Correlated Risks and the Value of Information for Agricultural Producers*

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Abstract

This paper considers a risk-neutral agricultural producer who faces two correlated risks: a risk on the level of output and a risk on the price of the output. It shows a case in which the value of information about the risk on output always increases with the coefficient of correlation (in absolute value) between the two risks, but it also shows a case where it may decrease with this coefficient.

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1 Introduction

In modern financial theory as in many other branches of economics or decision sciences, the ability to cope with situations involving multiple risks has represented a major progress (see, e.g., Gollier, 2001). Quite surprisingly the literature on the value of information - whether theoretical or applied - has been mostly developed under the assumption that the decision-maker faces a single source of risk.

In this paper, we consider a model with multiple risks and we examine the effect of the statistical correlation between these risks on the value of information. Intuitively, this effect should be positive. Indeed when the correlation (in absolute value) between two risks increases, receiving information on one of them offers indirectly a partial increase of information on the other risk. This suggests that one piece of information has more value since it reduces the ex post variance of two risks (a consequence of the Blackwell (1951)'s theorem), and not only that of one risk. Hence, one expects the value of information to increase with the correlation (in absolute value) between two risks. We show that this intuition is basically correct but that it does not account for the full relationship between correlation and information value. In fact, we specify a case where the intuition is fully satisfied and we identify a case where it is either incomplete or even wrong.

While our paper is mostly theoretical, it may have empirical implications. Consider for instance weather forecasts. Their value is most of the time determined through the information they yield on the future random output assuming the other elements of the decision-makers' environment are known with certainty (Adams et al., 1995, Bontems and Thomas, 2000, Chavas et al., 1991, Mjelde et al., 1998). However, a favorable climate not only raises the realized output for most producers above its planned level but it also puts a downward pressure on the market price since a majority of producers in a given area are affected in the same way. Hence a weather forecast gives not only a direct information on future output but it also gives an indirect information on future price through the correlation (negative in this case) between quantities and prices. Because this negative correlation tends naturally to stabilize total receipts (see, e.g., McKinnon, 1967), it reduces the risk faced by the agricultural producer which may reduce the value of information.[1] Hence this effect works in the opposite direction of the first

[1] There is no general result on the link between more riskiness and information value,

general positive effect described earlier.

Our paper is organized as follows. In the next two sections, we describe the model and its implications under quite general conditions. The following two sections are devoted to specific cases in order to better illustrate the forces at work. For instance, we analyze in section 4 the benchmark case of additive output risk. In this case, only the positive general effect is present and it yields a nice symmetric relationship between the degree of correlation and information value. In section 5, we turn to the more realistic case of a multiplicative output risk where we observe a lack of symmetry between the degree of correlation and information value. The last section concludes.

2 The model

We consider a risk-neutral mono-product farmer who faces two risks, a risk on output and a risk on price (see, e.g., McKinnon 1967, Eeckhoudt and Hansen, 1989). This farmer selects ex ante a single input level x yielding a random future output q given by

$$q(x, \widetilde{\varepsilon}), \tag{1}$$

where $\widetilde{\varepsilon}$ is a random element (e.g. climate conditions) with $E\widetilde{\varepsilon} = 0$. As is standard, the marginal productivity of x is positive ($q_1 > 0$) and non increasing ($q_{11} \leq 0$) for all realizations of $\widetilde{\varepsilon}$. By convention ε has a beneficiary effect on the realized output ($q_2 > 0$) and it may or may not affect the marginal productivity of x which is reflected by the sign of the second cross partial derivative of q (i.e. q_{12}).

To the extent that most producers of the same crop are affected in the same way by the random element $\widetilde{\varepsilon}$ they expect that in general the future unit price of their output will be (negatively) related to $\widetilde{\varepsilon}$ so that

$$\widetilde{p} = p_0(1 + \rho\widetilde{\varepsilon}), \tag{2}$$

where ρ is the correlation coefficient between perceived price and realized output and where p_0 is the expected price.[2] To avoid negative prices we assume that $\varepsilon \geq -1$.

except under linearity assumptions of the payoff function (see, e.g., Hess, 1982). This observation suggests that our result critically depends on the functional form of the payoff function, as we will show.

[2] A slightly more general model could have been considered: $\widetilde{p} = p_0(1 + \widetilde{\eta} + \rho\widetilde{\varepsilon})$, where

Although the story that motivates this model suggests that ρ is negative, we will consider throughout values of ρ ranging from -1 to $+1$ in order to guarantee the generality of the results.[3]

Given these specifications and without any information on the realization of ε the producer solves

$$\max_x \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)q(x,\varepsilon) - rx]dF(\varepsilon), \tag{3}$$

in which r is the certain unit price of x and $F(\varepsilon)$ is the cumulative distribution of ε.

The first and second order conditions (FOC, SOC) for a maximum are respectively

$$\int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)q_1(x,\varepsilon) - r]dF(\varepsilon) = 0, \tag{4}$$

and

$$D \equiv \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)q_{11}(x,\varepsilon)]dF(\varepsilon) < 0. \tag{5}$$

Note that since $(1+\rho\varepsilon)$ is never negative $q_{11} < 0$ is sufficient to satisfy (5).

The optimal solution to (4) will be denoted $\widehat{x}(\rho)$ showing the dependence of x upon ρ at the optimum. Differentiating (4) yields

$$\frac{\partial \widehat{x}}{\partial \rho} = \frac{\int_{-1}^{+\infty}[p_0\varepsilon q_1(x,\varepsilon)]dF(\varepsilon)}{-D}. \tag{6}$$

The sign of $\frac{\partial \widehat{x}}{\partial \rho}$ will play an important role later and it is easy to show using the Covariance rule that

$$q_{12} \lesseqgtr 0 \Longleftrightarrow \frac{\partial \widehat{x}}{\partial \rho} \lesseqgtr 0. \tag{7}$$

Because information value is a difference between optimal expected profits with and without information, we introduce the optimal value of x into (3)

$\widetilde{\eta}$ is independent from $\widetilde{\varepsilon}$ and is a zero-mean risk, $E\widetilde{\eta} = 0$. In this model, even when $\rho = 0$, there would still be a risk of price (independent from the risk on output). Yet, under risk-neutrality, only the correlated part of risk of price matters so that we can set $\widetilde{\eta}$ to its mean without loss of generality.

[3] To motivate the possibility of a positive value for ρ, suppose that $\widetilde{\varepsilon}$ stands for a demand level with a positive ε indicating an abnormally high level. As a result the price received by the producer will also be higher resulting in a positive correlation.

to express the expected level of profits in the absence of information ($E(\widetilde{\pi}_1)$):

$$E(\widetilde{\pi}_1) = \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)q(\widehat{x}(\rho),\varepsilon) - r\widehat{x}(\rho)]dF(\varepsilon). \tag{8}$$

If the decision-maker now receives perfect information on the realization of $\widetilde{\varepsilon}$ before selecting x,[4] then for each ε he maximizes

$$\pi = p_0(1+\rho\varepsilon)q(x,\varepsilon) - rx \tag{9}$$

yielding the following FOC and SOC

$$\frac{\partial\pi}{\partial x} = p_0(1+\rho\varepsilon)q_1(x,\varepsilon) - r = 0, \tag{10}$$

$$\frac{\partial^2\pi}{\partial x^2} \equiv R = p_0(1+\rho\varepsilon)q_{11}(x,\varepsilon) < 0. \tag{11}$$

From (10) it is obvious that with perfect information the optimal x (denoted x^*) depends both upon ρ and upon the announced ε. In the next section we use two results about the function $x^*(\rho,\varepsilon)$, namely

$$\frac{\partial x^*}{\partial\rho} = \frac{p_0\varepsilon q_1(x^*,\varepsilon)}{-R}, \tag{12}$$

$$\frac{\partial x^*}{\partial\varepsilon} = \frac{p_0[\rho q_1(x^*,\varepsilon) + (1+\rho\varepsilon)q_{12}(x^*,\varepsilon)]}{-R}. \tag{13}$$

Notice first that the sign of (12) is fully determined by that of ε. When ε is positive (negative) a larger correlation between the risks leads to a higher (lower) input demand. As far as (13) is concerned, matters are less easy. Notice however that if $q_{12} = 0$ the sign of $\frac{\partial x^*}{\partial\varepsilon}$ is fully determined by that of ρ.

To express the expected profit under perfect information ($E(\widetilde{\pi}_2)$), we introduce $x^*(\rho,\varepsilon)$ into (9) and we integrate on ε so that

$$E(\widetilde{\pi}_2) = \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)q(x^*(\rho,\varepsilon),\varepsilon) - rx^*(\rho,\varepsilon)]dF(\varepsilon). \tag{14}$$

Using (8) and (14) we will be able in the next section to express the value of information V and to analyze the impact of ρ on V.

[4] In order to simplify the notations we limit our analysis to the case of perfect information.

3 Information value

Because of the assumption of risk neutrality the concept of information value is uniquely[5] defined by

$$V = E(\widetilde{\pi}_2) - E(\widetilde{\pi}_1).$$

Using the envelope theorem twice - once for $E(\widetilde{\pi}_1)$ and once for $E(\widetilde{\pi}_2)$ - one obtains

$$\frac{\partial V}{\partial \rho} = p_0 \int_{-1}^{+\infty} \varepsilon [q(x^*(\rho, \varepsilon), \varepsilon) - q(\widehat{x}(\rho), \varepsilon)] dF(\varepsilon). \tag{15}$$

Equation (15) tells us how information value is affected by the correlation between the risks. Of course, in the case where information does not affect decisions, i.e. $x^* = \widehat{x}$, the value of information is zero and this sign is zero as well. Let us rewrite (15) as

$$\frac{\partial V}{\partial \rho} = p_0 \int_{-1}^{+\infty} \varepsilon h(\varepsilon) dF(\varepsilon),$$

where $h(\varepsilon)$ is the difference between the two output levels $q(x^*(\rho, \varepsilon), \varepsilon) - q(\widehat{x}(\rho), \varepsilon)$. Because $\widetilde{\varepsilon}$ has a zero expectation it is easy to prove using the Covariance rule that

$$\frac{\partial h}{\partial \varepsilon} \lesseqqgtr 0 \Longleftrightarrow \frac{\partial V}{\partial \rho} \lesseqqgtr 0.$$

Now

$$\frac{\partial h}{\partial \varepsilon} = q_1(x^*, \varepsilon) \frac{\partial x^*}{\partial \varepsilon} + [q_2(x^*, \varepsilon) - q_2(\widehat{x}, \varepsilon)]. \tag{16}$$

Since q_1 is always positive the sign of the first term on the right hand side of (16) is entirely determined by that of $\frac{\partial x^*}{\partial \varepsilon}$ (see equation (13) and the associated comment).

The second term - the one in brackets - is also sign ambiguous. Its sign is determined by that of q_{12} and the relative values of x^* and $\widehat{x}$. Notice however that if $q_{12} = 0$ the expression in brackets is zero since q_2 is not influenced by the level of x. It thus appears that without further specification on the production function it is extremely difficult to sign $\frac{\partial h}{\partial \varepsilon}$. This is why in the next two sections we consider specific cases in which the sign of $\frac{\partial h}{\partial \varepsilon}$ and hence that of $\frac{\partial V}{\partial \rho}$ can be analyzed and discussed.

[5] Under risk aversion, there are at least three possible definitions of the value of information (La Vallée, 1968). Here, these are all equivalent.

4 An additive production risk

In order to better characterize the impact of ρ on V, we consider first the case where the production risk is additive. Besides in order to obtain closed form solutions, we specify further the production function so that in the absence of information the problem is written

$$\max_x \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)(2x^{1/2}+\varepsilon) - rx] dF(\varepsilon). \tag{17}$$

Easy calculations show that

$$\widehat{x} = (\frac{p_0}{r})^2$$

and in this case the optimal demand for x is independent from ρ. Consequently

$$q(\widehat{x}, \varepsilon) = 2\frac{p_0}{r} + \varepsilon$$

and after obvious simplifications

$$E(\widetilde{\pi}_1) = \frac{(p_0)^2}{r} + \rho p_0 \sigma^2,$$

where $\sigma^2 \equiv E\widetilde{\varepsilon}^2$. Because profits are convex in ε, we obtain without surprise that $E(\widetilde{\pi}_1)$ is monotonically increasing in ρ. Indeed when ρ increases, the variability of the profits increases and since profits are convex in ε, $E(\widetilde{\pi}_1)$ increases with ρ.

When the producer receives perfect information on ε, it is easy to show that for each ε he selects

$$x^*(\rho, \varepsilon) = (\frac{p_0(1+\rho\varepsilon)}{r})^2$$

inducing

$$q(x^*, \varepsilon) = 2\frac{p_0(1+\rho\varepsilon)}{r} + \varepsilon.$$

Consequently the expected profit with perfect information is in this case

$$E(\widetilde{\pi}_2) = \frac{(p_0)^2}{r}(1+\rho^2\sigma^2) + \rho p_0 \sigma^2.$$

Substracting $E(\widetilde{\pi}_1)$ from $E(\widetilde{\pi}_2)$, we have in this case

$$V = \frac{(p_0)^2}{r}\rho^2\sigma^2. \tag{18}$$

Quite interestingly, V is here a perfectly symmetric function of ρ (as shown in figure 1) with a minimum value at $\rho = 0$.

Insert Figure 1

At $\rho = 0$, $V = 0$ because of the additive nature of the production risk. When ρ is different from zero, obtaining perfect information on the production risk indirectly offers partial information on the price risk. Since it is valuable to have joint information on two risks, V becomes strictly positive. Notice also that V monotonically increases with ρ (expressed in absolute term) which confirms the basic intuition described in the introduction. In the additive case only one effect is at work: it is the most natural one and it confirms the basic intuition.

Before turning to the multiplicative case, it is worth showing the link between the results obtained in this section and those obtained in the previous one.

Indeed from (18) we get

$$\frac{\partial V}{\partial \rho} = 2\frac{(p_0)^2}{r}\rho\sigma^2. \tag{19}$$

Now, if we introduce into (15) the value of $q(x^*, \varepsilon)$ and $q(\widehat{x}, \varepsilon)$ found for the additive case, it comes

$$\frac{\partial V}{\partial \rho} = p_0 \int_{-1}^{+\infty} \varepsilon[2\frac{p_0(1+\rho\varepsilon)}{r} + \varepsilon - 2\frac{p_0}{r} - \varepsilon]dF(\varepsilon),$$

which after obvious simplifications yields (19).

5 A multiplicative production risk

We now introduce a multiplicative specification for the production risk. As a result, in the absence of information, the optimization problem is

$$\max_x \int_{-1}^{+\infty} [p_0(1+\rho\varepsilon)(2x^{1/2})(1+\varepsilon) - rx]dF(\varepsilon),$$

with solution

$$\widehat{x}(\rho) = (\frac{p_0}{r})^2(1+\rho\sigma^2),$$

and

$$q(\widehat{x}(\rho),\varepsilon) = \frac{2p_0}{r}(1+\rho\sigma^2)(1+\varepsilon). \tag{20}$$

With perfect information, it is easy to show that

$$x^*(\rho,\varepsilon) = \frac{(p_0)^2(1+\varepsilon+\rho\varepsilon+\rho\varepsilon^2)^2}{r},$$

so that

$$q(x^*(\rho,\varepsilon),\varepsilon) = \frac{2p_0}{r}(1+\varepsilon+\rho\varepsilon+\rho\varepsilon^2)(1+\varepsilon). \tag{21}$$

Introducing (20) and (21) into (15), we then obtain[6]

$$\frac{\partial V}{\partial \rho} = \frac{2(p_0)^2}{r}(\sigma^2+\rho\sigma^2+E(\widetilde{\varepsilon}^3)(1+2\rho)+\rho(E(\widetilde{\varepsilon}^4)-\sigma^4). \tag{22}$$

This expression has many implications. First, at $\rho = 0$, $\frac{\partial V}{\partial \rho} \neq 0$, contrarily to what happens in the additive case. In fact

$$\frac{\partial V}{\partial \rho}|_{\rho=0} = \frac{2(p_0)^2}{r}(\sigma^2+E(\widetilde{\varepsilon}^3)),$$

an expression that is positive (see Appendix 1). This means that starting from $\rho = 0$, a fall in ρ, i.e. an increase in absolute values, reduces V in the multiplicative case (while V increases in the additive case). This illustrates the presence of another effect of ρ on V: in the multiplicative case a negative correlation stabilizes profits and thus the demand for information on $\widetilde{\varepsilon}$ is reduced. At $\rho = 0$, this effect is strong enough in the specific case to dominate the other one linked to the joint impact of information.

Second, from (22) we see that there exists a negative value of ρ such that $\frac{\partial V}{\partial \rho}$ is equal to zero, implying that V has a minimum at that point. This minimum value is obtained at a value $\overline{\rho}$ such that the right hand side of (22) is equal to zero, i.e.

$$\overline{\rho} = \frac{-(\sigma^2+E(\widetilde{\varepsilon}^3))}{\sigma^2+2E(\widetilde{\varepsilon}^3)+(E(\widetilde{\varepsilon}^4)-\sigma^4)}. \tag{23}$$

[6] A similar result could obviously be arrived at by a longer procedure, i.e., compute $E(\widetilde{\pi}_1)$ and $E(\widetilde{\pi}_2)$, express V as $E(\widetilde{\pi}_2) - E(\widetilde{\pi}_1)$ and then compute $\frac{\partial V}{\partial \rho}$. This is not difficult but tedious.

We prove in Appendix 2 that the denominator of $\overline{\rho}$ is necessarily positive, so that ρ itself is negative. As a result the relationship between V and ρ in the multiplicative case is given in figure 2.

Insert Figure 2

First observe that at $\rho = 0$, V is now strictly positive because of the multiplicative nature of the production risk.[7] Besides at $\rho = 0$, V is increasing in ρ.

It is important to notice that when ρ is positive and increasing, V is always increasing. This is because the two effects of ρ on V play in the same direction: the information on the production risk gives a better information on the price risk when ρ increases. Besides as the positive ρ increases, the profit becomes more volatile in terms of ε and this higher volatility stimulates the value of information. When ρ is negative however, the two forces work against each other. In fact for a negative ρ, an increase in ρ in absolute value stabilizes profits vis à vis ε and this has a depressing effect on V. At $\rho = \overline{\rho}$, the two effects neutralize each other.

While it is obvious from (23) that $\overline{\rho} < 0$, we still have to wonder if $\overline{\rho} > -1$. When the distribution of $\widetilde{\varepsilon}$ is symmetric, it is the case. Indeed for $E(\widetilde{\varepsilon}^3)$ equal to zero, $\overline{\rho}$ becomes

$$\overline{\rho} = \frac{-\sigma^2}{\sigma^2 + (E(\widetilde{\varepsilon}^4) - \sigma^4)}. \tag{24}$$

Since $E(\widetilde{\varepsilon}^4) > \sigma^4$ for any distribution (see also Appendix 2), it is obvious that $\overline{\rho}$ in (24) exceeds -1.

When the distribution of $\widetilde{\varepsilon}$ is not symmetric, it may be that V is everywhere increasing in ρ. For instance, suppose that $\widetilde{\varepsilon}$ is binary: with probability $2/3$ it takes $+0.375$ and with probability $1/3$ it takes -0.75. In this case, we have $\sigma^2 = \frac{9}{32}$, $E(\widetilde{\varepsilon}^3) = \frac{-27}{256}$ and $E(\widetilde{\varepsilon}^4) = \frac{243}{2048}$ so that $\overline{\rho} = \frac{-8}{5}$. Thus in the range $(-1, 0)$ increasing the coefficient of correlation (in absolute value) decreases the information value.

This example suggests that the stabilizing effect of an increase in ρ when it is negative can dominate the other effect at all values of ρ between -1 and zero.

[7] Remember that for the additive case $V = 0$ at $\rho = 0$.

6 Conclusion

This paper has considered a risk-neutral agricultural producer who faces two correlated risks. It has shown a case in which the value of information about the risk on output always increases with the coefficient of correlation (in absolute value) between the risks. However, it has also shown a case in which it may decrease with this coefficient. This example shows that the effect of a correlation coefficient in a model with multiple risks has not a clear effect on the value induced by information about one single risk. This may be thought counter-intuitive. Consider the attitude of an investor facing a portfolio of assets. One may think a priori that it is more valuable for him to learn about the distribution of one single asset when these assets are highly-correlated compared to the case where they are all independent. Indeed high correlation is one way to learn about the distribution of all assets as opposed to only learn about the distribution of one asset in the case of independence. Our paper has suggested that this intuition may not be correct since there may be a diversification effect induced by correlation. This effect may reduce the overall portfolio risk and thus may reduce the need for information of the investor.

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Appendix 1

To prove that $\sigma^2 + E(\widetilde{\varepsilon}^3)$ is positive when $\varepsilon \geq -1$, we write

$$\begin{aligned} \sigma^2 + E(\widetilde{\varepsilon}^3) &= \int_{-1}^{+\infty} (\varepsilon^2 + \varepsilon^3) dF(\varepsilon) \\ &= \int_{-1}^{0} \varepsilon^2 (1+\varepsilon) dF(\varepsilon) + \int_{0}^{+\infty} \varepsilon^2 (1+\varepsilon) dF(\varepsilon). \end{aligned} \tag{25}$$

The second integral on the RHS of (25) is clearly positive and so is the first one because $1+\varepsilon$ exceeds zero on the range $(-1, 0)$.

Appendix 2

The two results in this appendix are direct consequences of Jensen's inequality. Because $(\varepsilon^2)^2$ is striclty convex in ε^2 we have

$$E(\widetilde{\varepsilon}^2)^2 > (E(\widetilde{\varepsilon}^2))^2,$$

or

$$E(\widetilde{\varepsilon}^4) > \sigma^4.$$

Similarly,

$$E(\widetilde{\varepsilon}(1+\widetilde{\varepsilon}))^2 > (E(\widetilde{\varepsilon}(1+\widetilde{\varepsilon}))^2,$$

or

$$\sigma^2 + 2E(\widetilde{\varepsilon}^3) + E(\widetilde{\varepsilon}^4) > \sigma^4,$$

which proves that the denominator in (23) is positive.

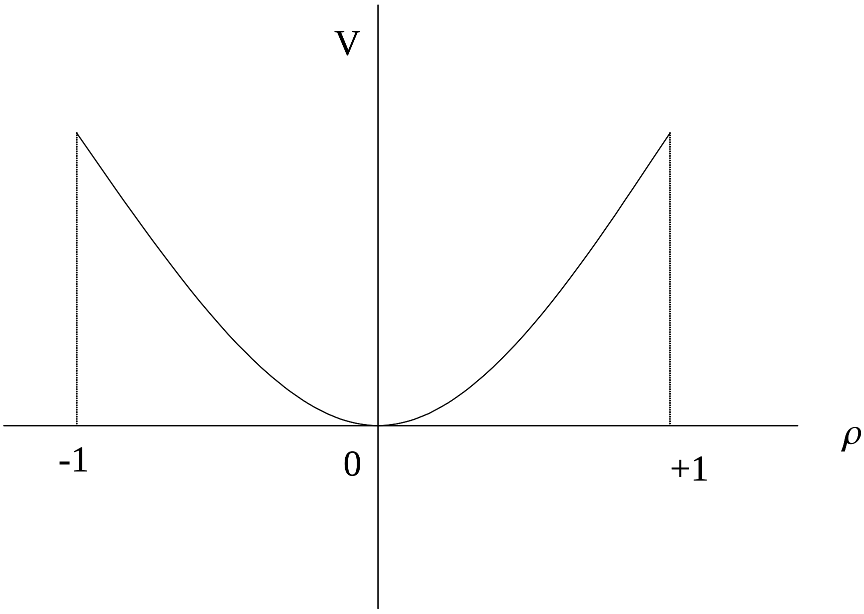

Figure 1:

Additive case: Information value V is increasing in the coefficient of correlation $|\rho|$

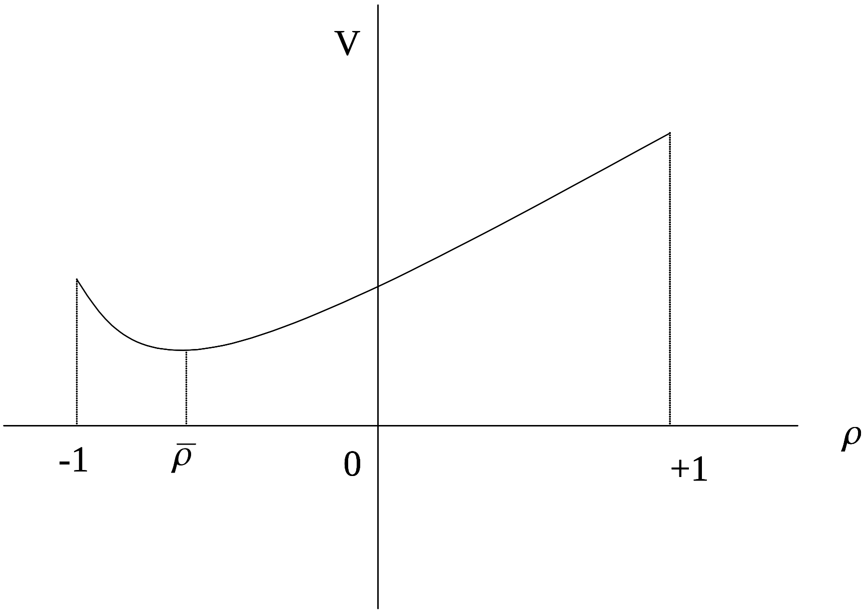

Figure 2:

Multiplicative case: Information value V is decreasing in the coefficient of correlation $|\rho|$ on the range $(0, \overline{\rho})$