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UNIVERSITÀ DEGLI STUDI DI SIENA

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

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Computation of Consistent Price Indices

n. 556 – Febbraio 2009

Abstract -The price index, a pervasive long established institution for economics, is a number issued by the Statistical Office that should tell anyone the ratio of costs of maintaining a given standard of living in two periods where prices differ.
For a chain of three periods, the product of the ratios for successive pairs must coincide with the ratio for the endpoints. This is the *chain consistency* required of price indices.
A usual supposition is that the index is determined by a formula involving price and quantity data for the two reference periods, as with the one or two hundred in the collection of Irving Fisher, joined with the question of which one to choose and the perplexity that chain consistency is not obtained with any. Hence finally they should all be abandoned. This situation reflects '*The Index Number Problem*'.
Now with any number of periods consistent prices indices are all computed together to make a resolution of the 'Problem', proved unique and hence never to be joined by others to make a Fisher-like proliferation.

Key words price index, price level, index numbers (economics), index number problem

JEL Classification C43, E31

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1 Introduction

The price index, a pervasive long established institution for economics, is a number issued by the Statistical Office that should tell anyone the ratio of costs of maintaining a given standard of living in two periods where prices differ.[1]

For a chain of three periods, the product of the ratios for successive pairs must coincide with the ratio for the endpoints. This is the *chain consistency* required of price indices.

A usual supposition is that the index is determined by a formula involving price and quantity data for the two reference periods, as with the one or two hundred in the collection of Irving Fisher, joined with the question of which one to choose and the perplexity that chain consistency is not obtained with any. Hence finally they should all be abandoned. This situation reflects '*The Index Number Problem*'.

Now with any number of periods consistent prices indices are all computed together to make a resolution of the 'Problem', proved unique and hence never to be joined by others to make a Fisher-like proliferation.

It may be in a way natural to discover beginnings after coming to the end. That happened with the new book where tacked onto the MS at the last moment is an appendix that shows those features. That is where, more logically, this paper starts, to see how the subject should be presented and to give a simplified overview. Theorems can be stated with reference elsewhere for a proof, and while computational method is sketched, the long drawn out mathematical treatment of the inequalities for price level determination is omitted, so also is the approach to approximation in the case of inconsistency with a relaxation of efficiency to partial efficiency so the fit of utility to data is loosened.[2] It is satisfactory to be able to explain arrival at the new formula in a single paragraph or half-page.

2 The Formula

With some m time periods, or countries or in any case *references* listed as 1, …, m, the initial data has the form of some m demand elements

$$(p_t, x_t) \ \left(t = 1, \dots, m\right)$$

giving row and column vectors of prices and quantities for some n goods demanded at the prices. For the efficient consumer, or for the utility to *fit* the data, the cost of consumption obtained should coincide with the minimum cost of a consumption that provides the same standard of living; if a unique consumption then the fit is *strict*.

[1] With *standard of living* based on consumption as its utility, this is where 'utility' enters unrestricted. The *cost of a standard of living* is the minimum cost of a consumption that provides it.

[2] More generally in Afriat (1968), (1973).

The initial data scheme is:

m number of references
n number of goods
p $m \times n$ price matrix, rows p_s
x $n \times m$ quantity matrix, columns x_t
$c = px$ $m \times m$ cross-cost matrix, elements $c_{st} = p_s x_t$

The first step is to compute the matrix L of *Laspeyres indices*

$$L_{st} = p_s x_t / p_t x_t$$

s being index for the *current reference* and t for the *base reference*.[3] Hence divide column t of c by diagonal element $p_t x_t$ to form the $m \times m$ Laspeyres matrix L with these elements.

Let P_{st} denote the price index from period s to period t. The number must apply equally to everyone experiencing the price change whatever their standard of living. Hence an expenditure M_s in period s at whatever level must be replaced by

$$M_r = P_{rs} M_s$$

in period r to obtain a consumption that with minimum cost maintains the same standard of living.

In particular, in period s according to the data, $p_s x_s$ is spent on the bundle x_s to buy the standard of living presumed at minimum cost, so

$$M_s = p_s x_s$$

is the cost of that standard of living. By this requirement the utility *fits* the data, by representing the consumer as *efficient*, from buying the received standard at minimum cost.

This cost in period s, as follows by application of the price index, becomes $M_r = P_{rs} M_s$ in period r for the same standard.

Since buying x_s itself is one available approach to buying the standard of living it supports, with expenditure $p_r x_s$, the minimum cost of buying that standard of living cannot exceed that expenditure, so

$$M_r \le p_r x_s ,$$

as follows from primitive free disposal.

We now have

$$P_{rs} = M_r / M_s \le p_r x_s / p_s x_s = L_{rs} ,$$

and hence

$$(O) \qquad P_{rs} \le L_{rs} ,$$

which conclusion marks a beginning for price index theory—and not far from the end, at least when following the main outline passing over issues to do with computation.

[3] In other words the inflation rate from base to current period of the base bundle of goods.

The argument had two steps, one the matter of the usual presumed consumer efficiency that defines the fit of utility to a demand, and the other free disposal.

Any price indices to be constructed have to satisfy the condition (*O*). But also chain consistency is required, by which they must have the form of ratios

$$(P) \qquad P_{rs} = P_r / P_s$$

of some numbers P_r so introduced with the role of *price levels.*

Combining constraint (*P*) with (*O*) we have the system of inequalities

$$(L) \qquad L_{rs} \geq P_r / P_s \,.$$

For the proposed *new formula*, therefore:

> **The system (*L*) serves to determine *price levels* P_r from which *price indices* P_{rs} are then obtained from (*P*) as their ratios .**

The simplicity and inevitability of this formula cannot be obscured by elaborations around it. Developments of theory and computational method have an account in Afriat (1981) and Afriat-Milana (2008) (A-M).

It has been seen that any consistent price indices must appear as an application of this formula. But that leaves open any distinction of results of such application. For a start, for any price index system obtained with this formula, chain consistency is automatic. Then there are 'truth' properties, in the language where truth means based on a utility that fits the data. To pursue that question terms are needed that come from an elaboration concerning properties of utility.

At this point it is suitable to observe that the existence of a utility that fits the data in every period, enabling relation (*O*) for the price indices, puts a condition on the given demand data. Let

$$D_{rs} = p_r x_s / p_s x_s - 1 = L_{rs} - 1 \,,$$

and

$$D_{rij\ldots ks} = \left(D_{ri}, D_{ij}, \ldots, D_{ks} \right).$$

Then this condition, the *cyclical preference test*, can be stated

$$(D) \qquad D_{r\ldots r} \leq 0 \Rightarrow D_{r\ldots r} = 0 \,.$$

This condition, according to the Utility Construction Theorem (UCT), is necessary and sufficient for the existence of such a utility.[4] The stricter condition

$$(H) \qquad D_{ri\ldots r} \leq 0 \Rightarrow x_r = x_i = \ldots$$

is the well-known condition of Houthakker (1950), with which the utility instead fits *strictly*.

We are about to encounter yet another condition on the data, the *Laspeyres cyclical product test* dealt with below, where it is observed to be a restriction of the cyclical preference test (*D*), and to be the sharper condition required for the existence of a

[4] Afriat (1960b) or (1964), or Fostel, Scarf and Todd (2003).

specifically conical[5] or constant returns utility that fits the data, as required for association with a price index, as about to be seen.

3 The cyclical test

In the *power algorithm* central to the method[6], the Laspeyres matrix is raised to powers L^r, $r = 1, 2, \ldots, m$, in a modified arithmetic where *plus* means *min*. The series terminates in a repetition not later than the *m*th, if at all, then repeated in succeeding powers up to the *m*th and beyond.

The termination condition is that the matrix $M = L^m$ have diagonal elements all 1. This matrix then defines the *derived Laspeyres matrix* with elements

$$(M) \qquad M_{st} = \min_{ij\ldots k} L_{si}L_{ij}\cdots L_{kt}\,.$$

With any *chain* described by a series of periods, or references,

$$s, i, j, \ldots, k, t$$

there is associated the *Laspeyes chain product*

$$L_{sij\ldots kt} = L_{si}L_{ij}\ldots L_{kt}\,,$$

the *coefficient* on the chain. Obviously

$$L_{r\ldots s\ldots t} = L_{r\ldots s}L_{s\ldots t}$$

A chain

$$t, i, j, \ldots, k, t$$

whose extremeties are the same defines a *cycle*. It is associated with the *Laspeyres cyclical product*

$$L_{tij\ldots kt} = L_{ti}L_{ij}\ldots L_{kt}$$

which is basis for the *Laspeyres cyclical product test*

$$L_{t\ldots t} \geq 1 \text{ for all cycles } t\ldots t$$

that all cycles have coefficient ≥ 1, which is *necessary and sufficient for existence of the derived Laspeyres indices M.*

But this is also *necessary and sufficient for the solubility of the system of inequalities (L)* for the determination of price levels, necessary for the existence of a system of consistent price indices subject to condition (*O*).

But now a further point can be added, that this is just *the condition on the data to admit construction of a fitting constant returns utility*, as provided by the Conical Utility Construction Theorem (CUCT), comparable with the ordinary UCT (for some "Afriat's Theorem") except that now utility has to be constant returns.

[5] A ray is a half-line with vertex at the origin, a cone is a bunch of rays, a function is conical if its graph is a cone, in other words, bringing in the here spurious concept of homogeneity in some degree, this is homogeneous in degree 1, or linearly homogeneous—a mouthful—while just homogeneous, degree unspecific, is ambiguous. A preferred alternative to conical is constant returns, a part of economic language. The popular term homothetic is unsuitable, as those who use it would know if they knew what it meant.

[6] See Bainbridge (1978), Edmunds (1973), Afriat (1979), (1980), (1981), (1982)b.

It has to be explained what a price index should have to do with such a utility, or any utility beyond the first unspecific reference. For there has been a reference to utility only in the argument for (*O*) where it is without restriction, and then there is arrival at the new formula without any further reference.

Propositions that have been stated just now without proof are dealt with in A-M Chapters 1 and 2 beside the 1960 and 1981 papers.

4 Utility properties

Given a utility function $\phi : \Omega^n \to \Omega$ [7], such as may govern demand, the cost at prices $p \in \Omega_n$ of attaining the the utility of consumption $x \in \Omega^n$ is

$$\rho(p, x) = \min\{py : \phi(y) \geq \phi(x)\}$$

ρ being the *utility-cost function*, depending on prices p and on consumption x through its utility value $\phi(x)$ that is representative of standard of living. This is the cost when prices are p of living at the standard represented by consumption x. The cost of living question, put in utility terms, is concerned with how this cost changes when prices change, for a given fixed standard of living.

By the condition

$$\rho(p, x) = px,$$

demand of x is *supported by prices* p, its cost being the minimum cost of obtaining its utility, and x is *supported* if supported by some *p*.

For concave utility, every x is supported since there is a supporting hyperplane to the graph at every point. For a continuous utility every p supports some x.

By definition, for all *p*, *x*

(a1) $\phi(y) \geq \phi(x) \Rightarrow py \geq \rho(p, x)$ for all *y*,

and, continuity provided,

(a2) $\phi(y) = \phi(x),\ py = \rho(p, x)$ for some *y*.

In particular,

(b1) $\rho(p, x) \leq px$ for all p, x.

and, for all p,

(b2) $\rho(p, x) = px$ for some x.

To be found now are implications of admissibility of *the special manner of resolution of the cost of living question by means of a price index*.

Let p_t be prices in period *t*, so $\rho(p_t, x)$ is the cost at those prices of living at the standard represented by *x*. In transition from period s to period r the cost changes from $\rho(p_s, x)$ to $\rho(p_r, x)$, in the ratio

$$\rho(p_r, x) / \rho(p_s, x)$$

---

[7] Ω the non-negative numbers, Ω^n column vectors, Ω_n row vectors.

in general depending on *x*.

In case this ratio is independent of x, a *price index* P_{rs} based on the utility is defined and given by this constant ratio, so giving satisfaction for all x of the conventional money conversion relation (P) depending on a price index, where now

$$M_r = \rho(p_r, x),\ M_s = \rho(p_s, x).$$

By this constancy condition the utility has the *price index property*, and P_{rs} is the *associated price index*, which provides

$$M_r = P_{rs} M_s$$

for all *x*.

Here represented in utility terms is the defining image obtained from the form of its use of the price index of ordinary practice. But to have this representation, the ratio to determine the price index has to be independent of the variable consumption bundle *x* the utility of which is the measure of standard of living, that is,

(i) $\rho(p_r, x)/\rho(p_s, x)$ is independent of x

This independence represents the special condition on the utility *by which it has the price index property* and *has a price index associated with it.*

This condition on utility, stated in terms of the utility cost function ρ, will be seen equivalent to the condition of *utility cost factorization* that requires

(ii) $\rho(p, x) = \theta(p)\phi(x)$

where *the utility cost function* ρ *factorizes into a product of a function* θ *of prices alone with a function* ϕ *of quantities alone*[8]. In this case the *associated price index,* or the *price index based on the utility*, is given immediately by

$$P_{rs} = \theta(p_r)/\theta(p_s).$$

THEOREM 1 For a utility to have the price index property, and so to have an associated price index, utility cost factorization is necessary and sufficient.

We have to show (i) $\Leftrightarrow$ (ii). Since (ii) $\Rightarrow$ (i) is immediate, it remains to prove (i) $\Rightarrow$ (ii).

Take any fixed $a \in C$. Then by the price index property (i),

$$\rho(p_r, x)/\rho(p_s, x) = \rho(p_r, a)/\rho(p_s, a)$$

for all *x*. Let

$$\theta(p) = \rho(p, a),$$

so now

$$\rho(p_r, x)/\rho(p_s, x) = \rho(p_r, a)/\rho(p_s, a) = \theta(p_r)/\theta(p_s),$$

and let

[8] Touched on in Afriat 1972, 1977 pp 101 ff or 2005 pp 87 ff ; also Deaton 1979 *JASA* 74, 365. The equivalence of (i) and (ii) is stated by Samuelson and Swamy (1974) p. 570.

$$\phi(x) = \rho(p,x)/\theta(p).$$

Then we have the utility cost factorization required by (ii), completing the proof.

THEOREM 2 For utility cost factorization it is necessary and sufficient that the utility be conical.

This Theorem may be a good candidate for the title "The Index Number Theorem" secured by Hicks for another purpose, and we are going to prove it now as before[9].

Given ϕ conical,

$$\begin{aligned}\rho(p,x) &= \min\{py : \phi(y) \geq \phi(x)\} \\ &= \min\left\{py(\phi(x))^{-1} : \phi\left(y(\phi(x))^{-1}\right) \geq 1\right\}\phi(x) \\ &= \theta(p)\phi(x)\end{aligned}$$

where

$$\theta(p) = \min\{pz : \phi(z) \geq 1\}$$

That shows the sufficiency. Since, for all *p*,

$$\theta(p)\phi(x) \leq px,$$

for all *x*, with equality for some *x*, as assured with continuous ϕ, it follows that

$$\theta(p) = \min_x px/\phi(x)$$

showing θ to be concave conical semi-increasing. Also for *x* demandable at some prices, as would be the case for any *x* if ϕ is concave, the inequality holds for all *p* with equality for some *p*, showing

$$\phi(x) = \min_p px/\theta(p)$$

which, in case every *x* is demandable at some prices, requires ϕ to be concave conical semi-increasing. But even when not all *x* are demandable, because they lie in caves and are without a supporting hyperplane, here is a conical function defined for all *x* that is effectively the same as the actual ϕ as far as any observable demand behaviour is concerned. So it appears that for the cost function factorization the utility function being conical is also necessary, beside being sufficient, as already remarked. Hence, with some details taken for granted, the Theorem is proved.

In consequence we have:

THEOREM 3 For a price index to be based on a utility it is necessary and sufficient that the utility be conical

This approach to the beginning of price index theory is one way of bringing forward the inevitablility of the association of a price index with conical utility.

There is another approach, where it appears that as soon as you start in any way about a price index, to do with utility, in the first moment you have constant returns

[9] Samuelson and Swamy (1974) p. 570 cite Afriat (1972).

utility. That should stand against protests and of course there is bound to be a penalty somewhere in dealing with such a restricted concept as the price index in the first place. If there is an *assumption* anywhere, it is the price index itself. There is, contrary to complaints, *no additional assumption* about utility being constant returns, only the implication.

Before leaving about factorization there should be notice about the factors.

From (ii) joined with (b1) and (b2) we have

(c1) $\theta(p) = \min_x px/\phi(x)$

which shows that $\theta(p)$, being the minimum of a family of homogeneous linear functions $px/\phi(x)$, *is concave conical*.

Also, for any x that is supported,

(c2) $\phi(x) = \min_p px/\theta(x)$.

Hence if ϕ, beside being anyway conical to have the factorization, is also concave, so every x is supported, then this holds unconditionally.

Functions that satisfy (c1) and (c2), both necessarily concave conical, define a *conjugate pair of price and quantity functions*. The symmetry here reflects a perfect symmetry throughout between price and quantity, having various manifestations.

5 Laspeyres and Paasche

The *Paasche indices* are given by

$$K_{ij} = 1/L_{ji} = p_i x_i / p_j x_i ,$$

forming the elements of an $m \times m$ matrix K, obtained by transposition of L and replacing each element by its reciprocal. The *Laspeyres-Paasche inequality*

(LP) $K_{ij} \le L_{ij}$

has significance for Laspeyres and Paasche indices as price index bounds, and for data consistency in respect to index construction..

Central to the proposed method is the system of inequalities

(L) $L_{ij} \ge P_i / P_j$.

This serves to determine *price levels* P_i from which the matrix P of *price indices*

$$P_{ij} = P_i / P_j$$

is derived.

Though a utility interpretation is not required, such price levels also enter into the construction of an hypothetical underlying utility which fits the given demand data and represents all these indices together as *true.*

The solubility of the system (L) imposes a condition on the given data, defining its *consistency.*

With any *chain* described by a series of periods, or references,

$$s, i, j, \ldots, k, t$$

there is associated the *Laspeyes chain product*

$$L_{sij\ldots kt} = L_{si}L_{ij}\ldots L_{kt}$$

termed the *coefficient* on the chain. Obviously

$$L_{r\ldots s\ldots t} = L_{r\ldots s}L_{s\ldots t}$$

A chain

$$t, i, j, \ldots, k, t$$

whose extremeties are the same defines a *cycle*. It is associated with the *Laspeyres cyclical product*

$$L_{tij\ldots kt} = L_{ti}L_{ij}\ldots L_{kt}$$

which is basis for the important *Laspeyres cyclical product test*

$$L_{t\ldots t} \geq 1 \text{ for all cycles } t\ldots t$$

which is *necessary and sufficient for consistency* of the given data, and is an *extension of the PL-inequality.*

Introducing the *chain Laspeyres* and *Paasche indices*

$$L_{sij\ldots kt} = L_{si}L_{ij}\cdots L_{kt}, \quad K_{sij\ldots kt} = K_{si}K_{ij}\cdots K_{kt},$$

the cycle test $L_{s\ldots t\ldots s} \geq 1$ is equivalently to

(chain *LP*) $K_{s\ldots t} \leq L_{s\ldots t}$

for all possible chains $\ldots$ the two occurrences here being taken separately. Hence, introducing the *derived Laspeyres* and *Paasche indices*

$$M_{st} = \min_{ij\ldots k} L_{si}L_{ij}\cdots L_{kt}, \quad H_{st} = \max_{ij\ldots k} K_{si}K_{ij}\cdots K_{kt},$$

subject to the now to be considered conditions required for their existence, where

$$H_{st} = 1/M_{ts},$$

this condition is equivalent to

(derived *LP*) $H_{st} \leq M_{st}$.

In this case

$$K_{st} \leq H_{st} \leq M_{st} \leq L_{st},$$

showing the relation of bounds for the *LP*-interval and the *narrower bounds* for the derived version that involves more data.

6 Basic solutions

The matrix *M*, and the matrix *H* constructed from it, in exactly the same way as the Paasche matrix *K* is constructed from the Laspeyres matrix *L*, is important in that *their columns provide a set of 2m solutions of the system of inequalities* (*L*). These are the *basic price level solutions*, coming in *m* pairs, from which other solutions are derived, even—now as a speculation—*all* other solutions. For from any given price level solutions, by taking their geometric mean with any given weights, element for element, a further price level solution is obtained. The unhappy irresolution from having many solutions, quite like all the equally true points in the Paasche Laspeyres interval, can be escaped by picking on one, the equally weighted geometric mean of the 2*m* basic solution, and the price index system formed from ratios of the elements. As applied to just two references this reduces simply to Fisher's index which is the geometric mean of Paasche and Laspeyres, without chain consistency when applied to more references. But for many references, as the elusive generalization, this is a price index system with chain consistency and truth of all indices in respect to a common underlying utility. The merit of the geometric mean is that this is how from given solutions, here termed true points, others are obtained, and of the equally weighted mean that from this we obtain a point comfortably in the interior of the true far away from the untrue, in fact: *a true point where minimum distance to the untrue is maximum* (distance suitably understood). How can you do better than that?

Beware of the easy mistake of, instead of going indirectly through price levels, going directly for an index by taking the geometric mean of the narrower chain index bounds obtained for the price index within the Paasche-Laspeyres interval, to obtain an index that certainly reduces to the Fisher for only two references but, just like the Fisher, is without chain consistency when applied to many references, let alone the common truth property.

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