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Ordinal vs. Cardinal Status: Two Examples

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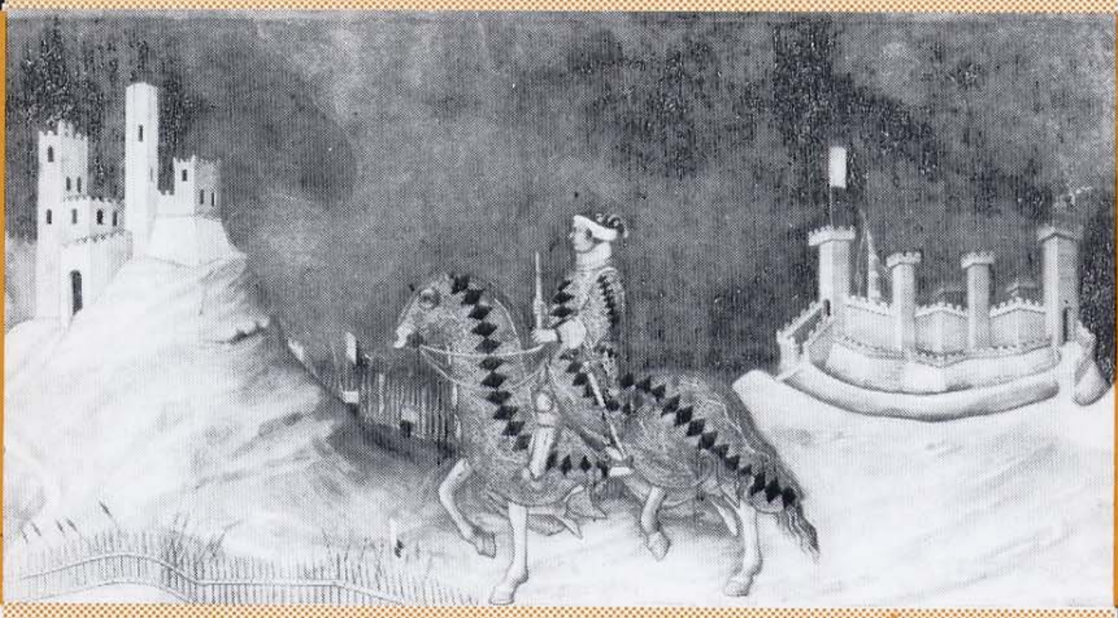

Abstract - We demonstrate that in models where agents have concerns for status the model predictions can drastically change depending on whether status is modelled as an ordinal or cardinal magnitude. As a proof, we show that two well known theoretical findings are not robust to the substitution of ordinal status with cardinal status (Frank (1985)) and viceversa (Clark and Oswald (1998)).

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1 Introduction

Since the work of Veblen (1899) the economic literature has been enriched by a large number of models where agents show concerns for status. Two main approaches to the modelling of status have been applied. The first follows the formalization of Duesenberry (1949) and sees status as the distance between one's own and others' possession of some status-bearing good or asset. In this approach status is intrinsically cardinal (see also Leibenstein (1950), Pollack (1976), Fershtman et al. (1996), Clark and Oswald (1998), Cooper et al. (2001), Bowles and Park (2005), Clark et al. (2008)). The second approach follows Frank (1985) and sees status as the rank in the distribution of the possession of the status-bearing good or asset. In this approach status is intrinsically ordinal (see for instance Robson (1992), Cole et al. (1992), Bagwell and Bernheim (1996), Corneo and Jeanne (1998), Hopkins and Kornienko (2004)).

The contributors to this stream of literature have rarely explained their choice to follow one approach or the other. A part from rare exceptions – e.g. Hopkins and Kornienko (2004), Clark et al. (2006) – the distinction between ordinal and cardinal status is not even acknowledged. Models incorporating concerns for status are usually presented as being related to both approaches (as an example see Frank (1985) who refers to Duesenberry (1949) but then defines status as ordinal and Cooper et al. (2001) who refer to Frank (1985) and then define status as cardinal). Most important, how results depend on whether status is ordinal or cardinal is never an object of discussion.

Such an omission can be explained only in two ways. Either results are substantially unaffected by the way status is modelled or economists working in this area of research have so far overlooked the issue. In this paper we show that the choice to model status as ordinal or cardinal does affect results in a substantial way. Intuitively, when status is ordinal the marginal benefit from spending in conspicuous consumption is lower the more dispersed the distribution of social competitors in the consumption of the relevant good. In the presence of cardinal status, instead, the reverse relation can hold; in particular, this happens if people evaluate small differences in conspicuous consumption less than proportionally with respect to large differences.

We provide two examples where switching from cardinality to ordinality and from ordinality to cardinality drastically change the model predictions (the examples are constructed using well known theoretical results obtained, respectively, by Frank (1985) and Clark and Oswald (1998)). We conclude that, in models where agents have concerns for their relative standing, the choice between cardinal and ordinal status should be more accurately motivated.

2 Cardinal and Ordinal Status

A simple and intuitive definition of ordinal and cardinal status is offered by the metaphor of the race scoring system. Let $\{1, \ldots, n\}$ be the set of cars participating in a race. We describe the final result of the competition by a vector, say $v \in \Re^n$, whose generic element v_i denotes car i's race time. Suppose that we have to define a scoring system for the annual car championship, that is, we have to specify a function, say $S(\cdot)$, which assigns to each car a number depending on vector v. Such a scoring system can naturally be defined as *ordinal* if $S(\cdot)$ only depends on the order of arrival induced by v. Instead, the scoring system can be defined as *cardinal* if $S(\cdot)$ depends on v otherwise.

This taxonomy can easily be applied to social status. Economic agents can be substituted for cars and the amount of the status-bearing good possessed for race times. So, $S(\cdot)$ can be reinterpreted as a social status function. This provides us with a distinction between ordinal and cardinal status which is based on how the prizes of the "social race" are assigned.

A natural question to ask at this point is which definition of status better represents people's concerns for relative standing. However, as far as we are interested in the conclusions that can be drawn from the introduction of status concerns, there is another question that logically comes first: does modelling status as ordinal or cardinal make any difference for the predictions of the model? Only if the answer to this question is yes then the issue of whether status should be modelled as an ordinal or cardinal magnitude becomes relevant. In the next two subsections we illustrate examples which prove that the answer to the latter question is yes.

2.1 Example 1: Status and Saving Behaviour

In his seminal paper Frank (1985) provides a proposition which states that, as income rises, the share of income spent in non-positional goods grows more rapidly (or decline less rapidly) in the presence of concerns for status (Proposition 3 pag. 105). Frank uses this finding to attempt an explanation of the saving behaviour of Americans which shows a cross-sectional positive correlation with income at any point in time but not over time – see Proposition $3'$ (pag. 105) and Section IV (pag. 109) in Frank (1985). In Frank's model the consumer problem is

$$
\begin{aligned}
\max_{x,y} \quad & U(x, y, R(x)) \\
\text{s.t.} \quad & P_x x + P_y y = M
\end{aligned} \tag{1}
$$

where x is the positional good, y is the non-positional one, P_x and P_y are the price of x and y, M is income and $R(x)$ is the status of the consumer. In particular, $R(x) = \int_{x_0}^{x} f(s)\mathrm{d}s$ where $f(\cdot)$ is the density function for x values and x_0 is the smallest value of x in the relevant population. This means that status is determined by the consumer's rank in the consumption of x and, as anticipated, it is ordinal. Finally, function U is supposed to satisfy standard concavity assumptions.

Manipulating the FOC obtained from problem (1) and introducing elasticities we get the following optimality condition (eq. $2''$ in Frank (1985))

$$
\frac{x}{y}\left(\frac{E_{U1}}{E_{U2}} + \frac{E_{U3}}{E_{U2}}E_{Rx}\right) = \frac{P_x}{P_y} \tag{2}
$$

where E_{Ui} is the elasticity of utility with respect to the i-th argument and E_{Rx} is the elasticity of $R(\cdot)$ with respect to x.

Since (2) differs from the standard condition just for the additional term $E_{U3}E_{Rx}/E_{U2}$, Proposition 3 holds only if such a term is decreasing in x. Frank assumes that this is the case because, he argues, it is very likely that E_{Rx} decreases monotonically in x. In particular he claims that, in a population where $x_0 > 0$, for any $f(\cdot)$ which is likely to be observed in practice the elasticity $E_{Rx} = f(x)/(R(x)x)$ is infinite for $x = x_0$ and then decreases monotonically to 0 as x moves towards the maximum value of its domain. Indeed, we have

$$
\frac{\mathrm{d}E_{Rx}}{\mathrm{d}x} = \frac{\left(f'(x)x + f(x)\right)R(x) - f^2(x)x}{R^2(x)} \tag{3}
$$

which is likely to be negative for most values of x if we assume that $f(x)$ is left skewed and approaches 0 as x approaches the maximum value of its domain.

However, even assuming such a "reasonable" $f(\cdot)$, Proposition 3 may be far away from reality if status is cardinal. This is easily seen by substituting $R(x)$ with the very simple form of cardinal status $C(x) = (x/\int_{x_0}^{x^1} sf(s)\mathrm{d}s)$, where x^1 is the maximum value of x in the relevant population. If status is $C(x)$ then the elasticity of status with respect to the consumption of the positional good is $E_{Cx} = 1$ and therefore the relative saving behaviour only depends on the ratio E_{U3}/E_{U2}. If, for instance, preferences are assumed to be homothetic as in Frank's Proposition $3'$, then the model provides quite different predictions, namely the neutrality of status concerns. In fact, even in the presence of status concerns, poor people save the same income share as rich people.

2.2 Example 2: Following Behaviour

In a well known article of a decade ago Clark and Oswald (1998) investigate which type of preferences induce a following behaviour – i.e. the desire to conform – in the presence of concerns for status. Their conclusion is that "models in which it is *assumed* that people have a taste for conformity can be justified at a more primitive axiomatic level by appealing to comparison-concave utility" (pag. 152). Such a result is obtained from the study of the following consumer problem

$$\max_a \quad U = sv(g(a, a^*)) + (1-s)u(a) - c(a) \tag{4}$$

where a is the consumer's choice variable (e.g. consumption), $v(\cdot)$ determines the benefits accruing from status, $g(\cdot)$ determines the consumer's status, a^* is an aggregate statistic of others' choice, the parameter s represents the relative importance of status concerns, $u(\cdot)$ determines all other benefits of a and $c(\cdot)$ all other costs. Finally, it is assumed that $U(\cdot)$ is strictly concave in a.

Clark and Oswald (1998) define follow behaviour as $\partial\hat{a}/\partial a^* > 0$ where $\hat{a}$ is the solution to the consumer problem. They investigate what makes people adopt a follow behaviour under two different types of cardinal status: $g_1 = (a - a^*)$ and $g_2 = a/a^*$. They find that the key preference parameter is concavity of $v(\cdot)$. Under g_1 consumers adopt a following behaviour if and only if $v(\cdot)$ is concave (Proposition 1 pag. 138) while under g_2 the follow behaviour is adopted if and only if the elasticity of marginal utility with respect to status exceeds unity (Proposition 3 pag. 141) which obtains for a function $v(\cdot)$ sufficiently concave. We claim that both results are true only if status is cardinal. In particular, we show that under ordinal status it may happen that people adopt a follow behaviour even if $v(\cdot)$ is strictly convex. Let us assume, for the sake of the argument, that there is a continuum of individuals. Suppose also that status is ordinal and, in particular, determined by $g_3 = \int_{a_0}^{a} f(r)\mathrm{d}r$ where a_0 is the minimum value of a in the relevant population and $f(\cdot)$ is the density function for the a values. For simplicity assume that $f(\cdot)$ is uniformly distributed and total population is normalized to 1. The FOC of consumer problem (4) becomes

$$\frac{\mathrm{d}U(f)}{\mathrm{d}a} = sv'\left(\frac{a-a_0}{a_1-a_0}\right)\frac{1}{a_1-a_0} + (1-s)u'(a) - c'(a) = 0 \tag{5}$$

where a_1 is the maximum value of a in the relevant population. Consider a generic individual whose optimal choice is $\hat{a}$ and suppose that all other individuals increase their consumption by an amount which is equal to half of the difference between their initial consumption and a_1. The resulting distribution of consumption levels f' is again uniform but on the smaller interval $[a_0 + (a_1 - a_0)/2, \bar{a}]$.[1] Let $\hat{a}'$ be the new optimal choice of the individual under consideration. Then it is easily seen that

$$\frac{\mathrm{d}U(f')}{\mathrm{d}a} > \frac{\mathrm{d}U(f)}{\mathrm{d}a} \Longleftrightarrow 2v'\left(\frac{2a - (a_0 + a_1)}{a_1 - a_0}\right) > v'\left(\frac{a - a_0}{a_1 - a_0}\right) \Longrightarrow \hat{a}' > \hat{a} \quad (6)$$

which means that follow behaviour can arise even if $v(\cdot)$ is strictly convex. The reason is that the anti-conformist effect due to the convexity of $v(\cdot)$ is more than compensated by the increased advantage of consuming which stems from the fact that now population is more dense and, therefore, more status is gained by one extra unit of consumption.

3 Conclusion

In models incorporating concerns for relative standing, the modelling choice between ordinal and cardinal status is never an object of discussion. We have provided two examples which prove that the two alternatives may lead to rather different predictions. In the light of this, we conclude that it is not legitimate to make such a choice without accurately motivating it.

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[1] More precisely, such a distribution is almost everywhere uniform since the individual with initial consumption $\hat{a}$ creates a negligible discontinuity.

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