Multi-stand Forest Management Under a Climatic Risk: Do time and Risk Preferences Matter?*

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Abstract

We propose a stochastic dynamic programming framework to model the management of a multi-stand forest under climate risk (strong wind occurrence). The preferences of the representative forest owner are specified by a non-expected utility in order to separately analyze intertemporal substitution and risk aversion effects. A numerical method is developed to characterize the optimal forest management policies and the optimal consumption-savings strategy. The stochastic dynamic programming framework is applied to a representative non-industrial private forest owner located in North-East of France. We show that the optimal decisions both depend upon risk and time preferences.

Keywords: Forest Economics, Stochastic Dynamic Programming, Non-expected Utility, Climate risk

JEL Codes: C61, D81, Q23

1 Introduction

The economic literature on forest management has been dominated for a long time by the Faustmann-Pressler-Ohlin (FPO) model, see Samuelson [1] and Mitra and Wan [2] among others. However, a number of recent studies have pointed out that some important features of the FPO framework still require further works, Johansson and Löfgren [3], Pukkala and Kangas [4] or Tahvonen and Salo [5]. Interdependences across age-class stands and forest owner's risk preferences constitute two good examples. A third dimension difficult to address within the FPO framework is the long-term consumption-savings tradeoff that must be solved, at least, by small private forest owners. For non-industrial private forest owners, the timber production is often viewed as an asset that must be managed to secure consumption over the long-run. Hence, the consumption-savings tradeoff should have an impact on the forest management, and *vice-versa*.

The main objective of our article is to develop a unified framework to analyze the linkage between consumption-savings, production decisions and risk management for a non-industrial forest owner facing a climatic risk. More precisely, we propose three extensions to the economic literature on optimal forest use in a stochastic environment. First, by using a non-expected utility function, we assess the impact of time and risk preferences on the optimal behavior of the forest owner. Second, we analyze a multiple age-class forest model. Last, we introduce saving as a decision of the forest owner at anytime. The forest economics literature has recognized the importance of each of these features without providing a framework incorporating all of them.

Production risk is known to be an important ingredient of forest management modeling (see the seminal work, Reed [6], on forest fires or the more recent article, Haight et al. [7], for wind storms). Some models have incorporated forest owners' risk aversion into stochastic control problems, see Taylor and Forston [8], Kangas [9], Pukkala and Kangas [4] or Willassen [10]. However, within the intertemporal setting of expected utility, the effect of time and risk preferences on decision making cannot be identified, see Epstein and Zin [11] or Knapp and Olson [12]. This is especially problematic due to the long-term horizon of forest managers. Only two studies, Koskela and Ollikainen [13] and Peltola and Knapp [14], have used a non-expected utility for characterizing forest owner preferences. They both conclude that risk aversion and temporal preferences have an impact on optimal forest policies. We extend this non-expected utility literature in forest management by introducing saving as a decision tool and by considering a multiple stand model.

Forest management has been traditionally addressed at a single stand level. This is a restrictive assumption as, in a stochastic environment, all forest stands may not be affected in the same way by the climatic risk. Some authors have incorporated age-class dynamics into forest models but these works don't include risk, see Tahvonen [15] or Uusivuori and Kuuluvainen [16] for recent surveys on age-class models.

Last, there exists only a few studies analyzing the consumption-savings tradeoff in the context of forest management (Tahvonen [17], Tahvonen and Salo [5], Salo and Tahvonen [18], Tahvonen [15], Uusivuori and Kuuluvainen [16]). These forest rotation models consider this tradeoff within a maximizing utility framework and a purely deterministic context. We propose to analyze the consumption-savings tradeoff within a stochastic framework.

The remaining of the paper is organized as follows. In Section 2, we describe the forest management model and we present the stochastic dynamic programming (SDP) method we will used. Section 3 deals with an empirical application to the case of a French representative private forest owner. We conclude by a brief summary of our findings.

2 A SDP framework for multi-stand forest management

2.1 Specification of the model

Forest age classes: Let us consider a forest owner who possesses a forest with a total area A (in ha). We denote by $A_{i,t}$ $\forall i \in \{1, \ldots, I\}$ and $t \in \{1, \ldots, \infty\}$ the land area allocated to age class, or stand, i at the beginning of date t. The following constraint:

$$\sum_{i=1}^{I} A_{i,t} + A_t^F = A \;\; \forall t \tag{1}$$

where A_t^F is the fallow land, must hold at each date.

An even-aged stand i of trees is characterized by the volume of timber per hectare denoted by V_i (in m^3/ha) and by the price of timber per cubic meter denoted by P_i (in thousand $euros/m^3$). Both V_i and P_i are assumed to be exogeneously given to the forest owner. The volume of timber per hectare and the timber price per cubic meter increase with age classes, $V_i > V_j$ and $P_i > P_j$ $\forall i > j$. We assume that the growth process is finite. There is no forest growth beyond the I^{th} age class, $V_i \equiv V_I$ $\forall i \geq I$. We also assume that the price per cubic meter remains constant for age classes older that the I^{th}, $P_i \equiv P_I$ $\forall i \geq I$. It follows that age classes older than I are strictly equivalent both in terms of growth and price.

Strong wind risk: The risky environment of the forest owner is described by the risk of windstorm occurrence and the risk of forest loss. We denote by p the probability of strong wind realization. Given strong wind occurrence, an age class i may or may not be destroyed. We denote by q_i the conditional probability of age class i destruction given a strong wind occurrence. As mentioned in Dhote [19], the wind resistance of forests varies very significantly among age classes. This explains why the conditional probability of forest loss depends upon the forest age class. For the i^{th} age class, the unconditional probability of forest loss is $p \cdot q_i$. The stochastic variable representing age class i loss is denoted by $\widetilde{\epsilon}_i$, with $\widetilde{\epsilon}_i = 1$ in case of forest loss and $\widetilde{\epsilon}_i = 0$ otherwise.

Timing of forest decisions: The timing of the model is the following. First, the forest owner chooses the area to be allocated to a new forest age class and the area to be harvested for each existing age class. Then the strong wind risk is realized. If no catastrophic event occurs, the forest age classes grow. If a catastrophic event occurs then, for each age class, the risk of production loss is realized and the existing forest plot may be lost. If a forest plot is destroyed by a strong wind, the fallow land is increased by the corresponding area.

Age class area's dynamics: We describe now more formally the dynamics of the area dedicated to each age class $i \in \{1, \ldots, I\}$.

At the beginning of period t, the forest owner can allocate some area to a new forest age class (planting) and harvest existing forest age classes. At the beginning of period $t+1$, the area allocated to age class 1 corresponds to the area planted with a new forest at the beginning of period t, if this age class has not been destroyed by strong winds during period t that is if $\widetilde{\epsilon}_{0,t} = 0$. The resulting dynamic is:

$$A_{1,t+1} = a_t^p \cdot (1 - \widetilde{\epsilon}_{0,t}) \tag{2}$$

where $A_{1,t+1}$ represents the area planted with age 1 trees at the beginning of period $t+1$. The term a_t^p corresponds to the area planted with a new age class during t (age zero at the beginning of period t).

Next, we consider age classes 2 to $I-1$ corresponding respectively to trees planted 2 and $I-1$ periods before the current one. The dynamics are:

$$A_{i,t+1} = (A_{i-1,t} - a_{i-1,t}^h) \cdot (1 - \widetilde{\epsilon}_{i-1,t}) \;\; \forall\, i \in \{2, \ldots, I-1\} \tag{3}$$

where $a^h_{i-1,t}$ represents the area harvested during period t with age class $i-1$ trees, with $a^h_{i-1,t} \leq A_{i-1,t}$. The previous dynamic equation states that if no catastrophic risk occurs or if the strong wind realization does not result in a production loss, the area at the beginning of $t+1$ planted with age i trees is equal to the area at the beginning of date t planted with age $i-1$ less the area harvested during that period, $a^h_{i-1,t}$.

Last, let us consider the dynamic associated with age I trees:

$$A_{I,t+1} = (A_{I-1,t} - a^h_{I-1,t}) \cdot (1 - \widetilde{\epsilon}_{I-1,t}) + (A_{I,t} - a^h_{I,t}) \cdot (1 - \widetilde{\epsilon}_{I,t}). \tag{4}$$

Since there is no growth beyond age I and since the price per cubic meter remains the same once age I has been reached, all age classes older than I are strictly equivalent. It follows that, if no forest loss risk is realized, the area allocated to age class I at the beginning of period $t+1$ is equal to the area allocated to age class $I-1$ at the beginning of t less the harvested area in t corresponding to this age class. It also includes the area allocated in t to age class I less the corresponding harvested area $a^h_{I,t}$.

Due to a high planting cost, forest owners may find profitable not to re-plant all their forest plots after a strong wind realization. The dynamic for fallow land is:

$$A^F_{t+1} = A^F_t + a^F_t + a^p_t \cdot \widetilde{\epsilon}_{0,t} + \sum_{i=1}^{I} (A_{i,t} - a^h_{i,t}) \cdot \widetilde{\epsilon}_{i,t} \tag{5}$$

where a^F_t represents the area which is converted during period t from forest to fallow land ($a^F_t > 0$) or from fallow land to forest ($a^F_t \leq 0$). At each period, we have:

$$a^F_t + a^p_t = \sum_{i=1}^{I} a^h_{i,t}. \tag{6}$$

Equation (6) states that at each time the area harvested must be equal to the area planted with a new age class and the area converted from forest to fallow land.

Wealth stock dynamic: Harvested trees are sold and the resulting revenue can be used either for consumption or for saving into a risk-free asset characterized by a given interest rate r. By denoting W_t the forest owner's wealth at date t, the wealth dynamic is:

$$W_{t+1} = W_t(1 + r) + \widetilde{\Pi_t} - c_t \tag{7}$$

where $\widetilde{\Pi}_t$ denotes the stochastic forest owner's annual profit from wood sales and c_t is the consumption of the forest owner, at date t. We assume that the consumption decision is taken once uncertainty about the catastrophic event realization is resolved.

Profits: The forest owner pays three types of cost. First, there is a cost for planting a new forest age class. We denote by $PC(a)$ the cost for planting an area a of land. Following Guo [20], this function is assumed to be linear with respect to land area, $PC(a) = k_1 \cdot a$. Second, there is an harvesting cost. The harvesting cost for an area a, denoted by $HC(a)$, is assumed to be linear, $HC(a) = k_2 \cdot a$. The last cost that must be paid by the forest owner is a recovering cost in case of strong wind occurrence. In the case of forest losses due to wind storm, the forest plots can be recovered and sold, but this requires additional expenses (due for instance to a more difficult access to the forest plots). The recovering cost for a land area a is denoted by $RC(a)$ and is assumed to be linear, $RC(a) = k_3 \cdot a$ with $k_3 > k_2$.

Using equations (2)-(4), the annual total cost for period t is:

$$PC(a_t^p) + \sum_{i=1}^{I} HC(a_{i,t}^h) + \sum_{i=1}^{I} RC(A_{i,t} - a_{i,t}^h) \cdot \widetilde{\epsilon}_{i,t} \tag{8}$$

The two left-hand side terms respectively represent the planting cost for a new forest age class and the harvesting cost for the existing ones. In the case of wind storm with age class i destruction, an area $A_{i,t} - a_{i,t}^h$ must be recovered. Hence, for age class i, the associated recovering cost writes $RC(A_{i,t} - a_{i,t}^h)$.

We now define the revenue generated by wood sales. At date t, the total revenue is:

$$\sum_{i=1}^{I} P_i \cdot V_i \cdot a_{i,t}^h + \sum_{i=1}^{I} P_i \cdot V_i \cdot (A_{i,t} - a_{i,t}^h) \cdot \mu \cdot \widetilde{\epsilon}_{i,t}. \tag{9}$$

The first term represents the revenue generated by age classes harvesting. The revenue from the recovered forest plots, in case strong wind occurrence, corresponds to the right hand-side term. We assume that, in the case of forest losses due to wind storm, the forest owner can recover and sell a proportion $\mu \in [0,1]$ of the forest area destroyed. Empirical evidences suggest that in the case of strong wind storm, the entire production is not definitively lost. We assume that a fixed proportion of the total forest plots is recovered and sold. This proportion, μ, does not vary across age classes. This assumption fits the behavior of small French forest owners after the 1999 strong wind but it can easily be extended to recovering coefficients μ varying across age classes. Of course, μ equal to 1 means that strong winds do not result in resource losses. The only effect is to impose harvesting at a time which may not be optimal. On contrary, μ equal to 0 means that no recovery is possible and a stand destroyed by the strong winds is definitively lost. From equation (9), it is clear that another interpretation could be given to the coefficient μ. One may consider that in case of strong wind occurrence all the timber volume $V_i \cdot (A_{i,t} - a_{i,t}^h)$ can be recovered but that it can be only sold at a lower price $\mu \cdot P_i < P_i$ (the market reacts to the increase in wood supply resulting from the storm or the wood quality is known to be altered).

Finally, the profit generated by the forest harvest is:

$$\begin{aligned} \widetilde{\Pi_t} &= \sum_{i=1}^{I} P_i \cdot V_i \cdot a_{i,t}^h + \sum_{i=1}^{I} P_i \cdot V_i \cdot (A_{i,t} - a_{i,t}^h) \cdot \mu \cdot \widetilde{\epsilon}_{i,t} \\ &\quad - PC(a_t^p) - \sum_{i=1}^{I} HC(a_{i,t}^h) - \sum_{i=1}^{I} RC(A_{i,t} - a_{i,t}^h) \cdot \widetilde{\epsilon}_{i,t} \end{aligned} \tag{10}$$

The profit is simply equal to the revenue resulting from wood sales less the forest management costs (planting, harvesting and recovering costs).

2.2 The stochastic dynamic optimization problem

The dynamic optimization problem consists at each period in determining if a forest age class must be harvested or not, in choosing the share of fallow land to be planted and in deciding if the revenue flow is used for consumption or saving such as the objective function of the forest owner is maximized. More formally, the stochastic control problem, $\mathcal{P}_1$, is:

$$\mathcal{P}_1 : \begin{cases} \max_{\{c_t, a_t^p, a_{.,t}^h\}_{t=1,\ldots,\infty}} & \mathcal{U}(\{c_t, a_t^p, a_{.,t}^h\}_{t=1,\ldots,\infty}) \\ \text{s.t.} & W_{t+1} = W_t(1+r) + \widetilde{\Pi_t} - c_t & \forall t \\ & A_{1,t+1} = a_t^p \cdot (1 - \widetilde{\epsilon}_{0,t}) & \forall t \\ & A_{i,t+1} = (A_{i-1,t} - a_{i-1,t}^h) \cdot (1 - \widetilde{\epsilon}_{i-1,t}) \ \ \forall\, i \in \{2, \ldots, I-1\} & \forall t \\ & A_{I,t+1} = (A_{I-1,t} - a_{I-1,t}^h) \cdot (1 - \widetilde{\epsilon}_{I-1,t}) + (A_{I,t} - a_{I,t}^h) \cdot (1 - \widetilde{\epsilon}_{I,t}) & \forall t \\ & A_{t+1}^F = A_t^F + a_t^F + a_t^p \cdot \widetilde{\epsilon}_{0,t} + \sum_{i=1}^{I} (A_{i,t} - a_{i,t}^h) \cdot \widetilde{\epsilon}_{i,t} & \forall t \\ & \sum_i A_{i,t} + A_t^F = A & \forall t \\ & (A_{.,0}, A_0^F, W_0) \text{ given} \end{cases}$$

where $\mathcal{U}(.)$ represents the objective function of the forest owner, $\widetilde{\Pi_t}$ is the profit defined by equation (10), $a_{.,t}^h = (a_{1,t}^h, \ldots, a_{I,t}^h)$ is the vector of harvested areas and $A_{.,t} = (A_{1,t}, \ldots, A_{I,t})$ is the vector of areas allocated to forest age classes.

Since the problem is dynamic and stochastic, the optimal decision path should both depend on forest owner risk and time preferences. Risk preferences refer to the forest owner desire to smooth consumption across states of the nature whereas time preferences reflect the forest owner propensity to limit consumption fluctuations over time. Historically, the expected utility model (EU) has been the most common way to integrate risk preferences into forest planning, and more generally into natural resource management problems. However, in the case of dynamic and stochastic environments, using the EU model raises a number of substantial issues. First, as stressed by Epstein and Zin [11], this framework does not allow to make a distinction between preference toward risk and toward time since both concepts are encompassed into the curvature of the utility function. This is especially problematic in the case of forest management since there is a priori no reason to believe that these preferences should be linked. A second drawback of the EU framework is that the decision-maker only takes into account the final consequences of his choices, neglecting the timing of uncertainty resolution: the individual's choice behavior is independent of the dynamic choice problem he is facing. Given the very long time horizon of forest owner, such a property of the EU model is clearly problematic. Last, as recently stressed by Chavas [21], concerns about the future likely vary with income level. It follows that discounting the future should both depend upon income and consumption levels. This suggests a move away from the standard time additive models, where utility of future consumption is discounted at a constant rate.

Recursive preferences have been proposed as a way to incorporate time and risk preferences into the decision-maker problem. Moreover, in a model with recursive preferences, the decision-maker integrates the timing of uncertainty resolution as a part of the decision problem. Following Epstein and Zin [11, 22], we use an isoelastic formulation of Kreps and Porteus preferences. Hence, the forest owner's objective function is given by its recursive utility at time t, U_t defined by:

$$U_t = \left\{ (1-\beta) \cdot c_t^{\frac{\sigma-1}{\sigma}} + \beta \left[\mathrm{E} U_{t+1}^{1-\alpha} \right]^{\frac{(\sigma-1)/\sigma}{1-\alpha}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{12}$$

where c_t represents the consumption of the forest owner, $\beta \in [0,1]$ is the subjective discount factor, $\alpha \in]0,\infty)$ is the Arrow-Pratt constant relative risk-aversion, $\sigma \in]0,\infty)$ is elasticity of intertemporal substitution (EIS) and E is the expectation with respect to all stochastic variables of the model. The left handside term in parentheses of equation (12) measures the utility directly derived from consumption at date t. The right handside term measures the expected future utility given the information available at date t. Aggregation of both terms then depends upon risk and time preferences of the decision-maker. Notice that the recursive preferences nest the EU model as a special case since by setting $\sigma = \frac{1}{\alpha}$, we get the familiar constant relative risk aversion expected utility function.

The stochastic control problem consists in choosing a sequence of decision rules that maximizes the objective function (12) subject to the previously defined equations of motion and constraints in $\mathcal{P}_1$.

2.3 Solving the stochastic dynamic programming problem

Since the problem is autonomous, we can drop the time subscript and make the value function of the problem solely dependent on the initial conditions for any period (all model parameters and functions are the same for all decision stages, which means that the problem is stationary). The stochastic dynamic recursive equation defining the optimal forest and saving management is:

$$J\left(A_{.}, W, A^F, \widetilde{\epsilon}\right) = Max_{\left\{c, a^p, a^h_{.}\right\}} \left\{(1-\beta)\cdot \mathrm{E}c^{\frac{\sigma-1}{\sigma}} + \beta\left[\mathrm{E}J^{1-\alpha}\left(A_{.}, W, A^F, \widetilde{\epsilon}\right)\right]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right\}^{\frac{\sigma}{\sigma-1}} \tag{13}$$

where $J(.)$ is the value function. We have to solve a stochastic dynamic programming problem. Since the discount factor is bounded, the mapping underlying the Bellman's equation is a strong contraction on the space of bounded continuous functions and, thus it yields to a unique value function. A value iteration method will be used for solving the SDP problem, see Judd [23]. As it will be discussed in the next section, it consists in assigning an initial guess for the value function, and then in recursively solving the maximization problem until the implied carry-over value function converges to an invariant approximation.

3 An application to spruce in North-East of France

3.1 Specification of the empirical model

Descriptions of the region and of the forest estate: The model is calibrated to represent the behavior of a representative non-industrial private forest owner located in Lorraine, a region located in North-East of France. The Lorraine's region produces around 10% of total French timber production. This region ranks second for forestry after the Aquitaine (located in the South-West of France). The private forest estate is assumed to produce spruce, one of the most common stands observed in the North-East of France. The spruce production in Lorraine represents 20% of the total timber production in France. With 25 million cubic meters on the ground in 1999, Lorraine was the second devastated French region by the December exceptional storms.

Forest age class characteristics: In order to be tractable, the number of forest age classes has been limited to 5, $i = 1, \cdots, 5$. Given the growth process of spruce, the time index t represents a 20 year interval. Hence, the age class i corresponds to trees of age $20 * i$ at the beginning of period t. The volumes per ha for each age class of the spruce species in North-East of France are derived from Vannière [24] and are presented in Table 1. The unit prices for each forest age class are derived from Guo [20]. In Table 1, the fourth column gives the annual gross return based on the production value (notice that this return does not include neither the planting nor the harvesting costs). A 12.48% annual return for the first age class means that the value of the forest increases on average by 12.48% each year from year 20 to 40. The annual gross returns should be compared to the interest rate for the risk-free asset, r, arbitrary fixed at 3%. Last, the cost functions associated with forest management (planting, harvesting and recovering cost) are derived from Guo [20]. The unit cost of planting and harvesting (k_1 and k_2) are respectively equal to 2.1038 and 0.0037 thousand euros per ha. The unit cost of recovering k_3 is 50% higher than the unit harvesting cost. Last, we assume that in case of strong wind and production loss, only 10% of the forest area can be recovered and sold (μ is equal to 0.1).

Distribution of the strong wind risk: We first need to define the probability of strong wind realization, p. According to Picard et al. [25], the annual probability of strong wind realization is $3.1^0/_{00}$ for France. As the time period in the SDP problem represents a 20 year interval, we must compute the probability associated with the occurrence of *at least* one strong wind during 20 years. By assuming that the occurrences of strong winds are independent and identically distributed, this probability can be computed using a binomial distribution:

$$p = \sum_{k=1}^{20} P(X = k) \quad \text{with} \quad X \backsim Bin(20, 0.0031). \tag{14}$$

The probability associated with the occurrence of at least one strong wind event during 20 years is equal to 5.86%. It is interesting to notice that, in the economic literature on forestry, the catastrophic event occurence has often been modelled using a Poisson process (Reed [6], Haight et al. [7]). We do not strictly depart from this literature since the binomial law converges in distribution toward a Poisson process (for a sufficient number of trials and a low probability).

When a strong wind occurs, the forest owner faces a conditional risk of production loss specific to each forest age class. It is difficult to evaluate this probability because it depends on the stem weight, the tree species, the soil type and the forest culture practices. From forest expert interviews, we have computed the conditional probability of overturning by age classes for spruce in Lorraine. As suggested by Dhote [19], this conditional probability increases with the age class. It varies from 1% for the first age class to 72% for the fifth. For each forest age class, the unconditional probability of loss is obtained by multiplying the probability of the strong wind realization by the unconditional probability of forest loss. This unconditional probability is presented in the last column in Table 1. For instance, with a probability of 3.79%, an age class 3 is destroyed by a strong wind event during each time period (20 years). Given the stochastic specification of the model, there are, at each period, 33 possible states of the nature. In the first state of the nature, the strong wind does not occur and the associated probability is $1 - p = 0.9414$. The $32 = 2^5$ remaining states of the nature correspond to the realization of the strong wind with or without production loss for each age class. The associated probabilities are directly computed using the probability of strong wind realization and the conditional probability of forest loss associated with each age class. They are used to derive the expectation in the Bellman's equation (13).

Forest owner's characteristics: In France, non-industrial private forest owners constitute the main type of forest ownership, representing roughly three quarters of the forest areas (10.9 millions of hectares). Approximately 68% of that area consists of small-scale forest holdings of less than one hectare. As a result, we have normalized the area of the forest estate to 1 ha, $A = 1$. This assumption is not restrictive and higher forest areas may be considered. Forest owners are also characterized by their time and risk preferences. There is currently no estimation available for these parameters in the case of small forest owner facing a climate risk. More generally, there is neither no consensus on the level of the EIS and the coefficient of risk aversion for economic agents. For instance, according to the study considered, the consumer EIS may range from zero, Hall [26], all the way to 0.87, Epstein and Zin [22]. For the risk aversion coefficient, α ranges from 0.82, Epstein and Zin [22] to 1.5, Normandin and Saint-Amour [27]. By reference to Epstein and Zin [11] and previous empirical studies, we have considered the following values $\beta = 0.98$, $\alpha = 0.9$ and $\sigma = 1/2$. These values are consistent with the range of reported estimates in the economic literature. They correspond to a low level of risk aversion and a high willingness to substitute consumption across time. This case will be termed the *recursive utility benchmark case* in the remaining of the article.

3.2 Solving the SDP problem

We need to estimate the value function $J\left(A_{.}, W, A^{F}, \widetilde{\epsilon}\right)$ defined by the recursive equation (13). Given the high non-linearity of the problem, no analytical solution of this equation can be found. Hence a numerical procedure must be implemented. We use a value iteration approach that is we seek a numerical approximation $\widehat{J}(.)$ to the infinite horizon value function that maximizes the value of the problem resulting from decisions carried out in the future. The main steps of the value iteration algorithm are presented in the chapter 12 of Judd [23]. We propose here to extend the approach followed by Howitt et al. [28] to the case of multi-dimensional state variables. In order to solve the Bellman's equation, a specific functional form for $\widehat{J}(.)$ must be chosen to approximate the solution to the infinite-horizon problem. Howitt et al. [28] have for instance used a Chebychev Polynomial form. The main difficulty with this class of polynomial approximation is that the number of parameters to be estimated exponentially increases with the number of state variables. As 7 state variables must be considered in our SDP problem, we have used a more simple polynomial approximation, namely a second-order polynomial approximation. As neither the objective function is quadratic nor the constraints are linear, we know that the value function is not quadratic. An important issue we will discuss in the next paragraph is to check ex-post that the second-order polynomial approximation is an accurate approximation of the unknown value function. The polynomial approximation simply writes:

$$\begin{aligned}\widehat{J}\left(A_{.}, W, A^F, \widetilde{\epsilon}\right) &= \sum_i \eta_i \cdot A_i + \eta_F \cdot A^F + \eta_W \cdot W \\ &+ \sum_i \eta_{ii} \cdot A_i A_i + \eta_{FF} \cdot A^F A^F + \eta_{WW} \cdot WW \\ &+ \sum_i \eta_{Wi} \cdot W A_i + \eta_{WF} \cdot W A^F \\ &+ \sum_i \eta_{Fi} \cdot A^F A_i \\ &+ \sum_{i,j/j>i} \eta_{ij} \cdot A_i A_j. \end{aligned} \tag{15}$$

The polynomial approximation is fully characterized by 35 parameters that must be estimated. In order to make the SDP numerically operational, the state variables must be discretized. The wealth's stock has been discretized in 3 points $\{10, 40, 70\}$. The areas allocated to each age class together with the fallow land have been discretized in 5 points $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$. Notice that as the total area is normalized to 1 ha, the forest state variables can be interpreted as the proportion of land allocated to each age class.

The value function iteration program has been written in GAMS. The code and the data are available from authors for replication of results.

3.3 The solution to the empirical SDP problem

3.3.1 Accuracy of the second-order polynomial approximation

The stabilization of value function parameters has been achieved after 260 iterations and a few hours of computing time. At the 260^{th} iteration, the sum of squared errors between parameter estimates is smaller than 0.00001. This means that the value iteration algorithm converges toward a stable approximation of the value function *within* the class of second-order polynomial forms. In order to evaluate the accuracy of the second-order approximation, the SDP has also been solved with a third-order polynomial form and we have compared the resulting estimated value functions at each grid point. The two polynomial approximations give similar results, the absolute relative difference being 2.15% on average (the maximum absolute relative difference at a given grid point is 6.25%). However, the computing time is more than 10 times higher with the third-order polynomial approximation (189 parameters must be estimated compared to 35 with the second-order approximation), the computing time for one iteration being 6 minutes on a computer with a pentium 1600 Mhz processor. In term of cost-benefit analysis, the precision loss due to the second-order polynomial approximation seems largely balanced by the increased speed of the value iteration algorithm. A last accuracy check of the second-order polynomial approximation has been to compute the residuals of the Bellman's equation (13) at each discretized point. The residuals are small enough for considering that the second-order polynomial form is a good approximation of the unknown value function. Although the true value function is not quadratic, the second-order polynomial approximation offers an accurate approximation.

3.3.2 The estimated value function

Value function coefficients: Table 2 gives the 35 coefficients of the value function of the SDP problem. The estimated value function possesses good concavity properties as it increases with each state variable, but at a decreasing rate.

The coefficients η_{Wi} reflecting the link between the wealth and the forest age class areas worth being investigated. The sign associated with coefficients η_{Wi} results from three effects. First, in case of a low level of wealth, the marginal value of harvesting forest plots is high as consumption is constrained. This has a negative impact on the value of forest plots. As harvesting is highly valued by customers, it is optimal to harvest important quantity of wood which can be achieved only if the value of forest plots is low. This *wealth effect* results in negative parameters η_{Wi}. Second, in case of a low level of wealth, the forest owner may prefer reducing risk exposition. This implies a low value associated with forest age class plots. This *risk exposition effect* results in negative parameters η_{Wi}. On the contrary, in case of a low level of wealth, harvesting forest plots implies low forest areas at the beginning of the next period and a high risk of lowering future consumption. In order to reduce the cost of a low future consumption, forest plots must be attributed a high value. This *continuation effect* results in positive parameters η_{Wi}. The optimal value of forest plots and the optimal harvesting and planting strategies balance these three effects. For age classes 1 to 4, the continuation effect dominates the two other effects (at a decreasing rate with tree age). For the fifth age class, the wealth effect and the risk exposition effect dominates. Keeping oldest tree is risky and harvesting is more likely to be the optimal strategy.

The value of forest plots: Next, we investigate how the forest plot value differs among age classes. In Figure 1, we have plotted the value of a given age class (assuming that no other age class is planted at the same time) as a function of forest owner wealth. For instance, the curve corresponding to A_1 gives the value of one hectare planted with the first age class. This value measures the expected flow of utility that will result from an optimal forest use *in the future*. Since fallow must be planted before being able to generate any positive profit, the value to be attributed to fallow is lower than the value of any forest class. Second, for a given level of wealth, the value of a forest plot increases with the age class. This is an intuitive result since the older is a forest age class, the higher is the profit per unit of area. Third, the difference between forest age class value functions appears to be finally quite limited. For example, the value of the age class 3 forest plot is less than 5% higher than the value associated with the age class 2 plot. This is surprising compared to the value per ha, more than three times higher for age class 3. One possible explanation is that the revenue from saving dominates the revenue from forest plots. Last, the difference between forest plot values decreases with the wealth stock. This is also an intuitive result. In the case of a low level of wealth, maintaining the consumption flow requires to harvest forest plots. Hence, the continuation effect is limited and the value of a forest plot is more driven by the its instantaneous return, that is by its wood productivity. In case of a high level of wealth, the consumption flow more largely depends on savings. The impact of the climatic risk on the value of forest plots is limited because consumption is secured by the high wealth stock and the continuation effect dominates, in such a case, the two other effects.

3.4 Assessing the impact of time and risk preferences on decisions

3.4.1 Time and risk preferences and optimal policies

The optimal forest decisions: We consider an initial land allocation where each forest class age is attributed the same area (20% of the total area). The optimal planting and harvesting strategies are then derived for different levels of wealth, see Table 3. We first focus on the optimal forest and consumption-savings strategies in the recursive utility benchmark case. In the case of a low wealth level ($W = 10$ or $W = 20$), age classes 3 to 5 are harvested. The wealth and the risk exposure effects dominate the continuation effect and it is optimal to harvest the young forest age classes. As the wealth level increases, consumption is less constrained, even in the case of strong winds, and the forest owner is ready to take more risks, that is to expose older age classes to the climatic risk (in order to get higher returns). For a very high level of wealth ($W = 60$), neither harvesting nor planting are optimal strategies. The consumption flow only relies on the saving stock. This explains why, in Figure 1, the value of the forest plots converges toward a unique equilibrium as the wealth level increases.

It is interesting to compare the optimal forest decisions in the recursive utility benchmark case to those obtained in an EU framework. For instance, in an EU context with a higher EIS ($\sigma = \frac{1}{\alpha} = 1.11 > 0.9$), it is optimal to harvest age classes 4 and 5 for an initial wealth equal to 20. Compared to the recursive utility benchmark case, the forest owner doesn't harvest age class 3 which corresponds to transferring a current consumption toward a future one. This is made possible by the higher EIS which means that substitution across periods is more desirable and that consumption can fluctuate over time. A symmetric result is obtained for an initial wealth level equal to 50. In that case, it is optimal to harvest age classes 4 and 5 in an expected utility context with a higher EIS whereas, in the recursive utility benchmark case, harvesting is restricted to the fifth age class. A higher EIS allows to transfer a future consumption toward current one if the wealth level is high enough.

The optimal consumption-savings decisions: Next we investigate, for different wealth levels, the consumption-savings tradeoff still assuming that each forest age class is attributed 20% of the total area. The optimal consumption-savings decisions are reported in Table 4. As the initial wealth increases, the consumption flow more heavily depends on saving. For the highest initial wealth level, $W = 60$, consumption is made possible exclusively using savings since no forest age class is harvested. As it could be expected, the flow of consumption increases with the initial wealth. Moreover, the higher is the initial wealth, the higher is the share of consumption to the available wealth (from less than 30% if $W = 10$ to around 50% in the higher initial wealth case, $W = 60$).

3.4.2 Time and risk preferences and the long-run equilibrium

In this subsection, we simulate the model over a very long time horizon (400 periods) assuming that no catastrophic event occurs. Such a situation allows us to analyze the long-run equilibrium of the dynamic model. In order to evaluate the impact of time and risk preferences on the long-run equilibrium, the SDP recursive equation has been solved for three levels of the Arrow-Pratt constant relative risk-aversion coefficient, α, and for three levels of EIS, σ, see Table 5.

Measuring the impact of forest owner preferences: For each (α, σ), all state variables converge toward a unique long-run equilibrium for which only the fifth forest age class is harvested. This result could be related to the high level of wealth at the equilibrium, higher than 50. At this wealth level, the continuation effect dominates the risk exposure and the wealth effects. Second, at the long-run equilibrium the area dedicated to age classes 1 to 4 is the same: $A_1^{LR} = \cdots = A_4^{LR}$. This result comes directly from the fact that these age classes are not harvested. Notice finally that, at the long-run equilibrium, the area allocated to age classes 1 to 4 differs from the fifth age class forest plot, A_5^{LR}. This may result from the fact that, as the growth process is finite, the dynamic of the last age class differs from other age classes dynamic. Without strong wind occurrence, the optimal forest management tends toward a normal forest structure where only the oldest forest age class is harvested. Notice that the convergence of optimal forestry programs toward a normal forest structure has recently received a considerable attention in the forest economics literature (see Salo and Tahvonen [18], and Uusivuori and Kuuluvainen [16] among others).

We wish now to investigate the impact of the forest owner time preferences on the long-run equilibrium. For a given level of the risk parameter, the lower is the EIS, the higher is the savings and the lower is the age class 5 forest area. The explanation is quite intuitive. Lowering the EIS implies that substitution across periods is less desirable. Hence, consumption should be stabilized over time. This can be first achieved though a high level of savings (with a risk-free return r) and second, through a lower final forest area exposed to the catastrophic risk. This second mechanism is made possible by harvesting a higher proportion of forest plots. In other words, a higher saving and a higher harvesting rate provide a greater insurance against the variation of consumption due to catastrophic events.

The nature of the impact of the risk preferences on the long-run equilibrium is less straightforward. In case of a high level of EIS ($\sigma = 2/3$), the higher is the parameter of risk aversion, the higher is the savings and the lower is the age class 5 forest area. This can be explained intuitively. A risk averse forest owner wants to secure income. This can be achieved by having a higher level of wealth and a lower area dedicated to the fifth age class. In case of a low level of EIS ($\sigma = 1/3$), the relationship between risk aversion and wealth (or forest plots) appears to be non monotonic.

The preferences for intertemporal substitution seem to have a much more important effect of the system than risk preferences. This is a striking result in view of the importance of risk aversion in static models of risk and uncertainty. We do not have a full explanation of this result but we conjecture that it may result from the difference in the structure of decision making between the static and dynamic cases. In the static case, decisions are made before uncertainty is resolved thus the net returns bear the full brunt of uncertainty. In the dynamic case, some decisions are taken after uncertainty is resolved (this is the case here for the consumption choice). It may be possible that some of the adverse effects of uncertainty in the static case are mitigated in the dynamic case, even before risk aversion is considered. Another explanation is that, at the long-run equilibrium without risk occurrence, the value of the forest plots may become too low compared to saving. In such a case, the catastrophic risk will only have a minor impact on forest owner wealth and risk does not matter as much as intertemporal substitution. Notice that in a one-period age class model, Peltola and Knapp [14] have also found that the intertemporal elasticity of substitution does significantly affect forest management. In particular, they have shown that with a low EIS, harvesting starts sooner as the forest owner does not want to postpone consumption for the sake of a higher biological productivity. With a high EIS, consumption is on contrary sacrificed at the beginning for a gain in future biological productivity.

Measuring the impact of uncertainty: In the previous analysis, the annual strong wind probability was 3.1 $^0/_{00}$. In order to assess the impact of uncertainty on forest owner behavior, the SDP problem has been solved with a higher probability, 5 $^0/_{00}$, and with a lower one, 2 $^0/_{00}$. In Table 6, we characterize the long-run equilibrium corresponding to the recursive utility benchmark case.

As expected, for a given level of time and risk preferences, the higher is the probability of strong wind occurrence, the higher is the precautionary saving. Moreover, the area allocated to the fifth age class decreases with the level of uncertainty, whereas the area allocated to age classes 1 to 4 increases. In the case of a high probability of strong winds, it is optimal to diversify risks toward younger age classes with a lower conditional probability of forest loss in case of strong wind realization. In a two-period one-class forest model, Koskela and Ollikainen [13] observe the same precautionary behavior. They report that a rise in the multiplicative forest growth risk increases current harvesting but decreases the future one. The fact that the long-run consumption increases with the probability of strong wind occurrence does not mean that a higher level of risk should be preferred. The long-run equilibrium values correspond to an hypothetical situation without strong wind realization. A higher probability of strong winds means that such an event will occur more often. It follows that the flow of consumption will more often deviate from the long-run equilibrium. This will have a negative impact on the utility of a forest owner wishing to stabilize consumption over time: a low risk environment will always be preferred to a high risk one. Last, the strong wind risk has an important impact on the optimal forest portfolio. For instance, when the strong wind risk probability goes from 2 $^{0}/_{00}$ to 5 $^{0}/_{00}$, the land allocated to the fifth age class goes from 0.625 to 0.520 ha.

3.4.3 Time and risk preferences and optimal dynamic paths

Next, we analyze the optimal management of forest age classes and wealth stock in a situation where the catastrophic risk may occur. The model has been simulated over 400 periods. At each date, the realization of the strong wind is drawn from the binomial distribution described by equation (14). We present these simulations for $\alpha = 0.1$ and σ varying from 1/3 (low EIS) to 2/3 (high EIS). Some simulations have also been conducted for other values of the constant relative risk aversion coefficient but as the qualitative results do not significantly differ, we only discuss the impact of the EIS on the optimal forest owner decisions. Figure 2 presents how the dynamic of wealth and consumption reacts to the occurrence of strong wind events.

We first focus on the recursive utility benchmark case. As it can be seen, a strong wind event is realized at date 69. The instantaneous impact of the strong wind event is to increase the forest owner wealth stock (as 10% of the forest age classes destroyed can be recovered and sold) from the long-run equilibrium, 55.01 to a higher level, 65.75. The realization of strong wind may appear to be "good news" for the forest owner as the wealth's stock initially increases but strong the wind realization implies a loss of surplus because first, it imposes a harvest at a date that may not be optimal and second, it generates higher future costs. At the same time some forest age classes are lost (age classes 4 and 5) and the proportion of fallow increases. The forest owner anticipates that the future consumption will have to be reduced (due to forest planting costs and since the new forest age class will be harvested later). Hence, the annual consumption falls from the long-run equilibrium level, 1.76, to 1.38. During the following period, all the fallow land (59% of the total area) is replanted with a new forest age class. As a consequence, the wealth stock decreases and at the same time the consumption increases. Finally, the dynamic of the system starts converging toward the long-run equilibrium. Figure 2 also allows us to analyze the impact of forest owner preferences for intertemporal substitution. As mentioned previously, a low value for σ corresponds to a low EIS which implies that substitution across periods is less desirable and that consumption should be stabilized over time. In the case of a low EIS ($\sigma = 1/3$), the forest owner tries to maintain a high level of precautionary saving (as mentioned previously the wealth long-run equilibrium decreases with σ). Once strong wind is realized, the high wealth level allows to maintain the consumption at a higher level (1.55 for $\sigma = 1/3$ compared to 1.38 for $\sigma = 1/2$ and 1.30 for $\sigma = 2/3$). In the same vein, the deviations from the long-run equilibrium, both in terms of wealth and consumption, increase with the EIS. On Figure 2, the consumption loss following a strong wind realization represents -0.33 monetary units for $\sigma = 1/3$ compared to -0.39 for $\sigma = 1/2$ and -0.43 for $\sigma = 2/3$.

These simulations suggest that time preferences profoundly affect both forest management decisions and the optimal consumption-savings dynamic path. Hence, the decision maker time and risk preferences should be viewed as important features of any framework aiming at modeling forest owner decisions facing a climatic risk.

4 Conclusion

We have used a non-expected utility approach (recursive preferences) for jointly analyzing the saving and forestry decisions of a small forest owner facing a climatic risk. This approach can contribute to forestry management as well as to natural resource economics under risk by providing an alternative structure to the usual expected utility model. Hence, the class of recursive preferences allows to separately analyze the impact of risk and time preferences on the forest owner optimal decision whereas these two distinct concepts are unattractively linked within the expected utility model. In order to solve the stochastic dynamic programming problem, we have extended the numerical approach followed by Howitt et al. [28] to the case of multi-dimensional state variables.

We have applied the stochastic dynamic programming model to the management of a French representative forest owner facing a strong wind risk. We have first shown that the linkage between consumption-savings decisions and forest management is a complex issue. This relationship is driven by a wealth, a risk-exposure and a continuation effects. The net impact of these effects crucially depends on the forest owner wealth. For instance, for a low level of wealth the risk-exposure effect and the wealth effect dominate the continuation effect. Hence, it is optimal to harvest young forest age classes. On contrary, for higher wealth level, the continuation effect dominates and only the oldest age classes are harvested. This suggests that concerns about the future likely vary with income level. Hence, discounting the future should both depend upon income and consumption levels. A move away from the standard time additive models, where utility of future consumption is discounted at a constant rate, seems justified. We have also demonstrate that time and risk preferences affect forest owner decisions (consumption, harvesting and planting) in a very different way. The numerical simulations have revealed that the impact of risk aversion on harvesting and saving decisions is not straightforward. The forest owner behavior is more driven by intertemporal substitution preferences than by risk aversion. This constitutes another important motivation from moving away from the expected utility framework.

There are several interesting directions for future researches. First, some form of insurance could be introduced into the model (private insurance, self-insurance or public funds). Another possible extension could be to introduce self-protection of forest owners into the model. In that case the conditional probability of forest loss is affected by forest owner decisions. The problem is no more stationary and the Bellman's equation can not be solved using conventional stochastic dynamic programming techniques. Last, there is now an important literature on non-market value of forests. Introducing these valuations could significantly alter the optimal duration of forest rotations. It has been shown, for instance, that the rotation length of tree stands is an effective way for managing the forest carbon budget. The forest owner should then balance longer rotations favorable to carbon sequestration with the risk increase.

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Tables

Table 1: Characteristics of forest age classes

Age class i	V_i^a (m^3/ha)	P_i^b $(10^3 euros/m^3)$	$P_i * V_i$ $(10^3 euros/ha)$	Annual return (%)	Unconditional Probability of age class loss^c (%)
1	24.60	0.0053	0.1303	12.48	0.06
2	112.20	0.0122	1.3688	9.50	1.75
3	353.50	0.0238	8.4133	4.94	3.79
4	601.40	0.0367	22.0713	1.71	4.14
5	694.70	0.0446	30.9836	0.14	4.20

^{a}: Adapted from Vannière [24].

^{b}: From Guo [20].

^{c}: Using Picard et al. [25] and based on forest expert interviews.

Table 2: Estimated parameters of the value function, $\alpha = 0.9$ and $\sigma = 1/2$

	Wealth	Age classes					Fallow
		1	2	3	4	5	
Linear terms	1.53	28.85	32.89	39.09	48.76	52.74	24.56
Cross terms							
Wealth	-0.01	0.34	0.29	0.18	0.03	-0.03	0.40
Age class 1		-1.93	-3.29	-1.72	-0.25	0.05	-4.46
Age class 2			-1.56	-1.65	-0.44	-0.25	-3.87
Age class 3				-0.46	-0.24	-0.26	-2.15
Age class 4					-0.41	-1.18	-0.37
Age class 5						-0.82	0.05
Fallow							-2.71

Table 3: Optimal harvesting and planting strategies, $\alpha = 0.9$ and $\sigma = 1/2$

	Initial wealth					
	10	20	30	40	50	60
a_1^{h*}	–	–	–	–	–	–
a_2^{h*}	–	–	–	–	–	–
a_3^{h*}	0.2	0.2	–	–	–	–
a_4^{h*}	0.2	0.2	0.2	0.2	–	–
a_5^{h*}	0.2	0.2	0.2	0.2	0.2	–
a_F^*	–	–	–	–	–	–
a^{p*}	0.6	0.6	0.4	0.4	0.2	–

Table 4: Optimal consumption-savings decisions, $\alpha = 0.9$ and $\sigma = 1/2$

	Initial wealth					
	10	20	30	40	50	60
Forest profit	13.54	13.54	10.74	10.74	5.78	0.00
Saving	18.06	36.12	54.18	72.24	90.31	108.36
Available wealth	31.60	49.66	64.93	82.99	96.09	108.36
– Used for consumption	9.15	13.73	19.63	28.40	39.40	51.98
– Used for saving	22.45	35.93	45.29	54.59	56.69	56.38

Table 5: Long-run equilibrium and forest owner preferences

α	σ	W^{LR}	c^{LR}	$A_{1,\cdots,4}^{LR}$	$a_{1,\cdots,4}^{h^{LR}}$	A_5^{LR}	$a_5^{h^{LR}}$	$A^{F^{LR}}$	$a^{F^{LR}}$
0.5	1/3	59.168	1.921	0.136	0	0.458	0.136	0	0
0.5	1/2	55.003	1.762	0.104	0	0.585	0.104	0	0
0.5	2/3	53.910	1.722	0.098	0	0.610	0.098	0	0
0.9	1/3	58.350	1.885	0.125	0	0.500	0.125	0	0
0.9	1/2	55.014	1.762	0.104	0	0.584	0.104	0	0
0.9	2/3	53.989	1.727	0.099	0	0.603	0.099	0	0
6	1/3	58.451	1.891	0.128	0	0.489	0.128	0	0
6	1/2	55.174	1.772	0.108	0	0.567	0.108	0	0
6	2/3	54.299	1.743	0.106	0	0.575	0.106	0	0

The superscript LR denotes long-run equilibrium value.

Table 6: Long-run equilibrium and uncertainty, $\alpha = 0.9$ and $\sigma = 1/2$

p	W^{LR}	c^{LR}	$A_{1,\cdots,4}^{LR}$	$a_{1,\cdots,4}^{h^{LR}}$	A_5^{LR}	$a_5^{h^{LR}}$	$A^{F^{LR}}$	$a^{F^{LR}}$
2 $^0/_{00}$	54.475	1.735	0.094	0	0.625	0.094	0	0
3.1 $^0/_{00}$	55.014	1.762	0.104	0	0.584	0.104	0	0
5 $^0/_{00}$	55.526	1.795	0.120	0	0.520	0.120	0	0

The superscript LR denotes long-run equilibrium value.

Figures

Figure 1: Value of forest age classes as a function of wealth, $J(A_i = 1, A_{-i} = 0, W)$

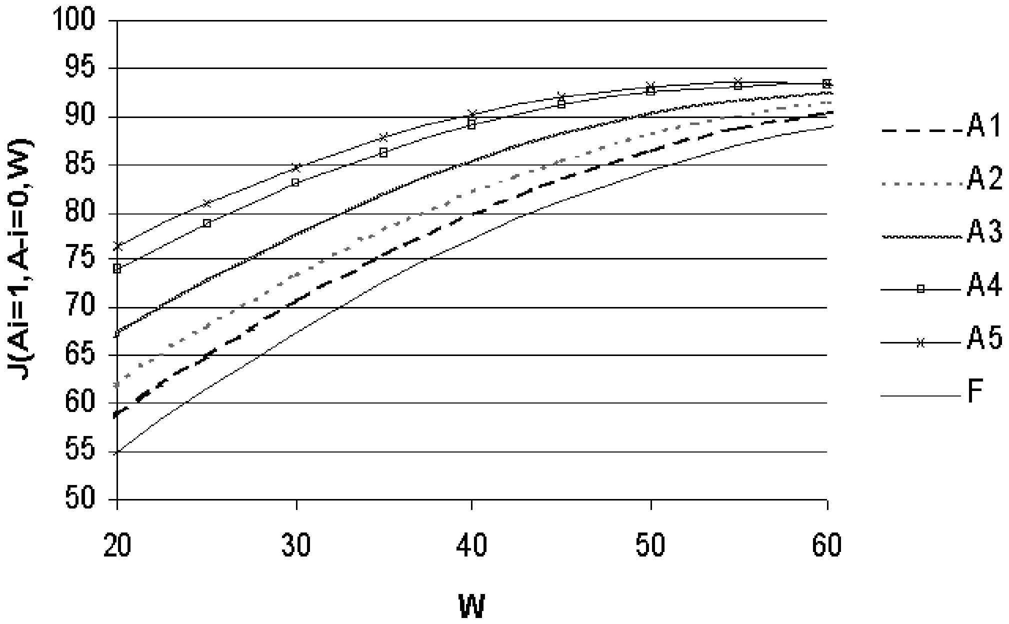
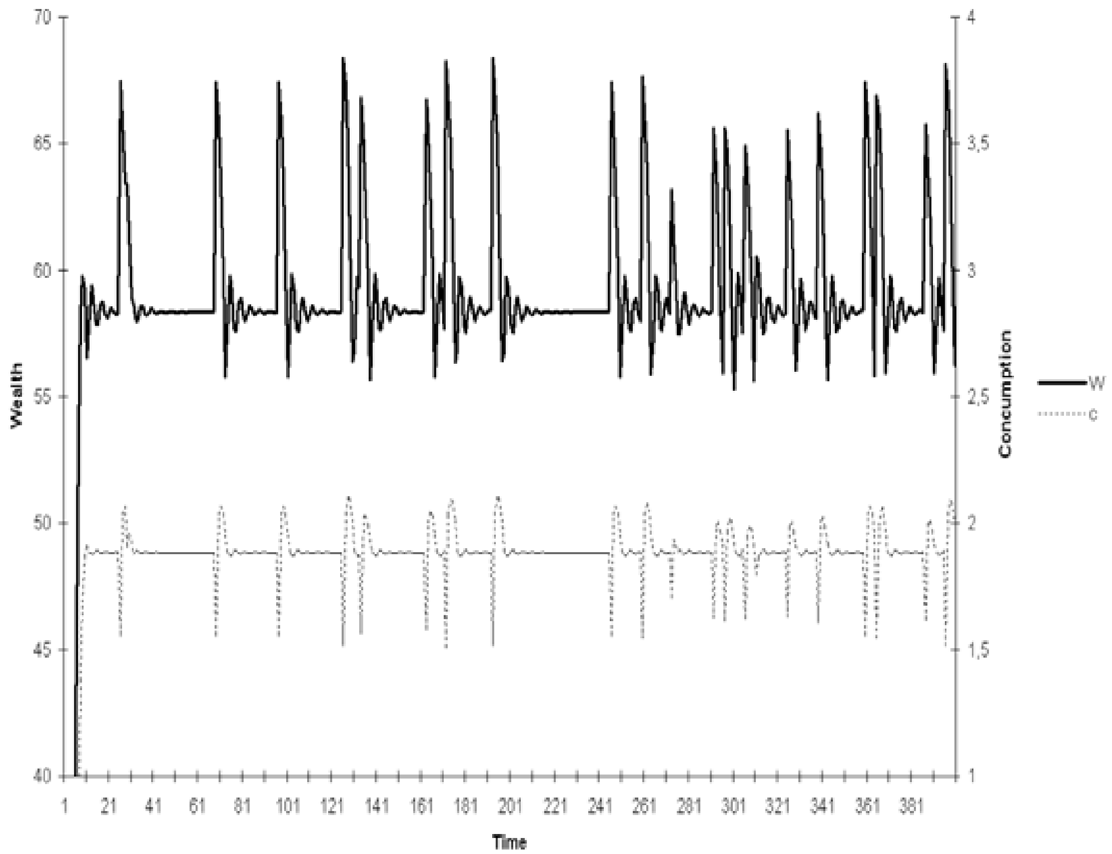
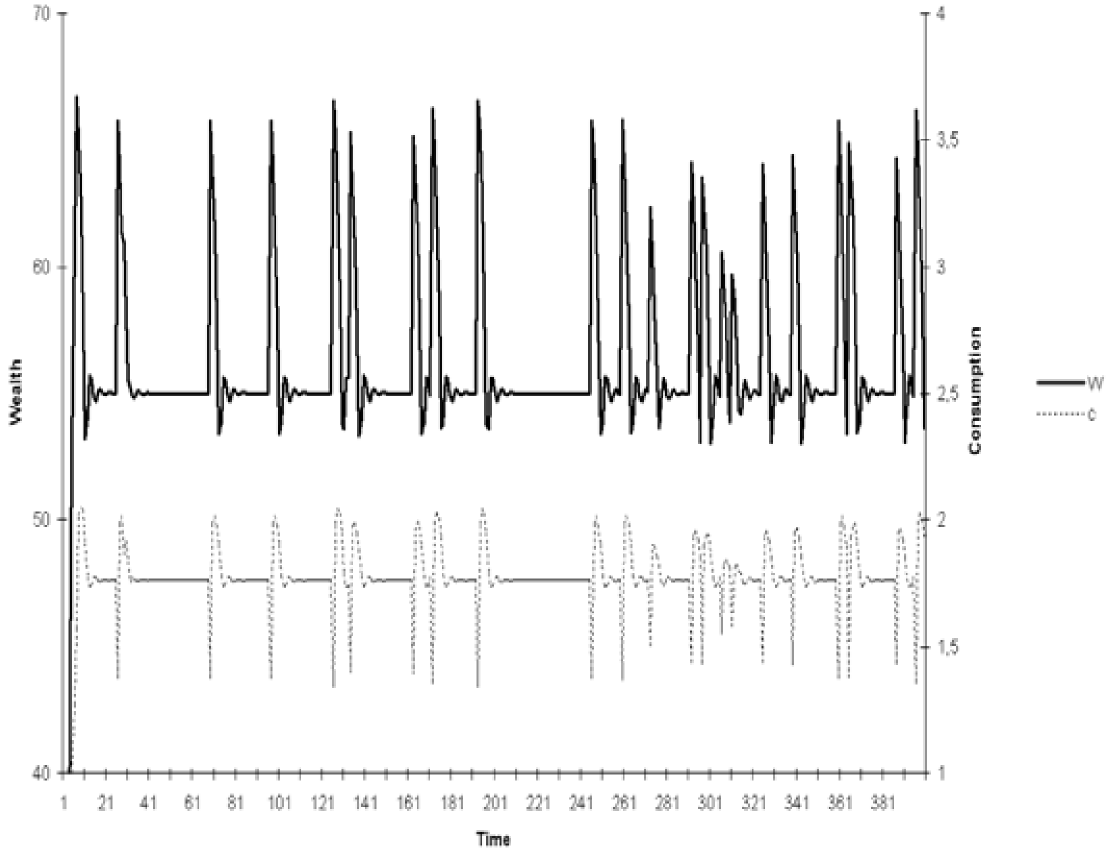


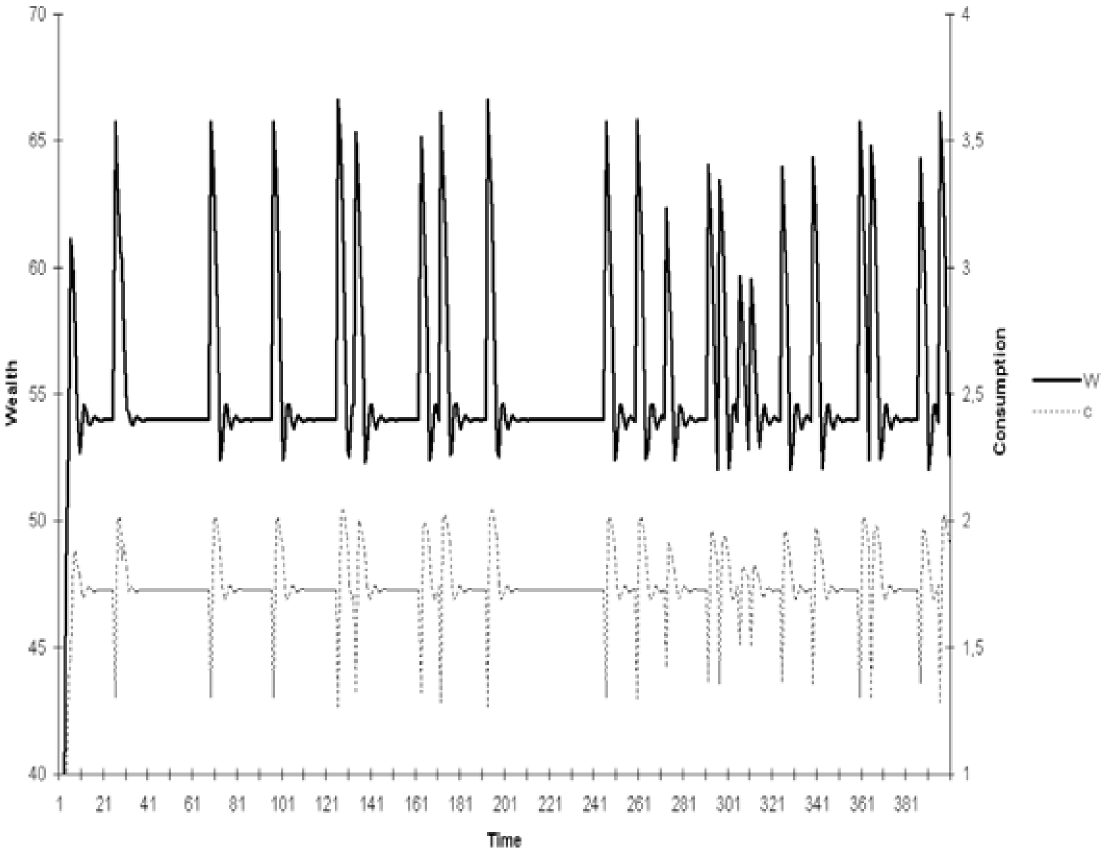
Figure 2: Wealth and consumption dynamic paths



(a) $\alpha = 0.9$ and $\sigma = 1/3$ (low EIS)



(b) $\alpha = 0.9$ and $\sigma = 1/2$ (recursive utility benchmark case)



(c) $\alpha = 0.9$ and $\sigma = 2/3$ (high EIS)