Should we discount the far-distant future at its lowest possible rate?

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Abstract

In this paper, we elaborate on an idea initially developed by Weitzman (1998) that justifies taking the lowest possible discount rate for far-distant future cash flows. His argument relies on the arbitrary assumption that when the future rate of return of capital (RRC) is uncertain, one should invest in any project with a positive expected net present value. We examine an economy with a risk-averse representative agent facing an uncertain evolution of the RRC. In this context, we characterize the socially efficient stochastic consumption path, which allows us in turn to use the Ramsey rule to characterize the term structure of socially efficient discount rates. We show that Weitzman's claim is qualitatively correct if shocks on the RRC are persistent. On the contrary, in the absence of any serial correlation in the RRC, the term structure of discount rates should be flat.

Keywords: Discount rate, term structure, certainty equivalent rate, Ramsey rule, sustainable development.

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1 Introduction

Important policy problems dealing with public investments that affect the distant future are blossoming. The problem of how many percentage points of growth we should sacrifice today in order to reduce the intensity of global warming is a typical example. Another example comes from the aging of the first generation of nuclear reactors in developed countries, yielding the question of the future of this technology of producing energy. The critical aspect of the question is how do we evaluate the cost of nuclear wastes disposals which will have to be checked for several thousands years. Other examples are the problem of deforestation, genetic manipulations, biodiversity and the like.

There are several aspects to these problems. In general, the future benefits of sacrifices that we could make today are uncertain. Also, most often, decisions are irreversible. The uncertainty affecting the future benefits and the irreversibility problem already got a correct treatment in the economics literature. But by far the most important aspect is the intertemporal dimension of costs and benefits. As is well-known, one should take into account of the date at which a cost or a benefit occurs by discounting it. Because discounting is exponential, a small change in the discount rate has a large effect on the discounted value of a cash flow occurring in a distant future.

The community of economists has no clear position on which discount rate should be used for such long time horizons for which financial markets do not provide any pricing rule. There is a tendency among decision makers to choose a discount rate for the very long term that is smaller than the one used to discount cash flows occurring in the short term. For example, since 2005, the French public institutions are required to use a 4% rate per year to discount cash flows up to thirty years, and to use a 2% rate for longer horizons. A similar term structure of discount rates is also used in the U.K..

Several recent papers provided economic arguments for a decreasing term structure of discount rates. Most of them rely on the Ramsey (1928) rule. Extending it to the case of an uncertain growth, the socially efficient discount rate has three components. The first one is the rate of pure preference for the present. The second one is a positive wealth effect: because one believes that one will consume more in the future, decreasing marginal utility implies that one more unit of consumption in the future has a smaller value than the same additional unit consumed today. The third component is a negative precautionary effect: because future consumption is uncertain, prudence should induce us to invest more for the future by reducing the discount rate. The term structure of the discount rates is decreasing if, when considering a longer time horizon, the wealth effect increases less than proportionally, or if the precautionary effect increases more than proportionally. In the most standard case with a power utility function and without any serial correlation in the growth rate of consumption, it is well known that the term structure of discount rates is flat, which means that the wealth effect and the precautionary effect are proportional to the time horizon. Gollier (2002) shows that the term structure is decreasing if the relative risk aversion of the representative agent is decreasing. Efforts have also recently been done to relax the assumption of the absence of serial correlation in the consumption growth process. Weitzman (2007) considers the case of an unknown volatility of the process. This uncertainty magnifies the long term risk, and the corresponding precautionary effect for long maturities, yielding a decreasing term structure. Gollier (2007) shows that the persistence of shocks on consumption growth rates justifies using a smaller discount rate for more distant cash flows.

Weitzman (1998, 2001) provided an interesting alternative argument that could be summarized as follows.[1] Suppose that the (continuously compounded) rate θ of return on capital for the next T periods will be known tomorrow, but is uncertain today. Consider a risk-neutral agent who must determine today whether to invest in a risk free project that yields a single payoff x at date t per euro invested today. He should do so if and only if the project has a positive Expected Net Present Value (ENPV), i.e., if

$$ENPV = -1 + xEe^{-\widetilde{\theta}t} \geq 0, \tag{1}$$

where $\widetilde{\theta}$ is the random variable describing the uncertainty of the rate of return on capital. The argument underlying the above condition is standard: As soon as the interest rate $\widetilde{\theta} = \theta$ is revealed, the investor can cash the future benefit of the investment on the credit market, which yields a present net benefit equaling $-1+e^{-\theta t}x$. Ex ante, the risk neutral investor would invest in the project if the expected net present benefit is positive. This is equivalent to using a term structure $r_p(t)$ of discount rates such that

$$r_p(t) = -\frac{1}{t}\ln Ee^{-\widetilde{\theta}t}. \tag{2}$$

[1]This argument was also developed by Pazner and Razin (1974).

It is easy to check that $r_p(t)$ is less than $E\widetilde{\theta}$, and that it tends to its lowest possible rate for large t.[2] This is the main message in Weitzman (1998). It corresponds to the idea that a risk-neutral agent likes the randomization of the per-period interest rate at which he will borrow. Because the net present value of a cash flow is a convex function of the interest rate, what he will be able to borrow on average will be larger. This raises his willingness to invest in the project. Equivalently, it lowers the critical internal rate of return at which he would like to invest in the project. Newell and Pizer (2003), Groom, Koundouri, Panipoulou and Pantelides (2007), and Gollier, Koundouri and Pantelides (2008) have estimated socially efficient discount rates based on equation (2).

As initially observed by Pazner and Razin (1975), and then by Gollier (2004), Hepburn and Groom (2007) and Buchholz and Schumacher (2008), rather than cashing at date 0 the future benefit on the credit market, the agent could transfer the cost of the investment to date t by borrowing it on the credit market. If θ is the interest rate over the period, the net future benefit would equal$-e^{\theta t}+x$. Ex ante, the risk neutral agent would invest in the project if the expected future net value is positive, i.e. if $-1+xe^{-r_f(t)t}$ is positive, where

$$r_f(t)=\frac{1}{t}\ln Ee^{\widetilde{\theta}t}. \tag{3}$$

This is the discount rate that is sustained the Expected Net Future Value (ENFV) approach. It is easy to check that $r_f(t)$ is larger than $E\widetilde{\theta}$, and that it tends to its largest possible rate when t tends to infinity. Clearly, using the ENFV and ENPV approaches yield opposite results, except in the special case of certainty. The point here is that one needs to find economic justifications to the evaluation model used in a context where the future interest rate is random. There is a huge literature on the term structure of interest rates that examines exactly this question. But most results in this literature rely on arbitrage, a technique that is mostly useless when considering distant time horizons.

In this paper, we reconcile the two approaches, and we link them to the Ramsey rule. To do this, we introduce risk aversion. We show that the present value approach and the future value approach lead to the same term structure of discount rate as soon as one takes care of the relative riskiness of the different risk transfer through time that are implicit in the

[2] See for example Hepburn and Groom (2007, Proposition 6).

two approaches. In fact, if the consumption/saving strategy is optimized, the investor should be indifferent to the allocation of the investment opportunity risk into consumption risk in different dates. This optimality condition implies that rules (2) and (3) adapted to risk aversion by using risk-neutral expectations will generate the same discount rates.

In short, we solve the puzzle by endogenizing the consumption path. The problem is that the optimal consumption path depends upon the return on capital, and upon the uncertainty that is associated to it. Once the growth process is derived from the exogenously given stochastic process for the return on capital, the term structure of socially efficient discount rates can be computed by the standard marginalist technique used to obtain the Ramsey pricing rule. Thus, this paper provides a general equilibrium foundation for the term structure of discount rates. It shows that the shape of the term structure depends heavily upon the persistence of the shocks on $\widetilde{\theta}$.

Other authors have attempted to solve the "Weitzman-Gollier puzzle". Hepburn and Groom (2007) showed that when the investor is risk neutral, the relevant problem is not about the allocation of risk through time, but rather about the choice of the evaluation date, which is arbitrary by nature. In their conclusion, they recognize that introducing risk aversion into the picture would provide a road to solve the puzzle. This is exactly what is done in this paper. Buchholz and Schumacher (2008) also recognize the necessity to introduce risk aversion into the analysis. They propose an interesting criterion in which investing at the discount rate yields the same expected utility than investing at the uncertain rate of return of capital: $u(\exp(r_{bs}(t)t)) = Eu(\exp(\widetilde{\theta}t))$. They conclude that if risk aversion is large enough, r_{bs} is decreasing with t. Our approach differs much from Buchholz and Schumacher's one because we do not assume that the benefit of the investment is consumed at the terminal date, and because we use the more standard marginalist approach to asset pricing.

2 The model

We consider the standard Discounted Expected Utility model with the following social welfare function:

$$W = \sum_{t=0} e^{-\delta t} Eu(c_t). \tag{4}$$

The utility function u on consumption is assumed to be three times differentiable, increasing and concave. Let c_t denote consumption at date t. At this stage, it is assumed to be exogenous. The rate of pure preference for the present is δ. Consider a marginal risk-free investment at date 0 which generates a single benefit e^{rt} at date t per euro invested at date 0. At the margin, investing in this project has the following impact on welfare:

$$\Delta W = e^{-\delta t} e^{rt} E u'(c_t) - E u'(c_0). \tag{5}$$

Because ΔW is increasing in r, there exists a critical rate of return denoted r_t, such that $\Delta W = 0$ for $r = r_t$. Obviously, r_t is the socially efficient discount rate, which satisfies the following standard pricing formula:

$$r_t = \delta - \frac{1}{t} \ln \frac{E u'(c_t)}{E u'(c_0)}. \tag{6}$$

This consumption-based pricing formula is universal. It must hold as soon as the Discounted Expected Utility model is assumed, independent of the structure of the risk affecting the consumption process, and of the structure of the economy.

If financial markets would be frictionless and efficient, r_t would be the equilibrium interest rate associated to maturity t. This formula is the standard asset pricing formula for riskfree bonds (See for example Cochrane (2001)). The Ramsey (1928) rule is obtained by assuming that $u(c) = c^{1-\gamma}/(1-\gamma)$, where γ is the index of relative risk aversion, and $c_t = c_0 e^{gt}$, where g is the constant growth rate of consumption. It yields $r_t = \delta + \gamma g$. The larger the growth rate of consumption, the smaller the future marginal utility of future consumption, the larger the socially efficient discount rate. This wealth effect is proportional to γ, which measures the speed at which marginal utility goes down when consumption increases.

This rule can be extended to the case of uncertainty. If $\ln c_{t+1} - \ln c_t$ is normally distributed with constant mean μ_c and volatility σ_c, one can prove that (see for example Gollier (2007))

$$r_t = \delta + \gamma g_c - 0.5\gamma(\gamma + 1)\sigma_c^2, \tag{7}$$

where $g_c = \mu_c + 0.5\sigma_c^2$ is the expected growth rate of consumption. The third term in the right-hand side of this extended Ramsey rule is the precautionary effect. The larger the uncertainty on future consumption, the larger the expected marginal utility of future consumption (because u' is convex), the smaller the socially efficient discount rate. As in the standard Ramsey rule, the term structure is flat in this case. The main objective of this paper is to reconcile this extended Ramsey rule to the Weitzman's rule (2).

3 Linking consumption growth, return on capital, and discount rates

3.1 The case of a permanent shock on the productivity of capital

In order to endogenize the growth process of consumption, let us suppose that the production function exhibits constant marginal productivity of capital: $Y_t = e^{\theta_t} K_{t-1}$. At date 0, the rate of return on capital undergoes a unique permanent shock in such a way that the it is permanently set to $\theta_t = \theta$ for $t = 1, ..., T$.[3] In this simple framework which covers the case considered by Weitzman (1998), the characterization of the optimal consumption path must be performed in two stages by backward induction. Let us suppose that the capital available at date 0 is K_0 and that the realization of $\widetilde{\theta}$ is θ. Conditional to $\widetilde{\theta} = \theta$, the representative agent solves the following program:

$$\max_{c_0,...,c_T} \sum_{t=0}^{T} e^{-\delta t} u(c_t)$$

$$\begin{aligned} s.t.\ \ K_t &= e^{\theta} K_{t-1} - c_{t-1} \geq 0 \text{ for all } t = 1, ..., T. \\ K_{T+1} &\geq 0. \end{aligned}$$

As is well-known, the Euler equation associated to this program is written as

$$u'(c_t) = \xi(\theta) e^{(\delta-\theta)t}. \tag{8}$$

where $\xi(\theta) = u'(c_0(\theta))$ is the Lagrangian multiplier associated to the intertemporal budget constraint. It is a positive function of θ. Condition (8) has an important meaning for our purpose. It states that the representative agent is indifferent at the margin about how to allocate an additional unit of wealth obtained at $t = 0$ into consumption along her remaining lifetime.

[3] We assume that T is finite. The case of infinite horizon would generate the same results, but it would require that the support of the distribution of $\widetilde{\theta}$ be in $]-\infty, \delta[$ in order to guarantee the boundedness of the solution.

Ex ante, it means that she is indifferent about how to allocate the capital risk generated by a marginal investment into a temporal consumption risk. This will imply that the choice of the investment evaluation date will be irrelevant to determine the investment value, contrary to what Hepburn and Groom (2007) obtained in a framework in which the consumption path is not optimized.

Using the pricing formula (6), it implies that the socially efficient discount rate after $\widetilde{\theta} = \theta$ is realized – that is for all $t > 0$ – equals $r_t = \delta - t^{-1}\ln(\xi e^{(\delta-\theta)t}/\xi) = \theta$.Without surprise, in the absence of any uncertainty ex post, the socially efficient discount rate equals the rate of return on capital. We are interested in characterizing the term structure that would prevail a few minutes before $t = 0$. At that time, $u'(c_t)$ and $u'(c_0)$ are random. Using the pricing formula (6) again yields

$$r_t = -\frac{1}{t}\ln\frac{E\xi(\widetilde{\theta})e^{-\widetilde{\theta}t}}{E\xi(\widetilde{\theta})}, \tag{9}$$

or, equivalently, using $\xi(\theta) = u'(c_0(\theta))$,

$$r_t = -\frac{1}{t}\ln E_0 e^{-\widetilde{\theta}t}, \tag{10}$$

where E_0 is a risk-neutral expectation operator defined in such a way that $E_0 f(\widetilde{\theta}) = Eu'(c_0(\widetilde{\theta}))f(\widetilde{\theta})/Eu'(c_0(\widetilde{\theta}))$ for all functions f. This characterization of the term structure of discount rates is not far from the one recommended by Weitzman (1998), i.e., from equation (2). In particular, the discount rate r_t is decreasing with the time horizon, and it tends to its smallest possible level when t tends to infinity.

Observe that, by replacing $\xi(\theta)$ by $u'(c_0(\theta))$, we have used date $t = 0$ as the evaluation date. Following Hepburn and Groom (2007) who considered the case of risk neutrality, one could alternatively use any date t between 1 and T, using the fact that $\xi(\theta) = u'(c_t(\theta))\exp((\theta - \delta)t)$. Equation (9) can then alternatively be rewritten as

$$r_t = \frac{1}{t}\ln\frac{Eu'(c_t(\widetilde{\theta}))e^{\widetilde{\theta}t}}{Eu'(c_t(\widetilde{\theta}))}. \tag{11}$$

Notice that $Eu'(c_t(\widetilde{\theta}))e^{\widetilde{\theta}t}/Eu'(c_t(\widetilde{\theta}))$ is the certainty equivalent increase in consumption at date t that yields the same increase in welfare than a sure unit payoff at date 0 that is invested in the economy until t. Condition (11) translates this finding into a socially efficient interest rate. It can be rewritten as

$$r_t = \frac{1}{t} \ln E_t e^{\widetilde{\theta} t}, \tag{12}$$

where E_t is an alternative risk-neutral expectation operator defined in such a way that $E_t f(\widetilde{\theta}) = Eu'(c_t(\widetilde{\theta}))f(\widetilde{\theta})/Eu'(c_t(\widetilde{\theta}))$ for all functions f. Whereas condition (10) looks like the Weitzman's ENPV rule (2), the equivalent rule (12) looks like the alternative ENFV rule (3). However, there is one important difference in this alternative rule. In equation (12), the risk-neutral probability distribution is a function of t, whereas it is not in equation (10). This is the nexus of the resolution of the Weitzman-Gollier puzzle. When one uses the present value approach, one always evaluates the investment opportunity risk borne by the agent in terms of an equivalent consumption risk at the *same* date, date $t = 0$. This implies that the risk-neutral pricing rule (10) is independent of t. When one considers the future value approach, one evaluates the investment opportunity risk in terms of an equivalent consumption risk at *different* dates. Because of the consumption risk at date t and the investment risk have a degree of correlation that is a function of t, the risk-neutral pricing rule (12) is sensitive to t.

It must be stressed that both the present value formula (10) and the future value formula (10) are right, and lead to the same term structure of discount rate. In a sense, contrary to our conclusion in Gollier (2004), both Weitzman (1998) and Gollier (2004) are right, as long as one takes care of risk aversion, and the right way to distort risk-neutral probabilities as a function of the evaluation date. However, because formula (10) relies on a single risk-neutral probability distribution, it is easier to use.

The only difference between the correct formula (10) and the Weitzman rule (2) is that the expectation must be computed using risk-neutral probabilities. Newell and Pizer (2003), Groom, Koundouri, Panipoulou and Pantelides (2007), and Gollier, Koundouri and Pantelides (2008) have used a certainty equivalent formula à la Weitzman (1998) to estimate discount rates for different time horizons based on an econometric estimation of the dynamic process of short term interest rates. An interesting question is to determine whether Weitzman's formula (2) overpredicts the socially efficient discount rate characterized by (10). An ingredient for the following useful Lemma is relative risk aversion, which is denoted $\gamma(c) = -cu''(c)/u'(c)$.

Lemma 1 *Suppose that the productivity of capital is subject to a single random shock that is permanent. It occurs at date $t = 0$. The consumption at date $t = 0$ is an increasing (resp. decreasing) function of the observed rate of return on capital if relative risk aversion is uniformly larger (resp. smaller) than unity.*

Proof: See the Appendix.

When relative risk aversion is larger than 1, the wealth effect generated by an increase in the productivity of capital is larger than the substitution effect, so that consumption at date 0 is increased. Now, observe that this implies that $\xi(\theta) = u'(c_0(\theta))$ is decreasing in θ. It yields

$$\widehat{E}_T \widetilde{\theta}^{-t} = \frac{E\xi(\widetilde{\theta})e^{-\widetilde{\theta}t}}{E\xi(\widetilde{\theta})} \geq \frac{E\xi(\widetilde{\theta})Ee^{-\widetilde{\theta}t}}{E\xi(\widetilde{\theta})} = E\widetilde{\theta}^{-t}. \tag{13}$$

Combining equations (10) and (13), we obtain that

$$r_t \leq -\frac{1}{t} \ln Ee^{-\widetilde{\theta}t} \tag{14}$$

This means that the Weitzman's formula (2) overpredicts the socially efficient discount rate when relative risk aversion is larger than 1. The other cases presented in Proposition 1 follow easily. Weitzman's formula is exact in the limit case of a logarithmic utility function.

Proposition 2 *Suppose that the productivity of capital is subject to a single random shock that is permanent. It occurs at date $t = 0$. Prior to that date, the term structure of discount rates is decreasing and tends to the smallest possible value when the maturity tends to infinity. Moreover, equation (2) characterizes the socially efficient discount rate if and only if the representative agent is logarithmic. It overpredicts (resp. underpredicts) the socially efficient discount rate if relative risk aversion is uniformly larger (resp. smaller) than 1.*

The intuition of the last result in this proposition is as follows: When relative risk aversion is larger than unity, the consumption level at date $t = 0$ is increasing in θ. It implies that the marginal utility of wealth $\xi(\theta) = u'(c_0(\theta))$ is decreasing in θ. But the net present value of the project is decreasing in θ. It implies that the additional discounted wealth generated by the investment project covaries positively with the marginal utility of wealth. Compare this situation to the case where the covariance is zero, as is the case with a logarithmic utility function leading to the optimality of Weitzman's rule. Obviously, this positive correlation provides an additional incentive for investing in the project. It follows that the discount rate is reduced. This is why the Weitzman's rule overpredicts the optimal discount rate when relative risk aversion is larger than unity.

3.2 The case of unpredictable future productivity of capital

Let us now consider the polar case in which there is no serial correlation in the rate of return on capital. More precisely, we assume that $Y_t = e^{\widetilde{\theta}_t} K_{t-1}$, where $\widetilde{\theta}_1$, $\widetilde{\theta}_2, ...$ are independent and identically distributed. In this section, we consider for simplicity the case of infinite time. The optimal consumption path is obtained by solving the following Bellman equation:

$$v(K) = \max_c \; u(c) + e^{-\delta} E v(e^{\widetilde{\theta}}(K - c)).$$

This yields the consumption policy function $c = c(K)$. The first-order condition associated to this program can be written as

$$u'(c) = e^{-\delta} E e^{\widetilde{\theta}} v'(e^{\widetilde{\theta}}(K - c)).$$

As is well-known, this problem has an analytical solution if we assume that $u(c) = c^{1-\gamma}/(1-\gamma)$. In that case, we know that $v(K) = hK^{1-\gamma}/(1-\gamma)$ and

$$\frac{c(K)}{K} = C = 1 - \left(e^{-\delta} E e^{(1-\gamma)\widetilde{\theta}}\right)^{\frac{1}{\gamma}}.$$

In this context, the consumption is proportional to wealth. It implies that the wealth accumulation process is such that

$$K_{t+1} = e^{\widetilde{\theta}_{t+1}}(1 - C)K_t,$$

which implies in turn that

$$\log c_{t+1} - \log c_t = \log(1 - C) + \widetilde{\theta}_{t+1}. \tag{15}$$

Suppose that $\widetilde{\theta}$ is normally distributed with mean μ_θ and volatility σ_θ. It implies that $\log c_{t+1} - \log c_t$ is normally distributed with volatility $\sigma_c = \sigma_\theta$ and mean

$$\begin{aligned} \mu_c &= \log(1-C) + \mu_\theta \\ &= \frac{\mu_\theta - \delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_\theta^2. \end{aligned}$$

We know that the pricing rule (6) has an analytical expression (7) in that case. Replacing μ_c and σ_c in this formula by their expression derived from the underlying stochastic process of capital productivity yields

$$r_t = \mu_\theta + (0.5 - \gamma)\sigma_\theta^2. \tag{16}$$

The following proposition sums up our finding.

Proposition 3 *Suppose that the rate of return on capital per period follows a random walk: $\widetilde{\theta}_1$, $\widetilde{\theta}_2$, ... are independent and normally distributed with mean μ_θ and volatility σ_θ. Suppose also that relative risk aversion is constant. It implies that changes in log consumption are independent and normally distributed, and yielding in turn that the socially efficient discount rate is independent of the time horizon.*

Compared to Proposition 2, this proposition exhibits a completely different shape for the term structure. The crucial difference between the two set of assumption is about the persistence of the shocks on the productivity of capital. When shocks are not persistent as assumed in this section, they have only temporary effects on the growth rate of consumption, as seen in equation (15). It implies that the precautionary effect per period remains constant with the time horizon, yielding a flat term structure. On the contrary, when shocks are permanent, consumption paths contingent to the size of the shock diverge exponentially, as seen from equation (8). This tends to magnify the long term risk relative to the short term one. This magnifies the precautionary effect for long horizons, yielding a decreasing term structure.

4 Conclusion

Following Weitzman (1998), we considered an economy in which the exogenous source of uncertainty affects the rate of return on capital. From the stochastic process that governs this rate, we derived the optimal stochastic process for the consumption of the representative agent. The final aim of this exercise was to use this consumption process to estimate the shape of the term structure of socially efficient discount rates, by using the standard consumption-based pricing rule for interest rates. This paper builds a bridge between two seemingly unrelated branches of the literature on discount rates: the one based on consumption growth, and the one based on the productivity of capital. For example, in the absence of any uncertainty on the productivity of capital, this methodology yields the standard Ramsey rule which is consumption-based, and the discount rate equals the rate of return on capital.

In order to explore the context implicitly considered by Weitzman (1998), we considered an economy facing a single permanent shock on the productivity of capital. We have shown that prior to the realization of the shock, the term structure of discount rate is decreasing and tends to the lowest possible rate of return on capital, as claimed by Weitzman. However, our model derives this result from a well specified social welfare function. However, the Weitzman's formula is correct only is the representative agent has a logarithmic utility function. When relative risk aversion is larger than unity, the Weitzman's formula overestimates the true socially efficient discount rates.

Our main contribution is to show that Weitzman's results rely heavily on the assumption that shocks on the rate of return on capital are permanent. We considered alternatively a model in which shocks are only transitory. In that alternative context, the term structure is flat. In that case, one should not discount the far-distant future at its lowest possible rate.

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Appendix: Proof of Lemma 1

Using first-order condition (8), we can rewrite the intertemporal budget constraint as follows:

$$\sum_{t=0} e^{-\theta t}\phi\left(\xi e^{(\delta-\theta)t}\right) = K_0, \tag{17}$$

where function ϕ is defined as $\phi(x) = u'^{-1}(x)$. Observe that risk aversion implies that ϕ is decreasing in its argument. Fully differentiating condition (17) with respect to θ yields

$$\xi'(\theta) = \frac{\sum_{t=0}\left[\xi t e^{(\delta-2\theta)t}\phi'(\xi e^{(\delta-\theta)t}) + te^{-\theta t}\phi(\xi e^{(\delta-\theta)t})\right]}{\sum_{t=0} e^{(\delta-2\theta)t}\phi'(\xi e^{(\delta-\theta)t})} \tag{18}$$

Because the denominator in equation (18) is negative, the sign of ξ' is opposite to the sign of its numerator. But this expression can be rewritten as

$$\sum_{t=0} te^{-\theta t}\phi(\xi e^{(\delta-\theta)t})\left[\xi e^{(\delta-\theta)t}\frac{\phi'(\xi e^{(\delta-\theta)t})}{\phi(\xi e^{(\delta-\theta)t})} + 1\right]. \tag{19}$$

Observe now that, by the definition of ϕ, we have that $x\phi'(x)/\phi(x) = -\left[\gamma(\phi(x))\right]^{-1}$, where $\gamma(\phi) = -\phi u''(\phi)/u'(\phi)$ is the index of relative risk aversion of u. We conclude that if γ is uniformly larger (resp. larger) than 1, the numerator in (18) is positive, so that ξ is decreasing (resp. increasing) in θ. Finally, observe that

$$c_0(\theta) = \phi(\xi(\theta)).$$

The result follows from the fact that ϕ is decreasing.■