Pumping Water to Compete in Electricity Markets

Claude Crampes* and Michel Moreaux†

February 7, 2008

*Toulouse School of Economics (GREMAQ and IDEI) ccrampes@cict.fr
†Toulouse School of Economics and IUF (IDEI and LERNA) mmoreaux@cict.fr

Contents

Abstract

The pump storage technique allows to use cheap thermal electricity at periods of low demand to restore water resources that can be used to generate electricity at periods of peak demand. When the thermal plant and the hydro plant are managed by the same operator, the two plants are used in an efficient way to substitute low cost fuel to high cost fuel. When there are two independent managers, competition requires careful analysis. The paper first analyses the optimal dispatch and the profit-maximizing dispatch of the thermal and hydro-generation units when water can be pumped up into reservoirs. We identify the demand and cost conditions where the pumping device must be used. We then switch to the case where the two plants are operated by separate owners. We analyse the Nash equilibrium of the game where the hydro unit is the client of the thermal unit at off-peak period, and compete against it at peak period under three alternative legal arrangements and/or technical constraints: i) the thermal producer cannot separate the energy sold to the hydroproducer from the energy sold to final consumers; ii) the thermal producer is obliged to sell at market price the energy demanded by the hydroproducer and iii) the hydroproducer is obliged to buy at market price the quantity of energy assigned by the thermal producer.

Keywords : water resource; pumping; hydroelectricity; Cournot competition

JEL classification: L12, L13, Q25, Q42

1 Introduction

Electricity can be generated from a large spectrum of heterogeneous technologies: nuclear thermal plants, gas, oil, brown-coal and coal burning installations, windmills, tidemills and river and mountain hydroplants, to quote only those which have proved to be not mere engineer dreams[1]. Within this large range of diverse technologies, some are complements and others are substitutes, complementarity and substituability being appreciated from both the user side of the market and its production side. For example, the nuclear technology is the cheapest way to satisfy base long-duration demand. By contrast, for short run and sufficiently spaced peak demand, gas turbine generation often happens to be the most efficient choice. Hence no technology is better than any other one in absolute terms. This is the reason why they are all operated simultaneously. A main point put forwards in electricity economics, is that electricity as such cannot be stored and must be produced at the very moment it is called for. But the storage problem must not be disconnected from the global analysis of the whole industry. The problem would be an unescapable one if all the inputs used in the generation process were themselves not storable. Fortunately all the non renewable inputs, carbon and nuclear fuels, are storable, either underground before extraction or overground after extraction, although at different costs. The main non storable inputs are renewable inputs like sun and wind energy. But in some cases the electricity output can be transformed into an input which is storable and may be used at a later date. In this paper we analyze the interaction between an hydroelectric generator and a thermal generator when there exists an hydroplant using power produced by the thermal generator.

The observation of real hydrosystems shows that hydrogenerators operate complex systems of dams where the water released and turbinated by an upstream dam can be stored in a downstream dam and either turbinated a second time or pumped to come back into the upstream dam. The economic analysis of such a complex system necessitates to construct a huge model to take into account all the technical restrictions imposed to the quantity of water that can be stored (at maximum and at minimum), released, turbinated, pumped, etc. in each dam. The objective of this paper is to qualify how water pumping changes the overall conditions of competition between heterogeneous technologies of generation[2]. Consequently, in the following pages,

[1] For complete pictures of the different energy sources and their transformations see for example Boyle (1996), Cassedy (2000) Edwards *et alii* (1999), or Johansson *et alii* (1993).

[2] The basic model of competition between a thermal plant and a hydroplant (without pumping) is given in Crampes and Moreaux (2001) and the extension to uncertainty in

we adopt strongly simplifying hypotheses as compared with the real world. In particular, we omit stock constraints and we focus on flow restraints.

While the electricity industry is commonly viewed as an industry where storage is totally impossible, economists and engineers know that the electricity produced during off-peak periods by fuel generators can be used to store water and this provides extra energy for peak periods[3]. According to Eurostat data, pumping represents 5.5 % of the generation capacity in the European Union. As shown in Table 1, the energy produced by means of this stored water is on average less than 1 % of the total production but it is an essential supply because it is called at peak periods, when thermal electricity becomes scarce and expensive. Within an integrated industry, the dispatching of the two types of plants at peak and off-peak periods is just a matter of efficiency. This part of the economic analysis has been clearly documented by Jackson (1973). But with the liberalization of the electricity industry, a new problem arises: how can competition in electricity markets work when hydroplants and thermal plants belong to separate owners? Indeed, the two types of generators are competitors at peak periods and potential partners at off-peak periods. This complex situation has not been addressed yet in the economic literature.

Table 1
Electricity Production in 2005 (TWh)

Countries	Total production	Hydraulic (H)	Pumped share TWh	Pumped share % of H
France	574.0	56.0	4.7	8.3
Germany	619.0	27.0	6.9	25.9
Italy	301.0	42.0	8.8	11.0
Spain	294.0	24.0	3.5	14.5
United Kingdom	400.3	7.3	2.9	39.7
European Union	3201.0	317.0	36.1	11.4
Japan	1111.9	103.6	9.1	8.8
North America	4908.0	660.0	24.0	0.4
Russia	932.0	160.8	1.8	1.1

Source: Worldwide electricity production from renewable energy sources. Eighth inventory Edition 2006. Observ'ER, EDF.

Dakhlaoui and Moreaux (2006). Competition between hydroproducers is analyzed by Ambec and Doucet (2003).

[3] "This pumped storage technique is particularly well suited to nuclear plants that cannot be "turned off" during low demand intervals. By using electricity for pumped storage, the nuclear generators can operate continuously at their most efficient output levels". Jackson (1973) p. 556.

Section 2 presents the features of the model where electricity can be produced either by a thermal plant or by a hydroplant supplied with water by electricity from the thermal plant. In section 3 we expose the characteristics of the efficient use of the two plants and we show how pump storage creates a strong complementarity between the two types of technologies. Section 4 determines and compares the decisions made by a benevolent planner and by a private integrated monopoly. In section 5, we switch to competition between two private operators, each controlling one technology. Actually, they mainly compete during the periods of peak demand. When the final demand for electricity is low, the hydroplant is the client of the thermal generator. We consider three different institutional settings where either the thermal producer of the hydroproducer is the decision maker on the pumped flow. Section 6 concludes.

2 Model setting

We consider a stationary cyclical electricity market which can be supplied by two types of perfectly substitutable generators, a thermal unit and a hydroplant.

Each 24-hour time interval, denoted by $\tau = 1, 2, ...$, is made of two periods, the night period labeled $t = 1$ and the day period labeled $t = 2$, of equal duration[4]. Hereafter, we will refer to "off-peak period" for $t = 1$, to "peak-period" for $t = 2$ and to "day" for the 24-hour time interval. Let $q_{\tau t}$ be the quantity consumed in period t of day τ. By a stationary market we first mean that the gross surplus that final users derive from consuming $q_{\tau t}$ depends on period t but not on day τ. Hence, deleting index τ, let $u_t(q_t)$ be the period t gross surplus generated by the consumption of q_t.

We assume that for any $t = 1, 2$, the gross surplus function $u_t : R_{++} \to R_+$ is first strictly concave and increasing over some internal $(0, \hat{q}_{tu}), 0 < \hat{q}_{tu} < +\infty$, and next constant over $[\hat{q}_{tu}, +\infty]$, that is:

$$\begin{array}{lll} u_t'(q_t) > 0 \text{ and } u_t''(q_t) < 0 \quad , \quad q_t \in (0, \hat{q}_{tu}) & , & t = 1, 2 \\ u_t(q_t) = \bar{u} > 0 \quad , \quad q_t \epsilon [\hat{q}_{tu}, +\infty] & , & t = 1, 2 \end{array}$$

[4]The assumption that $t = 1$ and $t = 2$ both have the same duration facilitates the graphical presentation of the results.

Period 2 is the peak-period, meaning that:

$$\begin{array}{ll} \hat{q}_{1u} < \hat{q}_{2u} & \\ u_1(q) < u_2(q) & , \quad q \in (0, +\infty) \\ u_1'(q) < u_2'(q) & , \quad q \in (0, \hat{q}_{2u}). \end{array}$$

Let $R_t(q_t) = u'(q_t)q_t, t = 1, 2$, denote the revenue function in period t and let $MR_t(q_t) \equiv R_t'(q_t) = u_t'(q_t) + u_t''(q_t)q_t$ be the corresponding marginal revenue function. We first assume that MR_t is strictly decreasing when positive. Thus there exists $\hat{q}_{tR}, 0 < \hat{q}_{tR} \leq \hat{q}_{tu}, t = 1, 2$, such that

$$MR_t(q_t) \begin{cases} > 0 & , \quad q_t \in (0, \hat{q}_{tR}) \\ \leq 0 & , \quad q_t \in [\hat{q}_{tR}, +\infty). \end{cases}$$

and:

$$MR_t'(q_t) = 2u_t''(q_t) + u_t'''(q_t)q_t < 0 \quad , \quad q_t \in (0, \hat{q}_{tR}).$$

Also we assume that, when positive, the marginal revenue in the peak period is higher than the marginal revenue in the off-peak period, for a same consumption level in each period. Thus:

$$\begin{array}{ll} \hat{q}_{1R} < \hat{q}_{2R} & \\ MR_1(q) < MR_2(q) & , \quad q \in (0, \hat{q}_{2R}). \end{array}$$

We assume that the operating cost function of the thermal plant is the same whatever the period. Let $c(q)$ be the cost function of the thermal plant which is assumed to be of class C^2 and strictly increasing and convex over the range of feasible production levels. We assume that there exists an interval $[0, \bar{q})$ such that both $u_t(q) - c(q) > 0$ and $u_t'(q) - c'(q) > 0, q \in (0, \bar{q}), \ \ t = 1, 2$ so that dispatching will always command to have positive generation for final consumers.

Concerning now the hydro-plant system, let us measure water in the dam in terms of energy units. The type of hydro-plant we have in mind is an Alpine mountain system in which the dam is located at some high altitude site whereas the turbine generators are located at a lower altitude site, so that the height of water stock in the dam itself can be neglected. Let $S_{\tau t}$ be the quantity of water available in the dam at the end of period t of day τ.

We assume that there exists no natural water inflow entering the system. All the water available in the dam must have been pumped from some source, using outside electric energy, that is energy generated by the thermal plant. Let $f_{\tau t}$ be the quantity of water pumped by the hydro-producer during period t of day τ. Assuming that the storage capacity of the dam is sufficiently large so that no storage constraint is ever active, assuming there is no loss of water within the system, and denoting by $q^H_{\tau t}$ the output of the hydro-plant in period t of day τ, absent any random event, we must have

$$S_{\tau t} \leqq \begin{cases} S_{\tau-1,t+1} + f_{\tau t} - q^H_{\tau t} & \text{if} \quad t = 1 \\ \\ S_{\tau,t-1} + f_{\tau t} - q^H_{\tau t} & \text{if} \quad t = 2 \end{cases}$$

Let us denote by $q^T_{\tau t}$ the thermal production of period t in day τ for final use within the same period. We limit our attention to cyclical stationary states in which for any τ and τ' and any t, we have:

$$f_{\tau t} = f_{\tau' t} \quad , \quad q^H_{\tau t} = q^H_{\tau' t} \quad , \quad q^T_{\tau t} = q^T_{\tau' t} \;\text{ and }\; S_{\tau t} = S_{\tau' t},$$

so that we must have $q^H_{\tau 1} + q^H_{\tau 2} \leq f_{\tau,1} + f_{\tau-1,2} = f_{\tau 1} + f_{\tau 2}$. Hence in what follows we drop the index τ.

Pumping water necessitates more energy than the pumped water can generate. Let $\alpha f_t, \alpha > 1$, be the quantity of electricity required to add f_t to the stock available in the dam at the beginning of the next period. The total production level of the thermal unit is $q^T_t + \alpha f_t$.

3 Efficient production schemes

When the decision whether to dispatch the thermal and the hydroplants and for how much is taken by one single entity, the mix of hydro and thermal electricity minimizes the total cost of electricity generation. We analyze the efficient production schemes in this section. In the next section, we will consider the final decision on the absolute values of dispatch depending on the objective of the decision-taker, either a public firm that maximizes social welfare or a private monopolist that maximizes profit.

How to produce efficiently a given pair (q_1, q_2) of electric energy for final consumers? The efficient dispatch $\{(q^H_t, q^T_t, f_t), t = 1, 2\}$ is the solution to problem P.1:

$$\text{P.1} \qquad \max_{\{(q^H_t, q^T_t, f_t),\ t=1,2\}} \quad - c(q^T_1 + \alpha f_1) - c(q^T_2 + \alpha f_2)$$

s.t.

$$
\begin{aligned}
q_t^T + q_t^H - q_t \geq 0 \quad &, \quad t = 1, 2 \qquad\qquad \gamma_t \qquad\qquad (3.1)\\
(f_1 + f_2) - (q_1^H + q_2^H) \geq 0 \quad & \qquad\qquad\qquad\qquad\quad \mu \qquad\qquad (3.2)\\
q_t^H \geq 0, \quad q_t^T \geq 0, \quad f_t \geq 0 \quad &, \quad t = 1, 2 \qquad\qquad \nu \qquad\qquad (3.3)
\end{aligned}
$$

Constraint (3.1) means that electricity cannot be stored and (3.2) recalls that the whole hydroresource comes from pumping. The first order conditions that characterize the solution to problem P.1 are detailed in the Appendix. From these conditions, we deduce that cost minimization commands constraints (3.1) and (3.2) to be binding. Actually, it would be wasteful to generate more energy than what is needed (constraint (3.1)) and to pump more water than what will be turbinated (constraint (3.2)). Hence, in what follows, we refer to (3.1) and (3.2) as equalities. Also, an immediate implication of the basic laws of thermodynamics, is that transforming first electricity into water in the dam and next water in the dam into electricity, both within the same period, is pure energy waste. Thus having both q_t^H and f_t strictly positive cannot be a component of a cost minimizing policy. Summing up:

Lemma 1 *Assume that $q_t > 0$, then cost minimization implies that:*

a - both constraints (3.1) and (3.2) must be satisfied as equalities;

b - q_t^H and f_t cannot be both strictly positive.

Proof: See Appendix A1

An immediate implication of Lemma 1-b is that the electricity produced by turbinated water within any period t is actually coming from pumped water within period $t-1$. But, because the marginal cost of the thermal plant is increasing, the period within which water has to be pumped is that period during which the consumption of the final users is the lowest one. Hence:

Lemma 2 *Whatever t and t' ($t, t' = 1, 2$ and $t \neq t'$), $q_t \geq q_{t'} > 0$ implies that $q_{t'}^H = 0$.*

Proof: See Appendix A1.

A straightforward implication of Lemma 2 is that if $q_t = q_{t'}$, then $q_{t'}^H = q_t^H = 0$ and consequently $f_{t'} = f_t = 0$. Actually, this specific result can be generalized in the following way. In the present setting, the hydro-system is a mere storage device or transfer device of the energy produced at one period to the next at a cost represented by the fraction of the energy lost during the transfer. Hence, this apparatus is to be used if and only if, without transfer, the marginal cost differential resulting from the production of quantity q_t exclusively during period t is higher than the marginal loss implied by the transfer. This is formally established in the following Lemma.

Lemma 3 *Suppose* $q_t \geq q_{t'} > 0,\ t, t' = 1, 2,$ *and* $t \neq t'$. *Then* $c'(q_t) \leq \alpha c'(q_{t'})$ *is a necessary and sufficient condition for* $f_t = f_{t'} = 0$.

Proof: See Appendix A1.

In order to illustrate how to determine the types of production profiles $(q_t, q_{t'})$ having to be supplied with hydrogeneration and thermal generation and those having to be supplied only by thermal generation, let us consider Figure 1. Let us assume that the marginal cost function is a linear function with $c'(0) \equiv lim_{q \downarrow 0} c'(q) > 0$. In the left quadrant, production levels q are measured along the vertical axis and the marginal costs c' and $\alpha c'$ are measured along the horizontal axis. In the right quadrant, q_t is measured along the vertical axis and $q_{t'}$ along the horizontal axis. Let $\underline{q}$ be that value of q such that $c'(\underline{q}) = \alpha c'(0)$. For any $q_{t'} \geq \underline{q}$ let us define $q_{tm}(q_{t'})$ as this value of q_t solving $c'(q_t) = c'(q_{t'})/\alpha$, and symmetrically for any $q_t \geq \underline{q}$ let us define $q_{t'm}(q_t)$ as this value of $q_{t'}$ solving $c'(q_{t'}) = c'(q_t)/\alpha$. For example in Figure 1 let us start from the value $\dot{q}_{t'}$ of $q_{t'}$,($\dot{q}_{t'} > \underline{q}$), then

D' in the left quadrant is the point $(\alpha c'(\dot{q}_{t'}), \dot{q}_{t'})$,

D'' within the same quadrant is the point $(c'(q_{tm}(\dot{q}_{t'})), q_{tm}(\dot{q}_{t'}))$

D''' in the right quadrant is the point $(q_{tm}(\dot{q}_{t'}), \dot{q}_{t'})$.

Now let us start from the value $\dot{q}_t$ of $q_t, \dot{q}_t > \underline{q}$. Then

C' in the left quadrant is the point $(c'(\dot{q}_t), \dot{q}_t)$,

C'' within the same quadrant is the point $(\alpha c'(q_{t'm}(\dot{q}_t)), q_{t'm}(\dot{q}_t))$,

C''' in the right quadrant is the point $(q_{t'm}(\dot{q}_t), \dot{q}_t)$.

Let q_t be any quantity of energy to be delivered in period t and $q_{t'} > q_{t'm}(q_t)$ be some quantity to be delivered in period t'. Assume first that both q_t and $q_{t'}$ are produced by the thermal plant within the delivery period. Then the thermal marginal cost of $q_{t'}$, that is $c'(q_{t'})$, is higher than the marginal cost of α additional units of energy produced in period t in the thermal plant, pumped into the dam and turbinated in period t' to deliver one unit of energy, which amounts to $\alpha c'(q_t)$. Thus for profiles $(q_t, q_{t'})$ located above the line $q_{tm}(q_{t'})$, hydrogeneration has to be used. The same argument clearly holds for profiles (q_t, q_t') located to the right of the $q_{t'm}(q_t)$ line. For profiles located below $q_{tm}(q_{t'})$ and above $q_{t'm}(q_t)$, the best solution is to use the thermal unit exclusively.

Clearly for $\alpha = +\infty$, the pumping cost in terms of energy lost is infinite so that no intertemporal smoothing device is available: q_{tm} is the vertical axis and $q_{t'm}$ the horizontal axis. At the other end of the spectrum, $\alpha = 1$, transfering energy from some period to the next is costless, and the only profiles for which hydrogeneration is not used are those profiles $(q_t, q_{t'})$ such that $q_t = q_{t'}$. Then $q_{tm}(q_{t'})$ and $q_{t'm}(q_t)$ are both the 45^o line. In such a case, whatever $(q_t, q_{t'})$, we must have $q_t^T = q_{t'}^T = (q_t + q_{t'})/2$, $q_t^H = 0$ if $q_t \leq (q_t + q_{t'})/2$, and $q_t^H = q_t - (q_t + q_{t'})/2$, if $q_t > (q_t + q_t')/2, t, t' = 1, 2$ and $t \neq t'$.

The exact shape of the no-hydro region (NH) depends on the properties of the marginal cost function. In Figure 1, we have assumed that marginal cost is linear, $c'(q) = c_0 + cq$. Under this specification, it is easy to derive $q_{tm}(q_{t'}) = (\alpha - 1)\dfrac{c_0}{c} + \alpha q_{t'}$. Therefore, the NH region expands with α and c_0 and shrinks with c.

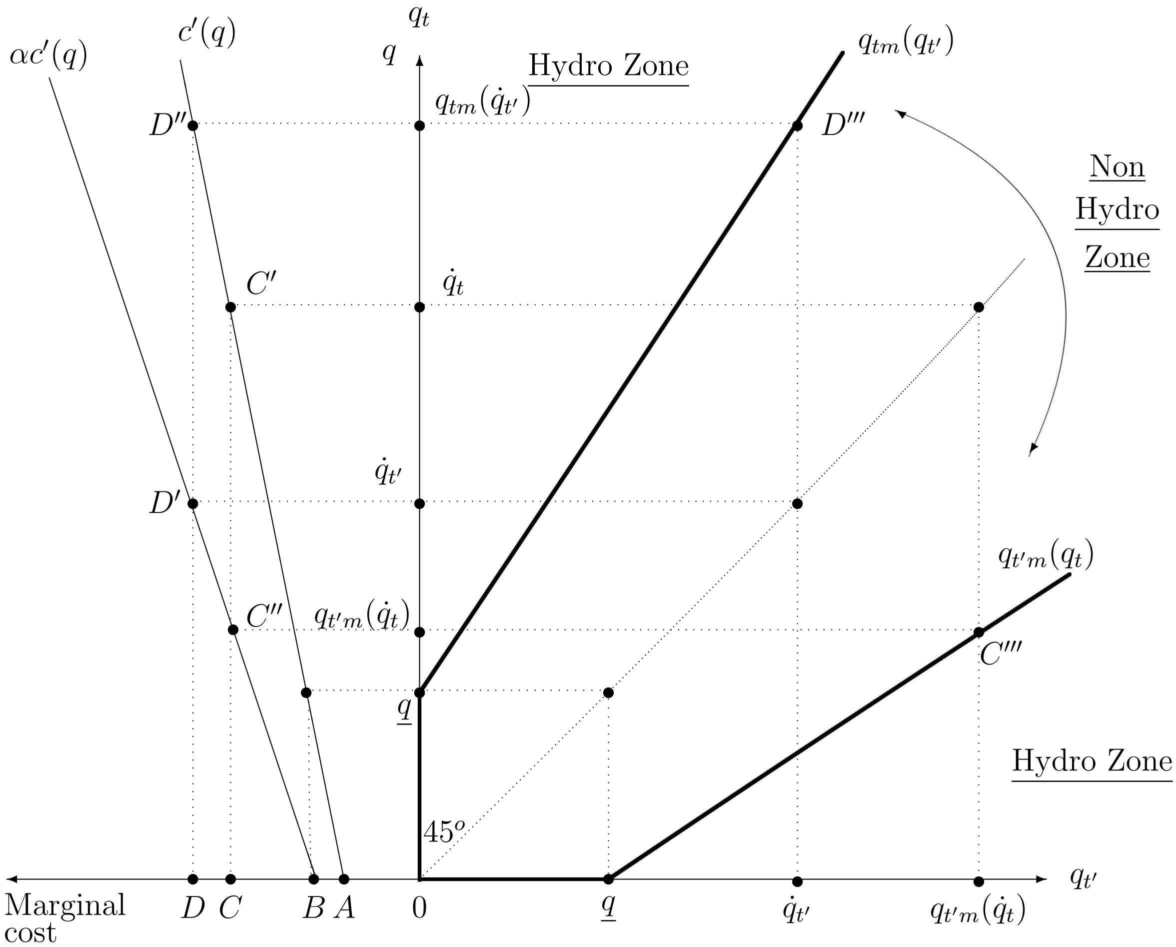

Figure 1: Determination of the production profiles $(q_t, q_{t'})$
to be produced with and without hydrogeneration

NB: $A = c'(0), B = \alpha c'(0), C = c'(\dot{q}_t) = \alpha c'(q_{t'm}(\dot{q}_t)),\ D = \alpha c'(\dot{q}_{t'}) = c'(q_{tm}(\dot{q}_{t'})), \underline{q} = q_{tm}(0) = q_{t'm}(0)$.

We see that, if $\alpha = 1$, $q_{tm}(q_t') = q_{t'}$ and the NH region vanishes since electricity can be transfered from one period to the other costless thanks to the hydroplant. When $\alpha = +\infty$ if the thermal cost is linear $(c_0 > 0, c = 0)$, it is always more efficient to rely only on instantaneous thermal generation rather than to use the storage possibility: the whole right quadrant is a NH zone. When $1 < \alpha < +\infty$ and the thermal cost is purely quadratic $(c_0 = 0, c > 0)$ the NH zone is a cone, the frontier of which only depends on the efficiency parameter α.

4 Integrated management

In this section, we successively consider the first best allocation, that is the dispatch that maximizes welfare, and the dispatch chosen by a private monopolist who controls the two plants. They only differ in the benefit function to maximize. They face the same technical constraints.

4.1 First best dispatch

The social planner solves the following problem[5]:

$$\text{P.2} \quad \max_{\{(q_t^H, q_t^T, f_t),\ t=1,2\}} \sum_{t=1}^{2} \{u_t(q_t^T + q_t^H) - c(q_t^T + \alpha f_t)\}$$
$$\text{s.t. (3.2) and (3.3).}$$

The first order conditions of P.2 are

$$q_t^T : u_t'(q_t^T + q_t^H) = c'(q_t^T + \alpha f_t) - \nu_t^T \qquad t = 1, 2 \qquad (4.1)$$
$$q_t^H : u_t'(q_t^T + q_t^H) = \mu - \nu_t^H \qquad t = 1, 2 \qquad (4.2)$$
$$f_t : \alpha c'(q_t^T + \alpha f_t) = \mu + \nu_t^f \qquad t = 1, 2 \qquad (4.3)$$

and the complementary slackness conditions (A5)-(A8) in the Appendix.

In section 2, we have assumed that $t = 2$ is the peak period. Intuition suggests that $q_2 > q_1$. This is proved in the Appendix A2. Since optimality implies efficiency, then by Lemma 2, we know that $q_1^H = 0$. Consequently, it is straightforward to obtain $f_2 = 0$[6]. Indeed, it would be inefficient to pump water at the peak period with the objective to keep it unused at the next period. The only question that is not yet solved is to know whether $f_1 = q_2^H$ is strictly positive or is nil.

Let $q_{ti}, (t = 1, 2)$, be the quantities produced and consumed at period t in a purely thermal system, that is in a system where $\alpha = +\infty$. In such a system, the optimal quantities, denoted by $q_{ti}^u, (t = 1, 2)$, are those values of q_{ti} solving:

$$u_t'(q_{ti}) = c'(q_{ti}) \quad t = 1, 2 \qquad (4.4)$$

[5]To simplify the problem, we already assume that (3.1) is an equality and we write the sum of the quantities generated by the two plants as the argument of the gross surplus functions.

[6]More formally, by (4.2) we have $\mu > 0$ so that $f_1 + f_2 = q_2^H$ by (A5) in the Appendix. Now, assume that $f_2 > 0$. By Lemma 1.b we would have $q_2^H = 0$, hence a contradiction.

We can now characterize the optimal policy in the complete system by comparison with what would prevail in a pure thermal system.

Proposition 1 *The first best dispatch must satisfy:*

- *if* $u_2'(q_{2i}^u) > \alpha c'(q_{1i}^u)$, *then:*

$$f_1 = q_2^H > 0 \quad and \quad f_2 = q_1^H = 0$$
$$u_1'(q_1^T + \alpha q_2^H) = c'(q_1^T + \alpha q_2^H)$$
$$u_2'(q_2^T + q_2^H) = c'(q_2^T) = \alpha c'(q_1^T + \alpha q_2^H)$$

- *if* $u_2'(q_{2i}^u) \leq \alpha c'(q_{1i}^u)$, *then:*

$$f_1 = f_2 = q_1^H = q_2^H = 0 \text{ and } q_t^T = q_{ti}^u \ , \ t = 1, 2$$

Proof of Proposition 1:

As we saw in Lemma 3 and in Figure 1, for a given pair of final consumptions, depending on the cost function and the loss coefficient α, the optimal solution will be either without hydrogeneration or a mix of hydro and thermal generation. The higher α, the more likely the no-hydro solution.

Starting from the only-thermal solution (q_{1i}^u, q_{2i}^u), we can define the threshold value

$$\alpha_u = \frac{c'(q_{2i}^u)}{c'(q_{1i}^u)} \tag{4.5}$$

such that if $\alpha \geq \alpha_u$ the no-hydro dispatch is optimal and if $\alpha < \alpha_u$, the best dispatch commands to mix the two technologies. Clearly if $\alpha c'(q_{1i}^u) < c'(q_{2i}^u)$, consuming the same quantity at each period but producing some part q_2^H of q_{2i}^u in period 1, that is producing thermally $q_{1i}^u + \alpha q_2^H$ in period 1 and $q_{2i}^u - q_2^H$ in period 2 would reduce the total cost provided that q_2^H is not too large. This is the case illustrated in Figure 2 for $\alpha = \alpha_1$ and it corresponds to the first part of Proposition 1. Symmetrically when $\alpha = \alpha_2$, $\alpha_2 c'(q_{1i}^u) > c'(q_{2i}^u)$ so that, it is better not to use the hydro system as established in the second part of Proposition 1. Equivalently for a given loss index α and a given cost function c, if the difference between u_1 and u_2 and u_1' and u_2' is not too large so that $q_{2i}^u - q_{1i}^u$ is small, then the hydro transfer system is useless; and for a given loss index α and a given utility function, if the marginal cost is slowly increasing, again the hydro system is not used. This case corresponds to the second part of Proposition 1.

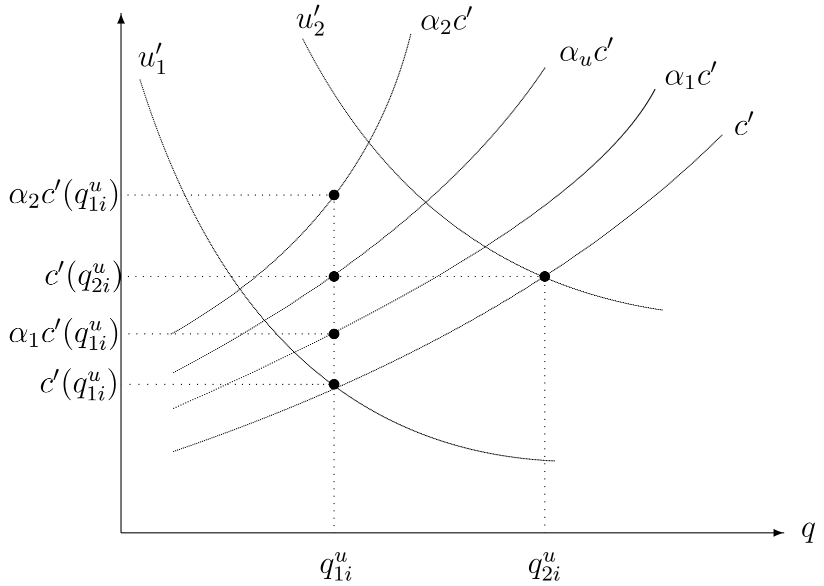

Figure 2: Deviations out of the all-thermal solution

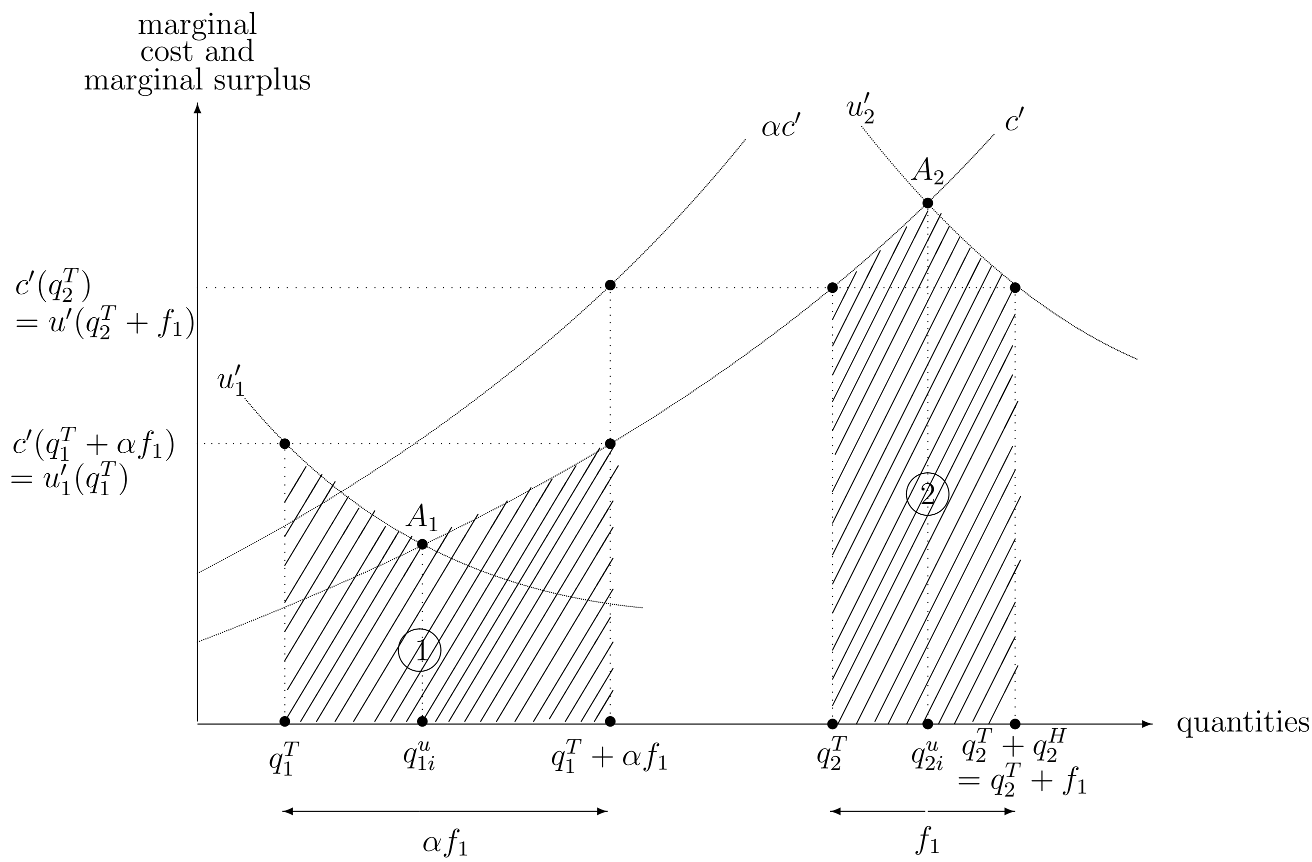

Figure 3: The welfare gains from pumping

The first best allocation is illustrated in Figure 3. The solution in a temporally isolated pure thermal system is illustrated by $A_t(t = 1, 2)$, for period t. Note that we have $q_1^T < q_{1i}^u$ that is the first best consumption in period 1 is lower than the consumption in period 1 in the temporally isolated pure thermal system, whereas the consumption in period 2, the peak period, is higher $q_2^T + q_2^H > q_{2i}^u$.

But the thermal production in period 1 is higher, and in period 2 is lower than in the pure thermal system. Compared to the pure thermal system there is a welfare loss in period 1, measured by the shaded area 1, more than compensated by a welfare gain in period 2, measured by the shaded area 2. If marginal utility is very inelastic, welfare gains from pumping are essentially technical. Otherwise, consumers incur a loss at night but their utility is increased at day.

4.2 The private monopoly

When the two plants are operated by a unique private firm, they are operated efficiently because the firm is minimizing its cost. Thus the difference between the optimally managed system and the unregulated private monopoly lies in the different objective functions each one wants to maximize. The program solved by the monopoly is like program P.2, substituting the revenue functions R_t for the gross surplus functions u_t at each period $t = 1, 2$. Let us call P.3 this program. The first order conditions are:

$$u_t'(q_t^T + q_t^H) + u_t''(q_t^T + q_t^H)[q_t^T + q_t^H] = c'(q_t^T + \alpha f_t) - \nu_t^T \quad , \quad t = 1, 2 \tag{4.6}$$

$$u_t'(q_t^T + q_t^H) + u_t''(q_t^T + q_t^H)[q_t^T + q_t^H] = \mu - \nu_t^H \quad , \quad t = 1, 2 \tag{4.7}$$

together with (4.3) and the complementary slackness conditions (A.5)-(A.8) in the Appendix.

By comparing the two dispatches solving respectively P.2 and P.3, we mean:

- First compare the consumption levels, period by period, in each setting;
- Second compare the types of production mix, thermal and hydro energies, used in each case.

Concerning the first point, the monopoly effect is the standard effect, that is a consumption level in each period lower than the optimal consumption level.

As regards the second it must be clear that anything may happen, that is:

- either the same type of mix in the two settings, both thermal and hydro energies in each one, or thermal energy only in each one;
- or different types of mix, either both energies at the optimum and only thermal under monopoly, or only thermal at the optimum but both energies under monopoly.

The reasons why we may have all these cases is illustrated in Figure 4 and with the following example.

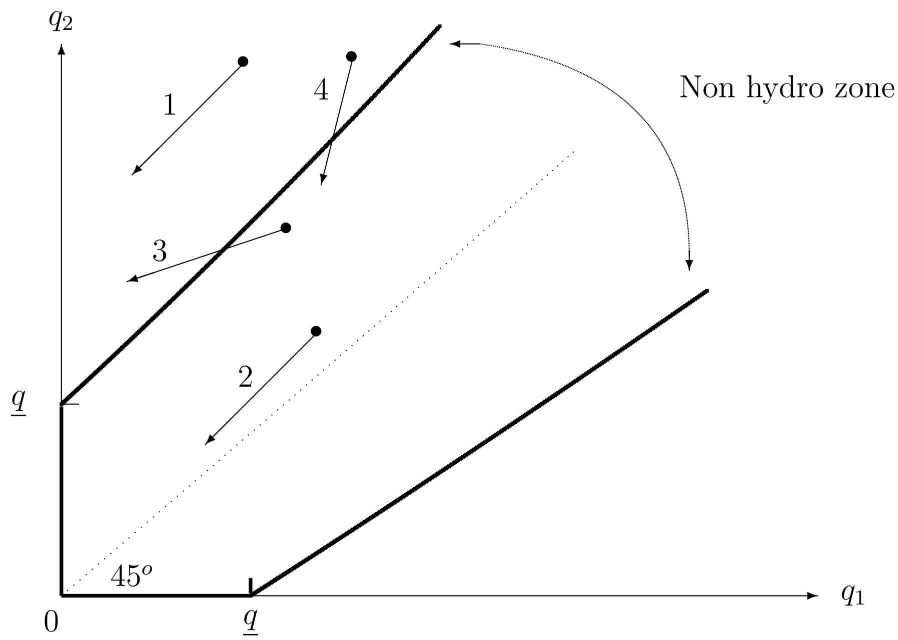

Figure 4: First best vs. private monopoly

Consider first Figure 4 which is the right hand side quadrant of Figure 1 with q_2, the peak-period production level, measured along the vertical axis, and q_1, the off-peak production level, measured along the horizontal axis. Since both optimal and monopolistic peak period production levels are higher than the off-peak period ones, both optimal and monopolistic production profiles are located above the 45^o line.

In Figure 4 each arrow illustrates a possible move from the optimal production profile toward the production profile chosen by the monopoly. With arrows 1 and 2, the types of production mix are the same in the two settings, using the two kinds of energy when moving along Arrow 1, using thermal energy only when moving along Arrow 2. With Arrows 3 and 4, the production mix changes when passing from the optimal profile to the monopoly profile. When moving along Arrow 3, thermal energy only is used at the optimum while both energies are used by the monopoly. On the contrary, when moving along Arrow 4, both energies are used at the optimum while thermal energy only is used by the monopoly.

The following example shows that any one among the moves just listed is possible.

Assume that $u_t'(q_t) = a_t - b_t q_t$ and $c'(q_t) = c.q_t, \;\; t = 1, 2$ where $a_2 > a_1 > c$.

The first-best non-hydro solution is given by $u_t'(q_t) = cq_t$, that is

$$q_{ti}^u = \frac{a_t}{b_t + c} \qquad\qquad t = 1, 2 \tag{4.8}$$

The non-hydro solution when outputs are chosen by a private monopoly is the solution to $u_t'(q_t) + q_t u_t''(q_t) = cq_t$, that is

$$q_{ti}^R = \frac{a_t}{2b_t + c} \qquad\qquad t = 1, 2 \tag{4.9}$$

With these values, it is easy to compute

$$\alpha_u = \frac{a_2}{b_2 + c} \frac{b_1 + c}{a_1} \quad \text{and} \quad \alpha_R = \frac{a_2}{2b_2 + c} \frac{2b_1 + c}{a_1}$$

which represent the threshold value of the loss due to pumping in the first best dispatch and in the monopoly dispatch respectively[7]. We see that

$$\alpha_u \lesseqgtr \alpha_R \;\; \text{according to} \;\; b_2 \lesseqgtr b_1$$

Since period 2 is the peak-period, we can assume that $b_2 > b_1$, denoting a demand less reactive to price variation. It results that

if $\alpha > \alpha_u$, the hydroplant is used neither by the social planner, nor by the monopolist (case 2 in Figure 4);

[7]The critical value of α corresponding to first best is defined in (4.5). The critical value that corresponds to the monopoly case is defined by $\alpha_R = \dfrac{c'(q_{2i}^R)}{c'(q_{1i}^R)}$ where q_{ti}^R is the value of q_{ti} solving $u_t'(q) + qu_t''(q) = c'(q), \; t = 1, 2$.

if $\alpha_u > \alpha > \alpha_R$, the monopolist does not use the hydroplant while it would be optimal to use it (case 4);

if $\alpha_R > \alpha$, the hydroplant is used by both the social planner and the monopoly (case 1).

Hereafter, we compare the optimal despatch and the monopoly's despatch when α is small enough for pumping to be used in both frameworks (case 1).

For $\alpha < \alpha_u$, from Proposition 1 we can write the first best dispatch as

$$\begin{aligned}
q_2^{Hu} &= \frac{(\alpha_u - \alpha)a_1 a_2}{\alpha^2 a_2 b_1 + \alpha_u b_2 a_1} \\
q_1^{Tu} &= \frac{\alpha_u a_1 b_2 + \alpha^2 a_2 b_1 - \alpha c a_2(\alpha_u - \alpha)}{(b_1 + c)[\alpha_u a_1 b_2 + \alpha^2 a_2 b_1]} a_1 = q_{1i}^u - \frac{\alpha c}{b_1 + c} q_2^{Hu} \\
q_2^{Tu} &= \frac{\alpha a_2[a_1 b_2 + \alpha a_2 b_1]}{(b_2 + c)(\alpha_u a_1 b_2 + \alpha^2 a_2 b_1)} = q_{2i}^u - \frac{b_2}{b_2 + c} q_2^{Hu}
\end{aligned}$$

As shown in figure 3, we observe that

$$q_1^{Tu} + \alpha q_2^{Hu} = q_{1i}^u + \alpha \frac{b_1}{b_1 + c} q_2^{Hu} > q_{1i}^u > q_1^{Tu}$$

and

$$q_2^{Tu} + q_2^{Hu} = q_{2i}^u + \frac{c}{b_2 + c} + q_2^{Hu} > q_{2i}^u > q_2^{Tu}$$

which means that at each period, the total production of electricity is larger than if the pumping plant were not available. At the off-peak period ($t = 1$) final consumption q_1^{Tu} is lowered by pumping but at the peak period ($t = 2$) final consumption $q_2^{Tu} + q_2^{Hu}$ is higher thanks to the extra energy provided by the hydroplant.

The same comments apply to the monopoly dispatch when $\alpha_R > \alpha$:

$$\begin{aligned}
q_2^{HR} &= \frac{(\alpha_R - \alpha)a_1 a_2}{2[\alpha^2 a_2 b_1 + \alpha_R a_1 b_2]} \\
q_1^{TR} &= q_{1i}^R - \frac{\alpha c}{2b_1 + c} q_2^{HR} \\
q_2^{TR} &= q_{2i}^R - \frac{2b_2}{2b_2 + c} q_2^{HR}
\end{aligned}$$

If we compare the monopoly dispatch with the first best when $\alpha_u > \alpha_R > \alpha$, it is straightforward to check that the monopoly does not use the hydroplant sufficiently as compared with first best: $q_2^{HR} < q_2^{Hu}$. This is trivially true when $\alpha_u > \alpha_R = \alpha$ since $q_2^{Hu} > 0$ while $q_2^{HR} = 0$.

It remains to compare the intensity of use of the thermal plant. We can compute that

$$q_1^{Tu} + \alpha q_2^{Hu} > q_1^{TR} + \alpha q_2^{HR} \tag{4.10}$$

and

$$q_2^{Tu} > q_2^{TR} \tag{4.11}$$

In other words, the possibility to combine the two technologies does not modify the behavior of the private monopoly: its thermal plant is used below the optimal level.

We will come back to the monopoly configuration in section 5.3.1. Figure 7 gives a graphical solution to profit maximization by an integrated firm.

5 Cournot Competition

We assume in this section that the thermal plant and the hydro plant are under the control of separate enterprises and we consider a "day-ahead" wholesale market for electricity in which:

i) the two agents announce to an independent Market Operator (MO thereafter) the quantities they intend to supply and/or demand in each period of the following day ; in other words, they play an open loop game.

ii) taking into account the demand functions of the final users the MO computes the price that clears the market in each period.

We neglect the markets where the price at which the electricity sold to the hydro producer or sold by the hydro producer is fixed by administrative rules as it is the case for generation from renewable resources in some countries. Therefore, the energy price is the same for final use and for intermediary use.

Competition between H and T is very dependant on who takes decisions as regards the flows f_t $(t = 1, 2)$. To keep things as simple as possible, we will assume from now on that $f_2 = 0$ at equilibrium, which is a trivial result given the hypothesis of peak demand at period 2. On the final market, the two distinct firms compete by fixing (q_1^H, q_2^H) for H and (q_1^T, q_2^T) for T.

The decision on the remaining variable, namely f_1, depends on the institutional setting and the network's topological characteristics. We will successively consider the three following cases :

1. Energy pool. Firm T produces the total quantity y_t^T at period t, from which q_t^T is consumed by final consumers and αf_t is consumed by H to store the quantity f_t of water.
2. Sale obligation. Firm H freely chooses to buy f_t at price p_t and firm T is obliged to serve it.
3. Purchase obligation. Firm T freely chooses to sell f_t at price p_t and firm H is obliged to buy it.

The three configurations have in common the way prices are fixed by the MO. Final consumers buy q_t at period t. Since they are price takers, their demand is the solution to

$$\max_{q_t} u_t(q_t) - p_t q_t \quad \text{where} \quad q_t = q_t^H + q_t^T, \qquad t = 1, 2.$$

From the first order condition, we can write the inverse demand function as

$$\tilde{p}_t(q_t) = u_t'(q_t^H + q_t^T) \quad t = 1, 2 \tag{5.1}$$

5.1 Energy pool

In this configuration, the thermal firm cannot separately decide on q_t^T and f_t. Its decision variable is the total quantity sold at each period:

$$y_t^T \overset{def}{=} q_t^T + \alpha f_t$$

In this setting, the hydroplant solves

$$\begin{array}{lll} \max\limits_{q_1^H, q_2^H, f_1} & \tilde{p}_1(y_1^T - \alpha f_1 + q_1^H)(q_1^H - \alpha f_1) + \tilde{p}_2(y_2^T + q_2^H) q_2^H & \\ s.t. & f_1 - (q_1^H + q_2^H) \geq 0 & \mu \\ & f_1 \geq 0 \;\;,\;\; q_t^H \geq 0 \;\;,\;\; t = 1, 2 & \nu \end{array}$$

We first prove that H will choose $q_1^H = 0$ independently of T's decisions.

The first order condition with respect to q_1^H is

$$p_1 + (q_1^H - \alpha f_1)\tilde{p}_1' + \nu_1^H - \mu = 0$$

and the first order condition with respect to $f_1 > 0$ is

$$-\alpha[p_1 + (q_1^H - \alpha f_1)\tilde{p}_1'] + \mu = 0$$

Consequently, if $q_1^H > 0$, then $\nu_1^H = 0$ and since $\alpha > 1$, the two above conditions can be satisfied only if

$$\mu = p_1 + (q_1^H - \alpha f_1)\tilde{p}_1' = 0$$

This implies $q_1^H - \alpha f_1 > 0$ by $\tilde{p}_1' < 0$. And since $\alpha > 1$, this implies in turn $q_1^H > f_1$, which clearly violates the resource constraint.

We conclude that $q_1^H = 0$, so that the hydroproducer's program can be rewritten as[8]

$$\max_{q_2^H} \ \tilde{p}_2(y_2^T + q_2^H)q_2^H - \tilde{p}_1(y_1^T - \alpha q_2^H)\alpha q_2^H$$

In order to sell one unit of electricity at period 2, H has to buy α units at period 1. Therefore, if the thermal producer sells to the final market quantities (q_1^T, q_2^T) such that $\tilde{p}_2(q_2^T) < \alpha\tilde{p}_1(q_1^T)$, H will fix $q_2^H = 0$.

Otherwise, its best response is $q_2^H = F_1(y_1^T, y_2^T)$ given by the unique solution of the first order condition

$$p_2 + \tilde{p}_2' q_2^H - \alpha[p_1 - \alpha\tilde{p}_1' q_2^H] = 0$$

under the assumption of a strictly concave profit function in q_2^H.

Differentiating the first order condition, we can write

$$sign\frac{\partial q_2^H}{\partial y_1^T} \equiv sign - \tilde{p}_1' + \alpha\tilde{p}_1'' q_2^H$$

$$sign\frac{\partial q_2^H}{\partial y_2^T} \equiv \quad sign \ \ \tilde{p}_2' + \tilde{p}_2'' q_2^H$$

[8] Because H controls both q_2^H and f_1 and it is costly to pump water, it is obvious that the constraint $q_2^H \leq f_1$ is binding.

Assuming that the curvature of demand functions is small at each period, we have that the hydroproducer will pump at period 1 and produce at period 2 a quantity increasing with the total output of T at period 1 and decreasing with its total output at period 2.

We now consider the program of T. It solves

$$\max_{y_1^T, y_2^T} \quad \sum_{t=1}^{2} \tilde{p}_t(q_t^H + y_t^T - \alpha f_t) y_t^T - c(y_t^T)$$

Taking for given that $q_1^H = f_2 = 0$, the first order conditions are

$$y_1^T.\tilde{p}_1'(y_1^T - \alpha f_1) + \tilde{p}_1(y_1^T - \alpha f_1) - c'(y_1^T) = 0$$

$$y_2^T.\tilde{p}_2'(y_2^T + q_2^H) + \tilde{p}_2(y_2^T + q_2^H) - c'(y_2^T) = 0$$

The best response functions are $y_t^T(f_1, q_2^H), t = 1, 2$ and we show in the appendix that, provided that the curvature of the demand functions be sufficiently low:

$$\frac{\partial y_1^T}{\partial f_1} > 0 \quad , \quad \frac{\partial y_1^T}{\partial q_2^H} = 0 \quad , \quad \frac{\partial y_2^T}{\partial f_1} = 0 \quad , \quad \frac{\partial y_2^T}{\partial q_2^H} < 0$$

The Nash equilibrium is then

$$f_2^N = q_1^{HN} = 0 \quad , \quad f_1^N = q_2^{HN} > 0 \quad , \quad q_1^{TN} > 0, \quad q_2^{TN} > 0$$

and consequently:

$$y_1^{TN} = q_1^{TN} + \alpha f_1^N, \quad y_2^{TN} = q_2^{TN}$$

where

$$y_1^{TN} = Y_1^T(f_1^N, q_2^{HN}), y_2^{TN} = Y_2^T(f_1^N, q_2^{HN}) \quad , \quad f_1^N = q_2^{HN} = Q_2^H(y_1^{TN}, y_2^{TN})$$

It is illustrated in Figure 5. The North-East panel shows the relationship between the demand of energy for pumping by H at period 1 as a function of the total output of T at the same period and the supply of T as a function of the demand for pumping. Both functions are increasing but the former is additionally parameterized negatively by the output of T at period 2 since all the energy bought at 1 will be used as an indirect input for the production of H at period 2. This illustrates how the client/provider relationship of period 1 is affected by competition at period 2 and depicted in the South-West panel. Here the decision variables are strategic substitutes with the best response function of the hydroproducer shifting upwards when T has a higher output at period 1 because this means more water in the reservoir.

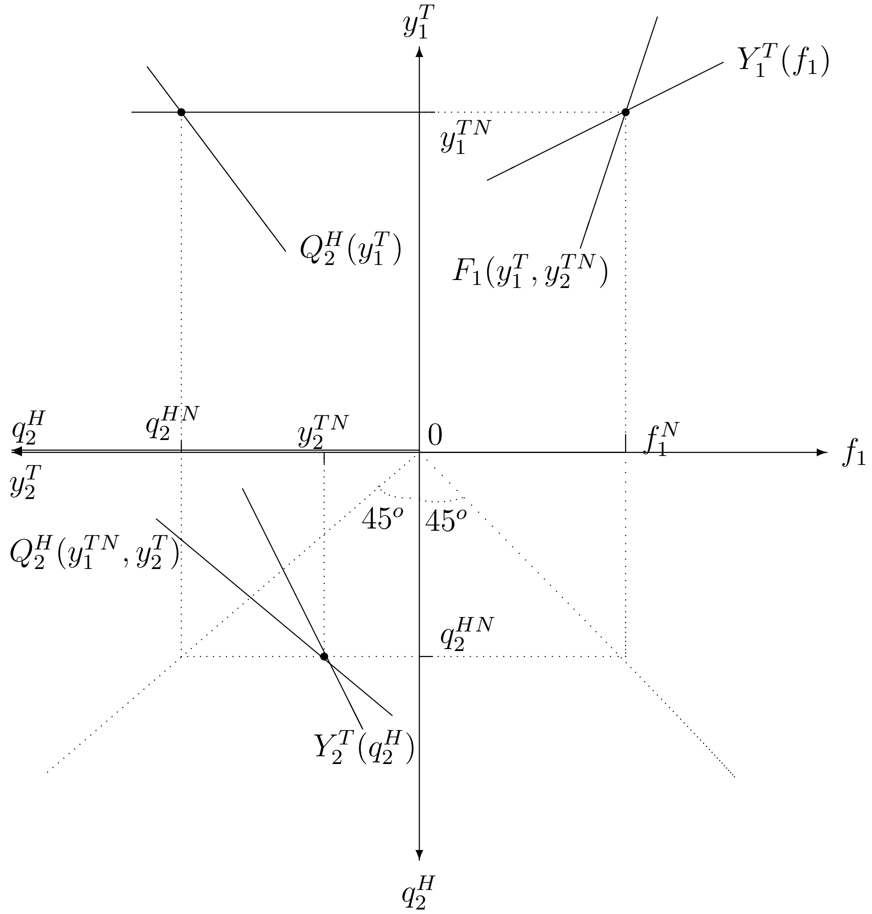

Figure 5: Nash equilibrium in the energy pool game

5.2 Sale obligation

In this institutional framework, we assume that the hydroplant is allowed to order energy from the thermal plant at market price. As we will see, this could be detrimental for financial equilibrium of the thermal plant, in which case we will assume that the energy delivered to the hydroproducer is rationed.

In this legal configuration, H chooses f_t and $q_t^H(t = 1, 2)$ and T chooses $q_t^T(t = 1, 2)$.

The objective of the thermal plant is

$$\max_{q_1^T, q_2^T} \quad \tilde{p}_1(q_1^T + q_1^H)(q_1^T + \alpha f_1) - c(q_1^T + \alpha f_1) + \tilde{p}_2(q_2^T + q_2^H)q_2^T - c(q_2^T)$$

The first order conditions for profit maximization are

$$\tilde{p}_1(q_1^T + q_1^H) + (q_1^T + \alpha f_1)\tilde{p}_1'(q_1^T + q_1^H) - c'(q_1^T + \alpha f_1) = 0 \tag{5.2}$$

$$\tilde{p}_2(q_2^T + q_2^H) + q_2^T \tilde{p}_2'(q_2^T + q_2^H) - c'(q_2^T) = 0 \tag{5.3}$$

The hydro-producer objective is

$$\begin{array}{lll} \max\limits_{q_1^H, q_2^H, f_1} & \tilde{p}_1(q_1^T + q_1^H)(q_1^H - \alpha f_1) + \tilde{p}_2(q_2^T + q_2^H)q_2^H & \\ s.t. & f_1 - (q_1^H + q_2^H) \geq 0 & \mu \\ & q_t^H \geq 0 \quad t = 1, 2 \quad , \quad f_1 \geq 0 & \nu \end{array}$$

The first order conditions are

$$p_1 + (q_1^H - \alpha f_1)\tilde{p}_1' - \mu + \nu_1^H \quad = 0 \tag{5.4}$$

$$p_2 + q_2^H \tilde{p}_2' - \mu + \nu_2^H \quad = 0 \tag{5.5}$$

$$-\alpha p_1 + \mu + \nu_1^f \quad = 0 \tag{5.6}$$

Note that $f_1 = 0$ would mean that the hydroplant stays out of the market. From now on, we assume[9] that $f_1 > 0$, so that $\nu_1^f = 0$.

With a given quantity f_1 in its reservoir, the hydro generator has to decide its allocation between the two periods. Although the hydrofirm could have a strong incentive to sell energy at the off-peak period 1 in order to lower the price it pays to fill its reservoir we first show that such an equilibrium is not stable. Hence we concentrate the analysis upon the properties of equilibria in which the hydrofirm sell electricity only in period 2.

5.2.1 The instability of equilibria with hydrogeneration at the off-peak period

Assume that, to determine the equilibrium, some tâtonnement process is used, each firm making alternate bids, and assume that there exists an equilibrium in which:

$$f_1 = q_1^H + q_2^H \quad , \quad q_1^H > 0 \quad , \quad q_2^H > 0 \quad , \quad q_1^T > 0 \quad \text{and} \quad q_2^T > 0.$$

[9]Note also that $\tilde{p}_1 = \tilde{p}_2$ would be a necessary condition for having both $f_1 > 0$ and $f_2 > 0$. Keeping the hypothesis that period 2 is the peak-period, we can assume that $\tilde{p}_2 > \tilde{p}_1$ at equilibrium, so that $f_2 = 0$ for sure. Indeed, pumping at period 1 and pumping at period 2 are perfect substitutes for the hydrogenerator and the best choice is a corner solution: all the electricity to store water will be bought at the off-peak period.

It means that starting from f_1' slightly smaller than $f_1, f_1' = f_1 - df_1, df_1 > 0$, and a $q_1^{H'}$ smaller than q_1^H by the same amount, $q_1^{H'} = q_1^H - dq_1^H, dq_1^H = df_1$, the hydrofirm could improve its profits by increasing its demand of electricity to the thermal firm in period 1 by df_1 and simultaneously increase its sales to the final users by the same amount. Let $q_1^{T'}, t = 1, 2$ and $q_2^{H'}$ be the values of the other variables at this stage of the tâtonnement process. Because $dq_1^H = df_1$ the derivative of the profit function of the hydroproducer corresponding to this change of policy is given by:

$$\frac{\partial \pi^H}{\partial q_1^H} + \frac{\partial \pi^H}{\partial f_1}$$

where $\pi^H = \tilde{p}_1(q_1^{T'} + q_1^{H'})(q_1^{H'} - \alpha f_1') + \tilde{p}_2(q_2^{T'} + q_2^{H'})q_2^{H'}$. Thus:

$$\frac{\partial \pi^H}{\partial q_1^H} + \frac{\partial \pi^H}{\partial f_1} = (1-\alpha)\tilde{p}_1(q_1^{T'} + q_1^{H'}) - \alpha f_1' \tilde{p}_1'(q_1^{T'} + q_1^{H'}).$$

If the hydroproducer is planning to sell a large quantity $q_2^{H'} = f_1' - q_1^{H'}$ at period 2 and/or if the price function of period 1 is very reactive ($\tilde{p}_1'$ large in absolute value), the second term of the right hand side in the former equation can overweight the first one. The hydroproducer has an incentive to increase its sales at the off-peak period, $dq_1^H > 0$, and simultaneously to buy more electricity at period 1, $df_1 > 0$. By doing that it intends to depress the price of period 1 at which it buys αf_1, that is to lower the cost to provide energy at the peak period.

Actually, the price will increase because the first period component of the reaction function of the thermal producer, denoted by $Q_1^T(q_1^H, f_1)$, is such that:

$$q_1^{H'} + df_1 + Q_1^T(q_1^{H'} + df_1, f_1' + df_1) < q_1^{H'} + Q_1^T(q_1^{H'}, f_1').$$

We can show it as follows. Assume that $q_1^{H'} + q_1^{T'} < q_1^H + q_1^T$. Then at the next step of the tâtonnement process the sales of period 1 are going farther from $q_1^H + q_1^T$ than $q_1^{H'} + q_1^{T'}$, and the equilibrium cannot be attained from any point located in the neighborhood of the equilibrium. To show it, let us compute $\frac{\partial q_1^T}{\partial q_1^H} + \frac{\partial q_1^T}{\partial f_1}$ from the first order condition (5.2) of the thermal producer. We obtain:

$$\frac{\partial q_1^T}{\partial q_1^H} + \frac{\partial q_1^T}{\partial f_1} = -\frac{\tilde{p}_1' + \tilde{p}_1''.(q_1^{T'} + \alpha f_1') + 2\alpha(\tilde{p}_1' - c'')}{\tilde{p}_1' + \tilde{p}_1''.(q_1^{T'} + \alpha f_1') + (\tilde{p}_1' - c'')} < 0.$$

Both the numerator and the denominator of the above ratio are negative because the second order condition of profit maximization with respect to q_1^H must be satisfied[10]. Furthermore, since $\alpha > 1$, then $2\alpha(\tilde{p}_1' - c'') < \tilde{p}' - c''$ so that:

$$\frac{\partial q_1^T}{\partial q_1^H} + \frac{\partial q_1'}{\partial f_1} < -1.$$

In words, when H increases q_1^H, it provokes a larger decrease in q_1^T and the total quantity delivered to the market decreases, so that the price increases.

Consequently any equilibrium with $q_1^H > 0$ is unstable.

5.2.2 Equilibrium analysis

We now fix $q_1^H = 0$, so that the resource's constraint is $q_2^H \leq f_1$. The best responses of H are given by the following conditions

$$\begin{aligned} \tilde{p}_2(q_2^H + q_2^T) + q_2^H \tilde{p}_2'(q_2^H + q_2^T) - \mu &= 0 \\ -\alpha \tilde{p}_1(q_1^T + \alpha f_1) + \mu &= 0 \\ q_2^H - f_1 &= 0 \end{aligned}$$

The best response function is $q_2^H(q_1^T, q_2^T)$ where

$$\frac{\partial q_2^H}{\partial q_1^T} = \frac{\alpha \tilde{p}_1'}{\Delta} \quad , \quad \frac{\partial q_2^H}{\partial q_2^T} = \frac{-(\tilde{p}_2' + q_2^H \tilde{p}_2'')}{\Delta}$$
$$\Delta = 2\tilde{p}_2' + \tilde{p}_2'' q_2^H - \alpha^2 \tilde{p}_1'$$

The concavity of the profit function of H requires that $\Delta < 0$, so that $\frac{\partial q_2^H}{\partial q_1^T} > 0$ and $\frac{\partial q_2^H}{\partial q_2^T} < 0$. In effect, when T increases its output at period 1, it decreases the market price which makes energy purchases less costly. Because its marginal cost decreases the hydrofirm increases its sales.

Figure 6 depicts the best choice of the thermal producer at periods 1 (in the left panel) and 2 (in the right panel), given q_2^H. At the off-peak period, the thermal producer faces total demand ${u_1'}^{-1}(p_1) + \alpha q_2^H$ which results in

[10]The second order condition with respect to q_1^H is:

$$\tilde{p}_1' + \tilde{p}_1''.(q_1^{T'} + \alpha f_1') + (\tilde{p}_1' - c'') < 0,$$

hence the denominator is negative. Because $\alpha > 1$, and $\tilde{p}_1' - c'' < 0$, then $2\alpha(\tilde{p}_1' - c'') < (\tilde{p}_1' - c'')$ and the numerator is also negative.

a discontinuous marginal revenue function depicted by the two-piece bold curve $MR_1(q_1)$. Actually, the intercept of the final demand curve with the vertical axis $p_1(0)$ can be viewed as a price cap for all buyers, including intermediate buyers. Therefore, when $q_1 \le \alpha q_2^H$, $MR_1 = p_1(0)$. At $q_1 = \alpha q_2^H$, marginal revenue jumps down since it is now derived from the demand of both final users and firm H. In Figure 6, we assume that the intercept of marginal revenue and marginal cost results in $Q_1^T(q_2^H) > 0$ where $Q_1^T(q_2^H)$ is the first period component of the reaction function of the thermal producer in the present context where $q_1^H = 0$. But, shifting upward the marginal cost function, it is easy to see that the best choice of firm T can be $Q_1^T(q_2^H) = 0$[11], which means that the final users receive nothing. In this case, the hydroplant can receive exactly what it demands, or it can be rationned[12], receiving less than αq_2^H. At the peak-period (see the right panel in Figure 6) as the hydro firm sells q_2^H, the marginal revenue of firm T is derived from the residual demand ${u_2'}^{-1}(p_2) - q_2^H$. The best choice of firm T is $Q_2^T(q_2^H)$ at the intersection of marginal cost and the residual marginal revenue.

[11] Again we write $Q_1^T(q_2^H)$ instead of $Q_1^T(0, q_2^H)$ to simplify the notation since $q_1^H = 0$.

[12] The reason why firm H can be rationed is that T would incur financial losses if it were obliged to serve H when its marginal cost is sharply increasing.

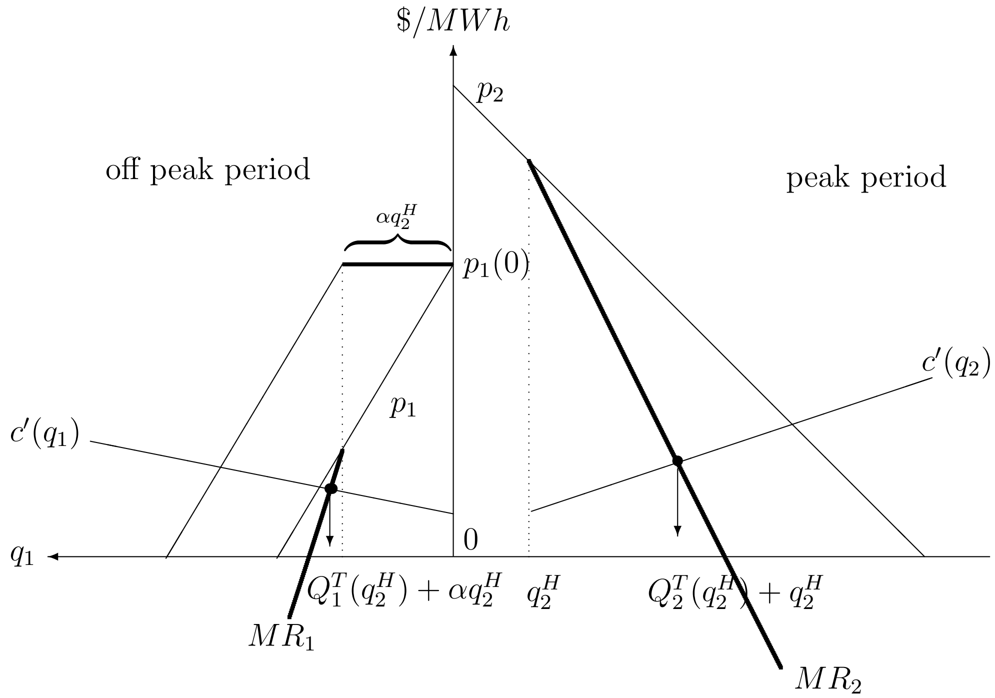

Figure 6: Best responses of the thermal plant

The T's canonical best responses are $q_t^T(f_1, q_2^H)$ such that (see proof in appendix A4)

$$\frac{\partial q_1^T}{\partial q_2^H} = 0 \quad , \quad \frac{\partial q_1^T}{\partial f_1} < 0 \quad , \quad \frac{\partial q_2^T}{\partial f_1} = 0.$$

Furthermore, provided that $\tilde{p}_2''$ be not too large if positive, then:

$$\frac{\partial q_2^T}{\partial q_2^H} < 0$$

The Cournot equilibrium under sales obligation (SO) is

$$q_t^{TSO} = Q_t^T(f_1^{SO}, q_2^{HSO}) \qquad t = 1, 2$$

$$f_1^{SO} = q_2^{HSO} = Q_2^H(q_1^{TSO}, q_2^{TSO})$$

It can be represented in a figure similar to figure 5 except that the total output of the thermal producer y_t^T is to be replaced by the quantity delivered to the final market $q_t^T (t = 1, 2)$. The consequence is that the curve $q_1^T(f_1)$ depicting the response of T at period 1 to changes in f_1 in the North-East quadrant is decreasing. In effect, when H increases its purchases, T must serve it (under the aforementioned provision that it does not incur financial losses). This increases marginal cost, which makes delivery to the final market more costly and T reacts by disminishing its output of final energy.

5.3 Purchase obligation

Whereas the former case is enforced in various countries, the case where the thermal producer can oblige the hydro producer to buy its electricity is mainly theoretical, but it allows to analyze a possible institutional arrangement with interesting economic features.

To keep things reasonable, we assume that T can oblige H to buy $f_1 \geq 0$ at period 1 but H can refuse if this would result in a negative profit. We also assume that $f_2 = 0$ since 2 is the peak period.

We first show how this setting differs from the integrated monopoly of section 4.2. Then, we determine the main features of the Nash equilibrium.

5.3.1 Differences with the monopoly case

At first sight it looks as if the thermal producer integrate the hydroproducer and use its hydroplant for developping a full monopoly power. However consider a monopoly solution in which the monopoly is using the hydroplant to produce electricity at period 2: $q_1^{Tm} > 0, q_2^{Tm} > 0, q_1^{Hm} = 0, q_2^{Hm} > 0, f_1^m = \alpha q_2^{Hm}$ and $f_2^m = 0$ where the supscrit m stands for "monopoly". This solution illustrated in figure 7 must satisfy (cf. (4.3), (4.9) and (4.10) supra):

- At period 1:
$$c'(q_1^{Tm} + \alpha q_2^{Hm}) = MR_1^m(q_1^{Tm}).$$
  The marginal cost at period 1 must be equal to the marginal revenue at the same period. (See points M_1 and M_1' in figure 7).

- At period 2:
$$\alpha c'(q_1^{Tm} + \alpha q_2^{Hm}) = c'(q_2^{Tm}) = MR_2^m(q_2^{Tm} + q_2^{Hm})$$

The marginal cost of hydro energy supplied at period 2 must be equal to the marginal cost of the thermal energy at the same period, and they must be equal to the marginal revenue. (See points M_2, M_2' and M_2'').

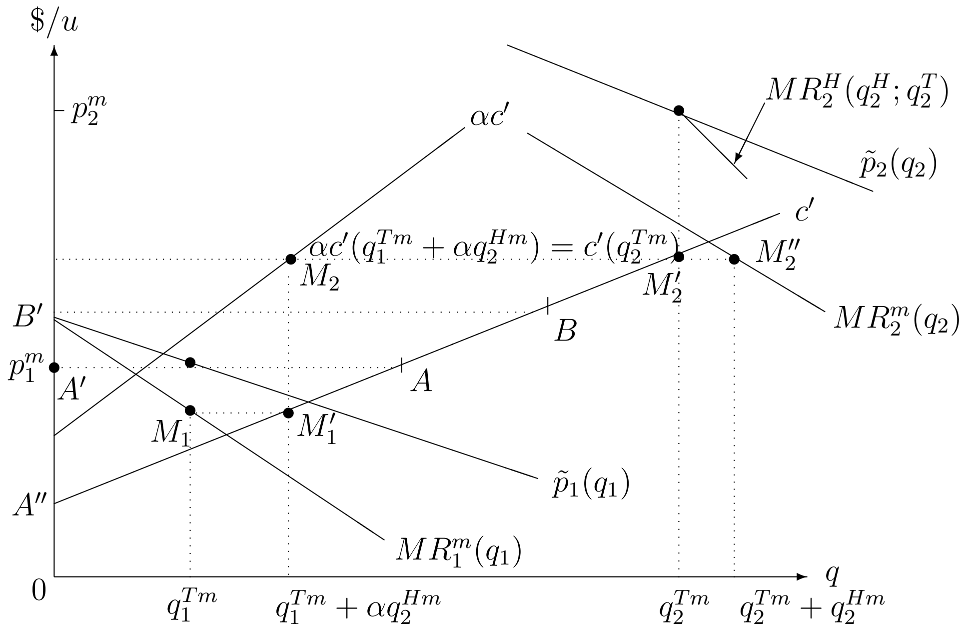

Figure 7: Monopoly equilibrium

Assume that the thermal producer try to mimic the monopoly solution in the purchase obligation setting by forcing the hydroproducer to buy $\alpha f_1 = \alpha q_2^{Hm}$ at period 1 while himself selling $q_t^{Tm}, t = 1, 2$ at period t. Figure 7 shows that the thermoproducer will be induced to depart from this solution. Given that H is forced to buy αq_2^{Hm} at period 1, and given that the thermoproducer sells q_t^{Tm} at period t, we have $MR_2^H(q_2^{Tm} + q_2^{Hm}) > p_1^m > MR_1^H(q_1^{Tm} + q_1^H), q_1^H > 0$, where MR_t^H is the marginal revenue of the hydrofirm. Thus the hydroproducer would mimic the hydroelectricity sale policy of the monopoly provided that its profits be non negative. For that it is sufficient that $\alpha p_1^m \leq p_2^m$ so that its profit margin per unit of hydroelectricity sold at period 2, which amounts to $p_2^m - \alpha p_1^m$, be non-negative. But the problem is that the first period revenue function of the thermal firm is now $\tilde{p}_1(q_1^T)(q_1^T + \alpha f_1)$ instead of $\tilde{p}_1(q_1^T)q_1^T$ for the monopoly. Thus even if the thermal producer were selling q_1^{Tm} at period 1, its marginal revenue with respect to αf_1 would be p_1^m so that it would choose the point A in the right hand side quadrant in Figure 7, at which its marginal revenue is equal to its marginal cost. But it is not clear that the thermal firm would even choose to sell q_1^{Tm} to the final users at period 1. By doing so its period 1 profits are given by the surface $AA'A''$ the difference between the horizontal p_1^m at which it sells to both the consumers and the hydrofirm, and the marginal cost curve c'. Would it sell nothing to the final users, the price at which it sells to the hydrofirm would be the period 1 choke price, and if it forces the hydrofirm to buy the quantity at which the marginal cost is equal to the choke price, corresponding to point B in the figure, its profits would be the surface $BB'A''$ which is larger than $AA'A''$. As we show in the next paragraph, it is never profitable to sell to the final users for the thermal producer at the sale obligation equilibrium, but the reverse is true for the hydroproducer.

5.3.2 Equilibrium analysis

The hydroproducer only controls q_1^H and q_2^H. Its first order conditions are given by (5.4) and (5.5). The thermal plant controls f_1, q_1^T and q_2^T. Its first order conditions are respectively

$$\begin{aligned} \alpha\tilde{p}_1(q_1^T + q_1^H) - \alpha c'(q_1^T + \alpha f_1) + \nu_1^f &= 0 \quad (5.7) \\ \tilde{p}_1(q_1^T + q_1^H) + (q_1^T + \alpha f_1)\tilde{p}_1'(q_1^T + q_1^H) - c'(q_1^T + \alpha f_1) + \nu_1^T &= 0 \quad (5.8) \\ \tilde{p}_2(q_2^T + q_2^H) + q_2^T\tilde{p}_2'(q_2^T + q_2^H) - c'(q_2^T) + \nu_2^T &= 0 \quad (5.9) \end{aligned}$$

To converge towards the equilibrium characteristics, we first establish a series of lemmas.

Lemma 4 *It is not profitable for firm T to sell to the market at period 1. Consequently, $q_1^T = 0$.*

Proof: See Appendix A5.

The intuition of this result is that T can sell any kWh either to H or to the market at period 1. But the former does not change $\tilde{p}_1(\cdot)$ whereas the latter decreases the selling price. Consequently, T will sell only to H at period 1.

Lemma 5 *It is profitable for firm T to sell energy to H at period 1. Consequently, $f_1 > 0$.*

Proof: See Appendix A5.

In effect, in the open loop framework, T does not take into account that f_1 will be used by its competitor at period 2. Therefore, it just compares the cost to produce f_1 and the instantaneous revenue from the sale of f_1.

Lemma 6 *It is profitable for firm T to sell to the market at period 2. Consequently, $q_2^T > 0$.*

Proof: See Appendix A5

Knowing that $q_1^T = 0$ by lemma 4, $q_2^T = 0$ would mean that T restricts its activity to generation and use H as a marketer. This could make sense if the generation cost at period 1, $c_1(.)$ was very small as compared with the cost at period 2, $c_2(.)$. But in our model, the cost function is the same at both periods and because of $\alpha > 1$, T should produce more energy at period 1 than what could be sold by H at period 2. This means that T would incur a higher cost than when it markets its own output at period 2. We show in the appendix that T would be better off when relying on H for marketing its energy only when H incurs financial losses. Therefore $q_2^T = 0$ cannot be observed at equilibrium.

Lemma 7 *It is profitable for firm H to sell to the market at period 2. Consequently, $q_2^H > 0$*

Proof: See Appendix A5.

Period 1 is the period of net expenses for firm H since $q_1^H \leq f_1$ and it buys αf_1 with $\alpha > 1$ at price p_1. It can also sell q_1^H at the same price. Therefore, the firm must produce at period 2 to recoup its expenditures.

Lemma 8 *The hydroproducer uses all the water made available by the thermal producer.*

Proof: See Appendix A5.

Selling electricity at period 1 allows H to decrease the price. Therefore, since it is forced to accept all the supply f_1 as long as it does not create financial losses, H has an incentive to use all the water resources pumped by the energy sold by T.

From this series of Lemma, we conclude that the best responses of T are

$$f_1 = F_1(q_1^H, q_2^H) \quad , \quad q_2^T = Q_2^T(q_1^H, q_2^H)$$

implicitly defined as the solution of:

$$\begin{aligned} \tilde{p}_1(q_1^H) - c'(\alpha f_1) &= 0 & (5.10) \\ \tilde{p}_2(q_2^T + q_2^H) + q_2^T \tilde{p}_2'(q_2^T + q_2^H) - c'(q_2^T) &= 0 & (5.11) \end{aligned}$$

It is easy to check that:

$$\frac{\partial f_1}{\partial q_1^H} = \frac{\tilde{p}_1'}{\alpha c''} < 0$$

In effect, an increase in q_1^H drives the price downward which makes the sale of f_1 less profitable. Note also that $\frac{\partial f_1}{\partial q_2^H} = 0$ because when deciding upon f_1, the thermal producer does not take into account the future use of f_1 by H (open loop strategy).

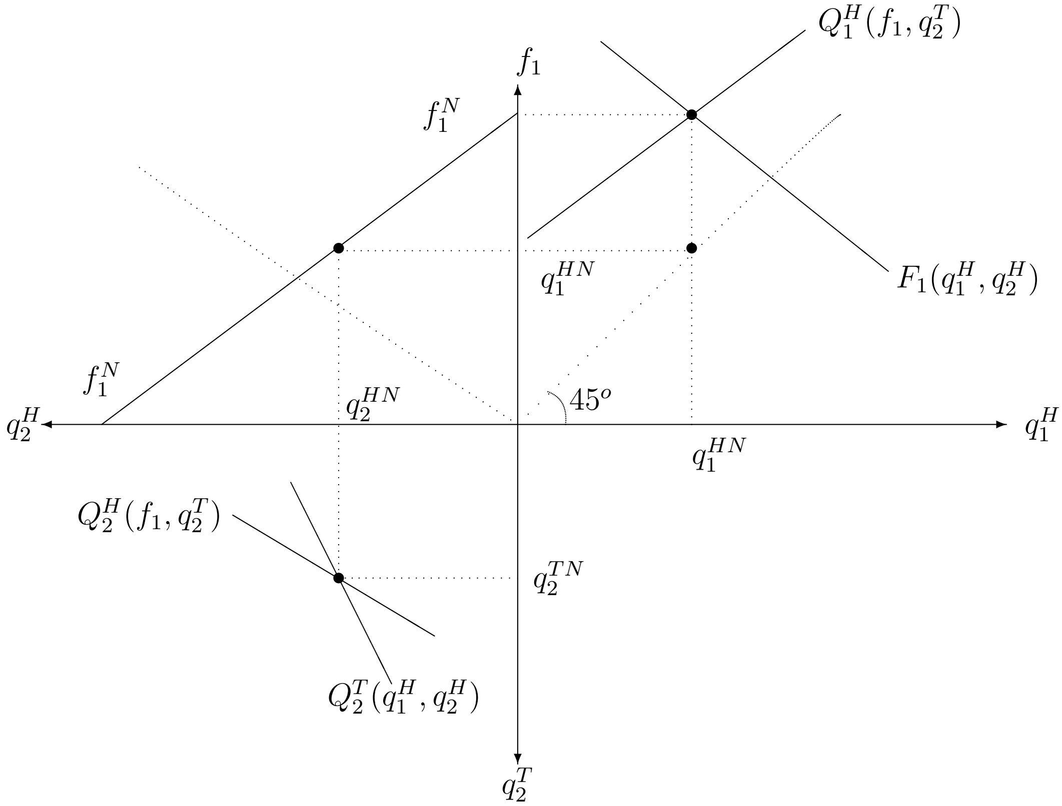

Figure 8: Nash equilibrium in the purchase obligation case

Also, $\dfrac{\partial q_2^T}{\partial q_2^H} < 0$ since the two firms are Cournot competitors at period 2 and $\dfrac{\partial q_2^T}{\partial q_1^H} = 0$ again because of the open loop framework.

The best responses of H are

$$q_1^H = Q_1^H(f_1, q_2^T)\ ,\ q_2^H = Q_2^H(f_1, q_2^T)$$

defined by

$$\tilde{p}_1(q_1^H) + (q_1^H - \alpha f_1)\tilde{p}_1'(q_1^H) + \nu_1^H = \tilde{p}_2(q_2^H + q_2^T) + q_2^H \tilde{p}_2(q_2^H + q_2^T) \quad (5.12)$$
$$q_1^H + q_2^H = f_1 \quad (5.13)$$

where ν_1^H is zero for $q_1^H > 0$.

From Lemma 4-8, we have two potential solutions at equilibrium depending on the decision of H as regards the market at period 1. If $\tilde{p}_1(0)$ is not too high, the equilibrium can be $q_1^H = 0, q_2^H = f_1$ where f_1 is the solution to

$$\tilde{p}_1(0) = c'(\alpha f_1) \text{ by } (5.10)$$

and q_2^T is given by (5.11).

But $q_1^H > 0$ is very likely in the case where $\tilde{p}_1(0)$ is high. The hydroproducer now faces the following trade-off: since $\dfrac{\partial f_1}{\partial q_1^H} < 0, \dfrac{\partial q_2^H}{\partial q_1^H} < 0$.

Therefore, the lower the price to buy water at period 1, the lower the quantity sold at period 2. From the total differentiation of equations (5.12) and (5.13), we can show that (see Appendix A6)

$\dfrac{\partial q_1^H}{\partial f_1} > 0$ because it allows to decrease the price of electricity bought at period 1.

$\dfrac{\partial q_2^H}{\partial q_2^T} < 0$ because the firms compete a la Cournot at period 2.

$\dfrac{\partial q_1^H}{\partial q_2^T} > 0$ because the competition expected at period 2 leaves more water available at period 1.

$\dfrac{\partial q_2^H}{\partial f_1} \lesseqgtr 0$ because on one hand more water available allows to sell more energy at period 2 but, on the other hand, it opens the opportunity to sell more at period 1 at the expense of period 2.

.

The Nash equilibrium $f_1^N, q_2^{TN}, q_1^{HN}, q_2^{HN}$ is obtained from the joint resolution of equations (5.10)-(5.13). It is illustrated in Figure 8.

6 Conclusions

Because hydroplants are more flexible than thermal plants to produce electricity, they play a very important role at peak hours. Indeed, in countries with hydro resources, the marginal technology (that is the last technology in merit order called to match demand) is very often the hydroelectricity technology during peak demand periods. The resource used at peak periods can come from pump storage, a technique that creates complementarity between off-peak thermal electricity and peak hydroelectricity. In this paper, we have analyzed this problem in two different frameworks: one is the joint decision framework where one single decision maker (either public or private) chooses the whole dispatch for the two types of plants for two periods in a steady state regime. In the second one, the two types of technology are under the control of separate private agents. In the first case, there is no hydroelectricity sale during the (off-peak) period of pumping. In the competition case, it may appear that the hydroplant would is better off when selling electricity and pumping simultaneously but this policy is profitable only in the legal framework where the hydroproducer is obliged to buy energy from the thermal producer. In the two other competition cases we have analyzed (when the thermal producer has a sale obligation or cannot separate the energy for final use from the energy for intermediary use), the hydroproducer does not buy and sell electricity during the same period of time.

The model can be extended in several directions, in particular by taking into consideration the hydro capacity constraints.

When the capacity of dams or the turbine capacity is binding, the best response functions of the hydroproducer are truncated and we obtain constrained equilibria. These technical constraints limit the flexibility of the hydroproducer. But in the relationship between these two types of agents, the technical constraints on the thermal producer are also more severe than what we have accomodated in our model. Warming-up delays and ramp rates can be so high that the firm is better off when keeping on producing, even though the instantaneous marginal net profit is negative. This is the case with nuclear plants. For these technical reasons, it is no longer true that the thermal firm performs static optimization. It is actually facing a dynamic optimization problem and the analysis of competition against a pumping hydrostation becomes more complicated.

7 References

Ambec S. and J. Doucet (2003), "Decentralizing Hydropower Production", *Canadian Journal of Economics*, 36(3), 588-607.

Boyle G. (1996), ed, *Renewable Energy*, Oxford: Oxford University Press.

Cassedy E. S. (2000), *Prospects for Sustainable Energy*, Cambridge: Cambridge University Press.

Crampes C. and M. Moreaux (2001), "Water Resource and Power Generation", *International Journal of Industrial Organization*, 19(6), 975-997.

Dakhlaoui A. and M. Moreaux (2006), "Trade-off between Hydro and Thermal Power Generation under Uncertainty", forthcoming in *Revista de Gestão e Economica*

Edwards B.K., S.J. Flaim and R.E. Howitt (1999), "Optimal Provision of Hydroelectric Power under Environmental and Regulatory Constraints", *Land Economics*, May, 75(2), 267-283.

Howes R. and A. Fainberg (1991), eds, *The Energy Sourcebook*, New York: American Institute of Physics.

Jackson R. (1973), "Peak Load Pricing Model of An Electric Utility Using Pumped Storage", *Water Resources Research*, vol 9, n^{o}3, 556-562.

Johansson T. B., H. Kelly, A. K. N. Reddy and R. H. Williams (1993), eds, *Renewable Energy*, London: Earthscan Publications, and Washington (D.C.): Island Press.

8 Appendix

A.1 Characterization of the solution to problem P.1

Let LP.1 be the Lagrange function of P.1

$$\begin{aligned}\text{LP.1} = \ & \sum_{t=1}^{2} \Big\{ -c(q_t^T + \alpha f_t) + \gamma_t (q_t^T + q_t^H - q_t) \\ & + \nu_t^H q_t^H + \nu_t^T q_t^T + \nu_t^f f_t \Big\} + \mu \left(\sum_{t=1}^{2} f_t - \sum_{t=1}^{2} q_t^H \right).\end{aligned}$$

The first order conditions of P.1 are

$$q_t^T : \qquad c'(q_t^T + \alpha f_t) = \gamma_t + \nu_t^T \qquad t = 1,2 \tag{A1}$$

$$q_t^H : \qquad \mu = \gamma_t + \nu_t^H \qquad t = 1,2 \tag{A2}$$

$$f_t : \qquad \alpha c'(q_t^T + \alpha f_t) = \mu + \nu_t^f \qquad t = 1,2 \tag{A3}$$

$$\gamma_t \geq 0 \ , \ q_t^T + q_t^H - q_t \geq 0 \ , \ \gamma_t (q_t^T + q_t^H - q_t) = 0 \ , \quad t = 1,2 \tag{A4}$$

$$\mu \geq 0 \ , \ f_1 + f_2 - q_1^H - q_2^H \geq 0, \ \mu (f_1 + f_2 - q_1^H - q_2^H) = 0 \tag{A5}$$

$$\nu_t^H \geq 0 \ , \ q_t^H \geq 0 \ , \ \nu_t^H q_t^H = 0 \qquad t = 1,2 \tag{A6}$$

$$\nu_t^T \geq 0 \ , \ q_t^T \geq 0 \ , \ \nu_t^T q_t^T = 0 \qquad t = 1,2 \tag{A7}$$

$$\nu_t^f \geq 0 \ , \ f_t \geq 0 \ , \ \nu_t^f f_t = 0 \qquad t = 1,2 \tag{A8}$$

Proof of Lemma 1

Assume that $q_t > 0$.

1. Suppose that (3.1) is satisfied as a strict inequality implying that $\gamma_t = 0$.

Since $q_t > 0$, then:

- either $q_t^T > 0$ so that $\nu_t^T = 0$,
- or $q_t^T = 0$, but then $q_t^H > 0$ so that $\nu_t^H = 0$.

1.a. Consider the first case $q_t^T > 0$ and $\nu_t^T = 0$. Then (A1) results in:

$$c'(q_t^T + \alpha f_t) = 0$$

Because $q_t^T > 0$, we have $c' > 0$, hence a contradiction and we are left with the second case.

1.b. When $q_t^T = 0, q_t^H > 0$ and $\nu_t^H = 0$, so that by (A2):

$$\mu = 0$$

Hence (A3) results in:

$$\alpha c'(\alpha f_t) = \nu_t^f.$$

Assume that $f_t > 0$, that is $\nu_t^f = 0$. The above equation is now:

$$\alpha c'(\alpha f_t) = 0$$

with $c' > 0$ because $f_t > 0$ and $\alpha > 0$; hence, a contradiction. Thus part b of lemma 1 is proved.

However, because $q_t^H > 0$ and $f_t = 0$, we must have:

- either $f_{t-1} > 0$ if $t = 2$
- or $f_{t+1} > 0$ if $t = 1$

in order to stay within the same day τ so that $\gamma_t = 0$ whatever the case.

Hence:

- if $f_{t-1} > 0$, then $\nu_{t-1}^f = 0$ so that (A3) results in:

$$\alpha c'(q_{t-1}^T + \alpha f_{t-1}) = 0,$$

  a contradiction since $f_{t-1} > 0$ implies $c' > 0$.

- if $f_{t+1} > 0$, then $\nu^f_{t+1} = 0$ so that (A3) results in:

$$\alpha c'(q^T_{t+1} + \alpha f_{t+1}) = 0,$$

again a contradiction.

2. Let us assume now that (3.2) is satisfied as a strict inequality so that $\mu = 0$, hence, by (A2), (A4) and (A6), both $\gamma_t = 0$ and $\nu^H_t = 0$. Since $\gamma_t = 0$ the proof runs along the same lines as in the proof of point 1.

■

Proof of Lemma 2

Suppose that $q_t \geq q_{t'} > 0$ and $q^H_{t'} > 0$. By Lemma 1.b we must have $f_{t'} = 0$, hence by lemma 1.a and (3.2) $f_t = q^H_t + q^H_{t'}$ implying that $f_t > 0$ and, by Lemma 1.b again, that $q^H_t = 0$. Thus $f_t = q^H_{t'}$ and from (3.1) as an equality by Lemma 1.a: $q^T_{t'} = q_{t'} - q^H_{t'}$ and $q^T_t = q_t$. From $q_t \geq q_{t'}$, we trivially get $q_t + \alpha f_t > q_{t'} - q^H_{t'}$. Now, note that

i) $q^H_{t'} > 0 \quad \Rightarrow \quad \nu^H_{t'} = 0$ so that, by $(A2)$, $\mu = \gamma_{t'}$

ii) Since $q^T_t = q_t$, then $\alpha c'(q^T_t + \alpha f_t) = \alpha c'(q_t + \alpha f_t)$. Futhermore:

$$f_t = q^H_{t'} > 0 \quad \Rightarrow \quad \nu^f_t = 0, \quad \text{so that, by (A3),} \quad \alpha c'(q_t + \alpha f_t) = \mu.$$

iii) From $f_{t'} = 0$ and $q^T_{t'} = q_{t'} - q^H_{t'}$, we get, by (A1):

$$\gamma_{t'} = c'(q_{t'} - q^H_{t'}) - \nu^T_{t'}$$

Summing up, we obtain:

$$\alpha c'(q_t + \alpha f_t) = \mu = \gamma_{t'} = c'(q_{t'} - q^H_{t'}) - \nu^T_{t'}$$

hence a contradiction since $\alpha > 1, \nu^T_{t'} \geq 0$ and $c'(q_t + \alpha f_t) \geq c'(q_{t'} - q^H_{t'})$ this last inequality resulting from the convexity of c and $q_t + \alpha f_t > q_{t'} - q^H_{t'}$.

■

Proof of Lemma 3

Necessity: Assume $f_t = f_{t'} = 0$. Since (3.2) is an equality we have that $q_t^H = q_{t'}^H = 0$ and, consequently, $q_t^T = q_t > 0$ and $q_{t'}^T = q_{t'} > 0$, hence $\nu_t^T = \nu_{t'}^T = 0$.

For period t, we can write from (A1) $c'(q_t) = \gamma_t$ so that $\mu = c'(q_t) + \nu_t^H$ by (A2). At date t', from (A3) we have $\alpha c'(q_{t'}) = \mu + \nu_{t'}^f$. Consequently, $c'(q_t) + \nu_t^H = \mu = \alpha c'(q_{t'}) - \nu_{t'}^f \Rightarrow c'(q_t) \leq \alpha c'(q_{t'})$.

Sufficiency: We already know by Lemma 1.b that $f_t > 0$ does not satisfy the first order conditions when $q_t > 0$. Let us prove that it is also true for $f_{t'} > 0$ as long as $c'(q_t) \leq \alpha c'(q_{t'})$.

Suppose that $f_t = 0$ and $f_{t'} > 0$. Then by (3.2) $f_{t'} = q_t^H + q_{t'}^H$, and by Lemma 1.b, $q_{t'}^H = 0$. Thus $q_{t'}^T = q_{t'}, q_t^H = f_{t'}$ and $q_t^T = q_t - q_t^H$.

Then:

$$\left.\begin{array}{l} f_{t'} > 0 \quad \Rightarrow \quad \nu_{t'}^f = 0 \\ \\ q_{t'}^T = q_{t'} \end{array}\right\} \Rightarrow \alpha c'(q_{t'} + \alpha f_{t'}) = \mu \ \text{ by (A3)},$$

$$\begin{array}{l} q_t^H > 0 \quad \Rightarrow \quad \nu_t^H = 0 \quad \Rightarrow \quad \mu = \gamma_t \ \text{ by (A2)}, \\ q_t^T = q_t - q_t^H \quad \Rightarrow \quad \gamma_t = c'(q_t - q_t^H) - \nu_t^T \ \text{ by (A1)}, \end{array}$$

from which we obtain

$$\alpha c'(q_{t'} + \alpha f_{t'}) = \mu = \gamma_t = c'(q_t - q_t^H) - \nu_t^T \leq c'(q_t - q_t^H).$$

Since $c''(.) > 0$, then $c'(q_{t'}) < c'(q_{t'} + \alpha f_{t'})$ and $c'(q_t - q_t^H) < c'(q_t)$. Thus we get:

$$\alpha c'(q_{t'}) < c'(q_t).$$

We conclude that $\alpha c'(q_{t'}) \geq c'(q_t)$ is sufficient for $f_{t'} = 0$.

■

A.2 Proof that the solution of P.2 must have $q_1 < q_2$.

Let (q_1, q_2) be a solution of the optimal dispatching problem $P.2$. Then from section 2 assumption $u_t(q) - c(q) > 0, q \int (0, \bar{q}), t = 1, 2$, we must have $q_t > 0, t = 1, 2$. Since any optimal policy must be efficient, then by Lemma 1.a:

$$q_t^T + q_t^H = q_t > 0 \quad , \quad t = 1, 2$$

so that:

$$\nu_t^H \nu_t^T = 0,$$

because either $q_t^H > 0$, or $q_t^T > 0$ or both, $t = 1, 2$. For the same reason we must have:

$$f_1 + f_2 = q_1^H + q_2^H.$$

Now assume that $q_1 \geq q_2$. Then by efficiency and Lemma 2, it must be the case that:

$$q_2^H = 0$$

Then by Lemma 1.b, either $q_1^H > 0$, in which case $f_1 = 0$, or $q_1^H = 0$. Hence we must have:

$$\begin{aligned} q_1 &= q_1^T + q_1^H = q_1^T + f_2 > 0 \\ q_2 &= q_2^T - \alpha f_2 = q_2^T - \alpha q_1^H > 0 \end{aligned}$$

where $f_2 \geq 0$.

Note that, whatever the value of f_2, $q_1 \geq q_2$ and the assumptions about u_t, $t = 1, 2$, imply that:

$$u_2'(q_2) \geq u_2'(q_1) > u_1'(q_1) \tag{A9}$$

If $f_2 = 0$, then $q_t^T = q_t > 0$, $(t = 1, 2)$, so that $q_1 \geq q_2$ and $c'' > 0$ together imply:

$$c'(q_1) \geq c'(q_2).$$

Since then $\nu_t^T = 0$, $(t = 1, 2)$, taking (A9) into account, we get by (4.1):

$$u_2'(q_2) > u_1'(q_1) = c'(q_1) \geq c'(q_2) = u_2'(q_2),$$

a contradiction.

Next, if $f_2 > 0$, then $q_2^T = q_2 > 0$ so that $\nu_2^T = 0$ and because $q_2^H = 0$, $\nu_2^H > 0$ is not necessarily excluded. Hence (4.1)-(4.2) for $t = 2$ result in:

$$u_2'(q_2) = c'(q_2^T + \alpha f_2) = \mu - \nu_2^H \leq \mu.$$

Since $f_2 = q_1^H > 0$, then $\nu_1^H = 0$ so that, for $t = 1$, (4.2) results in:

$$u_1'(q_1) = \mu.$$

Last taking (A9) into account, we obtain:

$$u_2'(q_2) \leq \mu = u_1'(q_1) < u_2'(q_2),$$

again a contradiction.

A.3 Analysis of T's best responses in the "Energy Pool" case

By total differentiation of the first order conditions

$$y_1^T \tilde{p}_1'(y_1^T - \alpha f_1) + \tilde{p}_1(y_1^T - \alpha f_1) - c'(y_1^T) = 0$$

$$y_2^T \tilde{p}_2'(y_2^T + q_2^H) + \tilde{p}_2(y_2^T + q_2^H) - c'(y_2^T) = 0$$

we deduce that

$$\frac{dy_1^T}{df_1} = \frac{\alpha(y_1^T \tilde{p}_1'' + \tilde{p}_1')}{y_1^T \tilde{p}_1'' + 2\tilde{p}_1' - c''} \sim \frac{\alpha \tilde{p}_1'}{2\tilde{p}_1' - c''} > 0$$

Similarly,

$$\frac{\partial y_2^T}{\partial q_2^H} = \frac{-(y_2^T \tilde{p}_2'' + \tilde{p}_2')}{y_2^T \tilde{p}_2'' + 2\tilde{p}_2 - c''} \sim \frac{-\tilde{p}_2'}{2\tilde{p}_2' - c''} < 0$$

.

And it is straightforward to obtain $\dfrac{\partial y_1^T}{\partial q_2^H} = 0 = \dfrac{\partial y_2^T}{\partial f_1}$

A.4 Analysis of T's best responses in the "Sale Obligation" case

From (5.2) and (5.3) in the text, the first order conditions of the thermal plant are

$$\tilde{p}_1(q_1^T) + (q_1^T + \alpha f_1)\tilde{p}_1'(q_1^T) - c'(q_1^T + \alpha f_1) = 0$$

$$\tilde{p}_2(q_2^T + q_2^H) + q_2^T \tilde{p}_2'(q_2^T + q_2^H) - c'(q_2^T) = 0$$

Total differentiation gives

$$\frac{\partial q_1^T}{\partial f_1} = -\frac{\alpha(\tilde{p}_1' - c'')}{\tilde{p}_1' + \tilde{p}_1''.(q_1^T + \alpha f_1) + (\tilde{p}_1' - c'')}$$

$$\frac{\partial q_2^T}{\partial q_2^H} = -\frac{(\tilde{p}_2' + q_2^T \tilde{p}_2'')}{2\tilde{p}_2' + q_2^T \tilde{p}_2'' - c''}$$

Therefore:

$\dfrac{\partial q_1^T}{\partial q_2^H} = 0 = \dfrac{\partial q_2^T}{\partial f_1}$ is trivial

$\dfrac{\partial q_1^T}{\partial f_1} < 0$ comes from $p_1' < 0$, $c'' > 0$ and the 2nd order conditions

$\dfrac{\partial q_2^T}{\partial q_2^H} < 0$ additionally requires $\tilde{p}_2''$ not too large when positive.

A5. The purchase obligation case

Proof of Lemma 4:

Suppose that $q_1^T > 0$. Then $\nu_1^T = 0$ and $\tilde{p}_1(.) - c'(.) = -(q_1^T + \alpha f_1)\tilde{p}_1'(.) > 0$. This would imply from (5.7) that $\nu_1^f < 0$, which is impossible. Consequently, $q_1^T = 0$.

Proof of Lemma 5:

Suppose that $f_1 = 0$. Then $q_1^H = 0$ and by (5.7) and Lemma 4, we have that

$$\tilde{p}_1(0) - c'(0) \leq 0$$

which contradicts our assumptions of section 2. Consequently, $f_1 > 0$.

Proof of Lemma 6:

Suppose that $q_2^T = 0$. Then from (5.9) we have that $\tilde{p}_2(q_2^H) - c'(0) \leq 0$. Also, we have $\tilde{p}_1(q_1^H) = c'(\alpha f_1)$ from (5.7) and Lemma 4. Consequently, $p_2 \leq c'(0) < c'(\alpha f_1) = p_1$. But if it were the case then $\pi^H = p_1.(q_1^H - \alpha f_1) + p_2 q_2^H < p_1.(q_1^H + q_2^H - \alpha f_1) < 0$ since $q_1^H + q_2^H \leq f_1$ and $\alpha > 1$.

Proof of Lemma 7:

Given that $f_1 > 0$, if $q_2^H = 0$, we would have that

$$\pi^H = p_1.(q_1^H - \alpha f_1) < 0 \;\text{ since }\; \alpha > 1 \;\text{ and }\; f_1 \geq q_1^H$$

Proof of Lemma 8:

Given the water constraint $q_1^H + q_2^H \leq f_1$ and given that $\alpha > 1$, the two first terms in the left hand side of (5.4) are strictly positive. Consequently $\mu > 0$ and the water constraint is binding.

A6. Best responses of *H* in the purchase obligation case

Let us rewrite equations (5.12) and (5.13) when $q_1^H > 0$

$$\begin{aligned} R_{m1} - \alpha f_1 \tilde{p}_1' - Rm_2 &= 0 \\ q_1^H + q_2^H &= f_1 \\ \text{where} \quad R_{m1} &\stackrel{def}{=} \tilde{p}_1(q_1^H) + (q_1^H - \alpha f_1)\tilde{p}_1'(q_1^H) \\ R_{m2} &\stackrel{def}{=} \tilde{p}_2(q_2^H + q_2^T) + q_2^H \tilde{p}_2'(q_2^H + q_2^T) \end{aligned}$$

Upon defining $\Delta \stackrel{def}{=} R_{m1}' - \alpha f_1 \tilde{p}_1'' + R_{m2}' < 0$, we obtain by total differentiation

$$\begin{aligned} \frac{\partial q_1^H}{\partial f_1} &= \frac{\alpha p_1' + R_{m2}'}{\Delta} > 0 \;\; , \;\; \frac{\partial q_1^H}{\partial q_2^T} = \frac{R_{m2}'}{\Delta} > 0 \\ \frac{\partial q_2^H}{\partial f_1} &= \frac{R_{m1}' - \alpha f_1 p_1'' - \alpha p_1'}{\Delta} \lesseqgtr 0 \;\; , \;\; \frac{\partial q_2^H}{\partial q_2^T} = -\frac{R_{m2}'}{\Delta} < 0. \end{aligned}$$