University of Essex

Department of Economics

Discussion Paper Series

No. 623 November 2006

Vulnerability, Unemployment and Poverty: A Class of Distribution and Sensitive Measures, Its Axiomatic Properties and Applications

Kaushik Basu and Patrick Nolen

Note : The Discussion Papers in this series are prepared by members of the Department of Economics, University of Essex, for private circulation to interested readers. They often represent preliminary reports on work in progress and should therefore be neither quoted nor referred to in published work without the written consent of the author.

Vulnerability, Unemployment and Poverty: A Class of Distribution Sensitive Measures, Its Axiomatic Properties and Applications*

Kaushik Basu[1] and Patrick Nolen[2]

[1] Cornell University, Ithaca NY 14853 `kb40@cornell.edu`
[2] University of Essex, Colchester UK CO4 3SQ `pjnolen@essex.ac.uk`

Summary. Measures of unemployment and poverty have tended to focus solely on those currently unemployed or below the poverty line. This approach has ignored the members of society that are *vulnerable* to becoming unemployed or falling into poverty. Current literature in this area has implicitly assumed that since someone who is vulnerable experiences pain from the chance of becoming unemployed or falling into poverty, our standard measures of unemployment and poverty do not accurately account for this pain. The implication is that vulnerability is a 'bad' and policies should aim to reduce the number of people who are vulnerable in a society. In this paper we argue that, at the macro level, vulnerability can be viewed as a 'good' because, with unemployment remaining constant, the presence of vulnerable people implies that there must also exist currently unemployed people who expect to find work in the near future. And a society where unemployment is more equitably shared is better than a society where the burden of unemployment is carried by only a few. Given this view of vulnerability we then suggest a class of measures that, unlike the standard unemployment rate, account for the amount of vulnerability that exists in a society. We show some attractive axioms that our measure satisfies, fully characterize our measure and apply it to data from the U.S. and South Africa.

Key words: Vulnerability, Unemployment, Poverty, Welfare

1 Introduction

Traditional measures of unemployment or poverty were only concerned with the total number of people unemployed or living in poverty. In recent years such measures have come under criticism for ignoring those who may not currently be poor or unemployed but are vulnerable, that is, they live under the risk of *becoming* unemployed or poor (see [14, 10, 28]). And alongside this criticism a small but rapidly growing literature is emerging that looks at various aspects of vulnerability and tries to measure it [17, 19, 2]. [3]

There is a presumption in much of this literature and the policy statements of international organizations and governments that since vulnerability is bad, we should craft policy to rescue people from being vulnerable. We argue in this paper that such a prescription is wrong, or, at best, misleading. Under a variety of 'normal' situations, having some people vulnerable to unemployment or to poverty make the *aggregate* problem of unemployment or poverty less severe (and more bearable).

The aim of this paper is to explain this normative stance of ours, to develop measures of unemployment and poverty that take account of this stance and then to apply it to U.S. and South African data.

The explanation of our normative position is not complicated and the general point can be made simply enough. Let us start by considering the case of unemployment. Suppose there is a society in which, currently, some people

---

[3] An important precursor of this literature is a body of writing that occurred around the theme of income mobility: see, for instance [24, 15, 12].

are unemployed and some people are vulnerable to unemployment (that is, there is a probability that they will become unemployed in the next period). The presumption in much of the literature and in many World Bank policy discussions (see, for instance, [29]) is that the standard measure of unemployment, which ignores the vulnerable, effectively underestimates the aggregate pain of unemployment (including the pain of its anticipation) in society. We, on the other hand, will argue that the standard measure of unemployment underestimates, not the *pain,* but the *inequity of the pain* of unemployment. Our argument is this – if unemployment holds constant over time and there are, currently, some people vulnerable to unemployment, then there must be some currently unemployed people who have a positive probability of becoming employed in the next period. If this is so, then an *aggregate* (that is, an economy-wide) measure of **effective** unemployment, while taking account of the pain of those who live under the risk of unemployment, must also take account of the hope of the currently unemployed who expect to find jobs soon. We will argue presently that in an overall measure of unemployment there is reason to treat the latter as more than offsetting the former. We should clarify that, contrary to the impression that the above sentences might create, we do not take a welfarist approach in this paper but use the above argument concerning the pain of living under the risk of unemployment as *motivation* for creating distribution-sensitive measures of unemployment.

Consider the point some would make, that we are not right to assume that just because there are some people who are vulnerable to unemployment, there

must be people currently unemployed but who have a positive probability of finding jobs in the next period. Our response to this is that if there were no such people, then having people who are vulnerable to unemployment is equivalent to saying that unemployment will rise tomorrow. If we then treat the situation as worse than what the standard measure captures, this does not show our valuation of vulnerability but the fact that the absolute amount of unemployment is about to rise. To isolate our attitude towards *vulnerability,* we must consider a case where the vulnerable population rises but the total number unemployed remains unchanged. But this compels us to assume that a vulnerable population will be matched by a population expecting a converse shift – *out* of unemployment.

To close the argument consider two societies, x and y, in which unemployment is the same, say 10%, and this remains constant over time. However, in society x no one is vulnerable to unemployment, while in y, 10% are vulnerable, that is they are currently employed but face a risk of unemployment. In other words, the total amount of the burden of unemployment to be shared in both societies is the same (10% of the people will have to be unemployed) but in y this burden is shared by 20% of the population, while in x this is borne entirely by only 10% of the population. The same way that, *ceteris paribus* (to use a term rapidly going into extinction), greater equality in the distribution of income and wealth ('good things', that is) is valued positively in most societies, we feel that there is reason to prefer a society where the 'bads', such as unemployment, are more equally distributed. It follows that,

starting with society x, if vulnerability is increased and we reach society y, then we must consider this a change for the better. Therefore, the effective unemployment must be considered to be less in society y than in x. [4] The next section formalizes the above idea by suggesting a new measure of effective unemployment.

The example just given is the idea that will be tracked in this paper. A full-blown analysis of individual vulnerability (as in [8, 11, 17]) entails 'dynamics' – how a person who may be employed today fares tomorrow. But in this paper we stay away from a full-blown dynamic account of vulnerability. What we are, instead, interested in is the common ground that lies between questions of vulnerability and questions of distribution that arise when we are attempting to create an *aggregate* measure of effective unemployment. Hence, the critical question that we are concerned with is the normative issue of how the "distribution of some fixed amount of unemployment time may be captured" with an inclination to favor "any trend towards a more equal distribution of unemployment" [26, p.5]

These concerns are shared in a parallel and large literature on unemployment durations (see, for instance [9, 1, 25]). However, the effort to bring these concerns under one aggregate measure of effective unemployment is quite rare

---

[4] Another case for better sharing of 'unemployment' can be made by arguing that, within each household, the unemployed are helped by the employed. In such a situation there arises the case for a better distribution of employment across households, as was argued Basu and Foster in [6] in the context of literacy.

([26, 18, 7] are the only ones that these authors can think of); and that is what is attempted in this paper.

2 Effective Unemployment

Consider a society with n persons. Let r_i be the fraction of a year during which person i is unemployed. Hence, by the measure of the "standard unemployment rate" this society's unemployment is

$$U \equiv \frac{r_1 + r_2 + ... + r_n}{n} \tag{1}$$

The standard unemployment measure that one encounters in newspapers is usually the above measure (often multiplied by 100, since the measure is generally stated in percentage terms).

From the discussion in the previous section it should be evident that we are looking for a measure of unemployment which is distribution sensitive. That is, if the same aggregate unemployment is unevenly shared in one society, we shall consider the **effective** unemployment to be greater in the more unequal society. We codify this later, in Axiom E, as the "equity axiom."

Let us define an **unemployment profile** of a society to be a vector $(r_1, r_2, ..., r_n)$ such that, for all i, $r_i \in [0, 1]$. Let Δ be the collection of all **unemployment profiles**. Hence, $\Delta = \{(r_1, r_2, ..., r_n) \mid n \in Z_{++}$ and $r_i \in [0, 1], \forall i\}$ where Z_{++} is the set of strictly positive integers.

Formally, a **measure of unemployment** (hereafter referred to as MOU) is a mapping

$$M : \Delta \to R_+$$

where R_+ is the set of non-negative real numbers.

The MOU that we propose in this paper, takes the following form:

$$M^\beta (r_1, ..., r_n) \equiv \frac{1}{\beta} - \prod_{i=1}^{n} (\frac{1}{\beta} - r_i)^{\frac{1}{n}} \qquad (2)$$

where $\beta \in (0,1)$

Since for every $\beta \in (0,1)$ we have a distinct measure M^β, what we have just proposed is a *class* of new measures of unemployment. We shall show that these measures have appealing properties, demonstrate, with some actual empirical examples, how using these new measures make a difference to the description of unemployment and then fully characterize these measures. Let us from now on call an MOU defined by (2), above, an **effective unemployment rate**.

One property of every member of the family of effective unemployment rates worth observing at the outset is that if $R = (r_1, r_2, ..., r_n)$ is such that $r_i = r, \forall i$, then $M^\beta (R) = r$. In other words, if the burden of unemployment is perfectly equitably shared by everybody then the effective unemployment rate is independent of $\beta \in (0,1)$ and equal to the standard unemployment rate defined in (1).

It is worth checking what the limits or boundaries of our class of measures look like. First consider the case where $\beta = 1$. This measure (which is not a part of the class we are recommending) is then represented by: $M^1(r_1, r_2, ..., r_n) = 1 - \prod_{i=1}^{n} (1 - r_i)^{\frac{1}{n}}$. Note that if, for some i, $r_i = 1$ (one

person is fully unemployed) then $M^1 = 1$. Hence, this measure makes no

distinction between the cases where 1 person is fully unemployed and where

10 persons are fully unemployed. It amounts to an extreme evaluation where

a tragedy for one is a tragedy for all. This is the standard 'multiplicative'

form of an evaluation function.

Now, what about the other limit, that is as β goes to 0? It can be shown

that as $\beta \to 0$, $M^\beta \to U$. That is as β goes to 0, our measure converges to the

standard unemployment rate as defined by (1). The first lemma establishes

this result. Since the standard measure is one in which individuals' unemploy-

ment are aggregated by simply adding up, this could be thought of as a kind

of utilitarian or additive representation of unemployment. Hence the class

of measures that we are proposing is bounded at one end by a multiplicative

representation and at the other end by an additive one.

Lemma 1. *For all $R = (r_1, r_2, ..., r_n) \in \Delta$, and for all $\beta \in (0, 1)$,*

$\lim_{\beta \to 0} M^\beta(R) = \frac{\sum_{i=1}^n r_i}{n}$

Proof.

$$\lim_{\beta \to 0} M^\beta(R) = \lim_{\beta \to 0} \{\frac{1}{\beta} - \prod_{i=1}^n (\frac{1}{\beta} - r_i)^{\frac{1}{n}}\}$$

$$= \lim_{\beta \to 0} \{\frac{1}{\beta}[1 - \prod_{i=1}^n (\beta)^{\frac{1}{n}} (\frac{1}{\beta} - r_i)^{\frac{1}{n}}]\}$$

$$= \lim_{\beta \to 0} \{\frac{1}{\beta}[1 - \prod_{i=1}^n (1 - \beta r_i)^{\frac{1}{n}}]\}$$

$$= \lim_{\beta \to 0} \{\frac{[1 - \prod_{i=1}^n (1 - \beta r_i)^{\frac{1}{n}}]}{\beta}\} = \frac{0}{0}$$

So we may now use L'Hôpital's Rule. Note that

$$\frac{\partial}{\partial \beta}\beta = 1,$$

and

$$\frac{\partial}{\partial \beta}[1 - \prod_{i=1}^{n}(1 - \beta r_i)^{\frac{1}{n}}] = -\sum_{k=1}^{n}\left(\frac{1}{n}\right)(1 - \beta r_k)^{\frac{1-n}{n}}(-r_k)\left(\prod_{i\neq k}(1 - \beta r_i)^{\frac{1}{n}}\right)$$

Taking the limit of this numerator we get

$$\lim_{\beta \to 0}\{-\sum_{k=1}^{n}\left(\frac{1}{n}\right)(1 - \beta r_k)^{\frac{1-n}{n}}(-r_k)(\prod_{i\neq k}(1 - \beta r_i)^{\frac{1}{n}})\} = -\sum_{k=1}^{n}\left(\frac{1}{n}\right)(-r_k)$$

$$= \frac{1}{n}\sum_{k=1}^{n}r_k$$

Thus by L'Hôpital's Rule

$$\lim_{\beta \to 0}\{\frac{[1 - \prod_{i=1}^{n}(1 - \beta r_i)^{\frac{1}{n}}]}{\beta}\} = \frac{1}{n}\sum_{k=1}^{n}r_k$$

Which implies that

$$\lim_{\beta \to 0}M^{\beta}(R) = \frac{1}{n}\sum_{k=1}^{n}r_k$$

$\square$

We shall now demonstrate how the effective unemployment rate, as characterized by (2), satisfies some attractive axioms. First of all consider two routine axioms.

Axiom O (Monotonicity Axiom): *An MOU, M, is said to satisfy the monotonicity axiom if for any* $R = (r_1, r_2, ..., r_n) \in \Delta$ *and* $R' = (r'_1, r'_2, ..., r'_n) \in \Delta$ *such that,* $\forall i$, $r_i \geq r'_i$ *and* $\exists j$ *where* $r_j > r'_j$, *then* $M(R) > M(R')$.

Axiom P (Population Replication Axiom): *An MOU, M, is said to satisfy the population replication axiom if for any $R = (r_1, r_2, ..., r_n) \in \Delta$ and $R^k = (r'_1, r'_2, ..., r'_{kn}) \in \Delta$, where R^k is a k-replica of R for some positive integer k (that is $r'_j = r_i$, $\forall j \in \{1 + (i-1)k, ..., ik\}$, $\forall i \in \{1, ..., n\}$), then $M(R) = M(R^k)$.*

These two axioms are standard and we would expect a good measure to satisfy them. Fortunately – as is easy to see – the effective unemployment rate that we have proposed satisfies both these axioms. Observe that, given the monotonicity axiom, coupled with the fact that $M^\beta(1, 1, ..., 1) = 1$, we now know that our measure ranges from 0 to 1. That is, $M^\beta(\Delta) \subset [0, 1]$.

Our measure, and the need to break away from the standard unemployment concept, was motivated by using an equity argument, namely, that it is superior to have a society where the burden of a certain amount of aggregate unemployment is more widely shared. So it is important to check that the effective unemployment rate satisfies equity. The simplest idea of equity may be formalized as follows.

Axiom E (Equity Axiom): *An MOU, M, is said to satisfy the equity axiom if for $R = (r_1, r_2, ..., r_n) \in \Delta$ and $R^* = (r^*, r^*, ..., r^*) \in \Delta$ such that $\sum_{i=1}^n r_i = nr^*$ and $R \neq R^*$, then $M(R) > M(R^*)$.*

It can be shown that M^β satisfies the equity axiom for every $\beta \in (0, 1)$. But instead of showing this directly, we will show that M^β satisfies another axiom and then show that the latter implies the equity axiom. This other axiom is the 'transfer axiom' widely used in the literature on poverty and inequality measurement (see [21] for instance). This, in the context of unemployment,

says the following. Suppose there are two people, one who is unemployed more than the other. Now if the more unemployed person becomes even more un-employed – say by ε amount of time – and the less unemployed person finds more work – again by ε amount of time – then the effective unemployment is higher. Formally,

Axiom T (Transfer Axiom): *An MOU, M, is said to satisfy the transfer axiom if for any $R = (r_1, r_2, ..., r_n) \in \Delta$ and $R' = (r'_1, r'_2, ..., r'_n) \in \Delta$ such that $r_k = r'_k \ \ \forall k \neq i, j$, $r_i \geq r_j$ and $r'_i = r_i + \varepsilon \leq 1$ and $r'_j = r_j - \varepsilon \geq 0$ (for some $\varepsilon > 0$), then $M(R') > M(R)$.*

Lemma 2. *For all $R = (r_1, r_2, ..., r_n) \in \Delta$, and for all $\beta \in (0, 1)$ every effective unemployment rate, M^β, satisfies the transfer axiom.*

Proof. $M^\beta(R') = \frac{1}{\beta} - \prod_{k=1}^{n} \left(\frac{1}{\beta} - r'_k\right)^{\frac{1}{n}}$

$= \frac{1}{\beta} - \left(\frac{1}{\beta} - r'_i\right)^{\frac{1}{n}} \left(\frac{1}{\beta} - r'_j\right)^{\frac{1}{n}} \prod_{k \neq i,j}^{n} \left(\frac{1}{\beta} - r'_k\right)^{\frac{1}{n}}$

$= \frac{1}{\beta} - \left[\left(\frac{1}{\beta} - r_i - \epsilon\right)\left(\frac{1}{\beta} - r_j + \epsilon\right)\right]^{\frac{1}{n}} \prod_{k \neq i,j}^{n} \left(\frac{1}{\beta} - r_k\right)^{\frac{1}{n}}$

$= \frac{1}{\beta} - \left[\left(\frac{1}{\beta} - r_i\right)\left(\frac{1}{\beta} - r_j\right) - (r_i - r_j)\varepsilon - \varepsilon^2\right]^{\frac{1}{n}} \prod_{k \neq i,j}^{n} \left(\frac{1}{\beta} - r_k\right)^{\frac{1}{n}}$

$> \frac{1}{\beta} - \left[\left(\frac{1}{\beta} - r_i\right)\left(\frac{1}{\beta} - r_j\right)\right] \prod_{k \neq i,j}^{n} \left(\frac{1}{\beta} - r_k\right)^{\frac{1}{n}}$, since $r_i \geq r_j$, $\varepsilon > 0$ and $\beta \in (0, 1)$

$= \frac{1}{\beta} - \prod_{i=1}^{n} \left(\frac{1}{\beta} - r_i\right)^{\frac{1}{n}} = M^\beta(R)$

$\square$

The fact that M^β satisfies the equity axiom follows from Lemma 2 and the following lemma.

Lemma 3. *If an MOU satisfies the transfer axiom, it must satisfy the equity axiom.*

Proof. Suppose M is an MOU that satisfies the transfer axiom.

Consider, $\widetilde{R} = (r_1, r_2, \ldots, r_n)$ and $R^* = (r^*, r^*, \ldots, r^*)$ which satisfy the hypotheses of the equity axiom. That is $\widetilde{R}, R^* \in \Delta$, $\widetilde{R} \neq R^*$ and

$\sum_{i=1}^{n} r_i = n r^*$

Define $S \subset \Delta$ such that $S \equiv \{ R = (r_1, r_2, \ldots, r_n) \in \Delta \mid \sum_{i=1}^{n} r_i = n r^* \}$.

Note that for any $R \neq R^*$, $R = (r_1, r_2, \ldots, r_n) \in S \backslash \{R^*\}$

So we can define $\bar{r}(R) \equiv \max_i r_i$ and $\underline{r}(R) \equiv \min_i r_i$.

Let $\varepsilon = \min\{\bar{r}(R) - r^*, r^* - \bar{r}(R)\}$

Now define a mapping $\Psi : S \to S$, as follows:

$\Psi(R^*) = R^*$or, if $R = (r_1, r_2, \ldots, r_n) \neq R^*$, then $\Psi(R) = R'$

where $R' = (r_1', r_2', \ldots, r_n')$ such that $r_k' = r_k$, $\forall r_k \neq \bar{r}(R), \underline{r}(R)$,

and $r_i' = \underline{r}(R) + \varepsilon$ for $r_i = \underline{r}(R)$

and $r_j' = \bar{r}(R) - \varepsilon$ for $r_j = \bar{r}(R)$

By the transfer axiom we know that $M(R) > M(\Psi(R))$.

Now look at the infinite sequence $\{R^1, R^2, \ldots\}$

such that $R^1 = \widetilde{R}$ and $R^{t+1} = \Psi(R^t)$ $\forall t > 1$.

There must exist some $\bar{t}$ such that $\forall t \geq \bar{t}$, $R^t = R^*$.

Thus $M(R^1) > M(R^t)$, $\forall t > 1$, and therefore $M(\widetilde{R}) > M(R^*)$

$\square$

In the light of this result, the next lemma is obvious and stated only for completeness.

Lemma 4. *Every effective unemployment rate, M^β, satisfies the Equity Axiom.*

While the measure being suggested here has attractive axiomatic properties, which particular β should one use when applying this measure? One possibility is to study the sensitivity of ranking societies with respect to changes in β. The other is to pick some salient values of β from the interval $(0, 1)$ and use those specific measures. This is the strategy that is often used vis-a-vis the Foster-Greer-Thorbecke family of poverty measures (see [13]).

For such salient β's an obvious one is the half-way mark, that is, $\beta = \frac{1}{2}$. There is another one, $\beta = \frac{8}{9}$, which appears unnatural at first sight, but has a natural explanation.

Consider a society of size n and suppose that x is the fraction of society that has to be unemployed. In other words, the total amount of jobs available is $(1 - x)n$. For matters of illustration we are ignoring the fact that $(1 - x)n$ may not be an integer. Let us fix x and consider different distributions of the total amount of unemployment nx, and their corresponding measures of effective unemployment. By using the equity axiom it is clear that effective unemployment is minimized if nx is distributed equitably, that is, if each person is unemployed a fraction x of her time.

Let $m(x)$ be the minimum effective unemployment rate for a society with a total burden of unemployment nx. It is easy to see this is independent of $\beta \in (0, 1)$. Hence, writing this as $m(x)$, with no mention of β, is fine. It is
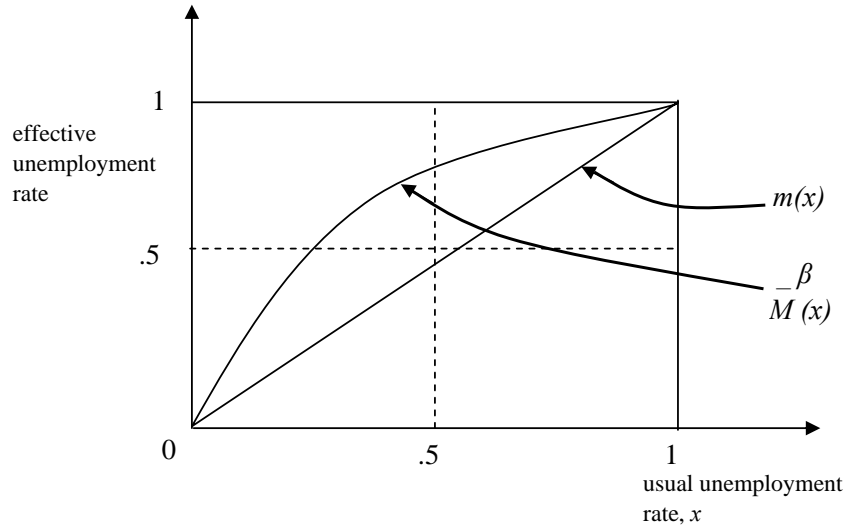
Fig. 1. This shows the relation between the usual and effective unemployment rate for a given value of β

obvious that $m(x)$, will be a 45°–line as shown in Figure One. Thus if half the society has to be half unemployed (i.e. $x = \frac{1}{2}$), the lowest value M^β takes is when every person is half-time unemployed. In that case, for all $\beta \in (0, 1)$, $M^\beta(x, \dots , x) = \frac{1}{2}$.

Here is an interesting question. Let us pick any $x \in [0, 1]$ and think of the worst distribution of this total burden of x unemployment (in the sense of the distribution that makes effective unemployment the maximum). By the transfer axiom, we know that this happens when some people are fully unemployed and the rest are fully employed. Hence, fix $\beta \in (0, 1)$, consider this worst-distribution for every x and define $\overline{M}^\beta(x)$ as the value of M^β for a given unemployment profile, $(r_1, r_2, \dots , r_n)$, which is the worst way to share

the burden of nx. Clearly $\overline{M}^{\beta}(x) \in (x, 1)$, $\forall x$. It is not hard to see that for a given β, $\overline{M}^{\beta}(x)$ will look something like the curve shown in Figure One. The higher the values of β, the higher the curve will be. And as β goes to 0, the line will converge to the $m(x)$ curve.

There are two ways of choosing β. One is to elicit this from individual choice. This involves asking individuals questions like: If you face a choice of two lotteries, one in which you will be unemployed all year with the probability $\frac{1}{4}$ or employed for the full year with probability $\frac{3}{4}$; and the other in which you will be employed for a fraction t of the year with certainty and unemployed for the remainder of the year, what value of t would you choose? This would be in the spirit of what is done by Ligon and Schechter in [17].

The other way to approach β is as a moral judgement of the policy maker. In the absence of data on individual risk-aversion, let us explore that moral approach here. Just to fix our thinking consider the case of $x = \frac{1}{2}$. We know that if every person is unemployed $\frac{1}{2}$ of the year then $M^{\beta}(\frac{1}{2}) = \frac{1}{2}$, $\forall \beta$. Now consider the worst distribution of this total burden. Clearly this is one where $\frac{n}{2}$ persons are fully employed and $\frac{n}{2}$ persons are fully unemployed. Let $R = (r_1, r_2, \ldots\ldots, r_n)$ signify such a distribution. We know that $M^{\beta}(R) \in (\frac{1}{2}, 1)$ as β varies from 0 to 1. We need to ask ourselves: what score we would like to give to $M^{\beta}(R)$? One simple strategy is to set this half-way in this interval. That is $M^{\beta}(R) = \frac{3}{4}$. In other words we are making the judgement that a society where half the people are fully employed and half are fully unemployed is equivalent to one where everybody is employed with certainty

for one quarter of the time. What would β have to be to yield this mid-way result?

The answer turns out to be, interestingly, $\frac{8}{9}$. To see this note:

$$M^{\beta}(R) = \frac{1}{\beta} - \left(\frac{1}{\beta} - 1\right)^{\frac{1}{n} \cdot \frac{n}{2}} \left(\frac{1}{\beta} - 0\right)^{\frac{1}{n} \cdot \frac{n}{2}}$$

$$= \frac{1}{\beta} - \left(\frac{1}{\beta} - 1\right)^{\frac{1}{2}} \left(\frac{1}{\beta}\right)^{\frac{1}{2}}$$

$$= \frac{1}{\beta} - \frac{(1 - \beta)^{\frac{1}{2}}}{\beta}$$

If $M^{\beta}(R) = \frac{3}{4}$, it follows that $\beta = \frac{8}{9}$. Hence, the $\frac{8}{9}$ rule. We shall use this in the empirical section, as one of the salient values. Before moving on we should point out that in practice β could well vary from one country to another. Depending on the policies that nations follow to support the unemployed, the trauma of unemployment can vary from one nation to another. We shall however ignore this complication here.

3 Simple Data Exercise

To provide an illustrative example of how our measure works we require certain information. Firstly, a history of how much someone was unemployed over a certain period of time. For this exercise we will use the weeks or months one was unemployed over a year. Second, we require a value for the parameter β. As explained earlier, for the purpose of illustration, we will use the values $\beta = \frac{1}{2}$ and $\beta = \frac{8}{9}$.

The March Current Population Surveys (hereafter referred to as the CPS) for the United States have the amount of weeks any member of the workforce was employed during the previous year. The Labour Force Surveys (hereafter referred to as LFS) for South Africa have the amount of months that one has been unemployed, if she was unemployed at the time of the survey, and when one started a job, if she is currently working at the time of the survey. These data allow us to illustrate the differences of the effective and the standard measures of unemployment. In the South African case one will have to make some assumptions to go from the available data to the measures we wish to calculate.

3.1 United States

We will begin with the case of the United States. The CPS contains how many weeks a survey participant had been employed during the previous year. Therefore, since we have the data for the March CPS from 1976 through 2003, we are able to calculate the usual yearly unemployment rate and the effective yearly unemployment rate for the years of 1975 through 2002 (excluding 1993).

To get measures of unemployment as accurate as possible we tried to exclude students and retired individuals by calculating the unemployment rates only for people between the ages of 25 and 54. Any persons who listed themselves as being unemployed for any of the following reasons were dropped from our data even if they were between the ages of 25 and 54: to take care of

house or family; ill or disabled; to attend school; retired. Anyone who claimed to have not worked for the year, but had spent less than four weeks searching for a job was also not included in our data set. Thus we were left only with people who were able to participate in the labor market during the full year and had been actively seeking work.

Table One shows the usual unemployment rate and the effective unemployment rates for $\beta = \frac{1}{2}$ and $\beta = \frac{8}{9}$ over the years for which we have data. Figure Two puts this information into graphical form. To begin our discussion it is useful to focus attention on three sets of years: 1987-1989; 1991-1992; and 1999-2000. These three periods illustrate how even though the usual unemployment rate stayed roughly constant our effective unemployment measures tell a different story in these periods. The period from 1987 to 1989 has a continuous decrease in the effective unemployment rate at $\beta = \frac{8}{9}$. During this period the usual unemployment rate, though, rose slightly from 1987 to 1988 and then back to slightly below the 1987 level in 1989. Thus, while the usual unemployment measure would rank 1987, 1988 and 1989 as being roughly equivalent, if one takes vulnerability into account – as we have defined it – then 1989 would be ranked as having an unambiguously better employment situation than either of the other two years, and 1988 would be ranked higher than 1987. The effective measure at $\beta = \frac{1}{2}$ shows the same trend but there is almost no difference between 1987 and 1988. Therefore during the years of 1987 to 1989, under President Reagan's administration, the burden

Year	Usual Unemployment Rate	Effective Unemployment Rate at $\beta = \frac{1}{2}$	Effective Unemployment Rate at $\beta = \frac{8}{9}$
2002	0.0529	0.0624	0.0803
2001	0.0457	0.0533	0.0669
2000	0.0382	0.0442	0.0544
1999	0.0382	0.0443	0.0547
1998	0.0490	0.0572	0.0721
1997	0.0431	0.0502	0.0630
1996	0.0530	0.0619	0.0782
1995	0.0585	0.0686	0.0874
1994	0.0612	0.0719	0.0922
1993	–	–	–
1992	0.0707	0.0834	0.1079
1991	0.0705	0.0822	0.1039
1990	0.0602	0.0697	0.0864
1989	0.0529	0.0613	0.0761
1988	0.0543	0.0633	0.0797
1987	0.0537	0.0632	0.0819
1986	0.0621	0.0734	0.0958
1985	0.0639	0.0756	0.0987
1984	0.0677	0.0807	0.1071
1983	0.0829	0.0994	0.1341
1982	0.0916	0.1092	0.1455
1981	0.0680	0.0800	0.1037
1980	0.0635	0.0744	0.0953
1979	0.0500	0.0579	0.0726
1978	0.0501	0.0583	0.0735
1977	0.0565	0.0663	0.0850
1976	0.0646	0.0763	0.0996
1975	0.0708	0.0836	0.1089

Table 1. Usual and efficient unemployment rates for individuals that were in the labor force, aged 25 to 54 and available for employment that entire year. ***Note:*** *the symbol " – " implies the unemployment rate could not be calculated for that period.*

of unemployment became more equitably shared according to the effective

unemployment measure with $\beta = \frac{8}{9}$.

United States Unemployment
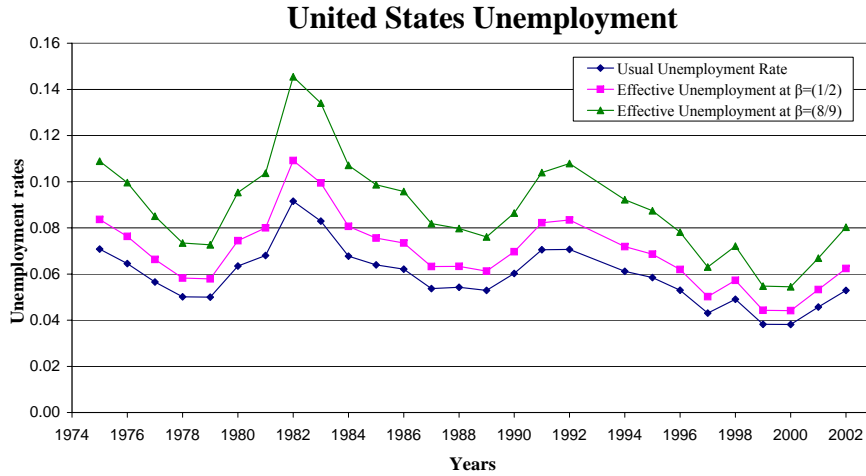
Fig. 2. The usual and effective yearly unemployment rates in the United States over the 1975 to 2002

The period of 1991-1992, under President George Herbert Walker Bush's

administration, our effective unemployment measures again deviate from the

rankings provided by the usual measure of unemployment. Again during this

period we have the usual unemployment rate staying roughly constant, but by

both the effective unemployment measure with $\beta = \frac{8}{9}$ and with $\beta = \frac{1}{2}$ effective

unemployment measures the burden of unemployment was being shared less

equitably in 1992 than in 1991. In contrast to these two examples, the period

of 1999-2000, during Bill Clinton's administration, shows that both the usual

unemployment measure and the effective unemployment measures rank the

years as being roughly equivalent. This shows that there was no significant change in the equity of how the unemployment burden was being shared.

These examples illustrate how our measure can be used to distinguish between years that might seem to be roughly equivalent in regards to unemployment, and shows how the role of vulnerability can cause a re-ranking of how one judges a country's unemployment situation over the years. This illustration uses only information from within one country. The effective unemployment measures could also be used to compare the unemployment situation between countries. Indeed, this will be done below.

3.2 South Africa

The South African LFS are twice yearly surveys that are representative of the Republic of South Africa. The survey collects data on people who are currently unemployed and those currently employed, but figuring out the yearly history of individuals is not as easy as with the CPS. Any person who is currently employed is asked when she began working at that job. Therefore we have a measure of her duration of current employment. Likewise, anyone unemployed is asked how long it has been since she last worked, if she had worked at all, giving us the duration of the current unemployment spell.

In South Africa the labor unions are rather powerful and, because of the historical situation, firing an individual is rather difficult. Therefore, job turnover is a rarer phenomenon than in the United States. Thus, the duration periods are probably an accurate measures of a labor force participant's

job status over the past year. With this in mind the usual and effective unemployment rates were calculated in this 'moderate' case. The durations are by no means guaranteed to be accurate representatives of the employment history, though, so we have also constructed bounds to the measures of unemployment.

To understand how these bounds were calculated let us look at a worker who was employed for six months at the time of the survey. We know that she has worked at least six months over the past year, and the employment pattern for the other six months is lost. In the worst case scenario she spent the previous six months searching for the one job she had at the time of the survey and was unemployed the rest of the time. This means that her duration of current employment is the worst history she could have. On the other hand, since we know she was only working at her current job for six months she could have found that job after only one month or less of unemployment – we count her as having had one month of unemployment even if it was less than that, though. Thus her best unemployment profile would be eleven months working and one month unemployed.

Likewise a currently unemployed person has a best and worse case scenario. If a person was unemployed for six months, then, in the best case, she was working for the six months before that and in the worst case she was working for only one month before her unemployment began. The usual and effective unemployment rates were therefore also calculated for both the 'worst' and 'best' case scenarios.

When comparing the usual unemployment rate under these three scenarios to that listed as the official unemployment rate in the LFS the moderate rate was almost identical in all five periods. [5] Therefore, in our discussion below we will use the results from the calculations done on the moderate data, though, using either of the other scenarios does not change our analysis.

South African Unemployment
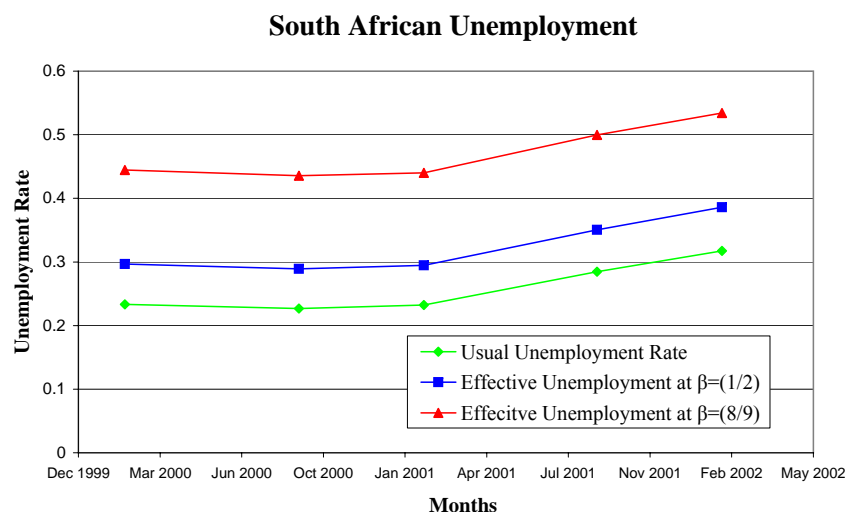
Fig. 3. The usual and effective unemployment rates in South Africa under the moderate scenario for the months of February 2000, 2001, 2002 and September 2000 and 2001

The usual and effective unemployment rates for the 'moderate' case are depicted graphically in figure three. The effective unemployment seems to be simply a horizontal increase of the usual unemployment. This may be because of the coarseness of the LFS data with respect to the CPS data or it may be

[5] The official unemployment rates according to the LFS are 26.2, 25.4, 26, 29.2 and 29 percent while the usual unemployment rate calculated using the 'moderate' estimates of r_i are 23, 23, 23.2, 28.5 and 31.7 for February 2000, September 2000, February 2001, September 2001 and February 2002 respectively.

because of a lack of change in the equity of unemployment. Comparing this graph to that of the United States, though, one can see that the jump from the usual measure of unemployment to the effective measures is much higher in both the $\beta = \frac{1}{2}$ and $\beta = \frac{8}{9}$ case for South Africa than for the United States. The effective unemployment rate at $\beta = \frac{1}{2}$, was, on average, 1.17 times the usual unemployment rate in the US and never above 1.199 the usual rate. In South Africa, though, the effective unemployment rate at $\beta = \frac{1}{2}$ is on average 1.25 times the usual unemployment rate and never below 1.22 times the usual rate. Likewise at $\beta = \frac{8}{9}$ the effective unemployment rate is on average 1.50 times and 1.83 times the usual unemployment rate in the US and South Africa respectively.

One may ask: What does this proportional difference show? Since our effective unemployment measure adjusts for the inequity of unemployment in a society then the higher jump from the usual to effective rate in the case of South Africa suggests that the burden of unemployment is shared less equitably in South Africa than in the United States. Given South Africa's history this is not a hard story to believe. To make this claim in a more rigorous manner, and not just to provide an illustration, one would have to test the confidence intervals of the proportions mentioned above. Given the size of the samples in both the LFS and the CPS the discussion above will probably still be valid. Furthermore, one could ask what is causing the difference in these proportions of unemployment. For example it may be that we are

only picking up frictional unemployment in the United States, but structural unemployment in South Africa.

4 Full Characterization of the Effective Measure

After seeing the usefulness of this measure and its applicability it is worth returning to the theoretical discussion and addressing a natural question. We have seen that our measure of unemployment satisfies several attractive axioms, but is there a set of axioms that exactly characterize our measure or, more precisely, the class of measures we proposed and is stated in (2)? This is what we set out to answer in this section.

A property that we have already discussed but needs to be stated formally is codified in the next axiom.

Axiom C (Coincidence): *A MOU, M, satisfies coincidence if $\forall R = (r_1, r_2, ..., r_n) \in \Delta$, such that $r_1 = r_2 = ...\ = r_n \equiv r$, $M(R) = r$.*

This axiom says that if in some society everybody is unemployed to the same extent as everybody else, then the society's unemployment rate should be equal to the individual's unemployment rate. This is a normalization axiom which says that if the distribution of unemployment is perfectly egalitarian then our measure coincides with the standard unemployment rate.

Another axiom that we will use and seems eminently reasonable is the following.

Axiom A (Anonymity): *The MOU, M, satisfies anonymity if* $\forall R = (r_1, r_2, ..., r_n) \in \Delta$ *and* $\forall$ *permutations* $\sigma : \{1, ..., n\} \rightarrow \{1, ..., n\}$, $M(R) = M\left(r_{\sigma(1)}, r_{\sigma(2)}, ..., r_{\sigma(n)}\right).$

And finally, a much stronger axiom.

Axiom R (Representation): *For every individual* i, $\exists$ *a utility function* $u_i : [0, 1] \rightarrow R_{++}$, *and* $\forall n \in Z_{++}$, $\exists$ *an aggregation mapping* $F : R_{++}^n \rightarrow R$, *such that,* $\forall R = (r_1, r_2, ..., r_n)$ *where* $r_i \in [0, 1]$, $M(R) = F\left(u_1(r_1), u_2(r_2), ..., u_n(r_n)\right)$ *and*

    (i) $\forall i$, u_i *is affine and decreasing,*

    (ii) F satisfies anonymity. That is $\forall u = (u_1, u_2, ..., u_n) \in R_{++}^n$ *and* $\forall$ *permutations* $\sigma : \{1, ..., n\} \rightarrow \{1, ..., n\}$, *then* $F(u_1, u_2, ..., u_n) = F(u_{\sigma(1)}, u_{\sigma(2)}, ..., u_{\sigma(n)})$. *And*

    (iii) F satisfies scale independence. That is $F(u_1, u_2, ..., u_n) \geq F(u_1', u_2', ..., u_n')$ *and* $(b_1, b_2, ..., b_n) \in R_{++}^n$ *implies that* $F(b_1 u_1, b_2 u_2, ..., b_n u_n) \geq F(b_1 u_1', b_2 u_2', ..., b_n u_n')$.

If an MOU satisfies axiom R, we shall call each u_i function and the F function referred to in the axiom as person i's utility function and the society's aggregation function, respectively.

What Axiom R(iii) says is that a change in the unit for measuring one person's utility must be of no consequence in our social evaluation of the economy. This not a normative axiom but an informational one, in the spirit of standard "invariance axioms"[20] used in social choice and bargaining theory and meant to capture the degree of measurability of an individual's utility.

Theorem 1: *An MOU, M, satisfies axioms A, C, O and R if and only if it belongs to the class described in (2). That is,* $\forall R \in (r_1, r_2, ..., r_n) \in \Delta$,

$$M\left(r\right) = \frac{1}{\beta} - \left[\prod_{i=1}^{n}\left(\frac{1}{\beta} - r_i\right)\right]^{\frac{1}{n}} \quad where \ \beta \in \left(0, 1\right)$$

Proof. ($\Rightarrow$) We have already seen that the MOU described in (2) satisfies axioms C and O. Axiom A is obvious. To see that it satisfies axiom R, consider $u_i = \frac{1}{\beta} - r_i$, for all i, and define the welfare mapping F as follows:

$$F = \frac{1}{\beta} - \left[\prod_{i=1}^{n} u_i\right]^{\frac{1}{n}} \tag{3}$$

It is easy to see that (3) satisfies axiom R.

($\Leftarrow$) Next assume that M is an MOU that satisfies axioms A, C, O, and R. By axiom R we know that $\exists$ a welfare mapping F such that, $\forall R = (r_1, r_2, ..., r_n) \in \Delta$, $M(R) = F\left(u_1\left(r_1\right), u_2\left(r_2\right), ..., u_n\left(r_n\right)\right)$. Now the proof will continue in a series of steps.

- Step 1: It will first be shown that F is a transformation of the product of the arguments. In other words, $\forall \ u = (u_1, u_2, ..., u_n) \in R_{++}^n$, $F(u) = \phi\left(\prod_{i=1}^{n} u_i\right)$. Consider a utility vector $u = (u_1, u_2, ..., u_n) \in R_{++}^n$. Then we know that $\left(u_1^{\frac{1}{n}}, u_2^{\frac{1}{n}}, ..., u_n^{\frac{1}{n}}\right) \in R_{++}^n$.

  Note $F\left(u_1^{\frac{1}{n}}, u_2^{\frac{1}{n}}, ..., u_n^{\frac{1}{n}}\right) = F\left(u_2^{\frac{1}{n}}, u_3^{\frac{1}{n}}, ..., u_n^{\frac{1}{n}}, u_1^{\frac{1}{n}}\right)$ by axiom R(ii).

  $F\left(u_1^{\frac{1}{n}} u_1^{\frac{1}{n}}, u_2^{\frac{1}{n}} u_2^{\frac{1}{n}}, ..., u_n^{\frac{1}{n}} u_n^{\frac{1}{n}}\right) = F\left(u_1^{\frac{1}{n}} u_2^{\frac{1}{n}}, u_2^{\frac{1}{n}} u_3^{\frac{1}{n}}, ..., u_{n-1}^{\frac{1}{n}} u_n^{\frac{1}{n}}, u_n^{\frac{1}{n}} u_1^{\frac{1}{n}}\right)$

  by axiom R(iii).

  $F\left(u_1^{\frac{2}{n}}, u_2^{\frac{2}{n}}, ..., u_n^{\frac{2}{n}}\right) = F\left((u_1 u_2)^{\frac{1}{n}}, (u_2 u_3)^{\frac{1}{n}}, ..., (u_{n-1} u_n)^{\frac{1}{n}}, (u_n u_1)^{\frac{1}{n}}\right)$

  $F\left(u_1^{\frac{2}{n}}, u_2^{\frac{2}{n}}, ..., u_n^{\frac{2}{n}}\right) = F\left((u_2 u_3)^{\frac{1}{n}}, (u_3 u_4)^{\frac{1}{n}}, ..., (u_n u_1)^{\frac{1}{n}}, (u_1 u_2)^{\frac{1}{n}}\right)$

by axiom R(ii).

$$F\left(u_1^{\frac{3}{n}}, u_2^{\frac{3}{n}}, ..., u_n^{\frac{3}{n}}\right) = F\left((u_1 u_2 u_3)^{\frac{1}{n}}, (u_2 u_3 u_4)^{\frac{1}{n}}, ..., (u_n u_1 u_2)^{\frac{1}{n}}\right)$$

by axiom R(iii).

Continuing in this manner we get

$$F\left(u_1^{\frac{n}{n}}, u_2^{\frac{n}{n}}, ..., u_n^{\frac{n}{n}}\right) = F\left((\textstyle\prod_{i=1}^{n} u_i)^{\frac{1}{n}}, (\textstyle\prod_{i=1}^{n} u_i)^{\frac{1}{n}}, ..., (\textstyle\prod_{i=1}^{n} u_i)^{\frac{1}{n}}\right)$$

It follows that if $u, v \in R_{++}^n$ such that $\prod_{i=1}^{n} u_i = \prod_{i=1}^{n} v_i$, then $F(u) = F(v)$. Hence, $\exists$ a function ϕ, such that $\forall\, u \in R_{++}^n$, $F(u) = \phi\left(\prod_{i=1}^{n} u_i\right)$.

- Step 2: Now it will be shown that F is a negative monotone transformation of the product of the arguments. That is $\exists\ \phi : R \to [0, 1]$ such that $x, y \in R$, $x > y$ implies $\phi(x) < \phi(y)$. Let $R,\ R' \in \Delta$ be such that $R = (r_1, r_2, ..., r_n)$ and $R' = (r_1', r_2', ..., r_n')$, $r_1' > r_1$ and $r_i' = r_i$, $\forall i \in \{2, 3, ..., n\}$. Then by axiom O, $M(R') > M(R)$. Now, by axiom R, in particular that u_i is a decreasing function for all i, we know that $u_1(r_1) \prod_{i=2}^{n} u_i(r_i) > u_1(r_1') \prod_{i=2}^{n} u_i(r_i)$. Furthermore, again by axiom R, we know:

$$M(R) = F(u_1(r_1), u_2(r_2), ..., u_n(r_n)) = \phi\left(u_1(r_1) \prod_{i=2}^{n} u_i(r_i)\right)$$

$$M(R') = \phi\left(u_1(r_1') \prod_{i=2}^{n} u_i(r_i)\right)$$

Since $M(R') > M(R)$, it follows that ϕ is a decreasing function.

- Thus with these two steps we know that there exists a decreasing function ϕ such that $\forall R = (r_1, r_2, ..., r_n)$

$$M(R) = \phi \left(\prod_{i=1}^{n} u_i\left(r_i\right) \right) \tag{4}$$

- Step 3: Now it will be shown that there is no loss of generality by requiring that every person's utility function is identical. Consider $R = (r^*, r_2, ..., r_n) \in \Delta$. By axiom A, $M\left(r^*, r_2, ..., r_n\right) = M\left(r_2, r^*, ..., r_n\right)$. Hence (4) implies that

$$M\left(r^*, r_2, ..., r_n\right) = M\left(r_2, r^*, ..., r_n\right)$$

$$\phi \left(u_1\left(r^*\right) u_2\left(r_2\right) \prod_{i=3}^{n} u_i\left(r_i\right) \right) = \phi \left(u_1\left(r_2\right) u_2\left(r^*\right) \prod_{i=3}^{n} u_i\left(r_i\right) \right)$$

and, since ϕ has been shown to be a decreasing function we have:

$$u_1\left(r^*\right) u_2\left(r_2\right) \prod_{i=3}^{n} u_i\left(r_i\right) = u_1\left(r_2\right) u_2\left(r^*\right) \prod_{i=3}^{n} u_i\left(r_i\right)$$

which implies $u_1\left(r^*\right) u_2\left(r_2\right) = u_1\left(r_2\right) u_2\left(r^*\right)$

$$u_2\left(r_2\right) = \frac{u_1\left(r_2\right) u_2\left(r^*\right)}{u_1\left(r^*\right)}$$

Likewise, using the same argument as above we have that $u_j\left(r_j\right) = u_1(r_j)\frac{u_j(r^*)}{u_1(r^*)}$, $\forall j = 1, ..., n$. Therefore, $\forall(r_1, r_2, ..., r_n) \in \Delta$, $\prod_{i=1}^{n} u_i\left(r_i\right) = \theta \prod_{i=1}^{n} u_i\left(r^*\right)$ where $\theta \equiv \frac{\prod_{i=1}^{n} u_i(r^*)}{u_1(r^*)^n} > 0$. It follows that if there is a decreasing function ϕ satisfying (4), there must exist a decreasing function Ψ, such that $\forall R = (r_1, r_2, ..., r_n) \in \Delta$, $M(R) = \Psi\left(\prod_{i=1}^{n} u_1\left(r_i\right)\right)$

For simplicity we will write $u(r_i)$ for $u_1(r_i)$ so we have

$$M(R) = \Psi \left(\prod_{i=1}^{n} u\left(r_i\right) \right) \tag{5}$$

- Step 4: We will now complete the proof. By axiom C, we know that

  $\forall r \in [0,1]$, $r = \Psi\left(u\left(r_i\right)^n\right)$. If we write $x \equiv u\left(r_i\right)^n$, then $\Psi\left(x\right) = u^{-1}\left(x^{\frac{1}{n}}\right)$.

  By axiom R(i), we can write $u\left(r\right) = A - Br$, where $B > 0$. Hence,

  $\Psi\left(x\right) = \frac{A}{B} - \frac{x^{\frac{1}{n}}}{B}$. Therefore, by using (5) we have

  $$M(r_1, r_2, ..., r_n) = \frac{A}{B} - \frac{1}{B}\left[\prod_{i=1}^{n}(A - Br_i)\right]^{\frac{1}{n}}$$

  $$= \frac{A}{B} - \left[\prod_{i=1}^{n}\left(\frac{A}{B} - r_i\right)\right]^{\frac{1}{n}}$$

  By writing β for $\frac{B}{A}$, we have

  $$M(r_1, r_2, ..., r_n) = \frac{1}{\beta} - \left[\prod_{i=1}^{n}\left(\frac{1}{\beta} - r_i\right)\right]^{\frac{1}{n}}$$

  Since $u : [0,1] \to R_{++}$, then $u\left(1\right) = [A - B \cdot 1] > 0$ which implies $A > B$.

  Therefore $1 > \beta$. Since, in addition $B > 0$ (by axiom R(i)), then $\beta > 0$.

  What we have left therefore is precisely the class of MOUs described in

  (2).   □

One advantage of a full axiomatization of the kind just undertaken is that it helps us evaluate the measure by factorizing it to its constituents. In this case the strong assumption is clearly axiom R. This requires individual utility to be cardinal but does not impose interpersonal comparability. This kind of an axiom is used to derive the Nash bargaining solution and is also widely used by Sen in social choice theory (see [20, 22] for instance). What may appear more contentious is the requirement that u_i be affine.

Some may treat this as reason to look for a different measure of unemployment, but there are two points worth keeping in mind. First, there are

alternate ways of axiomatizing the same measure. So there may be other ways of visualizing our measure that do not require one to use an affine utility function as an input.

Secondly, we must not think of the utility function of each person, u_i, as the person's own evaluation of her utility. Instead it should be viewed as society's evaluation of a person's employment status, which may well be different from the person's own utility evaluation (this is elaborated upon further in the next section). Once we take this approach and note that there are two steps to getting to a final measure (i) the assessment of each person's utility, u_i, and (ii) aggregation of these using a function, F, it becomes evident that the concavity of F acts as a substitute for diminishing marginal utility of the individual and that is the route we are taking here.

Moreover, our approach has some natural interpretational advantages. Consider person i's utility function: $u_i = A - Br_i$. Let Δu_i be the change in this person's utility if her status changed from fully unemployed ($r_i = 1$) to fully employed ($r_i = 0$). Clearly $\Delta u_i = B$. Now let $\widetilde{u}_i$ be this person's reservation utility, meaning the utility this person gets if she is without any work ($r_i = 1$). Clearly $\widetilde{u}_i = A - B$, i.e. a person without work has a utility of $A - B$. Hence, the ratio of the utility from other things (i.e. other than work) to utility from being able to work is given by $\frac{\widetilde{u}_i}{\Delta u_i} = \frac{A}{B} - 1$.

Hence, an increase in A denotes how the other things in life are more important than work. An increase in A is thus associated with moving to a society where there is reasonable social welfare and other sources of income

(for instance, through equity ownership) or where work is not as much a source of a person's social recognition. [6] Now note that since $\beta = \frac{B}{A}$, an increase in A is equivalent to β going towards zero. This, as we have already seen pushes us towards the additive, or utilitarian, case where egalitarianism in unemployment matters less in our MOU given by (2). Likewise as A becomes smaller, β becomes larger. In the limit employment achieves enormous importance and our MOU converges towards a multiplicative evaluation.

5 Discussion

Given the full characterization of the effective measure of unemployment and, before going on to discuss vulnerability in the context of poverty, it is useful to draw out some of the distinctions between the existing literature and our paper. First note that the bulk of the existing writing – the theoretical [5, 17] and the empirical [16] – mainly focuses on isolating individual vulnerability. It asks questions like: "Who is vulnerable to becoming unemployed or poor?" and "How do we estimate the number of vulnerable individuals?"

Our interest, on the other hand, is to assume that we know how many vulnerable or potentially unemployed persons (or people vulnerable to poverty, though, we have yet to address this) there are in society and then to develop an **aggregate** measure of **effective** unemployment, that is, to find a single number that captures the total unemployment – actual and potential. Among

---

[6] The 'social' cost of unemployment does not always get its due. But it is arguable that once our basic economic needs are satisfied, loss of face becomes a dominant cost of unemployment (see [23] for instance).

the few papers that share our concern with the aggregate are [25, 26, 18, 7].

[7] Borooah in [7] develops a measure, drawing on the work of Atkinson [3, 4] in which aggregate, effective unemployment is derived from an aggregation of separable individual utilities.

Our measure charts out a different course based on a rejection of this separability. Take a look at our proposed MOU again. Recall, $M^\beta(R) = \frac{1}{\beta} - \prod_{i=1}^{n} \left(\frac{1}{\beta} - r_i\right)^{\frac{1}{n}}$ and let us examine society's view of one person's unemployment load, or pain – as referred to by Borooah. Using r_1 as an example we can see that $\frac{\partial M^\beta(R)}{\partial r_1} = \frac{1}{n\left(\frac{1}{\beta} - r_1\right)} \left[\frac{1}{\beta} - M^\beta(R)\right]$ depends on the total effective unemployment as measured by $M^\beta(R)$. Hence, if total unemployment is higher, then $\frac{\partial M^\beta(R)}{\partial r_1}$ is lower. Therefore "the level of pain" that society associates with person i's unemployment depends on the level of effective unemployment in society. This essential relativity is not there in Borooah's measure.

Further, this paper takes the view that concepts like poverty, unemployment and even inequality cannot be reduced to pure welfarism. These are concepts that cannot be located entirely in the welfares of individuals and their aggregation. The same distinction that Sen (as in [21]) drew between ethical and descriptive features of inequality arise here in the context of unemployment and poverty.

---

[7] Interestingly, [26] is among the few papers that, like ours, uses an axiomatic approach, though the measure that he develops is different from ours.

We take the view that a greater amount of aggregate unemployment or aggregate poverty must not be **equated** with diminished aggregate social welfare. [8]

This leads to an important difference between our approach and that of much of the literature on vulnerability that uses the concept of 'certainty-equivalence' to evaluate vulnerability (see, for example, [17], 2003). Since the comment that follows applies equally to unemployment and poverty, we use these terms inter-changeably here, which also makes it a useful preamble to the next section which is purely on poverty measurement.

Suppose a person's poverty status can change in each month and on average he will be poor six months in a year (i.e. in each month the probability of being poor is $\frac{1}{2}$), and in the other six months he will be 100 dollars above poverty. Should he be counted as poor or not? According to the certainty-equivalence approach, we simply have to ask this person if he would prefer to change his position with that of another person who will be never poor for the 12 months but will be exactly on the poverty line at all times. If he says no, then this vulnerable person is effectively non-poor.

---

[8] Some economists would go even further and argue that a small amount of unemployment may reflect flexibility in the labor market and so be good for the economy overall. This is not to deny that there may be a mathematical isomorphism between the welfarist approach and our approach. This is evident from Theorem 1 if we interpret u_i's as each person's own evaluation of her utility and think of F as a welfare function. But such interpretations are not necessary and indeed we would resist them here.

This sounds like a very reasonable exercise if our interest is in welfare. But it is clear that the enormous literature on poverty measurement and unemployment measurement rejects such welfarism.

To understand this consider a society, x, with 12 persons, of whom six are poor (or unemployed) and six are each 100 dollars above the poverty line. Now transport all these 12 persons to a society, y, where they are exactly on the poverty line. Give each of them the choice of being born into society x without saying which position she will have. Let us say the probability that she will be poor (or unemployed) is $\frac{1}{2}$ and the probability that she will be non-poor with income 100 dollars above the poverty line is also $\frac{1}{2}$.

It is entirely possible that all 12 persons prefer society x to society y. Hence, in an <u>ex ante</u> sense x Pareto dominates y. Since there in no poverty (or unemployment) in y and everybody prefers x to y, if we were equating poverty (and unemployment) totally with welfare, we would be forced to say there is no poverty (and no unemployment) in x. But that would be absurd and indeed with six poor people in this society at all times poor, no one would say that x has no poverty.

Hence, in developing an aggregate measure of unemployment and poverty (treating these as descriptions of society) we may be justified in rejecting the welfarism inherent in the certainty-equivalence approach.

With this justification of the conceptual approach adopted in this paper, we can now proceed to discuss how the above method can be extended to measuring effective poverty.

6 Poverty

It should now be possible to extend our measure from the domain of unemployment to the domain of poverty. It we treat 'being poor' as a uniquely well-defined state, as is implicitly assumed in using the 'headcount rate' measure of poverty, then it is very easy to adopt our measure to this context. With this augmentation of vulnerability to our concept of poverty we again argue that vulnerability is again not necessarily a 'bad.' If the level of poverty, as measured by the headcount ratio, were to stay the same, then having people at risk of becoming poor means there exist people who are currently poor and are at *risk* of becoming non-poor. Therefore, under the headcount ratio, our measure would again provide some useful insights.

To begin, let poverty be measured using the headcount ratio, then define p_i as the proportion of a year that a person spends below the poverty line. Therefore if $p_i = 1$ then a person is poor for an entire year and if $p_i = 0$ the person is above the poverty line for an entire year. Now we have $(p_1, p_2, \ldots, p_n)$ defined as a **poverty profile** of a society where, for all i, $p_i \in [0, 1]$. Let Δ be the collection of all **poverty profiles**. That is, $\Delta = \{(p_1, p_2, \ldots, p_n) \mid n \in Z_{++} \ \& \ p_i \in [0, 1], \ \forall i\}$ where Z_{++} is the set of strictly positive integers. Then letting n be the number of people in a country we are able to define the 'usual headcount ratio of poverty' as follows:

$$H \equiv \frac{p_1 + p_2 + \ldots + p_n}{n} \tag{6}$$

If we define a **measure of poverty** (MOP) as a function $P : \Delta \to R_+$, then H is a MOP. Adapting our measure of unemployment to the present context we propose the following MOP

$$P^\beta (p_1, \dots, p_n) \equiv \frac{1}{\beta} - \prod_{i=1}^{n} (\frac{1}{\beta} - p_i)^{\frac{1}{n}} \tag{7}$$

where $\beta \in (0, 1)$

Here β would be our parameter of poverty aversion. Our MOP will satisfy all of the properties that are discussed above. This measure of poverty is bounded on one side by the 'usual' headcount ratio and on the side by a Rawlsian-type measure of poverty.

Ever since the celebrated work of Sen [21], poverty is however no longer treated as a binary concept. It is recognized that the depth of poverty a person suffers can vary and good measures of poverty ought to reflect this. One of the most widely used class of poverty measures is the Foster-Greer-Thorbecke (FGT) class of measures which make critical use of this. If we recognize that the depth of poverty can vary and people may be vulnerable to poverty of different depths, how can all this be combined into a single measure of *effective* poverty? There must be many ways of doing this but we shall outline a simple approach here which is based on treating each person at each point of time as a seperate entity. Once we use this idea of a 'timed individual,'

it is straightforward extending our measure of effective unemployment to this domain of poverty as well.

Imagine there is a society of n individuals, each of whom we observe over m time periods. Thus we have a total of $n \cdot m$ observations. Now, rather than averaging the Foster-Greer-Thorbecke (hereafter referred to as FGT) measure of poverty over time, consider the following. Take each individual i and her m observations. Let q_i be the number of periods that she was below the poverty line. Then calculate her FGT measure of poverty $p_i = \frac{1}{m} \sum_{j=1}^{q_i} (\frac{g_{ij}}{z})^\alpha$, where z is the poverty line, y_{it} is person i's income at time t, $g_{ij} = z - y_{ij}$ is person i's income gap at time j and α is the poverty aversion parameter.

One property of the FGT measure of poverty is that it is always between zero and one. Therefore we now have a value $p_i \in [0, 1]$ that takes account not only of whether person i is below the poverty line, but the depth of her poverty and we can now substitute this p_i into our MOP, defined by (7), above.

7 Conclusion

We have offered an alternative way to look at vulnerability to what is currently being discussed in the literature and by policy makers. That is, vulnerability, when viewed as a part of an aggregate measure, need not always be treated as a 'bad.' Given this perspective we have provided a way of measuring 'effective' unemployment. This measure is bounded on one side by the additive

or utilitarian measure and on the other side by a multiplicative measure. Furthermore, our measure satisfies axioms that most people would agree are what one would want from a measure motivated by equity concerns. We have fully characterized our measure and shown how the measure can be applied to data in both the US and in South Africa and what insights can be gained by comparing the usual and effective measures of unemployment.

This paper then serves two purposes. First, it suggests that the current debate on vulnerability needs to examine not only the effect of vulnerability on people currently employed, but also the *hope* that vulnerability provides to people who are currently unemployed. Secondly, this paper provides a way of taking account of these concerns in a single measure of unemployment and shows how the measure can actually be put to use.

References

[1] AKERLOF, G. A., AND B. G. M. MAIN (1980): "Unemployment Spells and Unemployment Experience," *American Economic Review*, 70(5), 885–893.

[2] AMIN, S., A. S. RAI, AND G. TOPA (2003): "Does Microcredit Reach the Poor and Vulnerable? Evidence from Northern Bangladesh," *Journal of Development Economics*, 70, 59–82.

[3] ATKINSON, A. (1970): "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–263.

[4] ——— (1983): *Social Justice and Public Policy*. MIT Press, Cambridge, MA.

[5] BANERJEE, A. (2000): "The Two Poverties," *Nordic Journal of Political Economy*, 26(2), 129–141.

[6] BASU, K., AND J. FOSTER (1998): "On Measuring Literacy," *Economic Journal*, 108(451), 1733–1749.

[7] BOROOAH, V. K. (2002): "A Duration-Sensitive Measure of the Unemployment Rate: Theory and Application," *Labour*, 16(3), 453–468.

[8] CALVO, C., AND S. DERCON (2005): "Measuring Individual Vulnerability," Oxford Department of Economics Discussion Paper Number 229.

[9] CLARK, K. B., AND L. H. SUMMERS (1979): "Labor Market Dynamics and Unemployment: A Reconsideration," *Brookings Papers on Economic Activity*, 1979(1), 13–72.

[10] CUNNINGHAM, W., AND W. F. MALONEY (2000): "Measuring Vulnerability: Who Suffered in the 1995 Mexican Crisis?," IBRD mimeo.

[11] DERCON, S. (2005): "Vulnerability: A Micro Perspective," paper presented at the ABCDE Europe conference, Amsterdam, May 2005.

[12] FIELDS, G. S. (1996): "The Meaning and Measurement of Income Mobility of Income Mobility," *Journal of Economic Theory*, 71, 349–377.

[13] FOSTER, J., J. GREER, AND E. THORBECKE (1984): "A Class of Decomposable Poverty Measures," *Econometrica*, 52(3), 761–766.

[14] GLEWWE, P., AND G. HALL (1998): "Are some Groups More Vulnerable to Macroeconomic Shocks Than Others? Hypothesis Tests Based on Panel Data from Peru," *Journal of Development Economics*, 56, 181–206.

[15] GROOTAERT, C., AND R. KANBUR (1995): "The Lucky Few Amidst Economic Decline: Distributional Change in Cote d'Ivoire As Seen Through Panel Datasets, 1985-1988," *Journal of Development Studies*, 31(4), 603–619.

[16] KAMANOU, G., AND J. MORDUCH (2002): "Measuring Vulnerability to Poverty," WIDER Discussion Paper No. 2002/58.

[17] LIGON, E., AND L. SCHECHTER (2003): "Measuring Vulnerability," *Economic Journal*, 113(486), C95–C102.

[18] PAUL, S. (1992): "An Illfare Approach to the Measurement of Unemployment," *Applied Economics*, 24, 739–743.

[19] PRITCHETT, L., A. SURYAHADI, AND S. SUMARTO (2000): "Quantifying Vulnerability to Poverty: A Proposed Measure, with Applications to Indonesia," SMERU Working Paper.

[20] SEN, A. (1974): "Informational Bases of Alternative Welfare Approaches," *Journal of Public Economics*, 3, 387–403.

[21] ——— (1976): "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44, 219–231.

[22] ——— (1977): "On Weights and Measures: Informational Constraints in Social Welfare Analysis," *Econometrica*, 45, 1539–1572.

[23] ——— (1997): "The Penalties of Unemployment," Bank of Italy, Rome Paper No. 307.

[24] SHORROCKS, A. F. (1978): "The Measurement of Mobility," *Econometrica*, 46(5), 1013–1024.

[25] ——— (1992): "Spell Incidence, Spell Duration and the Measurement of Unemployment," Discussion Paper 409, Department of Economics, University of Essex.

[26] ——— (1994): "On the Measurement of Unemployment," Discussion Paper 418, Department of Economics, University of Essex.

[27] SUZUMURA, K. (1999): "Consequences, Opportunities, and Procedures," *Social Choice and Welfare*, 16(1), 17–40.

[28] THORBECKE, E. (2003): "Conceptual and Measurement Issues in Poverty Analysis," Paper prepared for the Second Nordic Conference in Development Economics (NCDE-2) Copenhagen, Denmark, June 23-24.

[29] WORLD BANK (2002): "Pakistan Poverty Assessment. Poverty in Pakistan: Vulnerabilities, Social Gaps and Rural Dynamics," Report No. 24296-PAK.