Market Structure Analysis Using Birth and Asymmetric Growth of Products Based on a Mechanism of the 80/20 Law: Why and How the 80/20 Law Emerges

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Abstract

Consumers, companies and products differ among markets and change according to time; why does a universal structure – the 80/20 law – hold in any market and time, how does it emerge, and what is its meanings in marketing? This paper, after proving that the 80/20 law is universally observed in markets and time by consumer purchasing data, discusses its emerging mechanism and managerial meanings. First, a simulation model shows that the 80/20 law emerges only if a product occurs constantly and grows in process of time. Since birth and growth of products are natural in a market, the 80/20 law emerges inevitably and autonomously in any market. Next, this paper proposes market structure analysis method based on the mechanism of the law. While birth and growth of products are common to all markets, birth probability of a new product varies among markets, and a growth rate of existing products is asymmetric among products within a market (i.e. whether the growth rate of products increases or decreases according as products grow). By referring to the universal parameter, birth and asymmetric growth of products, companies compare directly any market and comprehend how products are born and grow in a market.

Key words: power models, the Pareto model, the Zipf model, the Mandelbrot model, market structure, market concentration, Monte Carlo simulation

1. Introduction

As information technology has advanced, companies have utilized easily more data for their management. One of contributions by marketing data analyses is that companies recognize valuable products, brands, customers, salespersons, transactions and other marketing components. In fact, values of marketing components differ widely. Only a few of these components produce the most of sales and profits; on the other hand, many of them yield small outcomes (Dubinsky and Hansen 1982; Hallberg 1995, Hise and Kratchman 1987; Schmittlein, Cooper, and Morrison 1993, Wolf 1996). That is, size distributions of marketing components are highly skewed. For example, the top 20% products rules 83% market shares in a 100% fruit juice market, 84% in an aluminum foil market, 87% in a coke market, and 89% in a detergent market (Japan, 1999)[1].

While these markets (e.g. a 100% fruit juice market) have little similarity except they are consumer packaged goods markets, they have a common or universal market structure: the product size distributions are highly skewed. Furthermore, this structure depends upon neither a particular product, market nor time, since many products always enter into and withdraw from markets but such structure is stable for years. In a beer market, for instance, the top 20% products occupied 86% (1990) and 87% (1999) market shares, and the top 30% products did 93% in both 1990 and 1999, even though 343 products were in the market in 1999 and the only 32 of them survived from

[1] Home scan panel data collected at Tokyo metropolitan area in Japan.

1990[2].

Companies sell products and consumers buy them freely in a market, and their behavior varies and changes. Hence, a market (the total sum of their behavior) may differ among markets and change as time passes. Nevertheless, universal structures or laws are observed regardless of markets or products[3]. One of such structures is the 80/20 law, since this law is derived from power models describing skewness common to various social and natural phenomena (Chen, Chong and Tong 1994; Lipovetsky 2009; Sanders 1987).

The 80/20 law describes skew phenomena in marketing. It shows that few marketing components (e.g. products, brands, customers, salespersons or transactions) produce the most of outcomes (e.g. sales or profits). That is, a very small proportion of company's marketing units (salespeople, products or customers) provide a large proportion of company profits, and other units add very little to (or detract from) the profits (Dubinsky and Hansen 1982); a small percentage of various aspects of its operations (customers, products, sales personnel and orders) account for a large percentage of a company's sales, on the other hand, a large percentage of them account for a small percentage of company's sales (Hise and Kratchman 1987); a large portion of the product's total purchases are made by a small fraction of all customers (Schmittlein, Cooper and Morrison 1993).

However marketing literature has long discussed the 80/20 law, only few investigations have empirically examined it (Dubinsky and Hansen 1982). As far as the author knows, marketing literature has seldom shown its structures or mechanisms. Hence, the 80/20 law's forming mechanism and theoretical meanings in marketing will be discussed. The purposes of the article are:

- showing that the 80/20 law holds universally in markets,
- discussing its mechanism (how it emerges, why it is commonly observed nevertheless markets differ and change), and
- validating empirically its forming mechanism, and proposing managerial implications by its theoretical meanings.

This paper's results are shown in advance as follows. First, after review of the 80/20 law and power models (2. Literature Review), data will show that the 80/20 law holds universally in markets. That is, home scan panel data collected in 31 markets during 2 years will show that the law formulated by the Zipf model, one of power models, is observed commonly in markets (3. Study 1: The 80/20 Law by the Pareto Model). Furthermore, data collected in 16 markets during 10 years will show that the law re-formulated by the Mandelbrot model, one of the general expressions of power models, is observed regardless of markets and time (4. Study2: The 80/20 Law by the Mandelbrot Model). These models' parameter describes theoretically birth probability of a new

[2] Home scan panel data collected at Tokyo metropolitan area in Japan.

[3] Universal structures or laws in marketing are externally uncontrollable phenomena common to markets, products, consumers, and/or companies (e.g. double jeopardy: Ehrenberg, Goodhardt, and Barwise 1990).

product and an asymmetric growth rate of existing products. The latter means that the growth rate of products increases according to product growth in some markets and decrease in other market, or that a growth rate of upper ranking products is larger than that of lower ranking products in some market and smaller in other markets.

Secondly, an emerging mechanism of the 80/20 law will be discussed. That is, a Monte Carlo simulation model in which a product occurs and grows virtually generates product size distribution data matching the home scan panel data collected in 16 markets during 10 years (5. Study 3: A Forming Mechanism by a Monte Carlo Simulation Model, Result 1). At this time, the 80/20 law emerges in any value of parameter in the simulation model (5. Study 3: A Forming Mechanism by a Monte Carlo Simulation Model, Result 2). That is, the law emerges, only if a product occurs constantly and grows in process of time. Since product birth and growth is natural in a market, the 80/20 law emerges inevitably and autonomously (or it organizes itself).

Thirdly, the 80/20 law's emerging mechanism will be empirically validated, and market analysis method will be proposed based on its mechanism (6. Managerial Implications). That is, since the 80/20 law holds universally and emerges inevitably in any value of the parameter in the simulation model, the parameter are universal among markets. On the other hand, the values of the parameter vary among markets. Hence, referring to the universal parameter calculated by the simulation model enables companies to compare directly markets and to comprehend them: whether a new product is born frequently or seldom, and whether a growth rate of existing product increase or decrease according to product growth. Finally concrete cases will validate empirically the value of the parameter and the market analysis method.

2. Literature Review

2.1. The 80/20 Law

First, research on the 80/20 law in marketing will be reviewed, then, power models formulating the 80/20 law will be reviewed. There are some possible approaches to research the 80/20 law in marketing literature. One is focusing on external skewness of marketing components in order to suggest managerial implication; another is discussing structures or mechanisms of the law from marketing viewpoints.

The former has focused on the external skew distribution when 80/20 law holds common to markets (Kumakura 1999, 2000b), and identified an individual marketing component to improve companies' performance and to increase their efficiency or productivity. That is, Dubinsky and Hansen (1982) has investigated 62 companies for their sales size, a number of salespeople, a number of costumers, and a number of products, and then shown marketing strategies how to improve companies' productivity. Hise and Kratchman (1987) have proposed concrete programs to improve performance and increase efficiency in various components (e.g. customer, sales personnel, products, orders, shipments, advertisements, geographical areas). The greatest common implication in those studies is that companies improve their performance by distinguishing profitability or efficiency of each component (e.g. customer), and focusing on better one and omitting worse one (Wolf 1996).

The latter studies have discussed structures or mechanisms of the 80/20. Schmittlein, Cooper and Morrison (1993) have considered the 80/20 law to be skew distribution of customers, and estimated the level of concentration among customers by panel data. At that time, they calculated the true (long run) level of concentration by the negative binominal distribution (NBD) model, since concentration among customer varies depending on observation length. And Anschuetz (1997) has argued that, if the law is determined by the purchase frequency distribution calculated by NBD, it is a predictable and regular feature of consumer behavior. Kumakura (2000a) has mathematically derived the law from a notional assumption that a larger size product requires several smaller size products for maintaining attractiveness of the larger size product. Furthermore, Kumakura (2001, 2002a, 2002b) has discussed a mechanism of the law by focusing on birth and growth of products, and generated it by a Monte Carlo simulation model. However, as far as the author knows, only few studies have discussed structures or mechanisms of the law.

2.2. Power Models

Next, power models will be reviewed, since the 80/20 law is formulated with the Pareto model (Chen, Chong, and Tong 1994; Lipovetsky 2009; Sanders 1987), and since the Pareto model is formulated with power models. Some typical models and phenomena following these models will be shown, and a forming mechanism of those phenomena will be briefly reviewed.

Models and Phenomena

Power models describe skew phenomena (Kisida 1988), and various models have been proposed (Haitum 1982a, 1982b, 1982c; Onodera 1988). These models have two mathematical expressions. The first is frequency distribution models, in which a horizontal axis is size and a vertical axis is frequency; the second is rank-size models, in which a horizontal axis is ranking and a vertical axis is size. These two expressions are approximately equal under a condition that size of all components differs (Kishida 1988).

Many natural and social skew phenomena following power models have empirically been shown (Chen and Leimkuhler 1986). In natural science, for example, Gutenberg and Richter (1944) have shown a famous empirical law about size of an earthquake and its frequency (a frequency distribution model),

$$f(E) = b_1 E^{-a_1}, \qquad (1)$$

where

E: the energy (size) of the earthquake,

$f(E)$: the frequency of earthquakes of which energy is E.

Equation (1) is linear in a log-log graph. Values of a_1 are nearly 1.5 regardless of regions in any region.

In social science, both bibliography and urban economics have paid special attention to power models. Lotka (1926), for instance, has shown the Lotka model (a frequency distribution model):

$$f(n) = b_2 n^{-a_2}, \qquad (2)$$

where

n: the number of academic papers (size),
$f(n)$: the frequency of chemists publishing n academic papers.

Then, the Zipf model, one of rank-size models, has shown a relationship between population size of a city and its ranking (Zipf 1949):

$$s(r) = b_3 r^{-a_3}, \qquad (3)$$

where

r: the ranking of a city,
$s(r)$: the population size of the r-th ranking city.

Values of a_3 are nearly 1 regardless of time and place. It means that size of the 2nd largest city is half of the largest city, and that of the 3rd largest city is 1/3 of the largest city[4]. These phenomena are observed not only in the U.S. at least since 1890 (Krugman 1996) but also in various times, countries and regions (Guseyn-Zade 1977; Kikuchi 1986; Rosen and Resnik 1980).

In economics, the Pareto law has been proposed as follows (Kimura 2005):

$$n(s) = b_4 s^{-a_4}, \qquad (4)$$

where

s: the income size of people or households,
$n(s)$: a number of people or households whose income is equal or larger than s.

This describes uneven distribution of wealth within a country, and it has been shown empirically in European countries. Namely, the UK (a_4=1.50 in 1843), the Prussia (1.89 in 1852), and the Basel (1.24 in 1887) (Kimura 2005). Incidentally, equation (4), the Pareto law, is mathematically the same as equation (3), the Zipf model, under an assumption that income size of all people or households differs.

Many other natural and social phenomena following power models have been observed. Namely,

- size s and the frequency $f(s)$ of companies whose size is s, or ranking r and the size $s(r)$ of companies whose ranking is r (Ijiri and Simon 1964; Simon 1955; Simon and Bonini 1958;

[4] For example, the population of New York was 8.36 million, that of Los Angeles was 3.83 million (almost half of NY's), and that of Chicago was 2.85 million (almost 1/3 of NY's) (July 2008, the United States Census Bureau).

Quandi 1966),

- the annual export/import ranking r of countries with a certain partner country and the export/import size $s(r)$ of countries whose ranking is r (Musha 1980),
- size s (the usage quantity) of words in an English written work and the frequency $f(s)$ of words whose usage quantity is s,
- size s (the volume of books which a library owns) and the frequency $f(s)$ of libraries whose size is s, or the ranking r of libraries and the size of libraries $s(r)$ whose ranking is r,
- the size s of snow slides and the frequency $f(s)$ of snow slides whose size is s,
- the size s of meteorites falling to the Earth and the frequency $f(s)$ of meteorites whose size is s,
- the size s of "genera" (the number of "species" composing a genus) and the frequency $f(s)$ of genera whose size is s,

and other phenomena (Chen and Leimkuhler 1986). In marketing literature, lifetime of a customer follows power models (Schmittlein, Morrison, and Colombo 1987). Furthermore, the relationship between product ranking r and the size $s(r)$ of product whose size is s follows power models (Kumakura 1999, 200a, 200b).

Forming Mechanisms

While the emerging mechanisms of those phenomena have not been generalized yet (Krugman 1996), a theoretical hypothesis has been proposed. Simon and his co-researchers (Ijiri and Simon 1964; Simon 1955; Simon and Bonini 1958)[5], for example, have proposed an emerging mechanism. That is, relationship between ranking and sales size of a company follows a rank-size model:

$$s(r) = b_5 r^{-a_5}, \qquad (5)$$

where

r: the ranking of a company,

$s(r)$: the size of the r-th ranking company,

if

1) a new company constantly emerges (probability that a new company enters into a market is larger than zero), and if
2) the frequency distribution of the growth rate of a company is independent from company size (the law of proportionate effect[6]).

5 Fujita, Krugman and Venables (1999), and Krugman (1996) have shown briefly the research of Simon and his co-researchers.

6 The law of the proportionate effect is a phenomenon that probability distribution of the growth rate of a component during a certain period is equal among all size sets, if a size set is composed of same size components (e.g. a size set is composed of cities having a same population), and if component size is larger than a certain critical size. Note that not the growth rate of *individual component* but that of a set (*the total sum of size of all components* in a set) is equal among all sets. That is, all size sets have the same probability

Furthermore, they suggested that power parameter a_5 relates to birth probability p_5 of a new company. Namely, $a_5 \cong 1 - p_5$ (Fujita, Krugman, and Venables 1999).

3. Study 1: The 80/20 Law by the Pareto Model

Since the 80/20 law has originated from the Pareto model (Chen, Chong, and Tong 1994; Lipovetsky 2009; Sanders 1987), one of the rank-size models, the 80/20 law will be first formulated with a rank-size model (the Zipf model). Then, its theoretical implication will be discussed. That is, parameter in the Zipf model relates to birth probability of a new product and market concentration. Next, home scan panel data collected in 31 markets during 2 years will show that the 80/20 law is observed regardless of markets and time.

3.1. Model

Formulation

Based on equation (4), and with the assumption that size of all products differs, the 80/20 law is formulated as follows,

$$s(r) = b_6 r^{-a_6}, \qquad (6)$$

where

r: the ranking of product sales size,
$s(r)$: sales size of the r-th ranking product,
$1 \leqq r \leqq N$,
N: a number of products (a positive integer, finite),
$0 < a_6$, b_6 (parameter).

Since this model is mathematically identical with equation (3), it is called the Zipf model.

Theoretical Implications

Next, theoretical implications of power parameter a_6 will be discussed. First, according to Simon (1955), a_6 relates to the birth probability of a component (e.g. company and city). Second, a_6 expresses market concentration as follows. The ranking r is assumed to be a positive continuous variable, regardless of whether it is a positive integer. With rankings r_1, r_2 ($r_1 < r_2$) and sizes s_1, s_2 ($s_1 > s_2$), equation (6) is

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distribution of the growth rate. For example, probability that a product chosen randomly from a set in which product size is 1 million grows 10% and probability that a product chosen randomly from another set in which product size is 1 billion grows 10% are equal. Simon and Bonini (1958) have exhibited that the law of constant returns causes theoretically the law of the proportionate effects. Incidentally, returns have not been constant according to PIMS (Buzzel 1981).

$$\frac{s_1}{s_2} = \left(\frac{r_1}{r_2}\right)^{-a_6} = \left(\frac{r_2}{r_1}\right)^{a_6}. \qquad (7)$$

Since

$$\frac{r_2}{r_1} > 1, \qquad (8)$$

this is assumed to be constant. As a_6 becomes larger, the left side of equation (7) becomes also larger. That is, when a_6 is larger, the ratio of the larger (upper ranking) product to the smaller (lower ranking) product becomes larger. Hence, a_6 expresses market concentration among products. In a log-log graph, if a_6 approaches 0, the slope of equation (6) also approaches 0 and the line of equation (6) becomes nearly parallel with axis x. Since the size of all products is almost the same at this time, regardless of their ranking, the market concentration is very small. If a_6 approaches infinite, the line of equation (6) is nearly parallel with axis y. At this time, the market concentration is very high. Above all, power parameter a_6 indicates the market concentration. On the other hand, a_6 relates to birth probability of a component (Simon 1955). Hence, concentration among products in a market depends on the market entry rate of a new product.

3.2. Method and Data

Method

Next, adapting equation (6) to home scan panel data will show that the 80/20 law is observed in markets. Parameter a_6 and b_6 were estimated using the linear least square method after the log linear transformation. Parameter was estimated in two cases:

- case 1) parameter was estimated with all products in a market, and
- case 2) a few small products in the bottom 5% market share were omitted.

Case 2) is justified. The reasons for this are as follows. Some products which should be discontinued exist sometimes in markets. However, market structure is well distinguished, when products which are circulated virtually in a market are analyzed. Furthermore, small-size components composing the lower end of the distribution are of little importance in the phenomena following power models (Hioki 1998).

Data

Home scan panel data was collected in the following 31 markets during 2 years[7].

[7] An outline of the home scan paned data is as follows:

- areas: cities, towns and villages within 30 kilometers from the Tokyo central station,
- sample (respondents): households in which housewives' age is under 59,

- food and beverages (15 markets): soy sauce, soybean paste, butter, ketchup, salad oil & sesame oil, seaweed with prepared rice, portable noodles (bag type instant noodles), regular coffee, 100% pure fruit juice, cola, salt, sauce, mayonnaise, curry roux, and boil-in-bag curry;
- commodities (16 markets): heavy detergent, aluminum foil, shampoo, tissue paper, toilet bowl cleaner, bathroom cleaner, hair conditioner, hair spray, body shampoo, wrapping film, insecticide, toothpaste, kitchen cleanser, household cleaner, air freshener, and mothballs;
- periods (2 years): January 1 to December 31 in 1994, and January 1 to December 31 in 1998.

In this study, a stock keeping unit (SKU) was used for a unit of a product (component), and sales size (purchasing dollar volume per 100 households) of an SKU was used for the product size. Incidentally, a brand (a product set under a identical brand name) was not used for a unit of a product, since the boundaries of a brand are sometimes obscure.

3.3. Results

Case 1 (All Products)

Parameter was estimated with the above method and data; the fitness of equation (6) was examined. Results of a heavy detergent market in 1998[8] are shown among others. In the first case, equation (6) was applied to all products and estimated as follows:

$$s(r) = 236{,}263r^{-1.886} ,$$

where

r: the ranking of product sales,

$s(r)$: purchasing dollar volume per 100 households for the r-th ranking product.

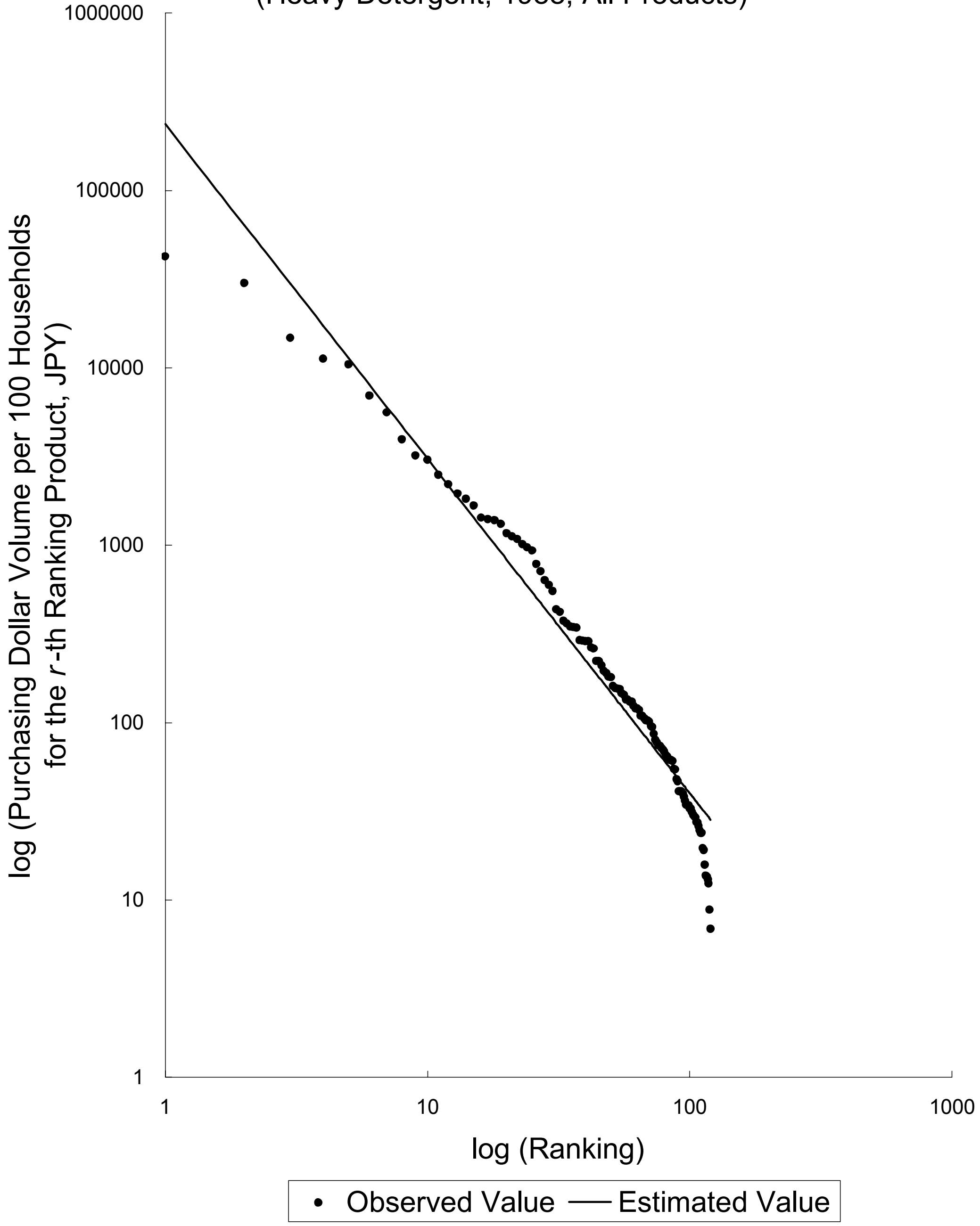
In this case, the sample size was 120. Since the coefficient of determination was R^2=0.957 (F(1, 118)=2620, p<0.001), and since the standard error of estimate was SEE=0.376 (the range of the response variable after log linear transformation was 1.929 to 10.658), equation (6) fitted data (Figure 1).

- sampling method: random two stage sampling,
- sample size: 2,500 households, and
- gathered by Tokyu Agency Co. Ltd. and Tokyu Research Institute.

[8] This market was illustrated as follows with data:

- the number of products: 120,
- the purchasing dollar volume per 100 households: ¥167,629.98 (Japanese Yen),
- the percentage of the purchasing household (penetration): 73.5%,
- the average prices of products: ¥342.76.

Figure 1
The Rank-Size Rule, Equation (6), the Zipf Model
(Heavy Detergent, 1988, All Products)



The regression results adapting equation (6) to the data were good in all markets and all periods according to both R^2 and *SEE* (Table 1-1 and 1-2). Hence, equation (6) fits the data. Incidentally, the maximum value of R^2 was 0.976 (tissue paper, 1994) and the minimum was 0.824 (salt, 1994).

Table 1-1

Regression Analysis, Equation (6), the Zipf Model (All Products, Foods and Beverages)

Market	Year	Parameter *a*	Parameter *b*	Sample Size	Coefficient of Determination*	Standard Error of Estimate*	Range of Response Variable*	
Soy Sauce	1994	1.673	133,294	156	0.916	0.482	1.932	10.406
	1998	1.608	93,547	193	0.957	0.324	1.908	10.096
Soybean Paste	1994	1.342	92,632	493	0.935	0.345	1.847	10.236
	1998	1.352	102,113	570	0.942	0.329	1.823	10.293
Butter	1994	1.894	81,266	53	0.963	0.334	3.338	10.094
	1998	2.106	121,015	48	0.924	0.543	2.517	9.983
Ketchup	1994	1.824	61,750	80	0.960	0.345	1.932	10.121
	1998	1.711	49,029	87	0.945	0.383	1.929	9.983
Salad Oil & Sesame Oil	1994	1.666	252,779	167	0.844	0.682	1.932	9.857
	1998	1.613	141,341	162	0.918	0.460	1.929	9.788
Seaweed with Prepared Rice	1994	1.588	20,614	85	0.952	0.332	1.932	9.438
	1998	1.670	28,575	104	0.972	0.267	1.235	9.700
Portable Noodles	1994	1.695	505,805	267	0.838	0.719	1.595	9.792
(Bag Type Instant Noodles)	1998	1.696	394,476	298	0.890	0.577	1.194	9.597
Regular Coffee	1994	1.352	66,505	271	0.892	0.453	1.952	8.494
	1998	1.375	96,168	325	0.874	0.506	1.131	9.270
100% Fruit Juice	1994	1.537	419,820	561	0.875	0.568	1.703	9.244
	1998	1.633	408,556	504	0.880	0.590	1.235	8.691
Cola	1994	2.104	247,708	78	0.863	0.772	1.567	10.333
	1998	2.284	221,306	62	0.920	0.615	1.654	10.767
Salt	1994	1.504	29,452	74	0.824	0.638	1.813	9.842
	1998	1.553	26,790	98	0.916	0.438	1.908	8.696
Sauce	1994	1.655	59,988	139	0.950	0.360	1.037	10.000
	1998	1.547	41,582	154	0.961	0.297	1.918	9.767
Mayonnaise	1994	1.876	198,896	76	0.867	0.676	0.867	0.867
	1998	1.740	113,170	89	0.914	0.497	0.914	0.914
Curry Roux	1994	1.799	454,242	225	0.931	0.470	1.932	9.993
	1998	1.886	515,554	181	0.890	0.632	1.929	9.719
Boil-in-Bag Curry	1994	1.452	91,282	191	0.866	0.547	1.952	8.720
	1998	1.482	105,781	286	0.898	0.482	1.845	9.197

* Log linear transformation

Table 1-2

Regression Analysis, Equation (6), the Zipf Model (All Products, Commodities)

Market	Year	Parameter *a*	Parameter *b*	Sample Size	Coefficient of Determination*	Standard Error of Estimate*	Range of Response Variable*	
Heavy Detergent	1994	1.889	382,258	113	0.921	0.518	2.635	10.951
	1998	1.886	236,263	120	0.957	0.376	1.929	10.658
Aluminum Foil	1994	1.552	13,235	75	0.956	0.305	1.825	8.937
	1998	1.596	11,914	67	0.940	0.368	1.802	8.293
Shampoo	1994	1.432	249,172	421	0.917	0.420	1.952	9.974
	1998	1.358	202,128	495	0.881	0.487	1.802	9.601
Tissue Paper	1994	1.648	95,225	272	0.976	0.248	1.442	10.826
	1998	1.673	111,454	264	0.969	0.287	0.880	10.606
Toilet Bowl Cleaner	1994	2.186	22,296	22	0.916	0.557	2.645	8.751
	1998	1.952	22,518	43	0.929	0.481	1.929	8.318
Bath Room Cleaner	1994	1.909	15,558	31	0.949	0.383	1.952	9.166
	1998	1.732	22,034	64	0.953	0.351	1.694	9.110
Hair Conditioner	1994	1.498	96,050	169	0.914	0.437	1.952	9.455
	1998	1.527	76,878	121	0.883	0.523	2.334	9.497
Hair Spray	1994	1.351	19,315	91	0.914	0.385	2.827	8.664
	1998	1.344	12,455	75	0.919	0.368	2.622	8.222
Body Shampoo	1994	1.329	30,702	150	0.927	0.355	2.252	8.891
	1998	1.440	62,013	197	0.893	0.478	1.707	8.905
Wrapping Film	1994	1.813	129,405	115	0.937	0.443	1.716	10.024
	1998	1.774	131,703	145	0.914	0.516	1.235	9.355
Insecticide	1994	1.394	6,984	53	0.962	0.249	2.938	7.785
	1998	1.347	17,143	94	0.839	0.548	-0.371	8.192
Toothpaste	1994	1.697	172,250	138	0.856	0.658	1.901	9.461
	1998	1.598	182,954	194	0.834	0.682	1.899	9.049
Kitchen Cleanser	1994	1.777	104,712	132	0.926	0.475	1.825	9.611
	1998	1.747	96,490	157	0.944	0.405	1.681	9.329
Household Cleaner	1994	1.822	20,803	34	0.881	0.585	2.410	8.592
	1998	1.762	27,767	53	0.879	0.589	1.908	8.175
Air Freshener	1994	1.024	5,549	192	0.856	0.401	1.952	7.194
	1998	1.156	10,143	206	0.868	0.431	1.899	7.042
Mothballs	1994	1.434	35,672	131	0.936	0.355	1.952	9.138
	1998	1.333	36,231	174	0.876	0.477	1.908	8.392

* Log linear transformation

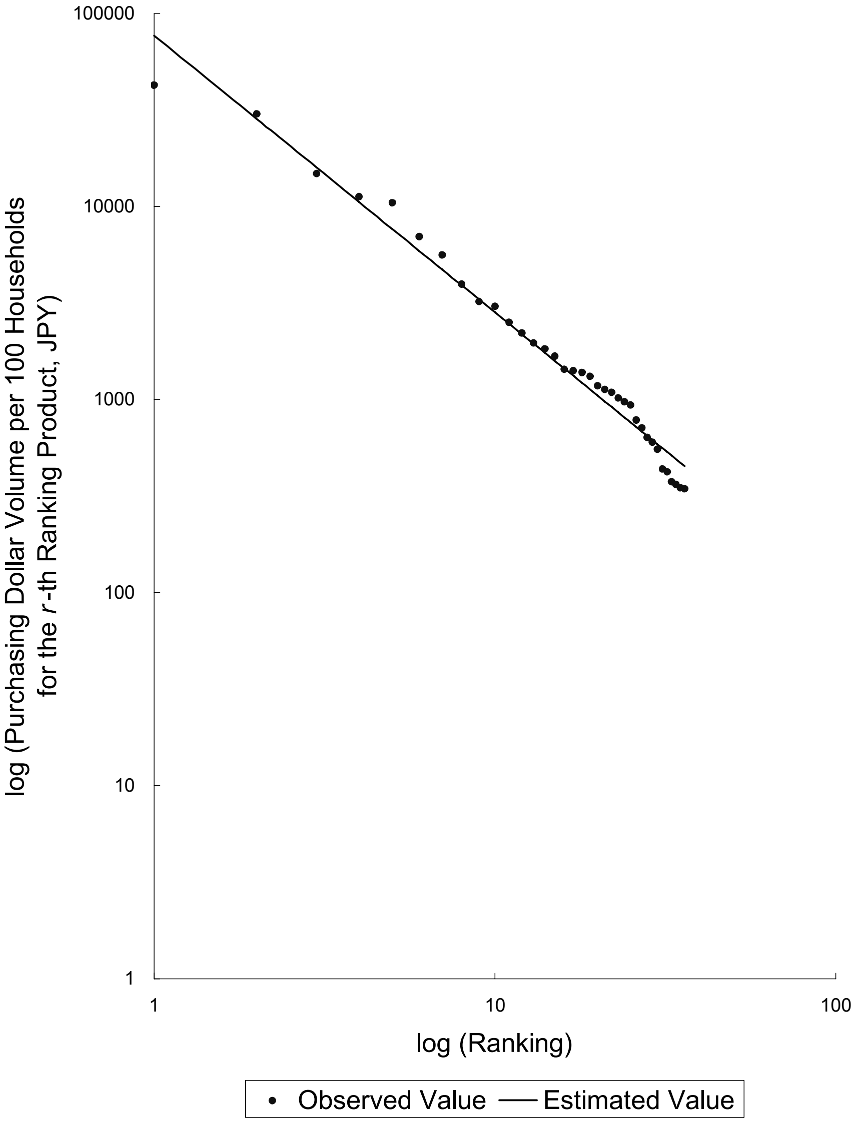
Case 2 (The Top 95% Products)

In the second case, small products in the bottom 5% market share were omitted, and equation (6) was applied to products ruling the top 95% market share. The data from a heavy detergent market in 1998 estimated parameter as follows:

$$s(r) = 76{,}848 r^{-1.434} .$$

The sample size was 36. The coefficient of determination was R^2=0.978 (F(1, 34)=1499, p< 0.001), and the standard error of estimate was SEE=0.190 (the range of the response variable after log

Figure 2
The Rank-Size Rule, Equation (6), the Zipf Model
(Heavy Detergent, 1988, Top 95% Products)



linear transformation was 5.842 to 10.658). Both became better than that in case 1. The geometrical fitness became better (Figure 2).

When equation (6) was applied to the data from other markets, both the value of R^2 and *SEE* became better (Table 2-1 and 2-2). The maximum value of R^2 was 0.994 (soy sauce, 1998) and the minimum was 0.887 (household cleaner, 1998). Above all, equation (6) fitted data in all the 31 markets, during the 2 years, and in both cases (all products and the top 95% products). Hence, it is not denied that the 80/20 law expressed by equation (6) holds in any packaged goods market.

Note that not the individual components (products) but structure (the 80/20 law) was focused on here. Even if market structure is robust regardless of time, it does not mean that the ranking or size of an individual product does not change. In heavy detergent market, for example, the regression coefficient in both 1994 and 1998 was almost the same. Furthermore, the market share of the top product was 25% in both periods, and that of the top 3 products was almost the same: 47% (1994) and 52% (1998). However, the products in the heavy detergent market changed completely between 1994 and 1998. That is, the top 3 products in 1994 were Kao "Attack 1.5 kg," Lion "High Top 1.5 kg" and P&G "Ultra Ariel 1.5 kg." In 1998, on the other hand, they were Kao "New Compact Attack 1.2 kg," Lion "Supper Top 1.2 kg" and P&G "Ariel Pure Clean 1.2 kg."

Table 2-1

Regression Analysis, Equation (6), the Zipf Model (Prodct in the Top 95%, Foods and Beverages)

Market	Year	Parameter *a*	Parameter *b*	Sample Size	Coefficient of Determination*	Standard Error of Estimate*	Range of Response Variable*	
Soy Sauce	1994	1.231	36,619	64	0.990	0.114	5.370	10.406
	1998	1.281	34,599	75	0.994	0.094	4.740	10.096
Soybean Paste	1994	1.075	31,864	255	0.976	0.163	4.226	10.236
	1998	1.096	35,309	290	0.980	0.152	4.008	10.293
Butter	1994	1.471	37,582	20	0.950	0.282	5.883	10.094
	1998	1.296	30,360	17	0.952	0.239	6.300	9.983
Ketchup	1994	1.433	26,207	27	0.988	0.138	5.221	10.121
	1998	1.369	21,489	36	0.984	0.151	5.088	9.983
Salad Oil & Sesame Oil	1994	1.043	39,811	80	0.926	0.272	5.539	9.857
	1998	1.144	36,391	73	0.980	0.150	5.385	9.788
Seaweed with Prepared Rice	1994	1.287	10,091	38	0.977	0.174	4.341	9.438
	1998	1.501	18,378	36	0.992	0.118	4.412	9.700
Portable Noodles	1994	1.012	50,108	124	0.952	0.213	5.588	9.792
(Bag Type Instant Noodles)	1998	1.131	56,080	127	0.957	0.225	5.107	9.597
Regular Coffee	1994	0.993	19,404	153	0.920	0.277	4.413	8.494
	1998	1.021	26,626	183	0.906	0.315	4.500	9.270
100% Fruit Juice	1994	1.064	60,217	275	0.931	0.280	4.644	9.244
	1998	1.056	42,172	222	0.920	0.298	4.491	8.691
Cola	1994	1.133	29,025	30	0.977	0.152	6.403	10.333
	1998	1.489	47,353	17	0.955	0.266	6.167	10.767
Salt	1994	1.011	9,392	41	0.916	0.271	5.028	9.842
	1998	1.150	9,752	51	0.974	0.169	4.382	8.696
Sauce	1994	1.427	31,006	53	0.993	0.109	4.686	10.000
	1998	1.271	18,724	65	0.985	0.143	4.538	9.767
Mayonnaise	1994	1.254	48,015	33	0.931	0.297	6.159	10.735
	1998	1.270	36,392	38	0.988	0.126	5.800	10.420
Curry Roux	1994	1.248	83,196	80	0.939	0.294	5.314	9.993
	1998	1.158	61,119	69	0.913	0.327	5.662	9.719
Boil-in-Bag Curry	1994	1.028	24,234	105	0.918	0.287	4.886	8.720
	1998	1.065	24,771	147	0.960	0.205	4.439	9.197

* Log linear transformation

Table 2-2

Regression Analysis, Equation (6), the Zipf Model (All Products, Commodities)

Market	Year	Parameter a	Parameter b	Sample Size	Coefficient of Determination*	Standard Error of Estimate*	Range of Response Variable*	
Heavy Detergent	1994	1.286	83,440	41	0.972	0.195	6.288	10.951
	1998	1.434	76,848	36	0.978	0.190	5.842	10.658
Aluminum Foil	1994	1.387	8,878	37	0.992	0.112	4.068	8.937
	1998	1.283	5,959	34	0.988	0.125	4.083	8.293
Shampoo	1994	1.091	66,953	215	0.980	0.150	5.043	9.974
	1998	1.002	48,036	277	0.960	0.198	4.767	9.601
Tissue Paper	1994	1.565	65,676	48	0.940	0.355	5.274	10.826
	1998	1.521	65,918	50	0.935	0.361	5.063	10.606
Toilet Bowl Cleaner	1994	1.460	9,113	10	0.904	0.371	5.076	8.751
	1998	1.401	8,765	18	0.888	0.411	4.968	8.318
Bath Room Cleaner	1994	1.795	12,547	12	0.944	0.348	5.056	9.166
	1998	1.493	12,980	28	0.986	0.152	4.589	9.110
Hair Conditioner	1994	1.198	38,183	88	0.980	0.160	5.067	9.455
	1998	1.100	23,987	65	0.959	0.207	5.163	9.497
Hair Spray	1994	1.040	8,937	54	0.959	0.195	4.582	8.664
	1998	1.060	6,390	47	0.983	0.124	4.532	8.222
Body Shampoo	1994	1.062	13,967	89	0.983	0.128	4.574	8.891
	1998	1.125	22,767	109	0.935	0.278	4.437	8.905
Wrapping Film	1994	1.389	43,054	43	0.969	0.223	5.410	10.024
	1998	1.321	36,728	63	0.952	0.269	4.918	9.355
Insecticide	1994	1.327	6,061	34	0.944	0.283	4.031	7.785
	1998	1.035	7,777	59	0.959	0.194	4.558	8.192
Toothpaste	1994	1.141	35,489	68	0.963	0.205	5.501	9.461
	1998	1.011	29,036	100	0.942	0.234	5.371	9.049
Kitchen Cleanser	1994	1.264	26,433	51	0.985	0.140	5.107	9.611
	1998	1.460	41,255	59	0.950	0.303	4.554	9.329
Household Cleaner	1994	1.156	7,262	17	0.888	0.337	5.022	8.592
	1998	1.158	8,529	26	0.887	0.353	4.909	8.175
Air Freshener	1994	0.808	2,758	139	0.941	0.192	7.194	7.194
	1998	0.910	4,536	139	0.917	0.258	3.530	7.042
Mothballs	1994	1.147	16,087	72	0.971	0.183	4.473	9.138
	1998	0.990	12,642	103	0.942	0.230	4.347	8.392

* Log linear transformation

3.4. Discussion

First, the validity of equation (6) will be empirically discussed with concrete cases. Then, the fitness of equation (6) will be explored again. That is, unfitness between the observed data and the estimated values in both the upper and lower ranking products will be discussed.

Empirical Validation

First, equation (6) will be empirically validated, if the estimated values of parameter a_6 are managerially appropriate. Namely, concrete cases show that the values of a_6 describe well both the market concentration and a new product birth. The value of a_6 was large, for example, in a salt market (a_6= 1.504 in 1994, and 1.553 in 1998). Here, only few products were distributed (a number of products were 74 in 1994, and 98 in 1998), and the market concentration was high (e.g. the sum of the top 3 products' market share was 49.1% in 1994 and 37.7% in 1998). On the other hand, a new product has only a small chance to enter into this market, since

- product differentiation was not easy (since salt is a typical commodity), and
- neither package extension nor size extension was available (since a full kind of packages and a

full kind of sizes were already provided).

On the other hand, the value of a_6 was small, for example, in an air freshener market (a_6=1.024 in 1984, 1.156 in 1998). Here, the market concentration was low (e.g. the market share of the top product was just 5.7% in 1994 and 4.1% in 1998), and many weak products were distributed (a number of products in the market was 192 in 1994 and 206 in 1998). Furthermore many new products emerged easily, since

- upper ranking products had changed every year,
- imitating other products was easy (since neither high technology nor prominent brand power was required for new products),
- companies could launch many products by slight modification, and
- consumers responded to such transient stimuli.

Above all, the values of a_6 describe well both birth probability of a new product and the market concentration. It validates the results estimated.

Unfitness in the Upper and Lower Ranking Products

Next, the fitness of equation (6) will be explored again. The fitness was good when equation (6) was adapted to the data. However, case 1 in which equation (6) was adapted to all products showed slight unfitness. That is, the estimated values were larger than the observed values in both ends of a regression line, in the upper and lower ranking products (Figure 1). Then, the fitness improved in case 2, since it omitted the data of lower ranking products of which residuals between the observed value and the estimated one were relatively large, and since the residuals in the upper ranking products decreased according as the slope of equation (6) became gradual. At that time, geometrical fitness improved.

The 80/20 law was formulated as equation (6). The first reason for this was that the 80/20 law has been originally formulated with the Pareto model. Second, the skew distribution of companies and the relationship between the size of a company and its ranking which are similar to the 80/20 law have usually been discussed with the Zips model (Ijiri and Simon 1964; Simon 1955; Simon and Bonini 1958). Third, the statistical fitness of equation (6) to data was good. Hence, it is reasonable that the 80/20 law was formulated with equation (6). However, the estimated values of both the upper ranking (larger size) products and the lower ranking (smaller size) ones were frequently larger than the observed values. This means that the relationship between the size of a product and its ranking is not liner but concave in log-log graph.

Furthermore, the value of a_6 should be between 0 and 1, since it describes birth probability p of a product ($a_6 \cong 1 - p$: Fujita, Krugman, and Venables 1999). However, some estimated values of a_6 were larger than 1. This requires another parameter in order to reduce the value of a_6. Hence, the 80/20 law will be formulated with the Mandelbrot model, one of the general expressions of rank-seize models, to improve fitness of a model in next study.

4. Study2: The 80/20 Law by the Mandelbrot Model

First, the 80/20 law will be formulated with the Mandelbrot model, one of the general expressions of rank-size models; its theoretical meanings will be discussed. Namely, the parameter in the Mandelbrot model will determine birth probability of a new product and the asymmetric growth rate of existing products. Next, the home scan panel data collected in 13 markets during 10 years will show that the 80/20 law is observed. Lastly, the validity of the model will be empirically discussed with concrete cases.

4.1. Model

Formulation

The 80/20 law is formulated with the Mandelbrot model, one of the general expressions of rank-size models:

$$s(r) = b_9(r + k_9)^{-a_9}, \qquad (9)$$

where

r: the ranking of product size,

$s(r)$: the size of the r-th ranking product,

$1 \leqq r \leqq N$,

N: the number of products (a positive integer, finite),

$0 < a_9$, b_9, $-1 < k_9$ (parameter).

Theoretical Implication

First, theoretical meanings of equation (9) are discussed. As mentioned above (e.g. Simon 1955), 1) if a new component (e.g. a product, a company or a city) occurs with constant probability, and 2) if the frequency distribution of the growth rate of a component is independent from its size (the law of proportionate effect), the Zipf model, equation (6), holds. On the other hand, equation (9) with parameter k_9=0 equals equation (6). Hence, if both assumptions 1) and 2) hold, k_9=0 in equation (9).

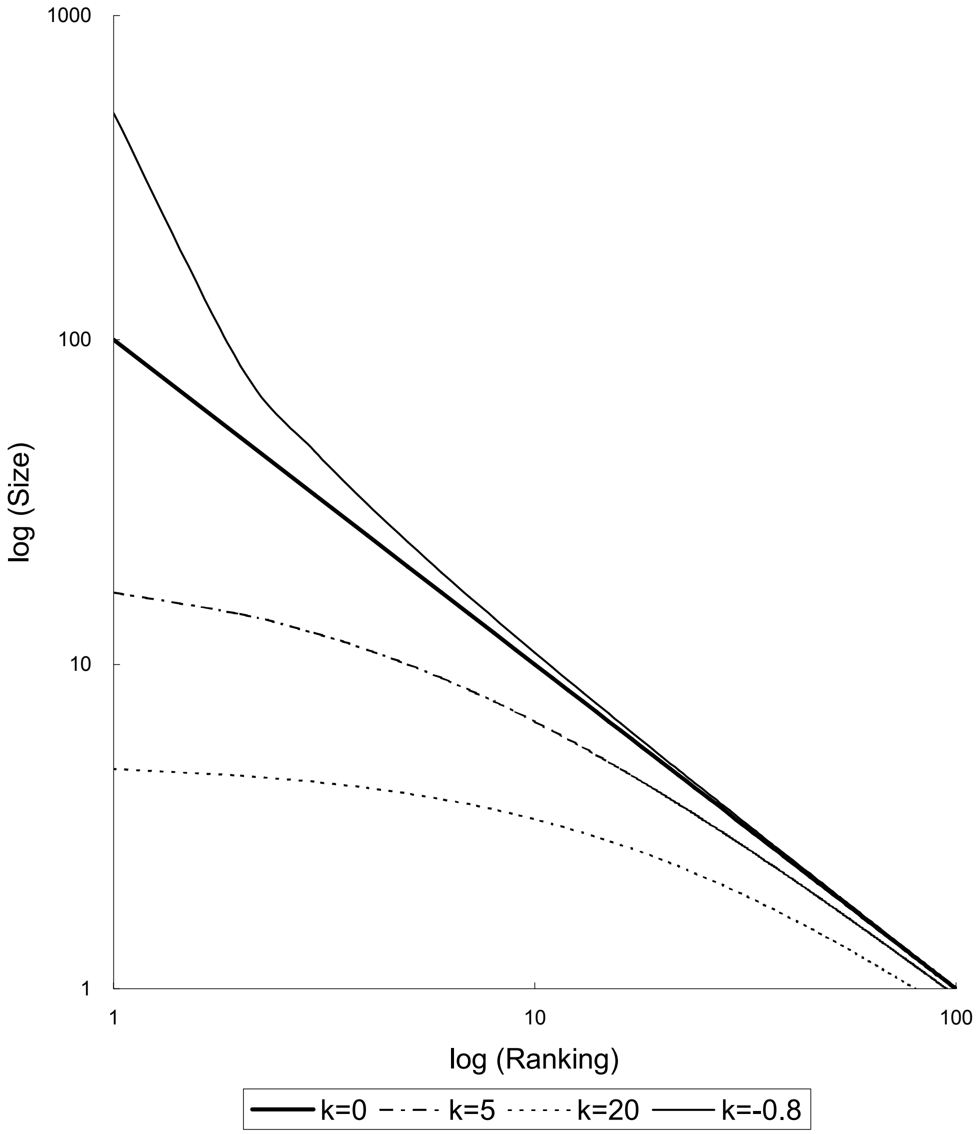
Next, a difference between equation (6) and (9) is discussed. While ranking r is a positive integer, it is regarded as continuous. The ratio of the size $S_Z(r)$ obtained from equation (6), the Zipf model, to the size $S_M(r)$ obtained from equation (9), the Mandelbrot model, is

$$\frac{S_Z(r)}{S_M(r)} = \frac{br^{-a}}{b(r+k)^{-a}} = \left(\frac{r+k}{r}\right)^a. \qquad (10)$$

Since of a>0, if k>0, then $S_Z(r)/S_M(r)$>1. Hence, the product size obtained from equation (6) is larger than that from equation (9). The ratio becomes larger, according as ranking r becomes smaller (product size becomes larger). On the other hand, if -1<k<0, then $S_Z(r)/S_M(r)$<1. Hence, the size obtained from equation (9) is larger than that from equation (6). And, the ratio becomes larger, according as ranking *r* becomes smaller (product size becomes larger).

What does make this difference between the size obtained from equation (6), $S_Z(r)$, and that obtained from (9), $S_M(r)$? The Zipf model, equation (6), holds under two assumptions that 1) the birth probability of a new component is constant, and that 2) the growth rate of an existing component is independent from its size. On the other hand, parameter *a* relates to the birth probability of a new component, and this parameter is common to both equations. Hence, it is reasonable to understand that the growth rate of an existing component results in the component

Figure 3
Change of the Rank-Size Rule According to Parameter *k*
Relating to the Asymmetric Growth Rate,
Equation (9), the Mondelbrot Model (*a* =1)



size gap between two equations, and that parameter k relates to the growth rate of an existing component.

This assumption helps us to understand an implication of parameter k:

- if k=0, then equation (9), the Mandelbrot model, is the same as equation (6), the Zipf model; it is linear in a log-log graph (a straight line in Figure 3). At this time, the growth rate of an existing product is independent from its size, or probability distribution of the growth rate is equal among all size sets when a size set is composed of same size products (the law of the proportionate effects). The law of constant returns causes theoretically this effects (Simon and Bonini 1958).
- if k>0 (a fine line in Figure 3), then the size of an upper ranking (a larger size) product in equation (9) becomes smaller than that in equation (6). It means that the growth rate of a product becomes smaller as ranking becomes higher (size becomes larger). Simon and Bonini (1958) imply it is caused by the law of diminishing returns. Equation (9) is concave in a log-log graph. On the other hand,
- if $-1<k<0$ (dotted lines in Figure 3), then the size of an upper ranking product in equation (9) becomes larger than that in equation (6). It means that the growth rate of a product becomes larger according as ranking becomes higher (size becomes larger). This implies the law of increasing returns. Equation (9) is convex.

4.2. Data and Method

Method

The parameter in equation (9) was estimated from the least squares method with the home scan panel data collected in consumer packaged goods markets, then the fitness of equation (9) to the data was examined. That is, the parameter minimizing the sum of the square residuals was estimated by numerical calculation with Solver. First, appropriate value was given to an initial value set of the parameter a_9, b_9 and k_9. Then, this value set changed in order to minimize residuals. Next, this solution was assigned to the new initial value set for the parameter, and a better solution was again searched for with Solver. This was repeated until parameter convergence. In this method, a converged value set depends sometimes on an initial value set. Hence, we had confirmed the validity of this method by numerical simulations in which different initial value sets converged with an identical solution through one search.

Data

In order to estimate the parameter in above way, equation (9) was adapted to the home scan panel data. Markets and periods were as follows:

- food and beverages (10 markets): sauce, catsup, butter, margarine, curry roux, boil-in-bag curry, instant coffee, cola, 100% pure fruit juice, and beer,
- commodities (3 markets): heavy detergent, shampoo, and aluminum foil;

・ periods (10 years): the data from Jan. 1 1990 to Dec. 31 1999 was added up each year so that there were 10 sets of the data.

That is, the 130 cases (the 13 markets and the 10 years) were examined.

4.3. Results

The parameter was estimated from the home scan panel data and the fitness was examined. The result of a heavy detergent market in 1998 is shown among others. Equation (9) was estimated as follows:

Figure 4
The Rank-Size Rule, Equation (9), the Mandelbrot Model
(Heavy Detergent, 1998, All Products)

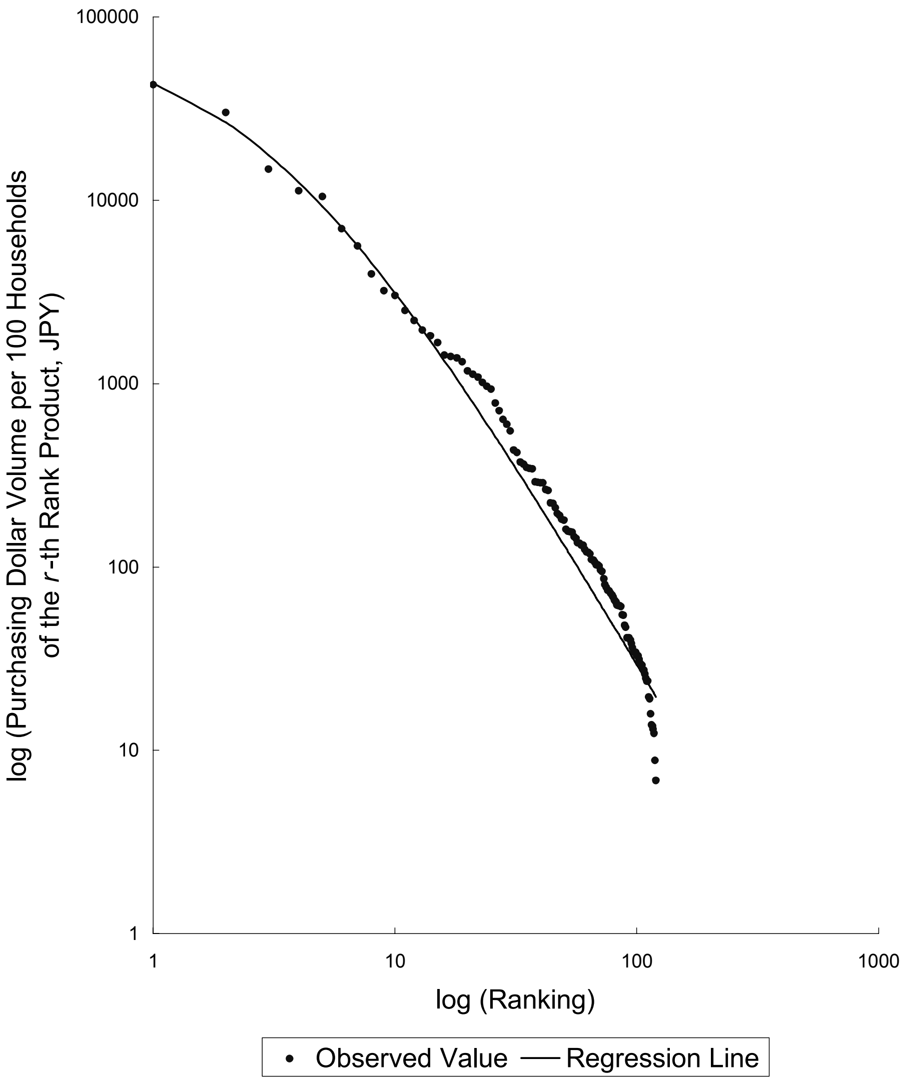


Table 3-1

Regression Analysis, Equation (9), the Mandelbrot Model (Food and Bevarage)

Market	Year	Sample Size	*Parameter*			Coefficient of	Standard Deviation	Range of	
			a	*b*	*k*	Determination	of Residuals	Response Variable	
Sauce	1990	137	1.343	30,411	0.421	0.990	196	19,093	2.7
	1991	150	1.445	46,764	0.803	0.988	215	20,283	2.1
	1992	157	1.412	39,441	0.607	0.991	184	20,371	5.5
	1993	150	1.419	41,180	0.563	0.992	187	22,087	5.5
	1994	139	1.444	39,902	0.517	0.994	173	22,027	2.8
	1995	130	1.270	22,364	0.055	0.988	226	21,058	7.1
	1996	156	1.590	49,134	0.874	0.993	150	18,238	3.5
	1997	149	1.556	48,739	0.841	0.994	140	18,997	6.2
	1998	155	1.398	29,916	0.481	0.988	177	17,445	0.7
	1999	159	1.248	18,475	0.028	0.992	141	17,965	1.9
Catsup	1990	82	1.800	73,374	0.692	0.990	346	28,261	9.0
	1991	88	1.467	41,539	0.305	0.996	225	28,155	6.9
	1992	83	1.570	52,323	0.577	0.991	313	25,484	6.6
	1993	76	1.693	72,914	0.827	0.999	134	26,286	8.8
	1994	80	2.039	107,646	1.046	0.994	243	24,851	6.9
	1995	93	1.084	11,504	-0.505	0.996	267	24,723	4.7
	1996	94	1.344	17,949	-0.187	0.999	113	23,676	9.0
	1997	84	1.415	19,641	-0.156	0.995	205	24,920	6.5
	1998	87	1.327	18,088	-0.129	0.993	213	21,663	6.9
	1999	94	1.493	24,571	0.142	0.982	309	19,997	5.6
Butter	1990	42	1.643	74,426	0.716	0.992	483	30,848	14.3
	1991	42	1.269	29,055	-0.036	0.982	695	30,703	10.6
	1992	38	1.661	90,017	0.975	0.961	1073	29,498	10.4
	1993	43	1.042	17,783	-0.424	0.983	739	31,776	6.9
	1994	53	1.335	34,984	0.330	0.985	458	24,199	13.2
	1995	55	1.334	34,162	0.239	0.983	513	25,994	10.7
	1996	66	1.789	150,513	1.991	0.996	216	21,421	13.2
	1997	51	1.918	171,667	1.825	0.990	405	23,542	13.8
	1998	49	1.831	139,507	1.777	0.990	370	21,664	3.4
	1999	52	1.427	35,366	0.320	0.995	250	23,874	16.1
Margarine	1990	87	1.513	96,963	1.335	0.961	717	25,811	9.0
	1991	84	1.249	69,444	1.161	0.975	599	27,606	8.0
	1992	78	1.172	50,043	0.633	0.982	535	28,797	3.7
	1993	85	1.331	68,568	1.066	0.986	424	26,711	4.1
	1994	76	1.513	96,963	1.335	0.990	381	27,197	8.7
	1995	63	1.339	51,972	0.757	0.995	261	24,602	9.8
	1996	69	1.121	26,473	0.066	0.988	379	24,910	3.5
	1997	62	1.287	41,618	0.462	0.994	283	25,618	7.2
	1998	69	1.213	35,003	0.354	0.991	328	24,449	10.9
	1999	67	1.148	27,171	0.131	0.986	402	23,858	6.3
Curry Roux	1990	251	1.337	189,774	4.536	0.989	228	19,767	8.0
	1991	263	1.071	63,595	1.242	0.982	337	28,125	4.3
	1992	233	1.122	79,992	1.641	0.978	407	28,672	6.5
	1993	237	1.399	218,507	3.891	0.990	260	25,312	6.9
	1994	225	1.754	785,556	6.614	0.992	236	21,871	6.9
	1995	221	1.887	1,432,451	8.878	0.996	149	18,323	7.0
	1996	219	1.876	1,591,324	10.326	0.989	236	16,257	6.3
	1997	217	1.957	1,978,578	9.993	0.989	253	16,853	7.2
	1998	181	1.901	1,561,620	9.900	0.988	265	16,628	6.9
	1999	179	1.671	537,587	6.669	0.988	262	17,158	6.4

Table 3-1 (Continued)

Regression Analysis, Equation (9), the Mandelbrot Model (Food and Bevarage)

Market	Year	Sample Size	*Parameter*			Coefficient of Determination	Standard Deviation of Residuals	Range of Response Variable	
			a	*b*	*k*				
Boil-in-Bag Curry	1990	105	4.297	60,305,892,755	44.357	0.974	157	5,275	8.0
	1991	131	1.865	996,907	16.176	0.981	132	5,756	6.9
	1992	137	2.155	3,885,024	20.141	0.991	99	5,517	11.1
	1993	161	1.704	513,481	11.894	0.994	87	7,156	8.8
	1994	191	1.913	1,537,697	17.809	0.992	88	6,123	7.0
	1995	198	2.218	6,240,013	22.393	0.997	56	5,879	7.0
	1996	230	1.508	210,933	10.688	0.991	71	5,279	7.0
	1997	264	1.317	93,745	6.865	0.990	81	6,486	7.2
	1998	286	0.993	21,763	1.363	0.982	119	9,869	6.3
	1999	364	1.303	81,463	6.007	0.992	61	6,703	5.0
Instant Coffee	1990	124	1.153	44,750	-0.307	0.997	481	68,428	6.1
	1991	128	1.202	60,535	0.019	0.986	716	59,773	6.9
	1992	126	1.213	62,940	-0.002	0.995	446	63,371	4.0
	1993	137	1.684	228,113	1.236	0.998	275	58,842	6.5
	1994	135	1.484	120,857	0.621	0.992	544	59,344	7.0
	1995	163	1.641	219,388	1.524	0.984	601	48,428	7.1
	1996	154	2.249	1,017,609	2.982	0.988	525	45,598	3.5
	1997	152	1.904	351,178	1.921	0.995	341	44,886	3.3
	1998	147	2.068	891,814	2.994	0.982	740	49,031	6.9
	1999	162	2.015	734,074	2.767	0.982	700	48,704	5.8
Cola	1990	34	0.957	16,783	-0.618	0.987	1119	42,247	62.1
	1991	46	3.785	32,008,713	4.823	0.982	943	41,412	4.0
	1992	57	1.657	132,301	1.391	0.990	488	31,593	7.0
	1993	58	1.990	236,995	2.058	0.973	664	24,965	3.4
	1994	78	0.970	19,295	-0.381	0.994	355	30,724	4.8
	1995	85	1.516	69,831	0.718	0.996	247	30,652	7.0
	1996	86	1.487	51,673	0.331	0.999	147	33,759	3.9
	1997	70	1.030	20,587	-0.502	0.988	695	42,361	5.0
	1998	62	0.931	14,828	-0.713	0.989	937	47,432	5.2
	1999	52	1.018	18,111	-0.564	0.986	900	42,247	3.1
100% Fruit Juice	1990	489	0.820	24,401	0.331	0.959	274	20,681	4.5
	1991	505	0.788	26,802	0.140	0.936	419	26,041	4.0
	1992	505	1.018	70,755	3.197	0.965	279	19,567	4.5
	1993	556	1.035	70,509	4.224	0.986	139	13,248	3.4
	1994	561	1.328	286,432	13.757	0.978	141	10,339	5.5
	1995	606	1.573	887,332	20.862	0.994	63	7,912	3.6
	1996	591	1.661	1,567,346	27.669	0.990	79	7,131	4.1
	1997	542	1.872	4,010,313	31.693	0.997	46	5,560	2.9
	1998	504	1.678	1,213,535	23.812	0.990	75	5,947	3.4
	1999	474	2.818	576,962,912	64.513	0.998	34	4,313	3.2
Beer	1990	240	1.617	4,654,889	4.916	0.994	2165	262,748	9.0
	1991	295	1.609	6,185,561	6.963	0.990	2435	229,656	10.3
	1992	260	1.982	23,601,784	10.349	0.979	3362	202,283	10.5
	1993	269	2.292	105,586,666	15.321	0.989	2369	165,298	6.9
	1994	280	3.173	4,336,984,090	23.246	0.983	2960	163,104	14.8
	1995	359	2.482	161,014,074	15.608	0.987	1923	141,031	7.1
	1996	351	1.724	5,246,264	6.486	0.995	1166	166,751	7.0
	1997	343	2.177	92,813,512	23.408	0.998	559	89,793	10.9
	1998	349	2.194	90,795,966	25.039	0.994	790	80,101	6.9
	1999	343	2.681	756,325,263	32.020	0.996	566	63,263	6.4

Table 3-2

Regression Analysis, Equation (9), the Mandelbrot Model (Commodities)

Market	Year	Sample Size	Parameter			Coefficient of	Standard Deviation	Range of	
			a	b	k	Determination	of Residuals	Response Variable	
Heavy Detergent	1990	130	1.134	57,618	-0.295	0.996	648	85,861	2.7
	1991	137	1.092	64,985	-0.188	0.991	766	82,187	8.2
	1992	141	1.442	172,571	0.841	0.997	434	71,571	11.9
	1993	122	1.483	193,935	1.184	0.993	568	61,356	13.6
	1994	113	1.279	88,586	0.415	0.995	432	57,010	13.9
	1995	136	1.815	576,913	4.063	0.976	609	30,578	1.4
	1996	153	1.158	52,690	0.455	0.969	620	33,706	3.4
	1997	128	1.931	373,856	1.991	0.981	699	44,115	7.2
	1998	120	2.261	1,046,259	3.086	0.991	478	42,542	6.9
	1999	125	2.802	7,616,378	5.876	0.964	813	31,549	6.4
Shampoo	1990	435	1.004	62,156	2.546	0.991	151	17,793	7.2
	1991	481	0.914	41,888	1.238	0.976	240	21,511	5.5
	1992	512	1.267	194,102	7.261	0.996	83	12,948	5.3
	1993	501	0.829	27,192	0.437	0.970	246	21,391	6.7
	1994	421	1.140	97,959	2.925	0.993	153	21,461	7.0
	1995	475	1.019	63,707	3.171	0.993	110	15,195	7.1
	1996	493	1.044	59,589	1.995	0.986	172	17,960	7.0
	1997	482	0.970	49,091	1.791	0.989	154	18,627	6.6
	1998	495	0.934	40,870	2.276	0.982	157	14,774	6.1
	1999	573	0.703	15,335	-0.171	0.954	236	18,188	6.4
Aluminum Foil	1990	81	2.150	139,776	3.040	0.995	68	7,008	8.9
	1991	86	1.490	18,713	0.869	0.989	102	7,452	3.4
	1992	88	1.908	45,996	1.571	0.987	112	7,490	3.3
	1993	85	1.933	52,889	1.874	0.979	134	6,765	6.2
	1994	75	1.423	10,743	0.275	0.998	46	7,606	6.2
	1995	64	2.463	213,397	3.672	0.959	156	4,588	6.3
	1996	71	2.476	283,040	4.390	0.948	159	4,118	4.8
	1997	64	2.327	164,108	3.804	0.969	123	4,123	5.5
	1998	67	2.176	88,358	3.092	0.986	76	3,998	6.1
	1999	71	1.778	22,417	1.796	0.984	68	3,566	3.7

$$s(r) = 1046259(r + 3.086),^{-2.261}$$

where

r: the ranking of product sales size,

$s(r)$: purchasing dollar volume per 100 households of the r-th ranking product.

The sample size was 120. Since the coefficient of determination was R^2=0.991 and the standard error of estimate was *SEE*=478 (the range of the response variable is 6.9 to 42542), equation (9) fitted well the data. The coefficient of determination in equation (9) improved from R^2=0.957 (all products were analyzed) or R^2=0.978 (the top 95% products were analyzed) in equation (6). Geometrical fitness improved also (Figure 4).

When the 13 packaged goods markets and 10 years are observed, the coefficients of determination and the value of standard error were good in all markets, in all periods (Table 3-1 and 3-2). The maximum of R^2 was 0.999 (ketchup, 1998) and the minimum of that was 0.954 (shampoo, 1999). Thus, equation (9) fitted well the data. Hence, it is reasonable to understand that the 80/20 law expressed by equation (9), the Mandelbrot model, is universally observed in markets.

4.4. Discussion

First, showing that the estimated values of k_9 are managerially appropriate will validate empirically equation (9). Then, the relationship between parameter a_6 in equation (6) and a_9 in equation (9) will be discussed.

Parameter k

The validity of equation (9) is examined by showing that the estimated values of k_9 are managerially appropriate. That is, it is empirically shown that k_9 managerially relates to the growth rate of a product. Note that, if $k_9=0$, the growth rate of a product is independent from its size (the law of the proportion effect) and equation (9) is liner. If $k_9>0$, the growth rate of a product becomes smaller as its ranking becomes higher (its size becomes larger) (equation (9) is concave); if $-1<k_9<0$, the growth rate becomes larger as its ranking becomes higher, (equation (9) is convex) (Figure 3). Incidentally, since $k_9>0$ in 114 cases out of 130 cases (Table 3-1 and 3-2), the growth rate decreases as the product size became larger (the law of diminishing returns) in most cases.

In a heavy detergent market from 1990 to 1991, for example, $k_9<0$ (k_9 =-0.295 in 1990 and -0.188 in 1991). That is, upper ranking products grew rapidly in those periods. The reason for this was as follows. Immediately prior to those periods, companies launched new "big" products that were so insightful that they changed totally the Japanese preference for a heavy detergent. They became rapidly upper ranking by stealing old products' share, and were still growing in 1990 and 1991. The value of k_9 (<0) corresponds with this reality.

On the other hand, k_9 became larger after 1995 (from k_9=-0.188 in 1991 to k_9=4.063 in 1995). The sale size of upper ranking products decreased during those periods (the sum of purchasing dollar volume per 100 households of the top 3 products decreased from ¥132,764 in 1990 to ¥68,669 in 1995). The reason for this was that products launched from 1987 to 1990 were still in upper ranking in 1995, but became unattractive because of their aging. Thus, the higher value of k_9 corresponds with this.

Furthermore, in 1996, the existing upper ranking products were discontinued; new attractive products were launched and grew rapidly to upper ranking. Hence, the growth rate of upper ranking products increased (the sum of purchasing dollar volume per 100 households of the top 3 products increased from ¥68,669 in 1995 to ¥86,313 in 1997), and k_9 decreased again (k_9 decreased from 4.063 to 1.991). Above all, it is reasonable to consider that the value of k_9 relates empirically to the growth rate of products.

Parameter a

Power parameter a_6 in equation (6) relates birth probability p of a new product (Ijiri and Simon 1964; Simon 1955; Simon and Bonini 1958). On the other hand, the relationship between a_9 in equation (9) and probability p has not been discussed, as far as the author knows. Since the Mandelbrot model, equation (9), is one of the general expressions of the Zipf model, equation (6), it is reasonable to understand that a_9 relates to p. Above all, a_9 relates to birth probability of a new product, and k relates empirically to the growth rate of products. Then, by focusing on birth of a new product and growth of existent products, a simulation model will be proposed to discuss a forming mechanism of the 80/20 law in the next study.

5. Study 3: A Forming Mechanism by a Monte Carlo Simulation Model

The 80/20 law formulated by equation (9), the Mandelbrot model, was observed universally in markets and time. At that time, the parameter a_9 and k_9 determines birth and asymmetric growth of a product. Hence, with focusing on them, a Monte Carlo simulation model to discuss a forming mechanism of the law will be proposed. That is, the model in which birth probability of a new product and a growth rate of an existing product are given will generate product size distribution data. Then, fitness of the generated data into the observed data will be examined. If the generated data fits the observed data (and observed data follows the law), an algorism of the model shows one of emerging mechanisms of the law (Result 1). Next, the simulation model will show that the law emerges in any market and time (Result 2). That is, the law emerges, only if a product occurs constantly and grows in process of time. Since product birth and growth is natural in a market, the law emerges inevitably and autonomously.

5.1. Model

A Monte Carlo simulation model will be proposed. It generates data following the 80/20 law, if birth probability of a new product and a growth rate of existing products are given. Simple local rules of the model are as follows. Incidentally, a product born at time t is called a new product, and a product born at or before t-1 is called an existing product.

Rule 1: Birth Probability of a New Product

When demand 1 unit occurs, a new product is born in probability p ($0 \leqq p \leqq 1$) and this product acquires sales size 1 unit, otherwise an existing product acquires sales size 1 unit in probability 1-p. Probability p is constant during time series T.

Rule 2:Growth Rate of Existing Products

Growth rate g of an existing product depends on growth "asymmetry" q ($0 \leqq q$). Namely, when d_t units of demand for existing products occurs at time t, growth rate $g_m^{\,t}$ of the m-th ranking product at time t is as follows.

$$g_m^t = \frac{\Delta S_m^t}{S_m^{t-1}} = \frac{\dfrac{(S_m^{t-1})^q}{\sum_{r=1}^{n}(S_r^{t-1})^q} d_t}{S_m^{t-1}}, \qquad (11)$$

where

$S_r^{\,t}$: the sales size of the r-th ranking product at time t ($t \geqq 2$),

n: the total number of products in a market (r: $1 \leqq m \leqq n$).

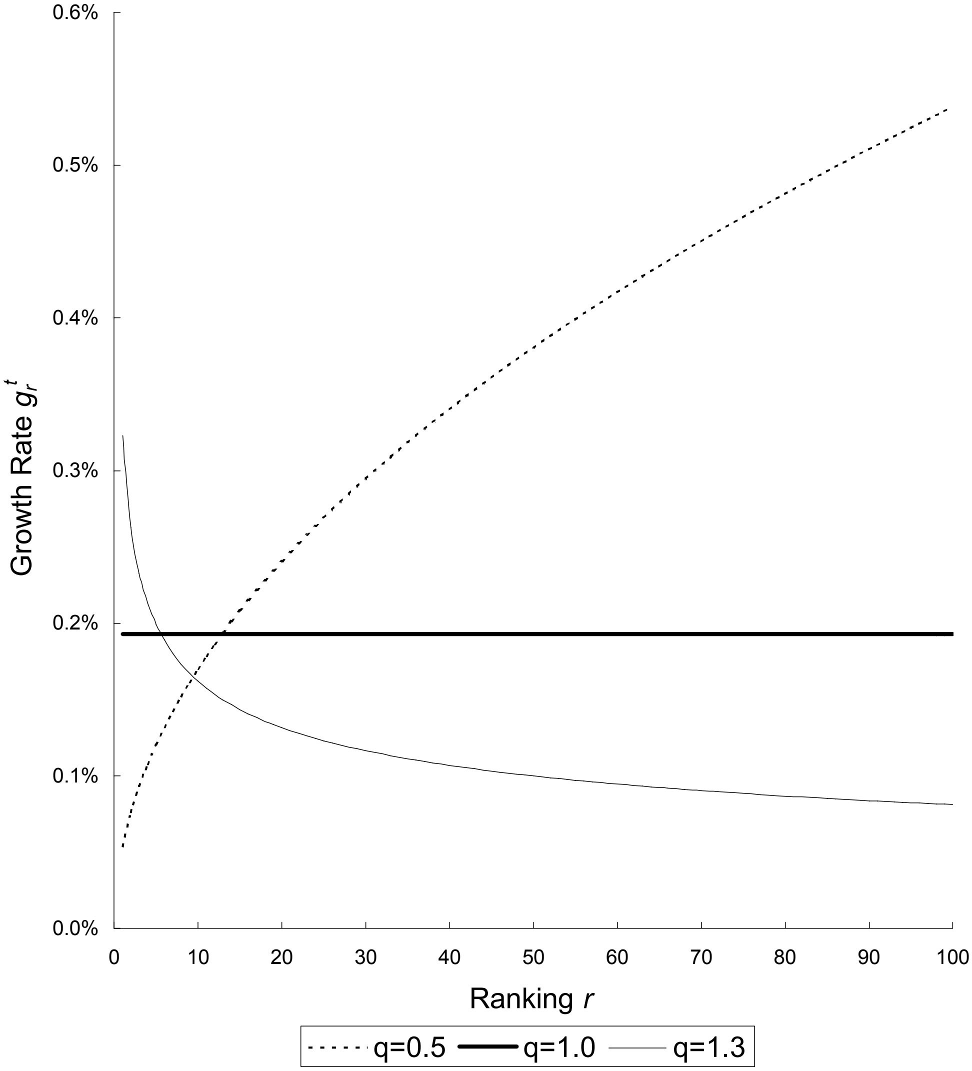
$S_r^{\,t}$ equals the total sum of sales of the r-th ranking product from time 1 to t. Parameter q is common to all products in a market, and constant during time series T.

Parameter q determines asymmetric growth rate $g_r^{\,t}$ of a product according to the size of the product. The relationship between q and g_r^t is as follows:

- If $0 \leqq q < 1$, growth rate g_r^t of an upper ranking (a larger size) product is smaller than that of a lower ranking (a smaller size) product (a dotted line in Figure 5). As q approaches 1, growth rate g_r^t of an upper ranking product approaches that of a lower ranking products.
- If q=1, the growth rate of all products is independent from their size (the law of the proportionate effect, a straight line in Figure 5).
- If q>1, the growth rate of an upper ranking product is larger than that of a lower ranking product (a fine line in Figure 5).

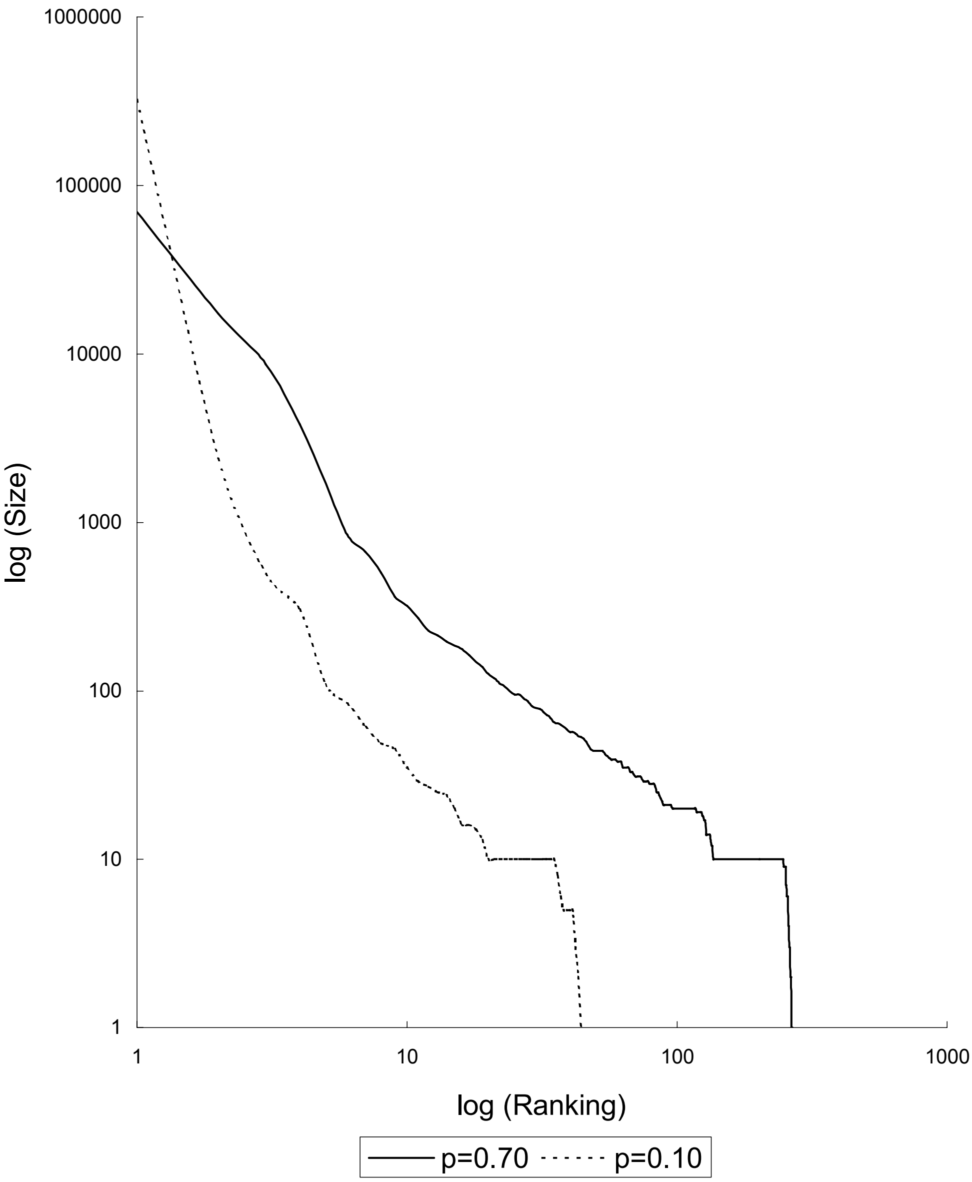
Figure 5

Asymmetric Growth Rate g_r^t of the r-th Ranking Product Determined by Parameter q ($S_r^{t-1}=100r^{-1}$, d_t=1)



Parameter q relates to degree of inertial behavior of consumers and companies. That is, it is reasonable to understand that, as q is larger, a consumer will purchase much the product that the consumer has much purchased. And (if product sales size is a function of marketing investments) a company will much invest the product that the company has much invested. The reasons for this are that consumers and companies sometimes prefer a prominent brand or a strong standard product ruling a market. Actually, as the author mentions later, q is large in an instant coffee and a shampoo

Figure 6
The Rank-Size Rule Determined by Birth Parameter *p*
(*q* =1.00, λ=100, *Price* =10)

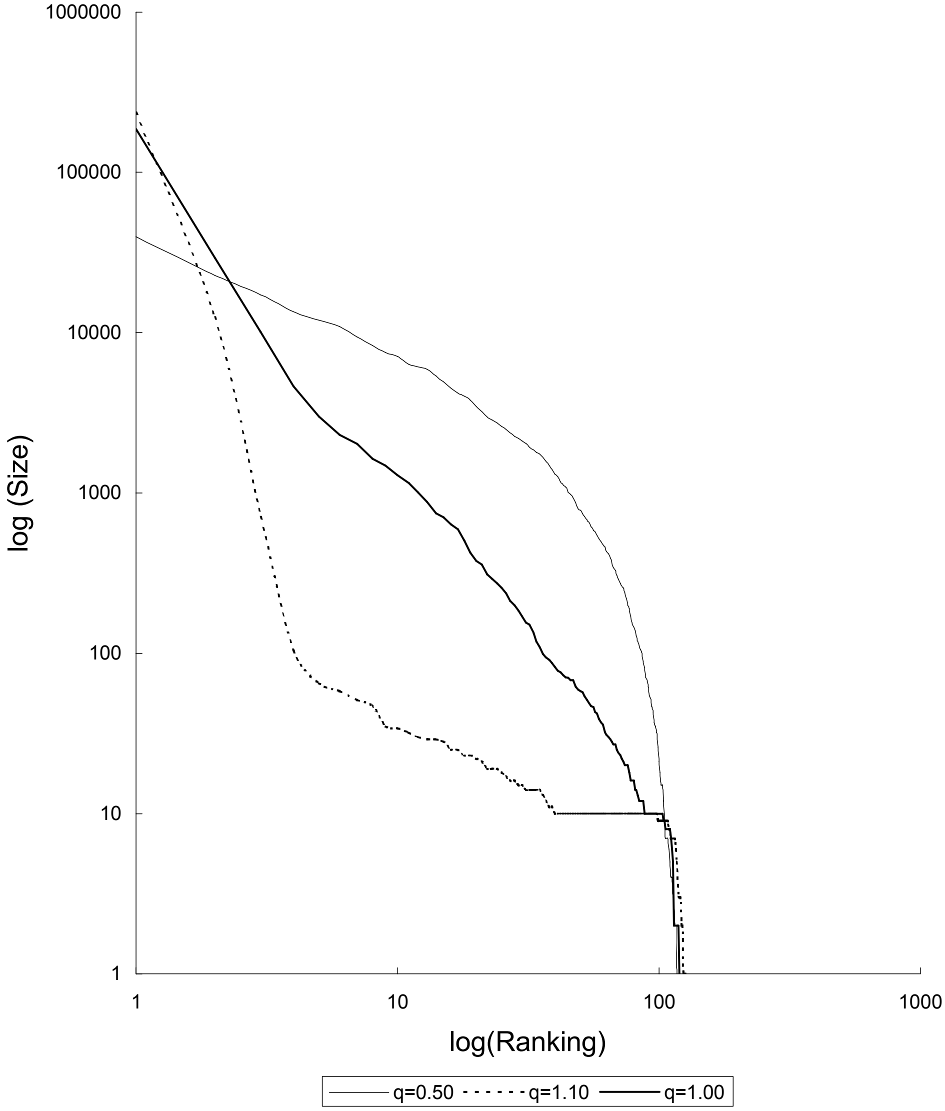


market in which brands are important; it is also large in a ketchup, a sauce, and an aluminum foil market in which strong standard products own a large share. Incidentally, both p and q were supposed to be constant during T, and time series T was assigned to one year. As to data, of course, the length of T will be optionally changed.

Geometrical Understanding of Parameter

In advance, it is shown what product size distribution data is generated in the model, and how the relationship between the ranking of product size and the size of the r-th ranking product

Figure 7
The Rank-Size Rule Determined by Asymmetric Growth Parameter q
(p =0.30, λ=100, *Price* =10)



changes according as p and q change.

- If p is large, many new products occur, and a market concentration ratio becomes small; the slope of a curb is gentle, and the tail of that is long (a solid line in Figure 6).
- If p is small, few new products occur, and a market concentration ratio becomes large; the slope of a curb is sharp, and the tail of that is short (a dotted line in Figure 6).

- If $0 \leqq q < 1$, the growth rate of upper ranking (larger size) products is smaller than that of small size (lower ranking) products; a curb is concave (a fine line in Figure 7). That is, the growth rate decreases according as a product grows.
- If $q=1$, the growth rate of products is independent from their size (the law of the proportion effect); a curb is liner (a bold line in Figure 7).
- If $q>1$, the growth rate of larger size products is larger than that of small size products; a curb is convex (a dotted line in Figure 7). That is, the growth rate increases according as a product grows.

5.2. Algorism

An algorism of the Monte Carlo simulation model generating data fitting the 80/20 law will be proposed (Figure 8). Parameter was determined to minimize the Cramer-Von Mises statistic[9] showing difference between the cumulative relative frequency distribution function of an observed data and that of the data generated by the model. Simulation ran in markets $i=\{1, 2,\ldots, m\}$ at periods $j=\{1, 2,\ldots, T_k\}$. Namely, market $i=\{m=13$ markets: sauce, catsup,…, and aluminum foil$\}$ and period $j=\{k=10$ years: 1990, 1991,…, and 1999$\}$ as mentions later. One period was supposed to be 1 year, 1 time series $T=\{1, 2,\ldots, t,\ldots, 365\}$. Since the algorism is common to markets i and periods j, indexes both i and j are omitted.

Step 1: Occurrence of Demand

Demand following Poisson distribution occurs at time t in time series $T=\{1, 2,\ldots, t,\ldots, 365\}$. Namely,

$$f(d) = e^{-\lambda}\lambda^{d} / d!, \qquad (12)$$

where

d: the amount of demand (unit) at time t ($t \geqq 1$),

$f(d)$: its probability.

$\hat{\lambda}$ = the average of purchasing unit per a day and per 100 households during a year in observed data[10].

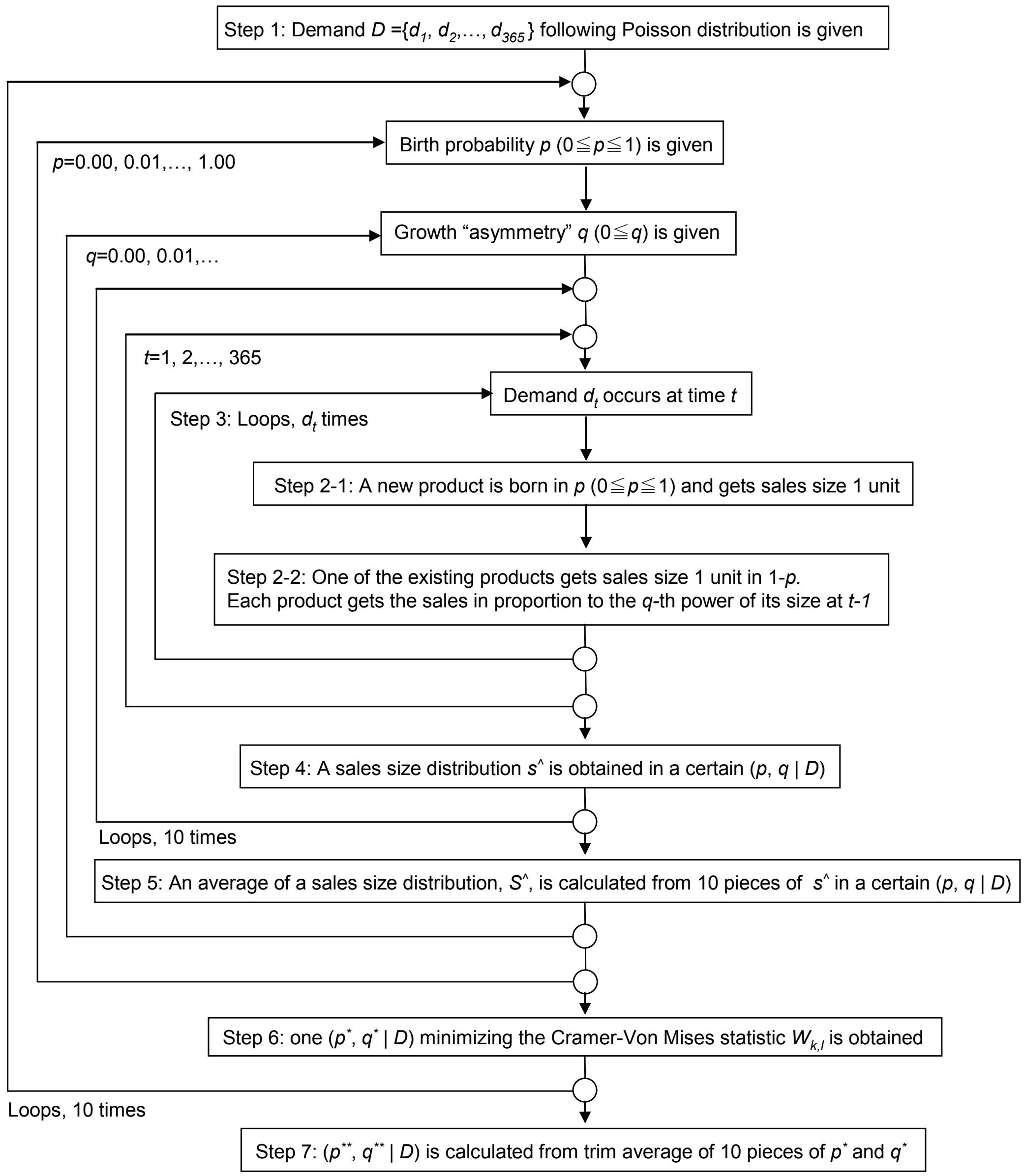
---

[9] The Cramer-Von Mises statistic, one of the nonparametric statistics, has some advantages (Hajek 1969; Siegel 1956; Sprent 1981). It is robust for dispersion of data, applicable to a small sample, and suits ranked data. Thus, this statistic fits product ranking data from consumer purchasing records.

[10] One tenth of this figure was used for $\hat{\lambda}$ in a beer market because of a limit on computing, and prices were enlarged ten times.

Figure 8

The Algorism of the Monte Carlo Simulation Model



Equation (12) generates Poisson random numbers d at t. It is considered this to be the amount of demand d_t at t (t = 1, 2,…, 365), and its time series to be $D=\{d_1, d_2,\dots, d_t,\dots, d_{365}\}$. The purchasing unit per 100 households in a cola market during 1999, for example, was 678.0; $\hat{\lambda}$=678.0/365=1.857.

Step 2-1: Birth of a New Product

When demand 1 unit occurs,

- a new product is born in probability p ($0 \leqq p \leqq 1$), and this new product acquires sales size 1 unit. Otherwise,
- one of the existing products acquires sales size 1 unit in probability 1-p.

At t=1, d_1 new product (s) of which sales size is 1 unit is/are born regardless of probability p.

Step 2-2: Growth of Existing Products

Existing products grow under equation (11). That is, if n pieces of existing products at t ($t \geqq 2$) are in a market, product M of which ranking is the m-th acquires 1 unit sales in probability $P^t(M=m)$:

$$P^t(M=m) = \frac{(S_m^{t-1})^q}{\sum_{r=1}^{n} \left(S_r^{t-1}\right)^q}, \qquad (13)$$

where

S_r^t: the sales size of the r-th ranking product at t ($t \geqq 2$),
n: a total number of products in a market (r: $1 \leqq m \leqq n$).

Table 4 shows an example of probability $P^t(M=m)$ in a cola market in 1999 (q=0.69, t=365).

Step 3: Loops by Demand

Step 2-1 and 2-2 are repeated d_t times (a total amount of demand) at t ($t \geqq 2$).

Step 4: A Sales Size Distribution $\hat{s}$ in a Certain (p, q | D)

Step 2-1, 2-2 and 3 are repeated during time series T. Then, sales size distribution $\hat{s}$ is generated in a certain (p, q | D).

Step 5: An Average of a Sales Size Distribution, $\hat{S}$, Calculated from $\hat{s}$

Step 2-1, 2-2, 3 and 4 are repeated 10 times. Then, sale size distribution average $\hat{S}$ is calculated from 10 pieces of sales size distribution $\hat{s}$ generated in a certain (p, q | D).

Step 6: Change of Values (p, q)

The steps from 2-1 to 5 are repeated according as p and q change (p=0.00, 0.01,…, 1.00; q=0.00, 0.01,…[11]). Then, a bunch of (p, q | D) which generates the sales size distribution average $\hat{S}$ is obtained. Among the sets of (p, q | D), one (p^*, q^* | D) minimizing the Cramer-Von Mises statistic $W_{k,l}$ is chosen. The Cramer-Von Mises statistic $W_{k,l}$ is the sum of the square of the

[11] However the value of q may vary in any positive value, an appropriate upper limit was assigned in advance.

Table 4
Probability $P^t(M=m)$:
Product M of Which Ranking is the m-th Acquires 1 Unit Sales
(Cola Market, 1999, q=0.69, t=365)

Ranking r	Sales Size S_r^{t-1}	$(S_r^{t-1})^q$	Prob. $P^t(M=m)$
1	38598	1461.3	0.227
2	12071	655.2	0.102
3	8968	533.8	0.083
4	6973	448.7	0.070
5	5713	391.1	0.061
6	4169	314.6	0.049
7	3374	271.9	0.042
8	2579	225.9	0.035
9	2219	203.6	0.032
10	1964	187.2	0.029
11	1680	168.1	0.026
12	1380	146.7	0.023
13	1230	135.5	0.021
14	1020	119.1	0.019
15	825	102.9	0.016
16	750	96.3	0.015
17	690	91.0	0.014
18	630	85.4	0.013
19	555	78.3	0.012
20	495	72.3	0.011
21	420	64.6	0.010
22	390	61.4	0.010
23	345	56.4	0.009
24	300	51.2	0.008
25	255	45.8	0.007
26	225	42.0	0.007
27	195	38.0	0.006
28	180	36.0	0.006
29	150	31.7	0.005
30	120	27.2	0.004
31	120	27.2	0.004
32	105	24.8	0.004
33	105	24.8	0.004
34	90	22.3	0.003
35	75	19.7	0.003
36	45	13.8	0.002
37	45	13.8	0.002
38	45	13.8	0.002
39	30	10.5	0.002
40	15	6.5	0.001
41	15	6.5	0.001
42	15	6.5	0.001
Total	99168	6433.3	1.000

difference between observed data's cumulative relative frequency distribution function $F_k(s)$ and generated data's cumulative relative frequency distribution function $G_l(\hat{S})$. Namely, $W_{k,l}$:

$$W_{k,l} = \sum_{i=1}^{k+l} (F_k(s) - G_l(\hat{S}))^2 , \qquad (14)$$

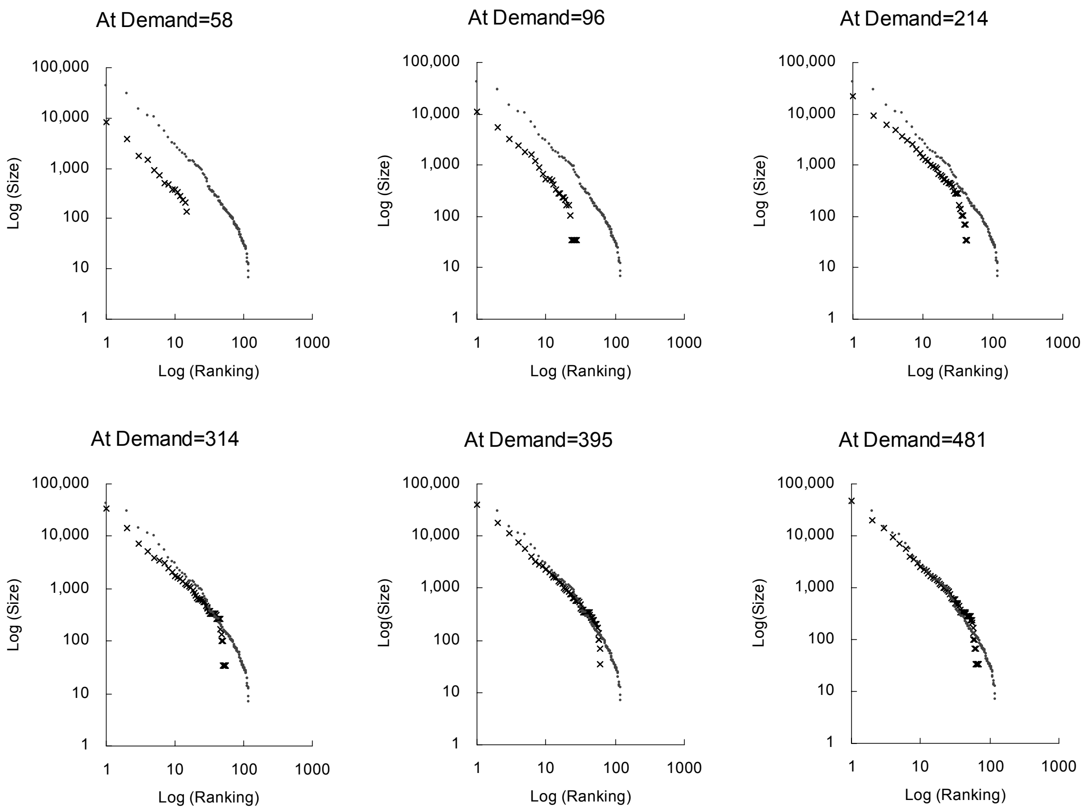
where

s: sales size distribution in observed data,
$\hat{s}$: sales size distribution generated from simulation,
$\hat{S}$: an average of 10 pieces of sales size distribution $\hat{s}$,
k: the number of products in distribution s,
l: the number of products in distribution $\hat{S}$.

Step 7: An Average of (p^*, q^* | D): (p^{**}, q^{**} | D)

The steps from 2-1 to 6 are repeated 10 times; 10 sets of (p^*, q^* | D) are obtained. Then, (p^{**}, q^{**} | D) is calculated from trim average of 10 pieces of p^* and q^* (the average of 8 values except both the maximum and minimum values).

Figure 9
Birth and Growth of Products in Process of Time
(Heavy Detergent, 1998, p=0.21, q=0.83, λ=1.340)



According to this algorism, the model calculates birth probability of a new product and the growth rate of existing products, and assigns demand/sale each product. Figure 9 shows birth and growth of product generated in the simulation model according as time passes and demand occurs (circles are observed data and crosses are generated data; p^{**}=0.21, q^{**}=0.83, λ=1.340 in a heavy detergent market, 1998). Then, it is investigated whether this generated sales size distribution reproduces accurately observed data. This procedure is repeated in market i={m=13 markets: sauce, catsup,…, and aluminum foil} and period j={k=10 years: 1990, 1991,…, and 1999}. Incidentally, we implicitly take withdrawal of a product into account by considering a product that has not acquired sales to be discontinued.

5.3. Data

The home scan panel data mentioned above was used; markets and periods were the same as the study 2 as follows:

- food and beverages (10 markets): sauce, catsup, butter, margarine, curry roux, boil-in-bag curry, instant coffee, cola, 100% pure fruit juice, and beer,
- commodities (3 markets): heavy detergent, shampoo, and aluminum foil;
- periods (10 years): the data from Jan. 1 1990 to Dec. 31 1999 was added up each year so that there were 10 sets of the data.

That is, 130 cases (13 markets and 10 years) were examined.

5.4. Result 1

The simulation model ran according to the above algorism to generate the data. The fitness of the generated data for the observed data was examined. The result from a cola market in 1999 is shown[12]. The data generated by the model fitted maximally the observed data, when p=0.11 and q=0.69. The number of products in the generated data was 42 and that in observed data was 52. Figure 10 shows cumulative relative frequency distribution function $F_k(s)$ of the observed data and $G_l(\hat{S})$ of the generated data. Since the value of the Cramer-Von Mises statistic $W_{k,l}$ was extremely small ($W_{k,l}$=0.0004), it was reasonable to understand that the sales size distribution of the data generated by the simulation model fitted that of the observed data. And the geometrical fitness was good (Figure 11). Furthermore, the cumulative market dollar shares calculated from both data were compared to investigate empirical validity of the model as follows:

[12] This market was illustrated with data as follows:

- the number of products: 52,
- the purchasing dollar volume per 100 households: ¥106,698,
- the percentage of the purchasing household: 61%,
- the average prices of products: ¥150.0.

	Observed Data	Generated Data
The Number of Products	52	42
Cumulative Market Dollar Shares		
Top 3 Products	59.9%	60.1%
Top 5 Products	72.3%	72.9%
Top 10 Products	87.3%	87.4%
Top 20 Products	96.7%	96.7%
Top 30 Products	99.2%	99.3%

Figure 10
Cumulative Relative Frequency Distribution Function,
the Observed Data vs. the Generated Data by the Model
(Cola, 1999)

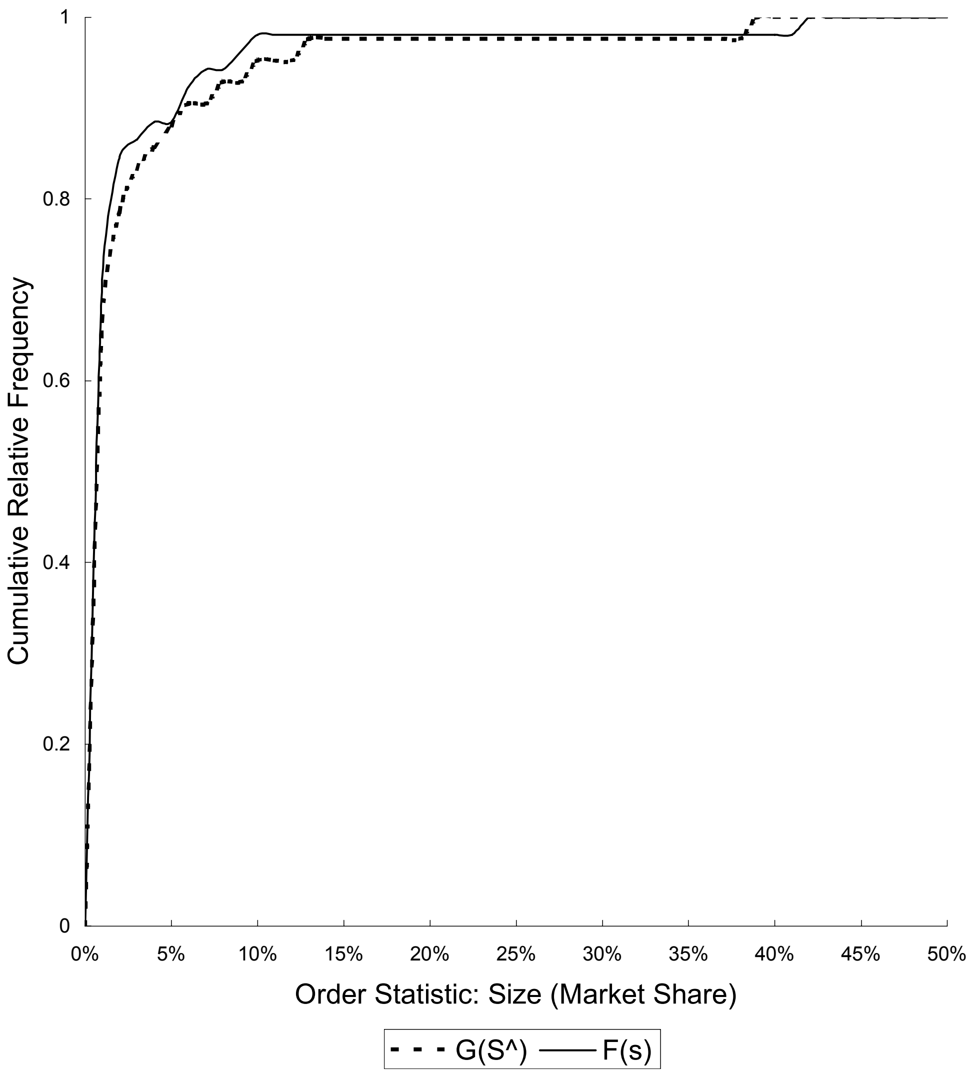
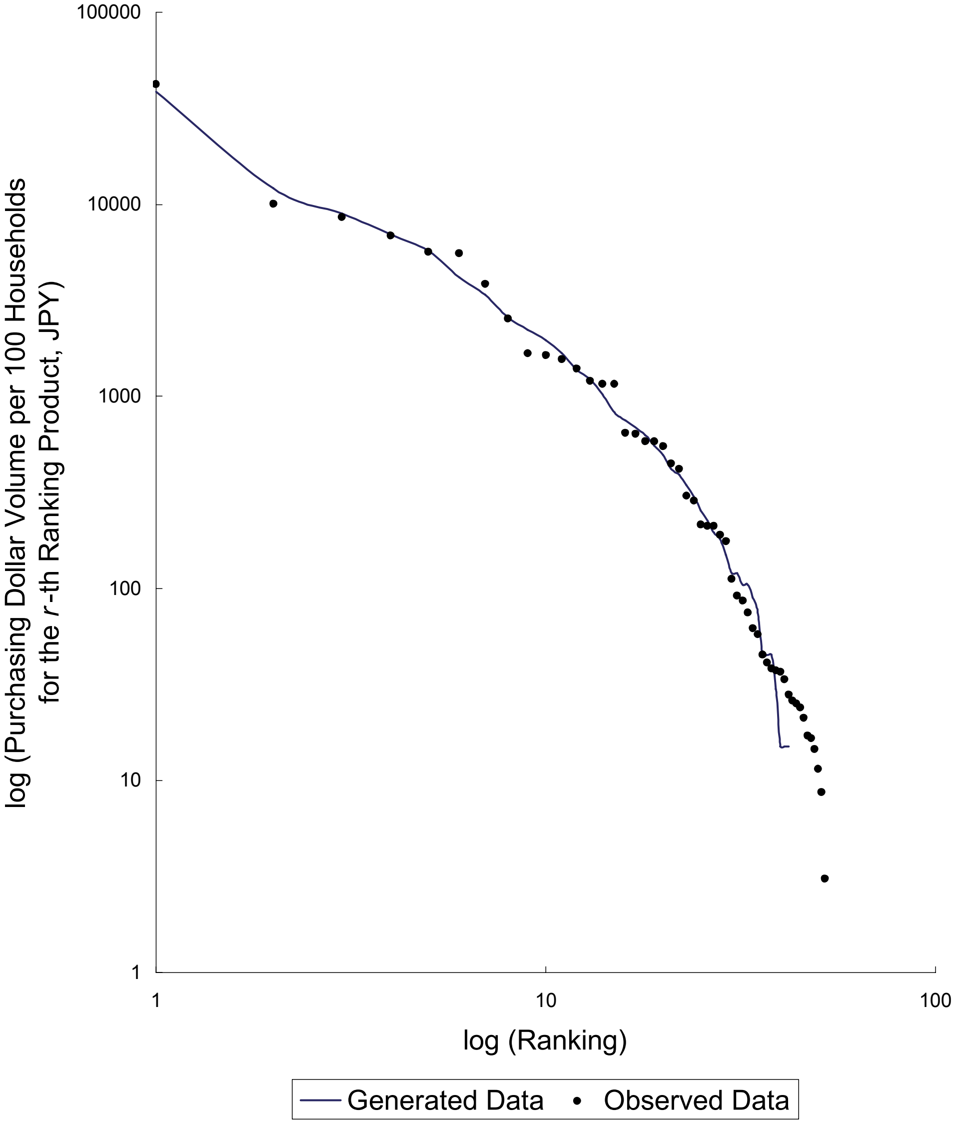


Figure 11
The Rank-Size Rule, the Observed Data vs. the Generated Data (Cola, 1999)



The cumulative market dollar share of the generated data fitted that of the observed data. While the number of products in the generated data was smaller than that in the observed data, it is not matter, since the top 30 products occupy more than 99% cumulative market share in both the observed data and the generated data.

According to the Cramer-Von Mises statistic, the data generated by the model fitted the observed data in all the 13 markets and all the 10 periods (Table 5-1 and 5-2). Furthermore, geometrical fitness was good. Incidentally, the minimum and maximum value of p were 0.06 and 0.72 respectively, and those of q were 0.40 and 1.16 respectively among all markets and years.

Table 5-1
The Result of the Simulation Model (Food and Bevarage)

Market	Year	Number of Products		Parameter		$W_{k,l}$
		Observed Data	Generated Data	p	q	
Sauce	1990	137	84	0.26	0.96	0.0003
	1991	150	87	0.30	0.96	0.0003
	1992	157	84	0.30	1.02	0.0004
	1993	150	81	0.28	0.99	0.0004
	1994	139	80	0.26	0.98	0.0004
	1995	130	82	0.28	1.06	0.0003
	1996	156	77	0.31	1.12	0.0005
	1997	149	89	0.30	1.07	0.0002
	1998	155	78	0.31	1.00	0.0006
	1999	159	79	0.30	1.06	0.0003
Catsup	1990	82	61	0.20	0.91	0.0003
	1991	88	60	0.18	0.92	0.0003
	1992	83	53	0.18	0.82	0.0006
	1993	76	55	0.19	0.87	0.0004
	1994	80	50	0.20	0.92	0.0005
	1995	93	58	0.23	0.97	0.0005
	1996	94	68	0.23	0.96	0.0001
	1997	84	58	0.20	0.96	0.0002
	1998	87	63	0.23	0.96	0.0002
	1999	94	62	0.24	0.94	0.0004
Butter	1990	42	35	0.11	0.63	0.0003
	1991	42	31	0.11	0.64	0.0003
	1992	38	29	0.09	0.54	0.0004
	1993	43	37	0.15	0.75	0.0004
	1994	53	45	0.15	0.69	0.0004
	1995	55	37	0.14	0.56	0.0003
	1996	66	44	0.16	0.71	0.0005
	1997	51	34	0.12	0.61	0.0002
	1998	49	39	0.12	0.60	0.0006
	1999	52	42	0.14	0.71	0.0003
Margarine	1990	87	71	0.17	0.59	0.0005
	1991	84	71	0.17	0.58	0.0004
	1992	78	65	0.17	0.60	0.0004
	1993	85	65	0.19	0.66	0.0008
	1994	76	57	0.16	0.63	0.0003
	1995	63	52	0.17	0.66	0.0004
	1996	69	60	0.17	0.65	0.0001
	1997	62	58	0.17	0.66	0.0002
	1998	69	60	0.18	0.66	0.0003
	1999	67	54	0.17	0.65	0.0007
Curry Roux	1990	251	169	0.42	0.68	0.0001
	1991	263	171	0.42	0.69	0.0003
	1992	233	152	0.35	0.68	0.0003
	1993	237	134	0.36	0.69	0.0005
	1994	225	130	0.34	0.74	0.0006
	1995	221	150	0.35	0.72	0.0002
	1996	219	124	0.32	0.64	0.0011
	1997	217	123	0.32	0.66	0.0009
	1998	181	124	0.29	0.60	0.0005
	1999	179	113	0.30	0.62	0.0010

Table 5-1 (Continued)
The Result of the Simulation Model (Food and Bevarage)

Market	Year	Number of Products		Parameter		$W_{k,l}$
		Observed Data	Generated Data	p	q	
Boil-in-Bag Curry	1990	105	92	0.30	0.50	0.0007
	1991	131	122	0.36	0.41	0.0002
	1992	137	105	0.35	0.40	0.0013
	1993	161	129	0.38	0.53	0.0002
	1994	191	144	0.42	0.58	0.0004
	1995	198	143	0.39	0.54	0.0004
	1996	230	157	0.48	0.63	0.0003
	1997	264	174	0.51	0.73	0.0003
	1998	286	188	0.51	0.89	0.0001
	1999	364	198	0.55	0.86	0.0002
Instant Coffee	1990	124	66	0.26	0.95	0.0006
	1991	128	69	0.25	0.89	0.0007
	1992	126	71	0.26	0.91	0.0006
	1993	137	79	0.26	0.93	0.0004
	1994	135	70	0.25	0.93	0.0006
	1995	163	84	0.31	0.94	0.0005
	1996	154	71	0.27	0.94	0.0010
	1997	152	70	0.27	0.94	0.0008
	1998	147	76	0.26	0.88	0.0008
	1999	162	74	0.27	0.88	0.0011
Cola	1990	34	28	0.06	0.52	0.0015
	1991	46	32	0.07	0.68	0.0014
	1992	57	43	0.09	0.57	0.0009
	1993	58	51	0.13	0.64	0.0003
	1994	78	69	0.16	0.70	0.0001
	1995	85	85	0.21	0.80	0.0001
	1996	86	68	0.18	0.82	0.0001
	1997	70	47	0.13	0.78	0.0005
	1998	62	46	0.11	0.74	0.0006
	1999	52	42	0.11	0.69	0.0004
100% Fruit Juice	1990	489	266	0.68	0.91	0.0001
	1991	505	270	0.66	0.82	0.0001
	1992	505	266	0.67	0.81	0.0002
	1993	556	285	0.71	0.82	0.0001
	1994	561	275	0.71	0.77	0.0002
	1995	606	276	0.72	0.84	0.0002
	1996	591	282	0.69	0.71	0.0002
	1997	542	262	0.67	0.68	0.0003
	1998	504	238	0.63	0.71	0.0002
	1999	474	217	0.59	0.65	0.0003
Beer	1990	240	145	0.38	0.80	0.0004
	1991	295	154	0.41	0.73	0.0005
	1992	260	147	0.38	0.72	0.0006
	1993	269	144	0.37	0.68	0.0010
	1994	280	140	0.37	0.73	0.0008
	1995	359	157	0.43	0.82	0.0007
	1996	351	154	0.42	0.88	0.0004
	1997	343	143	0.50	0.67	0.0014
	1998	349	131	0.53	0.75	0.0021
	1999	343	113	0.53	0.77	0.0022

Table 5-2
The Result of the Simulation Model (Commodities)

Market	Year	Number of Products		Parameter		$W_{k,l}$
		Observed Data	Generated Data	p	q	
Heavy Detergent	1990	130	86	0.26	0.95	0.0002
	1991	137	88	0.27	0.86	0.0003
	1992	141	93	0.28	0.90	0.0004
	1993	122	73	0.23	0.82	0.0009
	1994	113	77	0.23	0.80	0.0004
	1995	136	83	0.25	0.76	0.0011
	1996	153	89	0.29	0.80	0.0007
	1997	128	70	0.24	0.89	0.0007
	1998	120	69	0.21	0.83	0.0006
	1999	125	79	0.25	0.86	0.0006
Shampoo	1990	435	194	0.66	0.97	0.0002
	1991	481	202	0.72	0.97	0.0003
	1992	512	200	0.71	0.98	0.0003
	1993	501	203	0.72	1.11	0.0002
	1994	421	170	0.63	1.11	0.0003
	1995	475	203	0.73	1.15	0.0001
	1996	493	196	0.71	1.16	0.0001
	1997	482	211	0.69	1.13	0.0001
	1998	495	213	0.72	1.13	0.0002
	1999	573	226	0.76	1.13	0.0001
Aluminum foil	1990	81	57	0.26	0.92	0.0006
	1991	86	54	0.27	0.92	0.0013
	1992	88	68	0.27	0.96	0.0005
	1993	85	58	0.27	0.96	0.0007
	1994	75	56	0.26	0.97	0.0004
	1995	64	51	0.21	0.84	0.0009
	1996	71	51	0.24	0.87	0.0009
	1997	64	55	0.23	0.84	0.0007
	1998	67	46	0.26	0.89	0.0011
	1999	71	50	0.25	0.87	0.0010

Above all, the data generated by the Monte Carlo simulation model fitted the observed data. Since the 80/20 law formulated by equation (9) fitted the observed data, the generated data fitted the 80/20 law. Hence, the 80/20 law emerges, if simple local rules, a new product stochastically occurs and it grows as time passes, are given in the simulation model. On the other hand, birth and growth of a product are very natural in any market. Hence, the 80/20 law emerges in any market.

5.5. Result 2

The data generated by simple local rules (a new product stochastically occurs and it grows as time passes) in the simulation model fitted well the observed data following the 80/20 law. Since product birth and growth are universal market phenomenon, the 80/20 law emerges in any market. Then, it will be verified that the 80/20 law formulated by equation (9) emerges in any value of parameter p and q. That is, the law emerges, only if a new product stochastically occurs and it grows as time passes.

Artificial date sets generated in values of p ($0 \leqq p \leqq 1$) and q ($0 \leqq q$) ($\hat{\lambda}$=1000/365, Price=100) were applied to the 80/20 law formulated by equation (9). A result was as follows. If p=0, no product emerges; if p=1, all product acquire just 1 unit sales size. Hence, the 80/20 law does not emerge. On the other hand, if $0<p<1$, the artificial data sets fitted equation (9). The fitness was well according to a coefficient of determination R^2 and the standard error of estimate *SEE* (Table 6). The maximum and minimum values of R^2 were 0.994 (p=0.1, q=0.5 and 0.7) and 0.743 (p=0.9, q=0.3) respectively. The reason why R^2 was not good at p=0.9 was that the fitness of the lower ranking products was not good because of the shortage of demand. It will be better, if large demand occurs (this will be discussed again later). Thus, the artificial data generated by the simulation model, in any value of p ($0<p<1$) and q ($q \geqq 0$), fitted the 80/20 law formulated by equation (9), the Mandelbrot model. This shows that the 80/20 law emerges inevitably in any market.

Table 6
Regression Analysis Applying Artificial Data Generated by the Simulation Model to Equation (9)

(0<*k*, *Price* =100)

The Simulation Model		Number of	Equation (9)			R^2	*SEE*	Range of	
p	*q*	Products	*a*	*b*	*k*			Artificial Data	
0.1	0.1	115	1.436	223,526.799	13.098	0.969	177	5,900	10
0.1	0.3	112	1.379	164,202.194	10.555	0.980	152	6,310	10
0.1	0.5	119	0.928	19,502.023	0.614	0.979	231	13,000	10
0.1	0.7	110	1.069	26,262.309	0.405	0.994	166	18,500	10
0.1	0.9	111	1.443	34,753.443	0.001	0.994	266	35,270	10
0.3	0.1	333	1.029	42,935.829	23.388	0.970	51	2,020	10
0.3	0.3	333	0.945	24,831.541	12.810	0.979	48	2,380	10
0.3	0.5	333	0.723	7,850.678	2.367	0.968	67	3,680	10
0.3	0.7	333	0.742	7,833.707	0.572	0.987	58	5,830	10
0.3	0.9	333	0.898	12,536.716	0.382	0.996	47	9,280	10
0.5	0.1	522	0.763	10,231.718	23.152	0.962	29	1,200	10
0.5	0.3	539	0.703	6,922.882	13.373	0.961	31	1,320	10
0.5	0.5	539	0.690	6,119.151	7.938	0.971	30	1,620	10
0.5	0.7	539	0.610	3,888.879	1.826	0.970	35	2,320	10
0.5	0.9	533	0.629	3,950.812	0.001	0.990	27	4,070	10
0.7	0.1	730	0.474	1,827.413	9.867	0.931	21	650	10
0.7	0.3	716	0.492	2,001.918	7.827	0.934	23	780	10
0.7	0.5	744	0.463	1,638.066	2.779	0.926	26	1,000	10
0.7	0.7	733	0.479	1,785.609	3.119	0.942	23	980	10
0.7	0.9	732	0.466	1,622.651	0.518	0.958	22	1,350	10
0.9	0.1	912	0.236	419.245	0.357	0.751	18	370	10
0.9	0.3	924	0.232	406.701	0.527	0.743	18	350	10
0.9	0.5	924	0.249	444.658	0.001	0.752	19	440	10
0.9	0.7	916	0.246	440.185	0.001	0.766	18	420	10
0.9	0.9	912	0.273	510.515	0.001	0.799	19	560	10

5.6. Discussion

The Monte Carlo simulation model's algorism (i.e. a new product is born with probability p and it grows with growth rate g determined by growth "asymmetry" q in process of time) shows one of the emerging mechanisms of the 80/20 law, since the home scan panel data shows that the 80/20 law holds in packaged goods markets (Study 1 and 2), and since the simulation model generates the data fitting the observed data (Result 1 in Study 3). Therefore, the 80/20 law emerges, only if a new product is born and it grows in process of time. Since birth and growth of a product is universal, the law holds inevitably in any market. In fact, the 80/20 law emerged in any value of parameter p ($0<p<1$) and q ($q \geqq 0$) in the simulation model (Result 2 in Study 3). That is, the 80/20 law holds in any market and any time, although markets always changes and varies. At this time, since the 80/20 law emerges without any control from outside of a market, it emerges autonomously or organizes itself.

Note again that not individual products but the structure (the 80/20 law) is focused on here. Even if the structure is robust regardless of time, it does not mean that, as the author mentioned in study 1, the ranking or size of an individual product does not change.

The number of products in the simulation model

Lastly, a reason why a number of products generated by the simulation model was smaller than that in the observed data will be discussed. The model calculated sale size distribution average $\hat{S}$ from 10 pieces of sales size distribution $\hat{s}$ (step 5 in the algorisms). The minimum unit sales size of a product in $\hat{s}$ was 1; that in $\hat{S}$ was 0.1. On the other hand, there were many products of which sales size (purchasing units per 100 households in a year) was smaller than 0.1. For example, 24 products out of total 120 products were smaller than 0.1 (in purchasing units per 100 households) in a heavy detergent market in 1998. Thus, a reason why a number of products generated in the model was smaller than that in observed data was that the model was not able to generate products of which unit sales size was less than 0.1 in $\hat{S}$. The model generates a larger number of products with tiny unit sales size, if it calculates $\hat{S}$ from a larger number of $\hat{s}$. In fact, according as a number of $\hat{s}$ increased, a number of products in $\hat{S}$ increased (Case 1a, 1b and 1c, see below). Furthermore, change of a number of products according to change of λ is also shown (Case 2, λ determines total demands in market, in equation (12)).

Case	λ	A number of $\hat{s}$	A number of products in $\hat{S}$ (An average of 10 trials)
1a	1	10	130.5
1b	1	100	136.5
1c	1	500	141.8
2	10	10	193.6

(p=0.50, q=1.00)

The number of products generated by the model approached that of observed data, if the number of $\hat{s}$ generated by the model increased. However, it is not matter that the number of products is small, since the Cramer-Von Mises statistic was small and cumulative market shares of lower ranking products in observed data were enough small. Hence, it is reasonable to calculate sale size distribution average $\hat{S}$ from 10 pieces of sales size distribution $\hat{s}$.

6. Managerial Implications

Lastly, the 80/20 law's forming mechanism will be empirically validated, and market analysis method based on the mechanism will be proposed. The 80/20 law emerges inevitably in any value of parameter p and q in the simulation model ($0<p<1$, $q \geqq 0$). On the other hand, the values of them vary among markets and time. Hence, referring to values of p and q enables us to compare directly all markets, whether a new product is born frequently or seldom, and whether the growth rate of existing product increases or decreases according to product growth. This analysis will be applied to several markets, and if these results match our general knowledge about these markets, the mechanism of the law will be empirically validated.

6.1. Market Analysis Based on the Mechanism of The 80/20 Law

A market analysis method based on the mechanism of the 80/20 law will be proposed. It is managerially useful; the reason for this is as follows. The 80/20 law holds in any market, and emerges in any value of parameter p and q in the simulation model. On the other hand, their values vary among markets. Hence, p and q are universal among markets, and their values indicate comparable characteristics of markets. Hence, a market analysis proposed here provides us spatial comparisons of plural markets (6.3. Comparison among Markets), a time series analysis in a market (6.4. Time Series Analysis of Markets), and implications for portfolio strategies (6.5. Managerial Implications for Portfolio Strategies).

That is, first, referring p and q enables us to compare directly plural markets; whether a new product is born frequently or rarely, and whether larger size products grow rapidly or smaller ones grow rapidly. Market structure analysis methods that have been proposed in marketing are not opportune to compare directly plural markets. On the other hand, market concentration measures used in economics (e.g. the Gini coefficient, and the Herfindahl-Hirschman index) permit us to compare directly plural markets. However, these measurements are rarely used in marketing, since they are not able to illustrate market characteristics marketing requires. Thus, the method proposed here are better than these models and measurements, since it is based on the universal structure (the 80/20 law) and general marketing phenomena (i.e. birth and growth of products), and opportune to compare directly plural markets. Markets will be classified into four (product birth probability: it is high or low, and asymmetric growth rate: it increases or decreases according to growth), and their characteristics will be interpreted.

Secondly, the market analysis traces how market changes based on birth and growth of a product. Finally, comparing plural markets and tracing their time series changes provide companies managerial implications for portfolio strategies. That is, how companies should invest their finite resources in plural markets. Implications will be suggested based on the above market classification.

6.2. Understandings of Internal Characteristics of Markets

First, observing parameter p and q enable us to comprehend markets. Concrete examples are as follows. Instant coffee and beer are both luxury beverage but market concentrations are different. For example, the top 10% products had a 81% market dollar share in an instant coffee market in Japan (1999), on the other hand, that had a 72% market dollar share in a beer market. The reason for this was in the differences of birth probability of a new product and the growth rate of existing products. Namely, (p, q)=(0.27, 0.88) in an instant coffee market and (0.53, 0.77) in a beer market (1999).

These two markets are described as follows. A cohort effect affects highly consumer preferences to instant coffee (All Japan Coffee Association 1993). That is, a cohort who was born in 1952 prefers mostly instant coffee, since they were the first and main customers when it was first launched in Japan as a very fashionable beverage originated from western countries. When there is the cohort effect, it is important to maintain existing customers and products for them. Hence, growing of existing products was focused on; launching of new products was unheeded in a Japanese instant coffee market. In fact, Nestle had earned the most of its sales and profits at least for 10 years until 1999 by its only two brands. And their ads claimed their unchangeable brand images for long years. The small values of p and the large value of q match these phenomena.

On the other hand, since customers make much of "novelty" in a Japanese beer market, beer companies launch aggressively new products per quarter to stimulate consumers. This matches the large value of p and the small value of q. Above all, observing p and q allow companies to comprehend markets.

6.3 Comparison among Markets

Second, referring parameter p and q common to markets enable us to compare directly plural markets. Markets can be classified into four according to values of p and q (Figure 12), and each market is described as follows.

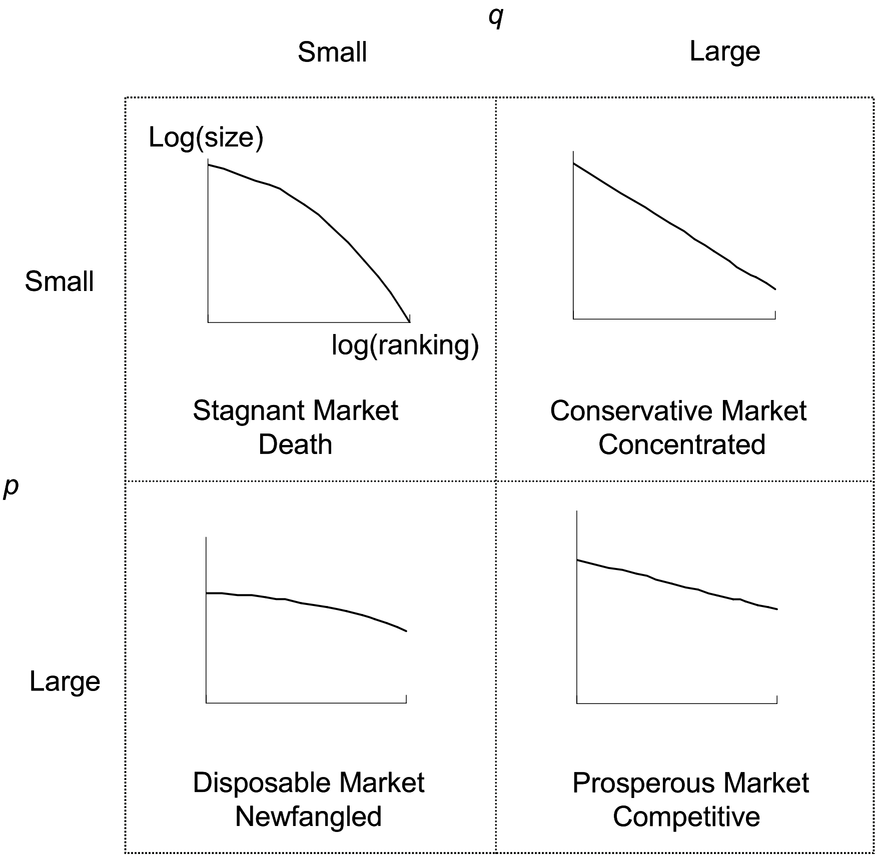
A Conservative Market: small p and large q

If p is small and q is large, birth probability of a new product is small, and the growth rate of upper ranking (larger size) products is larger than that of lower ranking (smaller size) products (it is caused by the law of increasing returns). Hence, a small number of huge size products (e.g. popular brands or standard goods) rule a market. Geometrically, the graph has a steep slope, a short tail, and convex line in a log-log graph (a horizontal axis is the product ranking and a vertical axis is its size). This is named a conservative or concentrated market.

A Disposable Market: large p and small q

If p is large and q is small, birth probability of a new product is large, and the growth rate of upper ranking products is smaller (it is caused by the law of diminishing returns). Since many new products are born and their growth rate decrease according as they grow, a huge size product rarely exists and many small products share their market share. A new product is easily born but grow hardly. The graph has a gentle slope, a long tail and concave line. This is named a disposable or newfangled market.

Figure 12
Market Classification Based on Birth and Asymmetric Growth of a Product



A Prosperous Market: large p and large q

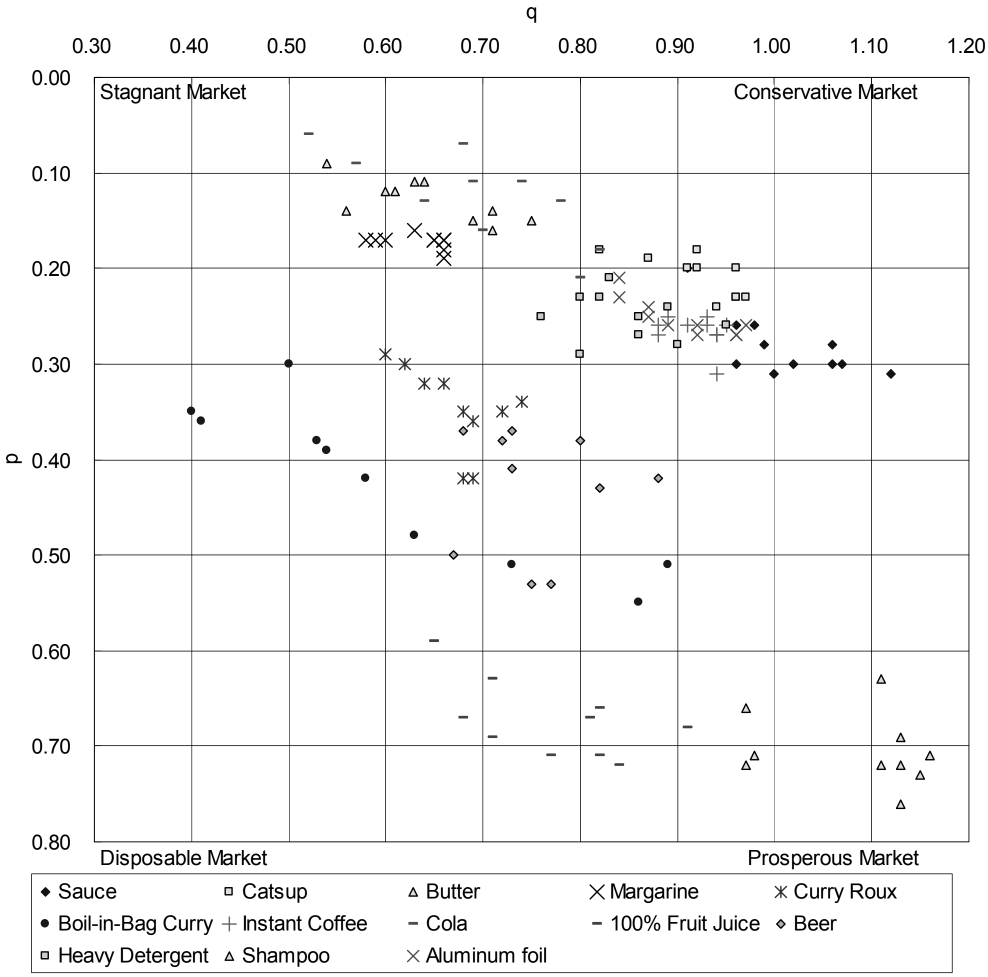
If *p* is large and *q* is large, birth probability of a new product is large, and the growth rate of upper ranking products is larger. Many new products are born, and most of them are discontinued, since the growth rate of lower ranking products is smaller. On the other hand, few products grow largely, if their size exceeds a threshold, since their growth rate increases according as they grow. However companies easily launch a new product, consumers prefer popular brands and standard goods. This market may be competitive. The graph has a gentle slope, a long tail and, convex line. This is named a prosperous market.

A Stagnant Market: p is small and q is small

If *p* is small and *q* is small, birth probability of a new product is small, and the growth rate of upper ranking products is smaller. A few products are launched, and their growth rate decreases according as they grow. This market may rarely change. The graph has a steep slope, a short tail and concave line. This is named a stagnant market.

Figure 13 shows p and q obtained from the 13 markets and 10 years. Since the average values of p and q are 0.33 and 0.81 respectively, markets can be classified into four at around these values. Concrete cases validate empirically this classification as follows.

Figure 13
Birth of a New Product vs. Asymmetric Growth of Existing Products:
Distribution of Parameter *p* and *q*



The Conservative Market

An example of the conservative market is a catsup market (1999) having small p=0.24 and large q=0.94. Companies had small chance to launch a new product, and consumers preferred standard goods. Namely it is as follows.

- A number of products in a catsup market was small (94 products in 1999),
- a huge size product (SKU), Kagome “Tomato Catsup Tube 500g,” occupied a 36.3%

market share, and

- a product had small chance to enter into this market. The reasons for this were as follows:
  - two mega brands (Kagome and Del Monte) ruled the market,
  - there were few opportunities for product extension (e.g. neither size nor package extension), since whole ranges of both size (12g and 10 pieces, 200g, 300g, 400g, 500g, 700g, 800g and 1kg) and package (tubes, bottles and plastic bottles) were provided, and
  - consumers generally were conservative for their taste of seasoning.

These phenomena match the conservative market in which the birth probability of a new product is small and the growth rate of upper ranking (larger size) products is larger than that of lower ranking (smaller size) products.

A Disposable Market

An example for the disposable market is a boil-in-bag curry[13] market (1999) having large p=0.55 and small q=0.86. This market had grown rapidly (total purchasing dollar volume per 100 households in the market had increased from ¥64,592 in 1990 to ¥102,888 in 1999), since consumers preferred its convenience/handiness. Many new products were born in this market, there was no standard product, and products in upper rankings were rapidly replaced with others. Namely, these were as follows:

- customers preferred gimmicks (e.g. the top and second ranking products in 1999 were merchandised by “Pocket Monster” which was very popular comic hero among children),
- since a boil-in-bag curry was packed for one meal and easy to cook, its main targets and a usage scene were children, students and young singles eating it alone. They sought variety fitfully.
- companies developed easily new products by changing just names/packages of existing products, or by merchandizing them with comic heroes/heroines, and
- many new products were launched because of the rapid growth of the market, variety seeking of consumers, and easiness of product development (a number of products had increased from 105 in 1990 to 364 in 1999).

These match the disposable market in which the birth probability of a new product is large and the growth rate of upper ranking products is smaller.

The Prosperous Market

An example for the prosperous market is a shampoo market (1999) having large p=0.76 and large q=1.14. Only few prominent brands ruled the market; on the other hand, many new products

[13] A boil-in-bag curry is a ready-cooked food packed for one meal; customers eat it by just heating. Curry with rice is the most popular home cooking in Japan.

were launched.

- Since consumers made much of brands as well as cosmetics, few prominent brands (e.g. P&G "Vidal Sassoon") occupied a huge market share,
- the market was finely segmented by products' benefits and purposes (e.g. damage care, scurf care, UV care, for men, and for children); companies created new segments and launched new products through technology development and evoking consumer needs, and
- since a main target (young females) not only made much of brands but also sought variety, companies launched regularly new products as well as promoted aggressively existing products and extended their lines to maintain their sales.

These match the prosperous market in which the birth probability of a new product is large and the growth rate of upper ranking products is larger.

The Stagnant Market

An example for the stagnant market is a curry roux market (1999) having small p=0.30 and small q=0.62. This market had reduced by demand switching to boil-in-bag curry (purchasing dollar volume per 100 households had decreased from ¥227,478 in 1990 to ¥182,105 in 1999). This market's characteristics were as follows.

- A number of new products had decreased (from 251 in 1990 to 179 in 1999), and
- sales size of upper ranking products had decreased (e.g. purchasing dollar volume per 100 households of the top product, House Foods "Vermont Curry, sweet taste, 250g," had decreased from ¥19,767 in 1990 to ¥17,158 in 1999).

These match the stagnant market in which the birth probability of a new product is small and the growth rate of upper ranking products is smaller.

6.4. Time Series Analysis of Markets

Tracing time series of parameter p and q enables companies to comprehend changes of a market. Concrete cases show coherence between parameter value changes and our general knowledge for market changes.

A 100% Fruit Juice Market

The value of q had decreased rapidly (from q=0.91 in 1990 to 0.65 in 1999) in a 100% fruit juice market. In fact, upper ranking (larger size) products with strong brand power had lost their sales (e.g. purchasing dollar volume per 100 households for the famous top SKU had decreased from ¥20,681 in 1990 to ¥4,312 in 1999), and (infamous) lower ranking ones had grown rapidly thanks to their price promotion, since a key for competition was totally changed from brands to in-store promotions. That is,

- huge market shares of prominent brands (e.g. Sunkist and Dole) in 1990 indicated that brands were important. On the other hand,
- in-store promotions (e.g. price promotions and displays) effected strongly sales in 1999,
  - since an average price had declined (from ¥166.6 in 1990 to ¥149.7 in 1999), and
  - since GMS/SM (general merchandising stores/supermarkets) promoting aggressively in-store promotions (especially a price promotion) had increased their shares among stores (from 61.4% in 1990 to 73.1% in 1999), and CVS (convenience stores) promoting negatively a price discount but positively brands had lost their shares (from 7.6% in 1990 to 4.6% in 1999).

During that period, upper ranking products lost their sales; on the other hand, lower ranking products grew. These match the decline of q.

A Heavy Detergent Market

An epoch-making heavy detergent, Kao “Attack,” was launched in May 1987, and was greatly welcomed by customers. Then, similar competitors were launched (Lion “High Top” in 1998 and P&G “Ultra Ariel” in 1990). These products ruled huge shares as standard products (market dollar shares in 1990 were 33.1% by Kao “Attack 1.5kg”, 11.6% by Lion “High Top 1.5kg”, and 6.5% by P&G “Ultra Ariel 1.5kg”). The phenomena that the larger size and upper ranking standard products grew even more and ruled the market match the large value of p=0.95 in 1990.

Many new products were still launched (e.g. Kao “Just” in 1991 and “Just Five” in 1993), however, they could not success and were soon discontinued. On the other hand, those three major products mentioned above occupied the top 3 rankings until 1994, however, they lost gradually their sales because of a drop in their attractiveness according to their aging (the sum of purchasing dollar volume per 100 households for the top 3 products had been still huge but decreased from ¥132,764 in 1990 and ¥105,334 in 1994). The phenomena that the sales of larger size and upper ranking products were struggling match the decline of q (from 0.95 in 1990 to 0.89 in 1994).

The three major products mentioned above were greatly renewed: Kao “New Compact Attack” and Lion “Enzyme Top” in 1995, and P&G “Ariel Pure Clean” in 1997. Since these new products grew as standard products, the growth rate of upper ranking (larger size) products increased again (the sum of purchasing dollar volume per 100 households for the top 3 products had increased from ¥68,669 in 1995 to ¥86,313 in 1997). This matches a rise in the value of q (from 0.76 in 1995 to 0.89 in 1997).

A sequence of these events (birth of the epoch-making products, their rapid growth to major products, their slackening after their growth, renewal of them, and their another growth to standard products) matches the change of the value of q.

Above all, the market analysis based on the emerging mechanism of the 80/20 law has analyzed several cases. At that time, since those market analyses matched our general knowledge and descriptions about those cases, the mechanism of the law were empirically valid.

6.5. Managerial Implications for Portfolio Strategies

The market classification by parameter p and q was validated by concrete cases. This classification shows managerial implications for marketing strategies. Referring to universal parameter describing birth and asymmetric growth of products enables a company investing in several markets to compare directly these markets and to comprehend them: a new product is born frequently or seldom, and the growth rate of existing products increases or decreases according to product growth. Implications for portfolio strategies (e.g. market choice and finite resource allocation) are as follows.

The Conservative Market

Launching a new product is not easy, and the growth rate of upper ranking (larger size) products is larger than that of lower ranking (smaller size) products. Hence, it is strategically important to own predominant brands or standard products. A company having such brands/products should concentrate its marketing investment on them. On the other hand, a company owning neither a excellent brand nor a standard product should grow an upper ranking product by its aggressive investment, or explore its retreat from a market, since the market is not attractive for a weak company.

The Disposable Market

A new product is easily launched; its growth rate decreases according as it grows. There is no predominant brand/product because of rapid changes in a market. A Company should focus on new product development, and should not hesitate launch a new product, even if a new product cannibalizes sales from its existing products.

The Prosperous Market

Many new products are launched, and the growth rate of upper ranking products is larger. This market is probably competitive but attractive because of its enlargement. A company should both launch aggressively new products and raise their products to predominant ones to success in this market.

The Stagnant Market

This market remains probably in decline. A new product has few opportunities; the growth rate of products decreases according as they grow. A company should count harvesting its products, while a company having upper ranking products should invest in them not to lose their shares or sales.

7. Conclusion

While the 80/20 law has long been discussed in marketing literature, only a few studies have investigated its mechanisms from marketing viewpoints, as far as the author knows. Hence, the 80/20 law's emerging mechanism and theoretical meanings in marketing were discussed. This article, after proving that the 80/20 law holds universally in markets, showed and validated empirically its emerging mechanism, and proposed the market structure analysis method by its theoretical meanings.

First, data showed that the 80/20 law holds universally in markets. That is, data collected in the 31 markets during the 2 years showed that the law formulated with the Zipf model (one of the power models) was observed commonly in markets. Furthermore, data collected in the 16 markets during the 10 years showed that the law re-formulated by the Mandelbrot model (one of the general expressions of power models) was observed commonly in markets. At that time, those models' parameter determined theoretically birth probability of a new product and the asymmetric growth rate of existing products within a market (i.e. whether the growth rate of larger products is higher or lower than that of smaller products).

Secondly, the emerging mechanism of the 80/20 law was discussed. That is, by focusing on birth and growth of a product, the Monte Carlo simulation model generated product size distribution data matching the observed data following the law. At that time, the law emerged in any value of the parameter, birth probability and the asymmetric growth rate of a product. That is, the law emerges only if a product occurs constantly and grows in process of time. Since product birth and growth is natural in a market, the law emerges inevitably and autonomously (or it organizes itself).

Thirdly, the 80/20 law's forming mechanism was empirically validated, and the market analysis method was proposed based on its mechanism. Since the 80/20 law holds universally and emerges inevitably in any value of the parameter in the simulation model, the parameter is common to markets. On the other hand, birth probability of a new product varies among markets, and the growth rate of existing products is asymmetric among products within a market. Hence, referring to the universal parameter in the simulation model enables companies to directly compare any market, whether a new product is born frequently or seldom, and whether the growth rate of products increases or decreases according as products grow. Finally concrete cases validated empirically the value of parameter and the market analysis method based on the value of the parameter.

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