Mixture Models and Convergence Clubs

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Abstract

In this paper we argue that modelling the cross-country distribution of per capita income as a mixture distribution provides a natural framework for the detection of convergence clubs. The framework yields tests for the number of component distributions that are likely to have more power than "bump hunting" tests and includes a natural method of assessing the cross-component immobility necessary to imply a correspondence between components and convergence clubs. Applying the mixture approach to cross-country per capita income data for the period 1960 to 2000 we find evidence of three component densities in each of the nine years that we examine. We find little cross-component mobility and so interpret the multiple mixture components as representing convergence clubs. We document a pronounced tendency for the strength of the bonds between countries and clubs to increase. We show that the well-known "hollowing out" of the middle of the distribution is largely attributable to the increased concentration of the rich countries around their component means. This increased concentration as well as that of the poor countries around their component mean produces a rise in polarization in the distribution over the sample period.

Key words: Convergence Clubs, Economic Growth, Mixture Models, Polarization.

JEL classification: D31; C14.

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1 Introduction

There has been a great deal of interest in the shape and evolution of the cross-country distribution of per capita income in recent years. Much of this interest arises from the relationship between those characteristics of the distribution and the neoclassical convergence hypothesis. That hypothesis states that initial conditions have no implications for long-run outcomes so that all countries will converge to a common level of GDP per capita regardless of where they begin.[1] The alternative hypothesis is that initial conditions do matter in the long run and that countries with similar initial conditions exhibit similar long-run outcomes so forming "convergence clubs" – groups of countries that converge locally but not globally. One possible manifestation of the presence of convergence clubs is multiple modes in the cross-country distribution of per capita income with each mode corresponding to a convergence club. Multimodality is, however, not enough to imply the existence of convergence clubs. That requires immobility within the distribution so that countries in the vicinity of a mode tend not to move to that of another mode.

Most of research investigating the shape of the cross-country distribution of per capita income has employed kernel estimation methods. See, for example, Quah (1996, 1997), Bianchi (1997), Jones (1997), Henderson, Parmeter and Russell (2007), and others. Bianchi (1997) and Henderson, Parmeter and Russell (2005) present various tests of the hypothesis of a unimodal distribution against that of a multimodal distribution. They are able to reject the null in most cases. Both papers also find little mobility between the modes they identify. Together, these findings support the existence of convergence clubs. Applications of mixture models, a semi-parametric alternative to the kernel approach, have been less numerous. The only application to the cross-country distribution of which we are aware is Paap and van Dijk (1998) although Tsionas (2000) uses mixture models to study the distribution of per capita output across the US states, while

[1] Durlauf, Johnson, and Temple (2005) provide a survey of the many guises taken by the convergence hypothesis and of their myriad empirical implementations.

Pittau (2005), and Pittau and Zelli (2006) use them to study the distribution of per capita incomes across EU regions. The findings of Paap and van Dijk are consistent with those of Bianchi (1997) and Henderson, Parmenter and Russell (2007).

The mixture approach offers several advantages over the kernel approach in the current application. Mixture models express the density of a random variable as the weighted average of a finite number of component densities with specified functional form. The parameters to be estimated are the number of, the weights attached to, and the parameters of, the component densities. In the growth context, multiple components, like multiple modes, can be indicative of multiple basins of attraction in the dynamic process describing the evolution of per capita income. Importantly, the mixture approach is able to detect the presence of multiple components in a distribution even if that multiplicity does not manifest itself as multimodality. As multimodality is not necessary for the existence of convergence clubs, used as part of a test of the convergence hypothesis, the mixture approach can thus provide a test with more power than the kernel approach. Moreover, the mixture approach provides integrated tests, such as a likelihood ratio test, for the number of components which represent improvements over the "bump hunting" methods employed in the kernel based studies. Indeed, Silverman (1986, p141) cautions that "[it] may be futile to expect very high power from procedures aimed at such broad hypotheses as unimodality and multimodality".

As with the kernel approach, the interpretation of multiple components as indicative of convergence clubs also requires an analysis of the mobility within the distribution, which in this case means that between the components. Again however, this can be accomplished quite naturally within the mixture model framework, providing another improvement on the relatively ad hoc methods of mobility analysis employed in the kernel based studies. The estimated mixture model parameters enable computation of the conditional probabilities that each entity belongs to each component. These probabilities can be used to assign entities to components as well as to gauge the strength of the affinity between the entity and the components. The propensity of entities to change their assigned components over time provides a measure of within-distribution mobility.

In this paper we use finite mixture models to investigate the number of components in the cross country distribution of per capita income over the 1960 to 2000 period. In addition to the improvements over the studies based on kernel estimation mentioned above, the primary contribution is the improvement in the methodology over that of Paap and van Dijk (1998) who choose the number of components to be two a priori based on the bimodality of histograms of their data. This procedure may not detect all components as components do not imply modes. Indeed, we find strong evidence of three rather than two components. The next section of the paper outlines our analytical framework as well as describing the data that we use. Section 3 presents our results and the final section offers our conclusions.

2 Analytical Framework and Data

The m-component mixture model specifies the density of a random variable as

$$f(x, m, \mathbf{\Theta}_m, \mathbf{\Pi}_m) = \sum_{j=1}^{m} \pi_j f_j(x, \boldsymbol{\theta}_j) \tag{1}$$

where $f_j(x, \boldsymbol{\theta}_j)$ is a probability density function with parameter vector $\boldsymbol{\theta}_j$, for $j = 1, \ldots, m$, $\mathbf{\Theta}_m = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_m)$, the π_j are the mixing proportions with $\pi_j > 0$ for $j = 1, \ldots, m$, $\sum_{j=1}^{m} \pi_j = 1$, and $\mathbf{\Pi}_m = (\pi_1, \pi_2, \ldots, \pi_m)$.

Given m and the functional forms of the component densities, $f_j(x, \boldsymbol{\theta}_j)$, the parameters of the model can be estimated by the method of maximum likelihood. We do so using an iterative fitting by maximum likelihood (ML) via the expectation-maximization (EM) algorithm (Dempster, Laird and Rubin, 1977). Each iteration comprises of an expectation step (E-step) followed by a maximization step (M-step). The EM algorithm seems to be superior to the other procedures in finding a local maximum of the likelihood function (McLachlan and Peel, 2000). We make the usual assumption that the component densities are normal so that $f_j(x, \boldsymbol{\theta}_j) = N(x; \mu_j, \sigma_j^2)$, the normal density function with mean μ_j and variance σ_j^2, for $j = 1, \ldots, m$. This is not, however, as restrictive as it may seem because any continuous density can be well approximated by a mixture of normal densities (Marron and Wand, 1992). Moreover, the normal distribution is especially easy to interpret in this application as μ_j is the mean per capita income in component j and σ_j^2 measures the within-component variation in per capita incomes.

We take two approaches to the selection of m, the number of components. The first is a likelihood ratio test (LRT) of the null hypothesis $m = m^*$ against the alternative $m = m^* + 1$. For this test, the distribution of the LRT statistic under the null hypothesis is estimated by bootstrap methods as the conditions necessary for the LRT statistic to have the usual asymptotic χ^2 distribution do not hold (McLachlan, 1987) [2].

For each m^*, B bootstrap samples are drawn from the mixture distribution $f(x, m^*, \widehat{\boldsymbol{\Theta}}_m^*, \widehat{\boldsymbol{\Pi}}_m^*)$ where the parameter values are those estimated using the original sample. An m^* component and an $m^* + 1$ component mixture model are estimated for each sample by the method of maximum likelihood and the usual LRT statistic is computed. The significance level of the sample LRT statistic is then computed as $1 - \frac{r}{B+1}$ where r is number of replications with an LRT statistic less than the sample LRT statistic. The second approach to selecting the number of components considers the goodness of fit of the estimated mixture model by comparison of a kernel estimate of the density of the data and its expected value under the null hypothesis that the population density is a mixture of m^* normal distributions. This comparison is made by computing the estimated integrated squared error (ISE) statistic

[2] Other approaches, like the modified LRT, derive a relatively simple asymptotic null distribution of the likelihood ratio test. See, distinctively Ghosh and Sen 1985, and more recently Chen, Chen and Kalbfleisch, 2004; Chen and Kalbfleisch, 2005. However, the implementation of such modified LR test does not alter the findings of this section.

$$\widehat{J} = \int_x \left[\widehat{f}(x) - \sum_{j=1}^{m^*} \widehat{\pi}_j \mathrm{N}(x; \widehat{\mu}_j, \widehat{\sigma}_j^2 + h^2) \right]^2 dx \tag{2}$$

where h is the bandwidth used to compute $\widehat{f}(x)$, the kernel estimate of $f(x)$, the true density of x. We select h using the Sheather and Jones (1991) method and compute $\widehat{f}(x)$ as a fixed, rather than adaptive, bandwidth estimate because of the difficulties in calculating the expected value of the kernel estimate in the current application (Bowman and Foster, 1993, p.535). While asymptotic results for the distribution of $\widehat{J}$ are available, we follow Fan (1995) because of our small sample size, and we estimate the distribution using a parametric bootstrap procedure in which the bootstrap samples are drawn from the mixture distribution $f(x, m^*, \widehat{\mathbf{\Theta}}_m^*, \widehat{\mathbf{\Pi}}_m^*)$ where the parameter values are those estimated using the original sample. The significance level of the sample $\widehat{J}$ is computed as $1 - \frac{r}{B+1}$ where B is the number of bootstrap replications and r is number of replications with $\widehat{J}$'s less than the sample $\widehat{J}$.

We apply both the LRT and ISE tests recursively beginning with the null hypothesis $m^* = 1$, continuing to that of $m^* = 2$ if the $m^* = 1$ null is rejected, and so on. We set m equal to the smallest m^* for which we are unable to reject the null hypothesis $m = m^*$. Once m is chosen, the parameter vectors $\mathbf{\Theta}_m$ and $\mathbf{\Pi}_m$ can be estimated enabling study of the properties of the m component densities. The π_j can be interpreted as the unconditional probability that X_i, observation i, is a draw from component j. The conditional probability of that event is given by

$$\zeta_{ji} = \frac{\pi_j f_j(X_i, \boldsymbol{\theta}_j)}{\sum_{j=1}^{m} \pi_j f_j(X_i, \boldsymbol{\theta}_j)} \tag{3}$$

These probabilities can be used to assign observations to components by assigning observation i to that component with the largest estimated ζ_{ji}, computed using equation (3) with the π_j and the $\boldsymbol{\theta}_j$ replaced by their estimates. Given a panel of data, mobility can be studied by noting the propensity of the assignment of entity i to change over time. The strength of the affinity between entity i and the components can be gauged.

The per capita income data used is real GDP per worker (RGDPWOK) from the Penn World Table (PWT) Version 6.1 (Heston, Summers, and Aten, 2002). As Durlauf, Johnson and Temple (2005) argue, GDP per worker accords more closely than GDP per capita with the dependent variable of interest in most growth models.[3] The sample consists of data on the 102 countries–all of those for which data is available for the entire 1960 to 1995 period. Following Durlauf, Johnson and Temple (2005) we exclude the middle-eastern oil producing countries and Luxemburg. For the year 2000 data we use data from 1998 for 98 of the 102 countries with that for the remaining four being extrapolated from the 1997 data.[4] The alternative, using the countries for which actual 2000 data is available, would reduce our dataset to 89 countries. We estimate a mixture model for each of the nine years 1960, 1965, ..., 2000. The variable used in our analysis is RGDPWOK relative to its workforce-weighted average over the 102 countries in the sample. Using the PWT 6.1 mnemonics, the workforce for each country, in each year, was computed as POP*RGDPCH/RGDPWOK.

3 Results

3.1 Number of Components

Table 1 reports the LRT statistics and and the corresponding bootstrapped p-values for testing the null hypothesis of $m = m^*$ components *versus* $m = m^* + 1$ in the mixture model for m^* ranging from 1 to 4. In each year the value of the LRT statistic implies rejection, at conventional significance levels, of the null hypotheses $m = 1$ and $m = 2$ but not that of $m = 3$. Moreover there is no tendency for the selected number of components to fall over time as would be

[3] The number of workers "... is usually a census definition based of economically active population". (Data Appendix to PWT 6.1 dated 10/18/02 p. 11)

[4] As the data for each year are analyzed independently, any errors caused by this extrapolation will be confined to the 2000 data.

Table 1: The choice of the number of components according to the likelihood ratio test.

	m^*=1		m^*=2		m^*=3		m^*=4	
year	LRT	p-value	LRT	p-value	LRT	p-value	LRT	p-value
1960	64.18	0.000	24.21	0.042	3.80	0.736	0.01	1.000
1965	56.14	0.000	34.91	0.026	3.33	0.804	0.00	0.998
1970	59.93	0.000	28.20	0.036	5.35	0.574	1.00	0.978
1975	51.89	0.000	37.52	0.022	3.72	0.642	0.01	1.000
1980	62.53	0.000	20.87	0.048	0.51	0.978	0.00	1.000
1985	47.06	0.002	35.08	0.028	2.17	0.932	0.00	1.000
1990	55.18	0.000	45.28	0.024	9.12	0.206	3.03	0.942
1995	64.47	0.000	45.17	0.020	10.17	0.192	0.58	0.992
2000	61.74	0.000	46.15	0.016	11.22	0.154	2.91	0.978

suggested by a tendency for the LRT statistics for the $m = 2$ null hypothesis to fall. To the contrary, if there is any tendency at all for the selected number of components to change, it is for a rise as evinced by the rise in the LRT statistics for the $m = 3$ null hypothesis although we are never able to reject this hypothesis.

Similarly, Table 2 presents the results of the statistical testing procedure using the goodness of fit test (ISE). These results are entirely consistent with those from the LRT procedure lending support to the conclusion that a mixture of three normal densities offers the preferred description of the cross-country distribution of output per worker.

The finding of three (or, more generally, more than one) mixture components is not enough to imply the existence of multiple convergence clubs in the cross country distribution of per capita income. That requires an additional analysis of the mobility of the basins of attraction which we undertake in Section 3.3 after a discussion of the evolution of the components over the sample period.

3.2 Evolution of Distribution and its Components

The estimation of the previous section produces estimates of the mean μ_j and variance σ_j^2 of per capita GDP for each of the three components which we label

Table 2: The choice of the number of components according to the goodness of fit test.

	m^*=1		m^*=2		m^*=3		m^*=4	
year	$\widehat{J}$	p-value	$\widehat{J}$	p-value	$\widehat{J}$	p-value	$\widehat{J}$	p-value
1960	10.76	0.000	2.11	0.000	0.21	0.736	0.04	0.960
1965	10.80	0.000	3.34	0.000	0.22	0.776	0.09	0.812
1970	10.61	0.000	1.19	0.006	0.38	0.365	0.11	0.713
1975	10.22	0.000	3.10	0.000	0.23	0.642	0.04	0.850
1980	9.28	0.000	2.23	0.000	0.06	0.954	0.05	0.849
1985	9.09	0.000	2.83	0.001	0.42	0.156	0.33	0.057
1990	11.58	0.000	3.55	0.000	0.50	0.192	0.17	0.579
1995	11.32	0.000	3.62	0.000	0.47	0.219	0.13	0.678
2000	11.51	0.000	2.42	0.000	0.48	0.112	0.14	0.673

The estimated ISE, $\widehat{J}$, is multiplied by 100

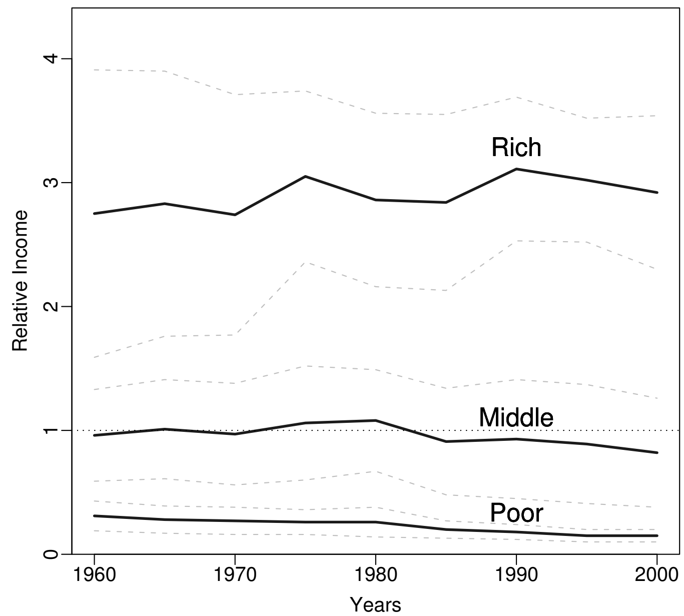
"poor", "middle", and "rich" according to the estimated means with $\widehat{\mu}_{\text{poor}} < \widehat{\mu}_{\text{middle}} < \widehat{\mu}_{\text{rich}}$.

Figure 1 plots the means against time (the solid lines) along with dashed lines that indicate the intervals containing 80% of the probability mass of each component. That is, the dashed lines are $\widehat{\mu}_{\text{poor}} \pm 1.282 \times \widehat{\sigma}_{\text{poor}}$, $\widehat{\mu}_{\text{middle}} \pm 1.282 \times \widehat{\sigma}_{\text{middle}}$ and $\widehat{\mu}_{\text{rich}} \pm 1.282 \times \widehat{\sigma}_{\text{rich}}$ where $\widehat{\sigma}_j$ is the estimated standard deviation of component j for $j =$"poor", "middle","rich".

As Figure 1 shows, over the sample period the mean of the poor component, $\widehat{\mu}_{\text{poor}}$, fell steadily so that, in 2000, it was about half of its 1960 value. Although because the 1960 value is so low – about 30% of the sample mean – this fall is small in absolute terms. The estimated means of the middle and rich components are slightly more volatile than that of the poor component with $\widehat{\mu}_{\text{rich}}$ having an upward trend over the sample period while $\widehat{\mu}_{\text{middle}}$ finishes the sample period slightly below where it began. The gap between the rich and poor components, measured as $\widehat{\mu}_{\text{rich}} - \widehat{\mu}_{\text{poor}}$, increases by about 14% over the sample period while $\widehat{\mu}_{\text{rich}} - \widehat{\mu}_{\text{middle}}$ increases by about 17%.

There also are important changes in the dispersion of the contries around the component means, especially in the case of the rich component. Over the

Figure 1: Groups' means over time.



sample period the estimated standard deviation for this component, $\widehat{\sigma}_{\text{rich}}$, falls by almost 50% with about half of the fall occurring between 1970 and 1975 and a further quarter occurring between 1985 and 1990. This is shown in Figure 1 by the narrowing of the interval containing 80% of the mass of this component to 60% of its 1960 value in 1975 and subsequently to 50% of its 1960 value in 1990.

This phenomenon, and the relative stability of the estimated standard deviation of the middle component, which rises by about 30% over the sample period, combine to open a region of low probability mass between the middle and rich components. This is evident in the successive panels of Figure 2 as the deepening of the antimode at a value of relative output per worker of about two. This figure shows the estimated kernel and mixture densities for GDP per worker in each year as well as the constituents of the estimated mixture distribution i.e., the $\widehat{\pi}_j f_j(x, \widehat{\boldsymbol{\theta}}_j)$ for $j =$"poor", "middle","rich"[5]. As panels (a), (b), (c) and (d) in Figure 2 show, the antimode is evident in 1960 and it remains substantially unchanged until 1975 when it becomes much deeper. Panels (e), (f) and (g) show that after 1975 the antimode was again substantially unchanged through 1980 and 1985 until it again become much deeper in 1990.

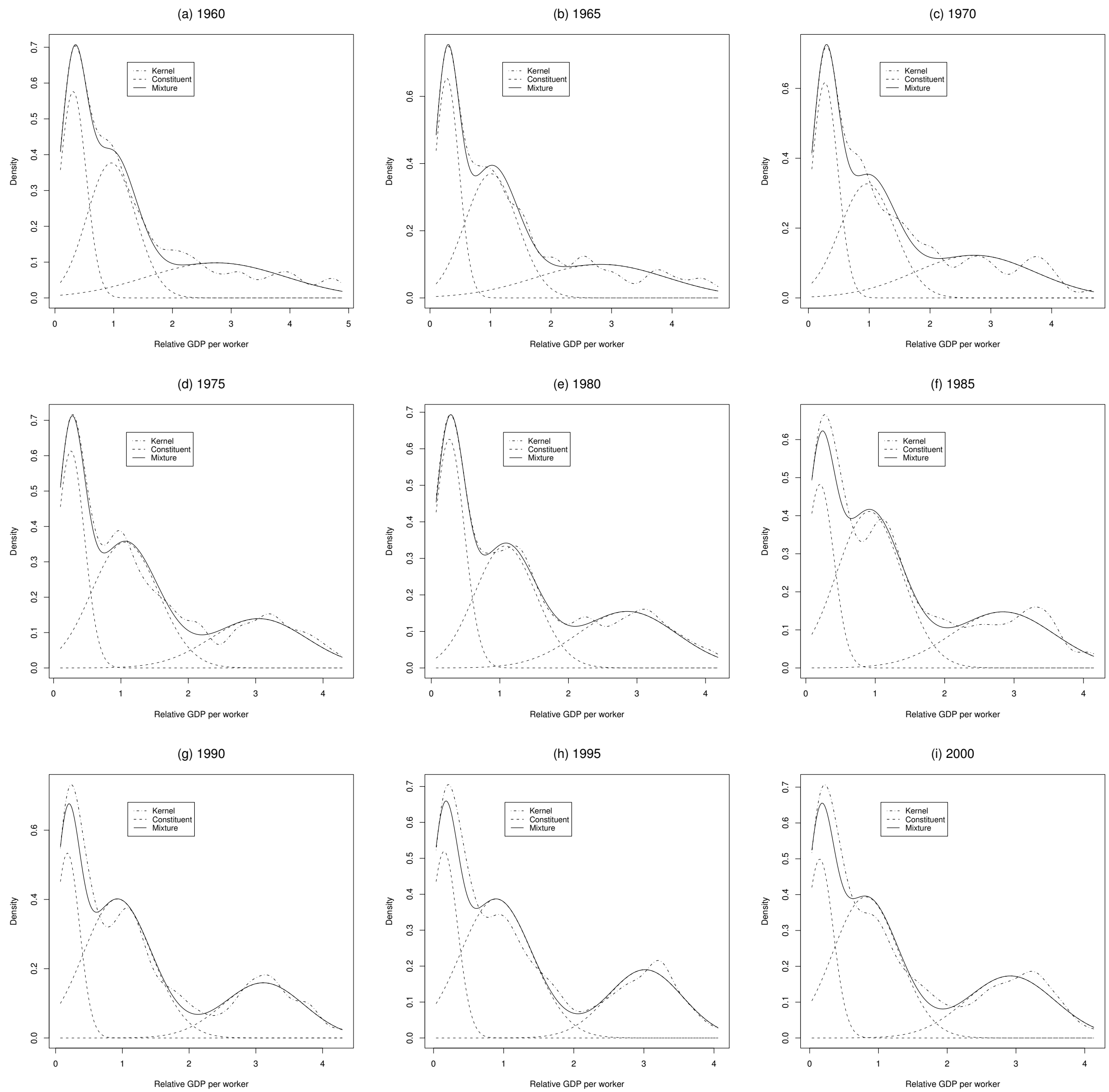
While the gap between the rich component and the middle component means does rise – by about 17% – over the sample period, the dominant cause of the observed "hollowing out" of the middle of the cross country distribution of output per worker seems to be the decrease in the within-component variation in the rich component. As this decrease could reflect, in part at least, compositional changes, we have more to say about it in Section 3.4 below after we discuss mobility across the components.

The variance of the poor component falls by almost 60%. As Figure 2 shows, the net effect of this and the smaller rise in the variance of the middle component is the appearance in 1965 of an antimode at a value of relative output per worker of about $\frac{2}{3}$. This antimode persists at various depths throughout the remainder of the sample period but is never very deep compared to the mode immediately to its right (at a value of relative output per worker slightly above unity). The importance of this phenomenon in the evolution of the distribution is much smaller that of the antimode discussed above – the magnitude of the former, both absolutely and relative to the modes on either side, is much smaller than that of the latter.

Our findings here are consistent with, for example, those of Beaudry, Collard and Green (2005) who document increases in the 15-85 and smaller percentile ranges of the cross-country distribution of output per worker along with reductions in the 10-90 and larger percentile ranges between 1960 and 1998. They provide evidence that these changes began in the mid 1970's. Our statistical

[5] These are the same densities used to compute the $\widehat{J}$ statistics discussed above. The $\widehat{\pi}_j f_j(x, \widehat{\boldsymbol{\theta}}_j)$ are not individually labeled due to space considerations but there ought not be any resultant ambiguity as $\widehat{\pi}_{\text{middle}} f_{\text{middle}}(x, \widehat{\boldsymbol{\theta}}_{\text{middle}})$ lies always to the right of $\widehat{\pi}_{\text{poor}} f_{\text{poor}}(x, \widehat{\boldsymbol{\theta}}_{\text{poor}})$ and so on.

Figure 2: Kernel density estimation and the three-components mixture model fit over the period 1960–2000.



explanation of their findings is a tightening of the component distributions at either extreme of the cross-country distribution of output per worker at that time which reduced the mass in the center of the distribution as well as in the tails.

More generally, in Section 3.5 we explain the often-discussed increase in the polarization of the cross-country distribution of output per worker since 1960 by an increase in the concentration of the poor and rich countries around their component means rather than by an increase in the gap between the means themselves.

3.3 Mobility between Components

As described above, we assign countries to components according to their maximal estimated conditional probability of belonging to each component. The $\widehat{\zeta}_{ji}$ for each country, each component and each year are given in Appendix B. Given these assignments, we are able to observe the implied transitions between components that occur when assignments change. Given that the link between multimodality and the existence of convergence clubs is tenuous, this seems to be a more natural definition of a "transition" than the crossing from one side of an antimode to another (as used by, for example, Bianchi, 1997, and Henderson, Parmeter and Russell, 2007). Moreover, it is not generally true that, if there is one, the antimode in a mixture distribution occurs at the point where the conditional probabilities of belonging to the two components are equal. That is, crossing from one side of the antimode to another need not imply a change in the component with the maximal conditional probability.

So defined, transitions are relatively rare events during our sample period and a small number of countries account for most of them so that immobility rather than mobility is the norm. Of the 714 possible transitions only 51, or about 7%, occur. Excepting the flurry of transitions in the mid-1980's, the transition rate is roughly constant over the sample period. Sixty-four of the 102 countries in our sample remain assigned to the same component throughout the sample period.[6] Of those that do transition from their initial component, 28 transition just once so that the remaining 10 countries account for over 40% of the observed transitions.

Of the countries that never leave their initial component, 18 are among the 26 initially rich countries while the other 8 initially rich countries (Argentina, Chile, Costa Rica, Mexico, South Africa, Trinidad and Tobago, Uruguay and Venezuela) move to the middle component where, with the exception of Argentina which returns to the rich component in 2000, they all remain until 2000.

A further 24 of the countries that remain attached to the same component over the entire sample period are among the 40 countries initially classified as belonging to the middle component. Of the 16 countries that transition from the middle component during the sample period, 3 countries (Angola, Central African Republic, Senegal) move to the poor component, 9 countries (Cyprus, Greece, Hong Kong, Japan, Korea, Mauritius, Portugal, Singapore and Taiwan) move to the rich component, 2 countries (Cameroon and Guinea) return to it after spending the 1970's and 1980's in the poor component, and 2 countries (Iran and Peru) return to it after visiting the rich component in 1970.

The remaining 22 of the countries that stay attached to the same component over the entire sample period are among the 36 initially poor countries. Of the 14 contries that leave the poor component, 12 (Bangladesh, Botswana, China, Republic of the Congo, Cote d'Ivorie, India, Indonesia, Pakistan, Romania, Ski Lanka, Thailand, and Zimbabwe) move to the middle component and remain there until the end of the sample period while 2 (Mauritania and Zambia) return to the poor component.

[6] They are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Ireland, Israel, Italy, The Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, The United Kingdom and The United States (all initially in the rich group); Bolivia, Brazil, Columbia, The Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Gabon, Guatemala, Guyana, Honduras, Jamaica, Jordan, Malaysia, Morocco, Namibia, Nicaragua, Panama, Papua New Guinea, Paraguay, Philippines, Syria, and Turkey (all initially in the middle group); and, Benin, Burkina Faso, Burundi, Chad, The Democratic Republic of the Congo, Ethiopia, The Gambia, Ghana, Guinea-Bissau, Kenya, Lesotho, Madagascar, Malawi, Mali, Mozambique, Nepal, Niger, Nigeria, Rwanda, Tanzania, Togo, and Uganda (all initially in the poor group).

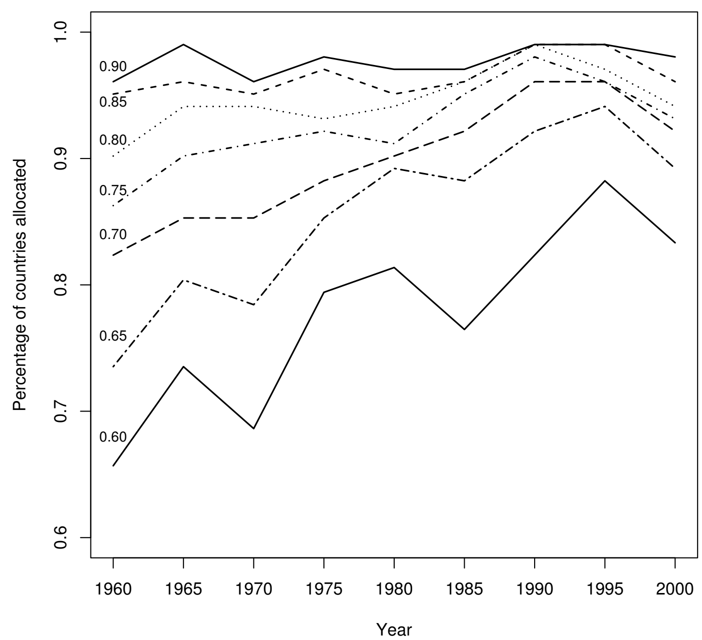
In sum, we conclude that the cross-component mobility during our sample period was low with transitions between components being relatively rare. While the transition rates that we find are low, they are somewhat higher than those documented in other studies such as Bianchi (1997) who finds that 3 of 238 (1.2%) possible transitions occur, Paap and van Dijk (1998) who find that 21 of 720 (2.9%) possible transitions occur, and Henderson, Parmeter and Russell (2007) who find that 12 of 291 (4.1%) and 19 of 414 (4.6%) possible transitions occur in the two per capita output datasets that they employ.[7] Our higher estimated transition rate is explained, in part at least, by our greater number of putative convergence clubs. Each of the studies cited above identifies two putative clubs whereas we find three so that we have twice as many between-club boundaries and hence twice as many points where a transition can occur. We would thus expect to observe a higher frequency of transitions given any degree of mobility within the distribution. Nonetheless, we conclude that the mobility between the components of the cross country distribution of per capita income is low.

In addition to allocating the countries among the components, the estimated conditional probabilities can be used to measure the strength of the affinity between countries and components. Higher probabilities indicate tighter bonds so we measure the overall tightness of the bonds between countries and components by counting the number of countries with a $\widehat{\zeta}_{ji}$ greater than a threshold, τ, for any j. Figure 3 plots the number of countries with a $\widehat{\zeta}_{ji} > \tau$ for any j for various values of τ from .6 to .9. The first feature evident in Figure 3 is the general strength of the bonding between countries and components. In 1960, for example, about $\frac{2}{3}$ of the countries have a $\widehat{\zeta}_{ji} > .9$ for some j and over 90% have a $\widehat{\zeta}_{ji} > .7$. The second, and arguably more important, feature displayed in Figure 3 is the evident increase in the strength of the affinity between countries and components over the sample period as shown by the general tendency for

[7]Henderson, Parmeter and Russell (2007) follow Bianchi (1997) and defines transitions as movements across the antimode between the two modes that they identify while Paap and van Dijk (1998) do as we do and count changes in component assignments based on maximal estimated conditional probabilities.

the lines in Figure 3 to rise over time. For example, in 1960, 74% of the countries had a maximal conditional probability greater than .85 whereas in 2000, 89%, were bound this tightly to a component. The tendency for the number of countries with a $\widehat{\zeta}_{ji} > \tau$ to rise between 1960 and 1998 holds for all values of τ although it is necessarily less pronounced for lower values. In sum, we conclude that most countries are bound very tightly to a component and that the tightness of the bonds has increased over time.

Figure 3: Percentage of countries allocated for different values of the threshold.



Our finding of low cross-component mobility leads us to interpret the multiple mixture components identified in Section 3.1 as representing multiple basins of attraction in the stochastic process describing the evolution of output per worker. That is, we regard that process as characterized by convergence clubs so that a country's initial level of output per worker plays an important role in determining its long-run level. Moreover, the role of initial conditions seems to be strengthening as the affinity of countries for clubs became stronger during the period that we have studied. It is important to note at this point that our results are subject to a version of the identification caveat discussed in Durlauf and Johnson (1995). As there, the behavior that we have documented is compatible with a model in which there are multiple steady states, or convergence clubs, as we have emphasized, as well as with a model in which countries transition though different stages of development before reaching a common (stochastic) steady state. In common with all of the empirical growth literature, differentiation between these two alternatives is hampered by the time span of our dataset.

3.4 Behavior within Components

Having discussed the mobility within the distribution we return now to the issue of the role of compositional changes in the reduction of the variance of the rich component over the sample period. Recall that this reduction occurs mainly in two steps viz., the fall between 1970 and 1975 and that between 1985 and 1990. While the latter is due in some part to the movement of Argentina, Mexico, South Africa, Trinidad and Tobago, and Venezuela out of the group, the role of such compositional changes in the former is small, as it is in the total reduction in the variance of the rich component over the sample period. To show this we consider the variation in output per worker across the 18 countries that remain assigned to the rich component throughout the sample period. Figure 6 plots the standard deviation of output per worker across this "always rich" group as well as that for rich component. Over the sample period the former fell by almost $\frac{2}{3}$ with about 75% of the decline occurring before 1975. By contrast, the estimated standard deviation of the rich component falls by about 50% over the sample period. This implies that the observed tendency for the rich component to become increasingly separated from the other two components is not due to compositional effects but rather to forces within the group causing the rich countries to become increasingly concentrated around the group mean. That is, the rich component did not become more concentrated around it's mean simply because some countries relatively far from the mean left the group. Instead, the rich countries tended to move closer to each other and in doing so increased their separation from the other countries in the world.

Figures 4 and 5 show analogous information for the poor and middle components respectively. As with the rich component, in both of these cases the behavior of the standard deviations of the group of countries always assigned to each component mirrors that of the corresponding estimated component standard deviation. Figure 4 shows that, as with the estimated standard deviation of the poor component, with exception of the late 1970's, the standard deviation of output per worker in the 22 "always-poor" countries fell steadily from 1960 to 2000. As with the rich component, the poor component did not become more concentrated around it's mean simply because some countries relatively far from the mean left the group. Rather, the poor countries tended to move closer to each other and in doing so also increased their separation from the other countries in the world. Figure 5 shows that both the estimated standard deviation of the middle component and the standard deviation of the income per capita across the 24 "always middle" countries both exhibit a slight upward trend over the 1960 to 2000 period.

Figure 4: Standard deviation of the poor group.

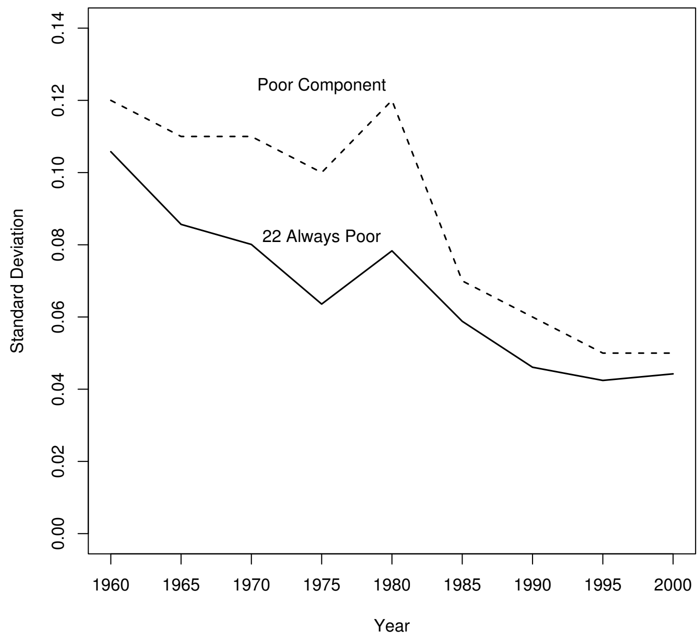


Figure 5: Standard deviation of the middle group.

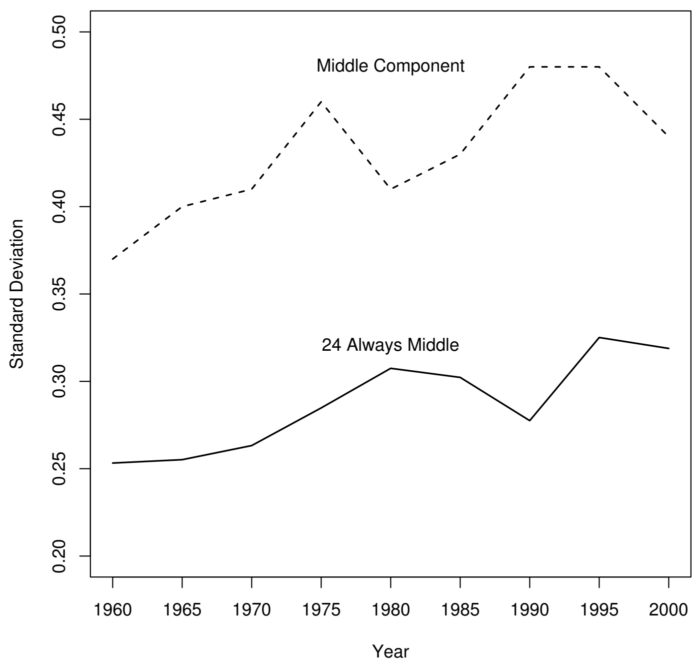
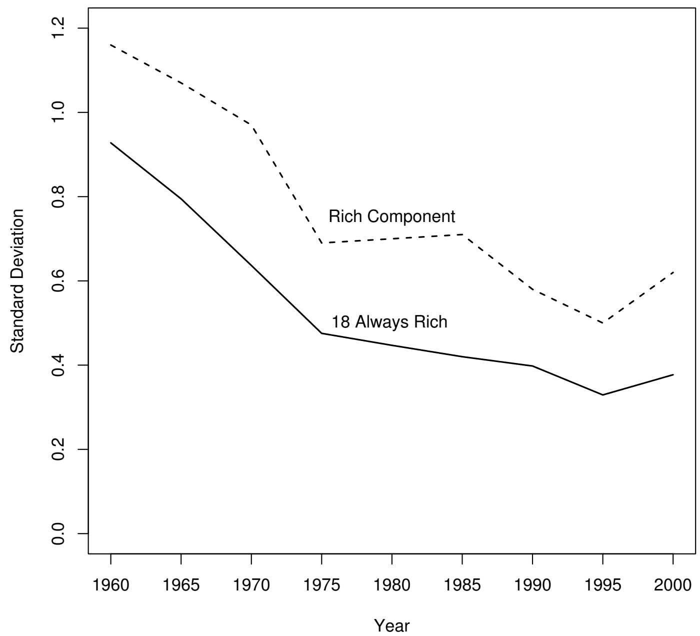


Figure 6: Standard deviation of the rich group.



3.5 Evolution of Inequality and Polarization

The evolution of the cross-country distribution of per capita income that we document above has implications for the degrees of inequality and polarization of the distribution. One way to formalize these implications is to compute the polarization measure proposed by Duclos, Esteban and Ray (2004). For a population with income distribution described by the density $f(x)$ this measure is

$$P_\alpha(f) = \int \int f(x)^{1+\alpha} f(y)|y - x| dy dx \tag{4}$$

were $\alpha \in [.25, 1]$ is a parameter that indexes the identification effect in the identification-alienation framework used by the authors. As they point out, $P_0(f)$ is twice the Gini coefficient although this value of α lies below the lower bound of .25 implied by their axioms. Table 3 shows estimates of $P_\alpha(f)$ for

$\alpha = 0, .25, .5, .75, 1$, computed with $f(x)$ replaced by the estimated 3-component mixture model of the cross-country distribution of per capita income for each of our sample years between 1960 and 2000. These measures indicate that, over this period, the inequality in the distribution, as measured by (twice) the Gini coefficient, $P_0(f)$, has fluctuated some, rising in the late 1960's and then falling in the early 1990's to finish the period virtually unchanged. In other words, at least as measured by the Gini coefficient, measured inequality in 2000 was about the same as it was in 1960. In contrast, for each value of $\alpha \geq .25$ that we use, measured polarization rises over this period. Both the absolute and proportional rises are increasing in α and, with the exception of the late 1970's and the late 1990's, these rises are monotonic. As we show below, this rise is driven by the tightening of the rich and poor component distributions around their respective means. The decreased dispersion within these distributions tends to increase within-club "identification" and so measured polarization because of the weight given to this effect in measuring polarization.

To study the statistical causes of the rise in polarization we compute $P^*_\alpha(f)$ which is a version of the computed $P_\alpha(f)$ with the component means held fixed at their estimated 1960 values. Comparison of $P^*_\alpha(f)$ and $P_\alpha(f)$ thus enables us to gauge the role of the changing component means in the rise in polarization. As noted in section 3.2, the gap between the rich and poor component means widens over that sample period – a phenomenon that would tend to increase inequality and polarization. We also compute $P^\sigma_\alpha(f)$ which is a version of the computed $P_\alpha(f)$ with both the component means and the mixing proportions held fixed at their estimated 1960 values. Changes in $P^\sigma_\alpha(f)$ thus reflect only the effects of the changes in the component standard deviations and comparison of $P^\sigma_\alpha(f)$ and $P^*_\alpha(f)$ isolates the effects of changes in the mixing weights. As noted in section 3.2, the standard deviations of the rich and poor components have fallen substantially over the sample period while that of the middle component has risen somewhat – phenomena that together would have ambiguous effects on polarization and inequality. Both $P^*_\alpha(f)$ and $P^\sigma_\alpha(f)$ are shown alongside

$P_\alpha(f)$ in Table 3 for each value of α and for each year.

In the case of the inequality measures, $P_0^*(f)$ fluctuates less than $P_0(f)$ and not always in the same direction – in both 1975 and 1990, $P_0(f)$ rises while $P_0^*(f)$ falls sharply. These two years saw large rises in $\widehat{\mu}_{\text{rich}}$ which increased the gaps between it and $\widehat{\mu}_{\text{poor}}$ and $\widehat{\mu}_{\text{middle}}$ by about 12% in each case – the largest changes in these gaps that we observe. This suggests that the changes component means can be an important source of the variation in inequality. However, despite the rises in both $\widehat{\mu}_{\text{rich}}-\ \widehat{\mu}_{\text{poor}}$ and $\widehat{\mu}_{\text{rich}}-\ \widehat{\mu}_{\text{middle}}$ over the entire sample period noted in Section 3.2, measured inequality is virtually the same at the end of the period as at the beginning.

Moreover, in 2000, $P_0^\sigma(f)$ is slightly greater than both $P_0^\sigma(f)$ in 1960 and $P_0(f)$ in 2000 showing that the net effect of the changes in the component standard deviations is also to (slightly) increase inequality. The reason that the rise in inequality is much less than the increased gaps between $\widehat{\mu}_{\text{rich}}$ and $\widehat{\mu}_{\text{poor}}$ and $\widehat{\mu}_{\text{middle}}$ alone would imply is the offset provided by the changes in the estimated mixing proportions, the $\widehat{\pi}_j$. While $\widehat{\pi}_{\text{rich}}$ is relatively constant at about .27 throughout the sample period, the behavior of $\widehat{\pi}_{\text{middle}}$ resembles a step function with a jump from about 0.4 to about 0.5 between 1980 and 1985, while $\widehat{\pi}_{\text{poor}}$ exhibits a corresponding fall from around .33 to .23. This large shift in mass towards the middle component dramatically reduces measured inequality.

For each value of $\alpha \geq 0$, the behavior of $P_\alpha^*(f)$ in most years is very similar to that of $P_\alpha(f)$ indicating that the net effect of the changes in the component means on measured polarization is small. This similarity tends to increase with α in that the differences between $P_\alpha^*(f)$ and $P_\alpha(f)$ decline as α rises.

As higher values of α increase the weight given to within-club "identification" this implies that it is increases in that aspect of the polarization measure that is a work here – a claim that is consistent with the differences in the behaviors of the inequality and polarization measures.

The computed $P_\alpha^\sigma(f)$ measures rise steadily through the sample period. Until 1980, they track the corresponding $P_\alpha(f)$and $P_\alpha^*(f)$ measures very closely implying that changes in the $\widehat{\sigma}_j$ are primarily responsible for the rise in polarization from 1960 to 1980. After 1980, however the paths of $P_\alpha^\sigma(f)$ and the other two polarization measures diverge with $P_\alpha^\sigma(f)$ rising more quickly than the others. The magnitude of this divergence increases with α. The divergence between $P_\alpha^\sigma(f)$ and $P_\alpha^*(f)$ implies that, while changes in the $\widehat{\sigma}_j$ remain an important factor in the rise in polarization after 1980, some offset is provided here by the changes in the estimated mixing proportions detailed above. The large shift in mass towards the middle component tends to reduce polarization and opens the gap between $P_\alpha^\sigma(f)$ and $P_\alpha^*(f)$ evident from 1985 onwards.

In sum, while inequality in the cross-country definition of per capita income, as measured by the Gini coefficient, is about the same in 2000 as it was in 1960, albeit after some fluctuations, the polarization in the distribution, at least as measured by $P_\alpha(f)$, rises steadily from 1960 to 2000. The primary proximate cause of this rise is the narrowing of the rich and poor component distributions. As the countries in the convergence clubs represented by those components become more concentrated around their respective club means they become more like each other and less like the countries in other convergence clubs. This increases cross-country polarization in the overall distribution

Table 3: Inequality and polarization measures.

	$\alpha = 0$			$\alpha = 0.25$			$\alpha = 0.5$			$\alpha = 0.75$			$\alpha = 1$		
	P_α	P_α^*	P_α^σ	P_α	P_α^*	P_α^σ	P_α	P_α^*	P_α^σ	P_α	P_α^*	P_α^σ	P_α	P_α^*	P_α^σ
1960	1.09	1.09	1.09	0.79	0.79	0.79	0.61	0.61	0.61	0.50	0.50	0.50	0.43	0.43	0.43
1965	1.12	1.09	1.12	0.82	0.80	0.82	0.64	0.63	0.64	0.54	0.53	0.54	0.47	0.47	0.48
1970	1.15	1.16	1.13	0.84	0.85	0.83	0.66	0.67	0.65	0.56	0.56	0.54	0.49	0.50	0.47
1975	1.17	1.06	1.13	0.87	0.80	0.85	0.69	0.64	0.68	0.58	0.54	0.58	0.51	0.49	0.52
1980	1.14	1.11	1.12	0.85	0.83	0.83	0.67	0.66	0.65	0.55	0.55	0.54	0.48	0.48	0.46
1985	1.12	1.09	1.17	0.83	0.82	0.87	0.66	0.65	0.73	0.55	0.55	0.65	0.49	0.48	0.62
1990	1.15	1.04	1.13	0.87	0.79	0.89	0.71	0.65	0.77	0.62	0.57	0.73	0.58	0.54	0.74
1995	1.11	1.05	1.12	0.86	0.82	0.90	0.72	0.69	0.81	0.65	0.63	0.80	0.63	0.61	0.84
2000	1.11	1.10	1.13	0.84	0.85	0.90	0.70	0.71	0.80	0.63	0.64	0.78	0.61	0.61	0.82

Note: P_α denotes the Duclos, Esteban and Ray index of polarization for a range of values of the parameter α. P_α for $\alpha = 0$ is equivalent to twice the Gini index of inequality. P_α^* measures the polarization with the component means held fixed at the estimated 1960 values. P_α^σ measures the polarization with the component means and mixing proportions held fixed at the estimated 1960 values.

4 Conclusions

We have argued that, despite the attention that it has received in the literature, multimodality of the cross-country distribution of per capita output is neither necessary nor sufficient for the presence of multiple basins of attraction, or convergence clubs, in the dynamic process describing the evolution of that distribution over time. Kernel estimation methods and the associated "bump hunting" approaches to the detection of multi pile modes are thus likely to be less informative, when investigating the convergence hypothesis, than approaches which model the distribution as a mixture of component densities. Each of these densities represents a putative convergence club and the mixture approach provides integrated tests for number of components. As tests of the convergence hypothesis, these tests have greater power than multimodality tests because the multiple components may not reveal themselves as multiple modes. Moreover, the estimated ex post probabilities that a country belongs to each of the components provides a natural metric for for assigning countries to components and, more generally, for measuring the strength of the affinity between countries and components. Comparison of such assignments over time provides a natural framework for the assessment of mobility between components which is important as low mobility is an essential part of convergence club view. Even if multiple components are detected, high mobility between them would be contrary to the claim that they represented multiple basis of attraction.

We implement the mixture approach using cross-country per capita income data for the period 1960 to 2000. In contrast to the commonly held view that the cross-country distribution of per capita income exhibits two modes, both of the statistical tests that we use indicate the presence of three component densities in each of the nine years that we examine over this period. For each year we thus estimate a 3-component mixture model and label the components as "poor", "middle", and "rich". We find that, while the gap between the mean relative per capita incomes of the rich and poor group has widened somewhat, the evident "hollowing out" of the middle of the distribution is largely attributable to the increased concentration of the rich and poor countries around their respective component means. This explanation is robust to the compositional changes brought by the few transitions out of these two groups that do occur. We track those transitions by using the estimated ex post probabilities of component membership to assign each country to a component in each year. While transitions do occur they are rare with only about 7% of the possible transitions actually occurring. Of the 102 countries in our sample, 64 remain assigned to the same component throughout the sample period and 28 transition just once, so that the remaining 10 countries account for over 40% of the observed transitions. This finding of low cross-component mobility leads us to interpret the multiple mixture components that we detect as representing convergence clubs. There is a pronounced tendency for the maximal ex post probability for each country to increase indicating a strengthening of the affinity of countries for the club in which they lie. Finally, we use our estimated mixture densities to compute measures of polarization and find that they have increased over the sample period - a phenomenon that we attribute primarily to the decreased variances of the poor and rich components.

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