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MAURO CAMINATI

Variety, Consumption and Growth

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Abstract - This paper attempts at fixing some guide-posts on the relation between variety, consumption and growth, while abstracting from the well known effect that variety may exert on productivity, through specialization. A mechanism is first described, through which preference for variety expressed by intertemporally-optimizing consumers perfectly predicting the endogenously growing future consumption opportunities can cause faster steady-state growth. The mechanism amounts to a substitution of future for present consumption causing a higher steady-state savings ratio and is most naturally, but not exclusively, embedded in the intertemporal-equilibrium approach to growth modeling. The paper shows that this growth enhancing effect of preference for variety may not be unambiguous, if the creation of new goods is endogenous and costly. Some of the results obtained in this part of the paper hinge upon the assumption that there are constant returns to the endogenous factor, all factors are producible and that each type of variety can be used both as a consumption good and as a intermediate good in the production of capital by competitive firms.
Dissatisfaction with the approach to preference for variety and innovation within the mechanism above is then motivated. The approach is oblivious of endogenous preference formation and the relation between innovation, consumption knowledge and consumption activities. Some research indications concerning long-term growth analysis in a world of endogenous preference formation are then drawn.

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Mauro Caminati, Dipartimento di Economia Politica, Università di Siena

1. Introduction

This paper attempts at fixing some guide-posts on the growth effects of the endogenous accumulation of variety, through its influence on consumption. We shall consider two different ways in which variety may affect the pattern of consumption.

The first case occurs with the introduction of radically-new goods responding to previously unmet needs. These goods convey new service characteristics, or at least a combination of previously unavailable characteristics. For instance, the creation of the internal-combustion-engine powered automobile offered a new mix of transportation services combining speed with flexibility of use in time and space and lack of animal-waste. Such a vector of service characteristics could not be supplied by the competing land-transportation-systems of the time based on trains and horses (Bresnahan and Gordon, 1997). A similar case is offered by the first introduction of domestic refrigerators bringing to previously un-imaginable levels the time-flexibility of fresh-food consumption. Holding to Becker's (1965) and Lancaster's (1971) models of a *fixed set of service characteristics* supplied in different degree and composition by the home-production of consumption services using home-labour and goods as inputs, Bresnahan and Gordon (1997) suggest that the innovation examples just given correspond to the creation of new inputs for consumption-service production, which enable this production process to meet 'objective (previously) unmet needs' (*ibidem*, p.11). It may be worth stressing how to the extent that such needs are 'objective', they are not induced by innovations or other endogenous changes in the economy, and can be specified, even if 'unmet'.

Other authors object that there is a process of learning and preference-formation associated with the creation of new goods which is not fully consistent with Becker's and Lancaster's approach, in that it can not be reduced to the creation of new productive inputs, while holding preferences unchanged. (Bianchi, 2002). The suggested relation between innovation and preference formation is not devoid of predictive implications that will turn out relevant to the present discussion. The theme will be however taken up only in the final sections of this paper.

Until then, it will suffice for our purposes identifying the first case in our list with the creation of a new consumption good which is *not a close substitute* of any other existing good. Whether creating a 'new need', or meeting a previously unmet 'objective need', the new good is not subject to the same demand constraints that would fall upon a perfect substitute of a mature good that is consumed in plenty and has an almost 'saturated demand' (low relative marginal utility). We shall assume that the new good in question is produced by a *new industry*.

The second possibility that we consider is the creation of a new consumption good which is not only radically new and meets the requirements of the first case above, but also has crucial complementarity effects. In addition to meeting a new or previously unsatisfied need, new goods open up a host of changes in the sphere of consumption, because they elicit qualitative changes in the output of other consumption industries or modify the preferences for existing goods through complementarity or external effects. Add to a car sophisticated digital instruments for audio and video communication and the experience of a driver at the wheel will not be the same as before. In this sense, the information-communication technology not only creates the home computer, but also modifies the definition of what is a car, a photographic camera, etc. and affects the utility of car-driving, picture taking, and the like. The construction-industry product innovation of supplying on a large scale non-luxury sub-urban dwellings with private garden not only brought this type of housing in the reach of the middle class, but also greatly increased the utility from having a car. Obviously enough, there are also examples of negative complementarities or externalities that may come to mind. The pleasure from shopping at the nearby grocery or from having half pint lager at the favourite pub may largely depend on the relations of acquaintance, friendship, solidarity with the clients usually met in that place; these relations, or the very possibility to meet the 'usual clients', may be destroyed by the diffusion of new 'life styles' (Earl, 1986) associated with the emergence of new goods, whether consumption goods or productive inputs. Thus, our second case is concerned with product innovations that are not perfect substitutes of any existing consumption good and, in addition, exert complementarity or external effects that increase or decrease the contribution to the personal well-being that may come from consuming traditional goods. We shall assume that also this type of innovation gives rise to a new industry.

There are of course innovations that produce close substitutes of existing consumption goods. We shall not be concerned with these innovations in the sequel, in that they are less interesting from the view-point of the long-term relation between variety, consumption and growth.

For the sake of simplicity, our treatment of exogenous preferences will rule out direct consumption complementarities between any specific couple of goods, to the effect that the marginal benefit from increasing the consumption of good j at t depends only on the quantity $c_{j,t}$ and on the total number of goods n_t that enter the consumption basket at t. Preference for variety makes sure that the distribution of the composite consumption flow $C_t = \int_{j=0}^{n_t} c_{j,t} dj$ across a larger number of goods makes the consumer better off, provided that C_t/n_t is sufficiently large. In particular, we shall restrict our attention to growth paths where consumption differentiation is only constrained by the available number of goods. Moreover, the availability of a larger number of goods makes a given increase in the total consumption flow C more desirable than it would have been the case otherwise. In this vein, the growth process is marked and sustained by the higher dynamism of the demand for the new goods, and there is a relative saturation of the demand for the old products (Kuznets, 1953; Pasinetti, 1981).

2. Technology of physical production

In the economy at time t there are n_t differentiated goods and one capital good. A differentiated good can be either consumed, or it can be used as intermediate input in capital good production. c_{jt} is the quantity of the differentiated good j consumed at t, x_{jt} is the quantity of the same good used as intermediate input at t.

Capital-good output at t is produced by perfectly competitive firms according to the constant returns to scale production function:

$$\dot{K}_t = n_t^{\frac{\alpha-1}{\alpha}} \left[\int_{j=0}^{n_t} x_{j,t}^{\alpha} d_j \right]^{\frac{1}{\alpha}} \tag{1}$$

where $0 < \alpha < 1$.

To emphasize the response of the production system to changes in demand, it is assumed that all the inputs to production are themselves producible. There are not 'fixed factors' in the economy.

The functional form of (1) and competition imply that the price p_j of the intermediate input j and the price p_K of one unit of the capital good are as follows:

$$p_{j,t} = n_t^{\alpha-1} \dot{K}_t^{1-\alpha} p_{K,t} x_{j,t}^{\alpha-1} \tag{2.1}$$

$$p_{K,t} = n_t^{(1-\alpha)/\alpha} \left[\int_{j=0}^{n_t} p_{j,t}^{1-\varepsilon} d_j \right]^{\frac{1}{1-\varepsilon}} \tag{2.2}$$

where $1-\varepsilon = -\alpha/(1-\alpha)$. It is worth observing how (2.1) implies that the demand for the intermediate input j by competitive firms has elasticity $-\varepsilon = -1/(1-\alpha)$ with respect to p_j. A lower α entails a less elastic demand-curve of the differentiated good j, *qua intermediate input*. Since the demand-curve of the differentiated good j, *qua consumption good*, will turn out to have price elasticity -1, a lower α is unambiguously related to a higher market power of the local monopolist producing good j.

Capital is the single physical input to differentiated-good production. Capital embodying a larger variety of ideas is not more productive. One unit of capital, if assisted by the appropriate blue-print of ideas, and no matter what is the number of intermediate-good varieties embodied therein, produces A units of differentiated goods, whatever their kind.

$$AK_{Y,t} = \int_{j=0}^{n_t} x_{j,t} dj + \int_{j=0}^{n_t} c_{j,t} dj \tag{3}$$

where K_Y is capital invested in physical output production.

It is a consequence of (1) that in a symmetric equilibrium where $x_{j,t} = x_t$, $j \in [0, n_t]$, the capital output-flow $\dot{K}_t$ obtained from the given total intermediate-input flow $X_t = \int_{j=0}^{n_t} x_{j,t} dj$ is independent of n_t, provided that X_t stays constant.

Taken together, (1) and (3) imply that variety does not affect productivity and the assumption is motivated by the goal of considering the growth-effects of variety exerted through consumption demand only.

Final output at t is $Y_t = \dot{K}_t p_{K,t} + \int_{j=0}^{n_t} c_{j,t} p_{j,t} dj$.

For the sake of later reference we observe that in a symmetric equilibrium where $p_{j,t} = p_t$, $x_{j,t} = x_t$ and $c_{j,t} = c_t$, $j \in [0, n_t]$, from (1) and (3) we obtain:

$$\dot{K}_t = n_t x_t \tag{4}$$

$$p_{K,t} = p_t \tag{5}$$

$$Y_t = p_t(\dot{K}_t + n_t c_t) \tag{6}$$

$$AK_{Y,t} = n_t(c_t + x_t) \tag{7}$$

(6) reveals that steady state investment and total-consumption expenditures grow at the rate:

$$g = \frac{\dot{n}}{n} + \frac{\dot{p}}{p} + \frac{\dot{c}}{c} \tag{8}$$

3. Preference for variety and the representative family inter-temporal plan

The representative family maximizes lifetime utility[1]

$$\max \int_{t=0}^{\infty} u_t e^{-\rho t} dt \tag{9}$$

subject to the flow budget constraint that asset accumulation $\dot{a}$ is constrained by current income less consumption expenditure: $\dot{a}_t = r_t a_t - \int_{j=0}^{n_t} c_{j,t} p_{j,t} dj$. In this expression asset price is implicitly normalized to 1 throughout and p_j can be interpreted as the price of good j relative to asset price.

The results of the paper crucially depend on the functional form for instantaneous utility. This intends to capture the basic idea that consumers have a preference for variety, to the effect that they increase their satisfaction by differentiating a given total consumption expenditure

[1] Here and elsewhere in the paper, e is understood to be the base of natural logarithms.

$E = \int_{j=0}^{n} c_j p_j dj$ across the highest possible number of goods consistent with the attained variety level n provided that E/n is not too small. and/or with the lower bound $b \geq e$ to the divisibility of goods. In particular, it is assumed:

$$u_t = n_t^{-\theta} \int_{j=0}^{n_t} \log c_{j,t} dj \, ; \tag{10}$$

$$n_0 = 1; 0 \leq \theta < b < 1.$$

where $(1 - \theta)$ measures the intensity of preference for variety. We may notice that in a symmetric equilibrium where the composite consumption flow C is uniformly distributed across the n goods, a larger number of goods is desirable[2], as long as $C/n > e^{\frac{1}{1-\theta}}$.

The reasons for deviating from the representation of preference for variety afforded by the more conventional, and to some extent realistic, Dixit-Stiglitz utility function (Grossman and Helpman, 1991, ch. 3) are twofold. The first is a quest for simplicity. The second, but not independent reason is that the model aims at preparing the ground for the treatment of product innovations that are used both as consumption goods (that is, inputs to the household production of consumption services) and as inputs to the factory production of some final good (capital, in our case) by competitive firms. The much debated case of the personal computer offers a relevant example of this double possibility of use. Unlike the Dixit-Stiglitz utility function, the representation (10) will enable separating the effects of a parametric change in the preference for variety from the effects of a parametric change in monopoly market power.

Expression (10) fully abstracts from the features, realistic as they may be, that make the contribution to instantaneous utility coming from consumption of a differentiated good, depend on the time interval elapsed since the good in question was first introduced[3]. Such features are inessential to the argument made in this paper.

Let μ_t and λ_t the discounted and undiscounted shadow price of the state variable a_t in the present-value and current-value Hamiltonian (respectively) associated to (9): $\lambda_t = e^{\rho t}\mu_t$. Necessary conditions for an interior optimum[4] are:

$$n_t^{-\theta} \lambda_t^{-1} = c_{j,t} p_{j,t} \tag{11}$$

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[2] Massimo De Francesco pointed out to me the need to qualify the statement that, according to the preferences (10), a larger differentiation of consumption is desirable. e is here understood to be the base of natural logarithms.

[3] These features are responsible of the logistic diffusion curves that are observed empirically and are explicitly introduced in Aoki and Yoshikawa (2002). As argued in section 8 below, a more thorough and satisfactory analysis of such features can be obtained only at the cost of removing the assumption of exogenous preferences, to consider the relation between novelty, preference for variety and the accumulation of consumption knowledge.

[4] Here and in the sequel the attention is focused on those growth paths where the optimal differentiated consumption flow $c_{j,t}$ is bounded away from zero for all j in $[0, n_t]$ and all dates $t \geq 0$.

$$\dot{\mu}_t = -\mu_t r_t \quad (12)$$

$$\lim_{t\to\infty} \mu_t a_t = 0 \quad (13)$$

(11) implies that consumption expenditure is uniform across varieties and total consumption expenditure at t is:

$$E_t = \frac{1}{\lambda_t} n_t^{1-\theta} \quad (14)$$

(11) and (12) yield the consumption growth equation:

$$\frac{\dot{c}_{j,t}}{c_{j,t}} = r_t - \rho - \theta \frac{\dot{n}_t}{n_t} - \frac{\dot{p}_{j,t}}{p_{j,t}} \quad (15)$$

In symmetric equilibrium with $p_{j,t} = 1, j \geq 0, t \geq 0$, (15) boils down to

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - \theta \frac{\dot{n}_t}{n_t} \quad (16)$$

4. Intertemporal equilibrium with exogenous innovations

In this section we consider some qualitative results, recently stressed in Aoki and Yoshikawa (2002), which refer to the model economy where innovations are costless, exogenous and markets are perfectly competitive. For the sake of simplicity we fully abstract from adjustment costs and their influence on capital utilization[5].

In this economy varieties grow at the exogenous rate g_n and the interest rate, as well as asset depreciation (see (12) above), is fixed by technology:

$$r_t = \frac{\dot{\mu}_t}{\mu_t} = r = A \quad (17)$$

In the symmetric equilibrium where $p_t = 1, 0 \leq t$, we have $p_{K,t} = 1, 0 \leq t$ and $p_{K,t}\dot{K}_t = \dot{a}_t$. Holding to this restriction, the transversality condition (13), together with (7), (16) and (17), imply the steady-state restriction:

$$\frac{\dot{a}}{a} = \frac{\dot{c}}{c} + g_n = A - \rho + (1-\theta) g_n < A \quad (18)$$

that is,

$$(1-\theta) g_n < \rho \quad (19)$$

[5] The influence of adjustment cost is instead prominent in Aoki and Yoshikawa (2002).

In this economy all capital is invested in physical production ($K_Y = K$). Thus, for $s \equiv c/x$, (4) and (7) reveal that

$$\frac{\dot{K}_t}{K_t} = \frac{1}{A(s_t + 1)}$$

where $1/A$ and $1/(s + 1)$ can be interpreted as 'capital-output ratio' and 'savings propensity', respectively.

A discrete, once and for all, increase of innovation growth at time t, if consistent with (19), has the effect that consumers want to increase their future consumption at a faster rate. They substitute future for present consumption. This requires a higher flow of saving and investment which in a symmetric equilibrium is instantaneously achieved through a discrete rise of μ_t and a corresponding discrete fall of c_t. The economy instantaneously attains the higher steady-state (symmetric) equilibrium growth-rate $A - \rho + (1-\theta)g_n$.

Likewise, a once and for all increase in the preference for variety $(1 - \theta)$ does not interfere with the technologically determined interest rate A and, if consistent with (19), instantaneously brings the economy to a higher steady-state growth rate.

5. Intertemporal equilibrium with endogenous innovations

In this section it is shown that the growth-enhancing effects of preference for variety considered above may not hold if the new goods result from a purposeful and costly innovation effort and do not affect productivity. For the sake of simplicity, technology of the R&D sector is described by the deterministic equation

$$\dot{n}_t = \delta K_{n,t} = \delta z_{n,t} K_t \tag{20}$$

where $K_{n,t}$ is the capital stock invested in R&D and $z_n \equiv K_n/K$. Since $0 \leq z_n \leq 1$, equation (20) implies the steady-state restriction[6]:

$$g_n = g_K \tag{21}$$

Using (21), from (4) and (7) we derive the further steady-state restriction:

$$g_x = g_c = 0 \tag{22}$$

(21), (22) and (16) yield the symmetric equilibrium, steady-state growth rate:

$$g_n = g_K = \frac{r - \rho}{\theta} \tag{23}$$

[6] Variables without the time subscript will henceforth indicate steady-state magnitudes.

We are left with the task of studying the endogenous determination of the interest rate in this economy and its relation with the savings propensity and the allocation of capital between physical-output production and ideas production. We shall consider both steady-state and transitional equilibrium paths.

The right of producing the differentiated good j comes from the acquisition of the corresponding infinite-life patent, which has market value $V_{j,t}$ at time t. Patent acquisition represents a fixed cost for the producer of the differentiated good j, which is the local monopolist j. His flow profit $\pi_{j,t}$ is determined by current revenue $p_{j,t}(x_{j,t}+c_{j,t})$ minus flow-cost $[(x_{j,t}+c_{j,t})/A]p_{K,t}r_t$. At any date in a symmetric equilibrium such that $0<s_{j,t}<\infty$ and

$$p_{j,t}=p_t=1, 0\le j\le n_t, 0\le t \tag{24}$$

we have:

$$\pi_{j,t}=x_t(1+s_t)\left(1-\frac{r_t}{A}\right) \tag{25}$$

(11) implies that the price elasticity of consumption demand for the differentiated good j is -1. Using this property, and the demand for the intermediate good j implied by (2.1), in symmetric equilibrium[7] the first order condition for monopoly-profit maximization yields:

$$r_t=\frac{\alpha A}{(1-\alpha)s_t+1}$$

Since $0 < \alpha < 1$, and $s_t > 0$, $r_t < \alpha A$. Conversely, in symmetric equilibrium:

$$s_t=\frac{\alpha A-r_t}{r_t(1-\alpha)}\equiv s(r_t) \tag{26}$$

It is worth recalling that in equilibrium the fraction of income which is not consumed can be written $1/(s_t+1)$; thus the equilibrium propensity to save is fully determined by the rate of interest and we are informed by (26) that there is a positive relation between the two variables. In other words, the local monopolists' maximizing behaviour fixes the relation $1/(s(r_t)+1)$ between the interest rate at t and the equilibrium composition of output between investment and consumption at the same date. Preferences can be interpreted as affecting the equilibrium composition of output through their effect on the interest rate.

Innovation value at t is $V_{j,t}=\int_t^\infty \pi_{j,\tau}\exp(-\int_t^\tau r_u du)d\tau$. In a steady-state symmetric equilibrium, $x_{j,t}=x, 0\le j, 0\le t$; steady-state innovation value can be written:

[7] Symmetric equilibrium is henceforth understood to be an equilibrium such that (24) holds.

$$V_{j,t} = V = x(1+s)\left(\frac{A-r}{Ar}\right) \tag{27}$$

Capital is instantaneously transferable across sectors. Free entry in R&D implies that at any date *t* the rate of return on capital invested in R&D is equal to the rate of interest:

$$\frac{\dot{n}_t V_t}{K_{n,t} p_{K,t}} = r_t = \delta V_t \tag{28}$$

This yields the steady-state, symmetric equilibrium restriction $\delta V = r$, or, using (27):

$$\delta x(1+s)\left(\frac{A-r}{Ar}\right) = r \tag{29}$$

Recalling that $K_t(1-z_{n,t}) = K_{Y,t}$, from (4), (7), (20), (23) and (29) we obtain the steady-state conditions:

$$1 - z_n = \frac{(r-\rho)\alpha(A-r)}{A\theta r(1-\alpha)} \tag{30}$$

$$1 = \theta \frac{z_n}{(1-z_n)} \frac{r}{(A-r)} + \frac{\rho}{r} \equiv F(r) \tag{31}$$

$$\frac{K_t}{n_t} = \frac{r-\rho}{\theta\delta z_n} \tag{32}$$

The expressions from (25) to (32) are well defined for $\alpha A > r > \rho$. The restriction is justified by the following proposition.

Proposition 1: A economically acceptable steady-state solution r to (31) is such that $\alpha A > r > \rho$*;* $1 > z_n(r) > 0$.
Proof: From (26), $r > \alpha A$ is inconsistent with $s \geq 0$, and $r = \alpha A$ would produce $s = 0$, which is inconsistent with consumers plans (11), (12), (13) at finite prices. $r < \rho$ yields $z_n(r) > 1$, and $r = \rho$ is not acceptable because it is inconsistent with equilibrium: at $r = \rho$ $g_n = g_K = 0$, hence $x = 0$ and capital is entirely demanded to produce consumption goods. Since capital good production is zero, the equality (5) ceases to hold and the local monopolist producing good *j* would set p_j to infinity, leaving his revenue unchanged but minimizing his cost. We conclude that at $r = \rho$ there is an excess supply of capital.□

Proposition 2: Let r_1 *and* r_2 *the real values of the interest rate that satisfy* $1 - z_n = 1$ *in equation (30). The following inequality holds:* $0 < \rho < r_1 < r_2 < A(1-H)$*, where* $H \equiv \theta(1-\alpha)/\alpha$.

$\lim_{H\to 0} r_1 = \rho$ *and* $\lim_{H\to 0} r_2 = A$. *If $r_1 < \alpha A$, there exist r^*, $\rho < r^* < r_1$, that satisfies (31). Moreover, if $r_2 < \alpha A$*[8] *and $F(\alpha A) > 1$, there exists also r^{**}, $r_2 < r^{**} < \alpha A$, that satisfies (31). Since the solution r^* is relatively close to ρ, it meets the transversality condition (13) for 'large' set of parameter values. The same transversality condition (13) will impose much stronger restriction on parameters at r^{**}.*
Proof: see appendix.

It is a direct implication of proposition 2 that the same point in parameter space, in particular, the same state in the preference for variety and technology of R&D, may be consistent with a 'low' or a 'high' value of the interest and growth rates. Moreover, the proposition suggests that these high or low rates may not map in a straightforward way to the share of resources invested in R&D. The same proposition implies that there is a non negligible set of parameter values such that only the 'low-growth' regime exists.

Proposition 3 (Transitional dynamics): Let r be a steady-state interest rate identified in proposition 2 and consider the corresponding steady state path of the economy. At the initial date $t = 0$ the stocks K_0 and n_0 are pre-determined and fix a transitional-equilibrium path converging to steady-state, with the following properties.

$$r_t = r, 0 \le t; V_t = r/\delta, 0 \le t$$

$$z_{n,t} = 1 - \frac{r^2}{(A-r)\delta}\frac{n_t}{K_t}$$

$$\frac{\dot{n}_t}{n_t} = \delta\frac{K_t}{n_t} - \frac{r^2}{A-r}$$

$$\frac{\dot{K}_t}{K_t} = \frac{A}{(1+s)}\frac{r^2}{(A-r)\delta}\frac{n_t}{K_t}$$

$$\frac{\dot{c}_t}{c_t} = r - \rho - \theta\left(\delta\frac{K_t}{n_t} - \frac{r^2}{A-r}\right)$$

Proof: see appendix.

Proposition 3 implies that the ratio between the stocks of physical capital and ideas, from any arbitrarily given initial condition converges monotonically to its steady-state value determined by (32). On the assumption that the initial value of this ratio is higher than at steady state, then the growth rate of physical capital $g_{K,t}$ converges to $g = (r-\rho)/\theta$ from below and the growth rate of ideas $g_{n,t}$ converges to $g = (r-\rho)/\theta$ from above. During the transition, the share of resources invested in R&D is larger than at steady state. In other words, this share converges to its steady-state value from above.

[8] A sufficient condition for this is $A - H \le \alpha A$, that is, $\theta \ge \alpha$.

Proposition 2 points to the possibility of multiple steady-state equilibria. The next proposition reveals that the local comparative-statics properties of the model, at the low interest rate equilibrium r*, may not confirm the growth sustaining effect of a parametric increase in the preference for variety that was found in the previous section.

Proposition 4. (Comparative-statics effects of a change in the preference for variety): Fix a given point $\mathbf{q}$ *= (A, α, δ, θ, ρ) in parameter space, such that r* defined in proposition 2 is sufficiently close to ρ; this implies, in particular, that r* meets the transversality condition and* $z_n(r)$ *as well as F(r) are decreasing at r*. For the 'small' parameter change* $\mathbf{q}$*'–* $\mathbf{q}$ *= (0, 0, 0, θ' – θ, 0), θ' – θ < 0, consider the effects on the steady-state interest rates r* and r**. We obtain:*

$$r^*(\mathbf{q}') < r^*(\mathbf{q})$$
$$g^*(\mathbf{q}') = \frac{r^*(\mathbf{q}') - \rho}{\theta'} \geq or \leq \frac{r^*(\mathbf{q}) - \rho}{\theta} = g^*(\mathbf{q})$$

Proof: see appendix.

The possibility that *g*(*$\mathbf{q}$*')* < *g*(*$\mathbf{q}$*)* is shown in the following.

Example: Let $\mathbf{q}$ = *(2, 0.8, 1, 0.5, 0.02),* $\mathbf{q}$*' = (2, 0.8, 1, 0.4, 0.02).* This yields: *r*(*$\mathbf{q}$*') = 0.0203757, r*(*$\mathbf{q}$*) = 0.0204729, g*(*$\mathbf{q}$*') = 0.00093925, g*(*$\mathbf{q}$*) = 0.00093925.*

The implications of proposition 4 concerning the local steady-state effects of changes in the preference for variety at the low interest rate regime *r** are worth emphasizing, because we are informed by propositions 2 and 4 that there is a non negligible set of parameter values such r* is the only steady-state solution to the model.

Proposition 5 (Comparative-statics effects of a change in the productivity of R&D): Fix a given point $\mathbf{q}$ *= (A, α, δ, θ, ρ) in parameter space and consider the steady-state effects of the parameter change* $\mathbf{q}$°– $\mathbf{q}$ *= (0, 0, δ° – δ, 0, 0), δ° – δ > 0. A small increase in the productivity of R&D leaves the steady-state interest rate locally unaffected and causes a fall of the steady-state ratio* K_t/n_t.
Proof: Direct inspection of (30), (31) and (32) yields the stated results .

Proposition 6 (Comparative-statics effects of a change in market power): Fix a given point $\mathbf{q} = (A, \alpha, \delta, \theta, \rho)$ *in parameter space, such that* r_1 *is sufficiently close to ρ. Consider the local steady-state effects of the parameter change* $\mathbf{q}^{\S} - \mathbf{q} = (0, \alpha^{\S} - \alpha, 0, 0, 0), \alpha^{\S} - \alpha < 0$, *on the interest rates r* defined by proposition 2. We obtain:*

$$r^*(\mathbf{q}^{\S}) > r^*(\mathbf{q})$$

Proof: see appendix.

6. Remarks on variety and leisure

A special feature of the model outlined in sections 3, 4 and 5 is that leisure does not enter the utility function and capital is the only input to production in both the R&D and physical-output sectors. The special feature moulds the stated effects of preference for variety in various ways, but two in particular are worth emphasizing here.

The first implication is that the intra-temporal substitution effects of preference for variety are confined to set of consumption goods existing at the same date and are for this reason separable from the inter-temporal substitution effects. If instead well being depends also on leisure, preference for variety would generally affect the rate at which agents are prepared to substitute at any given date the current consumption of goods for the current consumption of leisure. With labour entering the physical-output and R&D production technologies, this intra-temporal substitution effect would also have inter-temporal repercussions. The above separability between intra-temporal and inter-temporal substitution effects would not be any longer at hand.

The second implication is concerned with the conditions enabling the existence of a steady state. With leisure entering the utility function, the existence of a steady state may require further restrictions on the technology of consumption activities and on the rate at which agents are prepared to substitute goods for leisure.

Indeed, preference for leisure may be approached on the plausible hypothesis that well being depends not only on the physical amounts of goods actually consumed, but also on the 'ease' with which the corresponding activity is performed, where such an ease depends on the fraction of time invested in it. To the extent that the agent consuming a given quantity of good j in a smaller fraction of time is more hurried, she is worse off. But there may be an upper bound on the time that may be rewardingly invested in a consumption activity. Likewise, having more time at one's disposal may be only boring, if there is no way of investing the extra time in some activity. This is a way of saying that leisure consumption and goods consumption are complements and the specific complementarity relation depends upon the technology of the corresponding activities.

These quite obvious remarks bring to the fore the very restrictive nature of the conditions enabling the existence of a steady state with growing per-capita consumption.

The first restriction is that the fraction of time *necessary* to consume any given quantity is not bounded away from zero by a separate time constraint on consumption activities[9]. In fact, since the fraction of time devoted to consumption activities must be constant in steady state, the expanding per-capita consumption resulting from variety growth will hardly be plausible, unless more varieties $c_j,...,c_i,...,c_h,...$ can be jointly and simultaneously consumed, in that they add up to a more sophisticated composite good. Think of driving a car while listening to the music reproduced by a CD driver and stereo equipment. Alternatively, it must be assumed that consumers like having more and more goods at their disposal, however small is the fraction of time they can invest in the specific consumption activity corresponding to any of them. Think again of driving a car equipped with several audio, video, computational and information-communication digital devices, but using only one device at one time.

The second restriction is that the elasticity of substitution between consumption and leisure is constant[10] (see appendix A.2 for an example).

7. Fix-price inter-temporal equilibrium with endogenous innovations

In this section we introduce crucial modifications to the model outlined in section 5, while retaining the same assumptions on technology and market structure.

The crucial modifications are that the interest rate is exogenously fixed (we may think of the monetary authority controlling the level of *r*), the assumption of perfect foresight is dispensed with. As a result, the role of inter-temporal preferences will require further examination.

At any date *t* consumers' choices are consistent with maximization of u_t (specified by (10) and reflecting a preference for variety), subject to the consumption-goods prices $p_{j,t}, 0 \leq j \leq n_t$ and to a consumption budget E_t resulting from the time-*t*-updating of their inter-temporal choices. Our presentation will first leave the restrictions resulting from these inter-temporal choices in the background, with the aim of stressing that, at the exogenous interest rate *r*, restrictions from technology, profit maximization and arbitrage are sufficient to determine the conditions for steady-state growth. Inter-temporal choices will be brought back again in the final part of this section and more thoroughly discussed in the next.

[9] On the effects on consumers' choice flowing from the introduction of a separate time constraint, in addition to the conventional budget constraint, see Steedman (2000) and Metcalfe (2001).
[10] Ortigueira and Santos (1997) discuss problems related to instantaneous utility depending on leisure in a Lucas-type endogenous-growth model.

As before, the specification (10) implies that consumption of good *j* has price-elasticity − 1 and in symmetric equilibrium at *t* the consumption budget $E_t = \int_{j=0}^{n} c_{j,t} p_{j,t} dj$ is uniformly distributed across the n_t goods. Moreover, in the present simplified framework of no physical capital depreciation, no adjustment costs and constant prices, the user-cost of capital is the interest rate. In turn, this brings with it an unmodified symmetric-equilibrium relation (26) between *s* and *r*, as a result of the local-monopolist's profit maximization. If in the model of section 5 *s* and *r* were simultaneously determined in equilibrium, now the exogenously fixed interest rate and the macroeconomic symmetric equilibrium relations fix the 'propensity to save' $1/(s+1)$ as well, leaving the consumer with no degree of freedom in this respect. In the present framework, consumer's preferences can impinge on the equilibrium consumption/output ratio only at the cost of making the interest rate endogenous.

Aggregate capital accumulation is determined through an accelerator-type equation that can be easily 'micro-founded'.

$$\dot{K}_t = \frac{1}{A} g^e_X n_t (c_t + x_t) + \frac{1}{\delta} g^e_n \dot{n}_t \tag{33}$$

where g^e_y is the uniformly expected growth rate of the variable *y* and $X \equiv \int_{j=0}^{n} (c_j + x_j) dj$ is the aggregate demand for differentiated goods.

Restricting (7) to steady state, differentiating with respect to time and using (4) we obtain:

$$g_n (1+s) = A(1 - z_n) \tag{34}$$

Substituting from (4), (34) and (29) into (33) and using the steady-state, warranted-growth condition $g^e_X = g^e_n = g_n = g_K$ we obtain the following expression of the warranted growth rate *g*:

$$g^2 (1+s) \frac{A-r}{r^2} + g(1+s) - A = 0 \tag{35}$$

$$g = \frac{-r + \sqrt{r^2 + 4Ar(1-\alpha)/\alpha}}{2(1+s(r))} \equiv g(r) \tag{36}$$

where the function *s(r)* is defined by (26).

Notice that $g(r)$ is increasing in its argument. To be consistent with steady-state-equilibrium, growth expectations must be positively tuned with the exogenous interest rate, for this positively affects both the output $x(1+s)$ of each differentiated good and the investment share of this output which simultaneously preserves the ongoing equilibrium on the goods markets and the full capital-stock utilization in material and non-material production[11].

[11] We may also notice, in passing, how at the given interest rate *r* the relation between the growth rate *g* and the output/capital ratio *A* (in the differentiated good sector) is ambiguous, because a parametric rise of the latter increases the equilibrium value of *s*, as determined by the function $s(r)$ (see (26) above).

The steady- state capital share invested in R&D is

$$z_n = 1 - g\frac{1+s(r)}{A}$$

It is immediate consequence of (36) that at higher steady-growth rates of final output and of the number of varieties, a lower share of resources is invested in R&D. The result is related to the particular technology assumed for the R&D sector, which is extremely intensive in the input produced by the final output sector.

A further remarkable feature of (36) is its complete independence of the preference for variety θ, and indeed of any preference parameter whatsoever. Preferences conjure to arrive at the result (36) only in that the demand for consumption good *j* has elasticity – 1 with respect to p_j and the expenditure on each consumption good is uniform[12]. As stressed in the previous section, these features, together with the local monopolists' maximizing behaviour, fixes the relation $1/(1+s(r))$ between the interest rate and the symmetric-equilibrium composition of output between investment and consumption. Whereas in the previous section the market effects of consumers' choices conjured to determine the equilibrium interest rate, now, to the extent that *r* is exogenous, preferences do not have the same scope for action. But since equilibrium must include the notion that agents are satisfied with what they are doing, we are forced to conclude that a state of equilibrium will be one in which the interest rate management is appropriately tuned with consumers' preferences. The interest rate ceases to be exogenous and exogenous preferences apparently re-emerge as the prime mover.

To illustrate this point, let me suppose, that the assumption of perfect foresight ruling in section 5 is now temporarily replaced with subjectively certain expectations. The illustrative, thought-experiment nature of the exercise is worth emphasising. In fact, the assumption of subjectively certain expectations seems particularly unsuited to the strongly Harrodian flavour of the model outlined in this section. Harrod himself was inclined to hold the view that the distant future is 'violently uncertain'[13]. Having thus stated the necessary qualifications, let me assume that consumers formulate subjectively-certain expectations on choice sets and parameters at all future dates and, on this ground, hold to the objective functional (9). If at the exogenous interest rate *r* the warranted growth rate determined by (36) and (26) does not happen to coincide with the unique steady-growth rate consistent with consumer optimising behaviour, namely $r - \rho/\theta$, then a full steady-state equilibrium does not exist. Indeed, on a steady-growth path like (36) fixed by an exogenously given interest rate, consumers correctly forecasting $g_X^e = g_n^e = g$ would not be

[12] It is related to preference for variety and the shadow price of capital by (11).
[13] Cf. Harrod (1971), pp. 175-76.

generally satisfied with what they are doing[14]. Thus, full steady-state equilibrium entails the simultaneous fulfilment of a twofold knife-edge condition. Economic agents are required to correctly forecast the growth rate *g,* prices, and the characteristics of future commodities; the monetary authority to choose the 'appropriate' level of the interest rate[15].

8. Speculations on endogenous preferences in a long-term growth perspective.

The crucial characteristic of preference for variety, as outlined in section 3, is that it is a well defined preference ordering over a time-varying choice set, which is known ex-ante. The approach, as further examined in sections 4, 5 and 7, brings to the fore the following implications for equilibrium growth:

(i) At any given interest rate r_t and growth rate of varieties $g_{n,t}$ a higher preference for variety (lower θ) causes a higher desired growth rate of consumption (16) of each differentiated good, and a corresponding higher savings flow at *t*, because the optimising agent prefers to postpone consumption at dates in which she will be able to benefit from the opportunity of a wider choice set. The particular model structure separates the above inter-temporal-substitution effect from other intra-temporal substitution effects that arise when leisure affects well being and labour is an input to technology (see section 6 and appendix A.2).

(ii) In the full-fledged general equilibrium model with complete markets, constant returns to the endogenous factor capital and no labour input, the steady state is unique if variety growth is exogenous. In this case, higher preference for variety, or faster arrival of new goods, are unambiguously associated to faster growth of per-capita output, even if the productivity-enhancing effects of greater differentiation are ruled out ex hypothesis. When the creation of new goods is again unrelated to productivity, but is the outcome of a purposeful and costly innovation effort, the above results can not be take for granted. The paper builds a model example where the possibility arises that the same state of preference for variety maps to two steady-growth (and interest rate) regimes: a slow-growth and a fast-growth regime. In the slow growth regime a higher preference for variety is locally conducive (in a comparative-statics sense) to a lower interest rate and

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[14] They would not be expanding their consumption at the desired rate $\dot{c}_t / c_t = r - \rho - \theta g$ unless, possibly by a fluke, $r - \rho - \theta g = 0$.

[15] We may observe, in passing, that a higher preference for variety rising the warranted growth rate under such ideal conditions, may still not be able to accelerate the actual growth of the economy, if Harrod's considerations on the local instability of growth expectations in the neighbourhood of *g(r)* do apply.

may possibly cause also slower growth. In the fix-price economy driven by subjectively-certain expectations the main implication of our model example is that equilibrium would be contingent not only upon the correct expectation formation by the agents, but also on the appropriate choice of the interest rate by the monetary authority.

As a matter of interpretation, a stronger preference for differentiation in consumption, corresponding to a lower level of the parameter θ, can be thought of as resulting from more radical qualitative differences between goods, hence from a higher and perfectly foreseen novelty content of the innovation flow. Surprise, learning and endogenous preference formation, that are so characteristic of consumption innovation, are ruled out by definition from the above representation. In this sense we are inclined to interpret the effects *(i)* and *(ii)* as related to the *perfectly foreseen component* of variety growth, as based on the given-preference approach. In a long-term framework innovation phenomena become part of a normal state of affairs and up to some extent their effects can be predicted. But it is a logical consequence of innovation as a carrier of true novelty that there must be a large *unforeseen component* of variety growth.

The formation of preferences for truly new goods entails learning and knowledge acquisition processes that mostly occur in the course of consumption activities (Bianchi, 1998; Loasby, 1998; Scitovsky, 1992; Swann, 1999; Witt, 2001) or of interactions with other heterogeneous consumers (Dosi et al., 1999) and in any case not before the relevant information or reinforcement signals are released. Preference for variety is often the outcome of an experience-based discovery of consumption complementarities (Bianchi, 1998, 2002). To this extent, consumers mostly become aware of their preference for variety only after the new goods are marketed. Self-perception of preference for variety may entail surprise[16] and its effects can not be thoroughly recounted within the strait jacket of an equilibrium framework where, paradoxically, novelty is fully anticipated.

The argument above suggests that the demand effects of a variety-innovation flow will largely depend upon the prevailing foreseen or unforeseen nature of the flow. Unforeseen substitution effects are triggered by the diffusion within the population of agents of the knowledge about the consumption opportunities disclosed by innovations that have already taken place. The consumers newly reached by the diffusion process have both motives and knowledge for

[16] A consumer who is truly and favourably surprised at time t_0 by the acquired consumption knowledge on the number and service characteristics of the new goods available, and who is not expecting further favourable surprises in the future, may wish doing more than simply modifying the planned composition of her consumption basket (partly substituting the new goods for the old ones). She would wish at time t_0 to increase her consumption at dates close to t_0 over and above what she had planned to do on the base of the wrong perception that such a wider and attractive consumption differentiation would be available only in a more distant future.

formulating a new inter-temporal consumption plan, conditional on their current information set, and on the awareness that further surprises may arrive in the future.

If a representation of the consumer's choice problem truly concerned with the emergence of novelty and the necessary updating of plans and expectations can not but refer to bounded rationality, a bounded rationality approach meets problems, when faced with the task of considering the *persistent growth effects* of endogenous preference formation. A compromise solution open to future research may come from avoiding unfulfilled expectations, while at the same time avoiding infinite planning horizons.

Moreover, the emphasis on knowledge creation and its influence on preference formation brings to the fore the relevance to the present discussion of the *information distribution* concerning the characteristics of the new goods. In this respect, the representative agent framework of sections 3, 4, 5 is particularly defective. In that framework we could disregard the asymmetric position held on the demand side by the consumer and the innovator. It seems to be more plausible to posit that the latter is the primary holder of the information concerning the nature and service characteristics of the new goods to be marketed. For this reason, he is normally[17] in a better position to correctly predict the demand for the variety which he is about to introduce. Information distribution is relevant to endogenous preference formation, and non-uniform knowledge distribution requires agents' heterogeneity.

Bringing the preceding remarks together, the concluding section of this paper suggests a departure from the approach followed in sections 3, 4 and 5 to the long-term-growth effects of consumption innovation and variety. The alternative approach, while more consistent with endogenous preference formation and non uniform knowledge distribution across agents, wants to avoid a full fledged bounded-rationality approach to consumption knowledge, because of its undesirable consequences on the determination of a long-term growth path.

9. Conclusions

A possibility open to the analysis focused on the *long-term growth* effects of consumption variety is to proceed on the bold assumption that the innovation effort is sustained by the innovators' self-fulfilling anticipation of consumers' preferences for the new set of goods that the innovation itself is about to deliver. On a equilibrium growth path innovators' expectations are fulfilled and consumers' choices are endogenous. An equilibrium innovation flow will be one based

[17] The qualification is needed to account for cases in which innovating firms were quite blind to the consumer-demand opportunities open to their innovations. The radio and the telephone are cases in point (see Metcalfe, 2001, p. 47 and the references there cited).

on the correct understanding of endogenous preference formation. Here the emphasis shifts from the effects flowing from the choice and the once-and-for-all changes of the parameters representing a given state of preferences, to the processes of knowledge accumulation, preference formation and the structural factors behind them. The processes in question select the properties of the equilibrium innovation flow.

It is here conjectured that an overlapping generations framework may offer a minimal model environment capable of reconciling endogenous preference formation, fulfilled innovators' expectations and the further requirement of a non-uniform knowledge distribution across agents. In particular, while exerting their R&D effort and human capital accumulation when young, agents accumulate knowledge consumption and more general kinds of knowledge and in so doing, they shape their preferences when old.

In a more general setting, the inclusion of the labour input and of leisure would bring with it new forms of intra-temporal and inter-temporal substitution. In particular, consumption variety would affect life styles, the stringency of income and time constraints on consumption activities (Metcalfe, 2001) and the arbitraging between the benefits from goods and leisure consumption on one hand, and between leisure and labour effort on the other.

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Appendix A.1

Proof of Proposition 2: r_1 *and* r_2 are the real solutions to the equation $\frac{(r-\rho)\alpha(A-r)}{A\theta r(1-\alpha)}=0$. It is easily seen that they meet the stated restrictions. The function $F(r)$ defined by (31) is continuous in the intervals (ρ, r_1) and (r_2, αA). Moreover, we have: $F(\rho) = +\infty$ and $F(r_1) < 1$. $F(r_2) < 1$. □

Proof of Proposition 3: Using $\dot{K}_t + n_t c_t = A(1-z_{n,t})K_t$ together with the equilibrium equality between income $(K_t + n_t V_t)r_t$ and expenditure $\dot{K}_t + n_t c_t + \dot{n}_t V_t$ and (28) we obtain:

$$(1-z_{n,t}) = \frac{n_t}{K_t}\frac{r_t^2}{\delta(A-r_t)} = \frac{\dot{K}_t}{K_t}\frac{(1+s_t)}{A} \tag{37}$$

Using the above results and (25) we obtain that at any date t in symmetric equilibrium:

$$\pi_{j,t} = \frac{r_t^2}{\delta} \tag{38}$$

Substituting from (38) into the asset equation $\dot{V}_t = -\pi_{j,t} + r_t V_t$ it yields that at any date t in symmetric equilibrium:

$$\dot{r}_t = 0; \dot{V}_t = 0; \dot{s}_t = 0 \tag{39}$$

n_t and K_t are predetermined at any date *t*. Thus, using (37) and (39) we derive the transition paths for $z_{n,t}$, n_t and K_t:

$$z_{n,t} = 1 - \frac{r}{A-r}\frac{1}{\delta}\frac{n_t}{K_t} \tag{40}$$

$$\frac{\dot{n}_t}{n_t} = \delta\frac{K_t}{n_t} - \frac{r}{A-r} \tag{41}$$

$$\frac{\dot{K}_t}{K_t} = \frac{A}{(1+s)}\frac{r^2}{(A-r)}\frac{1}{\delta}\frac{n_t}{K_t} \tag{42}$$

This completes the proof. □

Proof of Proposition 4: Let us compute the derivative: $\frac{d\left[z_n(r)/(1-z_n(r))\right]}{dr} = \alpha(1-\alpha)A\theta(r^2 - A\rho)\left[(r-\rho)\alpha(A-r)\right]^{-2}$. This proves the stated property of the function $z_n(r)$ and can be easily used to show that *F(r)* is decreasing at *r**, for *r** sufficiently close to ρ. Inspection of (30) shows that a parametric fall of θ causes a fall of $z_n(r)$ and *F(r)* at given *r*. This proves the proposition. □

Proof of Proposition 6: (30) implies that a ceteris-paribus parametric fall of α causes a discrete rise of $z_n(r)$ at given *r*, that is, $z_n(\mathbf{q}^{\S}, r) > z_n(\mathbf{q}, r)$ at given *r*. The functions $z_n(r)$, $F(r)$ are decreasing in the interval (ρ, r_1), under the stated assumptions. □

Appendix A.2:

In this appendix we modify the assumptions on preferences and technology introduced in sections 3 and 5 above in a way that makes leisure an argument of the utility function and labour an input to technology.

Let $l_{j,t}$ be the fraction of time spent consuming good *j* at date *t*. Equation (10) is replaced by:

$$u_t = n_t^{-\theta}\int_{j=0}^{n_t}\frac{\left(c_{j,t}^{\gamma}l_{j,t}^{1-\gamma}\right)^{1-\sigma}-1}{1-\sigma}dj \tag{43}$$

For σ = 1, (43) boils down to:

$$u_t = n_t^{-\theta}\left[\gamma\int_{j=0}^{n_t}\log c_{j,t}dj + (1-\gamma)\int_{j=0}^{n_t}\log l_{j,t}dj\right] \tag{44}$$

For the sake of simplicity the technology of physical goods production is left unchanged, but labour rather than capital is now the input to R&D. Equation (2) is then replaced with:

$$\dot{n}_t = \delta\left(1-\int_{j=0}^{n_t}l_{j,t}dj\right)L_t n_t^{\eta} \tag{45}$$

where L_t is population stock at *t*.

The flow budget constraint associated to (9) is now $\dot{a}_t = r_t a_t + \left(1 - \int_{j=0}^{n_t} l_{j,t} dj\right) w_t - \int_{j=0}^{n_t} c_{j,t} p_{j,t} dj$, which implies that, in addition to (11), (12) and (13) the present value Hamiltonian yields the further necessary condition:

$$n_t^{-\theta} e^{-\rho t} \mu_t^{-1} (1-\gamma) = w_t l_{j,t} \tag{46}$$

At every date t in symmetric equilibrium where $p_{j,t} = p_{K,t} = 1$, $c_{j,t} = c_t$, $l_{j,t} = l_t$, in addition to (16), we obtain:

$$\frac{c_t}{l_t} = \frac{\gamma}{1-\gamma} w_t \tag{47}$$

This yields the steady state, symmetric-equilibrium restrictions:

$$\frac{\dot{l}_t}{l_t} = -\frac{\dot{n}_t}{n_t} \qquad \frac{\dot{c}_t}{c_t} + \frac{\dot{n}_t}{n_t} = \frac{\dot{w}_t}{w_t} \tag{48}$$

Free entry in R&D, together with (27) and (47) gives the arbitrage condition:

$$\delta\, n_t^{\eta} x_t (1+s) \frac{A-r}{r} = w_t = \frac{1-\gamma}{\gamma} \frac{c_t}{l_t} \tag{49}$$

It is an immediate consequence of (48) and (49) that the existence of a symmetric, steady state equilibrium requires $\eta = 1$. Moreover, observing that the technology of physical output production implies:

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{x}_t}{x_t} + \frac{\dot{n}_t}{n_t} = \frac{A}{1+s_t} \tag{50}$$

using (50), (16) and (49), for $\eta = 1$ and the steady-state restriction $s_t = s$, we obtain:

$$1 = \frac{\rho + \delta(1-\theta)}{r} + \frac{\left[A(1-\alpha) - \alpha(1-\theta)\frac{1-\gamma}{\gamma}\right] - r\left[A(1-\alpha) - (1-\theta)\frac{1-\gamma}{\gamma}\right]}{\alpha\left(A-r\right)^2} \tag{51}$$

Additional restrictions on technology and preferences support the existence of a steady-state r meeting (51).

Proposition A.2.1: Assume $A(1-\alpha) < \alpha(1-\theta)\frac{1-\gamma}{\gamma}$, ρ and δ sufficiently close to zero. There exists a unique solution r to (51) in the interval $(\rho + \delta(1-\theta), \alpha A)$.