



## A REGIONAL APPROACH FOR OPTIMIZATION OF THE MUNICIPAL WASTE MANAGEMENT SYSTEM USING FUZZY SETS

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### Biographical notes

**Dorel Ailenei** is Professor at the Department of Economics and Economic Policies, Vice President of Romanian Regional Science Association, Vice Dean of Faculty of Economics from Academy of Economic Studies of Bucharest. His research activities focus on: macroeconomics, regional development, economic policy, quantitative methods for policy analysis, behavioural analysis of economic agents, etc. He has a broad expertise in these fields. He had published 21 books and numerous articles (38).

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### Abstract

In the context of complexity in permanent growing of the problems and standards, the area of municipal waste management is in an accelerated evolution process. As the number of municipal waste management alternatives increase, the selection of the best waste management system- that is to decide on the combination of collection, processing and disposal techniques that will best serve the present and future needs of a community- become a more difficult task for local authorities.

Consequently, mathematical modelling techniques have been introduced to waste management in order to help the decision makers from this area. This paper presents a fuzzy optimization model for the development of an integrated waste management system at the regional level. Based on the options investigated in the model the end-users will be able to select and develop investments projects at local and county level in the aim of achieving the targets of the region.

**Keywords:** municipal waste, fuzzy optimization model, regional approach

**JEL Classification:** R 53, C 02

## 1. Introduction

We live in a society in which production, consumption and use of products is inevitable. But it is not unavoidable that these activities result in mountains of waste that led to a gradual degradation of the environment.

The issues of municipal waste management are some of the most important challenges of the XXI century taking into account the increased quantities of waste and the worries they generate in the global community, the new technologies and the environmental restrictions.

The EU policies from waste field underline the importance of an integrated approach in waste management. In this sense a group of measures have evolved in the aim of reducing the waste generation as well as the negative impact on health and the environment resulting from waste generation; also coordination, regulation and the organization of collecting, sorting, recycling and revaluation activities, final disposal.

Romania is an average-size country comparatively with other European countries - the thirteenth country in Europe as size. By contrast, in Romania resource consumption and waste quantities are high, exceeding the carrying capacities of the natural environment.

Romania has to fulfil certain obligations on environment chapter 22 negotiated with the European Union concerning waste. The EU accorded in the case of Romania transition periods for:

- Packaging and Packaging Waste (Directive 94/62/EC, modified by the directive 2004/12/CE)
- Waste land filling (Directive 99/31/EC)
- Waste incineration (The Council Directive 2000/76/EC)
- Electrical and electronic waste (The European Parliament and The Council Directive 2002 /96 /CE, modified by Directive 2003/108/ CE)
- Import , export and waste transit (Regulation 259/ 3/ CE)

The necessity of these transition periods are due to deficiencies recorded in the field: precarious infrastructure for waste collection, transport and elimination, weak awareness of the citizens and economic agents about the adequate waste management, limited capacity of authorities to elaborate viable project proposals, the number of sites damaged by pollution caused by economic activities and unsuitable landfill of waste, the permissive regime of environmental standards application [10].

## **2. The model**

The option for a fuzzy optimization model is explained by the fact that the main problem of the local authorities is the distribution of financial resources in an efficient way, so that the quantitative targets regarding waste and also the desired level of services to be achieved, the population capacity to pay to be respected - a new waste management system will have tariff implications on final consumers.

The fuzzy optimization model for the development of an integrated waste management system at the regional level has evolved from a fuzzy algorithm for resource allocation created by American researchers A.M.D. Esogue and Vengalatur Romash. This algorithm was adapted to the needs and characteristics of the waste management area. This model is presented next.

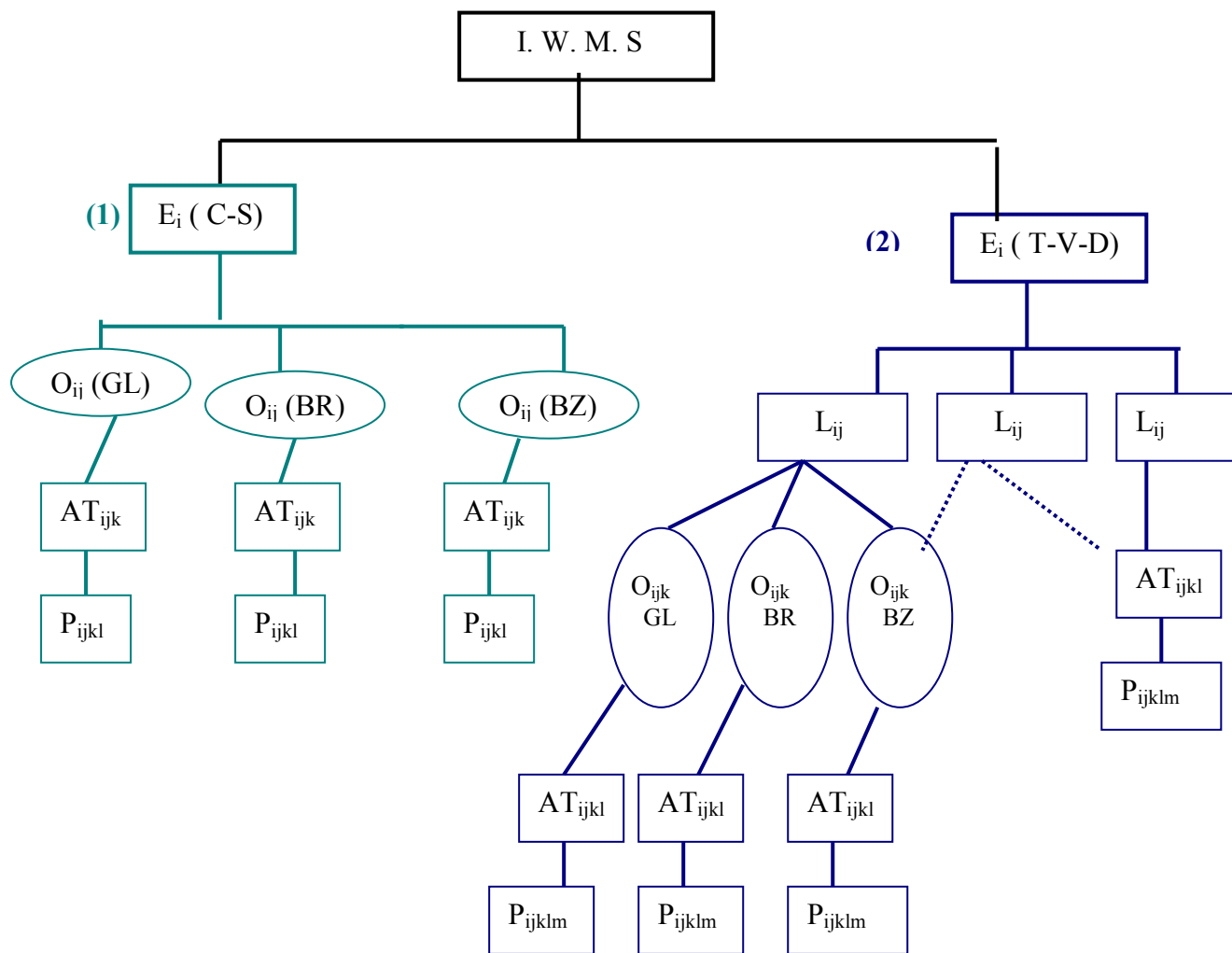
The objective is to develop an integrated waste management system at regional level. The problem consists in establishing the effective budget and also in setting the way in which this budget is allocated on subcomponents.

### **The model hypotheses**

- The integrated waste management system (IWMS) structure is known and also the set of available budget. The available budget for the development of an integrated waste management system = B.
- The size of sums assigned to investment projects, technical alternatives respectively stages of the waste management process that need to be developed in order that the integrated waste management system to be a success, is consider to be difficult to know, consequently imprecise.
- It can't be known with accuracy the development degree for the components of the system and in the case of the offers which of them will be selected.

These hypotheses lead us to believe that a fuzzy approach of the sums allocated for the components of the integrated waste management system is properly.

The structure of the system is presented in Figure 2.



**Figure 2. The system structure**

**The relations between the entities of block (1):**

1. Any town needs an integrated waste management system – **IWMS**.
2. All the stages of the waste management process –  $E_i$  - are important. In this block (1) on take into account two stages of the process, respectively collecting and sorting of waste.
3. for each stage of the waste management process in each town –  $O_{ij}$ - on consider more technical alternatives  $AT_{ijk}$  ; for testing the model we take into consideration three towns from South –East Region of Romania respectively Galați ( GL ), Brăila ( BR), Buzău ( BZ).
4. Each technical alternative it can be implemented using at least one offer.

On take into account a hierarchic level approach of the problem, in this sense it will be start with the inferior level. Consequently the model is formed of more sub-models connected by inputs and outputs.

For the **block (1)** on consider the next connection of sub-models: S<sub>1</sub>- S<sub>2</sub>-S<sub>3</sub>- S<sub>4</sub>.

**Sub-model S1: Selecting the offers  $P_{ijkl}$**

Let  $AT_{ijk}$  be the technical alternative. On consider that in each town for each technical option  $AT_{ijk}$  exists at least one offer that can be taking into account.

Let  $x_{ijk}$  be the budget estimated as necessary for the technical alternative  $AT_{ijk}$  and

$x_{ijkl}$  – the sum that could be allocated from this budget for the offer  $P_{ijkl}$

The relation between the two variables is:  $x_{ijkl} \leq x_{ijk}$ . Because it can't be known with precision how large will be the sum  $x_{ijkl}$  necessary for the offer  $P_{ijkl}$ , it can be expressed as a fuzzy set like this:

$$G_{ijkl} = \{ (x_{ijkl}), f_{G_{ijkl}}(x_{ijkl}) \mid x_{ijkl} \in (x_{ijkl}^{inf}, x_{ijkl}^{sup}) \} \tag{1}$$

Where the  $x_{ijkl}^{inf}$ ,  $x_{ijkl}^{sup}$  represent the lower limit and respectively the higher limit of the variation interval of the sum that can be allocated for the offer  $P_{ijkl}$ ,

$f_{G_{ijkl}}(x_{ijkl})$  represent the membership function of the  $x_{ijkl}$  at  $G_{ijkl}$  set.

The objective function of this sub-model considers the higher level, the technical alternative  $AT_{ijk}$  that take into account the characteristics of waste management process in each town. There are included in these characteristics: quantities of waste generated on streams, waste composition, frequency of waste collection services, tariff structure at local and county level, and so on.

The objective function can be expressed by:

$$g_{ijk}(x_{ijk1}, x_{ijk2}, \dots, x_{ijkl}, \dots, x_{ijkn}) = f_{G_{ijk1}}(x_{ijk1}) \vee f_{G_{ijk2}}(x_{ijk2}) \vee \dots \vee f_{G_{ijkl}}(x_{ijkl}) \vee \dots \vee f_{G_{ijkn}}(x_{ijkn}) \tag{2}$$

This mean it will be selected the offer  $P_{ijk1}$  or  $P_{ijk2}$  or ....  $P_{ijk1}$ .

The optimization of the model can be expressed by the following relation:

$$\text{Max } g_{ijk}(x_{ijk1}, x_{ijk2}, \dots, x_{ijkl}, \dots, x_{ijkn}) = \text{Max } \{ f_{G_{ijk1}}(x_{ijk1}) \vee f_{G_{ijk2}}(x_{ijk2}) \vee \dots \vee f_{G_{ijkl}}(x_{ijkl}) \vee \dots \vee f_{G_{ijkn}}(x_{ijkn}) \} \tag{3}$$

Under constraint:

$$\sum_{l=1}^n x_{ijkl} \leq x_{ijk} \tag{4}$$

Because on work with fuzzy sets the optimal conditions are expressed as:

$$\text{Max } g_{ijk}(x_{ijk1}, x_{ijk2}, \dots, x_{ijk1}, \dots, x_{ijkn}) = \text{Max} \{ \text{Max} [f_{G_{ijk1}}(x_{ijk1}), f_{G_{ijk2}}(x_{ijk2}), \dots, f_{G_{ijk1}}(x_{ijk1}), f_{G_{ijkn}}(x_{ijkn})] \} \quad (5)$$

Under constraint:

$$\sum_{l=1}^n x_{ijkl} \leq x_{ijk}. \quad (6)$$

The model will be settled using the dynamic programming. In this sense, are defined the following functions:

$f_{ijkl}(x_{ijk}) : G_{ijk} \longrightarrow [0,1]$  = the optimal membership degree of the offer  $P_{ijkl}$  to the technical alternative  $AT_{ijk}$  when are allocated for the technical alternative  $x_{ijk}$  monetary units.

The recurrence relationships are expressed as:

for  $l = n$ ,

$$f_{ijkn}(x_{ijk}) = \text{Max} \{ f_{G_{ijkn}}(x_{ijkn}) \}; x_{ijkn} \in G_{ijkn}; i=1,2; j=1,2,3; k=1,2. \quad (7)$$

for  $l=1, \dots, n-1$ ,

$$f_{ijkl}(x_{ijk}) = \text{Max} \{ f_{G_{ijkn}}(x_{ijkn}) \vee f_{ijkl+1}(x_{ijk} - x_{ijkl}) \} \text{ cu } x_{ijkl} \in G_{ijkl} \leq x_{ijk}; i=1,2; j=1,2,3; k=1,2. \quad (8)$$

In this way are valuated all the offers  $P_{ijkl}$  that accomplish the selection criteria for each town, so that to be selected the best technical alternative  $AT_{ijk}$ .

Is taking into account the identification of that level of the budget which can be assigned to the most adequate technical alternative  $AT_{ijk}$  for each town.

Through this sub-model on obtain an optimal estimation of the budgets allocated for each technical alternative  $AT_{ijk}$  in accordance with the stages  $E_i$  of the waste management process.

### **Sub-model $S_2$ : Choosing the technical alternative $AT_{ijk}$**

With the help of the sub-model  $S_1$  is obtained a previous selection of the offers  $P_{ijkl}$  corresponding with the technical alternatives considered.

Choosing the technical alternative is the next step. Waste can be managed in different way in each stage of the process but not the all technical alternatives accomplish in the same measure the needs of a town.

The estimated budget for the technical alternative  $AT_{ijk}$  can be expressed using the fuzzy set as:

$$G_{ijk} = \{ (x_{ijk}), h_{G_{ijk}}(x_{ijk}) \mid x_{ijk} \in (0, B) \} \quad (9)$$

The connection with the sub-model  $S_1$  is realised with the help of the membership function of this set like this:

$$h_{G_{ijk}}(x_{ijk}) = \text{Max} \{ f_{ijkl}(x_{ijk}) \}; i=1,2; j=1,2,3; k=1,2; l=1, \dots, n \quad (10)$$

The objective function of sub-model  $S_2$  take into account the town  $O_{ij}$  :

$$g_{ij}(x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{im}) = \bigcup_{k=1}^2 h_{G_{ijk}}^{(x_{ijk})} \quad (11)$$

The optimal conditions are expressed by:

$$\text{Max } g_{ij}(x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{im}) = \text{Max} \{ \text{Max} [h_{G_{ijk}}^{(x_{ijk})}] \} \quad (12)$$

under constraint:

$$\sum_{k=1}^2 x_{ijk} \leq x_{ij}. \quad (13)$$

The change into a dynamic programming model will be obtained using the function:

$f_{ijk}(x_{ij}) : G_{ij} \longrightarrow [0,1]$  = the optimal membership degree of the technical alternative  $AT_{ijk}$  to the town  $O_{ij}$  when  $x_{ij}$  monetary units will be allocated for the town  $O_{ij}$ .

for  $k=1,2$ , the recurrence relationships become:

$$f_{ijk}(x_{ij}) = \text{Max} \{ h_{G_{ijk}}^{(x_{ij})} \vee f_{ijk+1}(x_{ij} - x_{ijk}) \}; \quad i=1,2; j=1,2,3. \quad (14)$$

Will be selected those technical alternatives  $AT_{ijk}$  which offer the best solution to the needs of the town  $O_{ij}$  and to the features of the waste management process from it.

### **Sub-model S3: Dimensioning the necessary budget of the town $O_{ij}$**

Let  $G_{ij}$  be the fuzzy set of the budget estimated as necessary in the case of town  $O_{ij}$  for the development of the stages of waste management process.

$$G_{ij} = \{ (x_{ij}), h_{G_{ij}}^{(x_{ij})} \mid x_{ij} \in (0, B) \} \quad (15)$$

The connection with the sub-model  $S_2$  is realised with the help of the membership function of this set like this:

$$h_{G_{ij}}^{(x_{ij})} = \text{Max} \{ f_{ijk}(x_{ij}) \}; \quad i=1,2; j=1,2,3; k=1,2. \quad (16)$$

The objective function of the sub-model  $S_3$  takes into consideration the stages of the waste management process:

$$g_i(x_{i1}, x_{i2}, x_{i3}) = \bigcap_{j=1}^3 h_{Gij}^{(x_{ij})} \quad (17)$$

**The intersection relationship shows that at each town level is necessary to be developed both waste collecting and sorting for an efficient waste management system.**

The optimal conditions of the model will be:

$$\text{Max } g_i(x_{i1}, x_{i2}, x_{i3}) = \text{Max} \{ \text{Min} [h_{Gij}^{(x_{ij})}] \} \quad (18)$$

under constraint:

$$\sum_{j=1}^3 x_{ij} \leq x_i. \quad (19)$$

The change into a dynamic programming model will be obtained using the function:

$f_{ij}(x_i) : G_i \longrightarrow [0,1]$  = the optimal membership degree of the town  $O_{ij}$  regarding the stages  $E_i$  when  $x_i$  monetary units will be allocated for the stages of the waste management process.

for  $j=3$  the recurrence relationships become:

$$f_{ij}(x_i) = \text{Max} \{ h_{Gij}^{(x_i)} \}; i=1,2. \quad (20)$$

**Sub-model  $S_4$ : Choosing the degree of development of the waste management process stages -  $E_i$**

The budget estimated for the development of waste collecting and sorting can be expressed using a fuzzy set like this:

$$G_i = \{ (x_i), h_{G_i}^{(x_i)} \mid x_i \in (0, B) \} \quad (21)$$

The connection with the sub-model  $S_3$  is realised with the help of the membership function of this set like this:

$$h_{G_i}^{(x_i)} = \text{Max} \{ f_{ij}(x_i) \}; i= 1,2 ; j = 1,2,3 . \quad (22)$$

The objective function of the sub-model  $S_4$  is connected with the whole waste management system :



$$g(x_1, x_2) = \bigcap_{i=1}^2 h_{G_i}^{(x_i)} \quad (23)$$

**The intersection relationship shows that both wastes collecting and sorting will be developed, because all the stages of the waste management system are important.**

The optimal conditions of the model will be:

$$\text{Max } g(x_1, x_2) = \text{Max} \{ \text{Min} [h_{G_i}^{(x_i)}] \} ; i = 1, 2 \quad (24)$$

under constraint:

$$\sum_{i=1}^2 x_i < B \quad (25)$$

The model will be settled using the dynamic programming. In this sense, are defined the following functions:

$f_i(B) : B \rightarrow [0, 1]$  = the optimal membership degree of the waste management process stages  $E_i$  at the integrate waste management system I.W.M.S

for  $i = 1, 2$ , recurrence relationships become:

$$f_i(B) = \text{Max} \{ h_{G_i}^{(x_i)} \wedge f_{i+1}(B - x_i) \} ; i = 1, 2 . \quad (26)$$

$$x_i \in G_i < B . \quad (27)$$

In this way the algorithm will optimize the whole structure corresponding to the first stages of the waste management process considered in the **block (1)** respectively waste collecting and sorting.

The activities of collection and sorting of municipal waste from the region are organized differently depending on: the size of the locality, the endowment, type of property.

**The relations between the entities of block (2) are:**

1. Any town needs an integrated waste management system – **IWMS**.
2. All the stages of the waste management process – **E<sub>i</sub>** - are important. In this block (2) on take into account three stages of the process, respectively treatment, recycling and revaluation activities and final disposal of waste.
3. For the development of each stage of the waste management process are taking into consideration more locations **L<sub>ij</sub>** for the following situations:
  - a. The treatment, recycling and revaluation activities, final disposal of waste will be realized at local level, that is in each of town **Oijk**.

b. The treatment, recycling and revaluation activities, final disposal of waste will be realized in a common centre at the level of the three towns considered.

c. The treatment, recycling and revaluation activities, final disposal of waste will be realized both at local level and in a common centre.

4. for each stage of the waste management process in each town – Oijk- on consider more technical alternatives  $AT_{ijkl}$  ; for testing the model we take into consideration three towns from South –East Region of Romania respectively Galați ( GL ), Brăila ( BR), Buzău ( BZ).

5. Each technical alternative it can be implemented using at least one offer  $P_{ijklm}$ .

For the **block (2)** on consider the next connection of sub-models:  $S_1$ -  $S_2$ - $S_3$ -  $S_4$ - $S_5$ , that is:

$S_1$  - Selecting the offers,  $S_2$ - Choosing the technical alternative,  $S_3$ - sizing the necessary budget of the town,  $S_4$  – Choosing the location,  $S_5$  - Choosing the degree of development of the waste management process stages -  $E_i$  .

The equations for the sub-models  $S_1, S_2, S_3, S_5$  in the case of **block (2)** are expressed similar to the equations of **block (1)**.Consequently will be presented next only the equations for the sub-model  $S_4$  – Choosing the location.

#### **Sub-model $S_4$ : Choosing the location**

Let  $G_{ij}$  be the fuzzy set of the budget estimated as necessary in the case of location  $L_{ij}$  for the development of the stages of waste management process.

$$G_{ij} = \{ (x_{ij}), h_{G_{ij}}^{(x_{ij})} \mid x_{ij} \in (0, B) \} \quad (28)$$

The connection with the sub-model  $S_3$  is realised with the help of the membership function of this set like this:

$$h_{G_{ij}}^{(x_{ij})} = \text{Max} \{ f_{ijk}(x_{ij}) ; i= 1,2,3; j = 1,2,3 ; k= 1,..,3 \} . \quad (29)$$

The objective function of the sub-model  $S_4$  takes into consideration the stages of the waste management process:

$$g_i(x_{i1}, x_{i2}, x_{i3}) = \bigcup_{j=1}^3 h_{G_{ij}}^{(x_{ij})} \quad (30)$$

Will be selected the most adequate location corresponding to one of the three situations mentioned above. The optimal conditions of the model will be:

$$\text{Max } g_i(x_{i1}, x_{i2}, x_{i3}) = \text{Max} \{ \text{Max} [h_{Gij}^{(xij)}] \} \quad (31)$$

under constraint:

$$\sum_{j=1}^3 x_{ij} \leq x_i \quad (32)$$

The change into a dynamic programming model will be obtained using the function:

$f_{ij}(x_i) : G_i \longrightarrow [0,1]$  = the optimal membership degree of the location  $L_{ij}$  regarding the stages  $E_i$  when  $x_i$  monetary units will be allocated for the stages of the waste management process.

For  $j=3$  the recurrence relationships become:

$$f_{ij}(x_i) = \text{Max} \{ h_{Gij}^{(x_i)} \}; i=1,2,3. \quad (33)$$

For  $j=1, 2$ , the recurrence relationships become:

$$f_{ij}(x_i) = \text{Max} \{ h_{Gij}^{(x_i)} \vee f_{ij+1}(x_i - x_{ij}) \}; i=1,2,3; j=1,2,3. \quad (34)$$

### 3. Conclusions

Through the two blocks the model carry out both a selection of the offers for the technical alternatives corresponding to the stages of the waste management process an also an optimization of the structure for the whole waste management system. Also the model can point out the situations in which the initial budget proposed for the integrated waste management system is inadequate/insufficient.

The model allows sensitivity analyses; in this sense it can be used for testing the system „behaviour” in case the budget is modified. This is possible thanks to the connection between the sub-models which form the model. The recurrence relationships from different levels will be taking again with other values of the partial budgets in accordance with a new distribution of the total budget B.

The most difficult element of this model is represented by the estimation of the real form of membership functions. To obtain the membership functions we are taking into account applying the Delphi method. The results of this research will be presented in further articles.

## References

Ailenei, D., Târțiu, V.E, “A fuzzy approach for an integrated waste management system”, paper presented at the international conference “Economic growth and E.U. enlargement process”, Bucharest, 2008

Ailenei, D, *Market as Economic Space*, Didactica and Pedagogica Publishing House, Bucharest, 1999 (in Romanian)

Esogbue, A.M.D, Romash, Vengalatur, *Dynamic Programming Process and Fuzzy Allocation*, Technical Memorandum No. 202,1970

European Parliament, Resolution regarding thematic strategy on waste recycling,  
<http://www.europarl.europa.eu/sides/getDoc.do?pubRef=-//EP//NONSGML+TA+P6-TA-2007-0030+0+DOC+PDF+V0//RO->

Gherasim, O., *Uncertainty Mathematics*, Performantica Publishing House, Iasi, 2004 (in Romanian)

Gherasim, O., *Triangle Fuzzy Numbers Mathematics Applied in Management*, Performantica Publishing House, Iasi, 2005 (in Romanian)

Ghergut, D.I., Mindricelu, C., Petroaica, B. (eds), *Waste Release and Management in 2006*, INS Publishing House, Bucharest, 2008 (in Romanian)

Ministry of Environment, “ Environment Sectorial Operational Programme 2007-2013”, 2007

Ratiu-Suciu, C., *Economic Processes Modelling and Simulation*, 4th Edition, Economica Publishing House, Bucharest, 2005 (in Romanian)

Regional Plan For Waste Management – South-East Region [http://www.granturi-phare-deseuri.ro/docs/Plan\\_regional/PLANUL\\_REGIONAL\\_DE\\_GESTIONARE\\_A\\_DESEURILOR.pdf](http://www.granturi-phare-deseuri.ro/docs/Plan_regional/PLANUL_REGIONAL_DE_GESTIONARE_A_DESEURILOR.pdf)

<http://ec.europa.eu/environment/waste/>

<http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/07/303&format=HTML&aged=1&language=RO&guiLanguage=en> – MEMO / 07 / 303