QUADERNI

Università degli Studi di Siena
DIPARTIMENTO DI ECONOMIA POLITICA

LUIGI M. TOMASINI

Differences in Wealth and Altruistic Behaviour: A Model

n. 420 - Febbraio 2004

Abstract - In an economy with N individuals each individual is concerned, by assumption, with his own felicity as well as that of all or some others. If the society is composed of very few individuals with a high level of wealth and many with a low one and if altruistic behaviour of any two individuals toward each other decreases with their wealth difference, the largest contribution to the felicity of other individuals is made by those whose wealth is neither too high nor too low (middleclass).

Jel Classification: C00

Luigi M. Tomasini, Dipartimento di Economia Politica, Università di Siena

1. Protecting the environment, conserving natural resources, eliciting donations to charity, slowing arms races and promoting other aspects of the general good that demand collective effort and cooperation have been investigated by sociologist, economists and political scientists .

Economists have explored reciprocal altruism, a particular form of cooperative behaviour, constructing a variety of experiments to understand why individuals often cooperate with others to whom they are unrelated. It has been founded out that people reward those who act in a co-operative manner and punish those who do not, even if such behaviour costs them something personally, and even though there are no foreseeable future rewards from so behaving.

The results obtained in some of these experiments suggest that people sometimes value fairness over personal gains. The logic behind it is that, if people want to maximize their utility -as standard economics assumes- a recipient will accept any offer made by a donor while, conversely, the donor will always offer as little as possible. Acceptance and rejections appear to be strongly linked - in addition to material benefits - to feelings of fairness and reciprocity.

Moreover it has been found out that small groups of people are more likely to cooperate than larger ones and that cooperative attitudes increase in repeated iterations of a situation and when the participant. are allowed to communicate. Overall cooperation cannot however persist in groups that exceeded a critical size, the size depending on how long individuals expect to remain part of the group and on the amount of information available to them.

Gintis [1] has suggested that if policymakers want to achieve certain public goods, such as the sharing of common resources, it might be appropriate to provide opportunities for the public-spirited to punish the free riders in society. Punishment has the effect to substantially increase the amount that is invested in the public good. Altruism in other words emerges when people, to enforce a kind of *"fairness",* are willing to pay to punish someone with whom they would not interact again even when there are no foreseeable rewards or benefits.

There is no doubt that individuals will be better off if each and all of them adopt mutual aid behaviour. But such behaviour depends on the existence and the strength of the social norms in force in the societies where they live: if strong, the tendency to help others will be high while will decrease if and when weak. Social norms encourage altruism and contribute to the creation of an environment where altruistic behaviour can prosper.

A pure altruistic society presupposes perfect information among its members. Since anyone who cheats is certainly caught and punished, no rational individual will try to break the social norms governing that society. Imperfect information encourages cheating behaviour (an act of cheating may not be discovered by all other individuals) but is conducive to a stricter enforcement of social and legal norms.

2. In this kind of framework and as a first approach to the problem, I investigate a model of *altruistic behaviour* where the degree of altruism decreases with increasing social distance measured by the difference in wealth between two individuals. Assuming an "appropriate" distribution of wealth it is proved that the largest contribution to the felicity of some members of society is made by middleclass individuals.

Let us indicate with

(1) $F_i\,(x_1,\ldots,x_m)$

the felicity the *i.th* $(i \in N)$ individual has from the consumption of goods $(x_1,\ldots,x_m)$. In a purely individualistic society the *i.th* individual will maximize (1) so that the quantities $x_j\ (j=1,\ldots,m)$ are determined by the m equations

(2) $dF / dx_j = 0 \qquad\qquad (j = 1,\ldots,m)$

Furthermore, second order conditions require for a maximum that the *k.th* degree form

(3) $$\left(\sum_{j=1}^{m} b_j \frac{\partial}{\partial x_j}\right)^k F(x_1,\ldots,x_m)$$

must be negative definite.

Let us consider two individuals whose felicity will influence and be influenced by the felicity of the other. The felicity functions of the two individuals can be written as

(4) $F_j\,(x_{11},\ldots\, x_{1m}\,;\, x_{21},\ldots\, x_{2m}) \qquad (i = 1,2)$

As the individual behaviour is, in general, different the felicity function $F_1(\cdot)$ of the first individual will be different from that of the second $F_2(\cdot)$.

If the individual is egoistic (selfish) his behaviour is such that he will be concerned only with his felicity ignoring how his choice of the x_j affect the other individual felicity. He will then try to maximize his own F and hence we have

(5) $\partial F_i / \partial\, x_{ij} = 0 \qquad (i = 1,2\ ;\ j = 1,\ldots,m)$

together with the inequalities given by (3).

If an individual is concerned with the felicity of the other (altruistic behaviour) he will try to make $F_1 + F_2$ as large as possible i.e.

(6) $\partial(F_1+ F_2)/\partial x_{ij} = 0 \qquad (i = 1,2\ ;\ j = 1,\ldots,m)$

together with the inequalities given by (3).

Consider now an economy with N individuals, where we assume that N is very large. Let

(7) $N\,(w)\,dw$

be the number of individuals whose endowment (wealth) lies between the values w and $w+dw$ where we chose w so that it belong to the interval $[0,1]$, i.e. the lowest value in the economy is zero, while the highest is one. Then we have

(8) $$\int_0^1 N(w)dw = N$$

We assume that in our economy the *i.th* individual will be concerned both with his own felicity and with that of all or some others. We can assume that he acts so as to

(9) $$\max r_{i1} F_1(x) + r_{i2} F_2(x) + \ldots + r_{iN} F_N(x)$$

a linear combination of the felicity of all the individuals.

In (9) some r_{ij} may be zero and some may be even negative. The *i.th* individual is defined as an egoist (selfish) if $r_{ii} > 0$ but $r_{ij} = 0$. In this case he is only concerned with his felicity F_i. For the *altruistic behaviour*, all $r_{ij} > 0$ except $r_{ii} = 0$.

If we arrange the quantities r_{ij} we obtain a square matrix R

(10) $$\begin{vmatrix} r_{11} & \ldots\ldots & r_{1N} \\ r_{21} & & r_{2N} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ r_{N1} & \ldots\ldots & r_{NN} \end{vmatrix}$$

which tells us how people behave in our economy. This *social matrix* describes the existing level of altruism in a social system. Hence, if all elements in R except the diagonal (which may be chosen as equal to 1) are 0, the society is composed of egoistic individuals since each member considers only his own felicity. Thus a society of *selfish individuals* is characterized by

(11) $$\begin{vmatrix} 1 & 0 & \ldots\ldots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots\ldots & 1 \end{vmatrix}$$

On the other hand an economy of *altruists* (at the maximum) is characterized by

(12)
$$\begin{vmatrix} 0 & 1 & \ldots\ldots & 1 \\ 1 & 0 & & 1 \\ : & : & & : \\ : & : & & : \\ 1 & 1 & \ldots\ldots & 0 \end{vmatrix}$$

If the entries of R are all equal, the society is egalitarian. A society composed of altruistic individuals who will be concerned equally with their own felicity and with that of the others (i.e. if they try to make the sum of felicities $F_1 + F_2 + \ldots + F_N$ as large as possible) is characterized by

(13)
$$\begin{vmatrix} 1 & 1 & \ldots\ldots & 1 \\ 1 & 1 & & 1 \\ : & : & & : \\ : & : & & : \\ 1 & 1 & \ldots\ldots & 1 \end{vmatrix}$$

3. Let us consider the following felicity function (inverted parabola)

(14)
$$F_i = a_i\left(\sum_j c_{ji}x_j\right) - d_i x_i^2$$

where a and d are two coefficients and c_{ji} represent a fraction of the *j.th* individual consumption enjoyed by the *i.th* individual.

Putting $r_{ii} = 1$ the *i.th* individual will try to maximize

(15) $F_i^* = r_{i1}F_1 + r_{i2}F_2 + \ldots F_i + \ldots r_{ij}F_j + \ldots r_{iN}F_N$

The N equations given below will determine the values of x_i

(16) $\partial F_i^*/\partial x_i = 0$ $\qquad (i = 1,\ldots,N)$

Introducing (14) into (15) and the latter into (16) we obtain

(17) $r_{i1}\, a_1\, c_{1i} + \ldots\, a_i + \ldots r_{iN}\, a_N\, c_{N\,i} - 2d_i\, x_i = 0$

which gives

(18) $x_i = (a_i /2d_i\,)+(r_{i1}\, a_1\, c_1 + \ldots + r_{iN}\, a_N\, c_{N\,i}) \,/\, 2d_i$

Assuming (7) and (8) the coefficients a_i and d_i become now functions of *w*, *a(w)* and *d(w)*. The coefficient r_{ij} represents the *degree of altruism* of the *i.th* individual for the *j.th* one. In the continuous case, however, r_{ij} represents the degree of altruism of an individual of endowment *w* with respect to another with endowment *z*. Thus r_{ij} becomes a function *r(w,z)* of those two variables. Similarly, the c_{ji} become functions of those two variables : *c(z,N)*

Integrating (16) with respect to *z* , we have

(19) $$x(w) = \frac{a(w)}{2d(w)} + \left(\int_0^1 N(z) a(z) r(w,z) c(z,w) dz \right) / \, 2d(w)$$

We shall assume that there are very few individuals with a large wealth and many with a small one i.e.

(20) $N(w) = L\, e^{-sw}$

where *L* and *s* are two constants.

The coefficient *L* is determined from condition (8). Since

(21) $$L \int_0^1 e^{-sw} dw = (L/s)\left(1 - e^{-s}\right)$$

condition (8) requires that

(22) $$L = sN / \left(1 - e^{-s}\right)$$

Hence we have

(23) $N(w) = [s\,N\,/\,(1\text{-}e^{-s})]\;\; e^{-sw}$

4. Sorokin [4] suggested that an individual concern on another decreases with increasing social distance, in our case measured by wealth difference. Mathematically it is convenient to choose *r(w, z)* as

(24) $r(w, z) = e^{-h|w-z|}$
where *h* is a coefficient.

Let us now indicate with three constants

(25) $a(w) = a$; $d(w) = d$; $c(z,w) = c$.

Equation (19) now becomes

(26)
$$x(w) = \frac{a}{2d} + \left(\frac{ac}{2b}\right)\left(\frac{sN}{1-e^{-s}}\right)\int_0^1 e^{-sz}e^{-h|w-z|}dz$$

Because of the absolute value $|w-z|$ in (24) we see that

(27) $r(w, z) = e^{-s|w-z|}$ for $w > z$

(28) $r(w, z) = e^{s|w-z|}$ for $w < z$

Therefore, integral in (26) may be written

(29)
$$\int_0^w e^{-sz}e^{-h(w-z)}dz + \int_w^1 e^{-sz}e^{h(w-z)}dz =$$
$$= e^{-hz}\int_0^z e^{-(s-h)z}dz + e^{hz}\int_z^1 e^{-(s+h)z}dz$$

Introducing the resulting integrals into (26), we find

(30)
$$x(w) = \frac{a}{2d} + \frac{acsN}{2d\left(1-e^{-s}\right)}\cdot$$
$$\cdot\left[\frac{1}{s-h}e^{-hw} - \frac{2h}{s^2-h^2}e^{-sw} - \frac{1}{s+h}e^{-s-h(1-w)}\right]$$

For $w=0$ the expression in brackets is

(31) $(1/(s+h))\,(1-e^{-(s-h)}) > 0$

For $w=1$ it is equal to

(32) $(1/(s-h))\,(e^{-h} - e^{-s})$

If $s > h$, then both $1/(s-h) > 0$ and the difference of the exponential is also positive. If $s < h$, then both $1/(s-h)$ and the difference of the exponentials are negative. Hence in both cases the expression in brackets of (30) is positive. For $s = h$ we find that the expression [ ] has the same value, $e^{-h} > 0$.

Though for very large values of w the value of $x(w)$ becomes negative, yet, within the range $[0,1]$ of w, $x(w) > 0$.

5. The first derivative of [•] of (30) is

(33) $$-\frac{h}{s-h}e^{-hw}+\frac{2sh}{s^2-h^2}e^{-sh}-\frac{h}{s+h}e^{-s-h(1-w)}$$

For $w=0$ this expression becomes

(34) $(h/(s+h))(1-e^{-(s+h)}) > 0$

For $w=1$, expression (33) becomes

(35) $(h/(s-h))(e^{-s}\ e^{-h})$

Using an argument similar to that one used to prove the positivity of (26) we can prove that expression (35) is negative.

Since the derivative of the expression in brackets in eq. (30) is positive for $w=0$ and negative for $w=1$, it follows that the expression in brackets, and hence $x(w)$ itself, has a positive maximum somewhere between $w=0$ and $w=1$.

Since $x(w)$ is the contribution of an individual of wealth w to the felicity of all the individuals we can say that in a society where wealth is distributed according to eq. (23) and in which altruistic behaviour of any two individuals toward each other decreases with their wealth-distance according to eq. (24), the largest contribution to the felicity of other individuals is made by individuals whose wealth is neither too high nor too low *(middleclass).*

6. Some more remarks can be made considering equation (30). If $h \to \infty$ the function $r(w, z) \to 0$. This means that very large values of h correspond to a very small concern of an individual about others (*egoistic behaviour*). As h increases, eq. (30) tends to $a/2d$, which is its smallest value. *Altruistic behaviour* increases x (w).

Let us consider now the value of s. As $s \to \infty$, the relative number of individuals with low wealth increases. A large value of s means a small privileged class of wealthy people. With increasing s, $x(w)$ decreases and tends to $a/2d$.

For values of $h=0$, when each individual is interested equally in the felicity of all the individuals, $x(w)$ has the value

(36) $(a/2d)(1 + cN)$

which is independent of the wealth and its distribution. If $h > 0$ but $s = 0$, i.e. when there is the same number of individuals in each wealth bracket, then

(37) $x(w) = a/2d + (acN/2dh)(2- e^{-hw}- e^{-h(1-w)})$

In this case the derivative of the expression [•] in brackets of (37) is

(38) $h\, [e^{-hw}- e^{-h(1-w)}]$

which is equal to zero for $w=1/2$. Hence in this case the maximum contribution to the felicity of the whole society is given by individuals with $w=1/2$.

7. It is important to note that in the transition from eq. (16) to eq. (19) we assumed that the coefficients, a_i and d_i are functions of the wealth only; that the coefficients r_{ij} are functions of the difference in wealth only and that those are the same functions for all individuals.

Coefficients a_i and d_i may not however depend on wealth but rather on some individuals' innate biological characteristics distributed in an economy. They may also depend on the occupations that can be different within the same wealth bracket.

Within these limitations we can speculate that, in general, the concern of an individual with the felicity of another - the *degree of altruism* -depends on the social distance as well as on the physical distance, decreasing as this increases.

References

1.Gintis, H., *Game Theory Evolving*, Princeton University Press, Princeton, N.J.2000.

2.Kurz, M., Altruistic Equilibrium, in *Economic Progress, Private Values and Policy*, 1977.

3.Nowak, M. and Sigmund, K., Evolution of Indirect Reciprocity by Image Scoring*, Nature, vol.393, June 1998.*

*4.*Sorokin, P., *Social and Cultural Dynamics*, American Book Co. New York,1937.

5.Trivers, R.L., The Evolution of Reciprocal Altruism, *Quarterly Review of Biology, 1971, vol.46.*