed by one unit. It is assumed to be equal across assets issued by the same country. Going again through the steps discussed in Section 2 leads to the following expression for the risk premium on dollar-denominated risky assets under capital controls, for $s = t$:

$$\begin{aligned} E_t(1 + r_{t+1}^{*m}) - (1 + r_{t+1}^{*f}) &= -(1 + r_{t+1}^{*f})Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r_{t+1}^{*m}\right\} \\ &\quad - \frac{v_{t+1}^m(1 + r_{t+1}^{*f})}{B_t^{*m}u'(C_t - X_t)} \\ &= \rho_{t+1}^m - \Theta_{t+1}^m \end{aligned} \quad (52)$$

where ρ_{t+1}^m is the risk premium we derived in Section 2 under free capital mobility and

$$\Theta_{t+1}^m = \frac{v_{s+1}^m(1 + r_{t+1}^{*f})}{B_t^{*m}u'(C_t - X_t)}.$$

An analogous expression can be obtained for the risk premium on local currency-denominated risky assets:

$$\begin{aligned} E_t\left[\frac{e_t^m}{e_{t+1}^m}(1 + r_{t+1}^m)\right] - (1 + r_{t+1}^{*f}) &= -(1 + r_{t+1}^{*f})Cov\left\{\frac{\beta u'(C_{t+1} - X_{t+1})}{u'(C_t - X_t)}, r_{t+1}^{*m}\right\} \\ &\quad - \frac{e_s^m v_{s+1}^m(1 + r_{t+1}^{*f})}{B_t^m u'(C_t - X_t)} \\ &= \rho_{t+1}^m - \Theta_{t+1}^m \end{aligned} \quad (53)$$

where the third equality derives from the assumptions that the risk premium under free capital mobility ρ_{t+1}^m is equal across country m's assets and that $B_s^{*m} = B_s^m/e_s^m$.

The new expression for the probability of devaluation of country m's currency under capital controls, $\overline{\pi}_{t+1}^m$, is, therefore:

$$\overline{\pi}_{t+1}^m = F[\alpha\overline{\pi}_{t+1}^m + \varphi(\rho_{t+1}^m - \Theta_{t+1}^m) - \phi_{t+1}^m] < \pi_{t+1}^m \quad (31')$$

since $F[\cdot]$ is an increasing function. ■

Proposition 8 implies that the introduction of capital controls in the form of administrative measures can help stabilize the international financial system through a reduction of the risk premium and, in turn, a reduction of the probability of currency devaluation. In fact, this leads to a decreasing contagion because as a country's probability of devaluation is shocked, the effect on the risk premium on other countries' assets will be damped by the term Θ deriving from capital controls.

3.2 Tobin tax

We now apply our model to answer the following question: do indirect capital controls in the form of a Tobin tax, i.e. a foreign exchange transactions tax, reduce the probability of contagion of financial crises?

One trivial effect of the introduction of a Tobin tax is to reduce the crisis probability through an increase in the fiscal revenues of the emerging market's government. This can be seen from equations (28) $-$ (31): *ceteris paribus*, an increase of fiscal revenues will decrease the government deficit and thus improve the fundamentals; this, in turn, will reduce the crisis probability expectations formulated by private agents. However, this is not an effect peculiar to the Tobin tax since it may be exerted by any measure of fiscal consolidation adopted by the policymaker. It is far more interesting, therefore, to detect what is the influence of a Tobin tax on private investors' equilibrium behavior.

Proposition 9 *The introduction of a Tobin tax does not affect the risk premium demanded on risky assets, and thus does not affect the probability of devaluation.*

Proof. Remembering that the budget constraint is expressed in dollars, levying a tax with rate τ^m on each foreign exchange transactions modifies the international investor's budget constraint (2) as follows:

$$\begin{aligned} B_{s+1}^{*f} + \sum_{m=1}^{M} x_{s+1}^{*m} B_s^{*m} + \sum_{m=1}^{M} x_{s+1}^{m} (1-\tau^m) \frac{B_s^m}{e_s^m} \quad &= \quad (1 + r_s^{*f}) B_s^{*f} + \\ &+ \sum_{m=1}^{M} x_s^{*m} \left(I_s^{*m} + B_s^{*m} \right) \\ &\sum_{m=1}^{M} x_s^{m} \left[\frac{(1-\tau^m) \left(I_s^m + B_s^m \right)}{e_s^m} \right] \\ &-C_s \end{aligned} \tag{54}$$

and the FOCs (3) and (4) remain unchanged since:

$$\frac{e_{s+1}^m (1-\tau^m) \left(I_{s+1}^m + B_{s+1}^m \right)}{e_s^m (1-\tau^m) B_s^m} = \frac{e_{s+1}^m \left(I_{s+1}^m + B_{s+1}^m \right)}{e_s^m B_s^m}.$$

■

Proposition 9 implies that levying a tax on foreign exchange transactions leaves unchanged the equilibrium returns on the risky asset and, thus, the representative investor's equilibrium behavior. Therefore the risk premium and, in turn, the probability of currency devaluation and transmission of crises remain unchanged.

Interestingly enough, this result contradicts Cordella (2003) which asserts that in a "bank run" model controls in the form of a tax on short-term capital inflows can increase expected returns by preventing bank runs. This, the argument goes, may lead to an increase of gross investments in the emerging market. Conversely, in the international asset pricing framework here presented the same kind of tax does not affect rates of return implying that the marginal investment decisions are unchanged.

Advocates of the Tobin tax as a mean of stabilization of international financial markets argue that it should be universally and uniformly levied in order to be effective (Eichengreen *et al.* 1995). Again, in our framework a Tobin tax levied universally on all countries' securities and with a uniform rate is ineffective.

4 Conclusion

The existing literature presents an unsatisfactory partition of explanations of contagion between theories based on fundamentals and theories based on investors' behavior. The model presented in this paper nests both the main sources of contagion of financial crises, and adds another dimension of non-linearity to the Jeanne-Masson model increasing the likelihood of self-fulfilling equilibria. It shows that financial crises can be transmitted across seemingly unrelated countries (e.g. Russia and Brazil) through the risk attitudes of international investors. Thus, to understand financial crises it is not sufficient to look at the countries in question, but also at the portfolios of international investors. International business cycle considerations through the wealth effects can also play a role in the incidence of financial crises. The model also suggests that bond spreads, in the event of a financial crisis, would change in emerging markets depending on the pattern of portfolio holdings of international investors. Better risk-management can help reduce the incidence of financial crises through diversification. The model can help better understand the transmission of crises across markets which do not seem to be directly related to each other by emphasizing the role of capital flows and thus, integrating international trade and finance considerations. Finally, an extension of the model to introduce a no-short-selling constraint shows that frictions in the international capital flows can help reduce the instability originating from the self fulfilling expectations of rational investors.

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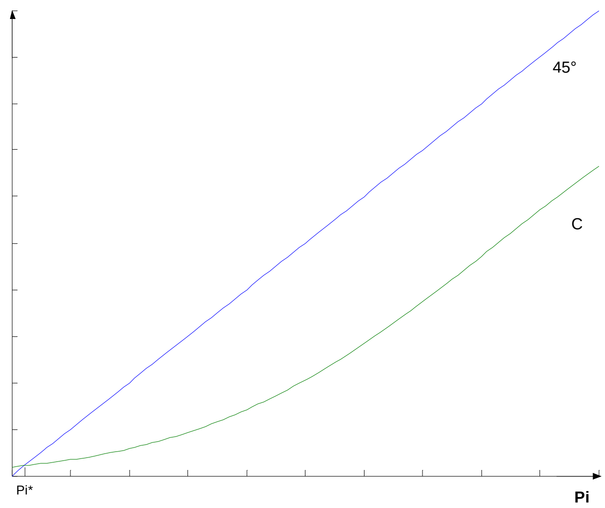

Figure 1. Unique equilibrium.

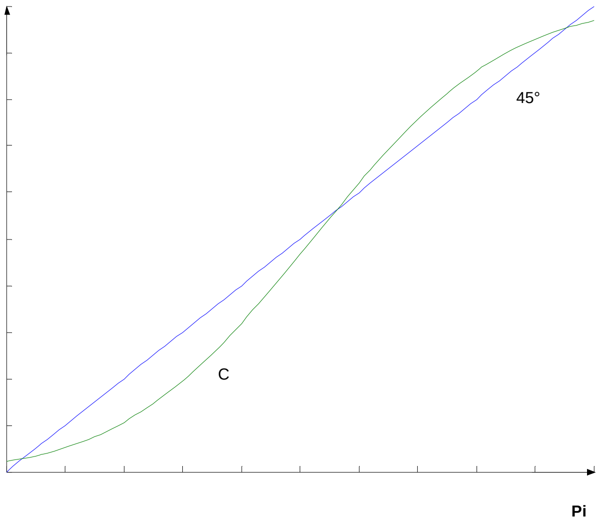

Figure 2. Multiple equilibria.

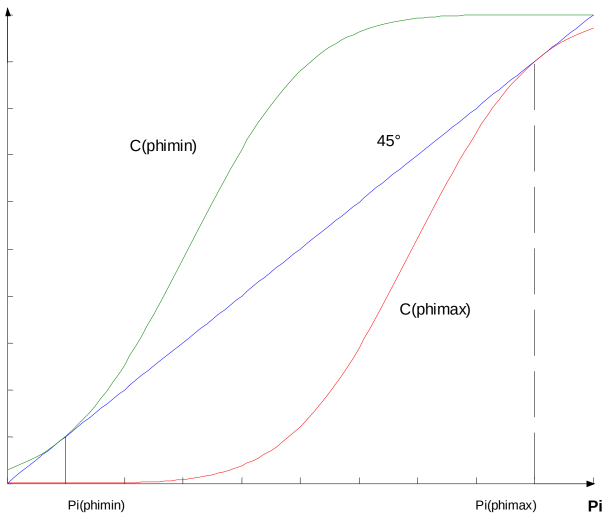

Figure 3. Critical values of fundamentals.