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INTER-COUNTRY GAPS IN INCREASING-RETURNS-TO-SCALE TECHNOLOGIES AND THE CHOICE AMONG INTERNATIONAL ECONOMIC REGIMES

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ABSTRUCT

The recent efforts of leading industrialized countries to reduce barriers on international transactions have been biased to trade in goods and capital and against labor migration. This paper examines the rationality of such an asymmetry in the liberalization policies by making a welfare comparison between countries under four different international economic regimes, (i)free trade in goods only, (ii)free trade in goods and labor force, (iii)free trade in goods and capital and (iv)free trade in goods and the factors of production in a general equililbrium model with the intra-industry trade which stems from monopolistic competition and increasing-returns-to-scale technologies using capital as a fixed input and labor as a variable input.

INTER-COUNTRY GAPS IN INCREASING-RETURNS-TO-SCALE TECHNOLOGIES AND THE CHOICE AMONG INTERNATIONAL ECONOMIC REGIMES

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I. Introduction

In the modern world with advanced means of communication and transportation, countries are economically tied through international commodity trade, international capital and labor movements. Although in some cases one of these international transactions is perfectly substitutable for another[1], in general all of them supplementarily serve to increase the efficiency of world production by reallocating factors of production from the uses of lower productivity to ones of higher productivity. Therefore, it will be rational for the world as a whole to try to remove barriers on these international transactions and move toward freer trade in goods and factors of production. In reality, however, there is a difference in the countries' efforts toward liberalization between trade in goods and capital and trade in labor force. This is especially prominent in advanced industrialized countries. Although they are trying to reduce barriers on commodity trade and international capital movement through global agreements such as the General Agreement on Tariffs and Trade and international institutions such as the International Monetary Fund, they continue to strictly restrict immigration of the labor force.

The purpose of this paper is to examine the rationality of such an asymmetry in world liberalization policies from a purely economic point of view. For this purpose, a one-sector, two-factor, two-country general equilibrium model is constructed with the intra-industry trade which stems from monopolistic competition in the markets of differentiated products as well as increasing-returns-to-scale technologies using capital as a fixed input and labor as a variable input[2]. Then we compare the welfare levels of each country as realized under the following four international economic regimes: T regime under which free trade in goods is allowed but international factor movements are prohibited; L-T regime under which free trade in goods and labor is allowed but international capital movement is prohibited; K-T regime under which free trade in goods and capital is allowed but international labor movement is prohibited; and LK-T regime under which free trade in goods, labor and capital is allowed.

The results obtained in this paper show that if both the efficiency levels of technology and the relative factor endowments are different between two countries, then (i) the LK-T regime cannot exist between them;(ii) the L-T or K-T regime stands first while the T regime invariably stands third in their welfare rankings; and (iii) whether the L-T or K-T regime is likely to stand first in the rankings depends on a combination of a country's technological superiority/inferiority and relative capital/labor abundance. If, for instance, a country with superior technologies is relatively capital-abundant and hence a country with inferior technologies is relatively labor-abundant, the L-T regime is likely to be the best choice and the K-T regime is likely to be the second best choice for both countries. With advanced industrialized countries being relatively capital-abundant, this result would imply that their failure to liberalize the quantity restrictions on labor immigration is not reasonable from the standpoint of their economic welfare. This result is also in contrast with the ones obtained in Suzuki(1989) in that it highlights the role of the inter-country technology gap in the choice between the L-T and K-T regimes. Suzuki(1989) studied the choice issue in a 2X2 Heckscher-Ohlin model characterized by particular types of intercountry difference in technology and derived the conclusions a la Heckscher-Ohlin that a country is better off under an international economic regime where the reward of its abundant factor is higher. In this model, thus, the two countries may prefer the same regime or different regimes, depending on the discrepancy in factor endowments between them. The technology gaps are certainly important in creating diversified economies but their role is vague in determining the choice.

The configuration of the present paper is as follows. In section II the framework of two countries' economies engaged in free commodity trade is presented and their welfare levels are studied. Section III shows that the L-T or K-T regime is generally better than the T regime for both countries and that the LK-T regime can exist only under a limiting condition on the technology gap and is then indifferent as to the L-T and K-T regimes. Section IV compares the welfare levels of the two countries under the L-T and K-T regimes to derive the main conclusions. Section V is devoted to concluding remarks.

II. Free Trade in Goods

Suppose that there are two countries, country 1 and country 2, which have the same consumers' preferences and market structures but not necessarily identical production technologies. Each national economy consists of one manufacturing sector within which firms produce the products with two factors of production, labor and capital, under increasing-returns-to-scale technologies and can costlessly differentiate their products. Labor and capital are respectively homogeneous and qualitatively identical between the two countries. The differentiated manufactured goods produced in the two countries are indexed by z which continuously runs from 0 to N, a large positive number[3]. Assume, for the sake of simplicity, that the products indexed by z which belong to a closed interval [0, n] are produced in country 1 and those indexed by z which belong to a closed interval [0*, n*], where n+n*=N, are produced in country 2. As will be shown below, n and n* are endogenously determined in the model.

Suppose that in the production of every variety of the manufactured goods certain amounts of capital, f, are needed as a fixed input to set up the production, and that some uniform units of labor, m, are required as a variable input to operate the unit production. Let x(z) be the output, K(z) the capital input, and L(z) the labor input of variety z. Then the production function of variety z is symmetric and represented for $z \in [0, n]$ by

$$\begin{aligned} x(z) &= 0 \qquad &&\text{for } K(z) < f \\ x(z) &= L(z)/m \qquad &&\text{for } K(z) \geq f. \end{aligned} \tag{1}$$

This production function is quasi-concave but not homothetic and originally used by Lawrence-Spiller (1983). Let w and r denote respectively the wage for a unit of labor and rental for a unit of capital. Then the total cost of variety z is obtained from (1) as rf+wmx(z). Obviously, the requirement of capital input is the source of economies of scale in the manufactured sector and at the same time acts as the barrier for firms to enter the market of variety z. The implication of increasing-returns-to-scale technology is that each firm produces only one variety different from those produced by the other firms. Suppose that firms producing the manufactured goods in country 2 have the production functions of the Lawrence-Spiller type with the parameters not necessarily identical to those in country 1. Let us distinguish the variables and parameters of country 2 from the counterparts of country 1 by attaching them an asterisk. Then the production function of variety z in country 2 is symmetric and represented by

$$x^*(z) = 0 \qquad \text{for } K^*(z) < f^*$$
$$x^*(z) = L^*(z)/m^* \qquad \text{for } K^*(z) \geqq f^*. \tag{2}$$

Each firm in country 2 also produces only one variety of the manufactured goods which is different from those produced by the other domestic and foreign firms.

In order to develop further the model of monopolistically competitive economies, it is necessary to introduce the demand side of the model before continuing to describe the producers' behaviors in the manufacturing sector. It is assumed, like in Dixit-Stiglitz (1977), that a consumer evaluates each variety of the manufactured goods symmetrically and has an identical CES utility function. Then the consumer in each country under the T regime consumes all the varieties of the goods produced domestically as well as the goods imported from abroad. Let $c(z)$ denote the per capita consumption of variety z in country 1 and b a positive constant with a value less than one, then a representative consumer's utility function is represented in country 1 by

$$U(c) = [\int_0^N c(z)^b dz]^{1/b} \qquad 0 < b < 1 \tag{3}$$

and in country 2 by

$$U(c^*) = [\int_0^N c^*(z)^b dz]^{1/b} \tag{4}$$

which is identical to the counterpart of country 1 in its form.

The demand functions for domestic and imported products are respectively derived from the consumer's utility maximization behavior under his budget constraint. Let us take country 2's labor as a numeraire, that is, $w^* \equiv 1$. Let $p(z)$ denote the price of z-variety manufactured good produced in country 1 and I a per capita income of the consumer in country 1, measured respectively with the numeraire. Then the demand functions for the domestic and imported products are respectively represented in country 1 by

$$c(z) = p(z)^{-1/(1-b)} IQ^{-1} \qquad z \in [0, n]$$
$$c(z) = p^*(z)^{-1/(1-b)} IQ^{-1} \qquad z \in [0^*, n^*] \tag{5}$$

where $Q \equiv \int_0^n p(z)^{-b/(1-b)} dz + \int_{0^*}^{n^*} p^*(z)^{-b/(1-b)} dz$ shows the influence on c(z) of the prices of other varieties than z. The demand functions for the imported and domestic products are respectively represented in country 2 by

$$c^*(z) = p(z)^{-1/(1-b)} I^*Q^{-1} \qquad z \in [0, n]$$
$$c^*(z) = p^*(z)^{-1/(1-b)} I^*Q^{-1} \qquad z \in [0^*, n^*]. \tag{6}$$

The equations (5) and (6) show that the consumption of a variety of the manufactured goods increases as a per capita income increases but decreases as the price of any variety rises. They also imply that the elasticity of demand for variety z is 1/(1-b) which is constant and symmetric for all varieties. Suppose that there are L consumers in country 1 and L* in country 2. Then the market-clearing conditions for the products produced in countries 1 and 2 are respectively represented by

$$Lc(z) + L^*c^*(z) = x(z) \qquad z \in [0, n]$$
$$Lc(z) + L^*c^*(z) = x^*(z) \qquad z \in [0^*, n^*]. \tag{7}$$

The firm producing monopolistically variety z of the manufactured goods, which hereafter is called firm z, maximizes its profits by setting the output level so as to equalize its marginal revenue to its marginal cost. Suppose that it has no monopsony powers in factor markets. Then the profit-maximization condition of the monopoly firm z in country 1 is represented by

$$p(z)b = wm \qquad z \in [0, n] \tag{8}$$

which is equivalent to saying that the free-trade prices of the manufactured goods produced in country 1 are symmetric and equal to wm/b. Similarly, the profit-maximization condition of the monopoly firm in country 2 determines the free-trade price of its product as

$$p_T* = m*/b \qquad z \in [0*, n*] \tag{9}$$

where the variable to which subscript T is attached denotes its equilibrium value under the T regime. Suppose that entry barriers to the production of variety z are so low that any positive level of profits in the industry will attract firms to the production of variety z. In an equilibrium where the entry of firms comes to an end firm z in country 1 has zero profits:

$$p_T x(z) = rf + wmx(z) \qquad z \in [0, n]. \tag{10}$$

The equations (8) and (10) imply that 1/b equals the index of economies of scale, the ratio of average to marginal cost, and they also determine the firm's free-trade output level as a symmetric variable with respect to factor prices:

$$x_T = bfr/[(1-b)wm] \tag{10'}$$

The corresponding conditions of the firm in country 2 determine its free-trade output level as

$$x_T* = bf*r*/[(1-b)m*] \qquad z \in [0*, n*]. \tag{11}$$

Consequently the demands for labor from the firms in countries 1 and 2, denoted by L_T and L_T* respectively, are symmetric.

Suppose that each consumer in country 1 (country 2) provides one unit of labor and k (k*) units of capital to domestic firms. Then his real income is represented in countries 1 and 2 respectively by

$$I = rk + w \qquad \text{and} \qquad I* = r*k* + 1 \tag{12}$$

because dividents from the domestic firms are zero in both countries. The total supplies of labor and capital become L and K ($\equiv$kL) in country 1 and L* and K* ($\equiv$k*L*) in country 2. This leads to the following full employment conditions for capital and labor: in country 1

$$\int_0^n K(z)dz = nf = K \tag{13}$$

$$L_T \equiv \int_0^n L(z)dz = nmx_T = L \tag{14}$$

and in country 2

$$\int_{0*}^{n*} K*(z)dz = n*f* = K* \tag{15}$$

$$L_T* \equiv \int_{0*}^{n*} L*(z)dz = n*m*x_T* = L*. \tag{16}$$

Notice that the number of goods produced in each country can be determined by the full employment condition for its capital.

When the product prices are symmetric, so are the per capita consumptions in both countries. Letting y=I/p, the real income in terms of the manufactured goods, and $q=n+n*(mw/m*)^{b/(1-b)}$, the index of country 1's terms of trade, and rewritting the demand functions (5) and (6) in terms of this notation, one has the following for country 1

$$\begin{aligned} c_T &\equiv c(z) = y_T/q \qquad z \in [0, n] \\ c_T' &\equiv c(z) = (y_T/q)(mw/m*)^{1/(1-b)} \qquad z \in [0*, n*] \end{aligned} \tag{5'}$$

and for country 2,

$$\begin{aligned} c_T* &\equiv c*(z) = (y_T*/q)(mw/m*)^{-1} \qquad z \in [0, n] \\ c_T*' &\equiv c*(z) = (y_T*/q)(mw/m*)^{b/(1-b)} \qquad z \in [0*, n*]. \end{aligned} \tag{6'}$$

The balance of trade condition of country 1 can be derived from (5'), (6'), and (7):

$$L*\int_0^n p(z)c*(z)dz - L\int_{0*}^{n*} p*(z)c(z)dz = 0.$$

The general equilibrium system of the two open economies consists of 20 equations from (1) to (16) with 19 unknowns of x, x*, L_T, L_T*, U(c), U(c*), c_T, c_T', c_T*, c_T*', I, I*, n, n*, p, p*, r, r* and w. Since the first equation in (7) can be obtained using (8), (10), (13), (15), (5'), and (6'), the system has 19 independent equations. The free-trade equilibrium value of wage rate in country 1 is determined by the commodity market-clearing conditions, and that of rental rate in country 1 is in turn determined by the full employment condition for its labor. They are respectively represented by

$$w_T = (m^*/m)^b (f^*/f)^{1-b} (k/k^*)^{1-b} \tag{17}$$

and

$$r_T = r_T^*(m^*/m)^b (f^*/f)^{1-b} (k^*/k)^b \tag{18}$$

where $r_T^*=(1-b)/(bk^*)$. These results show that if the production functions are internationally identical, that is, $m=m^*$ and $f=f^*$, then the wage rate of a capital-abundant country in the Heckscher-Ohlin sense is higher than that of a labor-abundant country while the rental rate of the former is lower than that of the latter, and that if the two countries have the same relative factor endowments, then both wage and rental rates of a country with superior technologies reflected by lower marginal and fixed costs are higher than those of a country with inferior technologies. Their relative heights equal the weighted average between the degree, whose weight is b, of the superiority in the marginal cost and the degree, whose weight is 1-b, of the superiority in the fixed cost.

The free-trade welfare level of the individual consumer in country 1 (U_T) can be obtained by substituting (5') into (3) as a product of the free-trade levels of y and $q^{(1-b)/b}$:

$$\begin{aligned} U_T &= y_T q_T^{(1-b)/b} \\ &= (1/m)[n\{1 + (\alpha w_T)^{-1}\}]^{(1-b)/b} \end{aligned} \tag{19}$$

where the second equality is derived by making a use of (12), (8), (17), (18), (13) and (15), and α denotes the relative scale of country 1's labor force, L/L^*. Since the welfare level of country 2's consumer is represented by U_T^*

$=y_T{*}q_T{}^{(1-b)/b}m{*}/(mw_T)$ it can be related to the counterpart of country 1 by

$$U_T{*} = U_T/w_T. \tag{20}$$

It can be easily proved by applying the analysis of Suzuki (1991) to this model that the intra-industry trade is beneficial to each participating country. Equation (20) shows that the international distribution of trade benefits depends on the relative value of country 1's wage rate realized under free trade. If it equals unity, then the wage rates are internationally equalized, and the gains of trade are evenly shared by the two countries in the sense that $U_T=U_T{*}$. If it takes a value other than unity, the share of a country with relatively higher wage rate is larger than that of a low-wage country.

III. Free Trade in Goods and Factors of Production

It was shown in the previous section that unless countries 1 and 2 are the same in their production technologies and relative factor endowments the factor prices are not internationally equalized under free trade in goods alone. Hence there are generally inducements of international factor movements besides commodity trade needed to increase the welfare in each country. The purpose of this section is to consider, in addition to the T regime, the other three international economic regimes presented in section I and to show that the introduction of one of the three regimes generally raises the welfare in every country.

(1) The L-T regime

When labor is allowed to move freely across countries' borders, its full-employment conditions for countries 1 and 2 are integrated into a condition for the world market:

$$\int_0^n L(z)dz + \int_{0*}^{n*} L*(z)dz = L + L* \equiv L^w \tag{21}$$

where L^w denotes the world supply of labor and L (or L*) the total quantity of labor owned by country 1 (or country 2) nationals. The wage rates in both countries are equalized in equilibrium,

$$w = w* \equiv 1. \tag{22}$$

Suppose that the wages of immigrants are remitted to the source country and spent on the consumption of manufactured goods there. Then, with L and L* consumers in countries 1 and 2 respectively, individual incomes under the L-T regime are of the same form as the ones represented by (12) in both countries, and the commodity market-clearing conditions under the T regime, (7), still hold under the L-T regime. Thus the conditions constituting the open economies in countries 1 and 2, (1) - (16), apply under the L-T regime as well, except for (14) and (16) which are replaced with (21) here. The commodity market-clearing conditions now determine the equilibrium ratio of the two countries' rental rates. Substituting (10') and (11) into (7), one can get

$$r/r* = (f*/f)(m*/m)^{b/(1-b)} \equiv \gamma . \tag{23}$$

This result tells us that the equilibrium rental ratio is independent of the factor endowments of the two countries and that the rental rate of a country with superior technologies is higher than that of a country with inferior technologies. How much higher is shown by the weighted sum of the degree, whose weight is the ratio of the elasticity of demand to the index of economies of scale, of superiority in the marginal cost and the degree, whose weight is unity, of the superiority in the fixed cost. The absolute values of the rental rates in the two countries are derived from (21) and (23):

$$r_L = \frac{(1 - b)\gamma L^w}{b(\gamma K + K*)} \qquad r_L* = \frac{(1 - b)L^w}{b(\gamma K + K*)} \tag{24}$$

where the variable attached with subscript L denotes its equilibrium value under the L-T regime.

As under the T-regime, the welfare level of the individual consumer in country 1 under the L-T regime, U_L, depends on the real per capita income in terms of the manufactured goods, y_L, the ratio of which to y_T hinges on x_L/x_T, and the index of its terms of trade effect, q_L, the ratio of which to q_T depends on w_L/w_T. If country 1 imports labor under the L-T regime, that is, if $k^*/k<\gamma$ is satisfied, then $w_T>w_L$, leading to the deterioration of its terms of trade after immigration. Its per capita real income, on the other hand, becomes larger because the scale of its representative firm increases as labor flows in. If country 1 exports labor its terms of trade is improved but its per capita real income is decreased under the L-T regime, as compared with the T regime. Therefore, to see U_L higher than U_T requires a direct comparison between them. The ratio of U_L to U_T can be derived using (18) and (24) as

$$\frac{U_L}{U_T} = \left(\frac{(1-b+\alpha)\gamma K+b\alpha K^*}{\alpha(\gamma K+K^*)}\right)\left(\frac{\gamma K+K^*}{\gamma K+K^*(\gamma K/(\alpha K^*))^b}\right)^{(1-b)/b} \tag{26}$$

For given values of b, γ and K/K^*, it can be viewed as a function of α which monotonically decreases from infinity to unity as α rises from zero to $\gamma K/K^*$ and increases from unity to some value larger than that as α rises from $\gamma K/K^*$ to infinity (Verification is given in Appendix A). This implies that U_L/U_T is higher than unity unless $\alpha=\gamma K/K^*$.

The welfare comparison for country 2 between the L-T and T regimes can be similarly made. The ratio of U_L^* to U_T^* is

$$\frac{U_L^*}{U_T^*} = \left(\frac{b\gamma K+(1+\alpha-b\alpha)K^*}{\gamma K+K^*}\right)\left(\frac{\gamma K+K^*}{K^*+\gamma K(\alpha K^*/(\gamma K))^b}\right)^{(1-b)/b} \tag{27}$$

It monotonically decreases from some value larger than unity to unity as α

rises from zero to $\gamma K/K*$ and increases from unity to infinity as α rises from $\gamma K/K*$ to infinity (Verification is given in Appendix A). Therefore, U_L*/U_T* is also larger than unity unless $\alpha = \gamma K/K*$. Noting that $k*/k = \alpha K*/K$ and both U_L/U_T and U_L*/U_T* equal unity for the same value of α, one can summarize the results:

Theorem 1. If $k*/k \neq (f*/f)(m*/m)^{b/(1-b)}$, the L-T regime is better than the T regime. If $k*/k = (f*/f)(m*/m)^{b/(1-b)}$, they are indifferent.

(2) The K-T regime

When capital is internationally mobile its full-employment conditions for countries 1 and 2 under the T regime are integrated into a condition for world markets:

$$nf + n*f* = K + K* \equiv K^W \tag{28}$$

where K^W denotes the world supply of capital and K (or K*) the total quantity of capital owned by country 1 (or country 2) nationals. The rental rates in both countries are equalized in equilibrium,

$$r = r*. \tag{29}$$

Suppose that the rentals earned in a country into which capital flows are remitted to the source country and spent on the consumption of manufactured goods there. Then individual incomes under the K-T regime are of the same form as the ones represented by (12) in both countries, and this leads the conditions which constitute the open economies in countries 1 and 2 to remain valid under the K-T regime, with the exception of (13) and (15) which are replaced with (28). Thus the wage rate realized in country 1 under the K-T regime is determined by the equilibrium conditions for the commodity markets, (7), as

$$w_K = (m*/m)(f*/f)^{(1-b)/b} \tag{30}$$

where the variable attached with subscript K denotes its equilibrium value under the K-T regime. Substituting (10'), (11) and (30) into (28) yields the equilibrium level of rental rates in both countries under the K-T regime

$$r_K = r_K* = (1 - b)(1 + \alpha \gamma^{(1-b)/b})L*/(bK^w). \quad (31)$$

As under the two regimes discussed previously, the welfare level of the individual consumer in country 1 under the K-T regime, U_K, equals a product of y_K and $q_K^{(1-b)/b}$. Although in this case the number of the varieties of the manufactured goods produced in each country is changeable corresponding to the amount of inter-country capital flows, q_K/q_T invariably depends on the relative wage rate, w_K/w_T, and y_K/y_T constantly depends on the relative firm size, x_K/x_T, both being affected in opposite directions to each other by international capital movements. Thus the efficiency of the K-T regime cannot be demonstrated until U_K is directly compared with U_T. The ratio of U_K to U_T can be derived using (30), (31), (17) and (18):

$$\frac{U_K}{U_T} = \left(\frac{(1-b)K+\alpha \gamma^{(1-b)/b}(K+bK*)}{\alpha K^w}\right)\left(\frac{K^w}{\gamma K+K*(\gamma K/(\alpha K*))^b}\right)^{(1-b)/b} \quad (33)$$

It can be verified that as a function of α, U_K/U_T resembles U_L/U_T in the shape of its diagram, being equal to unity when $\alpha = \gamma^{(b-1)/b}K/K*$ and higher than that otherwise (Verification is given in Appendix B).

The welfare comparison for country 2 between the K-T and T regimes can be made in the same fashion. The ratio of U_K* to U_T* is

$$\frac{U_K*}{U_T*} = \left(\frac{(1-b)(1+\alpha \gamma^{(1-b)/b})K*+bK^w}{K^w}\right)\left(\frac{K^w}{K*+\gamma K(\alpha K*/(\gamma K))^b}\right)^{(1-b)/b} \quad (34)$$

As a function of α、 U_K*/U_T* depicts an analogous diagram to that of U_L*/U_T* in shape, being equal to unity for the same value of α as U_K/U_T is and higher than that for the other values of α (Verification is given in Appendix B). The results can be summarized:

Theorem 2. If $k/k* \neq (m*/m)(f*/f)^{(1-b)/b}$, the K-T regime is better than the T regime. If $k/k* = (m*/m)(f*/f)^{(1-b)/b}$, they are indifferent.

(3) The LK-T regime

When labor and capital are internationally mobile the full-employment conditions for them are respectively represented by (21) and (28), and the equilibrium prices of the factors of production employed in the two countries are represented by (22) for labor and by (29) for capital. Using those full-employment conditions, one can get the equilibrium level of rental rates as

$$r_{LK} = (1 - b)L^{W}/(bK^{W}) \tag{35}$$

where the variable attached with subscript LK denotes its equilibrium value under the LK-T regime.

Given the full remittance to a source country of the wages and rentals earned in a host country, the open economies of countries 1 and 2 are constructed with conditions (1) - (12) together with (21), (28), (22) and (29). Then equilibrium firm sizes in the manufacturing sector are derived as $x_{LK} = fL^{W}/(mK^{W})$ in country 1 and $x_{LK}* = f*L^{W}/(m*K^{W})$ in country 2. These per-firm output levels require that the commodity market-clearing condition, (7), be satisfied if and only if $\gamma = 1$, a sufficient condition for which is that the technologies in the manufacturing sector are identical between the two countries.

When $\gamma = 1$, the welfare level of each country's individual consumer under the LK-T regime is as high as that under the L-T regime and under the K-T regime. For country 1 it is

$$\begin{aligned} U_{LK} &= (1/m)(K^{W}/f)^{(1-b)/b}[(1 - b)L^{W}k/K^{W} + b] \\ &= U_{L} = U_{K} \geq U_{T} \end{aligned} \tag{36}$$

where ≧ holds with equality if k=k*, and for country 2 it is

$$U_{LK}* = (1/m)(K^w/f)^{(1-b)/b}[(1 - b)L^w k*/K^w + b]$$

$$= U_L* = U_K* \geqq U_T* \qquad (37)$$

where ≧ holds with equality if k=k*. Considering Theorems 1 and 2 together, one can conclude:

Theorem 3. If $(m*/m)^b (f*/f)^{(1-b)} = 1$, the LK-T regime is indifferent to the L-T and K-T regimes but is preferable to the T regime for the countries with different relative factor endowments. If $(m*/m)^b (f*/f)^{(1-b)} \neq 1$, the LK-T regime cannot exist between the two countries with the economic conditions considered here.

Ⅳ. A Comparison of the Welfare Levels between the L-T and K-T Regimes

It is clear from the argument in the previous sections that countries prefer the L-T or K-T regime to the T regime and that the LK-T regime, which can exist only under a limiting condition for technology in this model, is indifferent as to the L-T and K-T regimes. The purpose of this section is to answer the questions as to which regime is preferable as an international economic regime and what conditions of technologies and relative factor endowments are the determinants of such a choice.

The comparison of each country's welfare levels between the L-T and K-T regime can be made by considering the ratio of U_L (U_L*) to U_K (U_K*) which is denoted by ϕ ($\phi*$). For country 1, ϕ can be obtained from (26) and (33):

$$\phi \equiv \frac{U_L}{U_K} = \left(\frac{\gamma K+K*}{\gamma K^w}\right)^{(1-2b)/b} \left(\frac{(1-b)\gamma K+\alpha(\gamma K+bK*)}{\gamma[(1-b)\gamma^{(b-1)/b}K+\alpha(K+bK*)]}\right) \qquad (38)$$

and for country 2, $\phi*$ can be obtained from (27) and (34):

$$\phi* \equiv \frac{U_L*}{U_K*} = \left(\frac{\gamma K+K*}{\gamma K^w}\right)^{(1-2b)/b} \left(\frac{b\gamma K+K*+(1-b)\alpha K*}{\gamma[(bK+K*)\gamma^{(b-1)/b}+(1-b)\alpha K*]}\right) \qquad (39)$$

For a given value of b, γ and K/K*, ϕ and $\phi*$ can be regarded as functions of α, the relative size of country 1's native labor force. It can be shown that if the technologies of the manufacturing sectors in the two countries are identical, that is, $\gamma \equiv (f^*/f)(m^*/m)^{b/(1-b)}=1$, then $\phi(\alpha)=\phi^*(\alpha)=1$; if country 1's technologies are superior to country 2's, that is, $\gamma>1$, then both $\phi(\alpha)$ and $\phi^*(\alpha)$ are decreasing functions of α with $\phi(0)>1$, $\phi^*(0)>1$, $\phi(\infty)<1$ and $\phi^*(\infty)<1$; if country 2's technologies are superior to country 1's, both $\phi(\alpha)$ and $\phi^*(\alpha)$ are increasing functions of α with $\phi(0)<1$, $\phi^*(0)<1$, $\phi(\infty)>1$ and $\phi^*(\infty)>1$ (Proof of these results is given in Appendix C).

The continuity and monotonicity of $\phi(\alpha)$ and $\phi^*(\alpha)$ assure that whether $\gamma>1$ or $\gamma<1$, there is a unique value of α, which is denoted by α_1, such that $\phi(\alpha)$ equals unity when $\alpha=\alpha_1$ and there is also a unique value of α, which is denoted by α_1^*, such that $\phi^*(\alpha)$ equals unity when $\alpha=\alpha_1^*$. It is possible to settle the domains in terms of the parameters, b, γ and K/K* for the critical values, α_1 and α_1^*, to lie by utilizing the results obtained in the previous section. Theorems 1 and 2 imply that if $\alpha=\gamma K/K^*$, then $\phi(\alpha)<1$ and $\phi^*(\alpha)<1$ because for any given positive values of K/K* and $\gamma\,(\neq 1)$, $U_L/U_T=U_L^*/U_T^*=1$, $U_K/U_T>1$ and $U_K^*/U_T^*>1$ at this value of α. They also imply that if $\alpha=\gamma^{(b-1)/b}K/K^*$, then $\phi(\alpha)>1$ and $\phi^*(\alpha)>1$ because in this case $U_K/U_T=U_K^*/U_T^*=1$, $U_L/U_T>1$ and $U_L^*/U_T^*>1$ at this value of α. The domains of α_1 and α_1^* can be determined by these properties of $\phi(\alpha)$ and $\phi^*(\alpha)$ as

$$\min[\gamma K/K^*,\ \gamma^{(b-1)/b}K/K^*] < \alpha_1,\ \alpha_1^* < \max[\gamma K/K^*,\ \gamma^{(b-1)/b}K/K^*] \quad (40)$$

The diagrams of $\phi(\alpha)$ in the cases where $\gamma>1$ and where $\gamma<1$ are shown in Fig. 1. Those of $\phi^*(\alpha)$ are not shown there but can be analogously drawn.

The economic implications of these outcomes are that a country's pref-

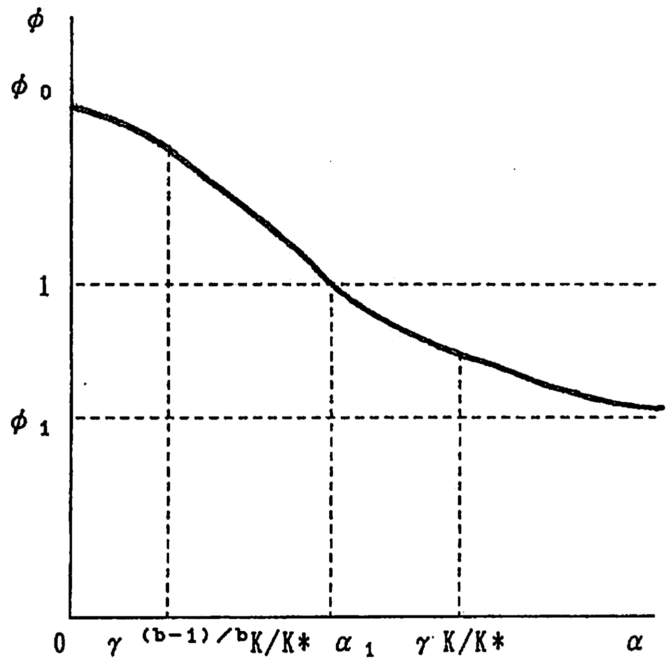

Fig. 1-a ($\gamma > 1$)

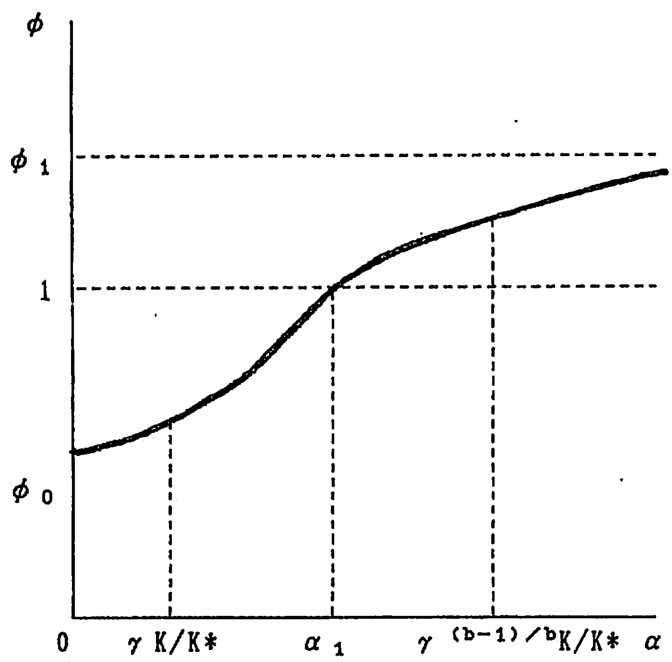

Fig. 1-b ($\gamma < 1$)

erence between the L-T and K-T regimes depends on its technological superiority and the relative size of its native labor force. If, for instance, country 1 is superior to country 2 in technologies of the manufacturing sector, then it prefers the L-T regime when the relative size of its native labor force is less than the critical value, α_1, it is indifferent between the two regimes when α is just equal to the critical value, and it prefers the K-T regime when α is larger than the critical value. The same thing is true for country 2, where the relative size of its native labor force and the critical value of this ratio are respectively represented by $1/\alpha$ and $1/\alpha_1*$. Therefore, it can be concluded that a country with superior technologies prefers the L-T regime when it has a relatively small supply of native labor and prefers the K-T regime when it has a relatively large supply of native labor.

If, as shown above, the technologies of the manufacturing sector are identical between the two countries, they are indifferent as to the K-T and L-T regimes for any given values of α, b and K/K*. But if the technologies are internationally different, each country is indifferent as to the two regimes only when the relative size of its native labor force takes its critical value, α_1 or α_1*. If b=1/2, or if the ratio of the elasticity of demand to the index of economies of scale equals unity, then the critical values of the two countries are identical and equal to K/K*. This means that if countries 1 and 2 have the same relative factor endowments they are indifferent as to the L-T and K-T regimes even if their technologies in the manufacturing sectors are different from each other. Of course, they are not if they have different factor endowments and technologies. In the case where country 1's technologies are superior to country 2's, $U_L > U_K$ and $U_L* > U_K*$ for $\alpha < K/K*$ while $U_L < U_K$ and $U_L* < U_K*$ for $\alpha > K/K*$. In the case where country 2's technologies are superior, $U_L<U_K$ and $U_L*<U_K*$ for $\alpha<K/K*$ while $U_L>U_K$ and $U_L*>U_K*$ for $\alpha>K/K*$. These results can be summed up as follows.

Theorem 4a. If b=1/2, or the ratio of the elasticity of demand to the index of economies of scale equals unity then

i) a country with superior technologies which is relatively capital-abundant and a country with inferior technologies which is relatively labor-abundant will both find that the L-T regime is the best and the K-T regime is the second best for them,

ii) a country with superior technologies which is relatively labor-abundant and a country with inferior technologies which is relatively capital-abundant will both find that the K-T regime is the best and the L-T regime is the second best for them, and

iii) countries with different technologies which have the same relative factor endowments will find no difference between the L-T and K-T regimes.

When b=1/2, the role of the marginal cost ratio and that of the fixed cost ratio are symmetric in determining the equilibrium price ratio of an immobile factor under each international economic regime, and at the same time α_1 equals α_1*. But if $b\neq 1/2$, they are asymmetric and α_1 is probably not identical to α_1*. Let $\alpha_{min}\equiv\min[\alpha_1, \alpha_1*]$ and $\alpha_{max}\equiv\max[\alpha_1, \alpha_1*]$. Then it is clear from the diagrams of ϕ and $\phi*$ that when country 1 is superior to country 2 in technologies of the manufactured goods,

$U_L > U_K$ and $U_L* > U_K*$ for $L/L* < \alpha_{min}$,

$U_K > U_L$ and $U_K* > U_L*$ for $L/L* > \alpha_{max}$,

$U_L\geqq U_K$ and $U_L*\leqq U_K*$ or $U_K\geqq U_L$ and $U_K*\leqq U_L*$ for $\alpha_{min}\leqq L/L*\leqq\alpha_{max}$

(with at least one inequality held);

when country 2 is superior to country 1 in the technologies of the manufactured goods,

$U_L^* > U_K^*$ and $U_L > U_K$ for $L^*/L < 1/\alpha_{max}$,

$U_K^* > U_L^*$ and $U_K > U_L$ for $L^*/L > 1/\alpha_{min}$,

$U_L^* \geqq U_K^*$ and $U_L \leqq U_K$ or $U_K^* \geqq U_L^*$ and $U_K \leqq U_L$ for $1/\alpha_{max} \leqq L^*/L \leqq 1/\alpha_{min}$

(with at least one inequality held).

It can be expected that when a country with superior technologies in the manufacturing sector is relatively capital-abundant, it is more likely for it to satisfy $L/L^* < \alpha_{min}$ or $L^*/L < 1/\alpha_{max}$ than for it to satisfy $L/L^* > \alpha_{min}$ or $L^*/L > 1/\alpha_{max}$.[4] Similarly, when a country with superior technologies is relatively labor-abundant it is more likely for it to satisfy $L/L^* > \alpha_{max}$ or $L^*/L > 1/\alpha_{min}$. Therefore, it can be concluded that:

Theorem 4b. If $b \neq 1/2$ or the ratio of the elasticity of demand to the index of economies of scale does not equal unity then

i) a country with superior technologies which is relatively capital-abundant and a country with inferior technologies which is relatively labor-abundant will both find that most likely the L-T regime will be the best and the K-T regime will be the second best for them and

ii) a country with superior technologies which is relatively labor-abundant and a country with inferior technologies which is relatively capital-abundant will both find that most likely the K-T regime will be the best and the L-T regime will be the second best for them.

V. Concluding Remarks

It has been made clear that in the model with increasing-returns-to-scale technologies characterized by the fixed input of capital and variable input of labor and with monopolistic competition in the markets of differentiated products of a constant price-elasticity, a country's preference among the L-T, K-T and T regimes depends upon a combination of its relative factor endowment and the superiority in its technology. In any case a regime which is one country's best choice is likely to coincide with the other country's best choice. It is reasonable to expect that such a regime is likely to become an international economic regime between the two countries. According to my analysis, it is likely to be the L-T regime between a capital-abundant country with superior technologies and a labor-abundant country with inferior technologies; it is likely to be the K-T regime between a capital-abundant country with inferior technologies and a labor-abundant country with superior technologies. These results imply that if the advanced industrial countries are labor-abundant relative to other countries their policies of liberalizing international trade in commodities and capital and restricting rigorously the immigration of labor are economically rational but if they are relatively capital-abundant such policies are irrational. These results also imply that there are likely to be no conflicts of interest between the countries for the choice of international economic regimes in this model. This is in contrast with the conclusions obtained by Suzuki(1989) where one country's most desirable regime is sometimes different from another country's so that conflicts of interest occur in such a choice.

The assumption that capital is a fixed input and labor is a variable input in the production of the differentiated goods is crucial to the results about a country's choice among the international economic regimes. If this assumption is reversed, that is, if it is assumed that labor is a fixed input and capital is a variable input in the differentiated-goods industry, then the K-T regime is likely to be chosen as the best international economic regime between a capital-abundant country with superior technologies and a labor-abundant country with inferior technologies; the L-

T regime is likely to be chosen as the best regime between a labor-abundant country with superior technologies and a capital-abundant country with inferior technologies. Therefore, the results concerning this point can be generalized that i) if a country with superior technologies is relatively capital-abundant and a country with inferior technologies is relatively labor-abundant, then the regime under which free trade in commodities and the factor used as a variable input in the production of differentiated goods is allowed (M-T regime) is likely to be the best, the regime under which free trade in commodities and the factor used as a fixed input in the production of differentiated goods is allowed (F-T regime) is likely to be the second best and the T regime is the third best for both countries; ii) if a country with superior technologies is relatively labor-abundant and a country with inferior technologies is relatively capital-abundant, then the F-T regime is likely to be the best, the M-T regime is likely to be the second best and the T regime is the third best for both countries.

Footnotes

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1. As Mundell (1957) showed in the two-sector, two-factor model, international capital or labor movement is a perfect substitute for commodity trade between countries which produce the two goods with internationally identical technologies.

2. The causes and implications of the intra-industry trade in differentiated products were originally studied by Krugman (1979, 1980).

3. The pioneering work about international trade in a continuum of goods is Dornbusch-Fisher-Samuelson (1977).

4. Suppose that $\gamma > 1$. Then $\gamma^{(b-1)/b}K/K^* < K/K^* < \gamma K/K^*$. Assume the distance between $\gamma K/K^*$ and α_{max} = the distance between α_{max} and α_{min} = the distance between α_{min} and $\gamma^{(b-1)/b}K/K^*$. Then probability[$K/K^* \geqq \alpha_{max}$]=1/3, probability[$\alpha_{min} \leqq K/K^* < \alpha_{max}$]=1/3, and probability[$K/K^* < \alpha_{min}$]=1/3. Note that $\alpha < K/K^*$ when country 1 with superior technologies in the manufacturing sector is relatively capital-abundant. When $K/K^* \geqq \alpha_{max}$, probability[$\alpha \geqq \alpha_{max}$]$\leqq$1/3, probability[$\alpha_{min} \leqq \alpha < \alpha_{max}$]$\geqq$1/3 and probability[$\alpha < \alpha_{min}$]$\geqq$ 1/3. Therefore, probability[$\alpha < \alpha_{min} < \alpha_{max} \leqq K/K^*$]$\geqq (1/3)^2 = 1/9$ and probabil-

ity[$\alpha_{min} \leqq \alpha$ and $\alpha_{max} \leqq K/K*$]$\leqq$(1/3)(2/3)=2/9. When $\alpha_{min} \leqq K/K* < \alpha_{max}$, probability[$\alpha \geqq \alpha_{min}$]$\leqq$1/2 and probability[$\alpha < \alpha_{min}$]$\geqq$1/2. Therefore, probability[$\alpha < \alpha_{min} \leqq K/K* < \alpha_{max}$]$\geqq$1/6 and probability[$\alpha_{min} \leqq \alpha \leqq K/K* < \alpha_{max}$] $\leqq$1/6. Since probability[$\alpha < \alpha_{min}$]=1 when $K/K* < \alpha_{min}$, probability[$\alpha < K/K* < \alpha_{min}$]=1/3 and probability[$\alpha \geqq \alpha_{min}$]=0. Therefore, probability[$\alpha < \alpha_{min}$]$\geqq$ 1/9+1/6+1/3=11/18 and probability[$\alpha \geqq \alpha_{min}$]$\leqq$2/9+1/6=7/18, which proves probability[$\alpha < \alpha_{min}$]>probability[$\alpha \geqq \alpha_{min}$]. This assertion can be proved similarly in the case where $\gamma < 1$.

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Appendix

[A] The diagrams of U_L/U_T and U_L^*/U_T^*

(1) U_L/U_T

The expression for U_L/U_T is given by (26) in the text. When α approaches zero, U_L/U_T approaches infinity. When $\alpha = \gamma K/K^*$, $U_L/U_T = 1$. When α approaches infinity, U_L/U_T approaches $(U_L/U_T)_1$, where

$$(U_L/U_T)_1 = [1 + K^*/(\gamma K)]^{(1-b)/b}[1 + bK^*/(\gamma K)][1 + K^*/(\gamma K)]^{-1} > 1.$$

This inequality relation can be proved by showing that when $K^*/(\gamma K)=0$, $(U_L/U_T)_1=1$, and

$$\frac{\widehat{(U_L/U_T)_1}}{\widehat{K^*/(\gamma K)}} = \frac{(1 - b)[1 - b + bK^*/(\gamma K)]K^*/(\gamma K)}{b[1 + K^*/(\gamma K)][1 + bK^*/(\gamma K)]} > 0.$$

The slope of U_L/U_T is:

$$\frac{\widehat{(U_L/U_T)}}{\hat{\alpha}} = \frac{(1-b)K^*(\gamma K/(\alpha K^*))^b}{\gamma K+K^*(\gamma K/(\alpha K^*))^b} + \frac{\alpha(\gamma K+bK^*)}{(1-b)\gamma K+\alpha(\gamma K+bK^*)} - 1$$

$$= \frac{(1-b)(\gamma K)^b[b\alpha^{-b}K^{*(1-b)}(\alpha K^*-\gamma K)+\gamma K\{(\alpha K^*)^{(1-b)}-(\gamma K)^{(1-b)}\}]}{[(1-b+\alpha)\gamma K+b\alpha K^*][\gamma K+K^*(\gamma K/(\alpha K^*))^b]}$$

$\lesseqgtr 0$ according as $\alpha \lesseqgtr \gamma K/K^*$.

(2) U_L^*/U_T^*

The expression of U_L^*/U_T^* is given by (27) in the text. When α approaches zero, U_L^*/U_T^* approaches $(U_L^*/U_T^*)_0$, where

$$(U_L^*/U_T^*)_0 = (1 + \gamma K/K^*)^{(1-b)/b}[b + (1 - b)(1 + \gamma K/K^*)^{-1}] > 1.$$

This inequality relation can be demonstrated in the same way as in the case of $(U_L/U_T)_1$. When $\alpha = \gamma K/K^*$, $U_L^*/U_T^*=1$. When α approaches infinity, U_L^*/U_T^* also approaches infinity. The slope of U_L^*/U_T^* is:

$$\frac{\widehat{(U_L^*/U_T^*)}}{\hat{\alpha}} = \frac{(1-b)(\alpha K^*)^b[K^*\{(\alpha K^*)^{(1-b)}-(\gamma K)^{(1-b)}\}+b(\gamma K)^{(1-b)}(\alpha K^*-\gamma K)]}{[K^*+(\gamma K)^{(1-b)}(\alpha K^*)^b][b\gamma K+(1+\alpha-b\alpha)K^*]}$$

$\gtreqless 0$ according as $\alpha \lesseqgtr \gamma K/K*$

[B] The diagrams of U_K/U_T and U_K*/U_T*

(1) U_K/U_T

The expression of U_K/U_T is given by (33) in the text. When α approaches zero, U_K/U_T approaches infinity. When $\alpha = \gamma^{(b-1)/b}K/K*$, $U_K/U_T = 1$. When α approaches infinity, U_K/U_T approaches $(U_K/U_T)_1$, where

$$(U_K/U_T)_1 = (1 + K*/K)^{(1-2b)/b}(1 + bK*/K) > 1.$$

This inequality relation can be proved in the same way as in the case of $(U_L/U_T)_1$. The slope of U_K/U_T is:

$$\frac{(\hat{U_K/U_T})}{\hat{\alpha}} = \frac{(1-b)K*(\gamma K/(\alpha K*))^b}{\gamma K+K*(\gamma K/(\alpha K*))^b} - \frac{(1-b)K}{(1-b)K+(K+bK*)\alpha \gamma^{(1-b)/b}}$$

$$= \frac{(1-b)[\gamma^b K^{(1+b)}\{(\alpha K*)^{(1-b)}-(\gamma^{(b-1)/b}K)^{(1-b)}\} +bK*(\alpha K*-\gamma^{(b-1)/b}K)(\gamma K/(\alpha K*))^b]}{[\gamma K+K*(\gamma K/(\alpha K*))^b][(1-b)\gamma^{(b-1)/b}K +\alpha(K+bK*)]}$$

$\gtreqless 0$ according as $\alpha \lesseqgtr \gamma^{(b-1)/b}K/K*$.

(2) U_K*/U_T*

The expression of U_K*/U_T* is given by (34) in the text. When α approaches zero, U_K*/U_T* approaches $(U_K*/U_T*)_0$, where

$$(U_K*/U_T*)_0 = (1 + K/K*)^{(1-b)/b}[b + (1 - b)(1 + K/K*)^{-1}] > 1.$$

When $\alpha = \gamma^{(b-1)/b}K/K*$, $U_K*/U_T* = 1$. When α approaches infinity, U_K*/U_T* also approaches infinity. The slope of U_K*/U_T* is:

$$\frac{(\hat{U_K*/U_T*})}{\hat{\alpha}} = \frac{(1-b)\gamma^{(1-b)/b}(\alpha K*)^b[K*^{(2-b)}\{\alpha^{(1-b)}-(\gamma^{(b-1)/b}K/K*)^{(1-b)}\}}{[(1-b)(1+\alpha\gamma^{(1-b)/b})K*+bK^W][K*}$$

$$+bK^*(\gamma K)^{(1-b)}(\alpha-\gamma^{(b-1)/b}K/K^*)]$$
$$\overline{+\gamma K(\alpha K^*/(\gamma K))^b]}$$

$$\lesseqgtr 0 \text{ according as } \alpha \lesseqgtr \gamma^{(b-1)/b}K/K^*$$

[C] The diagrams of ϕ and ϕ^*

The expression of ϕ is given by (38) and that of ϕ^* by (39). When α approaches zero, ϕ approaches ϕ_0, where

$$\phi_0 = \gamma[(\gamma K + K^*)/K^W]^{(1-2b)/b} \gtreqless 1 \text{ according as } \gamma \gtreqless 1.$$

This is because

$$\frac{\hat{\phi}_0}{\hat{\gamma}} = \frac{(1-b)\gamma K + bK^*}{b(\gamma K + K^*)} > 0.$$

When α approaches zero, ϕ^* approaches $\phi_0{}^*$, where

$$\phi_0{}^* = \left(\frac{(\gamma K + K^*)}{K^W}\right)^{(1-2b)/b}\left(\frac{b\gamma K + K^*}{bK + K^*}\right) \gtreqless 1 \text{ according as } \gamma \gtreqless 1$$

because $\phi_0{}^*$ is an increasing function of γ. When α approaches infinity, ϕ approaches ϕ_1, where

$$\phi_1 = \left(\frac{\gamma K + K^*}{K^W}\right)^{(1-2b)/b}\left(\frac{\gamma K + bK^*}{\gamma^{(1-b)/b}(K + bK^*)}\right) \lesseqgtr 1 \text{ according as } \gamma \gtreqless 1$$

because ϕ_1 is a decreasing function of γ. When α approaches infinity, ϕ^* approaches $\phi_1{}^*$, where

$$\phi_1{}^* = \gamma^{(b-1)/b}[(\gamma K+K^*)/K^W]^{(1-2b)/b} \lesseqgtr 1 \text{ according as } \gamma \gtreqless 1$$

because $\phi_1{}^*$ is a decreasing function of γ. The slope of ϕ is:

$$\frac{\hat{\phi}}{\hat{\alpha}} = \frac{\alpha(\gamma K + bK^*)}{(1-b)\gamma K+\alpha(\gamma K+bK^*)} - \frac{\alpha\gamma^{(1-b)/b}(K+bK^*)}{(1-b)K+\alpha\gamma^{(1-b)/b}(K+bK^*)}$$

$$= \frac{(1-b)\alpha K[(1-\gamma^{(1-b)/b})\gamma K+(1-\gamma^{1/b})bK^*]}{[(1-b)\gamma K+\alpha(\gamma K+bK^*)][(1-b)K+\alpha\gamma^{(1-b)/b}(K+bK^*)]}$$

$\lesseqgtr 0$ according as $\gamma \gtreqless 1$.

The slope of $\phi*$ is:

$$\frac{\hat{\phi}*}{\hat{\alpha}} = \frac{(1-b)\alpha K*[(1-\gamma^{1/b})bK+(1-\gamma^{(1-b)/b})K*]}{[b\gamma K+K*+(1-b)\alpha K*][bK+K*+(1-b)\alpha\gamma^{(1-b)/b}K*]}$$

$\lesseqgtr 0$ according as $\gamma \gtreqless 1$.