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Manifesto of Dynamic Social Economics

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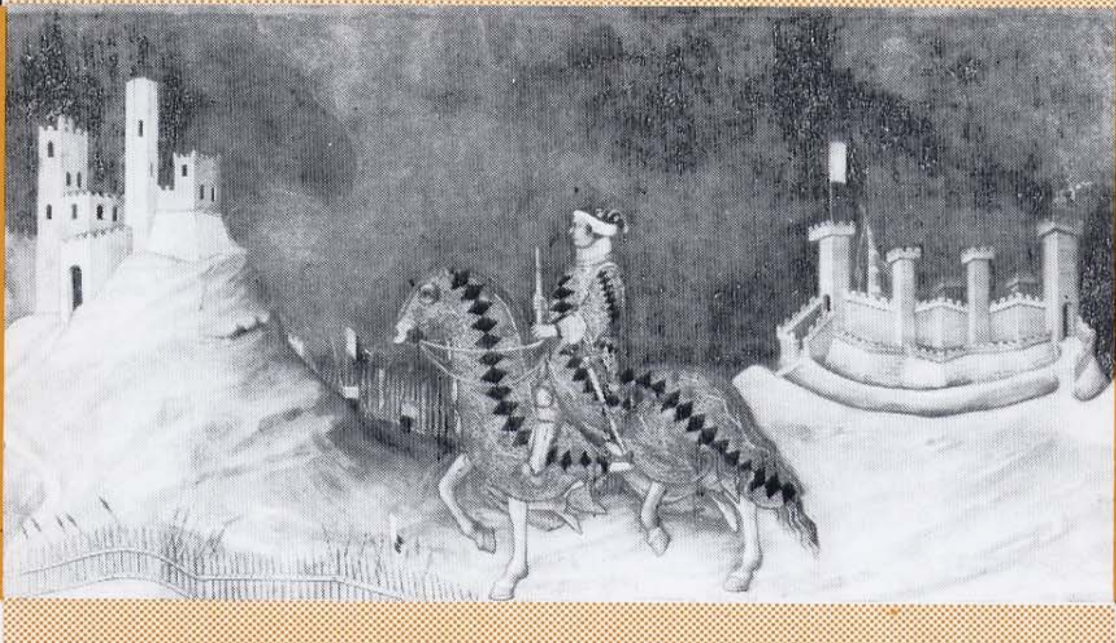

Abstract - Numerous contributions in the last decades are based on similar inspiratory principles, which, at least if considered altogether, are highly innovative with respect to the existing tradition in economics. This paper is a perspective and speculative guess aiming at identifying the gist of this ongoing change in economics and at promoting its accomplishment.
First, the features characterizing what I argue is a new approach to economic theorizing **-** labelled "dynamic social economics" **-** are presented and extensively discussed. Then, given the prominence of some mathematical techniques as general tools of analysis for this kind of models, a synthetic survey of the main relevant results is provided.

JEL classification: B0; D0
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1 Introduction

In relatively recent times an increasing number of economists has been writing works sharing a set of distinctive features. In particular, I would mention the literature placeable into those research streams known as evolutionary economics[1] - of which evolutionary game theory[2] can be thought of as the main formal expression - social interactions and agent-based modeling. The main ideas regarding rationality requirements, how reduction is carried out, the choice of objectives and instruments for economic modeling are distinguishing features of this arising way of doing theory with respect to traditional ones.

However, a precise identification of the set of works accomplishing this renewal is troublesome and not extremely useful. In fact, being this literature rather heterogeneous, the contribution which is relevant to my analysis is often affected by influences coming from distinct traditions. Hence some inconsistencies and strains among these works exist as a consequence of their different origins. Instead of being interested in finding out the features which are shared by some vaguely identifiable collection of works, my focus is on what would be presumably shared once the corrections due to the peculiarities of their backgrounds were applied. My guess is that several streams, if able to overcome the reticencies due to their origins, are going to merge in what is likely to be a new paradigm for economic theorizing. To use a methapor with a dynamic flavor, they are converging towards the same state in the space of features, but since they started from different initial conditions they are still spaced out. The first aim of the paper is to identify and comment the characteristics of that attracting state.

My prediction of a new paradigm in economics attracting some current research streams is highly speculative, and I will not try to sustain it through well-developed historiographic arguments, confining myself to some quotations and references by some of the main authors in the appropriate research fields. However, the features which I am going to discuss are, from my point of view, highly desirable for economic theory. Hence, if one disagrees with the descriptive nature of my prediction, then my contribution should be evaluated uniquely on a prescriptive ground.

As an anticipating example of subsequent contents, I would like to recall Schelling's dynamic model of residential segregation (Schelling, 1971). In

[1]Origins for evolutionary economics date back at least to Schumpeter (1934) and even earlier to Veblen (1898) (somewhat more controversial is the inclusion of Marx among evolutionary theorists, see Hodgson (1993) for a discussion), hence to times which are not recent at all. Here I refer to the new wave of evolutionary ideas originating mainly from the firm's maximization debate (Alchian, 1950; Friedman, 1953; Becker, 1962; Nelson and Winter, 1982).

[2]It can be worth recalling that evolutionary game theory appeared in a biological context thanks to the work of Maynard Smith and Price (1973) and Maynard Smith (1974) who introduced the concept of evolutionarily stable strategy.

that work the concern was on whether and how segregation can emerge out of uncoordinated decisions of a collection of agents, individually not aiming at segregated locations. Schelling analyzed the issue by taking agents from two populations and randomly placing them on the squares of a checkerboard, with each individual's neighborhood defined as the eight squares surrounding her location. Each individual is discontent if the relative proportion of neighbors belonging to her own population falls below a certain threshold. In each period a discontent agent can move to a free square and he will, if and only if there is any available which meets her demands to be content. Schelling carried out his analysis through simulations got through repeated trials by hand, arriving at the conclusion that with high probability agents will end up to be almost completely segregated by their population belonging.

More recently, a slightly modified version of Schelling's original model has been characterized analytically in its asymptotic behavior, rigorously deriving previous intuitive conclusions (Durlauf and Young, 2001, chap. 5). Nevertheless, Schelling's work contained already all - even if somewhat at a seminal stage - the core elements which I aim at pointing out in this paper: starting from a micro-structure of agents interacting through non-market relationships and endowed with limitive cognitive capabilities, the macro-properties of the resulting aggregate dynamic system are investigated. Moreover, I would like to stress that the lack of a deductive analysis in Schelling (1971)[3] is counterbalanced by the presence of an inductive reasoning grounded on simulation results. This particular method for drawing conclusions is the peculiar one in agent-based modeling and - I will argue in the following - it represents an important complementary instrument for doing theory in economics when mathematical deduction is impracticable.

After discussing the pre-analytical ground on which this approach to economic theorizing is rooted, the paper presents those mathematical techniques which naturally fit the role of tool for developing this kind of analysis. First, basic elements of Markov chain theory are introduced; then, some noise is added thus yielding a so-called perturbed Markov chain, whose long-run behavior converges to a stationary distribution irrespectively of the initial state. This distribution is, however, difficult to compute and hence it has become standard to focus instead on the limit of that stationary distribution for the amount of noise going to zero. Such limiting distribution is called stochastically stable distribution. The main results for its computation are provided.

A conclusive comment concerns the label selected for this new way of doing theory in economics, "dynamic social economics". The reason of such a choice is a mixture of minor reasons which I briefly mention. "The New

[3]Appropriate techniques for dealing with large statistical systems have been developed after Schelling's main contributions.

Social Economics" is the title of an article by Durlauf and Young where many features of this new approach are outlined; furthermore, that work belongs to a collection of many interesting papers called "Social Dynamics" (Durlauf and Young, 2001). Moreover, social interactions is an important research stream within this approach, but one of the main limitations it suffers is the lack of an extensive dynamic analysis. Therefore, the adjective "dynamic" in front of "social" remarks this point. I was also tempted to use "evolutionary" in the label but at the end prevented from, being such a term overloaded by misleading meanings rooted in its biological origin.[4] As it should be clear after reading the paper, the terms "dynamic" and "social" condense some important features of this approach. What about the role of "economics" in the label? Rather than to circumscribe the inquiry field - which instead is enlarged to social phenomena and not restricted to merely economic ones - its function is to recall the rigor of analysis and the type of categories used for representation, which are typical of economics.

The rest of the paper is organized as follows. Section 2 identifies the pre-analytical features of dynamic social economics. Section 3 presents the main analytical techniques employed to draw conclusions. Section 4 summarizes and briefly comments on the paper.

2 Identifying Features

The founding features of dynamic social economics are listed and explained in what follows.

Micro-structure. This issue touches a long and unsolved controversy regarding the role of reductionism in science and particularly in economics. By reductionism it is meant the idea that an explanation of a phenomenon must be given in terms of its elemental, constituent parts. I claim that in economics there are good reasons to use individuals as units for explana-

[4]In economics there arise difficulties to substantiate in a general way the principles of selection and inheritance, which are fundamental if some proper reference to how the term evolution is used in other disciplines is meant to be maintained. In biology there is a rather general agreement that genes are the units of selection and they are transmissible to one's own offspring. In economic environments instead there does not exist a general and meaningful way to identify what is selected and, above all, how its transmission works. Often some kind of mechanism which favors the spreading of successful behaviors is employed. However, many different specifications of this diffusion mechanism may be reasonably but not universally sustained, ranging for instance from popularity-based imitation to fictitious play, from pay-off based imitation to reinforcement learning. My point is not against the application of a properly adjusted Darwinian algorithm to economics; I simply find problematic the adoption of biological analogies because it can make people safe in using a generic evolutionary argument without fine-tuning all the details of transmission. This conviction about the questionable value of biological analogies is not shared, for instance, by Hodgson (1997).

tion, thus justifying the micro-foundation of dynamic social economics. My argument consists of three steps. First, any explanation of the functioning of an economy requires some reduction to more primitive elements. Second, the reduction procedure can in principle be prolonged up to the individual level. Third, there are reasons of convenience to stop right there.

Let me concentrate on the first step. The behavior of a system can in principle be explained by means of something outside. For instance, tides can be related to the distance between the earth and the moon and hence explained outside planet earth. This kind of explanation is however precluded if the system under analysis is closed, in the sense that it is not influenced by anything external to it. My point is that economic systems can be considered as closed, at least if they are spatially large enough and they are broadly meant so to include social interactions too. In more detail, when outer influences exist an enlargement of the system under analysis can always be considered. In economic settings many outer influences can be internalized by two types of enlargement. First, a spatial or horizontal enlargement: if two economic phenomena are interrelated, then the composition of the two can be analyzed. Second, a social enlargement: since strict economic behavior is affected by social relations, the latter should be included in the identification of the system to be studied. Of course, some external explanation is still possible. For instance a natural disaster may account for a slump in gross domestic product. However, the outer influences left over seem to me due to minor or extraordinary causes which cannot be internalized in any theory.

Drawing conclusions, if the behavior of a system cannot be explained by anything outside, an explanation must be sought after within its consitutent parts. Since dynamic social economics is interested in macroeconomic phenomena and considers a wide variety of social interactions, reduction is by far the main way for doing explanatory theory.

In the second step I argue that the reduction procedure can actually stop at the individual level. The reasoning is straightforward. The simple recognition that agents - just now not better specified individual entities - are part of socio-economy allows their identification within the system under consideration. This obviously counteracts a possible objection against reduction based on the inexistence of lower-level elements. In addition the above identification allows in principle, according to the common Aristotelean rules of logic, a meaningful difference operation to be taken between the whole system and agents. The rest of the difference operation represents complementary units of explanation and it can be kept rather aggregated or instead finely disaggregated, this not being relevant for the current issue.[5] Examples of what is left after the break-up of agents are (modeled

[5]There will be reasons of convenience of other types in favor of particular ways of disaggregating.

as) the kinds of relationships among them, public and private endowements of different nature, various types of constraints, outcome functions. From a certain perspective it can be asserted that what remains after individuals have been detected is the institutional setup, which translates into the rules of the game. It is worth noticing that the possibility of this decomposition does not entail independence between the behaviors of the identified elements. In general the evolution of the various components will be linked together determining an overall dynamics. In any case, it looks as logically true that agents can be employed as - even if not the unique - elementary units of explanation.

Finally, I address the third and last step. The first step has illustrated that any explanation of an economic system requires some reduction to its constituent elements. However, since a reduction procedure can in principle be iterated many times obtaining more and more elemental units, the question is where to stop a reductive chain and, consequently, what elements to choose as units for explanation. The second step has shown that agents can play this role, and now two reasons are suggested in favor of this choice. Before entering these motivations I state precisely what is to be meant by agents quoting one of the most prominent authors in the field of social interactions:

> In economic terms, agents are the units who interact with one another. The notion of an agent embraces persons, firms, and other entities such as nonprofit organizations and governments. The essential characteristic of an economic agent is not its physical form but rather its status as a decisionmaker.[6] (Manski, 2000, pp. 118)

The first reason in favor of agents as elemental units concerns the implementation of corrective interventions. Once a satisfactory explanation of an aggregate phenomenon has been provided showing the possibility for an overall improvement, there emerges the issue of what kind of intervention to implement. When units of explanation are decision-makers, a rather simple way to intervene exists since their behavior can be affected by modifying their incentives. This argument relies on the implicit assumption that individuals choose actions in order to attain an aim. As I will discuss in the following, this is strictly related to the concept of rationality.

The second reason concerns the realism of assumptions. Assumptions are usually made on elemental units of explanation as starting point for

[6]It might be argued that only persons can be assumed as agents, all other entities having to be reduced to the persons they are composed of. I prefer to skip this issue and I confine myself to notice that if the focus is on the status of decision-maker then the provided definition holds in any case, since the previous position can simply be rephrased by stating that firms and other organizations are not decision-makers.

deduction. Since economists themselves are individuals, introspection should represent a valuable channel of information helping them in the formulation of reasonable assumptions about individuals.[7]

I conclude this paragraph with a final note. I have argued how reduction to agents as elemental units is possible and useful, in principle. Now I mention a related possible shortcoming and how it can be dealt with. The fact that something is possible does not mean that it is easy to be accomplished too. The feature of micro-structure together with that of macro-relevance - which will be discussed in a following paragraph - makes the reductive chain extremely long and hence a correct reduction particularly difficult. This is a practical reason in favor of a shorter reductive chain. However, in my view it is not enough to give up a micro-foundation because of its numerous advantages which more than counterbalance this shortcoming. More simply, the likelihood of reduction mistakes, which may take to neglect significant factors, is an additional reason for including perturbations in the description of the system - as it will be more extensively treated in the final paragraph of this section.

Limited cognitive capabilities. It is worth discussing the concept of rationality before addressing the issue of human cognitive capabilities. As it should be hopefully clearer after reading this paragraph, the issue of rationality can be analyzed only in a motivational theory of individual behavior. Therefore agents are given an aim[8] and means to attain it. Rationality means consistency between aim and means from a decision-maker's point of view. If to an agent's knowledge a certain action allows to attain the aim to a greater extent with respect to another action, then rationality requires the latter action not to be selected. If no knowledge about the superiority of either action is at an agent's disposal, then no choice can be excluded by virtue of rationality. Notice that agents' ignorance may derive both from objective reasons - limited access to information - and from subjective reasons - limited computational capabilities which imply some cognitive simplification of problems. Suppose now that an agent knows the probability distribution of outer events whose occurrences determine which action performs better, she is sufficiently able in computation, and the aim is further specified so to become evaluable in expected terms. In this extreme case rationality requires the selection of the means maximizing in expectation the attainment of the aim.

[7] This argument holds to a smaller extent when decision-makers are not individual persons.

[8] The introduction of more aims would require some way to compare them, like utility for consumption bundles. This seems to me very close to define a unique aim with the above aims actually instruments for its achievement. Anyway, since these considerations have nothing to do with rationality and have the only effect to take attention away from the issue under consideration, I go on with the discussion assuming one aim only.

Previous considerations show that rationality is a requirement of a class of models of behavior and does not impose any bound to one's faculties. More precisely, a motivational theory of behavior must be rational in order to be meaningful. In fact, there would be a nonsense in explaining how people behave in terms of their aims if they were allowed not to try to pursue their aims. They can fail, but this is another point.

The definition that I have used deprives rationality from any intrinsic content reducing it to a matter of form, simply a way of doing theory. The validity of several statements in this paragraph about rationality clearly rests on such definition. Of course other definitions of rationality can be and have been used, usually including some requirement about human cognitive capabilities. However, it seems to me clearer to keep distinct the issue of consistency of a motivational model from that of the mental powers of its agents.

As already stated, rationality means motivational consistency from a decision maker's point of view. How demanding consistency is in terms of cognitive capabilities depends on the specification of this point of view, that is on the particular assumptions regarding agents' knowledge and capability in deductive chains. Once all constraints are explicitly specified then rationality means nothing but a check that the resulting model is indeed a motivational theory of behavior. To take this reasoning to its extreme, when also stupidity is considered as a constraint then a foolish behavior can be rational.

Therefore, the real issue concerns the type and degree of cognitive capabilities of human beings, to which I now turn my attention. In short since there is no surprise in that, agents are endowed with limited cognitive capabilities. Those branches of science regarding human brain, such as neuroscience, should provide directions for a correct specification of these capabilities. In some simple case agents will know every detail of the problem they face; for instance in a fair head-or-tails game. More often, given the complexity of situations agents usually encounter in their everyday life, their actual knowledge will be very poor if compared with their ignorance.

The following observation prepares the ground to possible critiques. If an agent does not know which of two actions is better and she is not an expected utility maximizer (since she is not able to evaluate uncertainty probabilistically or has some other limitation), then both actions are allowed as prediction. In general, a unique outcome of the decisional process is not ensured on the basis of motivational arguments. This may be seen as a shortcoming, but I do not share such a view. On the contrary, it seems to be in agreement with common sense that not all human behavior is explainable by motivations. When two alternatives cannot be evaluated in terms of their relative convenience, then some second order factor will be relevant for the decision. Cognitive science, psychology, sociology should help in determining this drift component. Examples of these factors are imitation, anti-conformism, habit.[9]

It is worth remarking that the theory of individual behavior which is emerging is motivational - in the sense that no inferior choice can be consciously made - but potentially incomplete. Such incompleteness is solved according to non-motivational arguments which play the role of tie-break rules.

Someone may feel vexed by the large variety of possible ways in which cognitive capabilities first and non-motivational factors then can be specified in principle. However, directions from other disciplines, as already stated, should reduce the range of possible specifications. Moreover, it is anyhow better to use case-specific assumptions which represent correctly human reasoning in a particular setting than false general ones. Finally, it should be noticed that the equilibrium refinements literature in game theory has shown how difficult is to obtain a unique prediction for human behavior under the assumption of unbounded cognitive capabilities as well, when interactive decision-making is considered.

Non-market relations. Neoclassical theory of general competitive equilibrium, which has been the core of mainstream economics for much of the twentieth century, circumscribed economic analysis to those interactions occurring in markets. The existence of non-market interactions relevant for economic phenomena was considered as a form of market incompleteness, and hence as a problem to face for the real world since being a possible source of inefficiency rather than as an aspect to be taken into account for doing good theory.

Dynamic social economics demands an explicit modeling of the relations among human beings. This opens to a large variety of different structures of interaction. The spreading of any kind of influence can in principle be subordinated to the existence of a certain relation. For instance, trading offers can be thought of as actually spreading only through purchase/sale channels, partially linking to each other sellers and buyers of a particular product.

Furthermore, the abandonment of impersonal markets as unique channel for human interactions allows an easier treatment of social relations, many of which cannot reasonably be thought of as occurring in markets. The recognition of the many interdependencies between social and strictly economic phenomena is the peculiar contribution of that stream of literature known as social interactions.[10] As a consequence of such a recognition, the inquiry field of dynamic social economics is enlarged in order to include

[9]It might be sustained the idea that these non-motivational factors are indeed in a sense instinctively motivational.

[10]Becker (1974) is the first work to my knowledge fully belonging to this research line. Manski (2000) provides an excellent reflection on the state of art of social interactions in economics.

social relations. I recall that by so doing an important source of outer influences is internalized in this emerging theory. Moreover, since dynamic social economics is based on a non completely motivational theory of individual behavior, the room left is mostly filled by social determinants. In more detail, the choice among alternatives which a decision-maker is unable to evaluate in terms of their relative convenience - because of her limited cognitive capabilities - is driven by many second-order factors of decision which are essentially of social nature. With a vague pretension to generality, I can refer to conformism or anticonformism (or some average behavior if the choice is quantifiable) with respect to successful or unsuccessful people (according to some parameter, for instance income) or to one's neighbors (from a cultural, racial, spatial, etc. point of view).

Macro-relevance. The aim of dynamic social economics is to understand how many individual decisions evolve into recognizable patterns of macro-behavior.

Schelling's prominent role for my discussion is already known from the reference to his model of residential segregation in the introduction. Here I will rely again on Schelling to sustain my point. The subject matter of his book "Micromotives and Macrobehavior" (Schelling, 1978) - and of dynamic social economics as well - is, as suggested by the title, that kind of analysis which

> ... explores the relation between the behavior characteristics of the *individuals* who comprise some social aggregate, and the characteristics of the *aggregate*. (Schelling, 1978, pp. 13)

As evidence of the diffusion of the feature of macro-relevance in economic theorizing I quote Blume and Durlauf who, to illustrate the characteristics of the models typical of social interactions, explain:

> The object of a typical exercise using these models is to understand the behavior of a population of economic actors rather than that of a single agent. (Blume and Durlauf, 2001, pp. 17)

Macro-relevance is no doubt a restriction on the field of application for this approach. However, I would remark how this is a restriction to the collectivity and as such it is more valuable than other kinds of restriction. In addition, I recall the role that this aggregate concern plays for the object of analysis to be considered as a closed system. Finally, I mention an additional advantage deriving from the concern on macro-properties. Often individual behavior can be hardly predictable, since influenced by many casual factors not includable in any theory. However, at the aggregate level some constraints or regularities can emerge and be exploited to get a macro-description. Relying again on Blume and Durlauf,

> In treating aggregate behavior as a statistical regularity it turns out that individual behavior need not be as tightly modeled as it is in traditional economic models. (Blume and Durlauf, 2001, pp. 18)

The focus on macro-features deriving from the aggregation of many individual choices suggests a brief digression about the issue of emergentism. According to a strict definition an aggregate property is emergent if it is irreducible to lower-level elements. In this sense, it can be emergent a real-world property with respect to some selected units of explanation, since some other relevant factor has been disregarded. However, it is impossible that something actually emerges within a model, every results obviously deriving from what assumed. Such a definition of emergentism is therefore inapplicable to a modeling approach. Suppose now that a model is built up, simulations are run showing unexpected outcomes, which even after repeated attempts are not successfully deduced by assumptions. Results still depend on the modeling assumptions, but it is not known how. If the requisite of irreducibility in the definition of emergentism is meant in a mathematical sense, then emergentism can be used to distinguish between proven (or deductively derived) and unproven (or inductively derived) aggregate properties. Moreover, if irreducibility is further specified so to be meant in a cognitive way, then also results which are proven but are not of immediate consequence from assumptions can be labelled as emergent. In my opinion, this last meaning is closest to the spirit of the old-time sentence “the whole is more than the sum of its parts”. Indeed, sometimes it is the product, meaning that aggregation can generate non-linearities. Social dynamic economics, as any potentially non-useless micro-based macro-theory, is emergent according to this last definition.

Out-of-equilibrium dynamics. Phenomena under examination by dynamic social economics are modeled as dynamic systems. A dynamic rule is obtained by the composition of all individual decision rules and then it is applied to the current state of the system in order to get the state at the next time. Since every decision requires time to be taken and to be put into effect, this representation is in no case a limitation. However, it is not useful either unless sufficiently many applications of the dynamic rule are considered so for the asymptotic behavior to become relevant. Indeed, many economic phenomena can be appropriately described as repetitions of a basic dynamics. In other words, stable forces and relations shaping a collection of similar individual decision problems can usually be detected as a consequence of the way human reasoning prevalently works. Facing a decision problem agents tend in fact to cognitively structure it by simplification and analogy with their past experience.

In all the cases where enough repetitions take place, given the finiteness of the state space which is usually assumed, an equilibrium or a cycle (or an ergodic set in case of a probabilistic setup of the model) is eventually reached. The evaluation of the state or the set of states to which the system moves is particularly relevant since once reached the system will stay there forever, hence justifying its adoption as prediction after that enough time is elapsed and as long as the model is considered a suitable description for the situation of interest. Moreover, since many different states are mapped into the same asymptotic behavior, the exact knowledge of the actual state may be not necessary to predict its long-run behavior. For instance, if it is known that when a certain behavior is less than 50% frequent in the population it is going to disappear, then a vague knowledge that such behavior is not widespread is sufficient to conclude about its disappearance.

The above considerations clearly stress the dynamic gist of dynamic social economics, as well as the role played by equilibria and other types of long-run behavior. Indeed, they are important as the lasting outcomes of a dynamic process. As a rule of thumb, their prominence may be measured by the relative largeness of their basins of attraction. Finally, it is worth remarking the importance of the precise knowledge of trajectories, since that allows prediction by connecting transient states with their final future projections.

Markov properties, mathematical deduction and simulations. This paragraph discusses Markov chains with the aim to point out how they are the natural representation when the previously analyzed principles are observed. This particular perspective from which Markov chains are looked at determines the inclusion of this paragragh within this section about pre-analytical features and a consequent informal style of presentation. A more formal treatment follows in the next section.

It has previously been remarked the importance for predictive purposes of the mapping of many transient states into the same asymptotic behavior. However, the greater the dependence of asymptotic behavior on initial conditions is, the more difficult prediction becomes, since it requires a more precise knowledge of the current state. Here the notion of stochastically stable distribution comes to assistance, proving to be a fundamenatal tool for the selection of a unique prediction. Consider a system whose current state univocally determines the probability with which the next period each possible state is reached. Such a process is a Markov chain. A deterministic dynamics can be represented as a special case by setting 1 to the probability of a certain transition and 0 to all other probabilities. The key insight here is that the addition of random perturbations affecting individual decision-making yields an irreducible and aperiodic chain.[11] In fact, that allows the

[11]The conditions under which a Markov chain is said irreducible and aperiodic are stated precisely in the next section.

application of a well-known result by which the system converges in the long run to a certain probability distribution - called unique invariant distribution - irrespectively of the initial condition. Unfortunately, in order to compute that unique prediction all transition probabilities are to be exactly specified and a system made of a probably enormous number of equations is to be solved. This possible empasse is overcome by the existence of i) the limit of the invariant distribution for the amount of perturbation going to zero and ii) relatively simple ways to get at least its support. Such limit distribution is called stochastically stable distribution.[12]

It is worth stressing how this analytical technique represents a rather general tool to make predictions. A very large variety of processes may be formalized so to be finite Markov chains under very weak conditions, which I now discuss. The first condition requires time to be discrete and states to be finite. In support of that, universe is probably finite according to the prevailing opinion in physics. Moreover, world population is surely finite and, because of human limited cognitive capabilities, agents would simplify in any case the decision problem they face so to get a finite representation of the alternatives among which to choose. Finally, the type of decisions agents take usually defines a time-unit in a natural way. Therefore this requirement does not look like a particularly demanding one, continuity instead probably being an approximation of reality used for analytical convenience.

The second condition, often called Markov property, requires the possibility to neglect any information about the past except the state of the system at the last time for predicting the state of the system at the next time. However, since states can be redefined so to be finite sequences of states, this condition actually amounts to the system having a finite memory. Because the system consists of agents with finite memory as a consequence of their limited cognitive capabilities, this second requirement should be safely accepted too.

Finally, the last and slightly more controversial condition requires time homogeneity, that is insignificance of time for predicting the future once the state at the last time is taken into account. In other words, what is here demanded is the constancy over time of the forces at play, so that being in a certain state at distinct times makes no difference for transition probabilities.

If finite Markov chains are accepted as representations, then what really matters in order to use the stochastically stable distribution is the existence of perturbations and the way they affect the system. Perturbations, more than as a technical device, appear in this framework as a conceptual requisite. A first reason for that actually applies to any theory, and especially to rough ones, as for the present theories of human behavior. In a model

[12]Applications of the stochastically stable distribution to economic issues are progressively spreading. For instance, in Young (1993b) the evolution of bargaining norms is extensively considered while in Young and Burke (2001) a reasonable explanation of the pattern of cropsharing contracts found in contemporary U.S. agriculture is provided.

the effects of a set of hypotheses are investigated and the usefulness of that rests on the possibility to extend drawn conclusions to real world situations. The presence of perturbations somehow checks the robustness of results with respect to shaking assumptions, therefore allowing a less problematic application of such results to reality.

The importance of perturbations is even greater in the framework of dynamic social economics for further reasons. As discussed in a previous paragraph about micro-structure, since analysis starts from micro-behavior to conclude about macro-properties, the explanatory chain is very long and hence many relevant elements are likely to be left out of the model or misrepresented. Moreover, individual decision rules are obtained by a particular specification of human cognitive capabilities and social influences. So doing there is no claim about general validity and no point in investigating the consequences of precisely that specification. In other words, there may be some sense in assuming exactly an extreme case in the space of cognitive capabilities, as standard models with highly-gifted agents do, since this is just a way to choose the relevant dimension. However, when the relevant dimensions have been chosen there is much less sense in assuming a very particular combination of them - an exact inner point surrounded by similar points - as models with humanly-gifted agents would do.

Perturbations of individual behavior have noteworthy implications on the concept of rationality. In a previous paragraph a behavior has been defined as rational if it is not in contrast with perceived convenience. The introduction of perturbations brings irrationality in this framewrok; sometimes people choose something they regard as inferior. This apparent discrepancy with the claimed motivational foundations of this theory of behavior disappears once it is taken into consideration that perturbations are inserted in a very small amount and, moreover, that they can be justified by the existence of some neglected motivation.

The existence of perturbations is however not enough. They ensure that people can make any choice, but what is indeed required is that any state must be reachable from any other state. This condition is called ergodicity and, given perturbations of individual behavior, it substantially amounts to the inexistence of irreversible processes.

Finally, there is a last and more subtle requirement for the application of the stochastically stable distribution, namely the correctness of the limit operation. My point is very simple. Since pertubations are not only a tool for selection but they find justification for their existence according to previous arguments, then particular care should be put in the limit operation. Some occurrences in the model may be extremely unlikely, so implying a very small probability for the transition from certain states to other states. This probability, however small it is, is bound to be much greater than the probability of a perturbation to occur, since the limit for the latter going to zero is computed to get the stochastically stable distribution. This is what necessarily occurs even when in our intuition the probabilities of a certain unlikely event and of a perturbation are comparable. Therefore, the choice to use this limiting technique for prediction imposes as limitation the task to represent extremely unlikely occurrences as impossible.

As extensively discussed, the stochastically stable distribution is by far the main instrument for deriving results from a set of assumptions in models of dynamic social economics. Other mathematical techniques for dynamic systems can also be used for contingent cases. However, systems composed of many interacting agents updating behavior by personal information often turn out to be intractable by any mathematical tool. At times known results can be applied in order to identify at least the support of the stochastically stable distribution. At other times that is not possible, the only available method remaining induction from data generated by computer simulations.[13] Moreover, simulations can help in identifying what is to be proven hence simplifying its formal demonstration.

Economics distinguishes from other social sciences for the rigor of analysis. This, which is a merit in itself, has brought about a snobbish attitude towards methods of analysis without formal proofs, including simulations. However, since economics is an applied discipline it seems to me natural to consider much better a good model where results are inferred by simulation than a model fully and elegantly characterized by theorems but scarcely applicable to reality.

The following quotation by some of the leading authors in agent-based modeling is a marvellous synthesis of the role of simulation in doing theory.

> Simulation in general, and [agent-based modeling] in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. (Axelrod and Tesfatsion, 2005, pp. 2)

After asserting the important role of this alternative tool of analysis, it is necessary to underline how having recourse to simulations should be

[13] As evidence of the increasing adoption of simulation techniques in economics - and of their complementary role with respect to the analytical derivation of the stochastically stable distribution - it is worth noticing how results in the already cited Young and Burke (2001) are complemented by computer simulations.

subordinated to mathematical deduction. A first reason is obviously the certainty of results deriving from proofs with respect to inductively drawn conclusions. However, since simulations allow enormous data sets and a full control on assumptions, this argument is not so strong as it could appear at a first glance. A second and more important reason concerns the deeper understanding of a model resulting from deduction. When hypotheses are worked out by means of chains of reasoning and results are eventually got, the dependence of the latter from the former is dissected.[14] Any conceivable model is a drastic simplification of reality. The choice of the more appropriate simplification is made easier by the grasp of the role played by assumptions, which can thus be evaluated functionally with respect to the real-world case under consideration.

3 Perturbed Markov Chains

Given the prominence of perturbed Markov chains as tool for representation and analysis, a brief theoretical survey is provided here.

Elements of Markov chain theory. I first introduce some basic definitions and findings about standard Markov chains before specifically addressing perturbed chains.

Definition 1 (Markov chain)**.** *A Markov chain in discrete time on a finite space is a couple* (S, T) *such that:*

1. S *is a finite set called* state space*;*
2. T *is a* transition matrix *on* S*, that is* $0 \leq T_{s_1 s_2} \leq 1$ *for any* $s_1, s_2 \in S$ *and* $\sum_{s_2} T_{s_1 s_2} = 1$ *for any* $s_1 \in S$*.*

The state space S is the set of all possible descriptions of the system under consideration. Every description - or state - specifies any detail which is relevant for the determination of the state the system will visit at the next time. For instance, a state can simply indicate the number of agents using each available action. However, if agents were heterogeneous under some significant characteristic, then the identity of the agents selecting every action would also be relevant and hence should be included in the description too. Again, if agents had remembrance of their previous actions and used somehow that remembrance for taking decisions, then past actions should be also part of a state. In brief, a lower bound to the largeness of the state space is imposed by the requisite for the dynamics to be Markovian on that state space, while an upper bound is usually established by aiming at simplicity, so disregarding any aspect which does not affect transition probabilities.

[14]This argument is stronger for constructive demonstrations and justifies their privileged use in economics.

The transition matrix T states the probability of moving from any given state in S to every state in S in the next period. Since every state is a complete description of all relevant aspects, then agents' decision problems are always properly defined. Given a specification of human cognitive capabilities and of non-motivational factors of choice, a behavioral rule turns out to be defined. The choices of all agents, applied to the current state of the system, determine the probability with which each state is reached at the next time.

Consider now the following notion of *accessibility.* A state s_2 is accessible from a state s_1 when there exists a non-negative integer r such that $T^r_{s_1 s_2} > 0$. Accessibility is a preorder since reflexive ($T^0_{ss} = 1$ for any s) and transitive ($T^{r_1}_{s_1 s_2} > 0$ and $T^{r_2}_{s_2 s_3} > 0 \Rightarrow T^{r_1+r_2}_{s_1 s_3} \geq T^{r_1}_{s_1 s_2} T^{r_2}_{s_2 s_3} > 0$). A preorder can be used to classify and partially order the elements of a set. The resultant equivalence classes are called *communication classes.* Maximal communication classes - those from which no other classes are accessible - are called *ergodic sets* (or absorbing sets or recurrent sets). Equivalently, a nonempty set of states is ergodic if it is a minimal set with respect to the property of being closed under dynamics T. States which do not belong to an ergodic set are called *transient.* The notion of basin of attraction has been used by the reference literature in two different ways which I find convenient to distinguish from one other. The *basin of weak attraction* of an ergodic set comprises all those states from which that ergodic set is accessible. The *basin of strong attraction* of an ergodic set comprises all those states from which that ergodic set is the unique accessible ergodic set. Notice that i-a) basins of weak attraction may intersect if the dynamics is not deterministic and ii-a) their union covers S entirely while i-b) basins of strong attraction never intersect and ii-b) their union may not cover S entirely if the dynamics is not deterministic.

A Markov chain (S, T) is said to be *irreducible* if S is the unique ergodic set. Let R_s be the set containing all those positive integers r such that there is a positive probability of moving from s to s in exactly r periods, $R_s \equiv \{r > 0 : T^r_{ss} > 0\}$. A Markov chain (S, T) is said to be *aperiodic* if for every state s the greatest common divisor of the elements of R_s is unity.

A probability distribution over S is a vector z collecting the probabilities to be in every state. The application of transition probabilities T to the current probability distribution z^t yields the probability distribution at the next time, $z^t T = z^{t+1}$. A vector z satisfying the condition $zT = z$ is called *invariant* or *stationary distribution* for intuitively obvious reasons. The following propositions are standard results about the asymptotic behavior of Markov chains.

Proposition 1. *If (S, T) has a unique ergodic set, then:*

1. *there exists a unique invariant distribution $\hat{z}$;*

2. *the relative frequencies with which states occur converges almost surely as time goes to infinity to $\hat{z}$, independently of the initial state.*

Proposition 2. *If (S,T) is irreducible and aperiodic then for any $s \in S$ the probability to be in s at time r converges as time goes to infinity to $\hat{z}$, independently of the initial state.*

Therefore the unique invariant distribution of any irreducible and aperiodic Markov chain can be interpreted both as the proportion of time spent in each state and as the probability of being exactly in each state, when time is sufficiently great. Moreover, this holds independently of the initial state.

Proposition 3 is due to Freidlin and Wentzell (1984) and provides a way to calculate the invariant distribution for irreducible Markov chains. Its main importance stems however from being the fundamental tool for deriving proposition 4. A couple of definitions prepare the ground.

Definition 2 (Rooted tree)**.** *Let X be a finite set and $x \in X$. An x-tree F_x on X is a collection of ordered pairs of elements of X, $F_x \subseteq X \times X$, such that:*

1. *For every $x_1 \in X$, $x_1 \neq x$, there exists a unique $x_2 \in X$ such that $(x_1, x_2) \in F_x$.*
2. *There is no $x_1 \in X$ such that $(x, x_1) \in F_x$.*
3. *For every $x_1 \in X$, $x_1 \neq x$, there exists a sequence $x_1, x_2, x_3, \ldots, x_k, x$ of elements of X such that $(x_1, x_2) \in F_x$, $(x_2, x_3) \in F_x$, ..., $(x_k, x) \in F_x$.*

Definition 3 (Likelihood of a rooted tree)**.** *Let (S,T) be a Markov chain. The likelihood of an s-tree F_s on S is*

$$L(F_s) \equiv \prod_{(s_1,s_2)\in F_s} T_{s_1 s_2}$$

Let $\mathcal{F}_s$ indicate the set of all s-trees on S. Proposition 3 establishes that the probability $\hat{z}_s$ of each state s is proportional to the sum of the likelihoods of all its s-trees.

Proposition 3. *Let (S,T) be an irreducible Markov chain with $\hat{z}$ its unique invariant distribution. Then:*

$$\hat{z}_s = \frac{\displaystyle\sum_{F_s\in\mathcal{F}_s} L(F_s)}{\displaystyle\sum_{s_1\in S}\sum_{F_{s_1}\in\mathcal{F}_{s_1}} L(F_{s_1})}$$

The addition of perturbations. Some noise is introduced in the form of a small amount of probability for agents to choose disregarding their behavioral rule. For a comment on the role of perturbations see the last paragraph of the previous section. Here just notice that such an introduction easily yields an irreducible and aperiodic Markov chain for any positive amount of noise.

Definition 4 (Perturbed Markov chain)**.** *A perturbed Markov chain*[15] *in discrete time on a finite space is a triple* $(S, T, T(\epsilon))$ *such that:*

1. *S is a finite state space;*
2. *T is a transition matrix on S;*
3. *$T(\epsilon)$ is a family of transition matrices on S indexed by $\epsilon \in [0, \bar{\epsilon})$ such that:*
   - *(a) $(S, T(\epsilon))$ is irreducible for each $\epsilon \in (0, \bar{\epsilon})$;*
   - *(b) $T(\epsilon)$ is continuous in ϵ with $T(0) = T$;*
   - *(c) there exists a function $r : S \times S \to \mathbb{R}^+ \cup \{\infty\}$ such that for all pairs of states $s_1, s_2 \in S$,*

$$\begin{cases} \lim_{\epsilon \to 0} \dfrac{T_{s_1 s_2}(\epsilon)}{\epsilon^{r(s_1,s_2)}} \textit{ exists and is strictly positive} & \textit{if } r(s_1, s_2) < \infty \\ T_{s_1 s_2}(\epsilon) = 0 \textit{ for sufficiently small } \epsilon & \textit{if } r(s_1, s_2) = \infty \end{cases}$$

Function r is usually called resistance *or* cost*.*

For any $\epsilon > 0$ the system converges to its unique invariant distribution $\hat{z}(\epsilon)$ solution of $z = zT(\epsilon)$. Since the presence of perturbations finds a pre-analytical justification, the ideal description of the long run behavior of the system would require the computation of $\hat{z}(\epsilon)$. However, that distribution is usually too difficult to be derived, even when proposition 3 is exploited. Therefore, it looks reasonable to use the limit for a vanishing amount of noise - if existing and easier to compute - as an approximation of the invariant distribution when perturbations are very rare.

Let $\mathcal{E}$ be the set of all the ergodic sets of the unperturbed Markov chain (S, T). The concept of rooted trees is now applied to set $\mathcal{E}$, with $\mathcal{F}_E$ indicating the set of all E-trees on $\mathcal{E}$. The notion of resistance (or cost) is extended to ergodic sets, letting $r(E_1, E_2)$ indicate the minimum sum of the resistances between states over any path starting in E_1 and ending in E_2, with E_1 and E_2 two distinct ergodic sets. Definition 5 provides a suitable potential function to be used in the following proposition 4 due to Kandori et al. (1993) and Young (1993a), which is the central result of this section.

[15] I call perturbed Markov chains what are regular perturbed Markov chains in Young's terminology (Young, 1998).

Definition 5 (Stochastic potential)**.** *Let $(S, T, T(\epsilon))$ be a perturbed Markov chain. The stochastic potential of an ergodic set E of the unperturbed dynamics is*

$$\gamma(E) \equiv \min_{F_E \in \mathcal{F}_E} \sum_{(E_1, E_2) \in F_E} r(E_1, E_2)$$

Proposition 4 (Stochastically stable distribution)**.** *Let $(S, T, T(\epsilon))$ be a perturbed Markov chain, with $\hat{z}(\epsilon)$ the unique invariant distribution for any positive ϵ. Then:*

1. $\lim_{\epsilon \to 0} \hat{z}(\epsilon) \equiv \tilde{z}$ *exists and is unique;*

2. $\tilde{z}$ *is an invariant distribution of (S, T);*

3. $\tilde{z}_s > 0 \Longleftrightarrow s \in E : E \in \arg\min_{\mathcal{E}} \gamma(E)$.

The limit distribution $\tilde{z}$ is usually called stochastically stable distribution and those states to which a positive probability is assigned are called *stochastically stable states*.

Proposition 4 establishes that the stochastically stable states are those easiest to reach from other states, with easiest interpreted as requiring the fewest mutations, that being measured by the stochastic potential.

The stochastically stable distribution provides a description of the very long run behavior of the system. One of the main limitations for its adoption as tool for prediction derives from the difficulties in quantifying how long is the waiting time for the stochastically stable distribution to become relevant. Proposition 5, due to Ellison (2000), deals with this issue establishing a bound to the expected waiting time until a stochastically stable state is reached for the first time. In addition, it provides a sufficient condition for identifying stochastically stable states. Some definitions are required in order to simplify the following expostion.

Let Ω be the union of one or more ergodic sets of the unperturbed Markov chain (S, T). Expanding a previously defined notion to unions of ergodic sets, let the basin of strong attraction of Ω be the set of states from which accessible ergodic sets belong uniquely to Ω, and indicate it with $D(\Omega)$. A *path* p from a set $X \subseteq S$ to a set $Y \subseteq S$ is a sequence of states $(s_1, s_2, \ldots, s_k)$ such that $s_1 \in X$ and $s_k \in Y$. Denote with $P(X, Y)$ the set of all paths from X to Y. In a perturbed Markov chain the *resistance* of a path $p = (s_1, s_2, \ldots, s_k)$ is $r(p) = \sum_{i=1}^{k-1} r(s_i, s_{i+1})$.

Definition 6. *Let $(S, T, T(\epsilon))$ be a perturbed Markov chain.*

1. *the* radius *of Ω is*

$$R(\Omega) = \min_{p \in P(\Omega, S - D(\Omega))} r(p)$$

2. *the* coradius *of* Ω *is*

$$CR(\Omega) = \max_{s \notin \Omega} \min_{p \in P(s,\Omega)} r(p)$$

3. *the* modified coradius *of* Ω *is*

$$CR^*(\Omega) = \max_{s \notin \Omega} \min_{p \in P(s,\Omega)} [r(p) - R(E(p))]$$

with $R(E(p)) = \sum_{i=2}^{r-1} R(E_i)$ *and* $E_1, E_2, \ldots, E_r$ *such that* $E_i \neq E_{i+1}$ *for* $i = 1, \ldots, r-1$ *the sequence of ergodic sets visited along the path* p.

Denote with $W(s, \Omega, \epsilon)$ the expected wait until a state belonging to Ω is first reached given s as initial state. The notation $f(x) = O(g(x))$ as $x \to 0$ is a shorthand for there exist $\bar{x} > 0$ and c such that $|f(x)| < c|g(x)|$ for any $x \in (0, \bar{x})$.

Proposition 5. *Let* $(S, T, T(\epsilon))$ *be a perturbed Markov chain and* Ω *a union of ergodic sets such that* $R(\Omega) > CR^*(\Omega)$. *Then:*

1. *the set of stochastically stable states is contained in* Ω*;*

2. *for any* $s \notin \Omega$, $W(s, \Omega, \epsilon) = O(\epsilon^{-CR^*(\Omega)})$ *as* $\epsilon \to 0$.

4 Conclusions

With reference to some recent developments in economics, sampled by the articles collected in "Social Dynamics", Durlauf and Young write:

> When this body of work is assessed as a whole, we do not think it is an exaggeration to say that a new social economics paradigm has begun to emerge. (Durlauf and Young, 2001, preface)

There is a widespread agreement that something new is arising and determining fundamental changes in economics. However, what exactly is emerging is a more controversial argument. The first aim of this paper has been to outline in perspective this new research direction, aware of the difficulties due to the ongoing state of research and hence having recourse to speculation, and to promote the change towards the identifed core of principles.

The identifying features for this new approach to economic theorizing - which I have labelled dynamic social economics - are listed and extensively discussed. What turns out is a theory looking at macro-properties of a perturbed dynamic system generated by a micro-structure of agents socially interconnected and endowed with limited cognitive capabilities. This seems to me a very natural way for examining in abstract terms the working of a social system. Indeed, it seems so natural to let me wonder why it has not gained a leading role in social sciences yet. I suspect a couple of reasons to have played an important role for hindering its diffusion.

First, the fact that agents are (or are referable to) human beings - even if helpful for formulating reasonable hypotheses - is a delicate point at risk of ideological influences. In more detail, accepting the possibility that individuals make systematic mistakes can (or can be thought to) partly deligitimate individual freedom. As a consequence, extremely high requisites have been imposed on human cognitive capabilities. This has in turn made decision problems very complicated hence requiring representative simplifications to get solutions, with the result that social relations have been ignored and economic relations mediated through impersonal markets.

The second reason concerns technical unmanageability. Society is a complex phenomenon and an appropriate representation of it - like, in my opinion, the one discussed in this paper - is likely to turn out too complex to be of any help. However, recent developments in analytical and computational techniques have been progressively overcoming technical difficulties, so making the suggested representation fruitful. I refer in particular to that kind of mathematical deduction represented by the stochastically stable distribution, and to computer simulations in an important complementary position.

The second aim of the paper has been to survey those mathematical techniques promising to become standard tools in this field. I pursued this objective first by introducing some basic elements of Markov chain theory and then by dealing with perturbed Markov chains, yielded through the addition of perturbations. The main results regarding the stochastically stable distribution (Kandori et al., 1993; Young, 1993a; Ellison, 2000) are presented and briefly discussed.

The next step in agenda for dynamic social economics is the setup of applied models for specific issues.[16] On one side this will reveal how useful this modeling technique can actually be. On the other side an important feedback can be obtained showing where theoretical or technical improvements are necessary.

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[16]As a first step along this line I would cite a couple of works of mine where efficiency of aggregate choice is investigated when individual choice is driven by only personal experience (Boncinelli, 2007a) and by both personal experience and imitation of others (Boncinelli, 2007b).

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