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A Coalitional Game-Theoretic Model of Stable Government Forms with Umpires

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Abstract - In this paper a government form is modeled as an effectivity function scheme (EFS) i.e. a parameterized family of effectivity functions having admissible (strong) weight-profiles as the relevant parameters. Working in a 2-jurisdiction outcome space we show that the existence of umpires is consistent with strong core-stability of a neo-parliamentary or mixed semi-presidential government form provided that the majority formation rule is collegial.

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1 Introduction

Is there any effective role for a constitutional umpire when the head of the executive is directly appointed by means of general elections? Is that role possibly consistent with stability of the corresponding government form in multiparty and/or two-party systems?

Such issues, and related ones, have been hotly debated in recent years in Italy as well in other Countries where constitutional reform has been on the public agenda. The general presumptions underlying those debates are apparently that

i) guaranteeing stability and effectiveness of the executive is a crucial priority,

ii) enhancing electoral control on coalition formation in multi-party parliamentary systems -and related ones such as semi–presidential systems where the cabinet if not the head of the executive herself can be demoted by a non-confidence vote- is also desirable, and

iii) direct election of the prime minister should be considered as a promising constitutional device to help achieve both.

Those views may be not compelling, but they are in any case popular enough to be definitely worth analyzing and probing from a mechanism design perspective.

Indeed, the foregoing problems of constitutional design -I submit- amount to well-defined *implementation problems*: therefore, they are in principle perfectly amenable to a mechanism-design treatment. As a matter of fact, a general implementation problem consists of three main components: a *success criterion*, a *game format*, and a *solution concept.* The *success criterion* specifies the choice rule to be implemented i.e. the performance to be achieved including of course the environment to be considered. The *game format* specifies the required type of description of the implementing mechanism, e.g. whether actions available to players are to be explicitly described and, if so, whether they should embody in an essential way a sequential structure or not. Finally, the *solution concept* specifies the way the games induced by the implementing mechanism should be solved, and amounts to a theory of well-adapted behaviour for the ensuing interactive situations. In the family of constitutional design problems we are about to consider, the success criterion amounts to non-empty-valuedness and Pareto efficiency of the choice correspondence as defined on a large domain of preference profiles of the relevant players (where preferences are suitably defined over the policy-outcome space). In order to focus on general guidelines concerning 'who can do what' to the effect of avoiding committment to specific strategy spaces while at the same time enhancing tractability we look for implementation by *coalitional game forms* which in turn strongly suggest reliance on so called 'cooperative' solutions such as the core and related notions.

In particular, in this paper *a government form is modeled as an effectivity function scheme (EFS)* i.e. a parameterized family of effectivity functions having admissible weight-profiles as the relevant parameters. In this setting, the 'stability problem' for government forms can be aptly re-formulated in terms of strong core-stability of the resulting effectivity functions over a large domain of preference profiles on the outcome/policy space. Thus, under the suggested interpretation the stability problem for government forms reduces to a (family of) implementation problems in a *coalitional format* with a refined version of the *core* we call 'strong core' as a solution concept and *non-emptiness of the resulting choice rule -i.e. 'stability'- as a success criterion.*

We know from some previous work of the present author (see Vannucci (2000, 2002)) that neo-parlamentary and mixed semi-presidential strongly-core stable government forms can be devised *even in multiparty environments* provided that a *collegial* majority formation rule is enacted, which means that there is (electoral appointment of) only one admissible minimal majority coalition for each legislature, i.e. in order-theoretic jargon the set of all powerful or 'winning' coalition is indeed a *principal order filter* as defined below.

The present paper extends the foregoing results to the case of constitutional umpires. A constitutional *umpire* (the President under most parliamentary or neo-parliamentary contemporary constitutions) is defined in our framework as a player that is endowed with veto power concerning early demotion of the executive including possibly its head and/or early legislature termination. Working in a 2-jurisdiction outcome space we show that the existence of umpires is consistent with strong core-stability of a neo-parliamentary or mixed semi-presidential government form provided that the majority formation rule is collegial.

2 Model and results

To begin with, let us recall some basic notions on effectivity-coalitional game forms that will be needed in the ensuing analysis.

Let N,X be two non-empty sets (denoting the player set and the outcome set, respectively), and $P(N)$,$P(X)$ their power sets. An *effectivity function (EF)* on (N,X) is a function $E{:}P(N) \to P(P(X))$ such that :

EF 1) $E(N) = P(X) \setminus \{\emptyset\}$; *EF 2)* $E(\emptyset) = \emptyset$; *EF 3)* $X \in E(S)$ for any $S \in P(N) \setminus \{\emptyset\}$; *EF 4)* $\emptyset \notin E(S)$ for any $S \in P(N) \setminus \{\emptyset\}$.

The following properties of an EF will be repeatedly taken into consideration:

N-Monotonicity: An EF E on (N, X) is $N-$ *monotonic* if for any $A \subseteq X$ and $S \subseteq T \subseteq N$, $A \in E(S)$ entails $A \in E(T)$.

X-Monotonicity: An EF E on (N, X) is $X-$*monotonic* if for any $A \subseteq B \subseteq X$, and $S \subseteq N$, $A \in E(S)$ entails $B \in E(S)$.

Monotonicity: An EF E on (N, X) is *monotonic* if it is both N-monotonic and X-monotonic.

Convexity: An EF E on (N, X) is *convex* if for any $S, T \subseteq N$, and $A, B \subseteq X, A \in E(S)$ and $B \in E(T)$ imply that either $A \cap B \in E(S \cup T)$ or $A \cup B \in E(S \cap T)$.

Clearly enough, the foregoing properties are not mutually independent. In particular, it is easily checked that convexity N-monotonicity.

The usually suggested interpretation of the statement '$A \in E(S)$' is that coalition S can "force" the outcome within subset A, according to the rules of of an underlying decision mechanism that is left unspecified. In particular, an EF

E on (N,X) is said to be *α-strategically playable* if there exists a strategic game form $G=(N,X,(S_i)_{i\in N},h)$ with (surjective) outcome function $h\in X^{\Pi_{i\in N}S_i}$ such that $E=E_\alpha(G)$, where $E_\alpha(G)(\emptyset)=\emptyset$ and for any *nonempty* $S\subseteq N$,

$$E_\alpha(G)(S)=\left\{\begin{array}{c} A\subseteq X:\text{there exists } s_S=(s_i)_{i\in S}\in\prod_{i\in S}S_i \\ \text{s.t. } h(s_S,t_{N\setminus S})\in A \text{ for any } t_{N\setminus S}\in\prod_{i\in N\setminus S}S_i \end{array}\right\}.$$

As mentioned above, coalitional stability of an EF will be assessed using the core as the basic solution concept.

Definition 1 *(*Core stability of an EF*) Let E be an EF on (N,X) and $D(N,X)$ a domain of preference profiles $\succ=(\succ_i)_{i\in N}$, where– for any $i\in N$ – $\succ_i$is a suitable binary relation on X. The* core *of E at $\succ$ –written $Core(E,\succ)$ – is the set of $(E,\succ)$-undominated outcomes i.e.*

$$Core(E,\succ)=\left\{\begin{array}{c} x\in X: \textit{for no } S\subseteq N, B\subseteq X, B\in E(S) \\ \textit{and } b\succ_i x \ \ \textit{for any } b\in B, i\in S \end{array}\right\}.$$

An EF E on (N,X) is (core-)stable *on $D(N,X)$ if $Core(E,\succ)\neq\emptyset$ for any $\succ\in D(N,X)$ and* unstable *otherwise.*

As it happens, core-stability of E cannot guarantee that an $(E,\succ)$-dominated outcome be dominated through some subset including some core-outcome of $(E,\succ)$ (see e.g. Abdou, Keiding(1991)).This motivates a *stronger* stability requirement first introduced by Demange(1987), namely:

Definition 2 *(*Strong core stability of an EF*). Let E be an EF on (N,X) and $D=D(N,X)$ a domain of preference profiles as defined above. E is said to be* strongly (core-)stable *over D if for any profile $\succ\in D(N,X)$ the following holds true: for any $x\in X\backslash C(E,\succ)$ a pair $S\subseteq N, B\subseteq X$ exists such that $B\cap C(E,\succ)\neq\emptyset$, $B\in E(S)$ and $b\succ_i x$ for any $b\in B, i\in S$.*

In particular, the ensuing analysis will rely on the following basic and very well-known facts:

i) convex EFs are strongly stable on a few large domains of preference profiles, including the domain of all preference profiles consisting of weak orders with a maximum (see e.g. Abdou and Keiding (1991)); ii) any superadditive and X-monotonic EF is α-strategically playable.

The '*president*' or *'head of the executive'* is denoted as player $\perp$,and the *'umpire'* as player $\top$. The $(n-player, h-sized)$ '*assembly*' $(\mathbf{v},h)\in Z_+^n\times Z_+\backslash\{0\}$–with weight profile $\mathbf{v}=(v_1,..,v_n)$ such that $\sum_{i=1}^n v_i=h$– comprises h seats which are divided among a finite number of parties, denoted as players $1,2,..,n$, with $n\geq 3$: the voting weight v_i of party i denotes the (non-negative, integer) number of seats under i's control. We posit $N=\{1,.,n\}$ and denote by $V(n,h)$ the set of all n-agent weight profiles for a h-sized assembly. Decisions are taken by the assembly using a rule that is implicitly defined by a *majority formation rule* $W^*:\Delta\subseteq V(n,h)\rightarrow P(P(N))\backslash\{\emptyset\}$ such that :

$(MF1)$: for any $\mathbf{v}\in\Delta$, $W^*(\mathbf{v})$ is an *order filter* of $(P(N),\subseteq)$ i.e. for any $S,T\subseteq N$, $S\in W^*(\mathbf{v})$ and $S\subseteq T$ entail $T\in W^*(\mathbf{v})$, and

$(MF2)$: $W^*(\mathbf{v})\subseteq W(\mathbf{v})$, where $W(\mathbf{v})$ is the set of all simple majority coalitions at $\mathbf{v}$, namely coalition $S\in W(\mathbf{v})$ if and only if $\sum_{i\in S}v_i\geq\lfloor h/2\rfloor+1$.

Thus, a majority formation rule $W^*(.)$ determines – for any admissible weight profile $\mathbf{v} \in \Delta$– the (non-empty) order-filter $W^*(\mathbf{v}) \subseteq P(N)$ of *admissible majority coalitions* .

A set $W^*(\mathbf{v})$ of admissible majorities for assembly $(\mathbf{v}, h)$ is said to be *collegial* if $W^*(\mathbf{v})$ is a *principal (or, equivalently, latticial) filter* i.e. a (majority) coalition $S \subseteq N$ exists such that $W^*(\mathbf{v}) = \{T \subseteq N : S \subseteq T\}$ (S is also referred to as the *basic* coalition of $W^*(\mathbf{v})$). Also, $W^*(\mathbf{v})$ is *minimal collegial* if it is collegial and its basic coalition is *minimal* (a majority coalition $S \in W(\mathbf{v})$ is said to be *minimal* if $T \notin W(\mathbf{v})$ for any $T \subseteq S, T \neq S$).

A *majority formation rule* $W^*(.)$ is said to be *(minimal) collegial* on Δ if $W^*(\mathbf{v})$ is (minimal) collegial at any $\mathbf{v} \in \Delta$, and *non-collegial* otherwise.

Moreover, without any real loss of generality, the outcome or policy space X is taken to be decomposable into two distinct non-overlapping jurisdictions, i.e. $X = Y \times Z$, where Y denotes the jurisdiction of the assembly and Z denotes the presidential jurisdiction. I also assume that two "special" outcome-components $y^* \in Y$, $z^* \in Z$ exist (whose exact interpretation will be made dependent on context as explained below) and that $\#A \geq 3$ for any $A \in \{Y, Z\}$, in order to avoid annoying qualifications. Moreover, following Vannucci (2000, and 2002) I shall be able to avoid some irrelevant complications by focussing on *strong assemblies* i.e.those assemblies whose two-coalitions partitions must include one majority. Strong assemblies are characterized by a *strong weight profile* as defined below.

Definition 3 *(*Strong weight profiles*) An $n-$dimensional weight profile* $\mathbf{v}$ *for a h-sized assembly is said to be* strong *if for any coalition $S \subseteq N$ either $\sum_{i \in S} v_i \geq \lfloor h/2 \rfloor + 1$ or $\sum_{i \in N \setminus S} v_i \geq \lfloor h/2 \rfloor + 1$. The set of all such $n-$dimensional $h-$sized strong weight profiles is denoted by $V^*(n, h)$.*

Notation 4 *The following notation will be used: a parameterized family $E(.)$ of EFs will also be referred to as an* EF-scheme (EFS) . *An EF-scheme $E(.)$ with parameter set V is said to enjoy property P on V if and only if $E(\mathbf{v})$ is P for any $\mathbf{v} \in V$. In particular, when referring to stability properties it will be said that $E(.)$ is (strongly) stable on (V, D) if and only if $E(\mathbf{v})$ is (strongly) stable on D for any $\mathbf{v} \in V$, and* unstable *otherwise.*

We are now ready to introduce our EF-based models of government forms with umpires. It should be noticed here that the stability problem of such a government form amounts to a parameterized version of a standard implementation problem by means of coalitional game forms. Indeed, solving the stability problem in the foregoing frameworks reduces to identifying an EF-scheme E such that for any $\mathbf{v} \in V$, $Core(E(\mathbf{v}), .) : D \to\to X$ is a *non-empty* (and Pareto-efficient) social choice correspondence.

As mentioned above, we regard a constitutional umpire as a player that is empowered to check those *unilateral* decisions on the part of the president or of the assembly that have a direct impact on the whole system as opposed to their respective jurisdictions. Hence in particular the constitutional umpire's support is required in order to demote the president or terminate the legislature i.e. replace the current assembly.

Let us start with a stylized version of (neo-)parliamentary government forms (PAGF). Here, PAGF are taken to be characterized by the power of the assembly –i.e. of its effective majorities–to remove the executive and/or block its actions. Moreover, we focus on the somewhat simplified 'symmetric'arrangement that requires appointment of a –possibly new–president *and a new umpire* whenever the "old" assembly is replaced. Thus, the weight-parameterized effectivity function of a PAGF such that election of a new assembly always involves election of a new president and a new umpire can be defined as follows:

Definition 5 *(* The EF-scheme $E_{PA}(W^*(.))$ of a symmetric neo-parliamentary government form with umpire*)*

Let $\mathbf{v}$ *be an* $n-$*dimensional weight profile for a* $h-$*sized assembly,* $W^*(\mathbf{v})$ *the non-empty set of effective majorities at* $\mathbf{v}$*,* $N^+ = N \cup \{\perp, \top\}, X = (Y \setminus \{y^*\} \times Z \setminus \{z^*\}) \cup \{(y^*, z^*)\}$*. Then,* $E_{PA}(W^*(\mathbf{v}))$ *is the EF on* (N^+, X) *as defined by the following rules: for any* $S \subseteq N^+, B \subseteq X$*,* $B \in E_{PA}(W^*(\mathbf{v}))(S)$ *iff one of the following clauses applies*

(i) $B \neq \emptyset$ *and* $S \supseteq M \cup \{\perp, \top\}$ *for some* $M \in W^*(\mathbf{v})$ *;*

(ii) $(y^*, z^*) \in B$*,* $\perp \in S$ *and* $S \cap M \neq \emptyset$ *for any* $M \in W^*(\mathbf{v})$*;*

(iii) $(y^*, z^*) \in B$*,* $\top \in S$ *and* $S \cap M \neq \emptyset$ *for any* $M \in W^*(\mathbf{v})$ *;*

(iv) $(y^*, z^*) \in B$ *and* $\{\perp, \top\} \subseteq S$ *;*

(v) $B = X$ *and* $S \neq \emptyset$*.*

In words, a coalition is all-powerful if and only if it comprises both an effective majority, the president, and the umpire. An effective majority has full control over the jurisdiction of the assembly (including of course the right to call for new elections) *provided it is supported by the umpire*. An effectively blocking coalition can also call for new elections and possibly remove the president (calling for new presidential elections) *if it is supported by the umpire*. Hence, the president only retains conditional control of her jurisdiction, given the possibility of being removed e.g. by means of a non-confidence vote.

Proposition 6 *Let* $V^*(n,h)$ *the set of strong weight profiles,* $W^*(.)$ *a* collegial *majority formation rule on* $V^*(n,h)$*,* $E_{PAU}(W^*(.))$ *a symmetric neo-parliamentary government form with umpire, and* D *the set of all profiles of weak orders with a maximum on the outcome space. Then* $E_{PAU}(W^*(.))$ *is* convex *–hence strongly core-stable– on* D*.*

Proof. To begin with, notice that $E_{PAU}(W^*(\mathbf{v}))$ is a monotonic EF, by definition. To check *convexity* of $E_{PAU}(W^*(\mathbf{v}))$ whenever $W^*(.)$ is collegial, i.e. –for any $\mathbf{v} \in V^*(n,h)$–$W^*(\mathbf{v}) = \{M \subseteq N : M \supseteq M^*\}$ is a (principal) latticial filter, with M^* as its unique generating (majority) coalition, we rely on a direct proof by enumeration of cases. Indeed, take $A, B \subseteq X, S, T \subseteq N, \mathbf{v} \in V^*(n,h)$ such that $(\alpha) A \in E_{PAU}(W^*(\mathbf{v}))(S)$, and $(\beta) B \in E_{PAU}(W^*(\mathbf{v}))(T)$. Several different cases are to be distinguished according to which of the defining clauses of $E_{PAU}(W^*(\mathbf{v}))$ underlie (α) and (β).

If both (α) and (β) rely on clause (i) -written $(\alpha : i, \beta : i)$ namely $S \supseteq M' \cup \{\bot, \top\}$ and $T \supseteq M'' \cup \{\bot, \top\}$ for some $M', M'' \in W^*(\mathbf{v})$,then $S \cap T \supseteq M^* \cup \{\bot, \top\}$ hence clearly $A \cup B \in E_{PAU}(W^*(\mathbf{v}))(S \cap T)$ (by clause (i) itself). Similarly, if $(\alpha : ii, \beta : ii)$ then $S \cap M^* \neq \emptyset \neq T \cap M^*$, $(y^*, z^*) \in A \cap B$ and $\bot \in S \cap T$ whence $A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$ (by clause (ii)).The case $(\alpha : iii, \beta : iii)$ implies without loss of generality(w.l.o.g.) that $S \cap M^* \neq \emptyset \neq T \cap M^*$with $\top \in S \cap T$ and $(y^*, z^*) \in A \cap B$ whence again $A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$, by clause (iii). In case $(\alpha : iv, \beta : iv)$ one has $(y^*, z^*) \in A \cap B$ and $\{\bot, \top\} \subseteq S \cap T$, hence both $A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$ and $A \cup B \in E_{PAU}(W^*(\mathbf{v}))(S \cap T)$ hold. Moreover, whenever $(\alpha : v)$ or $(\beta : v)$ is the case – hence $X \in \{A, B\}$– it follows that $\{A, B\} \ni A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$ by clause (v). Next consider the cases $(\gamma : i, \delta : ii)$, $\{\alpha, \beta\} = \{\gamma, \delta\}$ where without loss of generality $A \neq \emptyset, (y^*, z^*) \in B, S \supseteq M^* \cup \{\bot, \top\}, T \cap M^* \neq \emptyset$, and $\bot \in T$: here, $A \cup B \in E_{PAU}(W^*(\mathbf{v}))(S \cap T)$, by clause (ii). If $(\gamma : i, \delta : iii), \{\gamma, \delta\} = \{\alpha, \beta\}$ applies, i.e. w.l.o.g. $A \neq \emptyset, (y^*, z^*) \in B, S \supseteq M^* \cup \{\bot, \top\}, \top \in T$, and $T \cap M^* \neq \emptyset$, again $A \cup B \in E_{PAU}(W^*(\mathbf{v}))(S \cap T)$, by clause (iii). If $(\gamma : i, \delta : iv), \{\gamma, \delta\} = \{\alpha, \beta\}$ applies, i.e. w.l.o.g. $A \neq \emptyset, (y^*, z^*) \in B, S \supseteq M^* \cup \{\bot, \top\}$ and $\{\bot, \top\} \subseteq T$ whence $A \cup B \in E_{PAU}(W^*(\mathbf{v}))(S \cap T)$. Now consider the two remaining relevant classes of cases with $(\gamma : ii), \gamma \in \{\alpha, \beta\}$, namely with $(y^*, z^*) \in A$, $\bot \in S$ and -w.l.o.g.-$S \cap M^* \neq \emptyset$.If $(\delta : iii), \delta \in \{\alpha, \beta\} \setminus \{\gamma\}$ i.e. –w.l.o.g.–$T \cap M^* \neq \emptyset$, $\top \in T$, and $(y^*, z^*) \in B$, then –by each one of clauses (ii),(iii) and (iv)–$A \cap B \in E_{PAU}(W(\mathbf{v}))(S \cup T)$. Otherwise $(\delta : iv), \delta \in \{\alpha, \beta\} \setminus \{\gamma\}$, i.e. $(y^*, z^*) \in B$, and $\{\bot, \top\} \subseteq T$ whence again $A \cap B \in E_{PAU}(W(\mathbf{v}))(S \cup T)$ by clause (iv). Finally, we have the last two relevant cases $(\gamma : iii, \delta : iv), \{\gamma, \delta\} = \{\alpha, \beta\}$, namely–w.l.o.g.– $(y^*, z^*) \in A \cap B, \top \in S$, $S \cap M^* \neq \emptyset$, and $\{\bot, \top\} \subseteq T$, which jointly imply –by clause (iii) or (iv)– $A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$. ■

Let us now turn to the mixed semi-presidential case. Here we consider a semi-presidential government form (SPGF) with umpire with the following basic features: the president/head of the executive can terminate the assembly while keeping its office but only provided she gets support from the umpire. This arrangement is embodied in the following definition, namely

Definition 7 *(*The EF-scheme $EF_{SPU}(W^*(.))$ of a semi-presidential government form with umpire *). Let* $\mathbf{v}, W^*(\mathbf{v}), N^+$ *be as previously defined , and* $X^* = (Y \setminus \{y^*\} \times Z \setminus \{z^*\}) \cup (\{y^*\} \times Z)$. *Then,* $E_{SPU}(W^*(\mathbf{v}))$ *is the EF on* (N^+, X^*) *defined as follows : for any* $S \subseteq N^+, A \subseteq X^*$, $A \in E_{SPU}(W^*(\mathbf{v}))(S)$ *if and only if*

(i) $A \neq \emptyset$ *and* $S \supseteq M \cup \{\bot, \top\}$ *for some* $M \in W^*(\mathbf{v})$ *or*

(ii) $(y^*, z) \in A$ *for some* $z \in Z$ *, and* $\{\bot, \top\} \subseteq S$ *or*

(iii) $A \supseteq \{y^*\} \times Z$ *,* $\top \in S$ *and* $S \cap M \neq \emptyset$ *for any* $M \in W^*(\mathbf{v})$ *or*

(iv) $A = X^*$ *and* $S \neq \emptyset$.

We ask again the– by now–familiar question concerning stability of $E_{SPU}(W^*(\mathbf{v}))$ and its possible dependence on the features of $W^*(\mathbf{v})$. The answer is summarized by the following proposition:

Proposition 8 *Let $V^*(n,h)$ the set of strong weight profiles,* $W^*(.)$ *a* collegial *majority formation rule on $V^*(n,h)$, $E_{SPU}(W^*(.))$ the EF-scheme of a mixed semi-presidential government form with umpire, and D the set of all profiles of weak orders with a maximum on the outcome space. Then $E_{SPU}(W^*(.))$ is* convex *–hence strongly core-stable– on D.*

Proof. To begin with, notice that $E_{SPU}(W^*(\mathbf{v}))$ is a monotonic EF, by definition. To check *convexity* of $E_{SPU}(W^*(\mathbf{v}))$ whenever $W^*(.)$ is collegial, i.e. –for any $\mathbf{v} \in V^*(n,h)$–$W^*(\mathbf{v}) = \{M \subseteq N : M \supseteq M^*\}$ is a (principal) latticial filter, with M^* as its unique generating (majority) coalition, we rely again on a direct proof by enumeration of cases. Indeed, take $A, B \subseteq X, S, T \subseteq N, \mathbf{v} \in V^*(n,h)$ such that $(\alpha) A \in E_{SPU}(W^*(\mathbf{v}))(S)$, and $(\beta) B \in E_{SPU}(W^*(\mathbf{v}))(T)$. Several different cases are to be distinguished according to which of the defining clauses of $E_{SPU}(W^*(\mathbf{v}))$ underlie (α) and (β).

If both (α) and (β) rely on clause (i) -written $(\alpha : i, \beta : i)$ namely $S \supseteq M' \cup \{\bot, \top\}$ and $T \supseteq M'' \cup \{\bot, \top\}$ for some $M', M'' \in W^*(\mathbf{v})$,then $S \cap T \supseteq M^* \cup \{\bot, \top\}$ hence clearly $A \cup B \in E_{SPU}(W^*(\mathbf{v}))(S \cap T)$ (by clause (i) itself). Similarly, if $(\alpha : ii, \beta : ii)$ then $S \cap T \supseteq \{\bot, \top\}$,whence $A \cup B \in E_{SPU}(W^*(\mathbf{v}))(S \cap T)$ by clause (ii). If $(\alpha : iii, \beta : iii)$ then $A \cap B \supseteq \{y^*\} \times Z, \top \in S \cap T$, and w.l.o.g. $S \cap M^* \neq \emptyset \neq T \cap M^*$: therefore $A \cap B \in E(S \cup T)$ by clause (iii). Moreover, whenever $(\alpha : iv)$ or $(\beta : iv)$ is the case – hence $X \in \{A, B\}$– it follows that $\{A, B\} \ni A \cap B \in E_{SPU}(W^*(\mathbf{v}))(S \cup T)$ by clause (iv).

Next, consider the cases $(\gamma : i, \delta : ii)$, $\{\alpha, \beta\} = \{\gamma, \delta\}$ where without loss of generality $A \neq \emptyset, (y^*, z) \in B$ for some $z \in Z, S \supseteq M^* \cup \{\bot, \top\}, \{\bot, \top\} \subseteq T$: here, $A \cup B \in E_{SPU}(W^*(\mathbf{v}))(S \cap T)$, by clause (ii). Moreover, if $(\gamma : i, \delta : iii), \{\gamma, \delta\} = \{\alpha, \beta\}$ applies, i.e. w.l.o.g. $A \neq \emptyset, B \supseteq \{y^*\} \times Z$ $, S \supseteq M^* \cup \{\bot, \top\}, \top \in T$, and $T \cap M^* \neq \emptyset$, again $A \cup B \in E_{SPU}(W^*(\mathbf{v}))(S \cap T)$, by clause (iii). Finally, we have the last class of relevant cases $(\gamma : ii, \delta : iii), \{\gamma, \delta\} = \{\alpha, \beta\}$, namely–w.l.o.g.– $(y^*, z) \in A$ for some $z \in Z, B \supseteq \{y^*\} \times Z, \{\bot, \top\} \subseteq S$, and $T \cap M^* \neq \emptyset$ which jointly imply –by clause (iii) or (iv)– $A \cap B \in E_{PAU}(W^*(\mathbf{v}))(S \cup T)$ by clause (ii). ■

Remark 9 *The proofs of Propositions 6 and 8 provided above are direct. Alternative proofs could be devised relying on a general result on sufficient conditions for convexity of coalitional game forms as presented in Vannucci (2004).*

Remark 10 *It should be remarked that since for any $\mathbf{v} \in V(n,h)$, $E_{PAU}(W^*(\mathbf{v}))$ and $E_{SPU}(W^*(\mathbf{v}))$ are both monotonic and convex they are in particular α-* strategically playable *i.e. they are implementable by means of suitable mechanisms in strategic form (see e.g. Otten, Borm, Storcken and Tijs (1995)).*

3 Concluding remarks

It is sometimes claimed that whenever the premier or chief of the executive is appointed directly by means of general elections there is no possible significant role for a president/umpire. The foregoing results make quite clear that this is definitely not the case. Moreover, when combined with collegial majority formation rules, the existence of a constitutional umpire is consistent with the requirement of strong core-stability for government forms in a multiparty environment. Hence consistency between the existence of proper 'checks and balances' and stability of government forms seems to be in principle well within the reach of clever constitutional design.

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