Growth and convergence in a model with renewable and nonrenewable resources *

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Abstract

This paper analyzes the transitional dynamics in a model of economic growth with endogenous technological change and two alternative sources of energy: renewable and non-renewable resources. The conditions for the existence and saddle point property of the steady state are given. Finally, we present the estimation results on the data consisting of R&D energy, non-renewable energy consumption and renewable energy consumption.

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1 Introduction

Quite a number of models have been developed to analyze the impact of the pollution generated by using non-renewable resources which is considered as a standard energy (e.g., Barbier, 1999, Grimaud and Rouge, 2004, 2005). However, as Grimaud and Rouge (2006) have indicated, there are other resources which are renewable and generate less pollution (such as solar energy, hydrogen fuel cell, etc.) which should be taken into account. Moreover, because of concerns about global climate change and its impact on human well-being, it is important to study the mechanisms for reducing energy intensity and notably reducing energy consumption from non-renewable resources. Therefore, it is necessary to explore the possible contribution of environment and growth theory in providing models where crucial variables such as physical capital, labor, consumption, and technological level interact with renewable and non-renewable resources.

An important question that has captivated the attention of environmental economists is whether growth is sustainable in the presence of natural resource scarcity. The issue concerns both academics and public decision-makers, notably in the current context of increasing energy demands and the depletion of fossil fuels expected in the near future. The new growth theory gives the answer that with some technological properties, growth may be sustained in the long-run even if resource stock is finite. Examples include Barbier (1999) with a Romer-Stiglitz model, Aghion and Howitt (1998) and Grimaud and Rouge (2005) using the Schumpeterian models, in which they underlined that endogenous technological change can sufficiently sustain growth in the long run. This conclusion was also found in numerous studies in the literature and is successfully summarized in Smulders (2005):

> '...that a society willing to spend enough on R&D can realize a steady state of technological change sufficient to offset the diminishing returns from capital-resource substitution and sustain long-run growth.'

However, these studies do not pay enough attention to the empirical justification of their theoretical results. This is particularly due to the difficulty of building a testable version from a complex theoretical framework.

A recent strand of literature concerns the relationship between the economy and the environment. Studies are motivated by the search for empirical evidence for the proposed theoretical models. Pioneer work was done by Brock and Taylor (2004) and Alvarez et al. (2005). Brock and Taylor (2004) proposed a Solow growth model with pollution together with empirical justification. They found evidence of an environmental Kuznets curve (an inverted-U shaped relationship between emissions and income) for OECD countries. They also showed that pollution emissions have a convergence feature, like income convergence in empirical growth studies. Alvarez et al. (2005) provided a neoclassical growth model with pollution which is compatible with the empirical finding about pollution convergence in a panel of European countries.

In line with this strand of literature, our paper addresses a theoretical model of which the results may be tested with real data. We provide a framework where technological change is endogenized and the production employs the labor, physical capital and both type of renewable and non-renewable resources. We present a rigorous analysis of the existence and the characteristics of the optimal balanced growth path. Not only do we analyze the steady-state solution, we also characterize the transitional dynamics of the model and show that the balanced growth path has a saddle point of stability. Moreover, we prove the existence of an optimal solution of the social planner problem which is often assumed in the many papers in the literature (see d'Albis et al., 2008). Finally, we use the data from OECD countries to perform an empirical test based on the transitional equations of the model.

The paper is organized as follows. After the Introduction, in Section 2 we introduce a simple endogenous growth model in which the steady state is explicitly computed. The transitional dynamics show that the balanced growth path has a saddle point of stability. The existence of an optimal solution of the general model is also proven. Section 3 is devoted to the empirical results. We use data provided by the OECD and the International Energy Agency (IEA) on production, expenditures in energy R&D, and energy consumption to evaluate the theoretical model. Section 4 concludes.

2 The theoretical model

The model can be heuristically described as follows. The aggregate output produced from the labor, physical capital and two types of natural resources: non-renewable resources (e.g., fossil fuels) and renewable resources (solar, thermal, biomass, etc.). The final product is shared between consumption and accumulative physical capital. The representative consumer derives her utility from consumption. Therefore, the model correspond to a system with four state variables. As Kolstad and Krautkraemer (1993) remarked,

> '*...it is difficult or impossible to characterize the qualitative features of a dynamic model evolving three state variables without restrictive assumptions about the functional forms of important relationships...*',

we specify a set of restrictions imposed on preferences and production technology in order to have analytical results that may be tested with real data.

The production function takes the form

$$F(A, K, L_Y, Q, R) = A^{\theta} f(L_Y, K, Q, R)$$

where A, L_Y, K, Q and R represent the technological level, the labor input, physical capital input, non-renewable resources, and renewable resources. The production function includes the labor man-made capital as in Aghion and Howitt (1998), Barbier (1999), Bretschger and Smulders (2003), Grimaud and Rouge (2005), among others. Moreover, it includes the natural resources as in Grimaud and Rouge (2004).

Assume the rate of technological change is

$$\dot{A} = bA^{\phi} L_A \ , \phi > 0, \phi \leq 1, \tag{1}$$

where L_A is the labour employed for research and b is a positive constant. Normalizing the total flow of labor we have

$$L_Y + L_A = 1.$$

As in the other models of knowledge accumulation which are proposed in the literature (see Gastaldo and Ragot, 1996, Smulders, 1995, 2005, Grimaud and Rouge (2004, 2006)), we assume that human capital enters into the innovation process. The interesting points in our model is that the technological level is not only for human or physical capital but it can be also used for natural resources as the energy-saving technological[1].

The final output can be allocated between consumption and new capital accumulation

$$\dot{K} = F - C - \delta K$$

where $\delta \in (0, 1)$ is the depreciate rate.

It is standard that we assume the dynamics of stocks of renewable and non-renewable resources are

$$\dot{S}_{Q_t} = -Q_t \tag{4}$$

[1]For example, from our production function, if we assume $f(L_Y, K, Q, R)$ is the Cobb-Douglas function then

$$\begin{aligned} F(A, K, Q, R) &= A^{\theta} L_Y^{\gamma} K^{\xi} Q^{\alpha} R^{\beta}, & (2) \\ \gamma, \xi, \alpha, \beta &> 0, \gamma + \xi + \alpha + \beta = 1. & (3) \end{aligned}$$

Therefore, if $\theta = \alpha$ then $F(A, K, Q, R) = (AQ)^{\alpha} L_Y^{\gamma} K^{\xi} R^{\beta}$, technological progress used for non-renewable resource and if $\theta = \beta$, $F(A, K, Q, R) = (AR)^{\beta} L_Y^{\gamma} K^{\xi} Q^{\alpha}$, technological progress used for renewable resource.

and

$$\dot{S}_{R_t} = mS_{R_t} - R_t,\ m > 0. \tag{5}$$

The representative consumer's utility function is given by

$$U = \int_0^\infty u(C_t)e^{-\rho t}dt \tag{6}$$

From now on, as it is not necessary, the time index is not included for simplifying our notation.

3 Existence of the optimal solution

In this section, we shall prove the existence of optimal solution in the general model where the social problem is written as follows

$$\max \int_0^\infty u(C)e^{-\rho t}dt$$

subject to

$$\begin{aligned}
\dot{S}_R &= mS_R - R_t, && (7)\\
\dot{S}_Q &= -Q, && (8)\\
\dot{A} &= bL_A A^\phi, && (9)\\
\dot{K} &= A^\theta f(K, L_Y, Q, R) - C - \delta K, && (10)\\
1 &= L_A + L_Y, && (11)\\
C &\geq 0, K \geq 0, A \geq 0, 0 \leq L_Y \leq 1, 0 \leq L_A \leq 1, &&\\
& L_{Y0}, K_0, S_{Q_0}, S_{R_0}, \text{ given.} &&
\end{aligned}$$

The maximal Hamiltonian is not concave in every variable so the Arrow sufficiency theorem does not apply in our model.[2] The question of the existence of optimal solutions in infinite horizon optimization in continuous time is difficult since the feasible functions belong to a closed ball in L^1 space is not weakly compact. Moreover, even though the assumption that $u(C)$ is strictly concave and continuous, the objective function $\int_0^\infty u(C)Le^{-\rho t}dt$ is not always continuous. In our model, it is only upper-semi-continuous in the topology $\delta(L^1(e^{-\rho t}), L^\infty)$. Therefore, in many papers, the existence of solutions are assumed for the simplicity. Following the idea suggested in a recent paper of d'Albis et al. (2008), we will show that, under some assumptions, the social problem has the objective function which is uniformly bounded from above on the set of feasible control C . By using the Dunford-Pettis criterion, one can

[2]Note that θ may be greater than 1. The question of existence of optimal solution in the model with non-concave maximal Hamitonian is still a conjecture. See footnote 26, pp 93 in Groth and Schou, (2007) or Groth and Schou,(2002) for this discussion.

prove that the associated feasible sequences with C (maximizing sequences) will weakly converge in the topology $\sigma(L^1(e^{-\rho t}), L^\infty)$. This limit is feasible due to the Fatou lemma and we get the conclusion.

Recall that $L^1(e^{-\rho t})$ is the set of function f verifying the $\int_0^\infty |f(t)|\, e^{-\rho t} dt < \infty$ and the Dunfort-Pettis criterion is stated as follows:

Let B be a bounded subset of $L^1(e^{-\rho t})$. B is relative compact for the topology $\sigma(L^1, L^\infty)$ iff $\forall \varepsilon > 0, \exists \delta > 0$ such that $\int_K |f(t)|\, e^{-\rho t} dt < \varepsilon, \forall f \in B$ and $\forall K$ with $\int_K e^{-\rho t} dt < \delta$.

We make the following assumptions

H_1. The function $u(C)$ is strictly concave, strictly increasing and continuous.

H_2. Function f is continuously differentiable, concave, increasing on four arguments and

$$\begin{aligned} f(0,.) &= 0, \\ \lim_{K \to +\infty} f_K(K, 1, S_{Q_0}, S_{R_0}) &\leq 0, \end{aligned}$$

H_3. $\rho > \max\{b, m, b\theta\}$, $-\delta \leq \dot{K}/K$ and there exists a real number $\mu \neq +\infty$ such that

$$-\mu \leq \dot{S}_R / S_R.$$

H_4. $Q \leq S_{Q_0}$,$R \leq S_{R_0}$.

We say that the sequence $(Q, R, S_Q, S_R, A, K, C, L_Y, L_A)$ is feasible if it is satisfied the constraints (7)-(10) and

$$\begin{aligned} C &\geq 0, K \geq 0, A \geq 0, Q \geq 0, \\ R &\geq 0, 0 \leq L_Y \leq 1, 0 \leq L_A \leq 1. \\ & L_{Y0}, K_0, S_{Q_0}, S_{R_0}, A_0 \text{ given.} \end{aligned}$$

Proposition 1 *Under assumptions $H_1 - H_4$, the social planner problem has optimal solutions.*

Proof: The idea of proof as follows. Assumption H_1, H_2 implies objective function bounded from above.(The proof of upper-semi-continuous of objective function is similar to d'Albis et al, 2008). Thus consider feasible sequence $C(n)$ such that

$$sup \int_0^\infty u(C) e^{-\rho t} dt = \lim_{n \to \infty} \int_0^\infty u(C(n)) e^{-\rho t} dt$$

We will prove that assumptions H_2, H_3 imply that $C(n)$ and other associated feasible variables satisfy the Dunfort-Pettis criterion, i.e, these sequences weakly converge to limit points. This limit is feasible due to the Fatou lemma and we get the conclusion. First, we shall prove all variable sequences satisfy the Dunfort-Pettis criterion. Let us denote $(Q, R, S_Q, S_R, A, K)_n$ to be the feasible sequence which is associated with $C(n)$. We first need to prove that if $(Q, R, S_Q, S_R, A, K, C)$ is feasible from $S_{Q_0}, S_{R_0}, Q_0, R_0, A_0, K_0$ then they belong to the space $L^1(e^{-\rho t})$. Indeed, it follows from (8) that $\dot{S}_Q \leq 0$. That implies $0 \leq S_Q \leq S_{Q_0}$. By (7) we have $\dot{S}_R \leq mS_R$ or $-\mu \leq \dot{S}_R/S_R \leq m$. Thus, there exists $\overline{S}$ such that $\overline{S}e^{\mu t} \leq S_R \leq \overline{S}e^{mt}, \dot{S}_R \leq m\overline{S}e^{mt}$ and $0 \leq R \leq (m+\mu)\overline{S}e^{mt}$.

Since $\dot{A}/A^{\phi} = bL_A \leq b$, we will prove that there exist a constants $\overline{A}_1$such that

$$A \leq \overline{A}_1 e^{bt}.$$

Indeed, note that $\phi \leq 1$, if $A \geq 1$ then $\dot{A}/A \leq \dot{A}/A^{\phi} \leq b$.

If $A < 1$ then $\ln A < \ln 1$ and this implies $\dot{A}/A < 0 < b$. In both cases, there exists a constant $\overline{A}_1$such that

$$A \leq \overline{A}_1 e^{bt}.$$

Moreover, we have

$$0 \leq \dot{A} \leq A^{\phi} b \leq (\overline{A}_1)^{\phi} e^{\phi bt}.$$

Now, assumption H_3 implies that (Q, R, S_Q, S_R, A) belong to $L^1(e^{-\rho t})$ because

$$\begin{aligned}
0 &\leq \int_0^{\infty} Qe^{-\rho t}dt \leq S_{Q_0}\int_0^{\infty} e^{-\rho t}dt < +\infty, \\
0 &\leq \int_0^{\infty} Re^{-\rho t}dt \leq (m+\mu)\overline{S}\int_0^{\infty} e^{(m-\rho)t}dt < +\infty, \\
0 &\leq \int_0^{\infty} S_R e^{-\rho t}dt \leq \overline{S}\int_0^{\infty} e^{(m-\rho)t}dt < +\infty, \\
0 &\leq \int_0^{\infty} \left|\dot{A}\right| e^{-\rho t}dt \leq \overline{A}_1\int_0^{\infty} e^{(b-\rho)t}dt < +\infty, \\
0 &\leq \int_0^{\infty} Ae^{-\rho t}dt \leq \overline{A}_1\int_0^{\infty} e^{(b\phi-\rho)t}dt < +\infty
\end{aligned}$$

Since $\lim_{K\to+\infty} f_K(K, 1, S_{Q_0}, S_{R_0}) \leq 0$, for any $\zeta \in (0, \rho - b\theta)$ there exist a such that

$$f(K, L, Q, R) \leq B + \zeta K.$$

It follows that

$$\dot{K} \leq B + \zeta K.$$

Multiply by $e^{-\epsilon t}$ then we get

$$\int_0^t \frac{\partial(e^{-\zeta t}K)}{\partial t}dt \leq \int_0^t Be^{-\zeta t}.$$

This implies that there exists constant B' such that $K \leq B'e^{\zeta t}$. Moreover since $\dot{K} \geq -\delta K$, we then have there exist a constant B'' such that $\left|\dot{K}\right| \leq B''e^{\zeta t} <$

$B''e^{\rho t}$. Thus
$$\int_0^\infty Ke^{-\rho t}dt < +\infty, \int_0^\infty \left|\dot{K}\right| e^{-\rho t}dt < +\infty.$$

Because $\dot{K} \geq -\delta K$, we have

$$\begin{aligned} C &\leq A^\theta f(K, L, Q, R) \leq (\overline{A}_1)^\theta e^{b\theta t}(B + \zeta K) \\ &\leq (\overline{A}_1)^\theta e^{b\theta t}(B + \zeta B' e^{\zeta t}). \end{aligned}$$

Thus we can choose a constant $B'' \geq (\overline{A}_1)^\theta B + (\overline{A}_1)^\theta \zeta B'$ such that

$$C \leq B'' e^{(b\theta+\zeta)t} < B'' e^{\rho t}$$

which implies
$$0 \leq \int_0^\infty Ce^{-\rho t}dt < +\infty.$$
Hence, it is easy to see that the feasible sequences satisfy the Dunfort-Pettis criterion. Then they weakly converge to the limit points in L^1 for the topology $\sigma(L^1(e^{-\rho t}), L^\infty)$ which are feasible due to Fatou Lemma. (see d'Albis et al, 2008). These limit points are optimal solutions of our problem. ■

4 The optimal growth rates

In order to get the analytical solutions of the growth rates at the steady state, we suppose that the instantaneous utility function takes the following form

$$u(C) = \begin{cases} \frac{C^{1-\varepsilon}-1}{1-\varepsilon}, \text{ if } \varepsilon \neq 1 \\ \ln C \text{ if } \varepsilon = 1. \end{cases}$$

The program of the social planner can be written as

$$\max \int_0^\infty u(C)e^{-\rho t}dt$$

subject to

$$\begin{aligned} F &= A^\theta L_Y^\gamma K^\xi Q^\alpha R^\beta \\ \dot{S}_Q &= -Q \\ \dot{S}_R &= mS_R - R \\ \dot{K} &= F - C - \delta K \\ \dot{A} &= bA^\phi L_A \\ 1 &= L_A + L_Y \\ & \quad L_{Y_0}, K_0, S_{Q_0}, S_{R_0} \text{ given.} \end{aligned}$$

Let denote $g_X = \frac{\dot{X}}{X}$ be the growth rate of variable X We shall summarize the macroeconomic equilibrium in terms of the five variables: $x = F/K, y = C/K, z = Q/S_Q, u = R/S_R, q = L_Y A^{\phi-1}, r = A^{\phi-1}$ from which other equilibrium rates

$$g_F, g_K, g_C, g_{L_Y}, g_{L_A}, g_A, g_Q, g_{S_Q}, g_R, g_{S_R}$$

can be derived as the following proposition.

Proposition 2 *The optimal growth rates take the following values*

$$\begin{aligned}
g_A &= b(r-q), \\
g_K &= x - y - \delta, \\
g_C &= \frac{\xi x - \delta - \rho}{\varepsilon}, \\
g_{S_Q} &= -z, \\
g_{S_R} &= m - u
\end{aligned}$$

$$\begin{aligned}
g_Q &= -y + \frac{b\theta r}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} \\
g_R &= -y + \frac{b\theta r}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi} \\
g_{L_Y} &= -y + \frac{b\theta r}{\xi} + \frac{b\theta}{\gamma} q + \frac{m\beta + (1-\xi)\delta}{\xi} \\
g_F &= \xi x - y + \frac{b\theta r}{\xi} + \frac{m\beta + \delta(\alpha+\beta+\gamma)}{\xi} - \delta. \\
g_{L_A} &= \frac{q}{q-r} g_{L_Y}.
\end{aligned}$$

Proof: See Appendix ■

Before analyzing the full dynamic system we look at at the characterization of a balanced optimal growth path. A steady state satisfies that all rates of growth are constant and denoted by g_X^* for any variable X. Denote also by X^* the value at the steady state of X.

Proposition 3 *At the steady state, the growth rate take the following values*

$$g_Q^* = g_{S_Q}^* = -y^* + \frac{b\theta r^*}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} \tag{12}$$

$$g_R^* = g_{S_R}^* = -y^* + \frac{b\theta r^*}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi} \tag{13}$$

$$g_F^* = g_K^* = g_C^* = \frac{\xi x^* - \delta - \rho}{\varepsilon} \tag{14}$$

$$g_{L_Y}^* = g_{L_A}^* = 0 \tag{15}$$

$$g_A^* = b(r^* - q^*) \tag{16}$$

where if $\phi = 1$ *then*

$$\begin{aligned}
x^* &= \frac{b\theta + m\beta + \delta(1-\xi)}{\xi(1-\xi)} \\
y^* &= \frac{(\varepsilon - \xi)(b\theta + m\beta + \delta(1-\xi)) + \xi(1-\varepsilon)\delta + \rho]}{\varepsilon\xi(1-\xi)} \\
q^* &= [y^* - \frac{m\beta + (1-\xi)\delta + b\theta}{\xi}]\frac{\gamma}{\theta b} \\
r^* &= 1
\end{aligned}$$

If $\phi \neq 1$ *then* x^*, y^*, q^*, r^* *defined by*

$$\begin{aligned}
\frac{\xi x^* - \delta - \rho}{\varepsilon} &= x^* - y^* - \delta, \\
\xi x^* - y^* + \frac{b\theta q^* + m\beta + \delta(1-2\xi)}{\xi} &= x^* - y^* - \delta, \\
-y^* + \frac{m\beta + (1-\xi)\delta + b\theta q^*}{\xi} + \frac{b\theta}{\gamma}q^* &= 0, \\
r^* &= q^*.
\end{aligned}$$

Proof: See Appendix ■

Remark 1 *We have* $z^* = -g^*_{S_Q}$, $u^* = m - g^*_{S_R}$. *It follows from transversality conditions at the steady state and the Euler equation that*

$$\lim_{t\to+\infty} \mu S^*_Q e^{-\rho t} = 0 \text{ where } \mu = \mu(0)e^{-\rho t}$$

and

$$S^*_Q(t) = S^*_Q(0)e^{g^*_Q t}$$

we then obtain

$$\lim_{t\to+\infty} \mu(0)S^*_Q(0)e^{g^*_Q t} = 0.$$

This implies $g^*_Q < 0$.

Similarly, since $\lim_{t\to+\infty} \lambda S_R e^{-\rho t} = 0$ *where* $\lambda = \lambda(0)e^{(\rho-m)t}$, *we get*

$$\lim_{t\to+\infty} \lambda(0)S^*_R(0)e^{(g^*_R - m)t} = 0 \text{ or } g^*_R - m < 0.$$

In the next section, we study the dynamic behavior of the nonlinear system which is characterized by the behavior of the linearized system around the steady state.

5 Transitional dynamics

We shall summarize the macroeconomic equilibrium in terms of four stationary variables, $x = F/K, y = C/K, z = \frac{Q}{S_Q}, u = \frac{R}{S_R}, q = A^{\phi-1}L_Y,\ r = A^{\phi-1}$ from which other equilibrium rates can be derived as in Proposition 2.

Proposition 4 *If $\phi = 1$ then the Jacobien matrix has only one negative eigenvalue. If $\phi < 1$ then the Jacobien matrix has two negative eigenvalue if $\xi < \varepsilon$.*

Proof: Case1. If $\phi = 1$. In this case $r = 1$, we just need to analyze the dynamic system of x, y, z, u, q. By logarithmic differentiation the identities determine x, y, z, u, q and using the results obtained in Proposition 2 we get

$$\begin{aligned}
\dot{x} &= (g_F - g_K)x = [(\xi - 1)x + \frac{b\theta}{\xi} + \frac{m\beta + \delta(1-\xi)}{\xi}]x. \\
\dot{y} &= (g_C - g_K)y = [\frac{(\xi - \epsilon)x + (\varepsilon - 1)\delta - \rho}{\varepsilon} + y]y \\
\dot{z} &= (g_Q + z)z = (-y + \frac{b\theta}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} + z)z \\
\dot{u} &= (g_R + u - m)u = (-y + \frac{b\theta}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} + u)u \\
\dot{q} &= (g_{L_Y})q = (-y + \frac{b\theta}{\xi} + \frac{b\theta}{\gamma}q + \frac{m\beta + (1-\xi)\delta}{\xi})q
\end{aligned}$$

The dynamics of $\mathbf{h} = (x, y, z,\ u, q, r)$ are described by the system above. From the theory of linear approximation we know that in the neighborhood of the steady state the dynamic behavior of the nonlinear system is characterized by the behavior of the linearized system around the steady state $\dot{\mathbf{h}} = \mathbf{J}(\mathbf{h} - \mathbf{h}^*)$ where $\mathbf{h}^* = (x^*, y^*, z^*,\ u^*, q^*, r^*)$ and $\mathbf{J}$ is the Jacobien matrix evaluated at the steady state.

$$\mathbf{J} = \begin{pmatrix}
\partial\dot{x}/\partial x & \partial\dot{x}/\partial y & \partial\dot{x}/\partial z & \partial\dot{x}/\partial u & \partial\dot{x}/\partial q \\
\partial\dot{y}/\partial x & \partial\dot{y}/\partial y & \partial\dot{y}/\partial z & \partial\dot{y}/\partial u & \partial\dot{y}/\partial q \\
\partial\dot{z}/\partial x & \partial\dot{z}/\partial y & \partial\dot{z}/\partial z & \partial\dot{z}/\partial u & \partial\dot{z}/\partial q \\
\partial\dot{u}/\partial x & \partial\dot{u}/\partial y & \partial\dot{u}/\partial z & \partial\dot{u}/\partial u & \partial\dot{u}/\partial q \\
\partial\dot{q}/\partial x & \partial\dot{q}/\partial y & \partial\dot{q}/\partial z & \partial\dot{q}/\partial u & \partial\dot{q}/\partial q
\end{pmatrix}$$

Note that $x^*, y^*, z^*,\ u^*, q^*$ are stationary variables, if $\dot{x} = f(\mathbf{h})x$ then $f(\mathbf{h}^*) = 0$. Thus,

$$\frac{\partial\dot{x}}{\partial x}(\mathbf{h}^*) = \frac{\partial f(\mathbf{h}^*)}{\partial x}x^*.$$

Thus we get the Jacobian matrix

$$\mathbf{J} = \begin{pmatrix} (\xi-1)x^* & 0 & 0 & 0 & 0 \\ (\frac{\xi-\varepsilon}{\varepsilon})y^* & y^* & 0 & 0 & 0 \\ 0 & -z^* & z^* & 0 & 0 \\ 0 & -u^* & 0 & u^* & 0 \\ 0 & -q^* & 0 & 0 & \frac{b\theta}{\gamma}q^* \end{pmatrix}$$

The characteristic roots $\lambda_k (k = 1, .., 5)$ are the solutions of the characteristic equation $|\mathbf{J} - \lambda\mathbf{U}| = 0$ where $\mathbf{U}$ is the 5×5 unit matrix. We can write at $\mathbf{h}^*$ that

$$[\frac{b\theta}{\gamma}q^* - \lambda_5][u^* - \lambda_4][z^* - \lambda_3][y^* - \lambda_2][((\xi-1)x^* - \lambda_1] = 0$$

It is easy to see that there is only $\lambda_1 = (\xi - 1)x^* < 0$ while the others are positive.

Case 2. If $\phi \neq 1$, we must analyze the dynamic system of x, y, z, u, q and $r = A^{\phi-1}$. Since $\frac{\dot{r}}{r} = (\phi - 1)g_A$, we know that $g_A^* = 0$ which implies that r is a stationary variable. Moreover, $r^* = q^*$.We have

$$\begin{aligned}
\dot{x} &= (g_F - g_K)x = [(\xi-1)x + \frac{b\theta r}{\xi} + \frac{m\beta + \delta(1-\xi)}{\xi}]x. \\
\dot{y} &= (g_C - g_K)y = [\frac{(\xi-\epsilon)x + (\varepsilon-1)\delta - \rho}{\varepsilon} + y]y \\
\dot{z} &= (g_Q + z)z = (-y + \frac{b\theta r}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} + z)z \\
\dot{u} &= (g_R + u - m)u = (-y + \frac{b\theta r}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} + u)u \\
\dot{q} &= ((\phi-1)g_A + g_{L_Y})q = ((\phi-1)b(r-q) - y + \frac{b\theta r}{\xi} + \frac{b\theta}{\gamma}q + \frac{m\beta + (1-\xi)\delta}{\xi})q \\
\dot{r} &= [(\phi-1)g_A]r = [b(\phi-1)(r-q)]r.
\end{aligned}$$

It is easy to get

$$J = \begin{pmatrix} (\xi-1)x^* & 0 & 0 & 0 & 0 & \frac{b\theta}{\xi}x^* \\ \frac{(\xi-\epsilon)}{\varepsilon}y^* & y^* & 0 & 0 & 0 & 0 \\ 0 & -z^* & z^* & 0 & 0 & \frac{b\theta}{\xi}z^* \\ 0 & -u^* & 0 & u^* & 0 & \frac{b\theta}{\xi}u^* \\ 0 & -q^* & 0 & 0 & [-(\phi-1)b + \frac{b\theta}{\gamma}]q^* & [(\phi-1)b + \frac{b\theta}{\xi}]q^* \\ 0 & 0 & 0 & 0 & [-b(\phi-1)]r^* & [b(\phi-1)]r^*. \end{pmatrix}$$

The characteristic roots $\lambda_k (k = 1, .., 6)$ are the solutions of the characteristic equation

$$|\mathbf{J} - \lambda\mathbf{U}| = 0 \tag{17}$$

where $\mathbf{U}$ is the 6×6 unit matrix. Equation (17) is equivalent to

$$(z^* - \lambda)(u^* - \lambda)\det M = 0$$

where $M =$

$$\begin{pmatrix} (\xi-1)x^*-\lambda & 0 & 0 & \frac{b\theta}{\xi}x^* \\ \frac{(\xi-\epsilon)}{\varepsilon}y^* & y^*-\lambda & 0 & 0 \\ 0 & -q^* & [-(\phi-1)b+\frac{b\theta}{\gamma}]q^*-\lambda & [(\phi-1)b+\frac{b\theta}{\xi}]q^* \\ 0 & 0 & [-b(\phi-1)]r^* & [b(\phi-1)]r^*-\lambda \end{pmatrix}.$$

We then get two positive solutions, $\lambda = z^*, \lambda = u^*$ immediately. We have $\det M(\lambda) =$

$$\begin{aligned} & ((\xi-1)x^*-\lambda)(y^*-\lambda)\begin{vmatrix} (-(\phi-1)b+\frac{b\theta}{\gamma})q^*-\lambda & ((\phi-1)b+\frac{b\theta}{\xi})q^* \\ -b(\phi-1)r^* & b(\phi-1)r^*-\lambda \end{vmatrix} \\ & +(-1)^{1+4}\frac{b\theta}{\xi}x^*\begin{vmatrix} \frac{(\xi-\epsilon)}{\varepsilon}y^* & y^*-\lambda & 0 \\ 0 & -q^* & [-(\phi-1)b+\frac{b\theta}{\gamma}]q^*-\lambda \\ 0 & 0 & [-b(\phi-1)]r^* \end{vmatrix} \\ = \; & ((\xi-1)x^*-\lambda)(y^*-\lambda)\det N(\lambda) - \frac{b\theta}{\xi}\frac{(\xi-\varepsilon)}{\varepsilon}b(\phi-1)x^*y^*q^*r^*, \end{aligned}$$

where

$$\begin{aligned} \det N(\lambda) &= [(-(\phi-1)b+\frac{b\theta}{\gamma})q^*-\lambda][b(\phi-1)r^*-\lambda] \\ &\quad +((\phi-1)b+\frac{b\theta}{\xi})q^*b(\phi-1)r^*. \\ \det N(0) &= [(\frac{b\theta}{\gamma}+\frac{b\theta}{\xi})b(\phi-1)r^*q^*]. \end{aligned}$$

Hence $\det M(\lambda)$ is a polynomial degree of 4,

$$\det M(\lambda) = H(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda^2 + a_4\lambda + a_5$$

If this equation has solutions, then by Viete's theorem, we get

$$\begin{aligned} H(0) &= a_5 = \lambda_1\lambda_2\lambda_3\lambda_4 = \\ &= (\xi-1)x^*y^*[(\frac{b\theta}{\gamma}+\frac{b\theta}{\xi})b(\phi-1)r^*q^*] \\ &\quad -\frac{b\theta}{\xi}x^*\frac{(\xi-\varepsilon)}{\varepsilon}y^*b(\phi-1)q^*r^* \\ &= [b(\phi-1)r^*q^*b\theta x^*y^*][\frac{(\xi-1)(\xi+\gamma)}{\gamma}-\frac{(\xi-\varepsilon)}{\varepsilon}] \\ &= [b(\phi-1)r^*q^*b\theta x^*y^*][\frac{(\xi-1)\xi\varepsilon+(\xi-1)\gamma\varepsilon-\gamma(\xi-\varepsilon)}{\gamma\varepsilon}] \\ &= [b(\phi-1)r^*q^*b\theta x^*y^*][\frac{(\xi-1)\xi\varepsilon+\xi\gamma(\varepsilon-1)}{\gamma\varepsilon}] \\ &= [b(1-\phi)\theta r^*q^*bx^*y^*][\frac{\xi(\alpha+\beta)\varepsilon+\gamma)}{\gamma\varepsilon}]. \end{aligned}$$

If $\phi < 1$ then $H(0) > 0$.We can write

$$H(\lambda) = ((\xi - 1)x^* - \lambda)(y^* - \lambda)(n_1 - \lambda)(n_2 - \lambda) + \frac{b\theta}{\xi}\frac{(\xi - \varepsilon)}{\varepsilon}b(1-\phi)x^*y^*q^*r^*$$

where n_1, n_2 are solutions of the equation

$$\begin{aligned}
\det N(\lambda) &= [(-(\phi - 1)b + \frac{b\theta}{\gamma})q^* - \lambda][b(\phi - 1)r^* - \lambda] \\
&\quad +((\phi - 1)b + \frac{b\theta}{\xi})q^* b(\phi - 1)r^* = 0 \\
n_1.n_2 &= \det N(0) = [(\frac{b\theta}{\gamma} + \frac{b\theta}{\xi})b(\phi - 1)r^*q^*] < 0
\end{aligned}$$

It follows that

$$\begin{aligned}
H((\xi - 1)x^*) &= H(y^*) = H(n_1) = H(n_2) \\
&= \frac{b\theta}{\xi}\frac{(\xi - \varepsilon)}{\varepsilon}b(1-\phi)x^*y^*q^*r^* < 0.
\end{aligned}$$

This implies

$$\begin{aligned}
H((\xi - 1)x^*)H(0) &< 0, H(y^*)H(0) < 0, \\
H(n_1)H(0) &< 0, H(n_2)H(0) < 0,
\end{aligned}$$

which implies $H(\lambda) = 0$ have two negative equations and two positive solutions.
■

Remark 2 *Let us denote the eigenvectors corresponding to six eigenvalues $\lambda_i, i = 1, ..., 6$ with $\mathbf{v}_i$, we may write the general solution of $\dot{\mathbf{h}} = \mathbf{J}(\mathbf{h} - \mathbf{h}^*)$ where $\mathbf{h}^* = (x^*, y^*, z^*, u^*, q^*, r^*)$ as follows:*

$$\mathbf{h}(t) - \mathbf{h}^* = \sum_{i=1}^{6} a_i \mathbf{v}_i e^{\lambda_i t}$$

where parameters a_i are determined by the initial conditions $\mathbf{h}(0) - \mathbf{h}^* = \sum_{i=1}^{6} a_i \mathbf{v}_i$. The optimal path in the neighborhood of the steady state is located in the stable subspace which corresponds to the negative eigenvalues. Thus if $\phi = 1$, we can write $(\mathbf{h}(t) - \mathbf{h}^*) = a_1 \mathbf{v}_1 e^{\lambda_1 t}$ where $\lambda_1 < 0$.

6 Econometric estimation

6.1 Estimated equations

In this section, we discuss about the empirical implication of the theoretical model.

We concentrate our analysis on the growth rates $g_{F/K}$, g_Q, and g_R at the transition path. We only consider the case $\phi = 1$, where all of these growth rates only depend on $x \equiv F/K$ and $y \equiv C/K$. When $\phi < 1$, the model is not identified and then estimation becomes impossible unless some restrictions are imposed.[3] From the previous analysis for $\phi = 1$ (cf. Remark 2), as the components of $\mathbf{h}$ are independent of each other, we can write the approximation expressions for x and y as follows

$$\begin{aligned} x_t - x^* &= a_1 v_{1x} e^{\lambda_1 t} &(18)\\ y_t - y^* &= a_1 v_{1y} e^{\lambda_1 t}, &(19)\end{aligned}$$

where v_{1x} and v_{1y}, two components of $\mathbf{v}_1$. By using the initial conditions, we get

$$\begin{aligned} a_1 v_{1x} &= x_0 - x^* \\ a_1 v_{1y} &= y_0 - y^* \end{aligned}$$

Therefore, we obtain the following solution

$$\begin{aligned} x_t &= (1 - e^{\lambda_1 t}) x^* + e^{\lambda_1 t} x_0 &(20)\\ y_t &= (1 - e^{\lambda_1 t}) y^* + e^{\lambda_1 t} y_0, &(21)\end{aligned}$$

recall that we have

$$\begin{aligned} g_x &= (\xi - 1)x + \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} \\ g_Q &= -y + \frac{b\theta}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} \\ g_R &= -y + \frac{b\theta}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi}. \end{aligned}$$

We substitute (20) and (21) in these expressions and use the definitions $x_0 = F_0/K_0$, $y_0 = C_0/K_0$, and $g_x = g_{F/K} = \frac{1}{T}\ln(F_T/F_0) - \frac{1}{T}\ln(K_T/K_0), g_Q = \frac{1}{T}\ln(Q_T/Q_0)$ and $g_R = \frac{1}{T}\ln(R_T/R_0)$, which are respectively the average growth rates of F/K, Q, and R, between 0 and T. This results in the transitional dynamics of g_χ, $\chi = F/K, Q, R$ towards the steady-state of the economy.[4]

Firstly, $g_{F/K}$ is given by[5]

$$\begin{aligned} g_{F/K} &= (\xi - 1)x_T + \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} &(22)\\ &= \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} + (\xi - 1)(1 - e^{\lambda_1 T})x^* + (\xi - 1)e^{\lambda_1 T} x_0 &(23)\\ &= \alpha_0 + \alpha_1 F_0/K_0 + \varepsilon_{F/K} &(24)\end{aligned}$$

[3] This issue will be discussed deeply in a further work.

[4] The reason of using $g_{F/K}$ instead of g_F is that the approximation of the latter contains, in the right-hand side, both F/K and C/K which are highly correlated. Hence, regression of g_F on F/K and C/K will face a problem of multicolinearity.

[5] Index i is dropped to simplify notations.

where

$$\begin{aligned}
\alpha_0 &= \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} + (\xi - 1)(1 - e^{\lambda_1 T})x^* \\
\alpha_1 &= (\xi - 1)e^{\lambda_1 T}.
\end{aligned}$$

Concerning g_Q, we have

$$\begin{aligned}
g_Q &= -(1 - e^{\lambda_1 T})y^* - e^{\lambda_1 T}y_0 + \frac{b\theta}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} && (25) \\
&= \beta_0 + \beta_1 C_0/K_0 + \varepsilon_Q && (26)
\end{aligned}$$

where

$$\begin{aligned}
\beta_0 &= -(1 - e^{\lambda_1 T})y^* + \frac{b\theta}{\xi} + \frac{m\beta + (1-\xi)\delta}{\xi} && (27) \\
\beta_1 &= -e^{\lambda_1 T}. && (28)
\end{aligned}$$

Similarly, g_R is given by

$$\begin{aligned}
g_R &= -y + \frac{b\theta}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi} && (29) \\
&= -(1 - e^{\lambda_1 T})y^* - e^{\lambda_1 T}y_0 + \frac{b\theta}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi} && (30) \\
&= \gamma_0 + \gamma_1 C_0/K_0 + \varepsilon_R && (31)
\end{aligned}$$

where

$$\begin{aligned}
\gamma_0 &= \frac{b\theta}{\xi} + \frac{m(\beta+\xi) + (1-\xi)\delta}{\xi} - (1 - e^{\lambda_1 T})y^* && (32) \\
\gamma_1 &= -e^{\lambda_1 T}. && (33)
\end{aligned}$$

Equations (24), (26), and (31) represent three cross-sectional regressions where $\varepsilon_{F/K}$ and ε_Q and ε_R are the corresponding error terms.[6] These equations may be estimated by using Ordinary Least Squares. However, as underlined by Islam (1995) and subsequent studies on income convergence, we will lose lots of information included in the data sample as we only need observations of the initial and final dates, i.e. dates 0 and T, and the sample size is then equal to the number of countries.

An alternative approach is to transform the model in some panel structure. In particular, we can rewrite equations (24), (26), and (31) as follows:

$$\begin{aligned}
g_{(F/K)_{it}} &= \alpha_0 + \alpha_1 F_{i,t-1}/K_{i,t-1} + \varepsilon_{(F/K)_{it}} && (34) \\
g_{Q_{it}} &= \beta_0 + \beta_1 C_{i,t-1}/K_{i,t-1} + \varepsilon_{Q_{it}} && (35) \\
g_{R_{it}} &= \gamma_0 + \gamma_1 C_{i,t-1}/K_{i,t-1} + \varepsilon_{R_{it}} && (36)
\end{aligned}$$

[6]These error terms may correspond to omitted variables or unobserved factors affecting the growth rates $g_{F/K}$, g_Q and g_R. They can also represent measurement errors in these growth rates.

where $i = 1, ..., N$, and $t = 1, ..., T$. As in most studies on income convergence, we use data corresponding to the five year interval period, i.e. data from 1977, 1982, 1987, 1992, and 1997. Hence the length between t and $t-1$ is equal to 5 and $g_{\chi_{it}} = \frac{1}{5}\ln(\chi_t/\chi_{t-1})$. The purpose is to reduce the business cycle effect.

The sample size is 108 observations ($N = 27$, $T = 4$). In our panel data framework, the error terms $\varepsilon_{\chi_{it}}$, $\chi = F/K, Q, R$, include country and time effects, i.e. $\varepsilon_{\chi_{it}} = \mu_i + \lambda_t + u_{\chi_{it}}$ where μ_i and λ_t denote country heterogeneity and time heterogeneity respectively, and $u_{\chi_{it}}$ is the idiosyncratic error. Moreover, the model predicts that coefficients α_1, β_1, and γ_1 are negative.

Estimations of these equations can be obtained with standard panel methods (within estimation for fixed effects, Generalized Least Squares for random effects) which assumes the *strict exogeneity* of regressors, i.e. $E[(F_s/K_s)\varepsilon_{(F/K)_t}] = E[(C_s/K_s)\varepsilon_{Q_t}] = E[(C_s/K_s)\varepsilon_{R_t}] = 0$, $\forall s, t$. However, this assumption may be faulty as regressors can be correlated with some unobserved factors or with future values of the dependent variable. This is probably the case when we study macroeconomic data. For example, we may think that current consumption and capital stock may have some impacts not only on current income and energy consumption but also on their future values. This situation arises when regressors are *predetermined*, i.e. $E[(F_s/K_s)\varepsilon_{(F/K)_t}] = E[(C_s/K_s)\varepsilon_{Q_t}] = E[(C_s/K_s)\varepsilon_{R_t}] = 0$, $\forall s < t-1$. In this situation, the model can be estimated by using Generalized Methods of Moments (see, e.g., Baltagi, 2005, and Lee, 2002). This is also the approach adopted in our estimation strategy.

6.2 Data

The data concerns twenty-seven OECD countries for the period 1977-1997.[7] Data on non-renewable energy consumption Q and renewable energy consumption R are collected from the International Energy Angency (IEA). Non-renewable energy consumption, Q, is measured as the sum of consumption of gas, and liquid fuels (in metric tons oil equivalent, *toe*). We assume that renewable energy consumption, R, corresponds to the sum of nuclear energy, hydroelectricity, geothermal energy, renewable fuels and waste, solar energy, wind energy, and energy from tide, wave, and ocean (also in toe).

Table 1 here Figures 1-5 here

Data on production F, consumption C, physical capital stock K, and population are collected from the Penn World Table 6.1. We note that all figures,

[7] The data include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, and the United States.

except population, are expressed in PPP and 1996 prices for an international comparison purpose. We use the population series and the series on real GDP per capita in 1996 prices (series RGDPL) to produce the volume of real GDP in 1996 prices. We compute the product between real GDP and investment share of real GDP, on the one hand, and the product between real GDP and consumption share of real GDP, on the other hand, which correspond to the investment and consumption in real terms. Moreover, physical capital stock is based on the investment series (i.e. investment share of real GDP) and computed following the perpetual inventory method.[8] Data are used in per capita terms to neutralize the possible scale effect due to the difference in population size observed between countries.

Data descriptive statistics are summarized in Table 1. Evolutions of the averages of ratios F/K and C/K are displayed in Figure 1. These ratios have similar patterns, with two dips around 1982 and 1993, except that F/K has stronger variation than C/K. Series on consumptions of nonrenewable and renewable resources are presented in Figure 2. The average of consumption of renewable energies R, is much lower than that of nonrenewable energies Q, which increased over the whole period of the study whereas the latter considerably decreased in the late 70s to until the dip in 1983, to rising thereafter.

For the estimations, we take data corresponding to the five year interval period, (data from years 1977, 1982, 1987, 1992, and 1997) in order to eliminate business cycle effects as in most of empirical studies on economic convergence. Distributions of average annual growth rates (computed from these time intervals) of the output to capital ratio F/K, $g_{F/K}$, renewable energy consumption per capita, g_R, and nonrenewable energy consumption per capita, g_Q, are reported in Figures 3, 4, and 5, respectively. The distribution of these growth rates sensitivity changes over time. We also observe a particularity that the dispersion of g_Q and g_R diminishes throughout the period of study.

6.3 Estimation results

Estimation results by GMM are reported in Table 2. As GMM use the first-difference transformation, the intercept and country effects (which are not separately identified) are deleted from each regression and therefore not estimated. Furthermore, the assumption of predetermined regressors,

$$E[(F_s/K_s)\varepsilon_{(F/K)_t}] = E[(C_s/K_s)\varepsilon_{Q_t}] = E[(C_s/K_s)\varepsilon_{R_t}] = 0, \forall s < t-1,$$

[8]The perpetual inventory equation is $K_t = I_t + (1-\delta)K_{t-1}$ where I_t is the investment flow. The initial capital stock is given by $K_0 = I_0/(g_I + \delta)$ where g_I is the average geometric growth rate of investment from the initial date. The depreciation rate, δ, is often set to 4 to 6%. In our paper, changing δ from 4% to 6% does not modify the qualitative conclusion.

allows us to use all values of F_s/K_s and C_s/K_s, $s = 1, ..., t-2$ as possible instruments for F_t/K_t and C_t/K_t respectively. As a consequence, there are finally 81 observations used in the estimations.

Table 2 here

Empirical results based on OECD global data confirm the prediction of the theoretical model. Indeed, estimation results show that coefficients α_1, β_1, and γ_1 are negative as expected. Coefficients α_1 and γ_1 are statistically significant at the 5% and the 10% levels respectively, while β_1 is insignificant. The Wald test confirms the existence of time effects in all regressions. The Sargan specification test for over-identifying restrictions (relative to the use of instrumental variables) is always satisfied in either regressions as these over-identifying restrictions are not rejected.

It should be noted that the GMM estimator is consistent with an AR(1) process for the regression residuals but not consistent with an AR(2). We use the Arellano-Bond tests to study this issue. Test results reject the absence of autocorrelation of order 1 but do not reject that of order 2 for regressions with $g_{F/K}$ and g_R, suggesting that estimations are consistent in these cases. Concerning g_Q, the specification does not seem robust as Arellano-Bond test does not confirm, but only at the 10% level, the absence of an AR(2) process in the residuals.

7 Conclusion

The paper is a first attempt to explore theoretically and empirically the interaction between growth, technological level, and consumptions of renewable and non-renewable resources. We derive the optimal solution and characterize the BGP together with the transitional dynamics of the model. An empirical analysis of the model is also provided by using some panel econometrics on OECD data. Our estimation strategy is appealing since it accounts for country and time heterogeneities and it allows for a more flexible assumption about regressors than the usual assumption of strictly exogenous regressors in the standard framework. As underlined previously, it would be of particular interest, in another step, to study the identification issue of the empirical specification that can be derived from the theoretical model when $\phi < 1$.

Moreover, on the theoretical side, we do not consider externality from resource use in utility or production. It would therefore be interesting to include these factors in a future work to improve the realism of the modeling. Another research interest to consider is a more general model with very general functional forms for production, knowledge accumulation, utility, and natural resources dynamics. Competitive equilibrium and public policy will also require particular attention.

Appendix

Proof of Proposition 2

Proof: The current-value Hamiltonian is

$$\begin{aligned} H(C,K,Q,R,L_Y,A) &= u(C)+\lambda(mS_R-R)-\mu Q+\nu(F-C) \\ &= u(C)+\lambda(mS_R-R)-\mu Q \quad (37) \\ &\quad +\nu(F-C-\delta K)+\omega bA^{\phi}(1-L_Y). \quad (38) \end{aligned}$$

where λ, μ, ν, ω are four costate variables.

The first order conditions $\frac{\partial H}{\partial C}=0$, $\frac{\partial H}{\partial Q}=0$, $\frac{\partial H}{\partial R}=0$, $\frac{\partial H}{\partial L_Y}=0$ yield

$$\nu = U_C \quad (39)$$

$$\mu = vF_Q \quad (40)$$

$$\lambda = vF_R \quad (41)$$

$$\omega = \frac{vF_{L_Y}}{bA^{\phi}} \quad (42)$$

From Euler equations $\frac{\partial H}{\partial K}=\rho\nu-\dot{\nu}$, $\frac{\partial H}{\partial S_R}=\rho\lambda-\dot{\lambda}$, $\frac{\partial H}{\partial S_Q}=\rho\mu-\dot{\mu}$, and $\frac{\partial H}{\partial A}=\rho\omega-\dot{\omega}$ we get

$$\frac{\dot{\nu}}{v} = \rho-F_K-\delta \quad (43)$$

$$\frac{\dot{\mu}}{\mu} = \rho \quad (44)$$

$$\frac{\dot{\lambda}}{\lambda} = \rho-m \quad (45)$$

$$\dot{\omega} = (\rho-b\phi A^{\phi-1}(1-L_Y))\omega-vF_A.$$

By (39) and $\dot{A}/A=bA^{\phi-1}(1-L_Y)$ we get

$$\dot{\omega}=(\rho-\phi g_A)\omega-U_CF_A. \quad (46)$$

The transversality conditions are

$$\lim_{t\to+\infty}\lambda S_R e^{-\rho t}=\lim_{t\to+\infty}\mu S_Q e^{-\rho t}=\lim_{t\to+\infty}\nu K e^{-\rho t}=\lim_{t\to+\infty}\omega A e^{-\rho t}=0. \quad (47)$$

From the identities $\dot{S}_R=mS_R-R$, $\dot{S}_Q=-Q$ and $\dot{K}=F-C-\delta K$ we obtain

$$g_{S_Q} = -z, \quad (48)$$

$$g_{S_R} = m-u, \quad (49)$$

$$g_K = x-y-\delta, \quad (50)$$

$$g_A = b(r-q) \quad (51)$$

Since $F = A^\theta L_Y^\gamma K^\xi Q^\alpha R^\beta$, we have

$$F_K = \xi F/K = \xi x,$$

$$\begin{aligned}
\frac{\dot{F}_Q}{F_Q} &= \theta g_A + \gamma g_{L_Y} + \xi g_K + (\alpha-1) g_Q + \beta g_R, && (52)\\
\frac{\dot{F}_R}{F_R} &= \theta g_A + \gamma g_{L_Y} + \xi g_K + \alpha g_Q + (\beta-1) g_R, && (53)\\
\frac{\dot{F}_{L_Y}}{F_{L_Y}} &= \theta g_A + (\gamma-1) g_{L_Y} + \xi g_K + \alpha g_Q + \beta g_R && (54)
\end{aligned}$$

Equation (39) together with (43) yield

$$\rho - \frac{\dot{U}_C}{U_C} = F_K - \delta = \xi x - \delta. \tag{55}$$

It is easy to check that

$$\frac{\dot{U}_C}{U_C} = -\varepsilon \frac{\dot{C}}{C} = -\varepsilon g_C. \tag{56}$$

Thus,

$$g_C = \frac{\xi x - \delta - \rho}{\varepsilon}.$$

By logarithmic differentiation (40) and together with (44) we have

$$\frac{\dot{F}_Q}{F_Q} = \rho - \frac{\dot{U}_C}{U_C} = \xi x - \delta. \tag{57}$$

From (41) and (45) we get

$$\frac{\dot{F}_R}{F_R} = \rho - \frac{\dot{U}_C}{U_C} - m = \xi x - \delta - m. \tag{58}$$

From (42) and (46) we get

$$\begin{aligned}
\frac{\dot{U}_C}{U_C} + \frac{\dot{F}_{L_Y}}{F_{L_Y}} - \phi g_A &= \rho - \phi g_A - \frac{U_C F_A}{\omega} = \\
\rho - \phi g_A - \frac{U_C F_A}{U_C F_{L_Y}} A^\phi &= \rho - \phi g_A - \frac{\theta F/A}{\gamma F/L_Y} A^\phi = \\
\rho - \phi g_A - \frac{\theta}{\gamma} L_Y A^{\phi-1} &= \rho - \phi g_A - \frac{b\theta}{\gamma} q.
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\dot{F}_{L_Y}}{F_{L_Y}} &= \rho - \frac{\dot{U}_C}{U_C} - \frac{b\theta}{\gamma} q \\
&= \xi x - \delta - \frac{b\theta}{\gamma} q.
\end{aligned}$$

Now, it follows from (52)-(54) that we have a system of equations with three variables g_Q, g_R, g_{L_Y}

$$\gamma g_{L_Y} + (\alpha-1)g_Q + \beta g_R = \xi y - b\theta(r-q) + (\xi-1)\delta = T_1 \tag{59}$$
$$\gamma g_{L_Y} + \alpha g_Q + (\beta-1)g_R = \xi y - m - b\theta(r-q) + (\xi-1)\delta \tag{60}$$
$$(\gamma-1)g_{L_Y} + \alpha g_Q + \beta g_R = \xi y - \frac{b\theta}{\gamma}q - b\theta(r-q) + (\xi-1)\delta \tag{61}$$

From (59)-(60) we have $-g_Q + g_R = m$.

From (59)-(61) we get $\gamma g_{L_Y} = b\theta q + \gamma g_Q$.

Replace this equation into (59) and we have

$$(\gamma+\alpha-1)g_Q + \beta g_R = T_1 - b\theta q$$

We then have two equations to find g_Q, g_R.

$$\begin{aligned}
g_Q &= \frac{m\beta + b\theta q - T_1}{\xi} = \frac{b\theta r - \xi y + m\beta + (1-\xi)\delta}{\xi} \\
g_R &= g_Q + m = \frac{b\theta r - \xi y + m(\beta+\xi) + (1-\xi)\delta}{\xi}
\end{aligned}$$

and

$$g_{L_Y} = \frac{b\theta}{\gamma}q + g_Q = \frac{b\theta r - \xi y + m\beta + (1-\xi)\delta}{\xi} + \frac{b\theta}{\gamma}q.$$

By logarithmic differentiation the equation of $F = A^\theta L_Y^\gamma K^\xi Q^\alpha R^\beta$ we get

$$\begin{aligned}
g_F &= \theta g_A + \gamma g_{L_Y} + \xi g_K + \alpha g_Q + \beta g_R = \\
&= \gamma g_{L_Y} + (\alpha-1)g_Q + \beta g_R + \theta g_A + \xi g_K + g_Q \\
&= T_1 + \theta g_A + \xi g_K + g_Q \\
&= \xi y + (\xi-1)\delta + \xi(x-y-\delta) + \frac{b\theta r - \xi y + m\beta + (1-\xi)\delta}{\xi} \\
&= \xi x - y + \frac{b\theta r}{\xi} + \frac{m\beta + \delta(1-2\xi)}{\xi} \\
&= \xi x - y + \frac{b\theta r}{\xi} + \frac{m\beta + \delta(\alpha+\beta+\gamma)}{\xi} - \delta.
\end{aligned}$$

Finally, since $L_Y = \frac{q}{r}$,

$$g_{L_A} = \frac{\dot{L}_A}{L_A} = -\frac{\dot{L}_Y}{1-L_Y} = \frac{L_Y}{L_Y-1}g_{L_Y} = \frac{q}{q-r}g_{L_Y}.$$

■

Proof of Proposition 3

Proof: At the steady state, $g_A^* = b(r^* - q^*)$ is constant. Therefore, since g_C^*, g_K^* are constant, it follows that x^*, y^* is constant. It follows from g_Q^* constant that

r^* is constant. Thus q^*, z^*, u^* are also constant. Since $x = F/K, y = F/C$ we have $g^*_C = g^*_F = g^*_K$.Moreover $L^*_Y = q^*/r^*$ is constant which implies that L^*_A is constant. So we get

$$g^*_{L_Y} = g^*_{L_A} = 0.$$

Since $r^* = A^{*\phi-1}$ is constant, we have $(\phi-1)g^*_A = 0$ or

$$(\phi-1)b(r^* - q^*) = 0.$$

This equation together with $g^*_C = g^*_K, g^*_F = g^*_K, g^*_{L_Y} = 0$ yield

$$\begin{aligned}
\frac{\xi x^* - \delta - \rho}{\varepsilon} &= x^* - y^* - \delta \\
\xi x^* - y^* + \frac{b\theta r^* + m\beta + \delta(1-2\xi)}{\xi} &= x^* - y^* - \delta \\
-y^* + \frac{m\beta + (1-\xi)\delta + b\theta r^*}{\xi} + \frac{b\theta}{\gamma}q^* &= 0 \\
(\phi-1)(r^* - q^*) &= 0
\end{aligned}$$

a) If $\phi = 1$, note that $r^* = A^{*\phi-1} = 1$ we have

$$\begin{aligned}
\frac{\xi x^* - \delta - \rho}{\varepsilon} &= x^* - y^* - \delta \\
\xi x^* - y^* + \frac{b\theta + m\beta + \delta(1-2\xi)}{\xi} &= x^* - y^* - \delta \\
-y^* + \frac{m\beta + (1-\xi)\delta + b\theta}{\xi} + \frac{b\theta}{\gamma}q^* &= 0
\end{aligned}$$

Thus

$$\begin{aligned}
x^* &= \frac{b\theta + m\beta + \delta(1-\xi)}{\xi(1-\xi)} \\
y^* &= \frac{(\varepsilon - \xi)(b\theta + m\beta + \delta(1-\xi)) + \xi(1-\varepsilon)\delta + \rho]}{\varepsilon\xi(1-\xi)} \\
q^* &= [y^* - \frac{m\beta + (1-\xi)\delta + b\theta}{\xi}]\frac{\gamma}{\theta b}.
\end{aligned}$$

b) If $\phi \neq 1$ then we have $r^* = q^*$. Thus we have three equations which determine the optimal growth rates at the steady state

$$\begin{aligned}
\frac{\xi x^* - \delta - \rho}{\varepsilon} &= x^* - y^* - \delta \\
\xi x^* - y^* + \frac{b\theta q^* + m\beta + \delta(1-2\xi)}{\xi} &= x^* - y^* - \delta \\
-y^* + \frac{m\beta + (1-\xi)\delta + b\theta q^*}{\xi} + \frac{b\theta}{\gamma}q^* &= 0
\end{aligned}$$

Note that $A^* = (r^*)^{1/\phi-1}$.

Finally, since $\frac{\dot{S}_Q}{S_Q} = -\frac{Q}{S_Q}$ and $g^*_{S_Q}$ is constant, we have $g^*_Q = g^*_{S_Q}$.

Similarly, we have $g^*_R = g^*_{S_R}$. ■

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Table 1: Descriptive statistics, 1977–1997

Variable	Mean	Std. Dev.	Min	Max
Consumption per capita	10.558	3.473	2.757	21.505
Capital stock per capita	46.629	19.066	5.247	89.764
GDP per capita	16.592	5.591	4.325	37.917
Consumption-capital ratio C/K	0.254	0.098	0.129	0.730
Output-capital ratio F/K	0.391	0.121	0.190	0.911
Nonrenewable energy consumption Q	3.145	1.706	0.496	10.637
Renewable energy consumption R	0.864	1.097	0.004	5.467
Number of countries	27			
Number of years	21			

Notes: Data on energy consumption are collected from the IEA for the period 1977–1997. Consumptions of renewable and nonrenewable energies are expressed in metric tons oil equivalent (toe). Economic data are drawn from the Penn World Table 6.1 (see Heston et al., 2002). GDP, consumption, and capital stock are measured in thousands U.S. dollars and 1996 prices. All figures are in per capita terms.

Table 2: Estimation results

Equation	Variable	Coefficient	Sargan	AR(1)	AR(2)	Wald
$g_{(F/K)_t}$	$(F/K)_{t-1}$	-0.174**	0.229	-3.08**	-1.15	17.4**
		(0.060)				
g_{Q_t}	$(C/K)_{t-1}$	-0.015	0.883	-2.86**	1.41*	17.8**
		(0.067)				
g_{R_t}	$(C/K)_{t-1}$	-0.275*	5.7	-2.37**	-0.553	6.52*
		(0.166)				

Notes: Regressions include country effects and year effects. Over-identifying restrictions are tested by the Sargan test. AR(1) and AR(2) tests are the Arellano and Bond (1991) tests for serial correlation of order 1 and 2 respectively. The Wald test is for significance of year dummies. Estimation results are obtained by GMM with robust standard error *à la White* given in parentheses. * and ** represent significance levels of 10% and 5% respectively.

Figure 1: Averages of ratios F/K (solid line) and C/K (dashed line), period 1977–1997.

Figure 2: Average consumptions per capita of nonrenewable energies Q (solid line) and renewable energies R (dashed line), period 1977–1997.

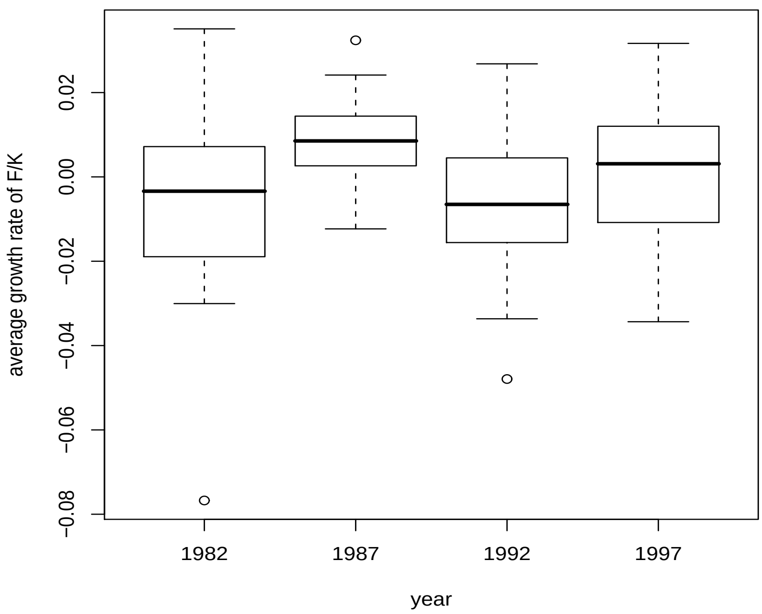

Figure 3: Average growth rate of F/K, period 1977–1997. The box plots relative to 1982, 1987, 1992, and 1997 represent the distribution of the average annual growth rate observed for periods 1977-1982, 1982-1987, 1987-1992, and 1992-1997, respectively.

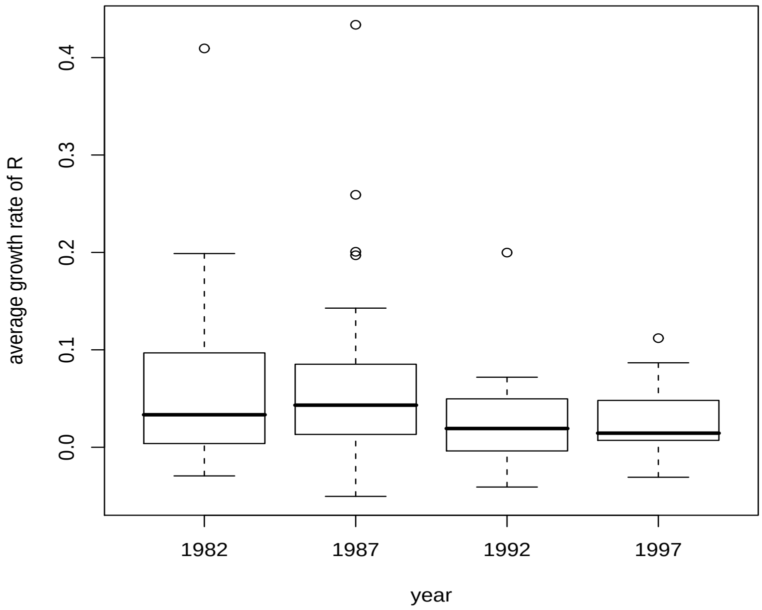

Figure 4: Average growth rate of renewable resource consumption per capita g_R, period 1977–1997. The box plots relative to 1982, 1987, 1992, and 1997 represent the distribution of the average annual growth rate observed for periods 1977-1982, 1982-1987, 1987-1992, and 1992-1997, respectively.

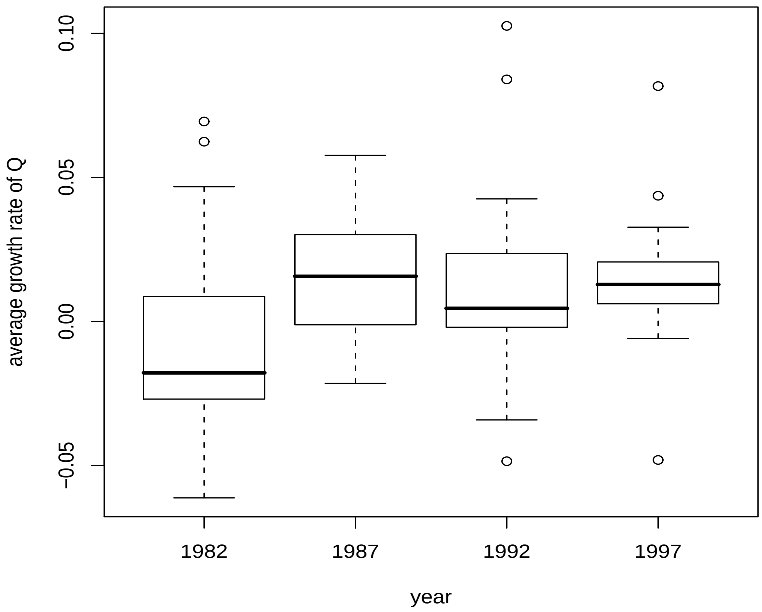

Figure 5: Average growth rate of nonrenewable resource consumption per capita g_Q, period 1977–1997. The box plots relative to 1982, 1987, 1992, and 1997 represent the distribution of the average annual growth rate observed for periods 1977-1982, 1982-1987, 1987-1992, and 1992-1997, respectively.