Ecological discounting

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Abstract

Which rates should we use to discount costs and benefits of different nature at different time horizons? We answer this question by considering a representative agent consuming two goods whose availability evolves over time in a stochastic way. We extend the Ramsey rule by taking into account the degree of substitutability between the two goods and of the uncertainty surrounding the economic and environmental growths. The rate at which environmental impacts should be discounted is in general different from the one at which monetary benefits should be discounted. We provide arguments in favor of an ecological discount rate smaller than the economic discount rate. In particular, we show that, under certainty and Cobb-Douglas preferences, the difference between the economic and the ecological discount rates equals the difference between the economic and the ecological growth rates. We also justify a decreasing term structure of the ecological discount rate on the basis of the large parametric uncertainty affecting the evolution of the environmental quality.

Keywords: Discounting, Ramsey rule, bivariate utility function, precautionary premium, sustainable development.

JEL Classification: G12, E43, Q51

1 Introduction

How much effort should we exert to improve the environmental quality that we will leave to future generations? This is a central question for a wide set of environmental contexts, as global warming, nuclear wastes, or biodiversity. Its answer depends upon our expectations about the quality of the environment and about the level of economic development that these future generations will face. For example, it is intuitive that our optimal effort should be relatively large if we believe that the environment will be much deteriorated in the future or/and if the economy will be ruined. The problem is made complex because of the considerable uncertainties that we face with respect to both the ecological and the economic evolutions of our societies.

There are two possible methods to evaluate the present monetary value of a sure future environmental impact. The classical one consists in first measuring the future monetary value of the impact, and second discounting this monetary equivalent impact to the present. This involves a pricing formula to value future changes in environmental quality, and an economic discount rate to discount these monetarized impacts. The second approach would consist in first discounting the future environmental impact to transform it into an immediate equivalent environmental impact, and then measuring the monetary value of this immediate impact. This involves an ecological discount rate, to discount environmental impacts. Of course, these two methods are strictly equivalent. As shown by Hoel and Sterner (2007) in the case of certainty, the two discount rates differ if the monetary value of environmental assets evolves over time.

The classical method is not well adapted to the case of uncertainty. Indeed, the value of environmental assets in the future depends upon their relative scarcity, which is unknown. This is a problem because the economic discount rate is useful to discount sure future monetary benefits. Because the monetary value of environmental impacts is uncertain, one needs to compute its certainty equivalent. This requires the use of a stochastic discount factor, which determines at the same time the risk premium and the economic discount rate. Standard pricing formulas exist that can be borrowed from the theory of finance, but there are seldom used in cost-benefit analyses of environmental projects because of their complexity. In this paper, we follow the alternative methods based on the ecological discount rate. The ecological discount factor associated to date t is the immediate sure environmental

impact that has the same impact on intergenerational welfare than a unit environmental impact at date t. The (shadow) price of an immediate environmental impact can then be used to value environmental projects. This alternative method is simpler because one does not need to compute certainty equivalent future values.

The aim of this paper is to characterize the determinants of the economic and the ecological discount rates. The efficient economic (resp. ecological) discount rate equals the marginal rate of substitution between future and present consumption (resp. environmental qualities). Since Ramsey (1928), we know that the socially efficient economic discount rate is driven by an economic growth effect: if aggregate consumption is growing over time, and if the marginal utility of consumption is decreasing, the marginal utility of consumption is decreasing with time, yielding a positive economic discount rate. A symmetric argument exists for the ecological discount rate: if the quality of the environment improves with time, and if the marginal utility of the quality of the environment is decreasing, this environmental growth effect justifies a positive ecological discount rate. On the contrary, if one believes that the quality of the environment will deteriorate over time, a negative ecological discount rate may be socially efficient. However, assuming that consumption is a substitute to the quality of the environment, the economic growth has a positive impact on the ecological discount rate, thereby potentially counterbalancing the effect of the deterioration of the environment. As observed for example by Traeger (2007), the possibility to substitute the deteriorating environment quality by other goods is at the core of the notion of sustainable development. If the substitutability is limited, the environmental deterioration effect dominates the economic growth effect, and the ecological discount rate should be small or negative, thereby inducing us to preserve environmental assets.

Following Weitzman (2007) and Gollier (2002, 2007), we consider a consumption-based theory of discount rates under uncertainty. Uncertainty adds three elements into the picture. Besides the growth effects and the substitution effect, there is a precautionary effect. The uncertainty associated to the future quality of the environment reduces the ecological rate if the marginal utility of the environment is convex in it. This assumption is very intuitive, as shown by the following thought experiment. Suppose that there are two equally-likely states of nature, one in which the environmental quality is much larger than in the other. Suppose that you must allocate to one of these two states an

environmental lottery that would preserve the mean environmental quality in the state in which this lottery is allocated. If the above mentioned assumption holds, one must prefer to put the lottery on the better state. In the terminology of Kimball (1993) and Eeckhoudt and Schlesinger (2006), a sure loss and a zero-mean risk on environmental quality are mutually aggravating.

We also exhibit a cross-precautionary effect. If the marginal utility of the environment is convex in consumption, the uncertainty on the economic growth has a negative impact on the ecological discount rate. This is because the substitution effect (which raises the ecological rate) becomes less reliable in this case. Following Eeckhoudt, Rey and Schlesinger (2007), this assumption on the preferences of the representative agent holds if a sure loss on environmental quality and a zero-mean risk in consumption are mutually aggravating. Finally, there is a correlation effect if the risks on economic growth and on the evolution of the environment are statistically related. If the marginal utility of the environment is supermodular, a positive correlation between the two variables tends to reduce the two discount rates. This is because this positive correlation tends to raise the aggregate future uncertainty, thereby inducing the representative agent to make more effort for the future.

Our analysis exhibits two arguments in favor of using an ecological discount rate smaller than the economic discount rate. Under certainty, we show that the difference between the economic and the ecological discount rates equals the difference between the economic and the ecological growth rates. A first argument is thus derived from the hypothesis that the growth of environmental quality is smaller than the economic growth. A second argument is based on the hypothesis that there is more uncertainty about the evolution of the environmental quality than on the evolution of the economy. The precautionary argument, which tends to reduce the discount rate, is thus stronger for the ecological discount rate.

An important question is to determine whether the ecological and the economic discount rates should be sensitive to the time horizon. Weitzman (2007) and Gollier (2007) have justified a decreasing term structure of the economic rate based on a learning effect in a model in which there is some parametric uncertainty affecting the growth process. We show that a similar result holds for the ecological discount rate in a model with a multi-attribute utility function when the sensitiveness of the environmental quality to changes in GDP per capita is uncertain. We believe that this argument is

particularly relevant for the ecological discount rate, because of the considerable parametric uncertainty underlying the evolution of the quality of the environment.

2   A model for efficient discount rates

We consider a simple aggregate model with two goods. The first one is an aggregate consumption good, whereas the second one is an aggregate environmental good. The latter can be seen as a quality index of the environment, which includes the comfort generated from the climate, the services extracted from the biodiversity, the morbidity due to various pollutions, or the life expectancy for example. The representative agent extracts felicity $U(x_{1t}, x_{2t})$ at date t by consuming x_{1t} when the quality of the environment is x_{2t}. We assume that the von Neumann-Morgenstern utility function $U : R^2 \rightarrow R$ is three times differentiable.

At date $t = 0$, the representative agent evaluates actions by using the following utilitarian social welfare function:

$$V = E\left[\int_0^\infty e^{-\delta t} U(x_{1t}, x_{2t}) dt\right], \tag{1}$$

where δ is an ethical parameter valuing future utils relative to current ones, and where E is the expectation operator that takes into account the fact that the pair (x_{1t}, x_{2t}) is uncertain at date $t = 0$.[1] The representative agent contemplates the possibility to sacrifice some current utility either to increase consumption at date t or to improve environmental quality at that date. The first problem refers to the choice of the economic discount rate, which discounts future consumption. The second problem refers to the choice of the ecological discount rate, which discounts future changes in the environmental quality.

We first examine the economic discount rate. Let us consider a simple marginal project that would increase consumption by a sure amount ε in period $[t, t + \Delta t]$, and that would reduce consumption by $\varepsilon e^{-r(t)t}$ in period $[0, \Delta t]$, leaving the environment unaffected by the action. Observe that this

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[1] In this paper, all expectations are taken conditional to the information available at date 0.

simple project has a sure internal rate of return $r(t)$. Implementing this marginal project would increase social welfare if

$$\left[-e^{-r(t)t}U_1(x_{10}, x_{20}) + e^{-\delta t}EU_1(x_{1t}, x_{2t})\right] \varepsilon \Delta t \geq 0,$$

or equivalently, if

$$r(t) \geq r_1(t) = \delta - \frac{1}{t} \ln \frac{EU_1(x_{1t}, x_{2t})}{U_1(x_{10}, x_{20})}. \tag{2}$$

In other words, the internal rate of return of the project must exceed a minimum threshold, $r_1(t)$, to be socially efficient. Thus, $r_1(t)$ defined by equation 2 is the socially efficient economic discount rate associated to time horizon t. It allows for the comparison of the value of different consumption increments at different dates.

Consider alternatively an investment project that increases the environmental quality by ε in period $[t, t + \Delta t]$. The standard way to include this environmental impact in the cost-benefit analysis would be to first express this impact in future monetary terms. The instantaneous value v_t of the environment at date t is measured by the marginal rate of substitution between consumption and the environment:

$$v_t = - \left. \frac{dx_{1t}}{dx_{2t}} \right|_U = \frac{U_2(x_{1t}, x_{2t})}{U_1(x_{1t}, x_{2t})}. \tag{3}$$

If the quality of the environment would be traded, v_t would be its equilibrium price, taking the aggregate consumption good as the numeraire. More generally, v_t is the instantaneous willingness to pay for improving environmental quality. Its evolution over time is uncertain, i.e., v_t is a random variable seen from $t = 0$. So is the future monetary benefit εv_t of the sure improvement of the environmental quality. Its certainty equivalent is

$$CE_t = \varepsilon \frac{Ev_t U_1(x_{1t}, x_{2t})}{EU_1(x_{1t}, x_{2t})}.$$

CE_t is the sure increase in consumption at date t that has the same effect on welfare than an ε increase in environmental quality at date t, seen from date 0. It would be the equilibrium future price P^f of an asset traded at date 0 that delivers one unit of the environmental good with certainty at date t

against the payment of P^f at that date. This certainty equivalent must then be discounted at the economic discount rate $r_1(t)$ to measure the net present monetary value of a sure future improvement of the environment.

A much simpler approach is obtained by defining an ecological discount rate. Consider a marginal project that would increase the environmental quality by a sure amount ε in period $[t, t + \Delta t]$, and that would reduce the environmental quality by $\varepsilon e^{-r(t)t}$ in period $[0, \Delta t]$. Implementing this project would be socially efficient if

$$r(t) \geq r_2(t) = \delta - \frac{1}{t} \ln \frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})}. \tag{4}$$

This equation define the ecological discount rate $r_2(t)$ associated to time horizon t. It allows to compare sure changes in the environment quality at different dates. Namely, an increase in environmental quality by ε at date t has the same effect on intertemporal welfare than an increase in current environmental quality by $\varepsilon e^{-r_2(t)t}$. In monetary terms, this is equal to $v_0 \varepsilon e^{-r_2(t)t}$.

The two methods value the environmental impact in the same way, since

$$e^{-r_1(t)t} CE_t = e^{-\delta t} \frac{EU_2(x_{1t}, x_{2t})}{U_1(x_{10}, x_{20})} = v_0 e^{-r_2(t)t}.$$

To sum up, the benefit of a unit increment in environmental quality at date t should be accounted for in the evaluation of a project as equivalent to an immediate increase in consumption by $v_0 e^{-r_2(t)t}$. This really means that environmental costs and benefits should be discounted at the ecological rate $r_2(t)$, which needs not to be the same than the economic discount rate $r_1(t)$. The potential discrepancy between the economic discount rate and the ecological discount rate takes into account of the stochastic changes in the relative social valuation of the environment.

Before examining the determinants of the ecological discount rate, let us discuss a few assumptions on the successive derivatives of the utility function that will be considered in this paper. First, we assume that U is increasing and concave in its two arguments. In addition, consider a sure loss l_i in variable i, $i = 1, 2$, and a zero-mean risk ε_j in variable j, $j = 1, 2$. Notice that a sure loss and a zero-mean risk are two "harms" for risk-averse agents. With Eeckhoudt and Schlesinger (2006, 2007), we hereafter assume that *the representative agent always prefer to incur one of the two harms for certain,*

with the only uncertainty being about which one will be received, as opposed to a 50-50 gamble of receiving the two harms simultaneously, or receiving neither. Following a terminology introduced by Kimball (1993), this means that pairs of harms are "mutually aggravating". As shown by Eeckhoudt and Schlesinger (2007), this implies that $U_{ij} \leq 0$ and $U_{ijk} \geq 0$ for all $i, j, k \in \{1, 2\}$. For example, U_{211} is positive if any zero-mean risk in consumption and any sure loss in environmental quality are mutually aggravating:

$$((x_1 + \varepsilon_1, x_2), 1/2; (x_1, x_2 - l_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1 + \varepsilon_1, x_2 - l_2), 1/2).$$

3 The determinants of the ecological discount rate

We can approximate the efficient ecological discount rate by performing a second-order Taylor expansion of $U_2(x_{1t}, x_{2t})$ around (x_{10}, x_{20}) in equation (4). It yields an "ecological Ramsey rule":

$$r_2(t) \approx \delta + R_{22} \left[g_{2t} - \frac{1}{2} P_{222} \sigma_{22t} \right] + R_{21} \left[g_{1t} - \frac{1}{2} P_{211} \sigma_{11t} \right] - R_{22} P_{221} \sigma_{12t}, \quad (5)$$

where

- $R_{ij} = R_{ij}(x_{10}, x_{20})$ where $R_{ij}(x_1, x_2) = -x_j U_{ij}(x_1, x_2)/U_i(x_1, x_2) > 0$ is the elasticity of U_i to changes in x_j. When $i = j$, this is the relative aversion to fluctuations (or inequality aversion) in dimension i. R_{21} and R_{12} are measures of the degree of substitutability, or correlation aversion.[2]

- $P_{ijk} = P_{ijk}(x_{10}, x_{20})$ where $P_{ijk}(x_1, x_2) = -x_k U_{ijk}(x_1, x_2)/U_{ij}(x_1, x_2) > 0$ is the elasticity of U_{ij} to changes in x_k. It is an index of prudence if $i = j = k$ or of cross-prudence otherwise;

- $g_{it} = (Ex_{it} - x_{i0})/x_{i0}t$ is the annualized expected growth rate of x_i in interval $[0, t]$;

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[2]Bommier (2005) discusses the notion of correlation aversion in the context of an intertemporally non-separable utility function.

- $\sigma_{ijt} = E(x_{it} - x_{i0})(x_{jt} - x_{j0})/x_{i0}x_{j0}t$ approximates the annualized covariance in $(x_{it}/x_{i0}, x_{jt}/x_{j0})$.

Beside the rate of pure preference for the present δ, the ecological discount rate has 5 determinants that are described by the 5 remaining terms in the right-hand side of equation (5). In the remainder of this section, we describe these determinants.

- $R_{22}g_{2t}$: This determinant of the ecological discount rate is based on the expectation about the evolution of the environmental good. Because the marginal utility of the environment is decreasing, any first-degree stochastic dominant shift in the distribution of the environmental quality raises r_2. For example, if we believe that the environment will improve in the future, a positive ecological return is necessary to compensate for the increased intergenerational environmental inequality that the implementation of the project would yield. This *environmental growth effect* is approximately equal to the product of the relative aversion to intergenerational environmental inequality R_{22} by the average growth rate of the environmental good g_{2t}. This effect is symmetric to the well-known economic growth effect in the Ramsey rule $r_1 \simeq \delta + R_{11}g_{1t}$, where g_{1t} is the average growth rate of consumption.

- $-0.5R_{22}P_{222}\sigma_{22t}$: By Jensen inequality, if U_2 is convex in x_2, any Rothschild-Stiglitz (1970) increase in risk on the future quality of the environment reduces the ecological discount rate. This result is intuitive, as future risk should induce us to perform more effort for this future. This result is symmetric to the notion of precautionary saving in the economic sphere. This *environmental precautionary effect* takes the form of reducing the expected environmental growth g_{2t} by the ecological precautionary premium $0.5P_{222}\sigma_{22t}$ (see Kimball (1990)).

- $R_{21}g_{1t}$: Suppose that consumption and the environment are substitutes, i.e., that U_2 is decreasing in x_1. Under this assumption, any first-degree stochastic dominant shift in future consumption raises the ecological discount rate. The intuition of this *economic growth effect* is that one should care less about the future environment if future generations will be able to compensate the environmental damages by

their better economic development. This economic growth effect is approximately equal to the product of the index R_{21} of substitutability by the average growth rate g_{1t} of consumption. When the degree of substitutability is limited, the economic growth effect is small, thereby inducing more environmental preservation.

- $-0.5R_{21}P_{211}\sigma_{11t}$: Symmetrically, if U_2 is convex in x_1, then any increase in risk on x_{1t} reduces the ecological discount rate. This is the *economic precautionary effect*, which reduces the expected economic growth g_{1t} by the economic precautionary premium $0.5P_{211}\sigma_{11t}$.

- $-R_{22}P_{221}\sigma_{12t}$: Suppose that U_2 be supermodular ($U_{221} \geq 0$), which is true if any zero-mean risk on environmental quality and any sure loss in consumption are mutually aggravating. Suppose also that the ecological risk and the economic risk are positively correlated. This would increase the global risk and the willingness to improve the future environmental quality. This is *the correlation effect*. In another context, Gollier (2007) formalizes the link between the supermodularity of a bivariate function and the positive statistical relationship of its variables. Suppose that an increase in x_{1t} yields a first-degree stochastic improvement in the conditional distribution of x_{2t}. Then it implies that, under the supermodularity of U_2, EU_2 is larger than if one would assume (x_{1t}, x_{2t}) to be independent with the same marginal distributions. This increase in EU_2 reduces the ecological discount rate.

A symmetric approximation can be derived for the economic discount rate, where the indexes $i = 1$ and $i = 2$ are exchanged. It yields

$$r_1(t) \simeq \delta + R_{11}\left[g_{1t} - \frac{1}{2}P_{111}\sigma_{11t}\right] + R_{12}\left[g_{2t} - \frac{1}{2}P_{122}\sigma_{22t}\right] - R_{11}P_{112}\sigma_{12t}. \quad (6)$$

4 Cobb-Douglas utility and lognormal distributions

In this section, we assume that the representative agent has a Cobb-Douglas utility function:

$$U(x_1, x_2) = kx_1^{1-\gamma_1}x_2^{1-\gamma_2} \quad (7)$$

in the domain $x_1 > 0$, $x_2 > 0$. The monotonicity of U with respect to x_1 and x_2 requires that

$$sgn(1 - \gamma_1) = sgn(1 - \gamma_2) = sgn(k).$$

The concavity of U with respect to x_1 and x_2 implies that γ_1 and γ_2 must be positive. Moreover, we obtain that the relative aversion to risk on x_i (R_{ii}) is a positive constant γ_i, whereas the relative correlation aversion R_{12} is a constant $\gamma_2 - 1$. We also have that $P_{iii} = 1 + \gamma_i > 0$, $P_{221} = \gamma_1 - 1$, and $P_{211} = \gamma_1$. If we assume that γ_1 and γ_2 are both larger than unity,[3] the representative agent considers pairs of harms as mutually aggravating, implying correlation aversion ($R_{ij} > 0$) and (cross-)prudence ($P_{ijk} > 0$).

We consider three different specifications for the dynamics of (x_{1t}, x_{2t}). In the first one, we suppose that it follows a bivariate geometric Brownian motion. It implies that for all t, $(\ln x_{1t}, \ln x_{2t})$ is jointly normally distributed with mean $(\ln x_{10} + \mu_1 t, \ln x_{20} + \mu_2 t)$ and variance-covariance matrix $\Sigma = (\sigma_{ij} t)_{i,j=1,2}$. The proof of the following propositions are relegated to the Appendix.

Proposition 1 *Suppose that $U(x_1, x_2) = k x_1^{1-\gamma_1} x_2^{1-\gamma_2}$ and that (x_{1t}, x_{2t}) follows a bivariate geometric Brownian motion. It implies that the ecological discount rate equals*

$$r_2(t) = \delta + \gamma_2 \left[g_2 - \frac{1}{2}(\gamma_2 + 1)\sigma_{22} \right] + (\gamma_1 - 1) \left[g_1 - \frac{1}{2}\gamma_1 \sigma_{11} \right] - (\gamma_1 - 1)\gamma_2 \sigma_{12}, \quad (8)$$

where $\sigma_{ij} = t^{-1} cov(x_{it}, x_{jt})$ and $g_i = t^{-1} \ln Ex_{it}/x_{i0} = \mu_i + 0.5\sigma_{ii}$. Symmetrically, the economic discount rate equals

$$r_1(t) = \delta + \gamma_1 \left[g_1 - \frac{1}{2}(\gamma_1 + 1)\sigma_{11} \right] + (\gamma_2 - 1) \left[g_2 - \frac{1}{2}\gamma_2 \sigma_{22} \right] - (\gamma_2 - 1)\gamma_1 \sigma_{12}. \quad (9)$$

These formulas extend the generalized Ramsey rule to an ecological economy. It shows that the approximations (5) and (6) are exact when the utility function is Cobb-Douglas and (x_{1t}, x_{2t}) are jointly lognormal. An important implication of this proposition is that the term structures of the economic discount rates and of the ecological discount rates are flat. In such an economy,

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[3]Solving the equity premium puzzle would require a relative risk aversion larger than 10. See also Drèze (1981), who suggests that relative risk aversion is around 4.

the random evolution of aggregate consumption and of the environmental quality does not justify to use a smaller rate to discount benefits occurring in a more distant future. Another immediate consequence of Proposition 1 is that

$$r_1 - r_2 = (g_1 - g_2) + (\gamma_2\sigma_{22} - \gamma_1\sigma_{11}) + (\gamma_1 - \gamma_2)\sigma_{12}. \qquad (10)$$

Interestingly enough, under certainty, the difference between the two discount rates is independent of the parameters of the Cobb-Douglas utility function. This equation provides two arguments in favor of $r_2 \leq r_1$. First, it is often suggested that the growth rate of environmental quality is smaller than the economic growth rate ($g_2 \leq g_1$), the first being potentially negative. Second, it seems that there is much more uncertainty surrounding the evolution of the environmental quality than the evolution of the economy itself ($\sigma_{22} \geq \sigma_{11}$). If the degrees aversion to risk on x_1 and on x_2 are not too heterogeneous, this would imply that $\gamma_2\sigma_{22} - \gamma_1\sigma_{11}$ be positive. Finally determining whether $(\gamma_1 - \gamma_2)\sigma_{12}$ is positive or negative is a more complex matter.

Because of the lack of time-series data about environmental quality, calibrating this specification is problematic. Various authors have argued in favor of a closer link between the environmental quality and economic growth than the one that we assumed in Proposition 1. Following this line, let us alternatively assume that the environmental quality is a deterministic function of economic achievement: $x_2 = f(x_1)$. Common wisdom suggests that the environmental quality is a decreasing function of GDP per capita, but this is heavily debated in scientific circles. The environmental Kuznets curve hypothesizes that the relationship between per capita income and the environmental quality has an inverted U-shape, but there is no consensus about it (see for example Millimet, List and Stengos (2003)). We hereafter hypothesize a monotone relationship by assuming that there exists $\rho \in R$ such that $x_2 = \eta x_1^\rho$, where ρ can be either positive or negative. If we assume that x_1 follows a geometric Brownian motion, we obtain an analytical solution for r_1 and r_2.

Proposition 2 *Suppose that $U(x_1, x_2) = k x_1^{1-\gamma_1} x_2^{1-\gamma_2}$, that $x_2 = \eta x_1^\rho$ and x_{1t} follows a geometric Brownian motion. It implies that the ecological discount rate equals*

$$r_2(t) = \delta + (\rho\gamma_2 + \gamma_1 - 1)\left[g_1 - 0.5(\rho\gamma_2 + \gamma_1)\sigma_{11}\right], \qquad (11)$$

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where $g_1 = t^{-1} \ln Ex_{1t}/x_{10}$ and $\sigma_{11} = t^{-1} Var(x_{1t})$. Symmetrically, the economic discount rate equals

$$r_1(t) = \delta + (\gamma_1 + \rho(\gamma_2 - 1)) \left[g_1 - 0.5(1 + \gamma_1 + \rho(\gamma_2 - 1))\sigma_{11} \right]. \qquad (12)$$

We also get here flat term structures of the socially efficient discount rates. In order to calibrate this model, let us assume that the rate of pure preference for the present δ is zero. We also assume that the relative aversion to risk on consumption is a constant $\gamma_1 = 2$, which is often considered as a reasonable estimation.[4] The parameter γ_2 of aversion to environmental risk is not easy to calibrate. Observe however that

$$\gamma^* = \frac{\gamma_2 - 1}{\gamma_1 + \gamma_2 - 2}$$

is the share of total consumption expenditures that the representative agent would use on environmental quality if environmental quality would be a tradable good.[5] Hoel and Sterner (2007) and Sterner and Persson (2008) suggested γ^* somewhere 10% and 50%, which yields γ_2 somewhere between 1.1 and 2 under our specification. We hereafter assume $\gamma^* = 30\%$, which implies $\gamma_2 = 1.4$.

Kocherlakota (1996) estimated the parameters of the growth process of consumption in the United States with yearly data between 1889 and 1978. He obtained $g_1 = 1.8\%$ and $\sigma_{11}^{1/2} = 3.6\%$. The choice of ρ depends upon how we define the environmental quality. In order to estimate ρ, we considered the SYS_LAN indicator contained in the Environmental Sustainability Index (ESI2005, Yale Center for Environmental Law and Policy, (2005)), which measures for 146 countries in 2005 the percentage of total land area (including inland waters) having very low or very high anthropogenic impact. The OLS estimation of the regression coefficients are as follows:

$$\ln x_2 = 1.93 - 0.10 \ln x_1 + \varepsilon$$

where x_1 is the country's GDP/cap[6] whereas x_2 is 3 plus the country's SYS_LAN indicator contained in ESI2005. The p-value for the slope-coefficient

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[4]See Drèze (1981) for example.

[5]Because the price elasticity equals -1 under this specification, this share remains constant over time.

[6]We used data from the World Economic Outlook Database of IMF, April 2008.

is -4.69, whereas the R2 coefficient equals 0.13. Plugging $\rho = -0.10$ in equations (11) and (12) yields $r_2 = 1.4\%$ and $r_1 = 3.2\%$.[7] The difference comes mostly from the large expected economic growth rate ($g_1 = 1.8\%$) compared to the expected environmental growth rate ($g_2 = \rho g_1 = 0.18\%$).

In the third specification for the dynamics of (x_{1t}, x_{2t}), we introduce some parametric uncertainty. Conditional to parameter θ, x_{1t} follows a geometric Brownian motion with drift $g_1(\theta)$ and volatility $\sigma_{11}^{1/2}(\theta)$, whereas $x_2 = \eta x_1^{\rho(\theta)}$. In this case, we obtain the following proposition.

Proposition 3 *Suppose that $U(x_1, x_2) = k x_1^{1-\gamma_1} x_2^{1-\gamma_2}$, that $x_2 = \eta x_1^{\rho}$ and x_{1t} follows a geometric Brownian motion. Suppose that the true value of triplet (g_1, σ_{11}, ρ) is uncertain at date 0 so that it depends upon some parameter θ whose cumulative distribution function is F. It implies that the ecological discount rate equals*

$$r_2(t) = \delta - \frac{1}{t} \ln \int \exp[-R_2(\theta)t] dF(\theta), \tag{13}$$

where $R_2(\theta) = (\rho\gamma_2 + \gamma_1 - 1)[g_1 - 0.5(\rho\gamma_2 + \gamma_1)\sigma_{11}]$. Symmetrically, the economic discount rate equals

$$r_1(t) = \delta - \frac{1}{t} \ln \int \exp[-R_1(\theta)t] dF(\theta), \tag{14}$$

where $R_1(\theta) = (\gamma_1 + \rho(\gamma_2 - 1)) [g_1 - 0.5(1 + \gamma_1 + \rho(\gamma_2 - 1))\sigma_{11}]$.

By Jensen inequality, this immediately implies that the term structures of r_1 and r_2 are decreasing. The short-term discount rate $r_i(t)$ equals δ plus the mean of R_i when t tends to zero, and it tends to δ plus the smallest possible value of $R_i(\theta)$ when t tends to infinity. These results generalize those obtained by Weitzman (2007) and Gollier (2007) to multiattribute utility functions. They both assumed that the economic growth rate was affected by parametric uncertainty. Suppose alternatively that g_1 and σ_{11} are known, but the elasticity ρ of environmental quality to changes in GDP is not. Rather than assuming that $\rho = -0.1$ as above, let us suppose that ρ is either -0.6 or $+0.4$ with equal probabilities. All other parameters remain

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[7]This solution is only marginally sensitive to the choice of γ_2. To illustrate, assuming $\gamma_2 = 2$ rather than 1.1 would yield $r_2 = 1.3\%$ and $r_1 = 3.1\%$.
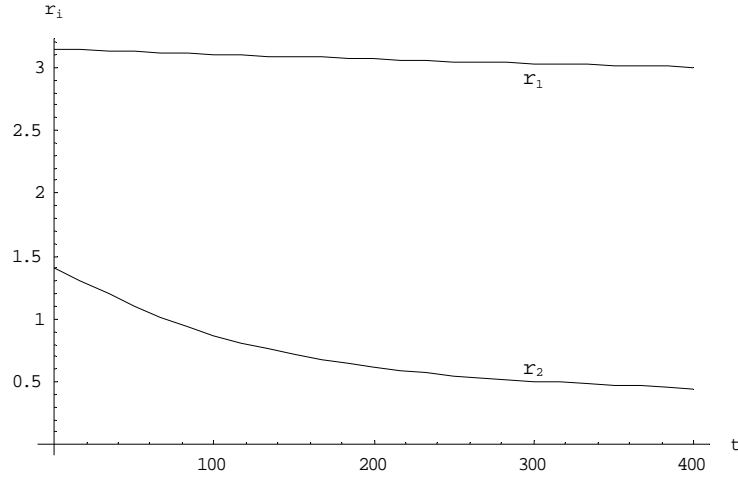
Figure 1: The term structures of the economic and ecological discount rates (in %), assuming $\delta = 0$, $\gamma = 2$, $\gamma_2 = 1.4$, $g_1 = 1.8\%$, $\sigma_{11}^{1/2} = 3.6\%$ and $\rho \sim (-0.6, 1/2; 0.4, 1/2)$.

unchanged. We draw the term structure of r_1 and r_2 in Figure 1. Whereas the economic discount rate is almost independent of time horizon, the ecological discount rate goes from 1.4% to 0.3% when t goes from 0 to infinity. The high uncertainty affecting the long-term evolution of the environment in this specification explains why the term structure of the ecological discount rate is decreasing.

5   Related literature

The Cobb-Douglas specification is the only one that yields an analytical solution for the integrals in (2) and (4) under a realistic description of expectations about (x_{1t}, x_{2t}) under uncertainty. Various authors have recently examined the term structure of the ecological discount rate when the economic growth rate is a constant g_1, i.e. $x_{1t} = x_{10}e^{g_1 t}$, and the environmental quality is a constant x_2. In that case, it is easy to check that equations (4)

and (2) simplify to

$$r_2(t) = \delta + g_1 \frac{\int_0^t R_{21}(x_{10}e^{g_1\tau}, x_2)d\tau}{t},$$

and

$$r_1(t) = \delta + g_1 \frac{\int_0^t R_{11}(x_{10}e^{g_1\tau}, x_2)d\tau}{t}.$$

These two equations immediately yield the following result.

Proposition 4 *Suppose that consumption grows at a positive constant rate and that the environmental quality is stable over time. Then,*

1. *(Gollier (2002)) the economic discount rate is decreasing (resp. increasing) with the time horizon if the relative aversion to consumption risk $R_{11}(x_1, x_2) = -x_1 U_{11}(x_1, x_2)/U_1(x_1, x_2)$ is decreasing (resp. increasing) with consumption.*

2. *the ecological discount rate is decreasing (resp. increasing) with the time horizon if the elasticity of the marginal utility of the environment with respect to consumption, $R_{21}(x_1, x_2) = -x_1 U_{21}(x_1, x_2)/U_2(x_1, x_2)$, is decreasing (resp. increasing) with consumption.*

The intuition for property 1 is that, under R_{11} decreasing, the rate of change of the marginal utility of consumption is decreasing in the degree of economic development, thereby reducing the size of the wealth effect. This result parallels the one obtained by Gollier (2002) with only one good. A parallel intuition holds for property 2: Under R_{21} decreasing, the rate of change of the marginal utility of the environment is decreasing in the degree of economic development, yielding a decreasing economic growth effect.

Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008) and Traeger (2007) obtained special cases of the results in Proposition 4 by considering the set of utility functions with constant elasticity of substitution (CES) characterized by

$$U(x_1, x_2) = \frac{1}{1-\alpha} y^{1-\alpha} \text{ with } y = \left[(1-\gamma)x_1^{\frac{\sigma-1}{\sigma}} + \gamma x_2^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad (15)$$

where $\sigma > 0$ is the elasticity of substitution, $\alpha > 0$ is relative aversion towards the risk on "aggregate good" y, and $\gamma \in [0, 1]$ is a preference weight

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in favor of the environment. When σ tends to unity, y tends to $x_1^{1-\gamma}x_2^{\gamma}$, and U tends to a Cobb-Douglas utility (7) with $1 - \gamma_1 = (1 - \alpha)(1 - \gamma)$ and $1 - \gamma_2 = (1 - \alpha)\gamma$. It is easy to check that

$$R_{11}(x_1, x_2) = \left(\alpha - \frac{1}{\sigma} \right)(1 - \gamma^*(x_1/x_2)) + \frac{1}{\sigma},$$

and

$$R_{21}(x_1, x_2) = \left(\alpha - \frac{1}{\sigma} \right)(1 - \gamma^*(x_1/x_2)).$$

with

$$\gamma^*(x) = \frac{\gamma}{(1 - \gamma)x^{\frac{\sigma-1}{\sigma}} + \gamma}. \tag{16}$$

Corollary 1 *(Guesnerie (2004), Hoel and Sterner (2007), Traeger (2007)) Suppose that the growth rate of consumption is a positive constant g_1, and that the environmental quality is stable. Suppose also that the utility function is characterized by (15). The economic discount rate and the ecological discount rate are decreasing (increasing) with the time horizon if $(\alpha\sigma - 1)(\sigma - 1)$ is negative (positive).*

When the elasticity of substitution σ tends to unity, the term structures are flat, which can be seen as a special case of Proposition 1.

6   Conclusion

Environmentalists are often quite skeptical about using standard cost-benefit analysis to shape environmental policies because environmental damages incurred in the distant future are claimed to receive insufficient weights in the economic evaluation. This may be due either because future environmental assets are undervalued, or because the economic discount rate is too large. In this paper, we address these two questions altogether by defining an ecological discount rate compatible with social welfare when the representative agent cares about both the economic and ecological environment faced by future generations. This ecological rate at which future environmental damages are discounted may be much smaller than the economic rate at which economic damages are discounted, because of the integration of the potentially increasing willingness to pay for the environment into the ecological

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discount rate. We have also shown in this paper that the uncertainties surrounding the evolutions of the environment and the economy tend to reduce the discount rates, in particular if they are positively correlated.

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APPENDIX

Proof of Proposition 1

Under the specification of this proposition, we can rewrite $EU_2(x_{1t}, x_{2t})$ as

$$EU_2(x_{1t}, x_{2t}) = k(1 - \gamma_2)E\left[\exp z_t\right],$$

where $z_t = (1 - \gamma_1)\ln x_{1t} - \gamma_2 \ln x_{2t}$ is normally distributed with mean

$$Ez_t = (1 - \gamma_1)(\ln x_{10} + \mu_1 t) - \gamma_2(\ln x_{20} + \mu_2 t)$$

and variance

$$Var(z_t) = \left((1 - \gamma_1)^2 \sigma_{11} + \gamma_2^2 \sigma_{22} - 2(1 - \gamma_1)\gamma_2 \sigma_{12}\right) t.$$

As is well-known, the Arrow-Pratt approximation is exact for an exponential utility function with a normally distributed random variable. It implies that

$$EU_2(x_{1t}, x_{2t}) = k(1 - \gamma_2)E\left[\exp z_t\right] = k(1 - \gamma_2)\exp\left(Ez_t + 0.5 Var(z_t)\right).$$

This implies in turn that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \exp\left((1 - \gamma_1)g_1 - \gamma_2 g_2 + 0.5\left(\gamma_1(\gamma_1 - 1)\sigma_{11} + \gamma_2(\gamma_2 + 1)\sigma_{22} - 2(1 - \gamma_1)\gamma_2 \sigma_{12}\right)\right) t,$$

where g_i is the expected growth rate of x_{it}: $Ex_{it} = x_{i0}e^{g_i t}$.[8] Applying (4) concludes this proof. A symmetric analysis can be made for $r_1(t)$. ∎

Proof of Proposition 2

We can rewrite $U_2(x_1, x_2) = k\eta^{-\gamma_2}(1 - \gamma_2)x_1^{1 - \gamma_1 - \rho\gamma_2}$, which implies that $EU_2(x_{1t}, x_{2t})$ be proportional to $E\exp\left[(1 - \gamma_1 - \rho\gamma_2)\ln x_{1t}\right]$. Again, since the Arrow-Pratt approximation is exact for an exponential utility function with a normally distributed random variable, we have that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \exp\left((1 - \gamma_1 - \rho\gamma_2)(\mu_1 t + 0.5(1 - \gamma_1 - \rho\gamma_2)\sigma_{11} t)\right), \quad (17)$$

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[8]Using Ito's Lemma or the property that the Arrow-Pratt approximation is exact in this framework yields that $g_i = \mu_i + 0.5\sigma_{ii}$.

with $\mu_1 = t^{-1}E\ln(x_{1t}/x_{10}) = g_1 - 0.5\sigma_{11}$. Applying (4) concludes this proof. A symmetric analysis can be made for $r_1(t)$, after noticing that $U_1(x_1, x_2) = k\eta^{1-\gamma_2}(1-\gamma_1)x_1^{-\gamma_1-\rho(\gamma_2-1)}$. $\blacksquare$

Proof of Proposition 3

We limit the proof to $r_2(t)$. Conditional to θ, equation (17) holds, which implies that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \int \exp\left[-R_2(\theta)t\right]dF(\theta).$$

Applying (4) concludes this proof. $\blacksquare$