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Where do Personal Experience and Imitation Drive Choice?

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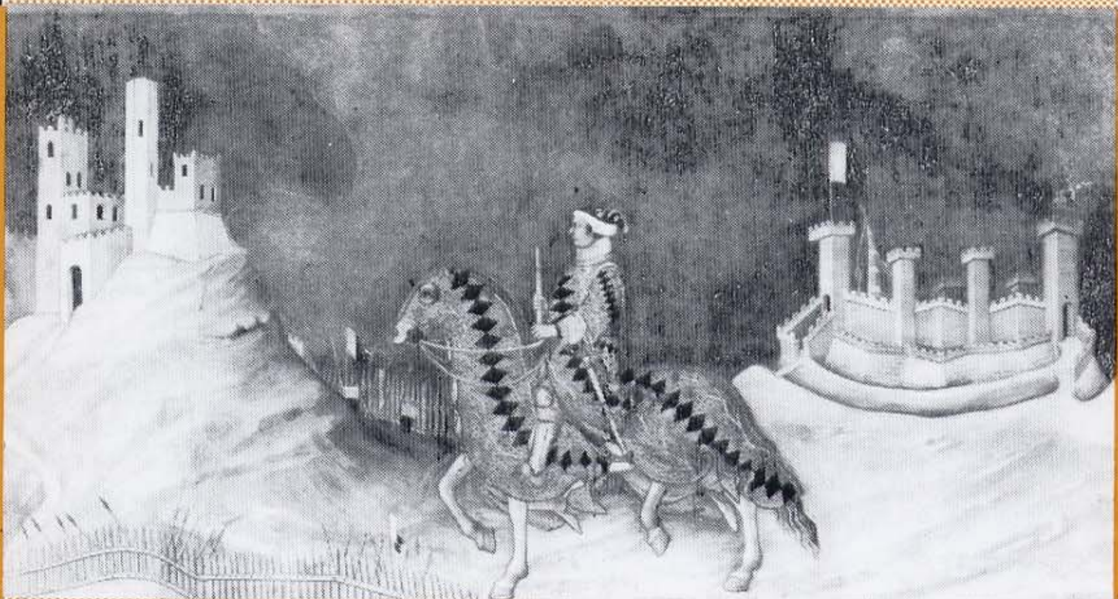

Abstract - This papers investigates the efficiency of aggregate choice in the long run when the individual decision is driven by both personal experience and imitation. Personal experience is represented by choice sets depending upon previous choices. Imitation is modeled first through popularity weighting and then through a network of social influences.
Intuition suggests imitation can work as a source of variety, spreading behaviors among which memory can make selection. However inefficiencies will persist in the stochastically stable distribution whenever the length of memory is not sufficiently long to stop inferior behaviors from moving perpetually along periodic cycles of social influences.

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1 Introduction

Nothing is easier than choosing what is best. The matter in choice is determining what is best. The amount of decisions an agent may be required to take is usually incredibly large, and even larger is the set of external factors which can affect the consequences of any decision. This makes the acquisition and elaboration of the whole of information required to assess the value of every alternative, and hence to select the best one, a very difficult task, reasonably too difficult for human cognitive capabilities. In a model of bounded rationality - or, better, of limited cognitive capabilities[1] - a specific rule, or a class of rules, is assumed for agents in the process of simplification of decision problems. In this paper a process of cognitive simplification based on personal experience and imitation is built up and investigated.

The starting point is Boncinelli (2007b), which is the direct reference for this paper, even for further clarification and reading about the representative structure.[2] In that piece of work the limited information agents have at their disposal comes solely from personal experience. Agents are able to assess the value only of those alternatives they have directly experienced - that is chosen at the previous time - and of those other alternatives which are similar to experienced ones, in that they share with the former much of the information for their evaluation. The alternatives which people are unacquainted with are supposed to be discarded from decision problems. The resultant decision rule provides that choice falls upon the maximum in the subset of selectable - or accessible, as often denoted - alternatives. Since any choice which is different from the previous one potentially modifies the set of accessible alternatives by modifying one's own experience, a new maximum is in principle available at any time; the outcome is a path of choices, ending where the new maximum coincides with the old one. Such an alternative is referred to as a local maximum. The described dynamics is then studied with the addition of perturbations, so obtaining a unique prediction of inefficiency. The subsequent enlargement of personal experience by letting people remember sequences of past choices shows to be a tool of selection that, together with the variety of behaviors ensured by perturbations, allows the best alternative to be selected and hence the system to reach efficiency.

Personal experience is clearly a very important channel for the acquisition of information, being however not unique. Observation of others is for instance another main source of information. The literature on information cascades (Bikhchandani et al., 1992, 1998) has analyzed the effects of learning from others in a fully rational framework, producing an explana-

[1] See Boncinelli (2007a) for a discussion about rationality (consistency between aim and means from a decision-maker's point of view) and agents' cognitive capabilities (the specification of this point of view).

[2] More general considerations about the background in which this paper takes root can be found in Boncinelli (2007a).

tion for many puzzling phenomena such as herding. Laboratory experiments (Anderson and Holt, 1996, 1997) have then tested those theoretical predictions with good results, so indirectly providing experimental evidence that people indeed rely on others for their decision processes. A fully rational approach may work rather well for an artificially simplified decision problem, but the requirements of cognitive capabilities become too demanding in a more realistic environment. Therefore the way I suppose people rely on others differs substantially from that of the literature on information cascades.

Some observations are worth considering. First, the introduction of imitation makes population irremovable from any model. If choice is based only on personal experience, then there is no need of others for an individual decision problem. In Boncinelli (2007b) a population is used instead of a single agent just for interpretative reasons. But when imitation is considered the existence of someone else to imitate obviously becomes a necessity. This intrinsically population-based nature of imitation causes some problems for the linearity of the system, thus requiring an adjustment of the state space.

Second, there does not exist a natural way to combine personal experience and imitation unless interpersonal comparisons are admitted. Consider a decision problem as the one is taking shape: once an agent knows that an alternative is better than other ones by personal experience and supposes that another alternative is not so bad since chosen by someone else, then how can she decide among the two? In economic models where imitation is adopted, agents usually imitate i) those who performed the best (Vega-Redondo, 1997), ii) those who performed better than them with a probability increasing in the payoff differential (Schlag, 1998, 1999), iii) whoever with a probabilty increasing in the observed payoff (Schlag, 1999). In any case this presupposes that people can observe others' evaluations of experienced alternatives and compare them. Such an hypothesis may suit circumstances where choices yield monetary payoffs, but it is in general not applicable.[3] In this paper agents are simply assumed to rely at times upon personal experience, by maximizing over what they know, and at other times upon others by copying out their choices.[4] Which are the times when imitation prevails over personal experience is first established probabilistically and then on the basis of a network of social influences when memory is

[3]An empirical analysis has been carried out in Offerman and Sonnemans (1998) where evidence has been found in favor of the hypothesis that people learn both from their own experience and from imitation of others. Moreover, they find that successful individuals are more likely to be imitated. Notice however that in their experiment interpersonal comparisons were possible, while in many real environments they can be precluded or hampered so making hard to establish what has been successful. In such cases it is likely that observed behaviors will be taken into account in a different way.

[4]In the way of modeling imitation this paper is closer to Ellison and Fudenberg (1993), who consider agents' decision rules incorporating popularity weighting, so that more popular choices are more likely to be adopted.

introduced. From a conceptual point of view, there exist some similarities between the transmission of behaviors in the present paper and the diffusion of infections in epidemiology. See Pastor-Satorras and Vespignani (2003) for a survey on epidemiology and networks.

Third, similarly to the remark about the role of population, the hypothesis of identical preferences among agents, even if assumed in Boncinelli (2007b) too, plays here a crucial role unlike when no imitation is considered. A choice by an agent reveals that an alternative is relatively the best, for herself. From that choice an observer gets information about her own preferences only when she knows the connection existing between their preference relations. Here it is assumed that all agents have the same preferences defined over the same set of possible alternatives.

In this paper a first version of the model where personal experience is represented by a single choice, the last one, is followed by a second version where agents remember a sequence of their own past choices. In any of the two cases the analysis is carried out without and with perturbations. The study of the perturbed system, following Foster and Young (1990), Young (1993), Kandori et al. (1993) and Ellison (1993, 2000), allows to select among equilibria when the ultra-long run is considered. The comparison of results in different setups clarifies the contributions of memory, imitation and perturbations and their reciprocal influences.

The paper is organized as follows. Section 1 has been introducing the subject matter and discussing the main hypotheses. Section 2 presents a no-memory version of the model, where agents choose repeatedly over time on the basis both of others' choices and of their own at the previous time. Section 3 enriches the model by allowing agents to store in memory a sequence of their own actions. Section 4 summarizes this contribution and makes some conclusive remarks. Formal proofs of any result throughout the paper are in appendix A.

2 Personal experience and imitation without memory

I begin this section by summarizing the basic model in Boncinelli (2007b) and presenting some modifications in order to accomodate imitation as codetermining factor for individual choice. Then both the unperturbed dynamics and the perturbed one are analyzed. Finally, a variation of the model is briefly considered and a critical note ends this section. A more detailed discussion about the underlying hypotheses and their interpretation is in the introduction.

The model. Let $\mathcal{C}$ be the set of available choices and n its cardinality. Let x be a vector representing how an infinite population is shared out among choices. Let agents' preferences be identical and representable by means of a linear order[5] $\succ$ defined on $\mathcal{C}$. Let A be the matrix of accessibility, that is an adjacency matrix such that $A_{ab} = 1$ if alternative b is accessible from a and $A_{ab} = 0$ otherwise. The notion of accessibility is aimed at representing the role of personal experience; any choice gives an agent some information and allows to evaluate those alternatives which are similar to the experienced one in the sense of sharing most of the information required to be evaluated. Following this reasoning the accessibility relation is assumed to be reflexive, $A_{aa} = 1$ for any $a \in \mathcal{C}$, and symmetric as well, $A_{ab} = 1 \Rightarrow A_{ba} = 1$ for all $a, b \in \mathcal{C}$. On the basis of personal experience, an agent selects the best alternative according to $\succ$ among those she is able to evaluate. Hence a matrix B, called of best accessibility, can be built in such a way that $B_{ab} = 1$ if b is the most preferred alternative among those accessible from a and $B_{ab} = 0$ otherwise. A choice a is called local maximum if $B_{aa} = 1$ and global maximum if it is the maximum according to $\succ$. From here on I refer to the efficient state as that configuration where the entire population is bound to choose the global maximum. In any following discussion efficiency is used as bench mark for the evaluation of results.

Throughout the paper I will use the notion of state with two different meanings, explicitly distinguishing only when necessary to avoid misunderstandings. At times I will refer to a population state, that is a profile of choices for all agents, at other times I will refer to a state in Markovian terminology. In the model under consideration a vector x is a generic population state while $\mathcal{C}$ is the space containing all the states of the Markov chain with (degenerate) transition probabilities B.

Starting from any population state x^0 the evolution according to the dynamic rule $x^{t+1} = x^t B$ has been easily and completely characterized in lemma 1 of Boncinelli (2007b). All the states where population is shared out only among local maxima are equilibria. The initial condition x^0 determines which particular equilibrium will occur.

In the above-described model agents rely only on personal experience to acquire information about the value of alternatives and hence to select the known best. I now suppose that with probability α each agent relies on imitation as source of information and guidance for choice. Let L be a matrix such that L_{ab} represents the probability that an imitating agent, having experienced alternative a at time t, will choose alternative b at time $t+1$. A very simple and useful hypothesis is popularity weighting, i.e. the probability to select an alternative by imitation is proportional to the fraction of population currently choosing that alternative, so being $L_{ab} = x_b$. The new dynamic rule, taking into account both personal experience and

[5] A linear order is a binary relation which is antisymmetric, transitive and total. Antisymmetry implies that agents cannot be indifferent between two distinct alternatives; this simplification allows a simpler analysis without modifying the gist of results, as it should be intuitively understood after reading the paper.

imitation, can be expressed as follows:

$$x^{t+1} = x^t(1-\alpha)B + x^t\alpha L$$

Notice that L is not a matrix of fixed numbers, its elements being variable with the composition of population choices. This is completely natural in a model where imitation is considered. While personal experience depends only on individual choices, imitation is necessarily determined by what others do. The model, as it stands, seems to be non-linear and this represents a severe technical complication. Fortunately, the hypothesis that the probability of imitation is proportional to the current distribution of population choices comes to assistance. Since a generic column b of L has all its elements equal to x_b, the b-th element of the vector x^tL is $x^t_b \sum_a x^t_a$, which is equal to x^t_b because $\sum_a x^t_a = 1$. The dynamic rule, which comes out to be linear, can therefore be re-written as in the following (1), where $B^\alpha \equiv [(1-\alpha)B + \alpha I]$ with I identity matrix.

$$x^{t+1} = x^t B^\alpha \tag{1}$$

By the comparison of B^α with B the effects of imitation are clarified. It may be useful to reflect upon them. If x_a is the mass of population currently choosing alternative a, then at the next time that mass can be thought as split into a fraction $(1-\alpha)$ moving to the best available alternative and a fraction α remaining in alternative a. Actual movements of people among alternatives are more complicated; however, if the concern is not on who does what but on how many do what then the representation may be simplified so to be linear. The starting model of choice based on both personal experience and imitation comes out to be formally equivalent to a model of choice of the known best, due to personal experience, with a component of inertia, whose intensity is measured by the same parameter α that measures the degree of imitation. Notice that the lines of B^α corresponding to local maxima are identical to those in B.

Proposition 1, which is intuitively established by the former considerations, characterizes the behavior of dynamic system (1). Let $\tilde{S}(b)$ be the set of alternatives containing b itself and all the alternatives from which b is reachable by some number of iterations of matrix B. Let $d_a(b)$ denote such number of iterations, with $a \in \tilde{S}(b)$. Finally, let d_a be the number of iterations required to reach a local maximum starting from alternative a, that is the highest number of iterations allowed from a along a path of best accessibility before reaching its end.

Lemma 1 (Equilibria)**.** *Let x^0 be the initial population state.*

1. *i) If $\alpha = 1$, then x^0 is an equilibrium.*

   ii) If $\alpha < 1$, then the system will converge to an equilibrium state $\tilde{x}$ such that for all $a \in \mathcal{C}$,

$$\tilde{x}_a = \begin{cases} 0 & \text{if } a \notin B(a) \\ \sum_{b \in \tilde{S}(a)} x_b^0 & \text{if } a \in B(a) \end{cases} \tag{2}$$

2. *i) If $\alpha = 1$ then no time must be waited for reaching an equilibrium.*

   ii) If $\alpha = 0$ the waiting time for reaching $\tilde{x}$ is $d(x^0) = \max_{x_a^0 > 0} d_a$.

   iii) If $0 < \alpha < 1$ the triangular distance between the population state at time t, with $t \geq \max d_a$, and the onwards equilibrium is so expressed:

$$\sum_{a \in \mathcal{C}} |x_a^t - \tilde{x}_a| = \sum_a x_a^0 \sum_{s=0}^{d_a - 1} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s \tag{3}$$

A main observation derives from the comparison between $\alpha = 0$ and $0 < \alpha < 1$.[6] The introduction of imitation as co-determining factor for individual choice (when $0 < \alpha < 1$) produces the only effect to slow down the process of convergence to equilibrium. More extensively, while in the case without imitation an equilibrium is reached in a finite number of periods - equal to the maximum distance to be covered by some initially existing fraction of population - with the addition of imitation the system tends towards an equilibrium without ever reaching it. Because of imitation inferior choices survive forever, only vanishing in the limit. The greater the parameter α the slower the process of convergence to equilibrium; when $\alpha = 1$ the system is so slow to be motionless and any state is an equilibrium. Notice that from any initial state the equilibrium which the system moves towards is the same with and without imitation. Since the path from any initial state towards its equilibrium is Pareto improving,[7] then the addition of imitation has produced a negative effect from a welfare point of view by slowing down the system.

Perturbations. With the main purpose of selection among possibly infinite equilibria I consider a perturbed version of dynamic system (1). With probability $n\epsilon$ agents make a mistake and in that case any alternative is supposed to be selected with the same probability. With the remaining probability $(1 - n\epsilon)$ agents follow the decision process expressed by B^α. A matrix $B^{\alpha,\epsilon}$ comes out to be defined as follows:

[6] When $\alpha = 0$ imitative behaviors disappear and the system is reduced to the one already characterized in lemma 1 of Boncinelli (2007b).

[7] Actually the path looks Pareto improving because opposite flows of people are compensated. However notice that under this dynamics, unlike the case without imitation, at the next time an agent may be worse off if an inferior behavior is imitated.

$$B_{ab}^{\alpha,\epsilon} = \begin{cases} \epsilon & \text{if } b \neq B(a) \wedge b \neq a \\ \alpha(1-n\epsilon)+\epsilon & \text{if } b \neq B(a) \wedge b = a \\ (1-\alpha)(1-n\epsilon)+\epsilon & \text{if } b = B(a) \wedge b \neq a \\ 1-n\epsilon+\epsilon & \text{if } b = B(a) \wedge b = a \end{cases}$$

Since $x^{t+1} = x^t B^{\alpha,\epsilon}$ is an irreducible and aperiodic Markov chain,[8] then it will converge to the unique solution $\hat{x} = \hat{x}B^{\alpha,\epsilon}$ independently of the initial condition. The behavior of the dynamic system under consideration is very similar to the behavior in case of no imitation, as revealed by comparing proposition 1 here in the following and the analogous proposition 1 in Boncinelli (2007b). Notice that the former is a generalization of the latter, which is obtained by setting α equal to zero.[9] Leaving aside the case when $\alpha = 1$ since of no particular interest,[10] let me restrict my attention to the cases when $\alpha < 1$. As previously dealt, from an overall perspective imitation is equivalent to inertia, hence for $\alpha > 0$ a correction due to the greater stickiness of the system is required in the invariant distribution. In particular, the equilibrium probability of the alternatives which are the best accessible choice from no alternative increases from ϵ to $\epsilon/[1-\alpha(1-n\epsilon)]$. A reduction instead occurs for local maxima since $(1-\alpha)(1-n\epsilon)/[1-\alpha(1-n\epsilon)]$ is less than $(1-n\epsilon)$. The variation of the equilibrium probability for the alternatives which are crossed by a path of best accessibility is ambiguous in general, being positive only when $n\epsilon$ is sufficiently small; this last condition should however be granted by an interpretative argument.

Point 2 of the theorem is got by a simple limit operation of the expressions in point 1 and comes out to be the same with and without imitation (unless $\alpha = 1$). The so-called stochastically stable distribution gives positive probability only to local maxima. In more detail, the ultra-long run frequencies of local maxima are equal to the relative size of their basins of attractions.

At last, point 3 provides a bound to the triangular distance between

[8] A Markov chain is said irreducible when there is a positive probability of moving from any state of the Markov chain to any other state in a finite number of periods, and it is said aperiodic when for every state s unity is the greatest common divisor of the set of all the integers r such that there is a positive probability of moving from s to s in exactly r periods.

[9] The expression here got when $\alpha = 0$ may look different from the one in Boncinelli (2007b), but they are indeed the same since

$$l(a) - \sum_{b \in \tilde{S}(a)} [1-(1-n\epsilon)^{d_b(a)}] = \sum_{b \in \tilde{S}(a)} (1-n\epsilon)^{d_b(a)}$$

with $l(a)$ the cardinality of $\tilde{S}(a)$.

[10] Uniform perturbations simply spread population uniformly over alternatives.

the system after a certain time and the equilibrium distribution.[11] A very simple remark is exploited to get this result: at any time people coming from different alternatives select the same choice with at least probability ϵ.

Proposition 1 (Equilibrium Distribution)**.** *Let $\hat{x}$ be the solution of $\hat{x} = \hat{x}B^{\alpha,\epsilon}$. Then:*

1. *for all $a \in \mathcal{C}$,*

   i) if $\alpha < 1$ then:

$$\hat{x}_a(\epsilon) = \begin{cases} \frac{\epsilon}{1-\alpha(1-n\epsilon)} \left[\sum_{b\in\tilde{S}(a)} \left(\frac{(1-\alpha)(1-n\epsilon)}{1-\alpha(1-n\epsilon)}\right)^{d_b(a)}\right] & \text{if } a \notin B(a) \\ \frac{1}{n} \left[\sum_{b\in\tilde{S}(a)} \left(\frac{(1-\alpha)(1-n\epsilon)}{1-\alpha(1-n\epsilon)}\right)^{d_b(a)}\right] & \text{if } a \in B(a) \end{cases} \tag{4}$$

   ii) if $\alpha = 1$ then:

$$\hat{x}_a(\epsilon) = \frac{1}{n} \tag{5}$$

2. *for all $a \in \mathcal{C}$,*

   i) if $\alpha < 1$ then:

$$\lim_{\epsilon\to 0} \hat{x}_a = \begin{cases} 0 & \text{if } a \notin B(a) \\ \frac{l(a)}{n} & \text{if } a \in B(a) \end{cases} \tag{6}$$

   ii) if $\alpha = 1$ then:

$$\lim_{\epsilon\to 0} \hat{x}_a(\epsilon) = \frac{1}{n} \tag{7}$$

3. *for any initial population state x^0,*

$$\sum_{a\in\mathcal{C}} |x_a^t - \hat{x}_a| \leq (1-n\epsilon)^t \sum_{a\in\mathcal{C}} |x_a^0 - \hat{x}_a| \tag{8}$$

[11] This is exactly the same bound, computed in the same way, as in the corresponding point of proposition 1 in Boncinelli (2007b).

A variation of the model. It may be useful to consider a modification of the previous model which produces simple but rather interesting results. I now suppose that with probability α each agent relies on others not by simply imitating them, but rather by having a conversation and then choosing what is best according to them. Matrix L now represents this learning process. Similarly to what previously done, the probability to meet and conversate with someone whose best alternative is b is assumed to be equal to the fraction of population who has chosen an alternative from which b is the known best; that is, $L_{ab} = xB_b$.

The system appears to be non-linear but, again, luck comes to assistance. Indeed, the b-th element of the vector $x^t L$ is $x^t B_b \sum_a x_a^t$, which is equal to $x^t B_b$ because $\sum_a x_a^t = 1$. Hence the system evolves over time according to the following expression:

$$x^{t+1} = x^t B \tag{9}$$

The dynamic rule in (9) is the same as for the model of choice based on personal experience only. The introduction of conversation allows agents sometimes to rely on others' experience rather than on their own. However, if this happens in a proportional way, then it is as if agents exchanged their experiences, and nothing actually changes if what interests is an aggregate representation of population choices. Therefore, lemma 1 when $\alpha = 0$ provides results about the behavior of the dynamic system in (9) as well. When perturbations are inserted the reference is instead proposition 1.

A critical note. Before entering the issue of memory, I feel the need to consider a possible critique to the above models. According to the story I have told in the introduction, agents require information in order to evaluate alternatives, and they receive information by personal experience and by imitation. Often the information got by personal experience and the information got by others concern sets of alternatives which are disjoint; in those cases it is plausible to suppose that with some probability an agent will rely upon either of the two sources of information. But when these sets have a non-empty intersection, then a more sophisticated mechanism is required for choice. In fact, an agent would reasonably never imitate someone choosing an alternative which she knows to be inferior by her personal experience. However, an attempt to build up a more realistic process of imitation in this setup would necessarily make the system non-linear. In the next section some substantial modifications to the model will be introduced in order to deal with memory and they will have the further effect to allow less restrictive imitation processes.

3 Personal experience and imitation with memory

Concepts are first presented and then formalized in a model. Communication classes are defined with reference to the network of social influences and some known results about them are recalled and subsequently exploited in order to characterize the behavior both of the unperturbed system and of the perturbed one. Some graphical representations are used as expositive device.

Introductive considerations. The effects of the introduction of memory have been analyzed in Boncinelli (2007b) by modifying the state space from choices to sequences of choices of length equal to the length of agents' memory. Now I want to set up a model which, preserving a linear form,[12] allows to deal with memory and imitation. As mentioned in the introduction and underlined in the previous section, non-linearity is naturally linked with imitation. However, when no memory is considered a linear form can be preserved by virtue of compensation phenomena between flows of people coming from different alternatives when appropriate imitation processes are used. No such opportunity is available in the presence of memory, since people keep trace of past choices and hence opposite flows cannot balance each other.

A particular device is usually exploited[13] to preserve linearity, and hence to keep on using Markov chains as representations. The same kind of trick allows me to avoid this difficulty: a state of the Markov chain is no longer defined as an individual description but as a collective description. In other words, the personal experiences of all agents are now required to identify a single state of the system. This representation device, together with the requirement of a finite state space for technical reasons, entails population to be finite (an hypothesis which certainly cannot be accused of unrealism). Such a state representation allows less restrictive assumptions, keeping the dynamics linear.

What remains to be established is the working of the imitation mechanism. Some kind of rule is needed to decide when people rely upon imitation rather than personal experience to solve their decision problems. As first possibility, consider a probabilistic rule similar to the one used in the previous section.

Suppose that every agent has a certain probability to imitate and the complementary probability to choose her known best. Suppose also that the behaviors agents copy out are established probabilistically too, with probabilities based on the existing fractions of behaviors or determined in

[12]Linearity is here to be taken nearly as a synonym of tractability of the model, and therefore it is much sought after.

[13]See for instance Young (1993, 1998).

some other way, and with personal experience in case preventing people from imitating the choices known to be inferior.

A brief reflection reveals that, unless particular degenerate probabilities, such a mechanism always gives positive probability to the survival of only one behavior within a finite time, since almost any conceivable sequence of behaviors based on imitation and personal experience is allowed for each agent. It is straightforward to establish that only the states where the entire population chooses the same local maximum are equilibria under this dynamics. Furthermore, notice that from any state a single perturbation can bring the global maximum into existence; after that, it can spread throughout population by imitation. On the contrary a single perturbation is never sufficient for the system to leave the efficient state because of memory, when personal experience prevents from the imitation of known inferior behaviors. This heuristic reasoning helps to understand that with such an imitation mechanism the stochastically stable distribution gives probability 1 to the efficient state.

However, I argue that a probabilistic imitation process is a questionable choice in this framework from a modeling point of view. In particular, I wonder whether homogeneous states are the only relevant ones when population is very large. Peculiar sequences of occurrences, or an extremely long time, are required for the system to end up in one of those states. If one agrees that a small but significant amount of noise should be actually present in the model, then the probabilities of those sequences of occurrences and of perturbations might be of the same magnitude, or the necessary amount of time sufficiently large for many perturbations to happen. In this case, the stochastically stable distribution would be a very poor prediction, since any probability is bound to be much greater than the probability of a perturbation when the limit for the latter going to zero is computed. Therefore, the choice to use the stochastically stable distribution - since easier to be computed - requires a particular care in establishing transition probabilities in order to obtain a meaningful prediction.

The above argument, together with an intuitive view of how people copy out behaviors from others, takes me to ground the imitation mechanism on a network of social influences. If an agent exerts an influence over another one, then I refer to the former as a model for the latter. An agent always imitates her models unless by personal experience she knows she makes a mistake by doing that, with this last condition in reply to the critical note at the end of the previous section. The choice whether to select the known best or to copy out an observed behavior is hence deterministic. The degree of imitation can be roughly measured by how many models agents have. As extreme cases, in a complete network a behavior can spread to the overall population in just one period, while in an empty network no imitation occurs. It should be remarked, however, that the network environment allows a large variety of imitation structures which cannot be summarized by a single parameter such as the degree of imitation. It is just by exploiting the richness inherent in netwoks that the main results of this paper are obtained.

The model. Let $\mathcal{M}$ be a finite set of players with cardinality m. Let G be a $m \times m$ adjacency matrix such that if $G_{ij} = 1$ then a behavior potentially flows from i to j, that is j is a potential imitator of i. No particular property is imposed on G but irreflexivity, $G_{ii} = 0$ for any agent i. Matrix G represents the existing structure of social influences. Let $(\mathcal{C}, A, \succ)$ be defined as in the previous section. A model is now a quintuple $(\mathcal{C}, A, \succ, \mathcal{M}, G)$.

A state s of the system is defined as a complete description of the sequences of choices - the current one and all the remembered past ones - for the entire population. Notice that now a population state coincide with a state of the Markov chain. The cardinality of the state space is $n^{(k^m)}$. Let z be a vector with $n^{(k^m)}$ components, each of them expressing the probability to be in the corresponding state. The efficient state is the state where the global maximum a holds any position in the sequence of choices of any agent.

Given a certain sequence of choices - or a memory, as sometimes called - define the accessible set as the collection of those alternatives which are accessible from at least one of the choices in the sequence. Formally, even if with some abuse of notation, define $A(b)$ as the set of alternatives accessible from b. Then the accessible set from a memory $(b_1, b_2, \ldots, b_k)$ is $\bigcup_{r=1}^{k} A(b_r)$. Consider the following individual rule of choice. An agent will select the known best alternative, that is the maximum in the accessible set according to $\succ$, unless some of her models is selecting an alternative which she does not know to be inferior. In the latter case the agent will imitate the model.[14]

In other words a choice is imitable for an agent if it is either equal to her known best or uncomparable with it, since outside the accessible set. If the choices of a few of agent i's models are imitable at the same time, then each of them is supposed to have a strictly positive probability to be transmitted. I will sometimes refer to the failure for some behavior to be passed on because of jams in the network of social influences as a congestion problem.

Formally, let $(b_1, \ldots, b_k)$ be i's memory at time t and let b be j's choice at time t. If $G_{ji} = 1$ and either $b = \max_{\succ} \bigcup_{r=1}^{k} A(b_r)$ or $b \notin \bigcup_{r=1}^{k} A(b_r)$, then i's choice at time $t+1$ will surely be determined by imitation and with positive probability it will be b.

The resulting transition matrix P^k is a $n^{(k^m)} \times n^{(k^m)}$ matrix such that for any couple of states (s_1, s_2), $P^k_{s_1 s_2} > 0$ if and only if any sequence of choices in s_2 is obtainable by the sequence in s_1 corresponding to the same agent

[14] In the model without memory the couple $(A, \succ)$ could be summarized by B. Here that is no longer the case, since some pieces of information given by the accessibility matrix A are not contained in B while they are used for establishing whether agents imitate.

by deleting the oldest alternative, moving all the other alternatives one step back in memory and adding as current choice an alternative to which the individual rule of choice assigns positive probability. Obviously $P^k_{s_1 s_2}$ is zero in any other case. The superscript k simply recalls how such matrix depends on the length of memory.

Unperturbed dynamics: analysis and results. I am interested in the following dynamics:

$$z^{t+1} = z^t P^k \tag{10}$$

The system in (10) is a Markov chain. Now I introduce some terminology used in Markov chain theory with the purpose of simplifying the following analysis.

Two states are defined as communicating if there is a positive probability to move from either state to the other one in a finite number of periods. Communication is an equivalence relation and therefore the state space can be partitioned into so-called communication classes. A communication class which once entered is never left is called an ergodic set. If an ergodic set contains only one element, such a state is called absorbing. A simple and well known result is that a Markov chain converges almost surely towards ergodic sets.

States where all agents' memories are full of the same local maximum are examples of absorbing states. The efficient state is therefore absorbing too. These kinds of states would be absorbing in the known best dynamics too and actual imitation phenomena are here excluded by the presence of only one behavior.[15]

The introduction of imitation brings about as distinctive effect the possibility of ergodic sets which are not singletons, as it is shown in figure 1, where three simple structures of social influences are represented as networks. Arrows are pointed from a model towards an influenced agent.

Consider case i). Suppose agent 1 has always been choosing a certain local maximum in the last k periods and agent 2 has always been choosing an inferior local maximum in the last k periods, with k the length of memory. Agent 2 will never change her behavior either by imitation, since she is not influenced by anyone, or by personal experience, since she has always been choosing the same local maximum and will never experience a new alternative. Agent 2 is the unique model for agent 1. Hence agent 1 imitates agent 2 the next time and whenever her personal experience does not prevent her from doing that. After experiencing the other alternative, agent 1 will come back selecting her previous superior choice thanks to memory. In addition

[15]Indeed, according to the working of the imitation mechanism all the agents influenced by models would imitate them, since the observed behavior is not inferior to any known. In this case, however, relying upon imitation instead of personal experience makes no difference.

$$
\begin{tikzcd}
1 & 2 \arrow[l]
\end{tikzcd}
\qquad
\begin{tikzcd}
1 \arrow[r] & 2 \arrow[d] \\
 & 3 \arrow[ul]
\end{tikzcd}
\qquad
\begin{tikzcd}
1 \arrow[r] & 2 \\
 & 3 \arrow[u]
\end{tikzcd}
$$

i) ii) iii)

Figure 1: Non-singleton ergodic sets.

memory allows her not to copy out her bad model until the unpleasant experience is consigned to oblivion. Summing up, agent 2 always chooses the same alternative while agent 1 cyclically selects for k periods her superior choice and then imitates her model once.

Consider case ii). Suppose the length of memory is two. Let memories be (a, a), (a, b) and (b, a) for agents 1, 2 and 3 respectively, with a a local maximum and b an inferior alternative, not necessarily a local maximum, such that $a \succ B(b)$. By convention, from here on more recent choices are represented more on the left in memory. The next time choice b flows from agent 3 to agent 1 and sequences in memory are so modified: (b, a), (a, a) and (a, b) for agents 1, 2 and 3 respectively. By an adequate relabeling of agents, each of them playing the same role in the structure of social influences, the initial state is found again, so determining the everlasting movement of the inferior choice b along the cycle of social influences. Notice that even if b is not a local maximum, it is not substituted by the best accessible alternative $B(b)$ since $a \succ B(b)$. Notice also that if the length of memory was greater than two, then no cycle would be possible.

Cycles are particular instances of non-singleton ergodic sets. Case iii) illustrates another possibility. Suppose the sequence in memory for agent 1 contains only local maximum a while the sequence in memory for agent 3 contains only local maximum b, with $a \succ b$. Obviously they keep on selecting the same local maximum forever, no social influence existing over them. Finally suppose that agent 2's memory has alternative a repeated for $k-1$ times and alernative b in the more recent position. Since $a \succ b$, memory prevents agent 2 from choosing b again for k periods. Hence alternative b gradually moves into the past for agent 2 till it is forgotten and only alternative a is in memory. The next period both choices have a chance to be copied out by agent 2. If b prevails, the above-explained k steps follow again. If a is transmitted, exactly the same state is found again with only a in agent 2's memory. After sufficiently long time the system can be in any of the $k+1$ described states and hence no cyclic movement is here present.

The concept of communication classes has been previously mentioned when introducing Markov chains. Now it is more deeply treated in application to the network of social influences. If $G_{ij}=1$ agent i has a direct social influence over agent j, as already defined.

Consider the notion of indirect social influence. If there exists a positive integer $r \geq 2$ such that $G^r_{ij}=c>0$, then agent i has an indirect influence over agent j through c sequences of intermediate agents of length $r-1$. Consider now the following relation: there exists a non-negative integer r such that $G^r_{ij}>0$. Here agent i can contact agent j directly or indirectly, or $i=j$ since $G^0_{ii}=1$. This relation is a preorder, since reflexive and transitive, and it can be used to classify and partially order the set of agents. The induced equivalence classes, called communication classes, are such that two distinct agents belong to the same class when they can exert an influence (directly or indirectly) over each other. Define a communication class as maximal when it is not influenced by any other class. Obviously, a plurality of maximal communication classes may exist.

Lemmata 2 and 3 have the main purpose to work as basic findings for proving all the results in the rest of the paper. Lemma 2 provides some standard results about communication classes.

Lemma 2 (Communication Classes)**.** *Let i and j belong to whatever communication class C. Let T_{ij} be the set of positive integers t such that agent i has a direct or indirect social influence of length t over agent j, $T_{ij} \equiv \{t \geq 1 : G^t_{ij} > 0\}$. Let p_i be the greatest common divisor of T_{ii}. Then,*

i) $p_i = p_j \equiv p$,

ii) communication class C can be partitioned into p cyclic classes indexed by $0, 1, \ldots, p-1$ such that if $j \in$ cyclic class u and $t \in T_{jr}$ then $r \in$ cyclic class $(u+t) \bmod p$,

iii) there exists $\hat{t}$ such that if i belongs to cyclic class u then for all $t \geq \hat{t}$, $t \in T_{il}$ for all l belonging to cyclic class $(u+t) \bmod p$.

In other words, after sufficiently long time a behavior may be taken by means of imitation from an agent to any other agent in the appropriate cyclic class; obviously, congestion problems and memory could prevent this from happening. I refer to p as the period of a communication class.

Lemma 3 provides straightforward results which, left aside the preventing effects of memory which should be excluded for each specific case, establish the positive probability of chains of imitations simply by exploiting the hypothesis that every imitable behavior always has a chance to spread from model to imitator.

Lemma 3 (Behavior Propagation)**.** *Suppose that agent* 1 *selects alternative* a *at time* t_1*, agent* 2 *selects alternative* a *at time* t_2*, ..., agent* l *selects alternative* a *at time* t_l*. Let* $\tilde{t} = \max\{t_1, t_2, \dots, t_l\}$*. Suppose also that memory never prevents alternative* a *from spreading.*

i) Then for all $t > \tilde{t}$*, there is a positive probability that every agent* j *such that* $G_{ij}^{t-t_i} > 0$ *for some agent* $i = 1, \dots, l$ *selects alternative* a *at time* t*.*

ii) Suppose also that $t_1 = t_2 = \ldots = t_l = \tilde{t}$ *and* $1, 2, \dots, l$ *are all the agents in cyclic class* u *of communication class* C*. Suppose that either* C *is maximal or all the agents of any communication class directly or indirectly influencing* C *choose alternative* a *forever. Then for all* $t > \tilde{t}$ *every agent* j *belonging to cyclic class* $(u + t - \tilde{t}) \bmod p$ *of* C *selects alternative* a *at time* t*.*

It is worth stressing that what happens in maximal communication classes affects the whole system. On the contrary, since a maximal communication class is not influenced by anything outside, then the Markov chain in (10) can be restricted to such a communication class keeping on to be a Markov chain. When I write about absorbing states or ergodic sets for maximal communication classes I refer to these restricted Markov chains. The following lemma states precisely an intuitive kind of influence which maximal communication classes have on the behavior of the system.

Lemma 4 (Maximal Communication Classes)**.** *Suppose that in every maximal communication class of* G *the global maximum is selected by all agents. Then the system will converge almost surely to the efficient state.*

Lemma 4 tells that, if the concern is on the achievement of the efficient state, then attention can be restricted to maximal communication classes. Lemma 5 helps in establishing what happens in the latter. A cycle of social influences is defined as a sequence of distinct agents $1, 2, \dots, s$ such that $G_{ii+1} = 1$, $1 \leq i < s$, and $G_{s1} = 1$.

Lemma 5 (Efficiency)**.** *Let* C *be a maximal communication class of* G*,* p *its period,* k *the length of memory,* a *the global maximum.*

i) if $k > n - ||A(a)||$*, and there exists an agent in* C *with alternative* a *as current or remembered choice and there does not exist in* C *a cycle of length greater than* k*, then efficiency in* C *is reached almost surely;*

ii) if there exists a cyclic class of C *with all the agents therein simultaneously choosing alternative* a *and there does not exist in* C *a cycle of length greater than* k*, then efficiency in* C *is reached in a finite time.*

A possible cause of inefficiency is the lack of accessibility of the global maximum; that happens if any one is not currently choosing it and neither an alternative in its basin of attraction. In such a case the global maximum cannot be discovered. Furthermore, even when the global maximum is chosen by someone, its survival is not guaranteed. Imitation can help it to spread to people unable to find it out by their own experience. However, imitation is also a possible source of extinction for the global maximum, helping the spreading of other alternatives too. Memory seems to favour the survival of the global maximum: when an unknown inferior alternative is copied out, memory allows the recall of the previous superior alternative and hence its recovery. But if unknown inferior alternatives are copied out uninterruptedly for a sufficiently long time, then the global maximum is consigned to oblivion. How many periods a sufficiently long time consists of is obviously determined by the length of memory in conjunction with the structure of accessibility.

Two simple observations are in order here. On one side imitation of unknown alternatives can induce people to give up the global maximum. On the other side congestion problems in transmission by imitation can hinder its diffusion.

The former observation is at the basis of the first sufficient condition in lemma 5 for the prevailing of efficiency in a maximal communication class. Once the global maximum is experienced, then it cannot be forgotten if sufficiently many alternatives do not exist. Whenever chosen the global maximum can spread to others with positive probability, since never prevented by memory, and then it will sooner or later be experienced, and hence never forgotten, by all agents.

The second sufficient condition in lemma 5 is based on the second observation. When all the agents of a certain cyclic class u choose at the same time t the global maximum, then no room is left for its extinction by virtue of point ii in lemma 3. To illustrate the gist of the reasoning, no congestion problems are possible since all the agents of cyclic class $u + 1$ are influenced only by agents in cyclic class u, and memory cannot prevent the spreading of the best alternative. Then the entire cyclic class $u + 1$ will select the global maximum at time $t + 1$. The same reasoning can be applied forever, so establishing that the best alternative can never disappear.

Once the survival of the global maximum is somehow guaranteed, still the possibility of inefficient configurations remains for a maximal communication class. Consider case ii) in figure 1 and suppose a is the global maximum. There exist three cyclic classes, each consisting of a single agent. Two of the agents have their memories full of the global maximum, so ensuring its everlasting survival. However the inferior alternative b survives forever too, hence keeping the system away from efficiency. One may think to rule out this possibility simply requiring $k \geq p$. However the following example in figure 2 shows this is not the case, suggesting a more complicated relation between the length of memory, the period of a maximal communication class and possible inefficiencies.[16]

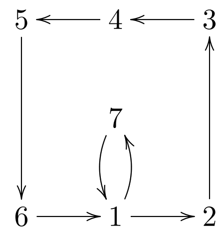

Figure 2: Room for inefficiencies.

The period of the communication class in figure 2 is clearly 2. Suppose a, b_1, a, b_2, a, b_3, b_1 are the current choices for agents 1, 2, 3, 4, 5, 6 and 7 respectively, with a the global maximum. Suppose also that no alternative is accessible from any other alternative. Finally let the length of memory be 3 and let the sequences in memory be (a, b_1, a), (b_1, a, b_2), (a, b_2, a), (b_2, a, b_3), (a, b_3, a), (b_3, a, b_1) and (b_1, a, b_2) for agents 1, 2, 3, 4, 5, 6 and 7 respectively. The next time the system will be characterized by the sequences (b_3, a, b_1), (a, b_1, a), (b_1, a, b_2), (a, b_2, a), (b_2, a, b_3), (a, b_3, a) and (a, b_1, a), in the same order as before. Another application of the dynamic rule transforms the sequences in (a, b_2, a), (b_2, a, b_3), (a, b_3, a), (b_3, a, b_1), (a, b_1, a), (b_1, a, b_2) and (b_2, a, b_3) respectively. By a simple relabeling of alternatives b_1, b_2 and b_3, which play the same role in this dynamic mechanism, the initial state is found again, so showing the existence of an endless cycle.

In both sufficient conditions of lemma 5 the survival of inferior alternatives is prevented by a rather strong assumption, based on the following observation: the existence of a cycle of social influences of length greater than the length of memory is a necessary condition for an inferior behavior to flee forever once some superior alternative is known. In fact, being the number of agents finite, a behavior necessarily has to come back to people having already experienced it. If no sufficiently long cycle exists, then agents will be prevented from imitating such a behavior by their memory.[17] A class of networks which naturally lends itself to satisfy this condition of shortness

[16] I think this kind of relation could be studied in deeper details in the attempt to characterize it precisely. However I guess the gain would not be worth the effort, at least if seen from a strictly economic point of view.

[17] In the attempt to establish an analogy, it may be useful to mention the two main categories of models in epidemiology: i) the SIR models, where an agent passes from being susceptible (S), to turning infected (I), to finally being recovered (R) and immune from other infections; ii) the SIS models, where an agents passes from being susceptible

of cycles is represented by *flowers*, where a central agent works as corolla, while the other agents form petals. An example is given in figure 3. Notice that petals can have different dimensions, that is different lengths of their respective cycles, and some Siamese petals can exist sharing an edge.

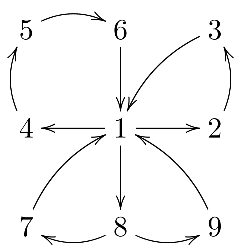

Figure 3: A flower.

By combining lemma 4 and lemma 5 some conclusions are drawn. Without imitation the network of social influences is empty and there are n maximal communication classes, each formed by a single isolated agent. In order to reach efficiency in such a case, for every agent the best accessible alternative from the initial sequence in memory must belong to the basin of attraction of the global maximum. In the presence of a structure of social influences lemma 4 allows to restrict attention for what concerns efficiency to a limited number of agents, those belonging to maximal communication classes. What happens in each maximal communication class can be extremely complicated. Each local maximum can prevail or cycles with non-singleton ergodic sets can emerge, that depending upon the initial condition and upon the network of social relationships. Lemma 5 provides a couple of intuitive sufficient conditions for reaching efficiency. This large variety of possible results is anyway unsatisfactory and calls for some selection of predictions, which will be provided in the next paragraph.

Perturbed dynamics: analysis and results. Results in the previous section help in determining what can happen, but they do not provide a unique prediction. The introduction of perturbations, sometimes called mu-

(S), to turning infected (I), to becoming again susceptible (S). In my model the diffusion mechanism of inferior choices through imitation falls in between the above-mentioned categories, so to deserve the SIRS label: an agent passes from being susceptible (S) to copy out an inferior choice when that choice is not prevented by memory, to turning infected (I) through its actual adoption, to being recovered (R) and temporarily immune from the same mistake, to finally becoming again susceptible (S) when the inferior choice is forgotten.

tations to remember the biological origins of this kind of models, allows to solve this drawback.

As a consequence of the addition of perturbations, each agent with probability $(1 - n\epsilon)$ behaves as in the unperturbed case, while with the remaining $n\epsilon$ probability she chooses a randomly selected alternative. The resultant transition matrix is denoted by $P^{k,\epsilon}$. The system into analysis is therefore characterized by the following rule of evolution:

$$z^{t+1} = z^t P^{k,\epsilon} \tag{11}$$

Such a system is an irreducible and aperiodic Markov chain and hence it converges to the unique $\hat{z}$ such that $\hat{z} = \hat{z}P^{k,\epsilon}$, called invariant distribution. The stochastically stable distribution is the limit of the invariant distribution as the probability of perturbations goes to zero. Some useful techniques[18] are availbale for determining which states belong to the support of this limiting distribution. Basically, only the states easiest to reach receive a positive probability in the stochastically stable distribution, with easiest interpreted as requiring the fewest perturbations, since as $\epsilon \to 0$ perturbations become much rarer than any other event. Propositions 6 and 7 establish how easy in terms of perturbations is to reach the efficient state and to leave it respectively. Combining these findings with known results proposition 2 is found.

Lemma 6 (Towards Efficiency)**.** *Consider the dynamics in (11) restricted to a maximal communication class C of G. Let p be the period of C. From any state s there exists a sequence of ergodic sets of the unperturbed dynamics $E_1, E_2, \ldots, E_p$, with E_p the efficient state, such that a single perturbation allows to go with positive probability from s to E_1 and if $p > 1$ from E_i to E_{i+1} for any $1 \leq i < p$.*

An intuitive explanation of lemma 6 is here provided. Let a be the global maximum. A single perturbation can induce an agent to choose a and then the imitation mechanism can spread that alternative so to be chosen by all the agents of a cyclic class at a same future time. If that happens an ergodic set of the unperturbed dynamics has been reached since all the agents of every cyclic class will keep on choosing the global maximum at least with period p. Analogously, another single perturbation can induce a cycle with a different timing. A sequence of no more than p steps is so built, the last ergodic set being the efficient state, having p different cycles of alternative a in motion.

Lemma 7 (Away from Efficiency)**.** *Consider the dynamics in (11) restricted to a maximal communication class C of G. Let p be the period of C, k the length of memory, a the global maximum.*

[18]Standard references are Young (1993), Kandori et al. (1993) and Ellison (2000).

i) Suppose $k = 1$ and $p = 1$. Then a single perturbation can move the system from the efficient state to another ergodic set if and only if there exists a local maximum different from a.

ii) Suppose $k > 1$ or $p > 1$ or both. Then a single perturbation can move the system from the efficient state to another ergodic set if and only if $A(a) \neq \mathcal{C}$ and $k < p$.

I now discuss lemma 7. Suppose to be in the efficient state. When $p = 1$ a single behavior, in case introduced by a single perturbation, eventually spreads to all agents at the same time, with $k = 1$ only requiring the prevailing behavior to be a local maximum. Consider the case when $k > 1$ and $p > 1$. A single perturbation can induce an agent to choose another alternative, which obviously must be outside a's accessible set to have some chance to survive. No other alternative can rise in the known best dynamics. In fact, when $k > 1$ the global maximum is never forgotten. When instead $p > 1$ the existence of more cyclic classes implies that either each potential imitator chooses a or all her models choose a; therefore a is the known best in the former case and the unique imitable choice in the latter case. As a consequence, the only possibility except coming back to the efficient state is an endless survival of the inferior alternative arisen from the perturbation. However, when $k \geq p$ the inferior alternative ends up coming back when it is still in memory, so determining its rejection by memory and hence its extinction. When $k < p$ with positive probability all the agents of a cyclic class end up choosing the inferior behavior at the same time, so generating an everlasting cycle by point *ii* of lemma 3.

Proposition 2 (Equilibrium Distribution with Memory)**.** *Consider the dynamics in (11). Let p_C be the period of the generic maximal communication class C and k the length of memory. Let E be the union of ergodic sets which have positive probability in the stochastically stable distribution. Let $W(s, E, \epsilon)$ be the expected wait until a state belonging to the set E is first reached when the system starts in state s. Then:*

1. *E contains only the efficient state if and only if in all the maximal communication classes of G one of the following conditions is satisfied:*

   i) $k = 1$ and $p = 1$ and there does not exist a local maximum different from the global one.

   ii) $A(a) = \mathcal{C}$ or $k \geq p$.

2. *For any state $s \notin E$, $W(s, E, \epsilon) = O(1/\epsilon)$ as $\epsilon \to 0$.*

By the application of lemmata 6 and 7 to standard mutations counting techniques a unique prediction for the system in (11) is obtained in point 1 of proposition 2. Suppose that agents have actual memory ($k > 1$) and that the global maximum does not allow a full accessibility ($A(a) \neq \mathcal{C}$). Then the period of every maximal communication class being not greater than the length of memory is a necessary and sufficient condition for the system to spend approximately all its time in the efficient state for a vanishing amount of noise. Therefore, how imitation influences population welfare is not a matter of intensity; as extreme cases, both the complete network and the empty one satisfy the condition for efficiency. Imitation works against efficiency only when a particular cyclic structure exists in at least one maximal communication class of the social network. When this kind of ciclicity is present, a single perturbation can move the system both towards and away from the efficient state. Notice that an increase in the length of memory may push the system towards efficiency, since making the inequality $k \geq p$ easier to be satisfied.

Point 2 of proposition 2 tries to establish a bound on the time of convergence. This kind of bounds are always useful when working with the stochastically stable distribution since a very long time may be required for its prediction to become relevant. By following Ellison (1993), let $W(s, E, \epsilon)$ denote the expected wait until a state belonging to the set E is first reached when the system starts in state s; its behavior as ϵ tends to zero is studied. By applying lemma 6 to theorem 2 in Ellison (1993), it is found that $W(s, E, \epsilon) = O(1/\epsilon)$ as $\epsilon \to 0$, which is a shorthand for there exist C, $\bar{\epsilon} > 0$ such that $W(s, E, \epsilon) < C(1/\epsilon)$ for all $\epsilon \in (0, \bar{\epsilon})$.

As an ilustration of the working of proposition 2 consider the following figure.

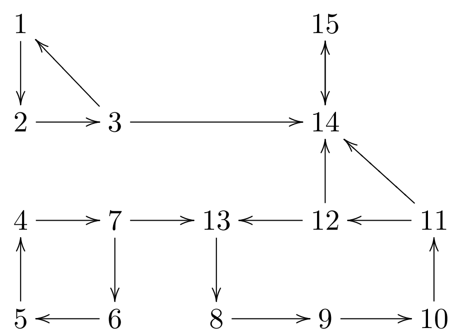

Figure 4: Efficient or not?.

In figure 4 there exist four communication classes: $\{1, 2, 3\}$, $\{4, 5, 6, 7\}$, $\{8, 9, 10, 11, 12, 13\}$, $\{14, 15\}$. Only the first two of them are maximal. Assume whatever set $\mathcal{C}$ of available choices and whatever preference relation on it, with the only condition that $A(a) \neq \mathcal{C}$ with a the global maximum. Suppose $k = 2$. Will the system reach efficiency in the very long run? No, since the period of both the maximal communication classes is larger than 2. Suppose now $k = 3$. Again inefficiencies will survive in the stochastically stable distribution because the period in the second maximal communication class is still larger than the length of memory. Efficiency will be granted if $k \geq 4$. But how can efficiency be got with $k = 3$? Two ways are available: i) reducing the period of the second maximal communication class by the addition of another link among them; ii) making that set of players no more a maximal communication class, by the removal of a link among them of by the addition of a link from an outer agent towards one of them. I conclude with a final remark. Consider figure 4 with $k = 4$. We know that the system will reach efficiency, however this result can be considered not very robust in the following sense: the removal of a single link - the one from 7 to 13 - is sufficient to allow inefficiencies. Notice that for $k \geq 6$ this weakness would be taken away.

4 Conclusions

Where do personal experience and imitation drive choice? This question is investigated by two main models. A common setup provides a population of agents choosing repeatedly over time within a finite set of alternatives. Personal experience allows the ordering in terms of preference of only some of the alternatives for lack of information required for evaluation. Agents at times choose their known best, at other times they copy out a behavior they observe.

In the first model agents' personal experience is assumed to be formed by their last choice, population is infinite and the criterion used to establish when relying upon personal experience and when upon observed behaviors is probabilistic. Moreover, if behaviors are copied out proportionally to their diffusion, then a linear form for the model is preserved in spite of the non-linear nature of imitation. In short, in such a context imitation is formally equivalent to inertia and has the unique effect to slow down the system with negative consequences for population welfare.

A slight variation is also considered, where imitating people do not copy out an observed behavior but select the best alternative accessible from that observed behavior (as if they had conversated). In this case imitation has no effect.

In both versions the analysis when perturbations are added provides a prediction of inefficiency.

In the second model agents are endowed with memory, which allows them to remember a sequence of past choices. Having a larger personal experience they are allowed to rank in preference more alternatives. However this enlarged personal experience brings about major problems of non-linearity that impose a change in the state space of the Markov chain, which in turn requires a finite population. Moreover, a remark about the risks of using the stochastically stable distribution as an approximated prediction for a system with a small but finite amount of noise suggests imitation to be modeled in a non-probabilistic way when population is finite. In particular, a directed network of social influences is used to describe channels through which behaviors can flow by imitation. An agent imitates some of the people she is influenced by whenever memory does not prevent her from doing that.

In the unperturbed dynamics many different outcomes are possible, among which cycles and other non-singleton predictions. The main results concern the fundamental role played by maximal communication classes of the network of social influences, and the identification of sufficient conditions for reaching efficiency.

When perturbations are added the stochastically stable distribution is used to get a unique prediction. The existence of a maximal communication class with a period longer than the length of memory represents the main source of inefficiency for the system.

Some conclusive speculative discussion follows. Imitation affects the dynamics of the system in a non trivial way. The evolution driven by only personal experience has the limit to be trapped in local maxima. The addition of perturbations brings variety, while the possibility for agents to store their personal experience in memory makes room for comparison and hence selection.[19] In an unperturbed dynamics, the inclusion of imitation as guidance for choice can play to some extent the same role of perturbations as source of variety. The limits of imitation compared to perturbations concern the incapacity to create inexistent behaviors, being only capable to spread existing ones, and its non-vanishing frequency of occurrence. The latter feature may keep on producing novelties from the point of view of single agents, so allowing the survival of inferior behaviors in particular cases even when perturbations are added.

More extensively, imitation has two constrasting effects on an inferior choice. From one side, imitation allows that choice to be copied out and hence to survive the selection by one's own memory. From the other side, the larger the diffusion of the choice the lower the number of next possible imitators, since personal experience makes agents immune from making the same mistake again for a number of periods equal to the length of memory. An increase in the length of memory has the consequence to strengthen the latter effect so making conditions for efficiency more easily satisfied.

My final observation is aimed at stressing the importance of the interaction structure, especially from the point of view of a public authority aiming

[19] This result is the main finding in Boncinelli (2007b).

at promoting efficiency: a policy of public intervention may take advantage of the existence of a network of social influences by concentrating its efforts on maximal communication classes or, if possible, properly modifying the structure of influences.

A Appendix

Proof of lemma 1. Point 1.i is immediately established since $B^\alpha = I$ when $\alpha = 1$.

By lemma 1 of Boncinelli (2007b) it is known that an equilibrium of dynamics B is reached within a finite time, $d(x^0)$. Therefore $B^t \equiv \hat{B}$ for $t \geq d(x^0)$. I now prove that if $\alpha < 1$, then

$$\lim_{t\to\infty} (B^\alpha)^t = \lim_{t\to\infty} B^t$$

Notice that

$$\begin{aligned}
(B^\alpha)^t &= \sum_{s=0}^{t} \binom{t}{s} \alpha^{t-s} I^{t-s} (1-\alpha)^s B^s = \\
&= \alpha^t I + \sum_{s=1}^{d(x^0)-1} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s B^s + \sum_{s=d(x^0)}^{t} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s \hat{B} =
\end{aligned}$$

$$\begin{aligned}
= \; & \alpha^t I + \sum_{s=1}^{d(x^0)-1} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s B^s + \\
& + \left(1 - \sum_{s=0}^{d(x^0)-1} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s \right) \hat{B}
\end{aligned}$$

As $t \to \infty$ in the last expression $\alpha^t I$ clearly tends to the null matrix, the second addend tends to the null matrix too since α^t dominates over $\binom{t}{s}$ for each term of the summatory, and the coefficient of $\hat{B}$ in the third addend tends to 1 for the just explained reasons. Therefore, $\lim_{t\to\infty} (B^\alpha)^t = \hat{B}$. By lemma 1 in Boncinelli (2007b) point 1.ii follows.

Point 2.i is trivial from previous point 1.i. Point 2.ii is point 2 of lemma 1 in Boncinelli (2007b). Consider point 2.iii. If $\alpha < 1$, starting from alternative a after d_a repetitions of the known best dynamics a local maximum is reached and never left. Therefore, after t periods the probability not to have reached the ownwards local maximum is given by the probability that less than d_a times matrix B has been applied. If $t \geq d_a$ and $\alpha > 0$ this probability is equal to $\sum_{s=0}^{d_a-1} \binom{t}{s} \alpha^{t-s} (1-\alpha)^s$. Considering how population is initially shared out among alternatives and how the triangular distance is computed the desired result is got. $\square$

Proof of proposition 1. I first prove point 1.*i*. Suppose $\alpha = 1$. Any alternative a can be reached from any alternative by perturbation and from a by inertia. Therefore in the invariant distribution,

$$\hat{x}_a = \sum_{b \in \mathcal{C}} \hat{x}_b \epsilon + (1 - n\epsilon)\hat{x}_a = \epsilon + (1 - n\epsilon)\hat{x}_a = \frac{1}{n}$$

The proof of point 1.*ii* is by induction. Suppose $\alpha < 1$. Notice that if an alternative a is the best accessible choice from no alternative, then $\hat{x}_a = \epsilon/[1-\alpha(n-\epsilon)]$, in accordance with (4). Any $\hat{x}_a$ relative to an alternative a which is not a local maximum can be expressed as follows:

$$\begin{aligned} \hat{x}_a &= \epsilon + \sum_{\substack{c:B(c)=a,\\ c \neq a}} (1-\alpha)(1-n\epsilon)\hat{x}_c + \alpha(1-n\epsilon)\hat{x}_a = \\ &= \frac{\epsilon + \sum_{\substack{c:B(c)=a,\\ c \neq a}} (1-\alpha)(1-n\epsilon)\hat{x}_c}{1-\alpha(1-n\epsilon)} \end{aligned} \tag{12}$$

Any $\hat{x}_a$ relative to a local maximum a can be expressed as follows:

$$\begin{aligned} \hat{x}_a &= \epsilon + \sum_{\substack{c:B(c)=a,\\ c \neq a}} (1-\alpha)(1-n\epsilon)\hat{x}_c + (1-n\epsilon)\hat{x}_a = \\ &= \frac{\epsilon + \sum_{\substack{c:B(c)=a,\\ c \neq a}} (1-\alpha)(1-n\epsilon)\hat{x}_c}{n\epsilon} \end{aligned} \tag{13}$$

Suppose that (4) holds for all the alternatives from which a is the best accessible choice. Then 12 can be rewritten as:

$$\hat{x}_a = \frac{\epsilon}{1-\alpha(1-n\epsilon)} \left[1 + \sum_{\substack{c:B(c)=a,\\ c \neq a}} \sum_{b \in \tilde{S}(c)} \left[\frac{(1-\alpha)(1-n\epsilon)}{1-\alpha(1-n\epsilon)} \right]^{d_b(c)+1} \right] \tag{14}$$

and (13) can be rewritten as:

$$\hat{x}_a = \frac{\epsilon}{n\epsilon} \left[1 + \sum_{\substack{c:B(c)=a,\\ c \neq a}} \sum_{b \in \tilde{S}(c)} \left[\frac{(1-\alpha)(1-n\epsilon)}{1-\alpha(1-n\epsilon)} \right]^{d_b(c)+1} \right] \tag{15}$$

Consider the following relations, which allow to transform (14) and (15) into the desidered parts of (4).

i) $d_b(c) + 1 = d_b(a)$ if $c \neq a$, $a = B(c)$ and $b \in \tilde{S}(c)$

ii) $\tilde{S}(a) = \left(\bigcup_{\substack{c:B(c)=a, \\ c \neq a}} \tilde{S}(c) \right) \cup \{a\}$

iii) $\left[\frac{(1-\alpha)(1-n\epsilon)}{1-\alpha(1-n\epsilon)} \right]^{d_a(a)} = 1$

Point 2 of the proposition is easily established by taking the limit of the previously found expressions as ϵ tends to zero.

Finally, as regards the bound to the rate of convergence in (8), it is got as in Boncinelli (2007b), to which I refer for the proof.

Proof of lemma 2. Each T_{ij} is non-empty since C is a communication class. Moreover, being each T_{ii} closed under addition, it contains infinite positive integers, of which the greatest common divisor is by hypothesis p_i. A finite subset of T_{ii} with p_i as greatest common divisor must exist. In fact, take two numbers in T_{ii} with $\tilde{p}$ their greatest common divisor. If all the other numbers in T_{ii} can be divided by $\tilde{p}$, then a finite sequence has already been found. If there exists a number which is not divisible by $\tilde{p}$, that number is added to the first two and their greatest common divisor is computed. The same reasoning applies, but since $\tilde{p}$ is finite it cannot be reduced too many times without reaching its lowest conceivable value of 1, so proving the initial assertion. Let n_1, n_2, …, n_r denote the elements of this finite subset.

I prove point i). Take two agents in C, say i and j. Let m_1 be in T_{ji} and m_2 be in T_{ij}. Obviously, $(m_1 + m_2)$ belongs to T_{jj}. Furthermore, $(m_1 + n_s + m_2)$ with $s = 1, \ldots, r$ belongs to T_{jj} too. Suppose $p_j > p_i$. Since $(m_1 + m_2)$ and any $(m_1 + n_s + m_2)$ are divisible by p_j, it follows that each n_s is divisible by p_j, against the hypothesis that p_i is the greatest common divisor of n_1, n_2, …, n_r. Therefore $p_j \leq p_i$. With an analogous reasoning $p_i \leq p_j$ is got, so implying $p_i = p_j$.

I now prove point ii). Take two agents in C, say i and j. Let m_1, m_2 be in T_{ij} and m_3 be in T_{ji}. Since both $m_1 + m_3$ and $m_2 + m_3$ belong to T_{ii} and are hence divisible by p_i, then $m_1 \equiv m_2 \pmod{p}$. Let t_{ij} be so defined, $t_{ij} \equiv m_1 \pmod{p}$, and define similar numbers for any other agent j. Then population turns out to be partioned into classes $0, 1, \ldots, p-1$, where j belongs to class u if $t_{ij} = u$. Now take any j belonging to class u and take any $\hat{t} \in T_{ij}$. For any $\tilde{t} \in T_{jr}$, since $(\hat{t} + \tilde{t}) \in T_{ir}$, it must be $t_{ir} = (\hat{t} + \tilde{t}) \bmod p = (u + \tilde{t}) \bmod p$, so proving what desired.

Finally I prove point iii). All the following are positive integers. Suppose b_1 and b_2 have 1 as greatest common divisor. Since they have completely different factorizations, $\alpha b_1 \equiv 0 \pmod{b_2} \Leftrightarrow \alpha = \beta b_2$. Because $\alpha b_1 \bmod b_2$

is different from zero for $\alpha = 1, \ldots, b_2 - 1$, there do not exist α_1 and α_2 such that $\alpha_1 < \alpha_2 < b_2$ and $\alpha_1 b_1 \equiv \alpha_2 b_1 \pmod{b_2}$, otherwise being $(\alpha_2 - \alpha_1) b_1 \equiv 0 \pmod{b_2}$. This implies that there exists α_{11} such that $\alpha_{11} b_1 \equiv 1 \pmod{b_2}$, or in other words there exist α_{11}, α_{12} such that $\alpha_{11} b_1 - \alpha_{12} b_2 = 1$. Now take n_1 and n_2. Let a_2 denote their greatest common divisor. There must exist b_1 and b_2 such that $n_1 = b_1 a_2$ and $n_2 = b_2 a_2$, with 1 greatest common divisor for b_1 and b_2. Hence, $\alpha_{11} n_1 - \alpha_{12} n_2 = a_2$. Now consider n_3. It is obvious that the greatest common divisor of n_1, n_2 and n_3 is equal to the greatest common divisor of a_2 and n_3. Call it a_3. With analogous resoning, there exist α_{21} and α_{22} such that $\alpha_{21} a_2 - \alpha_{22} n_3 = a_3$. So proceeding, it is finally got $\alpha_{r-11} a_{r-1} - \alpha_{r-12} n_r = a_r = p$. Define $\tilde{n} = n_1 \prod_{i=1}^{r-1} \alpha_{i1}$ and $\hat{n} = n_r \alpha_{r-12} + \sum_{i=1}^{r-2} n_{i+1} \alpha_{i2} \prod_{j=i+1}^{r-1} \alpha_{j1}$. Therefore, $\tilde{n} - \hat{n} = p$. Notice that $\tilde{n}$ and $\hat{n}$ belong to T_{ii} since it is closed under addition. Take $t = a\hat{n} + bp$, with $a \geq \hat{n} - 1$ and $0 \leq b \leq \hat{n} - 1$. Therefore, it can be written $t = (a - b)\hat{n} + b\tilde{n}$, so proving that any $t = \hat{n}(\hat{n} - 1) + cp$ belongs to T_{ii}. Define $\tilde{t_i} = \hat{n}(\hat{n} - 1)$. Define $\tilde{t}$ as the maximum over i of $\tilde{t_i}$, existing for the finiteness of the set of agents. Let i belong to cyclic class u. Define $\bar{t_{ij}}$ as the minimum in the set T_{ij}, existing since T_{ij} is a set of integers bounded from below, and define $\bar{t}$ as the maximum over i and j of $\bar{t_{ij}}$, existing by the finiteness of the set of players. Finally, $\hat{t} = \tilde{t} + \bar{t}$, and the desired result is got. $\square$

Proof of lemma 3. If agent 1 selects alternative a at time t_1 and memory does not prevent its spreading, then only congestion problems can hinder it. However, with positive probability a is imitated at time $t_1 + 1$ by any j such that $G_{1j} > 0$. Take all those agents j such that $G_{1j} > 0$. The same reasoning can be applied to each of them, with a specification. If there exist j_1, j_2 and r with $G_{1j_1} > 0$, $G_{1j_2} > 0$, $G_{j_1 r} > 0$ and $G_{j_2 r} > 0$, it is clear that both channels cannot be used for transmission. However, since both channels carry the same alternative a, it is indifferent which one prevails. Therefore, with positive probability all those agents j such that $G^2_{1j} > 0$ choose a at time $t_1 + 2$. Going on analogously since time $t > \tilde{t}$ is reached, it is got that with positive probability all those agents j such that $G^{t-t_1}_{1j} > 0$ choose a at time t. The same procedure is then repeated for agent 2. Now it may also happen for some $\hat{t}$, j_1, j_2 and r to have $G^{\hat{t}}_{1j_1} > 0$, $G^{\hat{t}}_{2j_2} > 0$, $G_{j_1 r} > 0$ and $G_{j_2 r} > 0$. Again, both channels cannot be used for transmission, this however not being a problem since the same alternative is carried. Going on analogously for agents $3, 4, \ldots, l$, it is finally got that at any time $t > \tilde{t}$ with positive probability all those agents j such that $G^{t-t_i}_{ij} > 0$ for some $i = 1, \ldots, l$ choose a.

Suppose now that at the same time $\tilde{t}$ all the agents of a cyclic class choose a. The procedure to obtain the proof is analogous to the one just followed. However, by virtue of point *ii* in lemma 2 all those agents of C influencing the choice at time t of the agents of cyclic class $(u + t - \tilde{t}) \bmod p$

choose a. In addition, it is assumed that any possible model belonging to another communication class always chooses a. Hence, since congestion problems only concern the transmission from different channels of the same alternative, a will be surely transmitted at any step. $\square$

Proof of lemma 4. Let a denote the global maximum. All the agents in every maximal communication class will never make a choice different from a, because there is no other imitable behavior and no superior alternative can be remembered, since non-existent.

Take a class C among second-ranked classes, which are those influenced only by maximal classes. Let p be its period.

By hypothesis, there exist two agents i and j, with i belonging to a maximal class, j belonging to C and $G_{ij} = 1$. Without loss of generality, suppose j belongs to cyclic class 0. By virtue of point *iii* in lemma 2, there exists $\tilde{t}$ such that for all $t \geq \tilde{t} : G^t_{jl} > 0$ for every agent l belonging to cyclic class $t \bmod p$. Obviously, $G^t_{il} > 0$ holds too for $t \geq \tilde{t} + 1$. Notice that memory can never prevent alternative a from spreading since, by hypothesis, a superior alternative does not exist. Consider that at time $0, 1, \ldots, p-1$ agent i selects a. Then, by virtue of point i in lemma 3, there is a positive probability that at time $\tilde{t} + p$ alternative a is selected by all the agents belonging to cyclic classes $(\tilde{t}+p-t_i) \bmod p$, with $t_i = 1, \ldots, p$. This amounts to say that all the agents in class C select a within a finite number of periods with positive probability. If that happens, no choice different from a will ever be made by virtue of point *ii* in lemma 3 applied to every cyclic class, so implying that everyone's memory will be soon filled by a only.

The same reasoning can be applied to every second-ranked class, and afterwards to third-ranked classes - which are those not influenced by classes different from first-ranked and second-ranked ones and which are not first-ranked or second-ranked classes themselves - and so on. The result is that, starting from a state s, with a positive probability q_s a state with all agents' memories containing only alternative a is reached in a finite number of periods $\hat{t}_s$. If that is not the case, another state where the hypotheses of this lemma are satisfied is reached in any case, since agents in maximal communication classes will never change their choice. Let q denote the minimum of q_s and $\hat{t}$ the maximum of $\hat{t}_s$, both existing by the finiteness of the state space. Therefore, the probability that the system will not converge to the efficient state in $r\hat{t}$ periods is bounded from above by $(1-q)^r$, which clearly tends to zero when $r \to \infty$. $\square$

Proof of lemma 5. Condition i is first considered. Suppose $k > n - ||A(a)||$. Call s the current state and suppose agent i's memory contains a. Each alternative different from a can never appear more than once in i's memory, so implying that a will never be forgotten since memory is long enough. This in turn implies that within k periods a will be chosen by i. Define $\bar{t_j}$ as the minimum in the set T_{ij}, existing since T_{ij} is a set of integers bounded from below, and define $\bar{t}$ as the maximum over j of $\bar{t_j}$, existing by the finiteness of the set of agents. By virtue of point i in lemma 3 there is a positive probability that after $\bar{t}$ periods all agents have experienced a. Call that probability q_s. Notice that with the complementary probability $1-q_s$ a state where agent i has a memory containing a is reached. As it happens for agent i, alternative a will never be forgotten by anyone once known. Since cycles longer than k do not exist, once all agents have experienced a any other alternative is no longer played after k periods and is forgotten within other k periods. Let q denote the minimum of q_s, existing by the finiteness of the state space. The probability that the system will not converge to the efficient state in $r(k+\bar{t})+2k$ periods is bounded from above by $(1-q)^r$, which clearly tends to zero when $r \to \infty$.

I now consider condition ii. Suppose there exists a cyclic class of C with all the agents therein simultaneously choosing alternative a. By virtue of point ii in lemma 3, the global maximum a will go on cycling among cyclic classes, so never disappearing. Since a cycle longer than k does not exist, the period of the cyclic class cannot be greater than k. Therefore, within k periods all agents have experienced alternative a, which is never forgotten by anyone. As before, the presence of a in memory and the inexistence of cycles longer than k imply that any alternative different from a is no longer played after k periods and is forgotten within other k periods. Therefore the efficient state is got within $3k$ periods. □

Proof of lemma 6. Take any agent belonging to cyclic class 0 at current time 0. A perturbation occurs and she chooses a, the global maximum. By virtue of point iii in lemma 2 and point i in lemma 3 - the latter can be applied since memory never prevents the global maximum from spreading - after $\tilde{t}$ periods all the agents of a certain cyclic class choose a. By point ii in lemma 3 alternative a will go on being cyclically selected by the entire appropriate cyclic class; therefore some kind of ergodic set E_1 with this feature has been reached. Now a perturbation occurs at time $t_1 \bmod p = 1$, inducing an agent belonging to cyclic class 0 to choose a. With an analogous reasoning an ergodic set E_2 is reached. Keep on with this procedure, perturbations affecting agents in cyclic class 0 at times $t_2 \bmod p = 2$, $t_3 \bmod p = 3$, $\dots$,$t_{p-1} \bmod p = p-1$. In the ergodic set E_p every agent belonging to cyclic classes $t \bmod p$, $(t-t_1) \bmod p = (t-1) \bmod p$, $(t-t_2) \bmod p = (t-2) \bmod p$, $\dots$, $(t-t_{p-1}) \bmod p = (t-p+1) \bmod p$ chooses the global maximum. Therefore, by point ii in lemma 2 it follows that E_r is the efficient state. □

Proof of lemma 7. I first prove point i. Assume $k=1$ and $p=1$. Suppose there does not exist a local maximum different from the global maximum a and that, starting from the efficient state, a single perturbation occurs inducing an agent to choose $b \neq a$. Observe that no agent can choose b twice in a row since memory prevents from doing that. Observe also that no new alternative can rise in the unperturbed dynamics since that would require that all the models of an agent choosing b chose b as well; however, since b can only be chosen by imitation (because there is no existing alternative from which b is the best accessible choice), some of i's models must have chosen b at the previous time, against the first observation. These two considerations ensure that the global maximum will never disappear. This in turn implies by virtue of point *iii* of lemma 2 and point i in lemma 3 that with positive probability the global maximum, whose spreading is never prevented by memory, will be chosen at the same time by all the agents of the unique cyclic class, that granting the reaching of the efficient state and proving the impossibility of leaving such absorbing state by a single perturbation.

Suppose now there exists a local maximum b different from the global maximum a and that, starting from the efficient state, a perturbation occurs inducing an agent to choose b. Since memory does not prevent its spreading, then point *iii* in lemma 2 and point i in lemma 3 apply establishing that with positive probability all the agents of the unique cyclic class will choose b at the same time. Obviously, that is an absorbing state, so completing the proof of point i.

I now prove point *ii*. Assume $k > 1$ or $p > 1$ or both. Starting from the efficient state a perturbation occurs inducing agent i belonging to cyclic class u to choose $b \neq a$ at time 0. If $A(a) = \mathcal{C}$ then memory prevents others from imitating b, while at time 1 agent i copies out a with the result that b is no longer chosen and it is forgotten within k periods, when the efficient state is reached again.

Suppose that $A(a) \neq \mathcal{C}$ and $b \notin A(a)$. No agent belonging to cyclic classes different from $(u + t) \bmod p$ can choose b at time t, since all the agents in the other cyclic classes will choose a by point *ii* in lemma 3. No other alternative can rise, because that would require personal experience to favour an alternative different from a, which is never the case, since a is never forgotten by anyone.

Suppose $k \geq p$. Suppose ad absurdum that an ergodic set different from the absorbing state has been reached. Then at time t all the agents belonging to cyclic class $(u+t) \bmod p$ must choose b, otherwise by point *iii* in lemma 2 and point i in lemma 3 a can spread with positive probability at a finite time $\tilde{t}$ to all the agents in cyclic class $(u + \tilde{t}) \bmod p$, so determining the reaching of the efficient state once b has left anyone's memory. This however requires that agents choose b with period p, that is when such inferior alternative is still in memory. Hence a contradiction has been got.

Finally, suppose $k < p$. Memory never prevents b from spreading because b cannot come back before p periods. The usual point *iii* in lemma 2 and point i in lemma 3 apply: with positive probability after $\tilde{t}$ periods all the agents of a cyclic class choose b, and from then on all the agents in the appropriate cyclic class will choose b by point ii in lemma 3. Therefore an ergodic set where at any time $p-1$ cyclic classes choose a and the remaining cyclic class chooses b is reached, completing the proof. $\square$

Proof of proposition 2. Consider all the ergodic sets $E_1, E_2, \ldots, E_s$ of P^k. The resistance r_{ij} between the ergodic sets E_i and E_j is defined as the minimum number of perturbations necessary to move from E_i to E_j with positive probability under P^k. Define an E_i-tree as a tree rooted in E_i using all the ergodic sets as nodes. The resistance of an E_i-tree is defined as the sum of all the resistances met when going from any terminal node to the root. Finally, the stochastic potential of E_i is the minimum resistance over all the trees rooted at E_i.

Young (1993) proved that the stochastically stable distribution for a regular perturbed Markov chain - as it is the model under analysis - exists and assigns positive probability to all and only those ergodic sets of the unperturbed dynamics which have minimum stochastic potential.

First, notice that each maximal communication class is independent of the rest of the system and when efficiency is reached in all the maximal communication classes no other perturbation is required to get overall efficiency with positive probability (actually lemma 4 tells more, ensuring that it will happen almost surely). Furthermore, notice that a stochastic potential cannot be lower than $s-1$, since the minimum resistance r_{ij} is 1 and the number of branches of any tree is $s-1$. Then by lemma 6 the stochastic potential of the efficient state is equal to $s-1$. Therefore the efficient state always receives positive probability in the stochastically stable distribution.

Suppose that in all the maximal communication classes of G condition 1.i or condition 1.ii in the terms of this proposition is satisfied. By lemma 7 a single perturbation does not suffice to move the system away from efficiency in each maximal communication class. By lemma 4 a perturbation outside maximal communication classes does not have any chance to affect the system permanently. Hence, whatever E_i-tree with E_i different from the efficient state, the branch coming from the efficient state has a resistance greater than 1, so implying a stochastic potential greater than $s-1$. The result is that the stochastically stable distribution assigns probability 1 to the efficient configuration.

Suppose now that in a maximal communication class neither condition 1.i nor condition 1.ii is satisfied. By lemma 7 a single perturbation can move the system away from efficiency for that maximal communication class and, consequently, for the entire system. Denote by E_j an ergodic set which the system can reach from the efficient state. Denote the efficient state by E_r. Consider an E_r-tree with resistance $s-1$. Delete the branch coming from E_j and add a branch from E_r to E_j. What comes out is an E_j-tree with resistance $s-1$. Therefore the stochastically stable distribution assigns positive probability to inefficient ergodic sets.

I finally prove point 2 of the proposition. Let $\tilde{E}$ be an ergodic set or an union of ergodic sets of P^k. The radius of $\tilde{E}$ is the minimum number of perturbations required to reach with positive probability a state in another ergodic set starting from a state in $\tilde{E}$. The coradius is the maximum over all states of the minimum number of perturbations required to reach a state in E_i with positive probability. The modified coradius - denoted by $CR^*(E_i)$ - is defined analogously to the coradius with the difference that the number of perturbations is modified by subtracting the radiuses of all the intermediate ergodic sets passed through.

Let E be the union of ergodic sets which have positive probability in the stochastically stable distribution. If no other ergodic sets exist then trivially $CR^*(E) = 0$. If other ergodic sets exist, then by lemma 6 $CR^*(E) = 1$. By proposition 2 in Ellison (1993), for any state $s \notin E$, $W(s, E, \epsilon) = O(\epsilon^{-CR^*(E)})$ as $\epsilon \to 0$. $\square$

References

L. Anderson and C. Holt. "Classroom Games: Information Cascades". *Journal of Economic Perspectives*, 10:187–193, 1996.

L. Anderson and C. Holt. "Information Cascades in the Laboratory". *American Economic Review*, 87:847–862, 1997.

W. B. Arthur, S. N. Durlauf, and D. A. Lane, editors. *The Economy as an Evolving Complex System II*, Menlo Park, CA, 1997. Addison-Wesley.

S. Bikhchandani, D. Hirshleifer, and I. Welch. "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades". *Journal of Political Economy*, 100:992–1026, 1992.

S. Bikhchandani, D. Hirshleifer, and I. Welch. "Learning from the Behaviors of Others: Conformity, Fads and Informational Cascades". *Journal of Economic Perspectives*, 12:151–170, 1998.

L. Boncinelli. "Manifesto of Dynamic Social Economics". Quaderni del Dipartimento di Economia Politica, University of Siena, 2007a.

L. Boncinelli. "Choice under Markovian Constraints". Quaderni del Dipartimento di Economia Politica, University of Siena, 2007b.

G. Ellison. "Basins of Attraction, Long Run Stochastic Stability, and the Speed of Step-by-Step Evolution". *Review of Economic Studies*, 67:17–45, 2000.

G. Ellison. “Learning, Local Interaction, and Coordination”. *Econometrica*, 61:1047–1071, 1993.

G. Ellison and D. Fudenberg. “Rules of Thumb for Social Learning”. *Journal of Political Economy*, 101:612–643, 1993.

D. Foster and H. P. Young. “Stochastic Evolutionary Game Dynamics”. *Theoretical Population Biology*, 38:219–232, 1990.

M. I. Freidlin and A. D. Wentzell. *Random Perturbations of Dynamical Systems*. Springer, New York, 1984.

D. Friedman and J. Yellin. “Evolving Landscapes for Population Games”, 1997. Mimeo. University of California, Santa Cruz.

F. Hayek. “Economics and Knowledge”. *Economica*, 4:33–54, 1937.

M. Kandori, G. J. Mailath, and R. Rob. “Learning, Mutation and Long Run Equilibria in Games”. *Econometrica*, 61:29–56, 1993.

S. Kauffman. *Origins of Order: Self-organization and Selection in Evolution*. Oxford University Press, New York, 1993.

S. Kauffman, J. Lobo, and W. G. Macready. “Optimal Search on a Technology Landscape”. *Journal of Economic Behavior and Organization*, 43: 141–166, 2000.

J. G. Kemeny and J. L. Snell. *Finite Markov Chains*. Springer-Verlag, 1976.

J. M. Keynes. “The General Theory of Employment”. *Quarterly Journal of Economics*, 51:209–223, 1937.

T. Offerman and J. Sonnemans. “Learning by Experience and Learning by Imitating Successful Others”. *Journal of Economic Behavior & Organization*, 34:559–575, 1998.

R. Pastor-Satorras and A. Vespignani. “Epidemics and Immunization in Scale-Free Networks”. In S. Bornholdt and H. G. Schuster, editors, *Handbook of Graphs and Networks*. Wiley-VCH, Berlin, 2003.

K. H. Schlag. “Why Imitate, and if so, How? A Boundedly Rational Approach to Multi-armed Bandits”. *Journal of Economic Theory*, 78:130–156, 1998.

K. H. Schlag. “Which One Should I Imitate?”. *Journal of Mathematical Economics*, 31:493–522, 1999.

G. Shackle. *Epistemics and Economics*. Cambridge University Press, Cambridge, 1973.

H. Simon. “A Behavioural Model of Rational Choice”. *Quarterly Journal of Economics*, 69:99–118, 1955.

F. Vega-Redondo. “The Evolution of Walrasian Behavior”. *Econometrica*, 65:375–384, 1997.

S. Wright. “Adaptation and Selection”. In L. Jepson, G. C. Simpson, and E. Mayr, editors, *Genetics, Paleontology, and Evolution*. Princeton University Press, 1949.

H. P. Young. *Individual Strategy and Social Structure*. Princeton University Press, Princeton, 1998.

H. P. Young. “The Evolution of Conventions”. *Econometrica*, 61:57–84, 1993.