Inventories and Endogenous Stackelberg Hierarchy in Two-period Cournot Oligopoly

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Abstract

Two-period Cournot competition between n identical firms producing at constant marginal cost and able to store has a unique asymmetric pure strategy Nash equilibrium (PSNE). All firms store simultaneously different quantities and release them on the market to form endogenously a Stackelberg hierarchy. When the number of firms competing over two periods goes to infinity, the PSNE is still asymmetric but converges to the competitive outcome. When entry occurs in second period only, incumbents producing in first period form a Stackelberg hierarchy thanks to different inventories, while entrants behave as Cournot oligopolists over the residual demand left by incumbents. Small setup costs of production leave the PSNE unchanged.

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1 Introduction

On many mineral or non-mineral commodity markets producers are in imperfect competition. The highly standardized nature of these products regularly triggers aggressive commercial behaviour: producers often modify their strategies to maintain their presence on the market through a high volume of sales relatively to their opponents. Business news report for example that the U.S. uranium spot market faced a large recession at the beginning of the 1990's[1], during which producers from Kazakhstan have been the object of an anti-dumping duty conducted by the U.S. Department of Commerce[2]. A similar inquiry has been conducted on the PC memories market (the DRAM market). On this market, South-Korean producers have been alleged to systematically dump their prices when the market was entering in a downturn, in order to enlarge their market share and sell a higher volume than their American competitors[3] with the objective to enlarge their customer base. On most if not all of these markets, storage behaviour of producers or consumers are playing a crucial role in the price formation process. On the uranium market for example, on ground storage (i.e. extracted from the mine but kept out of the market) has been one of the factors contributing to the decrease of spot prices in the 1990's. The impact of storage strategies on the DRAM prices has also been identified as important: market analysts emphasized the strategy of South-Korean producers, consisting in stockpiling DRAMs when demand started to decrease, to release these quantities in the subsequent periods, causing an even worse reduction of the market price. Unsurprisingly, applied economic research also recognized the link between market power and inventories: Amihud and Mendelson [1989] amongst others show that firms with greater market power hold larger inventories, using a sample of US firms from the good-producing manufacturing industries. However a reverted causal relationship, inventories allowing firms to enjoy larger market shares and consequently greater market power in an oligopolistic industry, has not been established from a theoretical point of view so far. This is the object of the present

[1]Consequence of the correction of the estimated growth of the number of nuclear powerplants, as well as of the large strategic inventories constituted by buyers in the 1980's to face the predicted and well advertised scarcity of this natural resource

[2]see U.S. department of Commerce (1999) for the summary of the investigations started in 1991.

[3]See the panel report of the World Trade Organization, WT/DS99/R 29 January 1999.

study.

Under constant returns-to-scale and in the absence of any capacity constraint or uncertainty, we first show that oligopolistic Cournot competition in which n firms build up inventories in a first period, before producing again and selling on the market in second period, has an essentially unique asymmetric pure strategy Nash equilibrium: the distribution of inventories, the distribution of market shares, the aggregate level of inventories and the aggregate level of sales are unique in equilibrium. Without any other incentive than strategic to use inventories, firms store different quantities in equilibrium and constitute endogenously a hierarchy of Stackelberg strategic agents, each of them acting as a leader over another firm given the inventories of firms acting as a leaders over it. Cournot competition with inventories is fiercer than standard static Cournot competition, and therefore the deadweight losses due to imperfect competition are smaller when firms are able to store than when they are not.

Second, we study the effect of free entry on the distribution of inventories and market shares in equilibrium. When the number of firms in competition during the two periods goes to infinity, the equilibrium converges to the competitive outcome, but firms are still storing different quantities: for n sufficiently large, the Nash equilibrium of the game is an asymmetric (quasi-)competitive equilibrium, even if firms are ex-ante identical. The Hirschman-Herfindhal Concentration index (HHI) is consequently bounded away from zero: leaders remain on the market no matter the number of firms, even if the price-cost margin tends to zero. When inventories are used for strategic purposes, theory therefore predicts it is possible to observe identical firms obtaining different market shares, with a leader enjoying at least half of the market and followers enjoying market shares falling down to zero, without almost any losses of Social Welfare. When the number of firms in competition increases in second period only, with n incumbents able to store before the entry of m new producers, the equilibrium of the game is still asymmetric. Incumbents store different quantities and act as a Stackelberg hierarchy, while entrants behave as Cournot oligopolists over the residual demand left by incumbents. When the number of entrants goes to infinity, incumbents do not modify their inventories but the price-cost margin reduces down to 0. The Nash equilibrium of the game is still an asymmetric (quasi-)competitive outcome.

Third and finally, we introduce small setup costs of production. When these fixed costs are sufficiently small, the equilibrium of the game is still asymmetric: firms build a Stackelberg hierarchy thanks to their inventories. A larger level of fixed cost may however reduce the number of firms active in equilibrium, compared to what would happen in static Cournot competition with the same fixed cost of production.

Our findings are closely related to Boyer and Moreaux ([1985], [1986]) and Robson [1990] who show that a market with an exogenous hierarchy of Stackelberg firms is competitive when the number of firms increases, market shares being different no matter the finite number of firms in competition. A striking result of the present paper is to show how the formation of a n firms Stackelberg hierarchy can result from the use of inventories in a simple model of two-period Cournot competition.

The commitment value of inventories in two-period Cournot duopolistic competition has been recognized in the literature since the path-breaking articles of Arvan [1985] and Ware [1985]. They both identify that once costs of production have been sunk, inventories form a capacity from which firms can sell while producing at a zero marginal cost, making them more aggressive on the market. Moreover inventories create an endogenous discontinuity in the marginal cost the firm is facing when deciding how much to produce and sell on the market. A firm may therefore be committed to sell exactly its inventories for a range of sales of its rival, i.e. its supply becomes locally inelastic to the supply of its opponent, and an adequate choice of inventories may result in a first mover advantage for this firm. This effect, obviously linked to Dixit [1980], implies the existence of asymmetric equilibria even if firms are identical with respect to their production technology, but contrary to Dixit's game asymmetric equilibria do not rely on an asymmetry in the timing[4]: firms produce and sell simultaneously in every period. In an independent series of papers, Saloner [1987] and Pal ([1991], [1996]) study duopolistic Cournot competition with advance production and constant returns-to-scale, to show that multiple equilibria may arise, but that this multiplicity may disappear with cost variations. More specifically Saloner [1987] shows that any outcome on the outer envelope of the Cournot reaction functions between the two Stackelberg equilibria is a Nash equilibrium in pure strategy of this game, and Pal ([1991], [1996]) shows how cost differentials may help to

[4]See Ware [1984].

select an outcome amongst the equilibria of Saloner's game. We generalize these results to the case of an oligopoly, we study the effect of free entry and of setup costs of production, and we show how alternative scenarii may help to select the equilibrium in Saloner's game. The resolution of this game is made by constructing the backward reaction mapping proposed by Novshek [1984] (see also Bamon and Frayssé [1985]), that is by constructing the best reply of each firm to the aggregate sales of the industry compatible with this firm optimization program. Due to the use of inventories, backward reaction mappings turn out to be different across firms even if they are ex-ante identical. For each distribution of inventories, the second period quantity competition sub-game has a unique Nash equilibrium in pure strategy that may be asymmetric, and we show how the aggregation of the non-differentiable backward reaction mappings leads to the determination of this equilibrium.

Our results are closely connected to the broader and more recent literature studying the industrial organization of commodity markets[5]. In their seminal article, Allaz and Vila [1993] show that forward markets may be used by Cournot duopolists to compete for the Stackelberg leadership. The equilibrium of the game is however symmetric, firms competing in advance for a larger share of the market. Thille and Slade [2000] introduce adjustment costs in Allaz and Vila setting to study their impact on market competitiveness. Mahenc and Salanié [2004] show that forward markets may be used to soften Bertrand competition, while Liski and Montero [2006] study the impact of a forward market on the possibility to collude. An important difference between all these studies and our paper lies in the asymmetry of our equilibrium. This is due to the type of commitment considered: while forward sales can be re-interpreted as a commitment which places the entire industry in front of a reduced residual demand in the period in which forward contracts are exerted, inventories are committing only the firm which is holding them, source of the asymmetry of the Nash equilibrium. The use of inventories and forward trading in imperfect competition has also been studied in the context of longer dynamic with uncertain demand and costs. In their seminal article, Kirman and Sobel [1974] study a dynamic Bertrand oligopoly with inventories. Thille ([2003], [2006]) studies how the use of inventories and of forward trading affect price volatility. The extension of our setting to a longer horizon is left for another study.

[5]See Anderson [1984] for earlier contributions.

Finally our results suggest some testable implications on markets for storable products that can be connected to the recent papers by Hendel and Nevo ([2002], [2005]), who show that demand elasticities for storable products can be over-estimated due to the presence of consumers storage. In presence of imperfect competition, we prove that a market with a fixed number of firms may be much more competitive when firms are able and do store their finished product than when they do not, and moreover we prove that the use of strategic inventories may result in higher concentration. These theoretical findings could be profitably confronted to real data.

The remaining of the paper is organized as follows: section 2 presents the model and shows that for any distribution of ordered inventories across firms, it exists a unique sub-game equilibrium in which market shares can be asymmetric. Then section 3 derives the equilibria of the game and study the effect of entry. Section 4 studies the robustness of our results to the introduction of a fixed setup cost of production and finally section 5 concludes.

2 The Model and Preliminary Results

We consider an homogenous market with n Cournot competitors indexed by i, $i \in I = \{1, ..., n\}$, $n \geq 2$, competing over two periods indexed by $t = 1, 2$. Let q_t^i be the production level of firm i in period t, $q_t = (q_t^1, ..., q_t^n)$ the production vector in period t, $Q_t = \sum_i q_t^i$ the aggregate output and $Q_t^{-i} = \sum_{j \neq i} q_t^j$ the aggregate output of firms $j \neq i$.

Production may be undertaken in any period, but the market opens in period 2 only. Let s^i be the quantity sold by firm i in period 2. Individual sales cannot be larger than the total output available in period 2, $s^i \leq q_1^i + q_2^i$. Finally we denote $s = (s^1, ..., s^n)$ the sales vector, $S = \sum_i s^i$ the aggregate sales level and $S^{-i} = \sum_{j \neq i} s^j$ the aggregate sales of firm $j \neq i$.

Firms have access to the same constant returns production technology and the same factor prices. Each one is a "small" buyer in the factor market, taking prices as given. Hence all firms have the same constant marginal cost of production denoted by c, $c > 0$.

We assume that pure inventory costs are nil, excepted the opportunity cost of working capital. Assuming that the capital market is perfectly competitive, and denoting by ρ the interest rate, the only opportunity cost is the cost of producing in period 1 rather than in period 2, that is $\rho\ c\ q_1^i$ in terms of value in period 2. Under a free disposal assumption insuring that inventories unsold at the end of period 2 can be disposed off at zero cost, the total cost of any production and sale plan is given by (C.1), in period 2 value:

<u>Assumption</u> *(C.1) For any firm $i \in I$ and any plan $\{(q_1^i, q_2^i, s^i) : s^i \leq q_1^i + q_2^i\}$, the total cost incurred in second period, $C_2^i(q_1^i, q_2^i, s^i)$, is given by:*

$$C_2^i(q_1^i, q_2^i, s^i) = (1+\rho)\ c\ q_1^i + c\ q_2^i.$$

The market demand function is assumed to be linear. Without loss of generality we assume that its slope is equal to -1. Thus:

<u>Assumption</u> *(D.1) Let $P(S)$ be the inverse demand function, then*

$$P(S) = \max\{a - S, 0\}, \quad a > 0.$$

For the ease of the analysis most of the discussion is lead under the two following assumptions:

<u>Assumption</u> *(A.1) The intercept of the inverse demand a and the marginal cost of production c satisfy*

$$c \leq a \leq 3\ c.$$

<u>Assumption</u> *(A.2) Firms are indexed by decreasing order of period 1 production levels,*

$$q_1^1 \geq q_1^2 \geq ... \geq q_1^{n-1} \geq q_1^n.$$

Assuming that any firm can observe all the period 1 production levels but cannot observe period 2 production and sale levels of its competitors, the strategy of firm i, denoted by σ^i, is a 3-uple:

$$\sigma^i = \{q_1^i, \widetilde{q}_2^i, \widetilde{s}^i\} \tag{1}$$

where:

$$q_1^i \in \mathbb{R}_+, \quad \widetilde{q}_2^i : \mathbb{R}_+^n \to \mathbb{R}_+, \quad \widetilde{s}^i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+ \tag{2}$$

with $\widetilde{s}^i$ satisfying the following condition:

$$\forall \ (q_1, q_2^i) \in \mathbb{R}_+^{n+1}, \ \widetilde{s}^i(q_1, q_2^i) \leq q_1^i + q_2^i. \tag{3}$$

To any n-uple of strategy σ, $\sigma = \{\sigma^1, ..., \sigma^i, ..., \sigma^n\}$, corresponds a payoff function π^i for firm i given by, in period 2 value:

$$\pi^i(\sigma^i, \sigma^{-i}) = P\left(\sum_{j \neq i} \widetilde{s}^j(q_1, \widetilde{q}_2^j) + \widetilde{s}^i(q_1, \widetilde{q}_2^i)\right) \ \widetilde{s}^i(q_1, \widetilde{q}_2^i) - C_2^i(q_1^i, \widetilde{q}_2^i, \widetilde{s}^i(q_1, \widetilde{q}_2^i)). \tag{4}$$

We describe now the sub-game equilibria for each vector of inventories chosen in period 1, q_1: proposition 1 presents the aggregate oligopolistic sales at the Nash equilibrium of each sub-game, and corollary 1 the corresponding equilibrium individual sales. As expected, individual and aggregate sales in equilibrium depend on the level of inventories produced by firms in period 1, and differ depending how large individual inventories are. The conditions on inventories characterizing each sub-game can be re-expressed in terms of differences between the marginal revenues obtained by firms when selling their inventories and firms *effective* marginal costs of production, as we shall now explain. The formal proof of these results is given in appendix A.1, but the reader can find hereafter the main steps leading to proposition 1 and its corollary.

The *effective* marginal cost of production of Cournot oligopolist storing their finished product is linked to, but differ from, the marginal cost of production as follows. As the cost of production of inventories is sunk when selling on the market in second period, inventories can be thought of as an exhaustible capacity from which firms can sell without producing as long as the quantity currently sold is smaller than the quantity previously stored. The marginal cost of supplying a unit from inventories is therefore equal to 0, as long as inventories are not exhausted. As soon as its sales exceed its inventories, a firm must produce again and suffers a positive marginal cost of production. Consequently bringing inventories of the finished product from one period to another modifies the economic behaviour of a firm by modifying its second period competitive supply. Although this effect of inventories has already been explained[6], and obviously relates to Dixit [1980], it is

[6] See Arvan [1985] and Ware [1985]

useful to present it differently from older studies by introducing formally the effective marginal cost of production of each firm, $\gamma^i(s^i, q_1^i)$, given by

$$\gamma^i(s^i, q_1^i) = \begin{cases} 0 & \text{if } s^i \leq q_1^i \\ c & \text{if } s^i > q_1^i. \end{cases} \tag{5}$$

Even if technologies are ex-ante identical, effective marginal costs differ across firms once inventories have been produced, and present a firm-specific jump at $s^i = q_1^i$. The set of economically feasible sales at a given market price is therefore larger when firms store their output than when they do not: firms are now ready to sell up to their inventories q_1^i if the market price p goes below their marginal cost of production c. Since producing today, storing and releasing these inventories tomorrow costs $(1+\rho)\ c\ q_1^i$, there is very little interest for firms in perfect competition to store their product if they enjoy constant returns-to-scale and face no uncertainty, as it is the case here: they would prefer not to store at all rather than suffering losses if they do. However, Cournot oligopolists integrating the effect of their decision on their future behaviour and on the equilibrium market price it will result in could find the possibility to store profitable. Indeed, by storing a quantity q_1^i large enough, a Cournot competitor may resist to an increase in its opponent sales by still preferring to release its inventories q_1^i on the market rather than reducing its sales. This effect of inventories can be derived by confronting $\gamma^i(s^i, q_1^i)$ to the marginal revenue $m^i(s^i, S^{-i})$,

$$m^i(s^i, S^{-i}) = a - 2\ s^i - S^{-i}, \tag{6}$$

to obtain the best response of a firm to an increase in sales of its competitors. Given the form of the effective marginal cost of production γ^i, the individual best reply shows the three different types of behaviour of firm i, depending on competitors sales S^{-i} and on initial inventories q_1^i. If given q_1^i, competitors sales S^{-i} are such that the marginal revenue to sell q_1^i, $m^i(q_1^i, S^{-i})$, exceeds the marginal cost c, then firm i produces again in second period and sells more than q_1^i. If $m^i(q_1^i, S^{-i})$ is lower than the marginal cost c but positive, then firm i sells exactly its inventories. Finally if $m^i(q_1^i, S^{-i})$ is strictly negative, then firm i is better off selling less than its inventories. Individual best responses are given by

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a - c - S^{-i}) & \text{if } S^{-i} \leq a - c - 2q_1^i \\ q_1^i & \text{if } S^{-i} \in [a - c - 2q_1^i, a - 2q_1^i] \\ \frac{1}{2}(a - S^{-i}) & \text{if } a - 2q_1^i \leq S^{-i} \end{cases} \tag{7}$$

Inventories therefore create firm-specific kinks in firms best replies, the firm best reply becomes locally inelastic to an increase in its competitors sales, and finally for any given level of sales of its competitors, the level of output a firm sells is higher when it owns inventories than when it does not. This is the source of the asymmetry of the Nash equilibrium in our game. To find all the equilibria of the game, we start to define the aggregate sales S compatible with the maximisation program of an individual firm i, by adding S^{-i} to the best response $\widehat{s}^i(S^{-i}, q_1^i)$. We denote this function $S_i^\bullet(S^{-i}, q_1^i)$, and we invert it to find aggregate sales of the competitors of firm i compatible with firm i maximisation program; we denote $S^{-i}(S, q_1^i)$ this function. Finally, we deduce the individual sales of firm i which best responds to aggregate sales S and which are compatible with firm i's optimization, by solving $S - s^i = S^{-i}(S, q_1^i)$ with respect to s^i. This last operation gives us the backward reaction mapping $s^i(S, q_1^i)$. We then aggregate all the backward reaction mappings, and we determine the fixed points of $\hat{S}(S, q_1^i) = \sum_{i=1}^n s^i(S, q_1^i)$. We draw the reader's attention on the fact that we describe the second period sub-game equilibria focusing on level of inventories lower than the quantity q^m a monopoly minimizing its cost of production would produce, where $q^m = (a - c)/2$. Indeed, due to the presence of the interest rate ρ and of $n - 1$ competitors, no firm stores more than the quantity a monopoly producing in second period would sell, and all the sub-games that follow inventory choices larger than q^m for all firms are trivially dominated. This restriction turns out to be useful when aggregating the individual best replies to derive the aggregate sales at the Nash equilibrium. To each fixed point of $\hat{S}(S, q_1^i)$ corresponds a unique n-uple of equilibrium individual sales, and the conditions on inventories vectors leading to each of the equilibrium sales appear to be linked to conditions on the marginal revenues and on the marginal costs, as we now discuss.

Let us give an example of conditions on inventories leading to a particular Nash equilibrium in the sales sub-game. To be in a sub-game equilibrium in which firms $1, 2, ..., k$ sell exactly their inventories, and firms $k + 1, ..., n$ sell strictly more than their inventories, it suffices that given equilibrium sales of competitors,

(1) the marginal revenue of firm $k+1$ when selling q_1^{k+1} is strictly higher than the marginal cost c (so will it be for firms $k+2, ..., n$ who own inventories lower than firm $k+1$),

(2) the marginal revenue of firm k when selling q_1^k is strictly lower than c (so will it be for firms $1, ..., k-1$ who own inventories higher than firm k) and

(3) the marginal revenue of firm 1 when selling q_1^1 is strictly positive (and so will it be for firms $2, ..., k$).

Figure 1 presents this case,

[INSERT FIGURE 1 HERE]

Indeed, when only firms $i = 1, ..., k$ sell exactly their inventories, then in equilibrium firms $k+1, ..., n$ sell the same quantity higher than their inventories. Firms $k+1, ..., n$ are therefore confronted to the same equilibrium sales of competitors (equal to the sum of inventories of firms $1, ..., k$ plus $(n-1)$ times the quantity sold by any of the firms $k+1, ..., n$). Consequently the marginal revenues of firms $k+1, ..., n$ are identical functions of individual sales s. Due to the fact that the marginal revenue is decreasing in individual sales s, then if firm $k+1$ (with the highest level of inventories q_1^{k+1} in the group of firms $\{k+1, ..., n\}$ selling more than their inventories) faces a marginal revenue higher than the effective marginal cost at $s = q_1^{k+1}$, then the same is true for all the other firms of this group, explaining condition (1).

On the contrary, the marginal revenues of firms $1, ..., k$ (who sell exactly their inventories) evaluated at the equilibrium sales of their competitors jump upward the higher the level of firm inventories. Indeed firm i is confronted to aggregate sales in equilibrium equal to the sum of inventories of firms $1, ..., i-1, i+1, ...k$, plus the identical sales of firms $k+1, ..., n$. This quantity is lower the higher the level of inventories of firm i, since the aggregate level of inventories of competitors selling these inventories is mechanically lower. The value of the marginal revenue at $s^i = q_1^i$ is however lower the higher the level of inventories of the firm we consider, by definition of the marginal revenue[7], which explains why focusing on the comparison

[7] One replaces $-q_1^i$ by $-2q_1^i$ in each expression.

between the marginal revenue of firm k when selling q_1^k and the marginal cost c is sufficient (condition (2)).

Finally under assumption (A.1), and restricting our attention to $q_1^1 \leq (a-c)/2$, condition (3) is verified as long as we consider $k < n$. However when we characterize the sub-game equilibrium in which all firms are selling exactly their inventories, i.e. such that the marginal revenue of firm n is lower than c when it sells exactly q_1^n, then condition (3) needs to be verified. If (3) is verified, then the marginal revenues of firms $1, ..., n-1$ are also positive, leading to the sub-game equilibrium we are searching for.

The same analysis can be done to characterize all the sub-game equilibria, in particular those in which some firms are selling exactly their inventories and some others less than their inventories. To present all the sub-game equilibria, we introduce the following set of notations. The sequence of sets $\{B(\ell)\}_{\ell=0,...,2n}$ characterizes for each ℓ the values of inventories leading to a sales sub-game in which some firms are selling more than, some firms are selling exactly, and some firms are selling less than their inventories. We define this sequence as follows:

- $B(0) = \{q_1 \mid m^1\left(q_1^1, (n-1)(a-c)/(n+1)\right) \geq c\}$ denotes the set of inventories such that all firms are selling in second period strictly more than their inventories,

- For $\ell \in \{1, ..., n-1\}$, $B(\ell)$ denotes the set of inventories such that firms $1, ..., \ell$ are selling exactly their inventories and firms $\ell+1, ..., n$ are selling strictly more. It is defined as
$B(\ell) = \{q_1 \mid m^\ell\left(q_1^\ell, (n-\ell)(a-c-\sum_{i=1}^{\ell} q_1^i)/(n-\ell+1) + \sum_{i=1}^{\ell-1} q_1^i\right) < c,$
$m^{\ell+1}\left(q_1^{\ell+1}, (n-\ell-1)(a-c-\sum_{i=1}^{\ell} q_1^i)/(n-\ell+1) + \sum_{i=1}^{\ell} q_1^i\right) \geq c\},$

- $B(n) = \{q_1 \mid m^n\left(q_1^n, \sum_{i=1}^{n-1} q_1^i\right) < c, m^1\left(q_1^1, \sum_{i=2}^{n} q_1^i\right) \geq 0\}$ denotes the set of inventories such that all firms are selling exactly their inventories,

- For $\ell \in \{n+1, ..., 2n-1\}$, $B(\ell)$ denotes the set of inventories such that firms $1, ..., \ell-n$ sell less than their inventories and firms $\ell-n+1, ..., n$ sell exactly their inventories. It is defined as

$$B(\ell) = \{q_1 \mid m^{\ell-n}\left(q_1^{\ell-n}, (\ell-n-1)(a-\textstyle\sum_{i=\ell-n+1}^{n} q_1^i)/(\ell-n+1) + \sum_{i=\ell-n+1}^{n} q_1^i\right) < 0,$$
$$m^{\ell-n+1}\left(q_1^{\ell-n+1}, (\ell-n)(a-\textstyle\sum_{i=\ell-n+1}^{n} q_1^i)/(\ell-n+1) + \sum_{i=\ell-n+2}^{n} q_1^i\right) \geq 0\},$$

- Finally $B(2n) = \{q_1 \mid m^n(q_1^n, (n-1)a/(n+1)) < 0\}$ denotes the set of inventories such that all firms sell less than their inventories.

As its proof shows (available in appendix A.1), this set of notations is sufficient to establish the following proposition and its corollary.

Proposition 1 *Under assumptions (A.1) and (A.2), restricting the attention to inventories lower than the quantity a monopoly minimizing its costs would produce, $q_1^1 \leq (a-c)/2$, aggregate sales S^* at the Nash equilibrium are given by:*

1. *if $q_1 \in B(0)$, then all firms sell more than their inventories, and $S^*(0) = \frac{n(a-c)}{n+1}$,*
2. *if $q_1 \in B(\ell)$ for $\ell \in \{1, ..., n-1\}$, then firms 1 to ℓ sell exactly their inventories and*
$$S^*(\ell) = \frac{(n-\ell)(a-c)}{n-\ell+1} + \frac{\sum_{i=1}^{\ell} q_1^i}{n-\ell+1},$$
3. *if $q_1 \in B(n)$ then all firms sell exactly their inventories and $S^*(n) = \sum_{i=1}^{n} q_1^i$,*
4. *if $q_1 \in B(\ell)$ for $\ell \in \{n+1, ..., 2n-1\}$, then firms 1 to $\ell - n$ sell less than their inventories and*
$$S^*(\ell) = \frac{(\ell-n)\ a}{\ell-n+1} + \frac{\sum_{i=\ell-n+1}^{n} q_1^i}{\ell-n+1},$$
5. *if $q_1 \in B(2n)$ then all firms sell less than their inventories and $S^*(2n) = \frac{n\ a}{n+1}$.*

As for a given level of industry sales there is a unique corresponding level of individual sales, there is a unique vector of individual sales in equilibrium. Individual sales in equilibrium are given in the corollary below.

Corollary 1 (to Proposition 1) *For each equilibrium level of aggregate sales $S^*(\ell)$, $\ell = 0, ..., 2n$, there is a unique equilibrium vector of individual sales $s^*(\ell)$ given by:*

1. *if $q_1 \in B(0)$, $s^{i*}(0) = \frac{a-c}{n+1}$ for any $i \in I$,*
2. *if $q_1 \in B(\ell)$ for $\ell \in \{1, ..., n-1\}$, then $s^{i*}(\ell) = q_1^i$ for $i = 1, ..., \ell$ and*
$$s^{i*}(\ell) = \frac{a-c}{n-\ell+1} - \frac{\sum_{i=1}^{\ell} q_1^i}{n-\ell+1} \text{ for } i = \ell+1, ..., n,$$
3. *if $q_1 \in B(n)$ then $s^{i*}(n) = q_1^i$ for all $i \in I$,*

4. *if* $q_1 \in B(\ell)$ *for* $\ell \in \{n+1,...,2n-1\}$*, then* $s^{i*}(\ell) = q_1^i$ *for* $i = \ell - n + 1, ..., n$ *and*

$$s^{i*}(\ell) = \frac{a}{\ell - n + 1} - \frac{\sum_{i=\ell-n+1}^{n} q_1^i}{\ell - n + 1} \text{ for } i = 1, ..., \ell - n,$$

5. *if* $q_1 \in B(2n)$ *then* $s^{i*}(2n) = \frac{a}{n+1}$ *for any* $i \in I$*.*

We turn now to the analysis of the equilibrium of the game.

3 Inventories in equilibrium and effect of entry

To start with, we state what cannot be an equilibrium of our game, to determine on which of the regions $\{B(\ell)\}_{\ell=0,...,2n}$ we can focus the search for a Nash equilibrium in pure strategies, if it exists. The lemma below shows first that it is not possible for more than one firm to store a quantity that requires to produce again in second period. This implies directly that firms will not produce in second period to sell the same quantity on the market. Remark that it does not rule out the fact that firms may produce and store the same quantity in first period. Second, firms cannot store more than what they sell in second period: as inventories are costly to produce, firms are strictly better off storing exactly what they sell. Third, the firm with the smallest level of inventories cannot store in equilibrium, as it is better off waiting for the second period to produce.

Lemma 1 *Any n-uple of inventories* $q_1 = (q_1^1, ..., q_1^n)$ *such that:*

(i) some firms are selling strictly less than their inventories, i.e. $q_1 \in B(\ell)$ *for* $\ell = n+1, ..., 2n$*,*

(ii) more than one firm are selling the same quantity strictly higher than their inventories, i.e. $q_1 \in B(\ell)$ *for* $\ell = 0, ..., n-2$*,*

(iii) firm n with the smallest level of inventories holds a quantity q_1^n *strictly positive, i.e.* $q_1 \in B(n)$*,*

cannot be an equilibrium.

Proof. Since inventories are costly to produce, if they were selling strictly less than their inventories, firms would be strictly better off reducing unilaterally their inventories. Situations in which some firms are selling less than their inventories cannot be an equilibrium: (i) holds.

To prove (ii), consider a situation in which firms 1 to k are selling exactly their inventories and firms $k+1$ to n are selling the same quantity $(a-c-\sum_{i=1}^{k} q_1^i)/(n-k+1)$ (described in corollary 1), that is the case of $q_1 \in B(k)$. We now prove that firm $k+1$ is better off increasing its inventories to make sure that it sells exactly q_1^{k+1} rather than the same quantity than firms $k+2, ..., n$. Indeed, if it sticks to q_1^{k+1} such that it sells $(a-c-\sum_{i=1}^{k} q_1^i)/(n-k+1)$, its profit is equal to

$$\pi^{k+1} = \frac{(a-c-\sum_{i=1}^{k} q_1^i)^2}{(n-k+1)^2} - \rho\ c\ q_1^{k+1}.$$

For ρ close enough to 0, this profit is constant in q_1^{k+1}. If firm $k+1$ increases its inventories to a level such that it sells exactly its inventories, as firms 1 to k do, but in which the remaining firms ($k+2$ to n) are still selling in second period and react optimally to inventories of firm $k+1$, then its profit is equal to

$$\pi^{k+1} = \frac{(a-c-\sum_{i=1}^{k+1} q_1^i)\ q_1^{k+1}}{n+k} - \rho\ c\ q_1^{k+1}.$$

This profit reaches a maximum when $q_1^{k+1} = (a-c-\sum_{i=1}^{k} q_1^i - (n-k)\ \rho\ c)/2$ that must be compared with the bounds of the region we consider, $q_1 \in B(k+1)$. For ρ sufficiently close to 0, this maximum is indeed higher than the lower bound $(a-c-\sum_{i=1}^{k} q_1^i)/(n-k+1)$. Since the profit is continuous, and for ρ sufficiently small, firm $k+1$ is better off storing

$$\min\{(a-c-\sum_{i=1}^{k} q_1^i - (n-k)\ \rho\ c)/2, a-c-\sum_{i=1}^{k} q_1^i - (n-k)\ q_1^{k+2}\}$$

than storing a lower quantity leading her to sell the same quantity than firms $k+2, ..., n$. This argument can be repeated for any $k = 1, ..., n-1$, and (ii) holds.

Finally (iii) is obviously granted: as $\rho > 0$ but small, firm n with the lowest level of inventories prefers to minimize its cost of production by setting $q_1^n = 0$ and producing its sales in second period.$\|$

This lemma has several important implications. Even if it does not rule out a priori equilibria in which all firms apart one select the same level of inventories, it directly leads us to search for a Nash equilibrium in the region $q_1 \in B(n-1)$, where equilibrium quantities are given by:

$$\begin{array}{l} S^*(n-1) = (a-c+\sum_{i=1}^{n-1} q_1^i)/2, \quad s^{i*}(n-1) = q_1^i \text{ for } i=1,...,n-1, \\ \text{and } s^{n*}(n-1) = (a-c-\sum_{i=1}^{n-1} q_1^i)/2. \end{array} \tag{8}$$

It therefore rules out the possibility for group of more than one firm to act as a non-cooperative leader over another group of more than one firm reacting identically in the second period: there is at most one ultimate follower. Consequently the equilibrium of the game, if existing, can only be asymmetric with (at least) one firm behaving as a follower with respect to $n-1$ leaders. Second, remark that in the range of inventories $B(n-1)$, sales of each firm are not modified by an increase in their opponent inventories: inventories commit firms behaviour on the second period market. The last ingredient we need is to prove that a firm facing a leader does not find profitable to increase its inventories in order to force him to sell less than its inventories: generalizing Arvan's terminology to the oligopoly case, any follower must not be able to put any of its leaders with redundant inventories.

Lemma 2 *Any firm k willing to force one of the leaders $1,...,k-1$ to sell less than its inventories, must store a quantity such that the second period market price falls below the marginal cost of production, c. No firm finds this strategy profitable.*

Proof. This result can be proved by simply looking at the case of firm 1 and 2 when $q_1 \in B(n-1)$. Imagine firm 2 wishes to force firm 1 to sell less than q_1^1. To do so, it must increase q_1^2 at a level such that $q_1 \in B(n+1)$, that is $q_1^1 \geq (a - \sum_{i=2}^n q_1^i)/2$ and such that $q_1^2 \leq (a - \sum_{i=3}^n q_1^i)/3$ to still sell its inventories on the market. The aggregate sales reached at this deviation are equal to $S^*(n+1) = (a - \sum_{i=2}^n q_1^i)/2$. Under assumption $(A.1)$ and $(A.2)$, we have seen in the proof of proposition 1 that $a-c-q_1^n \leq a-c \leq (a+c)/2 \leq a-q_1^1$. Aggregate sales $S^*(n+1)$ are by definition higher than $a-q_1^1$, and consequently higher than $a-c$. Therefore the second period market price must be lower than the marginal cost c for the strategy of putting firm 1 with redundant inventories to be successful. This strategy is obviously strictly dominated for any firm, as the profit is would obtain would be strictly negative. This argument applies to any firm wishing to force its leaders to sell less than their inventories.$\|$

As stressed in the proof above, this result follows directly from the assumption that consumers willingness to pay is not too large compared to the marginal cost of production. If it is not the case, asymmetric equilibria may still prevail, but instead of being extremely close to Stackelberg outcomes, the leader chooses a level of inventories smaller than the Stackelberg leader output and such that the follower cannot put him with redundant inventories[8].

We can discuss assumption (A.1) in the perspective of the previous literature. Although Arvan [1985] identifies the possibility for a leader to be put with redundant inventories in his duopoly setting, he does not show that for this threat to modify the behaviour of the leader, it must be the case that the market price stays at a level such that the follower, when increasing its inventories and threatening the leader, realizes a non-negative profit. The role of assumption (A.1) is here to rule out credible threats of putting leaders with redundant inventories, by forcing prices to be lower than the marginal cost every time a follower increases its inventories to force the leader to sell less than its inventories. The fact that redundant inventories (or production in their game) may matter, and the assumption ruling it out, are absent from the analysis of Saloner [1987], Pal ([1991], [1996]).

The remaining of the paper consists in determining the Nash equilibrium in pure strategies of our game, tackling that way the issue of its existence. This equilibrium turns out to be such that firms store different amounts and create a hierarchy of Stackelberg leaders. As we consider an interest rate ρ close to 0, the equilibrium does not depend on ρ. We discuss the role and the importance of this assumption at the end of this section.

Theorem 1 *The two-period Cournot oligopolistic competition with constant marginal cost of production c possesses a unique asymmetric Nash equilibrium in pure strategies, in which inventories and individual sales of firm of rank $i = 1, ..., n$ are given by*

$$q_1^{i*} = s^{i*} = \frac{a-c}{2^i}, \quad \textit{for all } i = 1, ..., n-1, \quad \textit{and } \; q_1^{n*} = 0, \; s^{n*} = \frac{a-c}{2^n}$$

[8] See Mitraille [2005]. In the duopoly game with ρ close to 0, this level is equal to the intersection between the curves $q_1^1 = \frac{a}{2} - \frac{1}{2}q_1^2$ and $q_1^2 = \frac{a-c}{2} - \frac{1}{2}q_1^1$ that is $(q_1^1, q_1^2) = (\frac{a+c}{3}, \frac{a-2c}{3})$. The assumption (2) is too restrictive on purpose: it would presumably be sufficient to assume that at the Stackelberg leader production, the second firm in the hierarchy cannot put the leader with redundant inventories.

Proof. From lemmas 1 and 2, we know that we can restrict our attention to inventories leading to (8). As firms are committed it suffices to optimize "sequentially" the payoffs, by starting with the choice of firm $n-1$ given the inventories owned by its competitors, then determine what firm $n-2$ chooses given the behaviour of firm $n-1$ ... and so on ... Each firm acts as a monopoly over the residual demand left by firms acting as leaders over it, integrating the influence on its followers. The result is straightforward to derive.‖

Aggregate sales and inventories are given by

$$S^* = (a-c)\left(1-\left(\frac{1}{2}\right)^n\right) \text{ and } I^* = (a-c)\left(1-\left(\frac{1}{2}\right)^{n-1}\right) \tag{9}$$

When the number of firms is finite, the welfare impact of inventories in Cournot competition can be derived by comparing aggregate sales in a static Cournot game with the one above. In static Cournot competition (indexed by SC), $S^{SC} = \frac{n(a-c)}{n+1}$. It is straightforward to verify that $S^* > S^{SC}$ for any number of firms in competition, and therefore the profit of the industry is lower in Cournot competition with inventories than in the standard static Cournot competition. Consumers surplus is therefore higher in Cournot competition with inventories than in static Cournot competition, and the welfare losses due to imperfect competition are smaller. Due to the strategic use of inventories to exert some leadership, competition is therefore fiercer.

We study now the consequence of free entry on our results, by analyzing two alternative scenarii. First, free entry of firms in period 1 competing therefore over two periods, and second, free entry in period 2 of m new firms competing with n firms already able to produce in period 1. From the analysis above, it is straightforward to analyze the consequence of the first scenario, free entry in period 1, and prove the corollary 2 below.

Corollary 2 *When the number n of firms in competition over the two periods increases, the aggregate sales S^* converges to the competitive sales $S^{CE} = a-c$ quicker than in the situation where Cournot competitors are unable to store. Consequently the Social Welfare converges quicker to the optimum when firms are able to store than when they are not.*

Proof. Consider the difference between the competitive outcome and our aggregate sales,

$$(a-c) - S^* = (a-c)\left(\frac{1}{2}\right)^n$$

and the same difference for the traditional static Cournot oligopoly;

$$(a-c) - \frac{n}{n+1}(a-c) = (a-c)\frac{1}{n+1}$$

It is immediate to check that $\left(\frac{1}{2}\right)^n$ converges to 0 quicker than $\frac{1}{n+1}$.‖

We may illustrate the speed with which Cournot competition with inventories converges to perfect competition by computing the ratio of aggregate Cournot sales over aggregate sales in perfect competition, with and without inventories. When the number of firms is equal to 4, Cournot competitors are selling an aggregate quantity equal to 80% of aggregate sales in perfect competition (that is of the social optimum here), while Cournot competitors able to store are selling an aggregate quantity equal to 93.75% of the social optimum!

More strikingly, the asymmetry of the Nash equilibrium of the game persists under free entry in period 1. This feature of Cournot competition with inventories can be proved by computing the limit of the Hirshman-Herfindahl concentration index (HHI) when the number of firms in competition in period 1 increases.

Corollary 3 *When the number of firms competing in period 1 tends to infinity, the equilibrium becomes competitive but firms still enjoy asymmetric market shares and leaders are present on the market as long as $n < +\infty$. The HHI concentration index is*

$$HHI = \lim_{n\to+\infty}\sum_{i=1}^{n}\left(\frac{s^i}{S}\right)^2 = \frac{1}{3} > 0.$$

Proof: It suffices to compute the limit of the summation in the corollary.‖

As the price converges to the marginal cost of production when the number of firms in competition in period 1 increases, we have proved that Cournot competition with inventories may result in an almost perfectly competitive outcome, that is a Lerner index close to 0, with concentration in market shares, that is an HHI strictly positive.

The results presented above are still valid in the case of free entry in period 2 only, as we shall now prove. Consider the case of n firms able to store in period 1 before the entry of m rivals in period 2, both groups of firms selling and producing simultaneously in period 2. Let us call "incumbents" the n firms present in period 1, and "entrants" the m firms producing and selling in period 2 only. First, with a finite number of entrants, theorem 1 still holds.

Theorem 2 *When there are n incumbent firms in competition in first and second period and m entrants able to produce and sell in second period only, the game possesses a unique asymmetric Nash equilibrium in pure strategies. Inventories and individual sales are given by*

$$q_1^{i*} = s^{i*} = \frac{a-c}{2^i}, \quad \textit{for all}\, i = 1, ..., n, \quad \textit{and} \;\; q_2^{j*} = s^{j*} = \frac{(a-c)}{2^n\ (m+1)} \;\; \textit{for all}\, j = 1, ..., m,$$

Proof: See appendix.||

The n incumbents use their inventories to form a hierarchy of Stackelberg strategic agents, and the m entrants act as Cournot competitors over the residual demand left by firms able to store. If the optimal inventory policy of the $n-1$ incumbents with the highest levels of inventories do not differ from what we obtained in theorem 1, the notable difference comes from the behaviour of the firm with the smallest level of inventories (the ultimate follower in the population of incumbents): instead of waiting until period 2 to produce and sell, incumbent n expecting entrants to produce in second period prefers to store in period 1 to exert some leadership upon entrants and force them to reduce their output in equilibrium. There are consequently two types of asymmetry in market shares on this market: an asymmetry within the incumbents, acting as a cascade of leaders-followers, and an asymmetry between incumbents and entrants, incumbents enjoying as a whole a larger market share than entrants. Entrants on the other hand obtain symmetric market shares in equilibrium, and share equally the residual demand left by incumbents. Aggregate sales of the two groups are

$$S_I^* = (1-2^{-n})(a-c) \text{ and } S_E^* = \frac{m\ (a-c)}{(m+1)\ 2^n} \tag{10}$$

When the number of entrants increases, it is obvious to verify the following corollary.

Corollary 4 *When the number of entrants in second period goes to infinity, the equilibrium becomes competitive but incumbents still enjoy asymmetric market shares, with leaders present on the market.*

It is worth noting that the role of entrants and the role of the interest rate relate directly to the issue of equilibrium selection in our game. As we proved above, a positive interest rate or the possibility of entry in second period modify the behaviour of the ultimate follower in the incumbent population. This ultimate follower is the economic agent responsible for the multiplicity of equilibria if we do not introduce the interest rate or the possibility of entry in the game.

Indeed, by making first period production slightly more costly than second period production, or by making competition more intense for the ultimate follower in second period if he chooses to wait, we are able to rule out of the set of equilibria the weakly dominated ones. If there were no interest rate and no entrants in second period, the ultimate follower would be indifferent between producing in period 1 and producing in period 2. But if he were producing for example the n-firms Cournot production in period 1, all the other incumbents would also be better off producing a Cournot outcome in period 1. Similarly if the ultimate follower were producing any quantity between the Cournot outcome $(a-c)/(n+1)$ and the ultimate follower sales $(a-c)/2^n$, all the other incumbents would choose to produce what maximizes their payoff given the positive inventories of the ultimate follower. The result we would obtain would simply be a generalization to the case of an oligopoly of the result obtained for a duopoly by Saloner [1987], who shows that any pair of inventories on the outer envelope of the Cournot reaction functions between the two Stackelberg outcomes[9] is a an equilibrium in pure strategies. The interest rate (or a cost of storage) is a selection device that forces the ultimate follower to wait until period 2 to produce: this can be seen as a special case of the selection criterium presented in the duopoly game by Pal ([1991], [1996])[10], who consider more generally cost differentials between periods. The presence of potential entrants in second period is a selection device that forces the ultimate follower to act as leader over the entrants

[9]Under some extra assumptions on demand and cost missing in Saloner's work relating to assumption (A.1) and to the fact that leaders may be put with redundant production, as we argued before.

[10]See also Robson [1990].

by storing its follower' sales in first period to be committed to it, and this is a different selection device to argue that only the Stackelberg outcomes should be considered as equilibria in Saloner's game.

4 Robustness to the presence of small fixed costs

We consider now the case in which firms suffer small fixed costs when their production is strictly positive[11]. We restrict our attention to fixed costs identical for all competitors, and such that at the unique pure strategy Nash equilibrium of a static Cournot oligopolistic interaction, n firms are active.

<u>Assumption</u> *(C.2) The per period cost of production of any firm $i \in I$ is given by*

$$C(q^i) = \begin{cases} F + c\, q^i & \text{if } q^i > 0 \\ 0 & \text{if } q^i = 0 \end{cases}$$

where $f = \sqrt{F}$ satisfies $f < (a-c)/2n$. The total cost of the production and sale plan incurred in second period is equal to

$$(1+\rho)\ C(q_1^i) + C(q_2^i).$$

Due to the presence of the fixed cost f, Cournot oligopolists owning inventories at the beginning of period 2 are committed not to produce in second period for a wider range of sales of their competitors compared to the situation where $f = 0$. The effect of inventories on Cournot competition when there are some fixed costs of production can be characterized by the effective cost of production in second period, $\Gamma^i(s^i, q_1^i)$:

$$\Gamma^i(s^i, q_1^i) = \begin{cases} 0 & \text{if } s^i \leq q_1^i \\ F + c \cdot (s^i - q_1^i) & \text{if } s^i > q_1^i \end{cases} \tag{11}$$

The effective cost of production is discontinuous when sales equal inventories, as the effective marginal cost given by equation (5) is. When deciding to produce again to sell more than its inventories, firm i must now make sure it can recoup its fixed costs, compared to the situation in which it sells its inventories only. The individual best response of firm i is consequently discontinuous for some aggregate sales of its

[11] We thank Eric Maskin for suggesting us this extension.

competitors, and at this discontinuity firm i is indifferent between selling more than its inventories and selling exactly its inventories, provided that inventories q_1^i are small enough. If $q_1^i \leq (a-c)/2 - f$, then

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a-c-S^{-i}) & \text{if } S^{-i} \in [0, a-c-2q_1^i - 2f] \\ q_1^i & \text{if } S^{-i} \in [a-c-2q_1^i-2f, a-2q_1^i] \\ \frac{1}{2}(a-S^{-i}) & \text{if } S^{-i} \geq a-2q_1^i \end{cases} \tag{12}$$

while if $q_1^i \geq (a-c)/2 - f$, then firm i never produces again and

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} q_1^i & \text{if } S^{-i} \in [0, a-2q_1^i] \\ \frac{1}{2}(a-S^{-i}) & \text{if } S^{-i} \geq a-2q_1^i \end{cases} \tag{13}$$

The derivation of the sub-game equilibria can be done as before. The main difference comes from the fact that the backward reaction mappings are not single-valued anymore for some levels of aggregate sales. Consequently there may be multiple equilibria in the sub-game, from which we have to choose one. To do so, we use Pareto dominance as a selection criterion: amongst the Nash equilibria derived for a given n-uple of inventories q_1, the Nash equilibrium of the sub-game is the n-uple of sales for which the industry profit is the highest. In this game of Cournot competition with constant marginal costs of production, this is equivalent to select as the Nash equilibrium the n-uple of sales that induces the smallest level of aggregate sales. We also conduct our analysis focusing on inventories different enough between two consecutive firms in a potential hierarchy,

$$q_1^i - q_1^{i+1} \geq f \text{ for } i = 1, ..., n-1. \tag{14}$$

The consequence of this restriction is to insure that only one firm may be indifferent between two level of sales at a time in the sub-game, instead of potentially n if we were considering smaller differences. For levels of fixed costs small enough, this will still allow us to discuss the existence of an asymmetric Nash equilibrium in pure strategies. Indeed assuming that the distance between inventories chosen is large enough rules out individual deviations making inventories more symmetric, but these deviations are dominated by the search for leadership in this quantity setting, for a level of fixed cost f is small enough. The reader may refer to appendix A.2 for the derivation of the sub-game equilibrium. We can establish the following result.

Proposition 2 *If the setup cost of production is small enough, $f \leq \widetilde{f} < (a-c)/2^n$, if we focus on level of inventories different enough, $q_1^i - q_1^{i+1} \geq f$ for $i = 1, ..., n-1$, the game possesses an asymmetric Nash equilibrium in pure strategies, in which all n firms are active and store different quantities to behave as a Stackelberg hierarchy.*

Proof: The proof consists of two stages. First, we show that if it exists, the equilibrium must be such that $n-1$ firms store and the ultimate follower does not. Second, we show that if an ultimate follower does not store, then firms do not store a quantity such that this ultimate follower does not sell in second period. To put it differently, the equilibrium described in theorem 1 must be immune to deviations of any of the firms but the ultimate follower that aim at forcing the ultimate follower to stay inactive in second period.

The first stage of the proof is straightforward: from the situation depicted in theorem 1, firms have now an incentive to produce in one period only instead of two, to suffer the setup cost only once. This reinforces the fact that inventories such that firms produce twice instead of once are dominated, as we proved before. Similarly, firms would be better off reducing their inventories if inventories were larger than sales, to save part of the cost of production. Finally no firm can put larger rivals with redundant inventories without letting the second period price fall below the marginal cost of production (we are still working under assumption (A.1)). An ultimate follower is better off producing in second period rather than in first for cost minimization reasons, and consequently it leaves us with the following region to explore,

$$S^* = \frac{a - c + \sum_{i=1}^{n-1} q_1^i}{2}, \; s^{i*} = q_1^i, \; s^{n*} = \frac{a - c - \sum_{i=1}^{n-1} q_1^i}{2}$$

which occurs if

$$0 \leq \frac{a - c - \sum_{i=1}^{n-1} q_1^i}{2} - f \text{ and } q_1^{n-1} + \frac{\sum_{i=1}^{n-2} q_1^i}{3} \geq \frac{a - c - 4f}{3}.$$

In this region, the equilibrium described in theorem 1 is obviously a candidate. The fixed cost must be such that $f \leq (a-c)/2^n$. To check whether the deviation forcing the ultimate follower to be inactive makes any of its leaders better off compared to the asymmetric candidate we already have, we need to check whether storing an inventory leading to $\sum_{i=1}^{n-1} q_1^i = (a-c) - 2f$ is profitable for any firm k amongst $1, ..., n-1$. Let us compare the profits.

When its inventories are given by $q_1^k = \frac{a-c}{2^k}$, firm k is earning

$$\pi_S^k = \left(a - c - \sum_{i=1}^{n} \frac{a-c}{2^i} \right) \frac{a-c}{2^k} - \rho\ c\ \frac{a-c}{2^k} - (1+\rho)\ F$$

while if it chooses $q_1^k = (a-c) - 2f - \sum_{i=1}^{n-1} \frac{a-c}{2^i} + \frac{a-c}{2^k}$, it earns a payoff equal to

$$\pi_P^k = 2f\ \left(\frac{a-c}{2^{n-1}} + \frac{a-c}{2^k} - 2f \right) - \rho\ c\ \frac{a-c}{2^k} - (1+\rho)\ F.$$

After some simplifications, we obtain

$$\pi_S^k = \frac{(a-c)^2}{2^{k+n}} - \rho\ c\ \frac{a-c}{2^k} - (1+\rho)\ F$$

and for ρ close to 0 we have

$$\pi_S^k = \frac{(a-c)^2}{2^{k+n}} - F \text{ and } \pi_P^k = 2f\ \left(\frac{a-c}{2^{n-1}} + \frac{a-c}{2^k} - 2f \right) - F.$$

The comparison gives $\pi_S^k \geq \pi_P^k$ if and only if

$$\frac{(a-c)^2}{2^{k+n}} - 2f\ \left(\frac{a-c}{2^{n-1}} + \frac{a-c}{2^k} \right) + 4f^2 \geq 0.$$

This last inequality asks to analyze the sign of a second degree polynomial expression in f. The discriminant is equal to $\Delta = (a-c)^2/(2^{2n-4} + 2^{2k-2}) > 0$, and the roots are given by

$$f' = \frac{1}{8} \left(2 \left(\frac{a-c}{2^{n-1}} + \frac{a-c}{2^k} \right) - \sqrt{\Delta} \right) \text{ and } f'' = \frac{1}{8} \left(2 \left(\frac{a-c}{2^{n-1}} + \frac{a-c}{2^k} \right) + \sqrt{\Delta} \right).$$

It is possible to check that $0 \leq f' \leq (a-c)/2^n$, so that for fixed costs small enough, $\pi_S^k \geq \pi_P^k$ for any k. There is no profitable deviation to the region in which the ultimate follower becomes inactive.$\|$

Remark that we have not investigated asymmetric equilibria in which less than n firms are active. This may occur for levels of fixed costs higher than $\widetilde{f}$ but still lower than the level of fixed cost such that n firms in static Cournot competition are active, here $(a-c)/2n$. This suggests that using inventories for predatory reasons is highly plausible: the aggregate level of sales should in that case higher than the level of sales of a static Cournot oligopoly, with a smaller number of firms being active.

5 Discussion and testable implications

Without any other incentive than strategic to build up inventories, and in a perfectly symmetric setting, this paper shows that it is possible to observe different levels of inventories and different market shares on an oligopolistic market, no matter the finite number of firms in competition. As pointed out in Boyer and Moreaux ([1985], [1986]) and Robson [1990], observing a leader on a market is not incompatible with perfect competition. However they assume that the hierarchy is exogenous, which does not address the important question of how firms may obtain the leadership. In this paper we prove that the leadership may be obtained through inventories: we show that firms may use large but different level of inventories to constrain their opponents and act as leaders even if the market has a large number of suppliers that are all in simultaneously interaction every time they take a decision. The commitment value of inventories does not disappear the higher the degree of competition.

The current results have been derived using a positive but close to 0 level of interest rate. If the interest rate increases, then the likelihood of observing strategic inventories diminishes. Introducing a cost of storage would have a similar effect. It is important to remark that the result we are pointing out is rather extreme in terms of the dispersion of market shares, and would be smoother but still present in a setting which includes a cost of storage. Moreover if we were to allow for uncertain demand and risk neutral firm, we would presumably obtain similar results.

The main assumption under which our results are derived is the observability of inventories. Relaxing it would of course affect our findings, but under reasonable assumptions the commitment effect of inventories we are characterizing would still play a role. In the context of Stackelberg duopolistic competition, Bagwell [1991] shows that the non-observability of the choice of the leader induces the leader and the follower to produce the Cournot outcome. The main assumption under which this result holds is a non-moving support assumption: the leader's commitment does not affect the support of the distribution of the signal received by the follower on the action chosen by the leader, that is any quantity can be randomly selected by the communication technology. To be a Nash equilibrium in pure strategies of the commitment game with imperfect observability, a pair of actions must be such that the equilibrium action of the follower is a best reply to the equilibrium action of the leader no matter the signal received by the follower, and such that the equilibrium action of the leader is a best reply to the equilibrium action of the follower. That is the pure strategy Nash equilibria of the noisy leader game and the simultaneous game must coincide. However this striking result is not valid when one looks for mixed strategy Nash equilibria: under the non-moving support assumption, Van Damme and Hurkens [1997] show that the noisy leader game has mixed strategy equilibria in which the leader puts a strictly positive probability weight on the leader output, no matter how large the noise on the leader's action is. Moreover Maggi [1998] shows that in presence of a private information, the follower does not neglect signals he receives on the leader output and therefore although affected, the value of a commitment does not disappear entirely when the commitment is not observable. Following the conclusions of these studies, the non-observability of inventories would not jeopardize our results if we were to introduce a communication technology precise enough: for example a high level of inventories should be associated with a signal randomly drawn on level of inventories higher than in the case of low inventories. To put it differently we should assume that the support of the distribution of the signal moves with the level of inventories. Relaxing the non-moving support assumption in the context of inventory competition is not unrealistic: business or industry analysts report on a very frequent basis the amount of inventories in hand of producers, so that assuming that inventory levels are communicated without too much error can be done without too much loss of generality.

Our results have presumably a certain number of testable implications. First, inventories are distributed over $[0, \frac{a-c}{2}]$ and market shares are asymmetric independently of the number of firms in competition, provided it is finite. The average level of inventories kept by a n-firm oligopoly is equal to

$$\overline{q}_1 = \frac{1}{n}\sum_{i=1}^{n-1} q_1^i = \frac{a-c}{n}\left(\sum_{i=0}^{n-1}\left(\frac{1}{2}\right)^i - 1\right) = \frac{a-c}{n}\left(1-\left(\frac{1}{2}\right)^{n-1}\right). \tag{15}$$

It converges to 0 when the number of firms increases. Remark that the usual Stackelberg leader quantity $\frac{a-c}{2}$ is stored with a strictly positive probability, although its likelihood in the population, $\frac{1}{n}$, decreases the larger the number of firms. The median level of inventories is given by

$$q_1^{median} = \begin{cases} \frac{a-c}{2^{(n+1)/2}} & \text{for } n \text{ odd} \\ \frac{1}{2}\left(\frac{a-c}{2^{(n-1)/2}} + \frac{a-c}{2^{(n+1)/2}}\right) & \text{for } n \text{ even} \end{cases} \tag{16}$$

Although the median level of inventories converges to 0, as $\overline{q}_1$ does, the difference $\overline{q}_1 - q_1^{median}$ is strictly positive for n large enough[12]. The distribution of inventories is therefore asymmetric, and as a direct consequence, the distribution of market shares presents also the same asymmetry.

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Appendices

A. **Proofs**

A.1 Proof of proposition 1

The proof is done in 3 steps. (*Step 1*) shows how to simplify each firm second period problem to enlighten the role of period 1 production (i.e. inventories) and derives the individual sales each firm chooses as a best reply to the aggregate sales of its competitors, $\widehat{s}^i(S^{-i}, q_1^i)$, which depends on inventories. First, as we study an oligopolistic competition, it is possible to restrict one's attention to period 1 inventories lower or equal to the individual production of a monopoly minimizing its cost of production, denoted q^m. Given the opportunity cost of producing in period 1 instead of period 2, and given the demand and costs parameters assumed before, this quantity is equal to $q^m = (a-c)/2$. Consequently, the vector of period 1 production satisfies assumption (A.2) and is such that $q_1 \in [0, (a-c)/2]^n$.

Second and most importantly, inventories create firm-specific kinks in firms best replies, source of the asymmetry of the Nash equilibrium of the game. Indeed as their cost of production is sunk when selling on the market in second period, inventories form a capacity from which firms can sell without producing. That is their effective marginal cost of production is given by

$$\gamma^i(s^i, q_1^i) = \begin{cases} 0 & \text{if } s^i \leq q_1^i \\ c & \text{if } s^i > q_1^i. \end{cases} \tag{17}$$

The best reply of firm i to aggregate sales of its competitors S^{-i} can be obtained by confronting the effective marginal cost $\gamma^i(s^i, q_1^i)$ to the marginal revenue

$$m^i(s^i, S^{-i}) = a - 2\ s^i - S^{-i} \tag{18}$$

If S^{-i} is such that $m^i(q_1^i, S^{-i})$, the marginal revenue firm i obtains from selling exactly its inventories, exceeds the marginal cost of production c, then firm i is better off producing again in period 2 to sell more than its period 1 inventories q_1^i. If S^{-i} is such that $m^i(q_1^i, S^{-i})$ is lower than the marginal cost c but strictly positive, then firm i does not produce in period 2 and sells exactly q_1^i on the market. Finally if S^{-i} is such that $m^i(q_1^i, S^{-i})$ is negative, firm i is better off selling less than its inventories q_1^i. For a given level of period 1 production, the best reply of firm i to aggregate sales S^{-i} of its competitors is therefore given by

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a - c - S^{-i}) & \text{if } S^{-i} \leq a - c - 2q_1^i \\ q_1^i & \text{if } S^{-i} \in [a - c - 2q_1^i, a - 2q_1^i] \\ \frac{1}{2}(a - S^{-i}) & \text{if } a - 2q_1^i \leq S^{-i} \end{cases} \tag{19}$$

As period 1 production of any firm i is lower than $(a-c)/2$, the bounds $a - c - 2q_1^i$ and $a - 2q_1^i$ are non-negative and consequently the best reply is continuous and has exactly two kinks: depending on competitors market behavior, each firm can either sell more than (first line of equation (7)), or sell exactly (second line of (7)), or sell less than (third line of (7)) its inventories.

In *Step 2*, we aggregate all the best replies to find the equilibrium aggregate sales of the industry. To do so, we construct the best reply of each firm to the aggregate quantity sold by the industry, $\widehat{s}^i(S, q_1^i)$, and we sum these functions over all firms to obtain the industry best reply to an aggregate sales level, $\sum_{i\in I} \widehat{s}^i(S, q_1^i) = \widehat{S}(S, q_1)$. To construct the best reply $\widehat{s}^i(S, q_1^i)$, also known as the backward reaction mapping (from Novshek 1984 terminology) we first determine the cumulative reaction to S^{-i} for firm i, $S_i^{\bullet}(S^{-i}, q_1^i) = \{s^i + S^{-i}/s^i = \widehat{s}^i(S^{-i}, q_1^i)\}$.

$$S_i^{\bullet}(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a - c + S^{-i}) & \text{if } S^{-i} \leq a - c - 2q_1^i \\ q_1^i + S^{-i} & \text{if } S^{-i} \in [a - c - 2q_1^i, a - 2q_1^i] \\ \frac{1}{2}(a + S^{-i}) & \text{if } S^{-i} \geq a - 2q_1^i \end{cases} \tag{20}$$

Then we invert it to obtain the inverse cumulative best response function $S_i^{-i}(S, q_1^i)$ for firm i. As $S_i^{\bullet}(S^{-i}, q_1^i)$ is strictly increasing it has a unique inverse,

$$S^{-i}(S, q_1^i) = \begin{cases} 2S - (a - c) & \text{if } S \in [\frac{a-c}{2}, a - c - q_1^i] \\ S - q_1^i & \text{if } S \in [a - c - q_1^i, a - q_1^i] \\ 2S - a & \text{if } S \geq a - q_1^i \end{cases} \tag{21}$$

Finally we solve for the individual sale s^i in $\{s^i/S - s^i = S^{-i}(S, q_1^i)\}$, to obtain the backward reaction mapping,

$$\widehat{s}^i(S, q_1^i) = \begin{cases} (a - c) - S & \text{if } S \in [(a - c)/2, a - c - q_1^i] \\ q_1^i & \text{if } S \in [a - c - q_1^i, a - q_1^i] \\ a - S & \text{if } S \geq a - q_1^i \end{cases} \tag{22}$$

where as in (7), given some industry sales S, firm i can either sell more than (first line of (22)), or sell exactly (second line of (22)), or sell less than (third line of (22)) its inventories q_1^i. Under assumptions (A.1) and (A.2), summing all the backward reaction mappings to obtain $\widehat{S}(S, q_1)$ can be done easily. Indeed for any level of industry sales S, all firms are either selling at least their inventories or selling at most their inventories, but it is not possible that some of them sell strictly more than their inventories, while some others are selling strictly less. To put it differently the cut-off values for S determining the reaction of an individual firm in (22) are "nicely" ranked across firms. To see this, first remark that from (A.2),

$$(a-c)/2 \leq a-c-q_1^1 \leq ... \leq a-c-q_1^n \leq a-c \tag{23}$$

and

$$a-q_1^1 \leq ... \leq a-q_1^n. \tag{24}$$

As justified in step 1, there is no loss of generality to restrict our attention to period 1 productions lower than $q^m = (a-c)/2$. Consequently the lower bound in the sequence of inequalities (24) can be minored, $a-q_1^1 \geq (a+c)/2$. It suffices to remark that as a consequence of (A.1), the upper bound of the sequence of inequalities (23) is lower than the lower bound of the sequence (24), i.e. $a-c \leq (a+c)/2$, to be able to rank across all firms the cut-off values at which there are kinks in the backward reaction mappings (22)

$$a-c-q_1^1 \leq ... \leq a-c-q_1^n \leq a-q_1^1 \leq ... \leq a-q_1^n \tag{25}$$

For a level of industry sales S lower than $a-c-q_1^1$, all firms are selling more than their period 1 production and consequently the sum of all the backward reaction mappings is simply equal to n times the expression in the first line of (22), $(a-c)-S$. For S higher than $a-c-q_1^1$ and lower than $a-c-q_1^2$, firm 1 is selling exactly its inventories and firms 2 to n are selling strictly more: the sum of the backward reaction mappings is equal to q_1^1 plus $n-1$ times $(a-c)-S$, ... and so on. For industry sales higher than $a-c-q_1^n$ and lower than $a-q_1^1$, all firms are selling exactly their inventories, and $\sum_{i\in I} \widehat{s}^i(S, q_1^i) = \sum_{i\in I} q_1^i = Q_1$. For S higher than $a-q_1^1$ and lower than $a-q_1^2$ firm 1 sells less than its inventories and firms 2 to n sell exactly their inventories: $\sum_{i\in I} \widehat{s}^i(S, q_1^i)$ is equal to $a-S+\sum_{i\geq 2} q_1^i$, ... and so on to complete the summation. To summarize, $\widehat{S}(S, q_1)$ is given by

$$\widehat{S}(S,q_1) = \begin{cases} n(a-c-S) & \text{if } S \in [(a-c)/2, a-c-q_1^1] \\ (n-k)(a-c-S)+\sum_{i=1}^{k} q_1^i & \text{if } S \in [a-c-q_1^k, a-c-q_1^{k+1}] \\ & \text{for } k=1,...,n-1 \\ ... & ... \\ \sum_{i=1}^{n} q_1^i & \text{if } S \in [a-c-q_1^n, a-q_1^1] \\ k(a-S)+\sum_{i=k+1}^{n} q_1^i & \text{if } S \in [a-q_1^k, a-q_1^{k+1}] \\ & \text{for } k=1,...,n-1 \\ ... & ... \\ n(a-S) & \text{if } S \geq a-q_1^n \end{cases} \tag{26}$$

Step 3 determines the fixed points of $\widehat{S}(S, q_1)$. As there are $2n$ cut-off values determining the different expressions of $\widehat{S}$, there are $2\ n+1$ different expressions and potentially $2n+1$ different sub-game equilibria to find. We index the consecutive lines from (26) by $\ell = 0, ..., 2\ n$: at line 0 all firms sell more than their inventories, at line 1 firm 1 sells exactly its inventories and the others more,... and so on. At line n all firms sell exactly their inventories and at line $n+1$ firm 1 sells less than its inventories while the others sell exactly their inventories, until line $2n$. We derive the fixed points of $\widehat{S}(S, q_1)$ line by line: for every line $\ell = 0, ..., 2n$, there is a unique equilibrium aggregate sales level $S^*(\ell)$. To this equilibrium industry sales $S^*(\ell)$ corresponds a unique set of period 1 inventories $B(\ell)$ such that if the vector of firms inventories q_1 belongs to $B(\ell)$, then the equilibrium is $S^*(\ell)$. Let us describe these fixed points and the sets that are associated to them. The equilibrium in which all firms sell more than their inventories (line $\ell = 0$) is characterized by $S^*(0) = n(a-c)/(n+1)$. Inventories must be such that $S^*(0) \leq a - c - q_1^1$ that is must belong to $B(0)$ given by

$$B(0) = \left\{q_1 \mid q_1^1 \leq (a-c)/(n+1)\right\}. \tag{27}$$

For $\ell = 1, ..., n-1$, equilibrium aggregate sales are $S^*(\ell) = ((n-\ell)(a-c) + \sum_{i=1}^{\ell} q_1^i)/(n-\ell+1)$, and q_1 must belong to $B(\ell)$ given by

$$B(\ell) = \left\{q_1 \mid q_1^\ell \geq (a-c-\sum_{i=1}^{\ell-1} q_1^i)/(n-\ell+2),\ q_1^{\ell+1} \leq (a-c-\sum_{i=1}^{\ell} q_1^i)/(n-\ell+1)\right\}. \tag{28}$$

For $\ell = n$, $S^*(n) = \sum_{i=1}^{n} q_1^i$ and q_1 must belong to $B(n)$

$$B(n) = \left\{q_1 \mid q_1^n \geq (a-c-\sum_{i=1}^{n-1} q_1^i)/2,\ q_1^1 \leq (a-\sum_{i=2}^{n} q_1^i)/2\right\}. \tag{29}$$

For $\ell = n+1, ..., 2n-1$, $S^*(\ell) = ((\ell-n)a + \sum_{i=\ell-n+1}^{n} q_1^i)/(\ell-n+1)$ and $q_1 \in B(\ell)$ such that

$$B(\ell) = \left\{q_1 \mid q_1^{\ell-n} \geq (a-\sum_{i=\ell-n+1}^{n} q_1^i)/(\ell-n+1),\ q_1^{\ell-n+1} \leq (a-\sum_{i=\ell-n+2}^{n} q_1^i)/(\ell-n+2)\right\}. \tag{30}$$

Finally for $\ell = 2n$, $S^*(2n) = na/(n+1)$ and $q_1 \in B(2n)$ such that

$$B(2n) = \left\{q_1 / q_1^n \geq a/(n+1)\right\}. \tag{31}$$

The intersection between (the interior of) two sets is empty, $B(\ell)\bigcap B(\ell') = \emptyset$ for $\ell \neq \ell'$, and the reunion of all sets $\bigcup_{\ell=0,\dots,2n} B(\ell)$ encompasses exactly all the cases for q_1 we are interested in. We complete this proof by expressing the conditions on inventories in terms of conditions on the marginal revenues to obtain our result. Obviously,

$$q_1^1 \leq \frac{a-c}{n+1} \Leftrightarrow m^1\left(q_1^1, \frac{n-1}{n+1}(a-c)\right) \geq c$$

and so on...

A.2 Sub-game equilibria with fixed costs in Cournot competition with inventories

In presence of fixed costs, the reaction functions of firms in Cournot competition with inventories differ from equation (7). The effective cost of production in second period is now discontinuous when sales equal inventories,

$$\Gamma^i(s^i, q_1^i) = \begin{cases} 0 & \text{if } s^i \leq q_1^i \\ F + c\cdot(s^i - q_1^i) & \text{if } s^i > q_1^i \end{cases} \tag{32}$$

while the effective marginal cost is still given by equation (5). When deciding to produce again to sell more than its inventories, firm i must now make sure it can recoup its fixed costs, compared to the situation in which it sells its inventories only. If $a-c-2q_1^i-2f \geq 0$, that is if $q_1^i \leq (a-c)/2 - f$ (case (a) hereafter), then

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a-c-S^{-i}) & \text{if } S^{-i} \in [0, a-c-2q_1^i-2f] \\ q_1^i & \text{if } S^{-i} \in [a-c-2q_1^i-2f, a-2q_1^i] \\ \frac{1}{2}(a-S^{-i}) & \text{if } S^{-i} \geq a-2q_1^i \end{cases} \tag{33}$$

while if $a-c-2q_1^i-2f \leq 0$, that is if $q_1^i \geq (a-c)/2 - f$ (case (b) hereafter), then firm i never produces again and

$$\widehat{s}^i(S^{-i}, q_1^i) = \begin{cases} q_1^i & \text{if } S^{-i} \in [0, a-2q_1^i] \\ \frac{1}{2}(a-S^{-i}) & \text{if } S^{-i} \geq a-2q_1^i \end{cases} \tag{34}$$

It is important to remark that the best reply of firm i to aggregate sales of its competitors shows a downward jumping discontinuity when S^{-i} reaches the value $a-c-2q_1^i-2f$, in the case where this quantity is positive. Indeed in that case producing to sell more than its inventories does not allow firm i to recoup its fixed cost, and consequently she prefers to sell its inventories rather than producing again to sell more. Remark also that under assumption (C.2), $f \leq (a-c)/2n$, and therefore $(a-c)/2 - f > 0$. It makes sense to consider inventories lower than $(a-c)/2 - f$.

We can follow the procedure introduced in the proof of proposition 1 to obtain the backward reaction mapping of firm i in both cases. While case (b) is obvious, case (a) must be solved cautiously.

Backward reaction mapping, case (a): $q_1^i \leq (a-c)/2 - f$. By adding S^{-i} to the best reply of firm i, we obtain

$$S_i^{\bullet}(S^{-i}, q_1^i) = \begin{cases} \frac{1}{2}(a - c + S^{-i}) & \text{if } S^{-i} \in [0, a - c - 2q_1^i - 2f] \\ q_1^i + S^{-i} & \text{if } S^{-i} \in [a - c - 2q_1^i - 2f, a - 2q_1^i] \\ \frac{1}{2}(a + S^{-i}) & \text{if } S^{-i} \geq a - 2q_1^i \end{cases} \tag{35}$$

We can now invert this relation, bearing in mind we are searching for a correspondence. We need again to distinguish two cases, $a - c - q_1^i - 2f \geq (a-c)/2$ (i.e. $q_1^i \leq (a-c)/2 - 2f$) and $a - c - q_1^i - 2f \leq (a-c)/2$ (i.e. $q_1^i \geq (a-c)/2 - 2f$).

Case (a.1): $q_1^i \leq (a-c)/2 - 2f$. In that case the inverse is given by

$$S^{-i}(S, q_1^i) = \begin{cases} 2S - (a-c) & \text{if } S \in [\frac{a-c}{2}, a - c - q_1^i - 2f] \\ \{2S - (a-c), S - q_1^i\} & \text{if } S \in [a - c - q_1^i - 2f, a - c - q_1^i - f] \\ S - q_1^i & \text{if } S \in [a - c - q_1^i - f, a - q_1^i] \\ 2S - a & \text{if } S \geq a - q_1^i \end{cases} \tag{36}$$

and the backward reaction mapping is given by

$$\hat{s}^i(S, q_1^i) = \begin{cases} (a-c) - S & \text{if } S \in [\frac{a-c}{2}, a - c - q_1^i - 2f] \\ \{(a-c) - S, q_1^i\} & \text{if } S \in [a - c - q_1^i - 2f, a - c - q_1^i - f] \\ q_1^i & \text{if } S \in [a - c - q_1^i - f, a - q_1^i] \\ a - S & \text{if } S \geq a - q_1^i \end{cases} \tag{37}$$

Case (a.2): $q_1^i \in [(a-c)/2 - 2f, (a-c)/2 - f]$. Since the aggregate sales cannot be lower than what firm i would sell if its competitors do not sell at all, that is $S \geq (a-c)/2$, the inverse is given by

$$S^{-i}(S, q_1^i) = \begin{cases} \{2S - (a-c), S - q_1^i\} & \text{if } S \in [(a-c)/2, a - c - q_1^i - f] \\ S - q_1^i & \text{if } S \in [a - c - q_1^i - f, a - q_1^i] \\ 2S - a & \text{if } S \geq a - q_1^i \end{cases} \tag{38}$$

and the backward reaction mapping is given by

$$\hat{s}^i(S, q_1^i) = \begin{cases} \{(a-c) - S, q_1^i\} & \text{if } S \in [(a-c)/2, a-c-q_1^i - f] \\ q_1^i & \text{if } S \in [a-c-q_1^i - f, a - q_1^i] \\ a - S & \text{if } S \geq a - q_1^i \end{cases} \tag{39}$$

Backward reaction mapping, case (b): $q_1^i \in [(a-c)/2 - f, (a-c)/2]$. In that case it is obvious to verify that the backward reaction mapping is given by

$$\hat{s}^i(S, q_1^i) = \begin{cases} q_1^i & \text{if } S \in [q_1^i, a - q_1^i] \\ a - S & \text{if } S \geq a - q_1^i \end{cases} \tag{40}$$

Aggregation of the backward reaction mappings. Amongst the cases to consider, we can examine the situation in which all firms are storing a quantity such that the backward reaction mapping of each of them corresponds to case (a.1). This is true if and only if the firm with the highest level of inventories, firm 1, stores a quantity lower than the bound of case (a.1),

$$q_1^1 \leq (a-c)/2 - 2f. \tag{41}$$

We must also consider the case in which firms $i = 2, ..., n$ store less than $(a-c)/2-2f$, and firm 1 store more than $(a-c)/2 - 2f$,

$$q_1^1 \geq (a-c)/2 - 2f \text{ and } q_1^2 \leq (a-c)/2 - 2f \tag{42}$$

There are of course many other cases to consider, but remark that this last case is the one in which we can find the equilibrium described in theorem 1, if it exists when there are some fixed costs of production. Indeed $(a-c)/4 \leq (a-c)/2 - 2f$ if and only if $f \leq (a-c)/8$, verified for $n \geq 4$ if we simply consider the constraint that the fixed cost is sufficiently small given the number of firms in competition, $f \leq (a-c)/2n$. To keep the number of discussions under control, we consider only very small fixed costs of production, so that we can focus on the two cases depicted above, and neglect the others.

Case I. Low level of inventories for all firms, $q_1^1 \leq (a-c)/2-2f$. In that case, the backward reaction mappings are given by equation (37) for all firms. As in the proof of proposition 1, we have

$$(a-c)/2 \leq a - c - q_1^1 - 2f \leq ... \leq a - c - q_1^n - 2f \tag{43}$$

and

$$a-c-q_1^n-f \leq a-q_1^1 \leq ... \leq a-q_1^n. \tag{44}$$

We restrict our attention to fixed costs small enough so that

$$a-c-q_1^1-2f \leq a-c-q_1^1-f \leq a-c-q_1^2-2f \leq ... \leq a-c-q_1^n-2f \leq a-c-q_1^n-f \tag{45}$$

which is equivalent to restrict our attention to

$$q_1^i \geq q_1^{i+1}+f \text{ for any } i=1,...,n-1. \tag{46}$$

Focusing on this situation will force us to rule out without a careful assessment possible deviations from very different to very similar levels of inventories when determining the period 1 production choices. This can be done without too much loss of generality, as the best response to a large commitment is a smaller commitment in this game of quantity competition. The nice consequence of this restriction is to have one firm at a time choosing between two level of sales in second period, and therefore one firm at a time conditioning the aggregate output of the industry. To see this, we may sum up the backward reaction mappings.

$$\widehat{S}(S,q_1)=\begin{cases} n(a-c-S) & \text{if } S\in[(a-c)/2, a-c-q_1^1-2f] \\ \{n(a-c-S),(n-1)(a-c-S)+q_1^1\} & \text{if } S\in[a-c-q_1^1-2f, a-c-q_1^1-f] \\ (n-1)(a-c-S)+q_1^1 & \text{if } S\in[a-c-q_1^1-f, a-c-q_1^2-2f] \\ ... & ... \\ \{(n-k+1)(a-c-S)+\sum_{i=1}^{k-1}q_1^i, & \text{if } S\in[a-c-q_1^k-2f, a-c-q_1^k-f] \\ (n-k)(a-c-S)+\sum_{i=1}^{k}q_1^i\} & \\ (n-k)(a-c-S)+\sum_{i=1}^{k}q_1^i & \text{if } S\in[a-c-q_1^k-f, a-c-q_1^{k+1}-2f] \\ & \text{for } k=1,...,n-1 \\ ... & ... \\ \sum_{i=1}^{n}q_1^i & \text{if } S\in[a-c-q_1^n-f, a-q_1^1] \\ k(a-S)+\sum_{i=k+1}^{n}q_1^i & \text{if } S\in[a-q_1^k, a-q_1^{k+1}] \\ & \text{for } k=1,...,n-1 \\ ... & ... \\ n(a-S) & \text{if } S\geq a-q_1^n \end{cases} \tag{47}$$

To obtain the aggregate output of the industry in equilibrium, it suffices to find the fixed points S^* in $S^*=\hat{S}(S^*,q_1)$. Since there may be several equilibria, we will need a selection criteria at this stage: we will keep in the set of sub-game equilibria only the ones which are Pareto-dominant from the viewpoint of the industry, that is the ones leading to the maximal profit of the industry. As we are dealing with quantity competition, if there is indeed a set of inventories leading to a multiplicity of equilibria in the sales sub-game, then we will keep the one leading to the smallest aggregate sales of the industry. As in the proof of proposition 1, we proceed case by case.

Solving $S = n(a-c-S)$ gives $S^* = n(a-c)/(n+1)$, and this turns out to be an equilibrium if $q_1^1 \leq (a-c)/(n+1) - 2f$.

The second case presents a potential multiplicity of equilibria: either the one derived previously, $S'' = n(a-c)/(n+1)$, or $S' = (n-1)(a-c)/n + q_1^1/n$ solution of $S = (n-1)(a-c-S) + q_1^1$. Clearly here $S' < S''$, and if for some levels of inventories both solutions co-exist, we will keep S'. S' belongs to the interval considered if $a - c - q_1^1 - 2f \leq S'$, that is if and only if $q_1^1 \geq (a-c)/(n+1) - 2nf/(n+1)$, and if $S' \leq a - c - q_1^1 - f$ that is if and only if $q_1^1 \leq (a-c)/(n+1) - nf/(n+1)$. Similarly S'' belongs to the interval considered if and only if $S'' \geq a - c - q_1^1 - 2f$, that is if $q_1^1 \geq (a-c)/(n+1) - 2f$, and if $S'' \leq a - c - q_1^1 - f$ that is if $q_1^1 \leq (a-c)/(n+1) - f$. Remark that

$$\frac{a-c}{n+1} - 2f \leq \frac{a-c}{n+1} - \frac{2nf}{n+1} \leq \frac{a-c}{n+1} - f \leq \frac{a-c}{n+1} - \frac{nf}{n+1} \tag{48}$$

Therefore if $q_1^1 \in [\frac{a-c}{n+1} - 2f, \frac{a-c}{n+1} - \frac{2nf}{n+1}]$ the unique solution is $S^* = n(a-c)/(n+1)$. If $q_1^1 \in [\frac{a-c}{n+1} - \frac{2nf}{n+1}, \frac{a-c}{n+1} - f]$ then both solutions are valid, but we keep the smallest one, $S^* = (n-1)(a-c)/n + q_1^1/n$. Finally if $q_1^1 \in [\frac{a-c}{n+1} - f, \frac{a-c}{n+1} - \frac{nf}{n+1}]$, then the unique solution is $S^* = (n-1)(a-c)/n + q_1^1/n$. To conclude,

$$S^* = \begin{cases} \frac{n(a-c)}{n+1} & \text{if } q_1^1 \in [\frac{a-c}{n+1} - 2f, \frac{a-c}{n+1} - \frac{2nf}{n+1}] \\ \frac{(n-1)(a-c)}{n} + \frac{q_1^1}{n} & \text{if } q_1^1 \in [\frac{a-c}{n+1} - \frac{2nf}{n+1}, \frac{a-c}{n+1} - \frac{nf}{n+1}] \end{cases} \tag{49}$$

If we consider now the third case. The equilibrium is given by $S^* = (n-1)(a-c)/n + q_1^1/n$, for inventories such that $q_1^1 \geq (a-c)/(n+1) - nf/(n+1)$ and such that $q_1^2 + q_1^1/n \leq (a-c)/n - 2f$.

We proceed now with the generic case in which there are two equilibria to consider. When S belongs to $[a - c - q_1^k - 2f, a - c - q_1^k - f]$, the aggregate sales in equilibrium can be either $S'' = (n-k+1)(a-c)/(n-k+2) + (\sum_{i=1}^{k-1} q_1^i)/(n-k+2)$, or $S' = (n-k)(a-c)/(n-k+1) + (\sum_{i=1}^{k} q_1^i)/(n-k+1)$. We have $S' < S''$, and therefore whenever both solutions co-exist, we will select S'. To belong to the interval we consider, $S' \geq a-c-q_1^k-2f$ that is $q_1^k \geq (a-c)/(n-k+2) - (\sum_{i=1}^{k-1} q_1^i)/(n-k+2) - 2(n-k+1)f/(n-k+2)$. Moreover, $S' \leq a-c-q_1^k - f$ is equivalent to $q_1^k \leq (a-c)/(n-k+2) - (\sum_{i=1}^{k-1} q_1^i)/(n-k+2) - (n-k+1)f/(n-k+2)$. Similarly $S'' \geq a-c-q_1^k-2f$ gives $q_1^k \geq (a-c)/(n-k+2) - (\sum_{i=1}^{k-1} q_1^i)/(n-k+2) - 2f$, and $S'' \leq a-c-q_1^k-f$ is equivalent to $q_1^k \leq (a-c)/(n-k+2) - (\sum_{i=1}^{k-1} q_1^i)/(n-k+2) - f$. The ranking between the bounds is similar to the one derived in the previous case, and by applying the selection criterion defined, we obtain

$$S^* = \begin{cases} \frac{(n-k+1)(a-c)}{n-k+2} + \frac{\sum_{i=1}^{k-1} q_1^i}{n-k+2} & \text{if } q_1^k \in [\frac{a-c}{n-k+2} - \frac{\sum_{i=1}^{k-1} q_1^i}{n-k+2} - 2f, \frac{a-c}{n-k+2} - \frac{\sum_{i=1}^{k-1} q_1^i}{n-k+2} - \frac{2(n-k+1)f}{n-k+2}] \\ \frac{(n-k)(a-c)}{n-k+1} + \frac{\sum_{i=1}^{k} q_1^i}{n-k+1} & \text{if } q_1^k \in [\frac{a-c}{n-k+2} - \frac{\sum_{i=1}^{k-1} q_1^i}{n-k+2} - \frac{2(n-k+1)f}{n-k+2}, \frac{a-c}{n-k+2} - \frac{\sum_{i=1}^{k-1} q_1^i}{n-k+2} - \frac{(n-k+1)f}{n-k+2}] \end{cases} \tag{50}$$

In the case where the aggregate backward reaction mapping is a function (and not a correspondence), the equilibrium is unique given by

$$S^* = (n-k)(a-c)/(n-k+1) + \sum_{i=1}^{k} q_1^i/(n-k+1) \tag{51}$$

if

$$q_1^k \geq \left(a-c-(n-k+1)f - \sum_{i=1}^{k-1} q_1^i\right)/(n-k+2) \text{ and } q_1^{k+1} \leq \left(a-c-\sum_{i=1}^{k} q_1^i\right)/(n-k+1)-2f \tag{52}$$

Sub-game equilibria in the case where some firms are selling less than their inventories are identical to what we derived in the proof of proposition 1. These results can be aggregated to give

$$S^* = \begin{cases} \frac{n(a-c)}{n+1} & \text{if } q_1^1 \leq \frac{a-c-2nf}{n+1} \\ \frac{(n-1)(a-c)+q_1^1}{n} & \text{if } q_1^1 \geq \frac{a-c-2nf}{n+1},\ q_1^2 + \frac{q_1^1}{n} \leq \frac{(a-c)-2(n-1)f}{n} \\ \frac{(n-2)(a-c)+q_1^1+q_1^2}{n-1} & \text{if } q_1^2 + \frac{q_1^1}{n} \geq \frac{(a-c)-2(n-1)f}{n},\ q_1^3 + \frac{q_1^1+q_1^2}{n-1} \leq \frac{(a-c)-2(n-2)f}{n-1} \\ \dots & \dots \\ \sum_{i=1}^{n} q_1^i & \text{if } q_1^n + \frac{\sum_{i=1}^{n-1} q_1^i}{2} \geq \frac{a-c}{2} - f,\ q_1^1 \leq \frac{a}{2} - \frac{\sum_{i=2}^{n} q_1^i}{2} \\ \dots & \dots \end{cases} \tag{53}$$

Case II. High level of inventories for the leader, $q_1^1 \geq (a-c)/2-2f$ and $q_1^2 \leq (a-c)/2-2f$. In that case the backward reaction mapping of firm 1 is given by case (a.2) or case (b), while all the others are given by case (a). We still consider that $q_1^i \geq q_1^{i+1} + f$, so that there is at most one firm hesitating between two level of sales.

First, consider the case $q_1^1 \in [(a-c)/2 - 2f, (a-c)/2 - f]$. In that case, the backward reaction mapping of firm 1 is given by expression (39). The aggregate backward reaction mapping is similar to what we derived previously, apart that inventories of firm 1 are too large to have all firms producing and selling more than inventories in second period. The first line of the aggregate backward reaction mapping is

$$\widehat{S}(S, q_1) = \{n(a-c-S), (n-1)(a-c-S)+q_1^1\} \text{ if } S \in [(a-c)/2, a-c-q_1^1-f]. \quad (54)$$

We can derive the possible equilibria in that case. Consider first $S' = n(a-c)/(n+1)$. For this equilibrium to occur, it must be the case that $q_1^1 \leq (a-c)/(n+1) - f$. However we must compare this bound with the bound of the region for q_1^1 we consider. Here, $(a-c)/(n+1) - f \geq (a-c)/2 - 2f$ if and only if $f \geq (n-1)(a-c)/2(n+1)$. However we considered small fixed costs, such that n firms are active in Cournot static competition: $f \leq (a-c)/2n$. Under this assumption, it is not possible that firm 1 produces and sells in second period when q_1^1 is larger than $(a-c)/2 - 2f$. Consequently $S^* = (n-1)(a-c)/n + q_1^1/n$ if $q_1^1 \leq (a-c)/(n+1) - nf/(n+1)$.

Similarly if $q_1^1 \in [(a-c)/2 - f, (a-c)/2]$, it is not possible now to have firm 1 indifferent between producing and selling more than its inventories in second period, or selling exactly its inventories in second period: its backward reaction mapping is given by expression (40), and therefore

$$\widehat{S}(S, q_1) = (n-1)(a-c-S) + q_1^1 \text{ if } S \in [q_1^1, a-c-q_1^2 - 2f]. \quad (55)$$

In that case the equilibrium is $S^* = (n-1)(a-c)/n + q_1^1/n$, if and only if $q_1^2 + q_1^1/n \leq (a-c)/n - 2f$, which is possible for some f.

Sub-game equilibria are consequently given by (53), apart for the fact that $S^* = n(a-c)/(n+1)$ disappears when $q_1^1 \geq (a-c)/2 - 2f$.

B. Figures

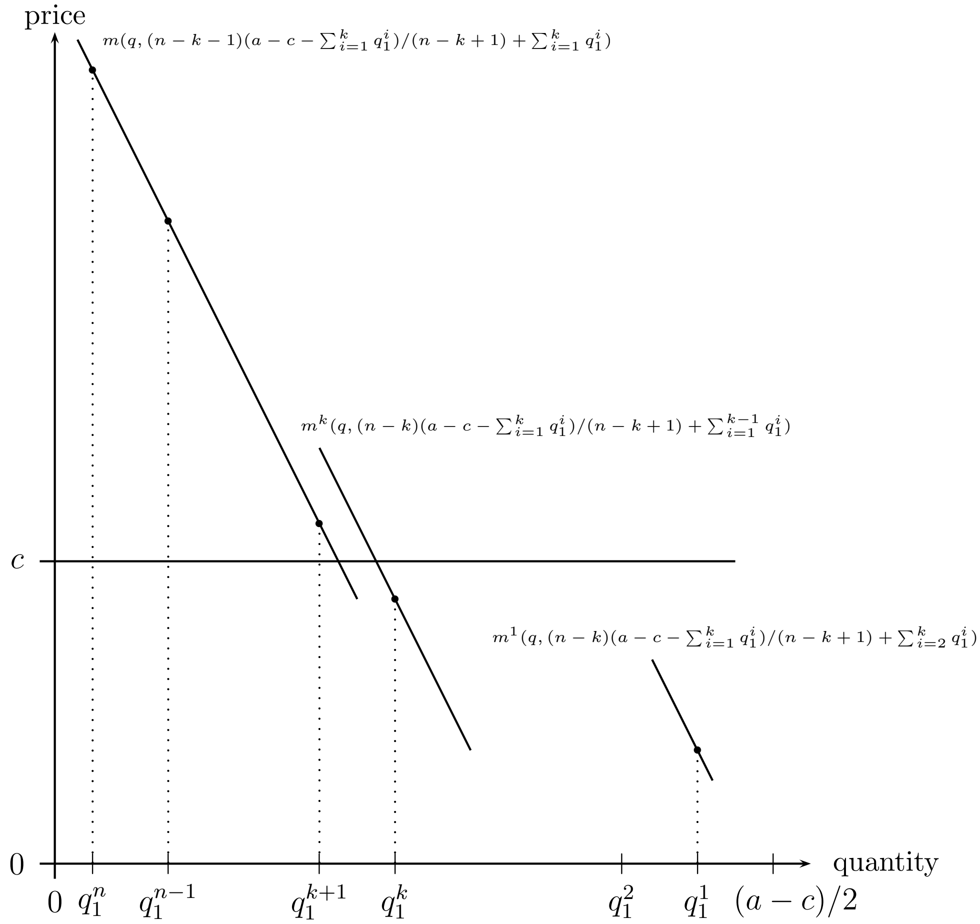

Figure 1: Graphical analysis of the sales sub-game

C. **Proof of theorem 2 (SUPPLEMENTARY MATERIAL)**

The proof uses the intermediate results leading to theorem 1. Entrants in second period act as standard Cournot competitors, that is they produce and sell so that their reaction is optimal to the choice of their competitors. Their individual reaction is

$$\hat{s}_E^j(S^{-j}) = \max\left\{\frac{1}{2}(a-c-S^{-j}), 0\right\} \text{ for any } j = 1, ..., m \tag{56}$$

where $S^{-j} = \sum_{i=1}^n s_I^i + \sum_{k \neq j} s_E^k$. The backward reaction mapping of this group is obviously given by

$$\hat{s}_E^j(S) = \max\{a-c-S, 0\} \text{ for any } j = 1, ..., m. \tag{57}$$

The Nash equilibrium of the sales sub-game is again obtained by first solving for the aggregate quantity sold in equilibrium, then by finding individual sales that support it. First, let us aggregate all the backward reaction mappings to form $\hat{S}(S, q_1) = \sum_{j=1}^m \hat{s}_E^j(S) + \sum_{i=1}^n \hat{s}_I^i(S, q_1^i)$ where $\hat{s}_I^i(S, q_1^i) \equiv \hat{s}^i(S, q_1^i)$ defined in equation (22) from appendix A.1. Given our assumptions, $a - c - q_1^n \leq a - c \leq a - q_1^1$ and consequently

$$\widehat{S}(S, q_1) = \begin{cases} (n+m)(a-c-S) & \text{if } S \in [(a-c)/2, a-c-q_1^1] \\ (n+m-k)(a-c-S) + \sum_{i=1}^k q_1^i & \text{if } S \in [a-c-q_1^k, a-c-q_1^{k+1}] \\ & \text{for } k = 1, ..., n-1 \\ ... & ... \\ m(a-c-S) + \sum_{i=1}^n q_1^i & \text{if } S \in [a-c-q_1^n, a-c] \\ \sum_{i=1}^n q_1^i & \text{if } S \in [a-c, a-q_1^1] \\ k(a-S) + \sum_{i=k+1}^n q_1^i & \text{if } S \in [a-q_1^k, a-q_1^{k+1}] \\ & \text{for } k = 1, ..., n-1 \\ ... & ... \\ n(a-S) & \text{if } S \geq a - q_1^n \end{cases} \tag{58}$$

The sub-game equilibria are given by

$$S^* = \frac{(n+m)(a-c)}{n+m+1} \text{ if } q_1^1 \leq \frac{a-c}{n+m+1}, \tag{59}$$

then

$$S^* = \frac{(n+m-k)(a-c)}{n+m-k+1} + \frac{\sum_{i=1}^k q_1^i}{n+m-k+1} \tag{60}$$

if

$$q_1^k \geq \frac{a-c-\sum_{i=1}^{k-1} q_1^i}{n+m-k+2} \text{ and } q_1^{k+1} \leq \frac{a-c-\sum_{i=1}^{k} q_1^i}{n+m-k+1}. \tag{61}$$

In the last case in which the entrants are producing are strictly positive quantity, we have

$$S^* = \frac{m(a-c)}{m+1} + \frac{\sum_{i=1}^{n} q_1^i}{m+1} \tag{62}$$

if

$$q_1^n \geq \frac{a-c-\sum_{i=1}^{n-1} q_1^i}{m+2} \text{ and } \sum_{i=1}^{n} q_1^i \leq a-c. \tag{63}$$

The rest of the sub-game equilibria is identical to proposition 1. We turn now to the description of the equilibria of the game. The argument of cost-minimization we used before guarantees that incumbents will not store quantities such that they sell strictly less than their inventories: they would be better reducing their sales to sell exactly their inventories. As this is true for every incumbent, the equilibrium must be such that $S^* \leq a - q_1^1$.

Start now with the case of n incumbents storing the same quantity leading to a symmetric equilibrium in the sales sub-game. Then firm 1 would be better off increasing its inventories to the level such that it sells exactly its inventories, as the profit comparison below shows. Indeed, if firm 1 sticks to level of inventories such that she sells the same quantity than the other incumbents and the entrants, $q_1^1 \leq (a-c)/(n+m+1)$, she earns

$$\pi^1 = \frac{(a-c)^2}{(n+m+1)^2} - \rho\ c\ q_1^1. \tag{64}$$

For ρ sufficiently close to 0, this profit is constant in q_1^1 when $q_1^1 \leq (a-c)/(n+m+1)$, and it has to be compared with the profit she obtains by storing a quantity $q_1^1 \geq (a-c)/(n+m+1)$ and such that $q_1^2 \leq (a-c-q_1^1)/(n+m)$. This other profit is equal to

$$\pi^1 = (a-c-S^*)\ q_1^1 - \rho\ c\ q_1^1 = \frac{a-c-q_1^1}{n+m}\ q_1^1 - \rho\ c\ q_1^1. \tag{65}$$

It has a maximum in q_1^1 when $q_1^1 = (a-c-(n+m)\ \rho\ c)/2$. This local maximum is higher than $(a-c)/(n+m+1)$ for ρ sufficiently close to 0, but it may be outside the region we consider for the deviation. Since the profit is continuous, firm 1 is still better off storing a quantity equal to $\min\{a-c-(n+m)\ q_1^2, (a-c-(n+m)\ \rho\ c)/2\}$, that is storing a quantity on the upper frontier of the region we consider, or storing the local maximum of its profit function if this solution is interior to the region.

Therefore all incumbents storing the same quantity in period 1 leading incumbents and entrants to sell the same quantity in period 2 cannot be an equilibrium of the game.

Analyze now the case in which incumbents $k+1, ..., n$ are selling the same quantity strictly larger than their inventories, and incumbents $1, ..., k$ are selling exactly their inventories. Then incumbent $k+1$ would be better off increasing its inventories as the profit comparison shows. If he chooses to sell a quantity such that $q_1^{k+1} \leq (a - c - \sum_{i=1}^{k} q_1^i)/(n + m - k + 1)$, and if $q_1^k \geq (a - c - \sum_{i=1}^{k-1} q_1^i)/(n + m - k + 2)$ then the profit obtained by incumbent $k+1$ is equal to

$$\pi^{k+1} = (a - S^*) \, \hat{s}_I^i(S^*) - (1+\rho) \, c \, q_1^{k+1} - c \, (\hat{s}_I^i(S^*) - q_1^{k+1}) = (a - c - S^*)^2 - \rho \, c \, q_1^{k+1} \quad (66)$$

giving

$$\pi^{k+1} = \frac{(a - c - \sum_{i=1}^{k} q_1^i)^2}{(n + m - k + 1)^2} - \rho \; c \; q_1^{k+1}. \quad (67)$$

This profit is constant if ρ is sufficiently close to 0. If on the other hand firm $k+1$ decides to increase its inventories so that it sells them entirely in period 2, that is if firm $k+1$ chooses $q_1^{k+1} \geq (a - c - \sum_{i=1}^{k} q_1^i)/(n + m - k + 1)$ and $q_1^{k+2} \leq (a - c - \sum_{i=1}^{k+1} q_1^i)/(n + m - k)$, then its profit is equal to

$$\pi^{k+1} = \frac{(a - c - \sum_{i=1}^{k+1} q_1^i) \; q_1^{k+1}}{n - m + k} - \rho \; c \; q_1^{k+1}. \quad (68)$$

This profit reaches a maximum when $q_1^{k+1} = (a - c - \sum_{i=1}^{k} q_1^i - (n+m-k) \; \rho \; c)/2$. For ρ sufficiently close to 0, this maximum is indeed higher than $(a - c - \sum_{i=1}^{k} q_1^i)/(n+m-k+1)$, and must be compared with the upper frontier of the region we consider. Since the profit is continuous, and for ρ sufficiently small, incumbent $k+1$ is nonetheless better off storing $\min\{(a - c - \sum_{i=1}^{k} q_1^i - (n + m - k) \; \rho \; c)/2, a - c - \sum_{i=1}^{k} q_1^i - (n + m - k) \; q_1^{k+2}\}$ than storing a lower quantity leading her to sell the same quantity than the entrants or the other incumbents with lower levels of inventories. This argument can be repeated for any $k = 1, ..., n-1$, and therefore we are left with the behaviour of incumbent n to examine.

As any other incumbent, incumbent n earns a constant profit (if ρ is sufficiently small) if it chooses to store a small quantity and sell the same quantity than the entrants. If he decides to increase its inventories, that is to store $q_1^n \geq (a - c - \sum_{i=1}^{n-1} q_1^i)/(m + 2)$ and $\sum_{i=1}^{n} q_1^i \leq a - c$, then its profit is equal to

$$\pi^n = \frac{(a - c - \sum_{i=1}^{n} q_1^i) \; q_1^n}{m + 1} - \rho \; c \; q_1^n. \quad (69)$$

This profit reaches a maximum when $q_1^n = (a - c - \sum_{i=1}^{n-1} q_1^i - (m+1)\ \rho\ c)/2$, a quantity that is now higher than $(a - c - \sum_{i=1}^{n-1} q_1^i)/(m+2)$ if ρ is sufficiently small, as long as there are some entrants in second period. Incumbent n is now better off storing $q_1^n = \min\{(a - c - \sum_{i=1}^{n-1} q_1^i - (m+1)\ \rho\ c)/2, a - c - \sum_{i=1}^{n-1} q_1^i\}$ rather than selling the same quantity than the entrants. In the case of potential entry, the behaviour of the ultimate follower is modified compared to the equilibrium described in theorem 1: the threat of entry can be used as a selection device in this game, as the interest ρ is.

Consider finally the situation in which the entrants are inactive, which occurs if incumbents are committed to sell their inventories entirely and if their production is large enough, that is if $\sum_{i=1}^{n} q_1^i \geq a - c$ and $q_1^1 \leq (a - \sum_{i=1}^{n-1} q_1^i)/2$. Inventories verifying these constraints cannot be an equilibrium: indeed the market price falls below the marginal cost of production, and any incumbent would be better off reducing its inventories to earn a non-negative profit. If the equilibrium exists, the entrants must be active.

We are left with the analysis of the region in which aggregate sales are given by $S^* = (m\ (a - c) + \sum_{i=1}^{n} q_1^i)/(m+1)$, for $q_1^n \geq (a - c - \sum_{i=1}^{n-1} q_1^i)/(m+2)$ and $\sum_{i=1}^{n} q_1^i \leq a - c$. We ruled out individual deviations leading incumbents to sell more than their inventories. If there are leaders emerging, we must rule out individual deviations forcing leaders to sell less than their inventories, that is putting them with redundant inventories. Consider that one incumbent tries to force incumbent 1 with the highest level of inventories to sell less than these inventories. Then this firm must increase its inventories at a level such that $q_1^1 \geq (a - \sum_{i=2}^{n} q_1^i)/2$ and if $q_1^2 \leq (a - \sum_{i=3}^{n} q_1^i)/3$, then only firm 1 will be put with redundant inventories. In that case, aggregate sales are given by $S^* = (a + \sum_{i=2}^{n} q_1^i)/2$. This level of aggregate sales is however, by definition of the backward reaction mapping, higher than $a - c$. The market price is therefore lower than the marginal cost of production c. Of course firms are individually better off doing a non-negative profit when they sell, and consequently it is not possible for such a deviation to occur. This argument applies to any firm which tries individually to put a group of competitors with redundant inventories. Consequently the equilibrium must be such that all incumbents choose inventories in such a way that they are committed to sell them on the market.

Considering ρ close to 0, then the sequential optimization of the firms profit gives directly that

$$q_1^i = \frac{a-c}{2^i} \text{ for i=1,...,n.} \tag{70}$$

Aggregate sales are given by

$$S^* = \left(m + \sum_{i=1}^{n} \frac{1}{2^i} \right) \frac{a-c}{m+1} = \left(m + 1 - \frac{1}{2^n} \right) \frac{a-c}{m+1}. \tag{71}$$

Sales of the entrants follows immediately,

$$s^{j*} = \frac{(a-c)}{2^n \ (m+1)}. \tag{72}$$