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Influence of Over-and Underconfidence on Marriage Market

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Abstract

This paper is an examination of the influence of an individual's self-confidence (overconfidence or underconfidence) on others in the marriage market. We consider a model in which there are three types of men and women according to marital charm, and some men/women overestimate/underestimate their own types. The result obtained is that the self-confidence of some single individuals affects not only themselves but also the marital behavior of other rational singles in the market. Furthermore, self-confidence improves the welfare of the economy if there are enough underconfident men/women or if there are sufficiently few overconfident men/women in the marriage market.

Key Words: marriage, search, overconfidence, underconfidence

JEL Classification Numbers: D82, D83, J12

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1 Introduction

Self-confidence (overconfidence or underconfidence) has been mainly researched in the field of psychology. For example, the classical psychological study of overconfidence is that by Svenson (1981), in which he reports that 88% of U.S. subjects overestimate themselves relative to others in the safety of their driving. Moreover, Baker and Emery (1993) show that people overestimate the likelihood of success in their marriage.

In economics, the concept of self-confidence has spread in recent years. Dubra (2004) analyzes the role of overconfidence in a labor search problem. He shows that overconfident agents tend to search longer since they overestimate their chances of finding a better offer. Santos-Pinto and Sobel (2005) investigate sufficient conditions for the appearance of a positive/negative self-image in subjective assessments of ability in a skill acquisition model. Köszegi (2006) shows how overconfidence regarding the appropriateness of a task occurs when individuals choose a task. Hoelzl and Rustichini (2005) demonstrate that, if tasks are easy (difficult) and familiar (unfamiliar), individuals tend to be overconfident (underconfident) of their relative skill in experimental situations.

In this paper, we investigate the influence of self-confidence on the marital pattern in the marriage market.[1] In the marriage market, it is likely that overconfidence and underconfidence exist: some people often overestimate/underestimate their market value when they seek a marital partner. In much of the marriage literature, an individual is assumed to choose a marriage partner based on his/her charm (type). Since charm is defined by various elements, including quality, attraction, intelligence, height, age, education, and family background, the meaning of the word "charm" might carry some ambiguity. Hoelzl and Rustichini (2005) report, using a verbal experiment, that self-confidence arises more easily when it is difficult for agents to interpret the meaning of the word.[2] Santos-Pinto and Sobel (2005) show that, when different people have different opinions about how skills determine ability, a subjective positive/negative self-image arises. Although they mainly analyze positive/negative self-image in a skill acquisition model, we can apply the same logic to our marriage market model.[3].

We consider the framework of Burdett and Coles (1997), which is two-sided search model. There are ex-ante heterogeneous singles in the marriage market. It is assumed that all singles in the market know the distribution of the type (charm) of each sex correctly in the marriage market. However, we assume that some people are overconfident/underconfident for their

[1] Our results also apply to the labor market. In the labor market, workers and employers seek each other as a working partner. Moreover, workers' productivity may differ according to the individual's ability and firms' productivity also differ according to their capital.

[2] They also show that, even if ambiguities in words are excluded as much as possible, self-confidence occurs in the course of experimentation.

[3] In such a case, individuals must invest in each component of charm to maximize their own charm before they enter the market. This assumption is also along the lines of those by Burdett and Coles (2001), where individuals increase his/her own pizzazz (charm) before they participate in the marriage market. Moreover, assuming that some individuals disagree about the contribution of each element of charm to effective charm, they overestimates or underestimates their own charm.

own type. An agent who has self-confidence behaves just like the type to which he/she believes that he/she belongs to and chooses the best strategy. All other singles, except people who have self-confidence, are rational and recognize their own types correctly. According to Burdett and Coles (1997), singles of either sex are partitioned into classes, and sorting arises in their framework. They term this sorting as a special kind of positive assortative mating. Positive assortative mating is a positive association in sorting the values of a partner's charm. Considering self-confidence, this two-sided aspect of the problem generates interesting results.

The results that we obtained are as follows. In our model, for simplicity, we assume that there are three types of men/women according to charm: high type, middle type and low type. First, to clarify the effect of self-confidence, we derive the conditions under which the economy is at a perfect sorting equilibrium, where only persons of the same type marry, if there are not any self-confident agents as a benchmark.[4] Next, we consider the case in which there are some high type women who are underconfident in the market. Underconfident high-type women believe that they belong to the middle type, and then accept offers from middle-type men. Some middle-type men can marry high-type women with underconfidence who turn down proposals from middle-type men if they do not underestimate their own type. Consequently, middle-type men change their decision: they turn down proposals from middle-type women and wait to meet high-type women with underconfidence. Given this, middle-type women also change their optimal strategies. They accept low-type men, as they prefer to marry rather than to remain single forever. Since low-type men can marry middle-type women, low-type men reject low-type women. Then, low-type women cannot marry. Thus, underconfidence by some high-type women changes the marital behavior of lower type singles. Underconfident high-type women marry earlier than in the case without underconfidence, since they accept offers from both high-type and middle-type men. Moreover, if the proportion of underconfident high-type women is sufficiently large, the total number of marriages and the welfare generated by marriages increase compared to those of the benchmark case.

Finally, suppose that there are some middle-type women who are overconfident in the market. They turn down proposals from middle-type men because women with overconfidence believe that they belong to the high type. Thus, middle-type men change their marriage behavior: they accept offers from low-type women. The reason for this is that some middle-type men, who marry middle-type women in the benchmark case, are rejected by some overconfident middle-type women. Since low-type women are accepted by middle-type men, low-type women also change their decision and reject low-type men. Therefore, low-type men cannot marry. Overconfident women cannot marry and keep seeking a marital partner forever in the market.[5] Moreover, if there are sufficiently few overconfident middle-type women in

[4] According to our assumption of three types, if all singles are unbiased, marrying a member of the opposite sex in the same class implies marrying a member of the opposite sex of the same type.

[5] In this paper, we assume that there is no learning. If agents learn their type from experience, there are agents who identify their correct types and those who do not. Agents who have already learned their type

the market, the total amount and the welfare of marriage rise relative to the case in which all agents are rational. Even if sex is reversed, these results are confirmed.

This paper is organized as follows. In Section 2, we describe the basic framework for our analysis. In Section 3, we first show the consequence of the benchmark case, in which all agents are rational. Next, we examine the underconfidence effect and the overconfidence effect and contrast them with the benchmark result. In Section 4, we analyze social welfare generated by marriage in the benchmark case, the underconfidence case, and the overconfidence case. Section 5 is the conclusion.

2 The basic model

In this section, we present a basic model for our analysis in this paper.

As the marriage problem is two-sided, a two-side search model is assumed. Throughout, we shall only consider a marriage market which is in the steady state. Consider a marriage market with a large and equal number of men and women. Let N denote the participating men/women in the marriage market. An agent in the market wishes to marry a member of the opposite sex.

It is assumed that it always takes a time cost to look for a marriage partner. It is difficult for agents to meet someone of the opposite sex in the market; therefore, contact happens every now and then. Let α denote the rate at which a single individual contacts a member of the opposite sex, where α is the parameter of the Poisson process.[6]

It is assumed that singles are ex-ante heterogeneous, which implies that all singles have the same ranking about a potential partner in the marriage market. Let x_j denote the type (charm) of a single man/woman j in the market and is assumed to represent the real number. When both sexes meet, each agent can instantly recognize the opponent's type x_i and then decide whether to propose or not. We assume that, if a couple marries, he/she obtains the utility flow which equals the spouse's type per unit of time and vice versa. Furthermore, utilities are non-transferable: there is no bargaining for the division of the total marital utility.

If both singles accept a proposal from an opponent, they marry and leave the marriage market. A steady state requires that the exit rate into the market equal the entry rate into the market for each type. In this paper, we assume that, if a pair marry and leave the market,

apply the same strategy as agents with no self-confidence, while agents who do not learn their types behave as self-confident agents in our model. Thus, the environment of a marriage market with learning agents is qualitatively the same as that in a market without learning. Even if overconfident middle-type women learn from their experiences, there remain overconfidence women in the marriage market. Therefore, middle-type men accept low-type women after all. Note that underconfident high-type women are not able to learn about their own type, since they marry before they identify their correct type.

[6] The encounter function is constant returns to scale: $\alpha = M(N,N)/N = M(1,1)$, where $M(N,N)$ is the function which indicates the number of encounters between men and women per unit of time as a the function of the stock of participating men and women. This function is assumed to be continuous and increasing in both components.

two identical singles enter the market at once (a *"clone"* assumption).[7] If at least one of the two decides not to propose, they return to the marriage market and look for another marital partner.

For the explanation, let us now regard agent j as a man and i as a woman. Suppose that x_j (x_i) follows $F_m(x)$ $(F_w(x))$, which denotes the distribution of types among men (women) in the market. Both $F_m(x)$ and $F_w(x)$ need not be symmetric among men and women. However, for simplicity, we assume that a certain agent's x_j corresponds to other agent's x_i regardless of sex if singles belong to the same type. Moreover, we assume that all singles correctly know $F_m(x)$ and $F_m(x)$.

Suppose that people live forever and divorce is not considered. Therefore, female agent i who marries male agent j obtains discounted lifetime utility x_j/r, where r is the discount rate. Since agent i obtains utility b_i per unit time while single, i earns utility $b_i\Delta$, where Δ is the small time interval. In addition, $\alpha_i\Delta$ is i's probability that she will meet a man who is willing to marry her. As $F_m(x_j)$ implies the probability that i will meet a man who generates no more than x_j per unit of time when they marry, the expected discounted lifetime utility of the single woman i is

$$V_i = \frac{1}{1+r\Delta}\left[b_i\Delta + (1-\alpha_i\Delta)V_i + \alpha_i\Delta E_i \max\left(V_i, \frac{x_j}{r}\right)\right], \tag{1}$$

where E_i is the expectation operator given x_j. Agent i will propose to agent if and only if the utility she obtains is at least as great as V_i. The optimal strategy of agent i is obtained from (1) given α_i and $F_m(x_j)$. This best strategy has the feature of the reservation utility level, that is, agent i will accept any offer $x_j \geq R_i$ from a man, where $R_i = rV_i$.

3 Analysis

First, we consider a two-side search model with rational agents, which is a benchmark case. In later sections, we study two cases with self-confidence, i.e., underconfidence or overconfidence, and compare these two cases with the benchmark case.

3.1 Benchmark result

To simplify the analysis, suppose that there are three types of men/women according to charm - high, middle and low. A participant in a marriage market belongs to one of these

[7] We can assume the exogenous inflow of new singles which is assumed in Burdett and Coles (1997). This exogenous entry flow is a more reasonable approach than that involving clones. However, to make the influence of underconfidence or overconfidence more obvious, we adopt the clone assumption. If an exogenous inflow is assumed, a couple who marries and leaves the market changes the composition of types in the market. As a result, the expected discounted utility of an unmatched single is changed. On the other hand, this externality does not occur once the clone assumption is assumed.

types and knows his/her own type correctly.[8] Let x_H/r denote the (discounted) utility of marrying a high-type agent; similarly, x_M/r and x_L/r represent the utilities of marrying a middle-type agent and a low-type agent, respectively. We assume that $x_H > x_M > x_L > 0$. Both sexes are assumed to obtain zero utility flow while they are single.

Let λ_H denote the proportion of high-type men/women in the marriage market. Similarly, λ_M and λ_L are the proportion of men/women who belong to the middle and low types, respectively, where $\lambda_H + \lambda_M + \lambda_L = 1$. Assume that λ_i $(i = H, M, L)$ of each sex are symmetric among men and women in the market. We now focus on a steady state, where N and λ_i $(i = H, M, L)$ are given. In any equilibrium, all singles would like to marry a high-type agent.

First, we investigate whether a high-type agent is willing to accept an agent of the opposite sex in the middle or low type. From (1), the value of being a single high-type agent V_H becomes

$$\begin{aligned} rV_H &= \alpha\lambda_H \left(\frac{x_H}{r} - V_H\right) \\ &+ \alpha\lambda_M \left[\max\left(V_H, \frac{x_M}{r}\right) - V_H\right] + \alpha\lambda_L \left[\max\left(V_H, \frac{x_L}{r}\right) - V_H\right]. \end{aligned} \tag{2}$$

A high-type agent $(i = H)$ meets another high-type agent of the opposite sex, and they marry with probability λ_H. However, if a high-type agent meets an agent of the middle or low type of the opposite sex with probability λ_M (λ_L), he/she compares x_M/r (x_L/r) and V_H and then decides whether to marry or not. By this comparison, we can obtain the reservation utility level of each type for rejecting other types. From this, 4 possible steady state equilibrium outcomes can be considered when all singles are unbiased. These outcomes depend on x_i $(i = H, M, L)$ holding all other parameters constant. In order to make the influences of the self-confidence more obvious, we restrict our attention to the next equilibrium in this article.

Definition 1 *In* the perfect sorting equilibrium*, high-type agents marry within their group, as do middle-type agents and low-type agents.*

In a perfect sorting equilibrium, men and women in the same type marry.[9] Therefore, we may consider that high-type agents who marry within their group form the first cluster of marriages, middle-type agents who marry within their group form the second cluster of marriages, and low-type agents who marry within their group form the third cluster of marriages in this equilibrium. We now define the following situation as a benchmark case: if all agents are rational, the perfect sorting equilibrium occurs. The following proposition shows the condition for the perfect sorting equilibrium when all singles are rational.

[8] If we consider a model with n types, our results do not change qualitatively. Then, the assumption of three types is only for analytical simplicity.

[9] The perfect sorting equilibrium corresponds to *the elitist Nash equilibrium*, which is called in Burdett and Coles (1997).

Proposition 1 *Suppose that all singles are rational. The economy is at the perfect sorting equilibrium if and only if*

$$x_M < R_H^* \equiv \frac{\alpha\lambda_H x_H}{\alpha\lambda_H + r} \quad \textit{and} \quad x_L < R_M^* \equiv \frac{\alpha\lambda_M x_M}{r + \alpha\lambda_M}.$$

Proof. Iff a high-type agent does not propose to a middle-type agent of the opposite sex, $V_H > \frac{x_M}{r}$. From (2), this high-type agent's discounted lifetime utility when he/she is single becomes

$$rV_H^1 = \alpha\lambda_H\left(\frac{x_H}{r} - V_H^1\right).$$

On the other hand, when a high-type agent accepts a middle-type agent and turns down a low-type agent, i.e., $\frac{x_M}{r} \geq V_H > \frac{x_L}{r}$, his/her value function is[10]

$$rV_H^2 = \alpha\lambda_H\left(\frac{x_H}{r} - V_H^2\right) + \alpha\lambda_M\left(\frac{x_M}{r} - V_H^2\right).$$

If $V_H^1 > V_H^2$ is satisfied, a high-type agent refuses a middle-type agent of the opposite sex. This inequality $V_H^1 > V_H^2$ means that

$$x_M < R_H^* \equiv \frac{\alpha\lambda_H x_H}{\alpha\lambda_H + r}. \tag{3}$$

If $x_M \geq R_H^*$, a high-type agent proposes to a middle-type agent.

Under inequality (3), we can obtain the condition for a middle-type agent to reject a low-type agent by the same process above. Consequently, we have

$$x_L < R_M^* \equiv \frac{\alpha\lambda_M x_M}{\alpha\lambda_M + r}. \tag{4}$$

■

If $x_M < R_H^*$ and $x_L < R_M^*$ are satisfied, $V_H > x_M/r > V_M > x_L/r > V_L$ holds. The parameter $\alpha\lambda_H$ implies the arrival rate of proposals for an agent to contact a high-type agent. If a high-type agent meets another high-type agent of the opposite sex, they marry. Similarly, $\alpha\lambda_M$ ($\alpha\lambda_L$) is the rate at which an agent meets a middle- (low-) type agent. The inequality (3) can be rewritten as the condition of the parameter λ_H. With constant α, if (iff) λ_H is large enough ($\alpha\lambda_H > \frac{rx_M}{(x_H - x_M)}$), a high-type agent turns down a middle- and a low-type agent in the market ($x_M < R_H^*$). Conversely, if (iff) there are sufficiently few high-type agents ($\alpha\lambda_H \leq \frac{rx_M}{(x_H - x_M)}$), a high-type agent will accept a middle-type agent when they meet ($x_M \geq R_H^*$). If (iff) $\alpha\lambda_H \leq \frac{rx_L}{(x_H - x_L)}$ holds, a high-type agent is willing to propose to any opposite sex ($x_L \geq R_H^*$). A similar discussion can be done for the parameter

[10] If a high-type agent proposes to a middle-type agent but turns down a low-type agent ($x_L/r < V_H \leq x_M/r$), the high- and middle-type agents receive at least the same number of offers. Hence, $V_H \geq V_M$, and then we have $V_M \leq x_M/r$. Namely, a middle-type agent wishes to marry another middle-type agent. Likewise, if a middle-type agent accepts a low-type agent ($V_M \leq x_L/r$), the middle- and low-type agents receive at least the same number of offers. Then, a low-type agent also wants to marry another low-type agent.

λ_M by the inequality (4). If λ_H and λ_M are small enough to satisfy $\alpha\lambda_H \leq \frac{rx_M}{(x_H - x_M)}$ and $\alpha\lambda_M \leq \frac{rx_L - \alpha\lambda_H(x_H - x_L)}{(x_M - x_L)}$, there is an equilibrium in which all singles obtain the same expected discounted lifetime utility $V_L = V_M = V_H < \frac{x_L}{r}$. In this case, all types accept each other, and then all singles marry the first agent of the opposite sex they meet.[11]

3.2 Underconfidence and Overconfidence

In this subsection, we introduce the notion of overconfidence and underconfidence into the benchmark case. Suppose that all singles correctly understand the type distribution $F_m(x_j)$ and $F_m(x_j)$. However, we assume that some women misunderstand their own types and behave as if they were an agent in the type they think they belong to.[12] Therefore, women who are underconfident or overconfident decide the optimal strategy from (1) as the agents in the type they think they belong to. Suppose that all singles know the proportion of the women who are underconfident or overconfident in the marriage market. In order to make our analysis explicit, in the following sections, we will consider the case in which the condition in Proposition 1 is satisfied: the economy is at the perfect sorting equilibrium if there are not any self-confident agents. Then, we assume that $x_M < R_H^*$ and $x_L < R_M^*$ are satisfied.

3.2.1 Underconfidence

Now, let us suppose that some women in the high type believe that they belong to the middle type. A proportion θ_{HM} $(0 < \theta_{HM} < 1)$ of high-type women are underconfident. They decide their own optimal strategy as middle-type women. Then, an underconfident high-type woman proposes to a high- or a middle-type man. On the other hand, middle-type men expect to marry underconfident women, as they know the existence of underconfident high-type women. Their optimal strategies are obtained in the next lemma.

Lemma 1 *Suppose that $x_M < R_H^*$, $x_L < R_M^*$ and θ_{HM} $(0 < \theta_{HM} < 1)$ of high-type women are underconfident. If*

$$\frac{\alpha\lambda_H\theta_{HM}x_H + \alpha\lambda_M x_M}{r + \alpha\lambda_M + \alpha\lambda_H\theta_{HM}} \equiv R_{Mm}^{U1} > (\leq)\ x_L,$$

a middle-type man rejects (accepts) a low-type woman. Moreover, if

$$\frac{\alpha\lambda_H\theta_{HM}}{(r + \alpha\lambda_H\theta_{HM})}x_H \equiv R_{Mm}^{U2} > (\leq)\ x_M,$$

a middle-type man rejects (accepts) a middle-type woman. In both of these cases, the reservation utility level of a middle-type man increases relative to the benchmark result, i.e., $R_M^* < R_{Mm}^{U1} < R_{Mm}^{U2}$.

[11] This equilibrium is *the mixing Nash equilibrium*, as called by Burdett and Coles (1999).

[12] This definition follows that of Santos-Pinto and Sobel (2005): agents have a subjective and egocentric self-image.

Proof. See Appendix. ■

This lemma means that, if some high-type women are underconfident, a middle-type man's reservation utility level increases relative to that of the benchmark result. This is because a middle-type man expects to marry an underconfident high-type woman. Therefore, a middle-type men decides whether to accept a middle- (low-) type women. The reservation utility level of a middle-type man $R^{U1}_{Mm} > (\leq)\ x_L$ can be rewritten as $\alpha\lambda_H\theta_{HM} > (\leq)\ \frac{(r+\alpha\lambda_M)x_L-\alpha\lambda_M x_M}{(x_H-x_L)}$. Further, we can rewrite $R^{U2}_{Mm} > (\leq)\ x_M$ as $\alpha\lambda_H\theta_{HM} > (\leq)\ \frac{rx_M}{(x_H-x_M)}$. [13] These inequalities mean that, with constant α, λ_M, and λ_H, if (iff) θ_{HM} is large enough to satisfy $\alpha\lambda_H\theta_{HM} > \frac{rx_M}{(x_H-x_M)}$, a middle-type man rejects a middle-type woman, as he wishes to accept only an underconfident woman. Conversely, if (iff) θ_{HM} is small enough ($\alpha\lambda_H\theta_{HM} \leq \frac{rx_M}{(x_H-x_M)}$), a middle-type man accepts a middle-type woman. If (iff) θ_{HM} satisfies $\frac{(r+\alpha\lambda_M)x_L-\alpha\lambda_M x_M}{(x_H-x_L)} < \alpha\lambda_H\theta_{HM} \leq \frac{rx_M}{(x_H-x_M)}$, a middle-type man accepts a middle-type woman and rejects a low-type woman. If (iff) θ_{HM} is small enough to satisfy $\alpha\lambda_H\theta_{HM} \leq \frac{(r+\alpha\lambda_M)x_L-\alpha\lambda_M x_M}{(x_H-x_L)}$, a middle-type man accepts a low-type woman and then will propose to any woman in the market.

It is noteworthy that underconfident high-type women marry earlier than rational high-type women, since all singles would like to marry high-type agents. The time until marriage of an underconfident woman can be calculated, i.e., $\frac{1}{(\alpha\lambda_H+\alpha\lambda_M)}$. In contrast, if a high-type woman recognizes her own type correctly, her time until marriage is $\frac{1}{\alpha\lambda_H}$. [14] Especially when $\alpha\lambda_H\theta_{HM} > \frac{rx_M}{(x_H-x_M)}$, an interesting Nash equilibrium occurs. Then, we obtain the following proposition. This equilibrium does not become the perfect sorting equilibrium due to the existence of underconfident women.

Proposition 2 *Suppose that $x_M < R^*_H$, $x_L < R^*_M$, and θ_{HM} $(0 < \theta_{HM} < 1)$ of high-type women are underconfident. If $\alpha\lambda_H\theta_{HM} > \frac{rx_M}{(x_H-x_M)}$, a middle-type man turns down a middle-type woman ($x_M < R^{U2}_{Mm}$). Therefore, a middle-type woman accepts a low-type man. As a result, a low-type man has the same reservation utility level as that of a middle-type agent in the benchmark case, i.e., $R^U_{Lm} = R^*_M$, and turns down a low-type woman. Hence, low-type women cannot marry.*

Proof. See Appendix. ■

Clearly, all agents except high-type agents are influenced by the underconfidence of a high-type woman. Proposition 2 means that high-type agents (including underconfident women) form the first cluster of marriages, middle-type men and underconfident women form the second cluster, and middle-type women and low-type men form the third cluster. A middle-type man prefers to continue to search for an underconfident woman than to marry

[13] It is noteworthy that $[x_L(r+\alpha\lambda_M)-\alpha\lambda_M x_{Mw}]/(x_H-x_L) < rx_M/(x_H-x_M)$, from $0 < x_L < x_M < x_H$.

[14] When a middle-type man accepts a middle-type woman and rejects a low-type woman, a middle-type man's rate of contact with women whom he wants to marry is $(\alpha\lambda_H\theta_{HM}+\alpha\lambda_M)$. Then, the amount of time until meeting that woman is $1/\left(\alpha\lambda_H\theta_{HM}+\alpha\lambda_M\right)$. Since that in the benchmark case is $1/\alpha\lambda_M$, a middle-type man's time until marriage is shorter than that in the benchmark case. A similar discussion can take place when a middle-type man accepts a low-type woman.

a middle-type woman, as there are a sufficient number of underconfident women. Then, a middle-type woman decides whether or not to marry a low-type man. However, she always proposes to a low-type man, since she prefers to marry than to remain single forever. Given this, a low-type man has the reservation utility for rejecting a low-type woman. Moreover, his reservation utility level becomes equal to that of a middle-type agent in the benchmark case. Consequently, a low-type man turns down a low-type woman as $R_{Lm}^U = R_M^* > x_L$. Then, although a low-type woman wishes to marry a low-type man, the opportunity never arises.

3.2.2 Overconfidence

Here, suppose that some middle-type women misidentify their type. Let us assume that proportion θ_{MH} $(0 < \theta_{MH} < 1)$ of middle-type women consider themselves to be high-type women. Hence, their optimal strategies are R_H^* from (3). We have the next Lemma for a middle-type man.

Lemma 2 *Suppose that $x_M < R_H^*$, $x_L < R_M^*$, and θ_{MH} $(0 < \theta_{MH} < 1)$ of middle-type women are overeconfident. If*

$$x_L < (\geq)\ R_M^O \equiv \frac{\alpha\lambda_M (1-\theta_{MH})\, x_M}{r + \alpha\lambda_M (1-\theta_{MH})},$$

a middle-type man rejects (accepts) a low-type woman. In this case, the reservation utility level of a middle-type man decreases, in contrast with the benchmark result, i.e., $R_M^* > R_M^O$.

Proof. See Appendix. ■

This lemma means that overconfidence in some middle-type women lowers the reservation utility level of a middle-type man for a low-type woman. The reason for this is that middle-type men, who marry middle-type women in the benchmark case, are turned down by overconfident middle-type women.

We can rewrite $R_M^O > (\leq)\ x_L$ as $\alpha\lambda_M(1-\theta_{MH}) > (\leq)\ \frac{rx_L}{(x_M - x_L)}$. This means that, with constant α and λ_M, if (iff) θ_{MH} is small enough to $\alpha\lambda_M(1-\theta_{MH}) > \frac{rx_L}{(x_M - x_L)}$, a middle-type man turns down a low-type woman due to his expectation to marry a rational middle-type woman. Conversely, if (iff) θ_{MH} is large enough $(\alpha\lambda_M(1-\theta_{MH}) \leq \frac{rx_L}{(x_M - x_L)})$, a middle-type man accepts a low-type woman.

Moreover, if $R_M^O > x_L$, the middle-type man's rate of contact with a woman whom he wishes to marry is $\alpha\lambda_M (1-\theta_{MH})$. Then, a middle-type man's time (duration) until meeting such a woman is $\frac{1}{\alpha\lambda_M(1-\theta_{MH})}$. Their time until marriage is delayed since that in the benchmark is $\frac{1}{\alpha\lambda_M}$. On the other hand, middle-type women with overconfidence can never marry. The reason for this is that their offers are turned down by high-type men as long as they are overconfident.

Given the above analysis, we can focus on an interesting Nash equilibrium and obtain the next Proposition.

Proposition 3 *Suppose that* $x_M < R_H^*$*,* $x_L < R_M^*$ *and* θ_{MH} $(0 < \theta_{MH} < 1)$ *of middle-type women are overconfident. If* $\alpha\lambda_M (1 - \theta_{MH}) \leq \frac{rx_L}{(x_M - x_L)}$*, a middle-type man accepts a low-type woman* $(R_M^O \leq x_L)$*. Consequently, a low-type woman has the same reservation utility level as a middle-type woman* $R_{Lw}^O = R_M^*$ *and, therefore, refuses a low-type man. Hence, a low-type man cannot marry.*

Proof. See Appendix. ■

Clearly, this equilibrium is not the perfect sorting equilibrium. The implications of Proposition 3 are as follows: high-type agents form the first cluster of marriages, middle-type men and rational middle-type women form the second cluster, and middle-type men and low-type women form the third cluster. Now, a middle-type man accepts a low-type woman, since there are a sufficient number of overconfident middle-type women in the marriage market from Lemma 2. Although a low-type woman is willing to marry a middle-type man, she may also decide whether or not to marry a low-type man. Hence, a low-type woman has the reservation utility for turning down a low-type man. However, her reservation utility level becomes equal to that of a middle-type agent in the benchmark case. Then, a low-type woman refuses a low-type man, and then a low-type man can never marry. Moreover, overconfident women cannot marry unless they identify their type correctly. It is noteworthy that the first cluster of marriages is not influenced by overconfidence.

4 Welfare

It is meaningful to investigate the effects of self-confidence on social welfare. Underconfidence or overconfidence may improve the welfare of the economy relative to the benchmark case. We investigate the amount of marriages and their welfare at any point in time.

First, we show welfare for the case in which all players are rational as the benchmark. Next, we investigate the welfare in the cases of underconfidence and overconfidence and contrast these two cases with the benchmark case.

4.1 Benchmark result

Suppose that all agents are rational in a marriage market. As $x_M < R_H^*$ and $x_L < R_M^*$, high-type, middle-type, and low-type agents marry within their groups. As a result, high-type agents form the first cluster of marriages, middle-type agents form the second cluster, and low-type agents form the third cluster from Proposition 1. Now, a high-type man/woman meets a high-type woman/man with probability $\alpha\lambda_H$, and there are $\lambda_H N$ number of high-type men/women in the market. Then, the number of marriages among high-type agents in a market is $\alpha\lambda_H^2 N$. In the same way, we obtain the number of marriages of middle-type $\alpha\lambda_M^2 N$ and low-type $\alpha\lambda_L^2 N$. Therefore, the total number of marriages in the marriage market T^* is

$$T^* = \alpha\lambda_H^2 N + \alpha\lambda_M^2 N + \alpha\lambda_L^2 N. \tag{5}$$

Moreover, we explore welfare. If a high-type man marries a high-type woman, each of them obtains the utility of marriage x_H. Hence, the aggregation of high-type agents' utilities from marriage is $2\alpha\lambda_H^2 x_H N$. Similarly, we obtain $2\alpha\lambda_M^2 x_M N$ for the middle type and $2\alpha\lambda_L^2 x_L N$ for the low type. As a result, the welfare of the whole society W^* is

$$W^* = 2\alpha\lambda_H^2 x_H N + 2\alpha\lambda_M^2 x_M N + 2\alpha\lambda_L^2 x_L N. \tag{6}$$

In the following subsections, we introduce self-confidence into the marriage market.

4.2 Underconfidence

Suppose that θ_{HM} $(0 < \theta_{HM} < 1)$ of high-type women are underconfident in the market and $x_L < R_M^* \le x_M < R_{Mm}^{U2}$ holds. Then, we restrict our attention to the case in which $\theta_{HM} \in (\frac{rx_M}{\alpha\lambda_H(x_H - x_M)}, 1)$ in this subsection. High-type men marry high-type women (including underconfident women) in the first cluster, middle-type men marry underconfident high-type women in the second cluster, and low-type men marry middle-type women in the third cluster from Proposition 2. A high-type man meets a rational high-type woman with probability $\alpha(1-\theta_{HM})\lambda_H$ and an underconfident woman with $\alpha\theta_{HM}\lambda_H$. Since there are $\lambda_H N$ number of high-type men, the number of marriages for them is $\alpha\lambda_H^2 N$. A middle-type man meets an underconfident women with probability $\alpha\theta_{HM}\lambda_H$. As there are $\lambda_M N$ number of middle-type men in a steady state, the number of marriages for them is $\alpha\theta_{HM}\lambda_H\lambda_M N$. In the same way, we obtain the number of marriages for low-type men $\alpha\lambda_M\lambda_L N$. [15] Therefore, we obtain the total number of marriages

$$\begin{aligned} T^U &= \alpha(1-\theta_{HM})\lambda_H^2 N + \alpha\theta_{HM}\lambda_H^2 N + \alpha\theta_{HM}\lambda_H\lambda_M N + \alpha\lambda_M\lambda_L N, \\ &= \alpha\lambda_H^2 N + \alpha\theta_{HM}\lambda_H\lambda_M N + \alpha\lambda_M\lambda_L N, \end{aligned} \tag{7}$$

and social welfare,

$$W^U = 2\alpha\lambda_H^2 x_H N + \alpha\theta_{HM}\lambda_H\lambda_M (x_H + x_M) N + \alpha\lambda_M\lambda_L (x_M + x_L) N. \tag{8}$$

Next, we contrast the number of marriages and welfare in the case of underconfidence with those in the benchmark case. The next lemma shows the necessary and sufficient condition

[15] By the same procedure, the number of marriages for rational high-type women is $\alpha(1-\theta_{HM})\lambda_H^2 N$. Likewise, that for underconfident women is $\alpha(\lambda_H + \alpha\lambda_M)\theta_{HM}\lambda_H N$, and that for middle-type women is $\alpha\lambda_M\lambda_L N$. Then, the total number of marriages for women corresponds to (7).

under which underconfidence raises the total number of marriages.

Lemma 3 *Suppose that* $x_L < R_M^* \leq x_M < R_{Mm}^{U2}$ *and* θ_{HM} *of high-type women are underconfident. Iff*

$$\theta_{HM} > (\leq)\ \theta^{UT} \equiv \frac{\lambda_M^2 - \lambda_L\lambda_M + \lambda_L^2}{\lambda_H\lambda_M}, \tag{9}$$

the number of marriages of an economy with underconfidence is higher (lower) than the benchmark result, i.e., $T^U > (\leq)\ T^*$.

Proof. See Appendix. ■

This lemma means that, if there are enough (few enough) underconfident women ($\theta_{HM} > (\leq)\ \theta^{UT}$), the number of marriages is higher (lower) in an economy with underconfidence women than in the benchmark economy. This threshold point depends only on λ_i ($i = H, M, L$). It is noteworthy that $\theta^{UT} > 0$ as the numerator of θ^{UT} is always positive. Whenever $\theta^{UT} > 1$, $\theta^{UT} > \theta_{MH}$ must hold. In this case, the number of marriages is not related with the proportion of underconfidence θ_{MH}.[16]

The number of marriages for the first cluster has the same value as the benchmark case. However, that for the second cluster depends on θ_{HM} and λ_i ($i = H, M$) in the marriage market. Iff $\theta_{HM} > (<)\ \theta_2^{UT} \equiv \lambda_M/\lambda_H$, there are more (fewer) underconfident women than middle-type women.[17] Hence, the number of marriages for the second cluster increases (decreases). Whenever $\lambda_M > \lambda_H$, then $\theta_2^{UT} > 1 > \theta_{HM}$. Therefore, the number of marriages for the second cluster decreases under any values of θ_{HM}. On the other hand, that of the third cluster depends on only λ_i ($i = M, L$) independently of the proportion of underconfident women. If (iff) $\lambda_M > (\leq)\ \lambda_L$, the number of marriages for the third cluster increases (decreases) from that of the benchmark case.[18] Moreover, $\lambda_M > (\leq)\ \lambda_L$ means that $\theta_2^{UT} > (\leq)\ \theta^{UT}$ holds. Therefore, if $\lambda_M > (\leq)\ \lambda_L$ and $\theta_{HM} > (<)\ \theta_2^{UT}$, both the number of marriages for the second cluster and that for the third cluster increase (decrease). Consequently, the total number of marriages increases (decreases).[19]

The next proposition shows the condition of raising (decreasing) the welfare from marriage.

[16] For instance, if $\lambda_H > \lambda_M > \lambda_L$, then $\theta^{UT} > 1 > \theta_{HM}$ and $T^U > T^*$ under any $\theta_{HM} \in (\frac{rx_M}{\alpha\lambda_H(x_H - x_M)}, 1)$.

[17] Moreover, θ_2^{UT} also corresponds with the threshold for increasing the number of marriages for middle-type men. If $\theta_2^{UT} < (\geq)\ \theta_{HM}$, the number of marriages for middle-type men is raised (lowered) relative to the benchmark case.

[18] If $\lambda_M > (\leq)\ \lambda_L$, the number of middle-type women is lowered (raised), and that of low-type men is raised (lowered) in the third cluster.

[19] Consider the case in which $\lambda_M > \lambda_L$. Hence, the number of marriages for the third cluster increases relative to that in the benchmark case. If $\lambda_M/\lambda_H > \theta_{HM} > \theta^{UT}$, that for the second cluster decreases. However, the total number of marriages increases. This is because the increasing effect on the number of marriages due to the third cluster is larger than the decreasing effect due to the second cluster. If $\lambda_M/\lambda_H > \theta^{UT} > \theta_{HM}$, the total number of marriages decreases. At this time, the increasing effect by the third cluster is smaller than the decreasing effect by the second cluster. Similar discussions can be carried out for $\theta^{UT} > \theta_{HM} > \lambda_M/\lambda_H$ and for $\theta^{UT} > \lambda_M/\lambda_H > \theta_{HM}$ in the case in which $\lambda_M < \lambda_L$.

Proposition 4 *Suppose that* $x_L < R_M^* \leq x_M < R_{Mm}^{U2}$ *and* θ_{HM} *of high-type women are underconfident. Iff*

$$\theta_{HM} > (\leq)\ \theta^{UW} \equiv \frac{\lambda_L^2 2x_L - \lambda_L\lambda_M (x_L + x_M) + \lambda_M^2 2x_M}{\lambda_H\lambda_M (x_H + x_M)}, \tag{10}$$

the welfare of an economy with underconfidence is higher (lower) than the benchmark result, i.e., $W^* < (\geq)\ W^U$.

Proof. See Appendix. ■

Proposition 4 implies that, if there are enough (few enough) underconfident women in the steady state ($\theta_{HM} > (\leq)\ \theta^{UW}$), the welfare increases (decreases) by underconfidence relative to the benchmark case. The threshold for increasing the number of marriages depends only on the distribution of types λ_i $(i = H, M, L)$, whereas that for increasing the welfare generated by marriage depends not only on λ_i but also on the utility of marrying a member of a type x_i $(i = H, M, L)$. Hence, an increase in the number of marriages does not necessarily mean an increase in welfare.

Here, we discuss the overall effect of underconfidence on the welfare from marriage and compare an economy with underconfidence with the benchmark economy. High-type agents, except underconfident women, are not influenced by underconfidence. The welfare of low-type women is always lowered, who cannot marry in the economy with underconfident women. In contrast, the underconfident women who marry middle-type men obtain lower utilities than rational women. However, their number of marriages increases, since they accept not only high-type but also middle-type men.[20] Consequently, the welfare of underconfident women is always higher than that in the benchmark case. The welfare of middle-type men depends on θ_{HM}, x_i and λ_i $(i = H, M)$. Middle-type men can get higher marital utilities than those in the benchmark case. Iff there are enough (few enough) underconfident women ($\theta_{HM} > (\leq) \frac{x_M\lambda_M}{x_H\lambda_H}$), the welfare of middle-type men is raised (lowered). Then, if $\theta_{HM} > \theta_2^{UT}$, middle-type men can marry underconfident women quickly, and their marriages and welfare increase.[21] If there are not many underconfident women ($\frac{x_M\lambda_M}{x_H\lambda_H} < \theta_{HM} \leq \theta_2^{UT}$), middle-type men's marriages are delayed, and then the number of their marriages decreases. However, in this case, since they can obtain sufficiently large marital utilities, their welfare is improved relative to that in the benchmark case. In addition, if (iff) $x_M\lambda_M > (\leq)\ x_L\lambda_L$, the welfare of middle-type women is lowered (raised), whereas that of low-type men is raised (lowered). If the difference between x_M and x_L is large enough ($x_M\lambda_M > x_L\lambda_L$ and $\lambda_M \leq \lambda_L$), the number of marriages for the third cluster is lowered, but the welfare of that cluster is raised

[20] The number of marriages for underconfident women is always raised, including that for underconfident women who marry high-type men. However, that of underconfident women who marry not middle-type men but high-type men is included in the number of marriages for the first cluster. That of underconfident women who marry middle-type men is included in the number of marriages for the second cluster.

[21] Iff $\theta_{HM} > (\leq)\ \theta_2^{UT} = \lambda_M/\lambda_H$, the number of marriages of middle-type men is raised (lowered). It is noteworthy that $\frac{x_M\lambda_M}{x_H\lambda_H} < \theta_2^{UT}$. However, $x_M\lambda_M > x_H\lambda_H$ is necessary for $\frac{x_M\lambda_M}{x_H\lambda_H} < 1$.

by the marital utilities. However, the welfare of the third cluster is lowered if $\lambda_M \geq \lambda_L$, or if $x_M\lambda_M < x_L\lambda_L$. In the case of $\lambda_M \geq \lambda_L$, the decline in the welfare of middle-type women is larger than the increase in that of low-type men. In the case of $x_M\lambda_M < x_L\lambda_L$, the increase in the welfare of middle-type women is much smaller than the decrease in that of low-type men, since there are many more low type men than middle type women. Therefore, the welfare of the third cluster is lowered in both of these cases.

Iff the numerator of θ^{UW} is positive and smaller than the denominator, the threshold value θ^{UW} is in $(0, 1)$.[22] However, θ^{UW} cannot be in $(0, 1)$. At this time, the welfare is increased or decreased by the value of both λ_i and x_i rather than by the proportion of underconfidence θ_{HM} in the market. Whenever the numerator of θ^{UW} is negative, then $\theta_{HM} > 0 > \theta^{UW}$ holds. Therefore, the welfare from marriage is always better under any θ_{HM}. However, it is necessary that $\lambda_L > \lambda_M$ for the negative numerator of θ^{UW}.[23] In contrast, if the numerator of θ^{UW} is larger than the denominator, then we obtain $\theta_{HM} < 1 < \theta^{UW}$. Then, the welfare from marriage is always worse under the any proportion of underconfident women.[24] Of course, $\theta_{HM} > (\leq)\ \theta^{UW}$ does not contradict $x_M < R^{U2}_{Mm}$ or $\theta_{HM} > (\leq)\ \theta^{UT}$.

If the distribution of types is the discrete uniform distribution, the next corollary is straightforward by Lemma 3 and Proposition 4. Once the discrete uniform distribution are assumed, the threshold proportion θ^{UW} depends only on the marital utilities.

Corollary 1 *Suppose that $x_L < R^*_M < x_M < R^{U2}_{Mm}$, θ_{MH} of middle-type women are overconfident. Moreover, we assume that the distribution of types is the discrete uniform distribution: $\lambda_L = \lambda_M = \lambda_H$. In this case, the total number of marriages of the economy with underconfidence is more than the benchmark result: $T^* < T^U$. Iff $\theta_{HM} > (\leq)\ \theta^{uw} = \frac{x_L + x_M}{(x_H + x_M)}$, the social welfare of the economy with underconfidence is higher (lower) than the benchmark case: $W^* < (\geq)\ W^U$.*

Proof. See Appendix. ■

The number of marriages and the welfare in the first cluster are the same values to those of the benchmark case. The number of marriages for the third cluster has also the same values as the benchmark case, since the proportion of middle-type agents is equal to that of low-type agents. However, the number of marriages for the second cluster always decreases by underconfidence as $\lambda_H\theta_{HM} < \lambda_M$. From these results, the total number of marriages always decreases.

The welfare of high-type agents except underconfident women is not influenced by the underconfidence. That of underconfident women is always higher than the benchmark case since they accept not only high-type but also middle-type men. The welfare of low-type women is always lowered. Iff there are enough (few enough) underconfident women ($\theta_{HM} >$

[22] For some examples, see Example 1 in Appendix.

[23] For example, when $x_M = 15x_L$ and $\lambda_L = 4\lambda_M$, the numerator of θ^{UW} becomes negative.

[24] For instance, if $\lambda_L > \lambda_M \frac{(x_L + x_M)}{2x_L}$ and $\lambda_M > \lambda_H \frac{(x_H + x_M)}{2x_M}$, we have $\theta^{UW} > 1$.

$(\leq)\ \frac{x_M}{x_H}$), the welfare of middle-type men is raised (lowered). These results are similar to those in Proposition 4. However, the welfare of middle-type women is always lowered since they marry low-type men. Whereas, that of low-type men is always raised compared to the bench mark case. However, the increase in the welfare of middle-type women and the decline in the welfare of low-type men are just offset. Therefore, the social welfare is not influenced by the third cluster of marriages. The overall effect of underconfidence on the social welfare depends on the threshold value θ^{uw}. If the difference among marital utility x_i $(i = H, M, L)$ is large (small) enough, θ^{uw} is close to zero (one).[25] Then, a small (large) amount of underconfident women can improve the social welfare from marriage.[26]

4.3 Overconfidence

Suppose that θ_{MH} $(0 < \theta_{MH} < 1)$ of middle-type women are overconfident in the market. They believe that their type is the high type. All singles, except women who are overconfident, correctly identify their types. Let us now assume that $R_M^O \leq x_L$ $(< R_M^* \leq x_M < R_H^*)$ and, therefore, we focus on the case in which $\theta_{MH} \in (\frac{\alpha\lambda_M(x_M - x_L) - rx_L}{(x_M - x_L)\alpha\lambda_M}, 1)$. Hence, high types marry within their group (they form the first cluster), and middle-type men marry rational middle-type (they form the second cluster) and low-type women (they form the third cluster) as in Proposition 3. Overconfident women and low-type women cannot marry. Therefore, we obtain the number of marriages,

$$T^O = \alpha\lambda_H^2 N + \alpha\left(1 - \theta_{MH}\right)\lambda_M^2 N + \alpha\lambda_M\lambda_L N, \tag{11}$$

and the welfare from marriage,

$$W^O = \alpha\lambda_H^2 2x_H N + \alpha\left(1 - \theta_{MH}\right)\lambda_M^2 2x_M N + \alpha\lambda_M\lambda_L\left(x_M + x_L\right)N. \tag{12}$$

Furthermore, we contrast these results in the case of overconfidence with those in the benchmark case.

Lemma 4 *Suppose that $R_M^O \leq x_L < R_M^* \leq x_M < R_H^*$ and θ_{MH} of middle-type women are overconfident. Iff*

$$\theta_{MH} < (\geq)\ \theta^{OT} \equiv \frac{\lambda_L\left(\lambda_M - \lambda_L\right)}{\lambda_M^2},$$

the number of marriages of the economy with overconfidence is more (less) than the benchmark result, i.e., $T^* > (\leq)\ T^O$.

Proof. See Appendix. ■

[25] If the difference between x_H and x_M is large (small), $rx_M/[\alpha\lambda_H\left(x_H - x_M\right)](< \theta_{HM})$ also becomes small (large).

[26] It is noteworthy that iff $x_H x_L \lesseqgtr x_M^2$, $\theta^{uw} \lesseqgtr \frac{x_M}{x_H}$.

This lemma means that, if there are few enough (enough) overconfident women ($\theta_{MH} \leq (>) \ \theta^{OT}$), the number of marriages increases (decreases) relative to the benchmark case. It is noteworthy that $\theta^{OT} < 1$ always holds. However, it is necessary and sufficient for $\theta^{OT} > 0$ that there be more middle-type agents than low-type agents ($\lambda_M > \lambda_L$). If $\lambda_M < \lambda_L$, then $\theta_{MH} > 0 > \theta^{OT}$ holds. That is to say, the number of marriages is always worse under any θ_{MH}. The number of marriages for the first cluster has the same value as that for the benchmark case. However, that for the second cluster is always worse, since middle-type men are turned down by overconfident women. On the other hand, that for the third cluster depends on the distribution of types λ_i ($i = M, L$). Now, low-type women reject low-type men and wait for middle-type men. Then, if (iff) $\lambda_M < (\geq) \ \lambda_L$, the number of marriages of the third cluster is worse (better). Therefore, in order to increase the number of marriages, $\lambda_M > \lambda_L$ is necessary.

The next proposition shows the condition for raising (decreasing) the welfare from marriage.

Proposition 5 *Suppose that $R_M^O \leq x_L < R_M^* \leq x_M < R_H^*$ and θ_{MH} of middle-type women are overconfident. Iff*

$$\theta_{MH} < (\geq) \ \theta^{OW} \equiv \frac{\lambda_L \lambda_M (x_L + x_M) - 2x_L \lambda_L^2}{2x_M \lambda_M^2}, \tag{13}$$

the welfare of the economy with overconfidence is higher (lower) than that in the benchmark case, i.e., $W^* > (\leq) \ W^O$.

Proof. See Appendix. ■

The intuition of Proposition 5 is as follows: if (iff) there are a small (large) enough number of overconfident women ($\theta_{MH} < (\geq) \ \theta^{OW}$), the welfare from marriage is better (worse) due to overconfidence relative to that in the benchmark case. Whereas θ^{OT} depends only on λ_i ($i = M, L$), θ^{OW} depends not only on λ_i ($i = M, L$) but also on the marital utility x_i ($i = M, L$). Similarly to Proposition 4, it is necessary and sufficient for $\theta^{OW} \in (0, 1)$ that the numerator be smaller than the denominator and that the numerator be positive. If the positive numerator is larger than the denominator in (13), then $\theta^{OW} > 1 > \theta_{MH}$ holds. In this case, the welfare of the economy with overconfident women is always higher than the economy of the benchmark case. If $\theta_{HM} > 0 > \theta^{OW}$ holds, the welfare from marriage is always worse under any value of θ_{MH}.[27] It is noteworthy that $\theta_{MH} > (<) \ \theta^{OW}$ does not contradict $x_M < R_{Mm}^{U2}$ and $\theta_{MH} > (<) \ \theta^{OT}$.

Let us discuss now the overall effect of overconfidence on the welfare and compare the economy with overconfidence with the benchmark economy. The welfare of marriage to

[27] However, $\theta_{MH} \leq \frac{rx_L - \alpha(x_M - x_L)\lambda_M}{(x_M - x_L)\alpha\lambda_M}$ is satisfied from the assumption $R_M^O \leq x_L$.

rational middle-type women has the same value to that of the benchmark case.[28] However, the welfare of overconfident women and that of low-type men in an overconfident economy are always worse, since they cannot get married. The welfare of marriages of middle-type men depends on θ_{MH}, x_i and λ_i $(i = M, L)$. The refusals by overconfident women lower the welfare of middle-type men. However, middle-type men accept both middle-type and low-type women in an economy with overconfidence. Then, if there are many more low-type women than overconfident women $(x_L\lambda_L > x_M\lambda_M\theta_{MH})$, the number of marriages for middle-type men increases, and then their welfare also increases as a whole.[29] Conversely, if there are fewer low-type women than underconfident women $(\lambda_L < \lambda_M\theta_{MH})$, or, if the marital utility x_M is larger enough than x_L $(x_L\lambda_L < x_M\lambda_M\theta_{MH})$ under $\lambda_L \geq \lambda_M\theta_{MH}$, the welfare of middle-type men is worse due to overconfidence relative to that in the benchmark case. The welfare of low-type women increases if x_M is sufficiently larger than x_L $(x_L\lambda_L < x_M\lambda_M)$ under $\lambda_M < \lambda_L$, or if $\lambda_M \geq \lambda_L$.[30] In contrast, if there are many more low-type agents than middle-type agents $(x_L\lambda_L > x_M\lambda_M)$, the welfare of low-type women is worse relative to that in the benchmark case. From these results, if $\theta_{MH} \leq \theta^{OW}$ and the marital utility x_M is larger enough than x_L $(x_M\lambda_M > x_L\lambda_L > x_M\lambda_M\theta_{MH})$, the social welfare increases. The reason for this is that the effects which raise the welfare of middle-type men and low-type women are larger than the above effects, which lower welfare.

If the distribution of types is a discrete uniform distribution, the next corollary is obtained. Similarly to Corollary 1, the threshold proportion θ^{OW} depends only on the values of the marital utility of each type.

Corollary 2 *Suppose that $R_M^O \leq x_L < R_M^* \leq x_M < R_H^*$, θ_{MH} of middle-type women are overconfident. Moreover, we assume that the distribution of types is the discrete uniform distribution: $\lambda_L = \lambda_M = \lambda_H$. In this case, the total number of marriages decreases compared to the benchmark result: $T^* > T^o$. Iff $\theta_{MH} > (<)\ \theta^{ow} \equiv \frac{x_M - x_L}{2x_M}$, the social welfare of the economy with overconfidence is lower (higher) than the benchmark case: $W^* > (<)\ W^O$.*

Proof. See Appendix. ∎

The number of marriages and the welfare for the first cluster are the same as those of the benchmark case. The number of marriages for the third cluster also has the same values as those of the benchmark case. However, the number of marriages for the second cluster is always lower in an overconfident economy than in the benchmark economy. This is because there are some overconfident women in the middle type who reject the proposals of middle-type men. Therefore, the total number of marriages is always lowered by overconfident women.

[28] The number of marriages to rational middle-type women also has the same value to that of the benchmark case.

[29] The number of marriages for middle-type men is raised (lowered) if $\lambda_L > (\leq)\ \lambda_M\theta_{HM}$. It is noteworthy that $\frac{x_L\lambda_L}{x_M\lambda_M} < \frac{\lambda_L}{\lambda_M}$ must hold.

[30] The number of marriages for low-type women is raised (lowered) if $\lambda_M > (\leq)\ \lambda_L$.

The welfare of high-type agents and that of rational middle-type women is the same value as those in the benchmark case. The welfare of overconfident women and that of low-type men in the overconfident economy are always worse. If (iff) the proportion of overconfident women is small (large) enough ($\frac{x_L}{x_M} > (\leq)\ \theta_{MH}$), the welfare of middle-type men increases (decreases) on the whole. This is because that middle-type men accept both middle-type and low-type women.[31] These results are similar to those in Proposition 5. However, the welfare of low-type women is always better relative to the benchmark case as they can obtain higher marital utility. Therefore, the overall effect on the social welfare depends on the threshold value θ^{ow}. If the difference between x_M and x_L is small enough, θ^{ow} is close to zero.[32] Then, a small number of overconfident women decrease the social welfare relative to the case in the benchmark economy. In this case, low-type women cannot obtain large marital utility, since the difference between x_M and x_L is relatively small. If the difference between x_M and x_L is sufficiently large, θ^{ow} is close to $\frac{1}{2}$ and $\frac{x_L}{x_M}$ is close to 0. In this case, even if overconfident women are enough to satisfy $\frac{x_L}{x_M} < \theta_{MH} < \theta^{ow}$, the social welfare can improve. The reason for this is that low type women obtain the large marital utility from the marriage to middle type men although the welfare of middle type men decreases.

5 Concluding remarks

A person often overvalues or undervalues his/her own type (charm), as early research on psychology and behavioral economics has shown. In this paper, we investigated the influence of self-confidence on a marriage market, which is a two-sided search model. Considering self-confidence, this two-sided aspect of the problem generates a significant interest. We show that the self-confidence of some single individuals (overconfidence or underconfidence) affects not only themselves but also the marital behavior of other rational singles when rational singles coexist with overconfident or underconfident singles in the marriage market. A single individual who is overconfident or underconfident is always worse off than if he/she is rational. If there are some underconfident high-type women in the marriage market, low-type women cannot marry low-type men, who primarily propose to them. On the other hand, if there are some overconfident middle-type women, low-type men cannot marry low-type women, who accept them in the case where all agents are rational. However, if there are enough underconfident women or if there are sufficiently few overconfident women in the marriage market, the social welfare increases by the existence of agents with self-confidence relative to the economy without self-confidence. It is noteworthy that, even if sex is reversed, these results are confirmed.

[31] The number of marriages of middle-type men is always raised. However, that of middle type men who marry rational middle type women is included in in the number of marriages for the second cluster. That of middle type men who marry a low type women is included in the number of marriages for the third cluster.

[32] At this time, $\frac{x_L}{x_M}$ is close to 1, and then $\frac{x_L}{x_M} > \theta^{ow}$ holds. If θ_{MH} is small but $\frac{x_L}{x_M} > \theta_{MH} > \theta^{ow}$, overconfident women decreases the social welfare relative to the benchmark economy. In this case, though the welfare of middle type men increases, low type women cannot obtain large marital utility.

This framework can be expanded. The model can be enriched to incorporate continuous types and to allow learning of agents with self-confidence. Our results also apply to the labor markets. In the labor market context, workers and employers seek each other as working partners. Moreover, workers' productivity may differ according to individuals' ability, and firms' productivity differ according to their capital.

All of our results require the assumption that there is egocentric self-confidence in the marriage market. Especially, although it is possible to theoretically analyze underconfidence in our model, overconfidence is a more popularly studied biased behavior than underconfidence in economic and psychological literature. By investigating experimental evidence of self-confidence in two-sided search, we will have a realistic marital behavior and a situation with self-confidence.

Appendix

Proof of Lemma 1 It is noteworthy that a rational high-type agent turns down a middle-type and a low-type agent from $x_M < R_H^*$. First, let us consider whether a middle-type man is willing to marry a low-type woman. His reservation utility for a low-type woman can be obtained as follows: if a middle-type man turns down a low-type woman, i.e., $x_M/r \geq V_M > x_L/r$, his discounted lifetime utility is

$$rV_M^1 = \alpha\lambda_H\theta_{HM}\left(\frac{x_H}{r} - V_M^1\right) + \alpha\lambda_M\left(\frac{x_M}{r} - V_M^1\right). \tag{14}$$

In contrast, if he proposes to a low-type woman, i.e., $V_M \leq x_L/r$,

$$rV_M^2 = \alpha\lambda_H\theta_{HM}\left(\frac{x_H}{r} - V_M^2\right) + \alpha\lambda_M\left(\frac{x_M}{r} - V_M^2\right) + \alpha\lambda_L\left(\frac{x_L}{r} - V_M^2\right).$$

Hence, $V_M^1 > V_M^2$ means that

$$\frac{\alpha\lambda_H\theta_{HM}x_H + \alpha\lambda_M x_M}{r + \alpha\lambda_M + \alpha\lambda_H\theta_{HM}} \equiv R_{Mm}^{U1} > x_L.$$

If $x_L \geq R_{Mm}^{U1}$, a middle-type man accepts a low-type woman. Compared with the benchmark case for the reservation utility level of a middle-type man for a low-type woman,

$$R_M^* - R_{Mm}^{U1} = \alpha\frac{\left(-\alpha\lambda_M\left(x_H - x_M\right) - rx_H\right)\lambda_H\theta_{HM}}{\left(r + \alpha\lambda_H\theta_{HM}\right)\left(r + \alpha\lambda_M\right)} < 0$$

is obtained from $x_H > x_M$.

Next, we consider whether a middle-type man wishes to marry a middle-type woman. When a middle-type man turns down a middle-type woman ($x_M/r < V_M$), his value function is

$$rV_M^3 = \alpha\lambda_H\theta_{HM}\left(\frac{x_H}{r} - V_M^2\right).$$

If a middle-type man proposes to a middle-type woman ($x_M/r > V_M \geq x_L/r$), his value function becomes (14). Hence, from $V_M^3 > V_M^1$,

$$\frac{\alpha\lambda_H\theta_{HM}}{(r+\alpha\lambda_H\theta_{HM})}x_H \equiv R_{Mm}^{U2} > x_M.$$

If $x_M \geq R_{Mm}^{U2}$, a middle-type man accepts a middle-type woman. $R_{Mm}^{U2} > R_{Mm}^{U1}$ as $V_M^3 > \frac{x_M}{r} > V_M^1$. □

Proof of Proposition 2 As a middle-type woman is rejected by a middle-type man, her option is to marry or to reject a low-type man. If she turns down a low-type man, her expected discounted lifetime utility is $V_m = 0$. Therefore, a middle-type woman proposes to a low-type man with

$$rV_{Mw} = \alpha\lambda_L\left(\frac{x_L}{r} - V_{Mw}\right).$$

Once a low-type man knows that he can be accepted by a middle-type woman, his option is to marry or to turn down a low-type woman. His optimal strategy can be obtained as follows: if he proposes to a low-type woman, that is, $V_{Lm}^1 \leq \frac{x_{Lw}}{r}$, his value function is

$$rV_{Lm}^1 = \alpha\lambda_M\left(\frac{x_{Mw}}{r} - V_L\right) + \alpha\lambda_L\left(\frac{x_L}{r} - V_L\right).$$

Conversely, if he turns down a low-type woman ($V_{Lm}^2 > \frac{x_{Lw}}{r}$) ,

$$rV_{Lm}^2 = \alpha\lambda_M\left(\frac{x_{Mw}}{r} - V_L\right).$$

Then, the reservation utility of a low-type man for a low-type woman is $R_{Lm}^U \equiv \frac{\alpha\lambda_M x_{Mw}}{r+\alpha\lambda_M} = R_M^*$. As $R_M^* > x_L$, $R_{Lm}^U > x_L$. Therefore, a low-type man turns down a low-type woman. As a result, low-type women are not accepted by any man and therefore never marry. It is noteworthy that, when all single individuals are rational (unbiased), there is no reservation utility of a low-type man. □

Proof of Lemma 2 The reservation utility of a middle-type man for a low type can be calculated as follows: if a middle-type man turns down a low-type woman ($V_M > x_L/r$), his value function becomes

$$rV_M^1 = \alpha\lambda_M(1-\theta_{MH})\left(\frac{x_M}{r} - V_M^1\right).$$

Conversely, when a middle-type man proposes to a low-type woman ($V_M \leq x_L/r$),

$$rV_M^2 = \alpha\lambda_M(1-\theta_{MH})\left(\frac{x_M}{r} - V_M^2\right) + \alpha\lambda_L\left(\frac{x_L}{r} - V_M^2\right)$$

holds. Hence, we have his reservation utility level for declining a low-type woman,

$$\frac{\alpha\lambda_M (1-\theta_{MH}) x_M}{r+\alpha\lambda_M (1-\theta_{MH})} \equiv R_M^O. \tag{15}$$

Compared to the benchmark case, we have

$$R_M^O < R_M^* = \frac{\alpha\lambda_M x_M}{r+\alpha\lambda_M}.$$

□

Proof of Proposition 3 As a low-type woman can be accepted by a middle-type man, she decides whether to accept or reject a low-type man. In this case, following the same procedure as above, the reservation utility of a low-type woman for a low-type man is

$$R_{Lm}^O \equiv \frac{\alpha\lambda_M x_M}{r+\alpha\lambda_M}.$$

It is noteworthy that $R_{Lm}^O = R_M^*$. As $x_L < R_M^*$, $x_L < R_{Lm}^O$ holds. Hence, a low-type woman turns down a low-type man. □

Proof of Lemma 3 From (5) and (7),

$$T^U - T^* = \alpha N \left[\lambda_L (\lambda_M - \lambda_L) + \lambda_M (\lambda_H \theta_{HM} - \lambda_M)\right] \tag{16}$$

holds. Then,

$$T^U \gtreqless T^* \Leftrightarrow \theta_{HM} \gtreqless \frac{\lambda_M^2 - \lambda_L\lambda_M + \lambda_L^2}{\lambda_H\lambda_M} = \theta^{UT}.$$

□

Proof of Proposition 4 From (6) and (8), we have

$$\begin{aligned} W^* - W^U &= \alpha\lambda_M \left[\lambda_M 2x_M - \lambda_H\theta_{HM}(x_H + x_M)\right] N \\ &\quad + \alpha\lambda_L \left[\lambda_L 2x_L - \lambda_M (x_L + x_M)\right] N. \end{aligned} \tag{17}$$

Hence,

$$W^U \gtreqless W^* \Leftrightarrow \theta_{HM} \gtreqless \frac{\lambda_L^2 2x_L - \lambda_L\lambda_M (x_L + x_M) + \lambda_M^2 2x_M}{\lambda_H\lambda_M (x_H + x_M)} = \theta^{UW}.$$

holds. □

Proof of Corollary 1 Substituting $\lambda_L = \lambda_M = \lambda_H = \lambda$ into (16), we have $T^* - T^U = \alpha N\lambda^2 (1-\theta_{HM}) > 0$. For social welfare, we have $W^* - W^U = \lambda^2\alpha N[(x_L + x_M) - (x_H + x_M)\,\theta_{HM}]$ from (17). Therefore, $W^* \lesseqgtr W^U \Leftrightarrow \theta_{HM} \gtreqless \frac{x_L + x_M}{(x_H + x_M)}$ holds. □

Proof of Lemma 4 From (5) and (11),

$$T^* - T^O = \alpha N \left[\left(\lambda_M \theta_{MH} - \lambda_L \right) \lambda_M + \lambda_L^2 \right] \tag{18}$$

holds. Then, we have

$$T^* \gtreqless T^O \Leftrightarrow \theta_{MH} \gtreqless \frac{\lambda_L \left(\lambda_M - \lambda_L \right)}{\lambda_M^2}.$$

□

Proof of Proposition 5 From (6) and (12),

$$W^* - W^O = \alpha N \left\{ 2x_L \lambda_L^2 + \left[2x_M \lambda_M \theta_{MH} - \left(x_L + x_M \right) \lambda_L \right] \lambda_M \right\}. \tag{19}$$

Hence,

$$W^* \gtreqless W^O \Leftrightarrow \theta_{MH} \gtreqless \frac{\lambda_L \lambda_M \left(x_L + x_M \right) - 2x_L \lambda_L^2}{2x_M \lambda_M^2}$$

is obtained. □

Proof of Corollary 2 Substituting $\lambda_L = \lambda_M = \lambda_H = \lambda$ into (18), we have $T^* - T^O = \alpha N \theta_{MH} > 0$. For social welfare, we have $W^* - W^O = \alpha N[2x_L + 2x_M \theta_{MH} - (x_L + x_M)]$ from (19). Therefore, $\theta_{HM} \gtreqless \frac{x_M - x_L}{2x_M} \Leftrightarrow W^* \gtreqless W^O$ holds. □

Example 1 The following examples illustrate that the necessary and sufficient condition for $\theta^{UW} \in (0, 1)$ can have meaningful interpretations.

- If the utility of marrying middle-type agents x_M is approximately fourteen times smaller than the utility of marrying low-type agents x_L ($x_M < (7 + 4\sqrt{3}) x_L$), the numerator of θ^{UW} is positive independently of the values of λ_H, λ_M, and λ_L.
- If there are not fewer middle-type agents than low-type agents ($\lambda_M \geq \lambda_L$), the numerator of θ^{UW} becomes positive with any constant x_H, x_M, and x_L.
- If the distribution of types is $\lambda_H > \lambda_M > \lambda_L$ with any constant x_i $(i = H, M, L)$, $\theta^{UW} \in (0, 1)$ holds.

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