DYNAMIC GAMES IN THE WHOLESALE ELECTRICITY MARKET

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Abstract: *In this paper, we analyze infinite discrete-time games between hydraulic and thermal power operators in the wholesale electricity market. Two types of games are considered: Cournot closed-loop game and Stackelberg closed-loop game. We consider a deregulated electrical industry where certain demand is satisfied by hydraulic and thermal technologies. The hydraulic operator decides the production in each season of each period that maximizes the sum of expected profit from power generation with respect to the stochastic dynamic constraint on the water stored in the dam, the environmental constraint and the non-negative output constraint. In contrast, the thermal plant is operated with quadratic cost function, with respect to the capacity production constraint and the non negativity output constraint. This paper is devoted to the numerical computations of equilibrium strategies and value function in each kind of games. We show that under imperfect competition, the hydraulic operator has a strategic storage of water in the peak season. Then, we quantify the strategic inter annual and intra annual water transfer in the both games and we compare the numerical results. Under Cournot closed-loop game, we show that the traditional principle of least-cost operation is inverted at the binding capacity constraint of thermal operator. Finally, under Stackelberg closed-loop game, we show that thermal operator can restrict the hydraulic output without compensation. The technical complementarities and Stackelberg competition may distorted the traditional "merit order" operating principal.*

Key Words: water resource, wholesale electricity market, thermal system, inflows uncertainty, Cournot closed-loop game and Stackelberg closed-loop game.

JEL Classification: Q4, Q25, C73, C61.

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1-INTRODUCTION

In the last decades, the electricity industry set out the very important structural reform by inserting a competition in generation of high voltage. The transmission and distribution of electricity have been considered by most economists as "natural monopoly". These changes have split the single state owned firm into several private owned firms that compete in a single market: the wholesale electricity market. The economic analysis has been turned toward some problems like as congestion of transmission electric power network, concentration measures, horizontal market power, electricity spot market, bilateral transaction, nodal pricing in a simple and a complex network, etc; and which has been made in a static framework compatible with thermal power generation system. Thus, there are few analyses concerning the restructuring of the electricity industry with heterogeneous technologies. This paper takes part in the new research field concerning the mixed hydrothermal system operating problems under deregulated industry. The objectives of the paper are also to compute numerical equilibrium strategies in two kinds of games and to find the conditions under which the imperfect competition breeds a distortion to traditional least-cost rule used in the mixed system operating.

In several countries, the liberalization process had led to different asymmetric duopoly. The first asymmetric case concerned the British electricity supply industry which has been transferred to two successors companies: National power and Gen power. In this case, Green and Newbery (1992) have shown that "... in an asymmetric case, less output would be sold at higher price, and industry operating costs will be further raised for any level of output since the stations will no longer operate in "merit order". The second asymmetric case is spreading across countries owned with significant hydraulic resources like as Norway, New Zealand, Western United States, etc. In the economic literature, there are few studies on the strategic behavior of hydraulic and thermal operators after the reform of the electricity industry. In the New Zealand case, Scott T. J and E. G. Read (1996) have developed a dual dynamic programming approach in order to characterize an optimal hydraulic schedule for a strategic firm that controls all the storage hydro capacity in a market with other Cournot producer that controls the thermal generation. They show that «... there is relatively little loss in coordination efficiency if, but only if, there is a high level of contracting and / or a high effective elasticity". The same problem has been analyzed in J. Bushnell (1998) model which developed a sub-game perfect equilibrium of a multi-period Cournot game between strategic producer who controls both hydro and thermal technologies. In the latter, the author has shown that hydraulic releases are directly done in contradictory to the principle of least-cost production since this decision appears as profitable for certain firms. Also, C. Crampes and M. Moreaux (2000) have analyzed the strategic behavior of thermal and hydraulic operators in closed-loop game and open-loop game with two periods. The authors have shown how the presence of the hydroelectric station changes the optimal as well as the market equilibrium outputs of the thermal station. They found that the thermal plant facing hydro plants has to be managed as if it were dynamically connected. The interrelation between heterogeneous technologies can imply other results. Indeed, the technological complementation may incite, under some conditions, operators to behave strategically in the electricity market and in the inverse to the traditional "merit order" operation.

None of theses models take into account the uncertainty in hydraulic resource evolution. The objective of this paper is to develop a multi-periods game between thermal and hydraulic operators in an uncertain framework. We based our analyses on the C. Crampes and M. Moreaux model in order to develop a more realistic asymmetric case. Indeed, the main characteristic of the thermal plants is the possibility to supply certain but an expensive output. Regarding hydraulic plant, its output is linked to a sub-renewal resource with intrinsic uncertain profile. Before the inflows realization, hydraulic power is provisory an exhaustible resource and any additional release reduces the residual water quantity stored in the dam. But, once the inflows are realized, any quantity of water devoted for power generation can be totally or partially renewed by the natural process. We think that this kind of uncertainty and the technical complementation may be in favor of strategic behavior particularly in higher priced peak market. Consequently, this may distort the traditional "merit order" rule and which is against the target of the structural change in the electricity industry.

The rest of the paper proceeds as follow. In the section 2, we present the model. Then, in section 3, we deal with a Cournot closed-loop game between thermal and hydroelectric operators in the wholesale electricity market. Section 4 is devoted to the numerical analysis of Stackelberg closed-loop game between the two operators. Next, we compare the numerical results of imperfect competition. We conclude in section 5.

THE MODEL

Consider a multi-periods model where electric power is generated by two technologies: thermal technology (T) and hydraulic technology (H). The total output is used to satisfy a known demand. Since electric power demand is characterized by fluctuation, then we divide time into infinite years indexed by $t = 0,1,...$; and the year is divided into two seasons ($j = \{h,l\}$, the peak season (h) and the off-peak season (l). Let $P_j\left(q_{jt}\right)$ be the inverse demand function in the season j. It is assumed to be a linear demand function: $P_j\left(q_{jt}\right) = a_j - q_{jt}^H - q_{jt}^T$, where q_{jt}^H is the electricity demand satisfied by the hydroelectric technology, q_{jt}^T is the electricity demand satisfied by the thermal technology and a_j is a positive constant representing the demand characteristic in each season. From year to year, we take the assumption of the stationary of electrical demand function. Since in each season the output of two plants is homogenous then total electric power generation is written as the some of thermal plant output, q_{jt}^T and hydroelectric plant output q_{jt}^H : for a period t and a season j, $q_{jt} = q_{jt}^H + q_{jt}^T$.

The hydraulic plant uses as input a sub-renewable resource stored in a dam. In every year t, the total output must satisfy the following technical constraint: $S_{t+1} = S_t - q_t^{hH} - q_t^{lH} + f_{t+1}$. We denote by S_t the current stock and f_{t+1} the inflow assumed to be observed at the end of the period t. To simplify we do not consider the technical limit in terms of water turbine[1] and we assume that hydraulic generation is costless. The inflows of period t are denoted by a random variable f_{t+1} with density probability function $\phi'\left(f_{t+1}\right)$ defined on $\left[0,\overline{F}\right]$, where $\overline{F}$ is the maximum inflow. In addition, we assume that inflows are represented by a random and a stationary process, $\left\{f_{t+1}\right\}_{t=0,1,2,...}$ defined on $\left[0,\overline{F}\right]$. In addition, the hydroelectric operator must satisfy an environmental constraint: his storage at the end period must be at a known level S^*. Where S^* is the level at which if it is not respected, the producer must pay a cost assumed to

1 Bernard J. T. and J. Chatel (1984) are analyzed operating and investment problem in a mixed hydrothermal system with technical limit in terms of water turbine.

be quadratic: $-\frac{1}{2}\left(S^{*}-S_{t+1}\right)^{2}$. This assumption is compatible also with a mobile dam[2] and a fixed dam. First, if the dam is mobile, then S^{*} represents the maximum capacity of the fixed part which to simplify is confounded with maximum alert level. Consequently, any current stock S_{t} beyond the level S^{*} represents the supplementary water resource stored by closing the gates in order to avoid flood. Second, if the dam is a fixed one, then the penalty cost paid by producer is due to bad release policy not compatible with flooding period. This case is observed in July1996 in Kénogami dam located in sanguinary region in Quebec. During this period, the planner observed an abundant in flows in three days which has affected the water level. The water level attends 166,08 m which exceeds the maximum capacity of the dam equals to 165,67 m. Since the planner does not adopt a release policy compatible with this flooding period, then this over taking of maximal capacity cause a flood with enormous damages. Consequently, the quadratic form penalty cost compatible with the area flooded around the dam. Now, if the expected storage at the end of a period t is less then the level S^{*}, then it causes a negative externality on other activity around the dam such as navigation, agriculture...etc.

The technical characteristics of the thermal plant are the following. At each season j during the period t, the generation of q_{jt}^{T} units of energy from thermal plant needs a total cost equals to $C\left(q_{jt}^{T}\right)$. We assume that the thermal operating cost function is quadratic on $\left[0,\overline{q}^{T}\right]$: For all $q_{jt}^{T}\in\left[0,\overline{q}^{T}\right]$ and for all season $j\in\{h,l\}: C\left(q_{jt}^{T}\right)=\frac{1}{2}c_{j}\left(q_{jt}^{T}\right)^{2}$. Where c_{j} is a positive constant for all season $j\in\{h,l\}$ and $\overline{q}^{T}$ is a known and a positive constant representing the installed capacity. By this hypothesis we neglect the depreciation problem linked to thermal plant operating.

We consider a wholesale electricity market with duopolistic structure. We assume that each operator controls one process. The new industrial structure is represented as following:

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2 More information about mobile dam and a fixed dam are available on line at http://retd.edf.fr/futur/publications/disponible.htm.

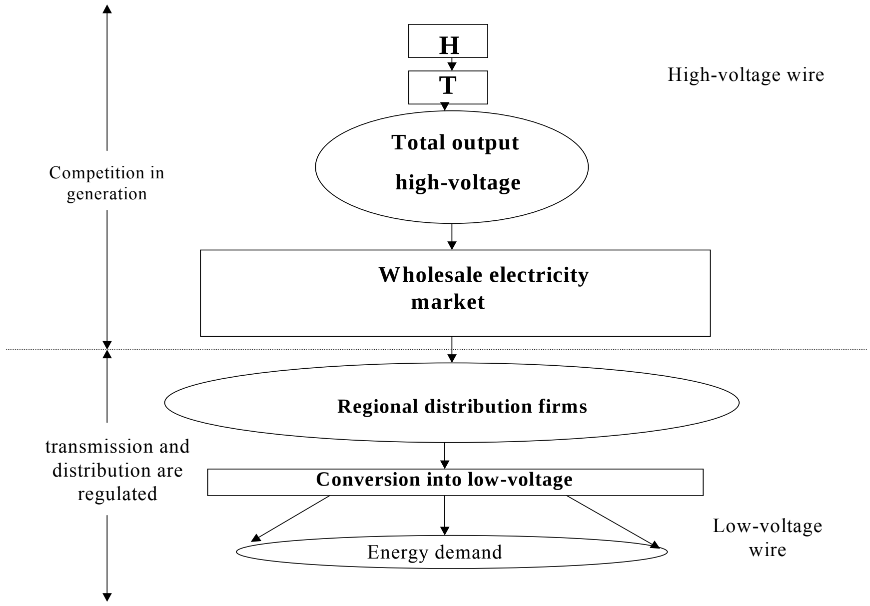

Fig 1: The new structure of electricity industry.

COURNOT CLOSED-LOOP GAME

In this section, we assume that the two operators compete in Cournot closed-loop game in the electricity market.

The thermal operator chooses the output in the peak and off-peak season of every period that maximizes his profit and where the hydroelectric operator output q_{jt}^H, for $j \in \{l,h\}$ is taken as given. The thermal operator solves the following problem:

$$\max_{\{q_{lt}^T, q_{ht}^T\}} P_h\left(q_{ht}^T + q_{ht}^H\right)q_{ht}^T - C_h\left(q_{ht}^T\right) + P_l\left(q_{lt}^T + q_{lt}^H\right)q_{lt}^T - C_l\left(q_{lt}^T\right) \qquad (1)$$

With respect to the capacity constraint and the non negativity constraint:

$$0 \le q_{jt}^T \le \overline{q}^T \quad \text{, for all } j \in \{h,l\} \text{ and all } t = 0,1,... \qquad (2)$$

The thermal operator problem is a static one. The first order conditions are:

$$\frac{\partial P_j}{\partial q_{jt}^T}\left(q_{jt}^H + q_{jt}^T\right)q_{jt}^T + P_j\left(q_{jt}^H + q_{jt}^T\right) - \frac{dC_j}{dq_{jt}^T}\left(q_{jt}^T\right) - \mu_{jt}^T + \theta_{jt}^T = 0 \qquad (3)$$

$$\mu_{jt}^T\left(\overline{q}^T - q_{jt}^T\right) = 0 \quad , \quad \theta_{jt}^T q_{jt}^T = 0 \qquad (4)$$

$$\mu_{jt}^T \ge 0 \quad , \quad q_{jt}^T \ge 0 \quad , \quad \theta_{jt}^T \ge 0 \qquad (5)$$

Where μ_{jt}^T and θ_{jt}^T are the multipliers associated respectively to the capacity constraint and the non-negativity constraint.

The first order conditions give the reaction function of the thermal operator in each season of the period t:

$$R_{jt}^{T}\left(q_{jt}^{H}\right)=\frac{a_j-q_{jt}^{H}}{2+c_j} \qquad (6)$$

Let $R_{jt}^{T}\left(q_{jt}^{H}\right)$ be the reaction function of thermal operator in season j of period t. Given the assumption on the inverse demand function and the cost function, then the reaction function is a strictly decreasing one: $\frac{dR_{jt}^{T}}{dq_{jt}^{H}}=-\frac{1}{\left(2+c_j\right)^2}<0$.

The hydraulic operator maximizes the sum of expected profit from power generation under the stochastic dynamic constraint on the water stored in the dam, the environmental constraint and the non-negativity output constraint. The hydraulic operator solves the following problem, where S_t, q_{jt}^{T} are taken as given, for all $j\in\{h,l\}$:

$$\max_{\left\{q_{lt}^{H},q_{ht}^{H}\right\}_{t=0}^{+\infty}} E_0\left\{\sum_{t=0}^{+\infty}P_h\left(q_{ht}^{T}+q_{ht}^{H}\right)q_{ht}^{H}+P_l\left(q_{lt}^{T}+q_{lt}^{H}\right)q_{lt}^{H}-\frac{1}{2}\left(S^{*}-S_{t+1}\right)^2\right\}$$

With respect to the dynamic constraint:

$$S_{t+1}=S_t-q_t^{hH}-q_t^{lH}+f_{t+1},\quad \text{for all } t=0,1,...$$

S_0 is given.

The closed-loop strategy of hydraulic operator satisfies the Bellman equation:

$$J^{c}\left(S_t\right)=\max_{\left\{q_{lt}^{H},q_{ht}^{H}\right\}_{t=0}^{+\infty}}\sum_{t=0}^{+\infty}P_h\left(q_{ht}^{T}+q_{ht}^{H}\right)q_{ht}^{H}+P_l\left(q_{lt}^{T}+q_{lt}^{H}\right)q_{lt}^{H}+E_t\left\{J^{c}\left(S_{t+1}\right)-\frac{1}{2}\left(S^{*}-S_{t+1}\right)^2\right\}$$

Where $E_t(\cdot)$ is the mean operator conditional to the information at period t which includes S_t and $J^{c}(\cdot)$ is the current value function associated to the stochastic dynamic program. It is assumed to be quadratic with unknown coefficients b^c and B^c:

$$J^{c}\left(S_t\right)=b^{c}S_t-\frac{1}{2}B^{c}\left(S_t\right)^2$$

In the stochastic dynamic programming terminology, this solution is called the closed-loop strategy. Under the above hypothesis, the equilibrium strategy exists and it is unique[3].

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[3] See Blume L. Easley D. and M. O'Harra (1982).

The first order condition in the season j at period t is:

$$\frac{\partial P_j}{\partial q_{jt}^H}\left(q_{jt}^H + q_{jt}^T\right)q_{jt}^H + P_j\left(q_{jt}^H + q_{jt}^T\right) = V^c{}_{t+1}$$

$$P^j\left(\frac{1}{\eta_{jt}} + 1\right) = V_{t+1}^c$$

Where $V_{t+1}^c = E_t\left(J'^c\right) - E_t\left\{S_{t+1} - S^*\right\}$ represents the net marginal value of the hydraulic resources in stock under the decentralized industry. Since J^c is a concave function then V_{t+1}^c is an increasing function in q_t^{jH} and a decreasing function in S_t [4]. We denote η_{jt} the demand elasticity in season j of period t. We assume that η_{jt} is negative and $\eta_{ht} < \eta_{lt}$.

The combination of the two first order conditions implies

$$q_{jt}^H = -\frac{1}{3+B^c}q_{jt}^T + \frac{1+B^c}{3+B^c}S_t - \frac{1+B^c}{3+B^c}q_{j't}^H + \frac{a_{jt} - b^c - S^* + \left(1+B^c\right)\overline{f}}{3+B^c}$$

For all $j \neq j'$ and $\left(j, j'\right) \in \left\{h,l\right\} \times \left\{h,l\right\}$; where $\overline{f}$ represents the inflows mean.

This equation gives the hydroelectric output in the peak season as a function of the thermal operator output in the same season, the current state of the hydraulic storage and the water release in the off-peak season at the period t.

The resolution of Bellman equation by undetermined coefficient method gives the equilibrium strategy of each player in period t [5]. The results are presented in the following proposition. The resolution of this equations system determines the hydraulic operator strategy in each season as a function of the current stock: $q_t^{Hc*}\left(S_t\right) = \left[q_{ht}^{Hc*}\left(S_t\right), q_{lt}^{Hc*}\left(S_t\right)\right]$. Next, we replace this strategy in the reaction function of thermal operator; we find that this latter is written as a function of the current stock of hydraulic operator $q_{jt}^{Tc} = R_{jt}\left[q_{jt}^{Hc}\left(S_t\right)\right]$. The equilibrium strategy of thermal operator in the period t is written as: $q_t^{Tc*}\left(S_t\right) = \left[q_{ht}^{Tc*}\left(S_t\right), q_{lt}^{Tc*}\left(S_t\right)\right]$.

4 $\frac{\partial V_{t+1}^c}{\partial S_t} = E_t\left\{\frac{d^2 J^c}{S_{t+1}^2}\right\} - 1 < 0$ and $\frac{\partial V_{t+1}^c}{\partial q_t^{jH}} = -\frac{\partial V_{t+1}^c}{\partial S_t} > 0$.

Proposition 1

In a closed-loop game, the thermal and the hydroelectric operators output in the peak season and in the off-peak season is given by the following equations:

$$q_{ht}^{Hc*} = \frac{25\left(1+B^c\right)}{97+52B^c}S_t + \frac{4\left(8+3B^c\right)a_{ht} - 10\left(1+B^c\right)a_{lt} - 25\left[\left(1+B^c\right)\overline{f} - S^* - b^c\right]}{97+52B^c}$$

$$q_{ht}^{Tc*} = -\frac{5\left(1+B^c\right)}{97+52B^c}S_t + \frac{\left(13+8B^c\right)a_{ht} + 2\left(1+B^c\right)a_{lt} + 5\left[\left(1+B^c\right)\overline{f} - S^* - b^c\right]}{97+52B^c}$$

$$q_{lt}^{Hc*} = \frac{27\left(1+B^c\right)}{97+52B^c}S_t + \frac{2\left(14+5B^c\right)a_{lt} - 12\left(1+B^c\right)a_{ht} - 27\left[\left(1+B^c\right)\overline{f} - S^* - b^c\right]}{97+52B^c}$$

$$q_{lt}^{Tc*} = -\frac{9\left(1+B^c\right)}{97+52B^c}S_t + \frac{\left(23+14B^c\right)a_{lt} + 4\left(1+B^c\right)a_{ht} + 9\left[\left(1+B^c\right)\overline{f} - S^* - b^c\right]}{97+52B^c}$$

$$B^c = 2.0527 \text{ and } b^c = 2.96\times10^{-4}\overline{f} - S^* - 1.29a_{ht} - 0.37a_{lt} \blacksquare$$

We remark that both thermal operator strategy and hydraulic operator strategy depend on current stock S_t, on the inflows mean $\overline{f}$, on the storage level S^* and on the electric demand in the peak season and off-peak season. The comparative static in the regulated industry (m)[6] and the Cournot competition case (c) is given by the following table:

t	*Systems*	q_{ht}^{H*}	q_{ht}^{T*}	q_{lt}^{H*}	q_{lt}^{T*}
S_t	*c*	0,374	-0,075	0,405	-0,134
	m	0,322	-0,08	0,48	-0,24
a_{ht}	*c*	0,118	0,169	-0,349	0,115
	m	0,6	0,1	-0,165	0,3
a_{lt}	*c*	-0,195	0,038	0,187	0,269
	m	-0,11	0,22	0,4	0,3
$\overline{f}$	*c*	-1,9□10□^{3}	-0,03	-0,4□10□^{3}	-3,05.10□□
	m	0,4	-0,1	0,6	-0,3
S^*	*c*	-1,098	0,221	-1,193	0,397
	m	-0,32	0,08	-0,48	0,24

Table 3: Comparative static in the Cournot competition case.

[5] See Basar T. and Olsder G. J (1995).
[6] See Dakhlaoui A. and M. Moreaux (2004).

From this table, we remark that any increase in water resource encourages the hydraulic operator to make an intra-annual transfer from peak season to off-peak season. We compare the Cournot solution with the first best solution; we conclude that the intra-annual transfer sense is maintained also with the structural change in the electricity industry. There is a change only in the volume of transfer. Indeed, the intra-annual transfer is appraised at 0,158 ΔS_t under the regulated industry and it is appraised only at 0,031 ΔS_t under Cournot competition. This show that, for a given period, the use of supplementary resource in the peak season under deregulated system is greater than those under regulated system. Thus, the storage effect ($SE_t = \Delta S_t - \Delta q_t^H$)[7] of competition solution is greater than the storage effect of the first best solution[8]. In addition, the intra-annual transfer affects the thermal operator strategy. Indeed, this latter cannot decrease his output only with a quantity equals to 0,075 ΔS_t units in the peak season and with a quantity equal to 0,134 ΔS_t units in the off-peak season. Consequently, the substitution of expensive technology output with costless. Technology output under competition is less than the substitution under regulated system[9]. Regarding the satisfaction of any increase in the peak demand, both the thermal operator and the hydraulic operator increase their output in different proportions. The hydraulic producer prefers to make an intra-annual transfer from off-peak season to peak season. Indeed, the release increase in the peak season and it is appraised at 0,118 Δa_{ht} units, but the release decrease in the off-peak season is appraised at 0,349 Δa_{ht} units. For the thermal operator, he increases the output in both seasons in order to let possible the storage of water resource in the off-peak season. We remark that the hydraulic operator does not use all the intra-annual transfer to satisfy the supplementary demand because the storage in the off-peak season is greater than the release in the peak season. Thus, there is storage of water resource appraised at 0,231 Δa_{ht} units that will be used for the satisfaction of electricity demand for the following periods.

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[7] Storage effect is defined as the interaction between the increase in inter annual transfers comes from preceding periods (ΔS_t) and the increase in the release from the stock at the end of period t.

[8] If the renewable effect is nil, then $SE^c = 0.221\Delta S_t$ and $SE^m = 0.198\Delta S_t$.

[9] The substitution of thermal output by hydraulic output is equals to 0,299 ΔS_t under competition and it is equal to 0,322 ΔS_t under regulated industry.

The numerical approximation lets possible both the quantification of water transfer and the substitution between technologies. We conclude the structural reform in the electricity industry not only changes solution greatness, but also it saves the least cost operating principle if the thermal operator does not hurt a binding capacity constraint problem.

Now, we assume that the thermal operator has a binding capacity constraint problem in the peak-season. Implicitly, we consider a large capacity constraint which is impossible to be bound in the off-peak season. In this case, the thermal operator cannot produce only $\overline{q}^{T}$. The hydraulic operator must satisfy the residual demand: $q_{ht} - \overline{q}^{T}$, where q_{ht} is the total demand in the peak-season of period t. The hydraulic operator can choose between Cournot equilibrium output or to exert a market power on the residual demand with an output q_{ht}^{T} less than q_{ht}^{Tc}. The strategic behavior of hydraulic operator leads to an increase in energy price in the peak-season. This behavior affects the strategy of two players in the off-peak season. Thus, the output decreasing is in favor of intra-annual transfer from peak season to off-peak season. Besides the transfer under constrained Cournot competition is greater than the transfer under unconstrained Cournot competition, it is not used to increase the off-peak season supply and consequently to substitute the thermal energy. The hydraulic operator tends to storage much water resource in order to push the thermal operator to produce more than the Cournot equilibrium output. This strategic behavior is, then, in favor of an important storage effect. This hydraulic storage is done in a strategic motivation in the peak-season of following periods. At the binding capacity constraint, the peak-season thermal plant operating is greater than those of hydraulic plant even with abundant water resource. Consequently, we can conclude that the binding capacity constraint is among the conditions under which the operating system rule is inverted in a deregulated industry.

STACKELBERG CLOSED-LOOP GAME

We analyze an other kind of quantity game in the wholesale electricity market such as the Stackelberg game between hydroelectric and thermal operator. We assume that the thermal operator constitute a dominant firm in the market. At each period the thermal operator first decides its production also in the peak and the off-peak season. Then, the hydroelectric operator decides its output with respect to the current stock of water, the stochastic futur storage in the dam and the output of the thermal operator.

Given the thermal strategy, the hydroelectric player decides the energy output that maximizes the expected sum of its annual profit with respect to the stochastic dynamic of the water stock stored in the dam.

The value function $J_t^s\left(S_t\right)$, evaluated at the period t must satisfy the Bellman equation:

$$J_t^s(S_t) = \underset{\left\{q_{ht}^H,q_{lt}^H\right\}_{t=0}^{+\infty}}{Max} \sum_{t=0}^{+\infty} P_h\left(q_{ht}^T + q_{ht}^H\right)q_{ht}^H + P_l\left(q_{lt}^T + q_{lt}^H\right)q_{lt}^H + E_t\left\{J_{t+1}^s\left(S_{t+1}\right) - \frac{1}{2}\left(S^* - S_{t+1}\right)^2\right\}$$

Where $E_t\left(\cdot\right)$ is the mean operator conditional to the information at period t which include S_t .

At each season j of the period t, the hydroelectric operator decides the quantity produced given the thermal operator strategy and the current stock of water. The reaction functions of the hydroelectric operator are solutions of theses equations:

At the peak season of the period t:

$$\frac{\partial P_h}{\partial q_{ht}^H}\left(q_{ht}^T + q_{ht}^H\right)q_{ht}^H + P_h\left(q_{ht}^T + q_{ht}^H\right) - E_t\left\{S^* - S_{t+1}\right\} - E_t\left\{\frac{dJ_{t+1}^s}{dS_{t+1}}\left(S_{t+1}\right)\right\} = 0$$

At the off-peak season of the period t :

$$\frac{\partial P_l}{\partial q_{lt}^H}\left(q_{lt}^T + q_{lt}^H\right)q_{lt}^H + P_l\left(q_{lt}^T + q_{lt}^H\right) - E_t\left\{S^* - S_{t+1}\right\} - E_t\left\{\frac{dJ_{t+1}^s}{dS_{t+1}}\left(S_{t+1}\right)\right\} = 0$$

Theses equations give the peak season hydroelectric production as a function of the water release at the off-peak season, the thermal output at the peak season and the stock of water available at the period t. The same resolution at the of-peak season:

$$q_{ht}^H = Q_{ht}^H\left(q_{lt}^T, q_{ht}^T, S_t\right)$$

$$q_{lt}^H = Q_{lt}^H\left(q_{lt}^T, q_{ht}^T, S_t\right)$$

The thermal profit of season j at the period t is equals to the total receipt of energy purchase comes from thermal plant mines the total cost of production:

$$\pi_{jt}^T\left(q_{jt}^H, q_{jt}^T\right) = P_j\left(q_{jt}^H + q_{jt}^T\right)q_{jt}^T - C_j\left(q_{jt}^T\right)$$

Given the hydroelectric operator strategy at current period, $q_{ht}^H = Q_{ht}^H\left(q_{lt}^T, q_{ht}^T, S_t\right)$ and $q_{lt}^H = Q_{lt}^H\left(q_{lt}^T, q_{ht}^T, S_t\right)$, the thermal operator decides to supply energy that maximizes the profit with respect to the capacity constraint and the non-negativity output constraint. The reaction function of the thermal operator is a solution this problem:

$$\underset{q_{ht}^T,q_{lt}^T}{Max}\, \pi_t = P_h\left(q_{ht}^H + q_{ht}^T\right)q_{ht}^T - C_h\left(q_{ht}^T\right) + P_l\left(q_{lt}^H + q_{lt}^T\right)q_{lt}^T - C_l\left(q_{lt}^T\right)$$

With respect to the constraints:

$$q_{ht}^{H}=Q_{ht}^{H}\left(q_{lt}^{T},q_{ht}^{T},S_{t}\right)$$

$$q_{lt}^{H}=Q_{lt}^{H}\left(q_{lt}^{T},q_{ht}^{T},S_{t}\right)$$

The thermal operator problem is a static one. The Lagrange function is written as:

$$J^{s}=\underset{q_{ht}^{T},q_{lt}^{T}}{Max}\,P_{h}\left(Q_{ht}^{H}\left(q_{lt}^{T},q_{ht}^{T},S_{t}\right)+q_{ht}^{T}\right)q_{ht}^{T}-C_{h}\left(q_{ht}^{T}\right)+\theta_{ht}^{Ts}q_{ht}^{T}+P_{l}\left(Q_{lt}^{H}\left(q_{lt}^{T},q_{ht}^{T},S_{t}\right)+q_{lt}^{T}\right)q_{lt}^{T}-C_{l}\left(q_{lt}^{T}\right)+\theta_{lt}^{Ts}q_{lt}^{T}$$

Where θ_{jt}^{Ts} is the multiplicator of non negativity output constraint of the thermal plant in period j at the period t, for all $j\in\{h,l\}$.To simplify, we neglect the turbinate capacity of thermal plant.

The reaction function of the thermal operator is characterized by the Kuhn-Tucker conditions. For the season $j\in\{h,l\}$ at period t:

$$\frac{\partial P_{j}}{\partial q_{jt}^{T}}\cdot q_{jt}^{T}+\frac{\partial Q_{jt}^{H}}{\partial q_{jt}^{T}}\cdot\frac{\partial P_{j}}{\partial q_{jt}^{H}}+P_{j}-\frac{dC_{h}}{dq_{jt}^{T}}+\theta_{ht}^{T}=0\,,\theta_{jt}^{T}q_{jt}^{T}=0\,,\ q_{jt}^{T}\geq 0\ \text{and}\ \theta_{jt}^{T}\geq 0\,.$$

If the multiplicator of non-negativity output constraint is nil, the thermal operator strategy is a solution of following system: $\frac{dC_{h}}{dq_{jt}^{T}}=\frac{\partial P_{j}}{\partial q_{jt}^{T}}\cdot q_{jt}^{T}+\frac{\partial Q_{jt}^{H}}{\partial q_{jt}^{T}}\cdot\frac{\partial P_{j}}{\partial q_{jt}^{H}}+P_{j}$

The resolution of the equations system gives the thermal operator strategy at the two seasons as a function of the current water stock of the thermal operator: $q_{jt}^{Ts}=Q_{jt}^{T}\left(S_{t}\right)$, for all $j\in\{h,l\}$.

We replace the thermal operator strategy at the first order conditions of the hydroelectric operator. In the season j at the period t :

$$\frac{\partial P_{j}}{\partial q_{jt}^{H}}\left(Q_{jt}^{T}\left(S_{t}\right)+q_{jt}^{H}\right)q_{jt}^{H}+P_{j}\left(Q_{jt}^{T}\left(S_{t}\right)+q_{jt}^{H}\right)=E_{t}\left\{S^{*}-S_{t+1}\right\}+E_{t}\left\{\frac{dJ_{t+1}^{s}}{dS_{t+1}}\left(S_{t+1}\right)\right\}$$

The resolution of theses equations system gives the hydroelectric operator strategy at the equilibrium as a function of the current water stock: $q_{jt}^{Hs}=Q_{jt}^{H}\left(S_{t}\right)$.

We replace this strategy in the Bellman equation; we can find the value function associated to the hydroelectric operator in the Stackelberg competition with the thermal operator:

$$J^s(S_t)=P_h\left[q_{ht}^T(S_t)+q_{ht}^H(S_t)\right]q_{ht}^H(S_t)+P_l\left(q_{lt}^T(S_t)+q_{lt}^H(S_t)\right)q_{lt}^H(S_t)$$
$$+E_t\left\{J^s\left(S_t-q_{lt}^H(S_t)-q_{lt}^H(S_t)+f_{t+1}\right)-\frac{1}{2}\left(S^*+q_{lt}^H(S_t)+q_{lt}^H(S_t)-f_{t+1}\right)^2\right\}$$

NUMERICAL APPROXIMATION OF EQUILIBRIUM: LINEAR-QUADRATIC CASE

We consider a linear inverse demand function at the electricity market: $P_j\left(q_{jt}^H+q_{jt}^T\right)=a_j-q_{jt}^H-q_{jt}^T$. Let's $J^s(S_t)$ the value function associated to the hydroelectric operator problem under Stackelberg competition between the two producers. We assume that the value function is quadratic: $J^s(S_t)=b^sS_t-\frac{1}{2}B^sS_t^2$, where b^s and B^s are unknown coefficients.

The hydroelectric operator strategy at the two seasons verifies the Bellman equation:

$$J_t^s(S_t)=\left(a_h-q_{ht}^H-q_{ht}^T\right)q_{ht}^H+\left(a_l-q_{lt}^H-q_{lt}^T\right)q_{lt}^H-\frac{1}{2}E_t\left\{\left(S^*-S_t+q_{ht}^H+q_{lt}^H-f_{t+1}\right)^2\right\}$$
$$+E_t\left\{b^s\left(S_t-q_{ht}^H-q_{lt}^H+f_{t+1}\right)\right\}-\frac{1}{2}B^sE_t\left\{\left(S_t-q_{ht}^H-q_{lt}^H+f_{t+1}\right)\right\}$$
$$-\frac{1}{2}B^sE_t\left\{\left(S_t-q_{ht}^H-q_{lt}^H+f_{t+1}\right)^2\right\}$$

Given $J_t^s(S_t)$, the first order conditions are given by the following equations:

$$J_t^s(S_t)=\left(a_h-q_{ht}^H-q_{ht}^T\right)q_{ht}^H+\left(a_l-q_{lt}^H-q_{lt}^T\right)q_{lt}^H-\frac{1}{2}E_t\left\{\left(S^*-S_t+q_{ht}^H+q_{lt}^H-f_{t+1}\right)^2\right\}$$
$$+E_t\left\{b^s\left(S_t-q_{ht}^H-q_{lt}^H+f_{t+1}\right)\right\}-\frac{1}{2}B^sE_t\left\{\left(S_t-q_{ht}^H-q_{lt}^H+f_{t+1}\right)^2\right\}$$

The combination of the two latter equations gives the hydroelectric operator output at the peak season as a function of the thermal operator output at the peak and the off-peak season:

$$q_{ht}^H=L_1^Hq_{ht}^T+L_2^HS_t+L_3^Hq_{lt}^T+L_0^{hH}$$

Where

$$L_1^h=-\frac{3+B^s}{4\left(2+B^s\right)},L_2^H=\frac{1+B^s}{2\left(2+B^s\right)},L_3^H=-\frac{1+B^s}{4\left(2+B^s\right)}\text{and}$$

$$L_0^{hH}=\frac{\left(3+B^s\right)a_{ht}-\left(1+B^s\right)a_{lt}+2\left[\left(1+B^s\right)\overline{f}-S^*-b^s\right]}{4\left(2+B^s\right)}.$$

For the hydroelectric output at the off-peak season:

$$q_{lt}^{H} = L_1^H q_{lt}^T + L_2^H S_t + L_3^H q_{ht}^T + L_0^{lH}$$

Where $L_0^{lH} = \dfrac{\left(3+B^s\right)a_{lt} - \left(1+B^s\right)a_{ht} + 2\left[\left(1+B^s\right)\overline{f} - S^* - b^s\right]}{4\left(2+B^s\right)}$.

The thermal operator decides the output given the hydroelectric operator strategy. The thermal operator problem is written as:

$$\begin{aligned}\underset{q_{lt}^T, q_{ht}^T}{Max}\,\pi_t &= \left(a_{ht} - L_1^H q_{ht}^T - L_2^H S_t - L_3^H q_{lt}^T - L_0^{hH} - q_{ht}^T\right)q_{ht}^T - \frac{1}{2}c_h\left(q_{ht}^T\right)^2 \\ &+ \left(a_{lt} - L_1^H q_{lt}^T - L_2^H S_t - L_3^H q_{ht}^T - L_0^{lH} - q_{lt}^T\right)q_{lt}^T - \frac{1}{2}c_l\left(q_{lt}^T\right)^2\end{aligned}$$

The first order conditions give the thermal operator output as a function also of his production in the other season and the current water stock: $q_{ht}^T = \theta_1^T q_{lt}^T + \theta_2^T S_t + \theta_0^{hT}$ and $q_{lt}^T = \theta_1^T q_{lt}^T + \theta_2^T S_t + \theta_0^{lT}$.

Where $\theta_1^T = \dfrac{1+B^s}{5+3B^s+2\left(2+B^s\right)c_h}$, $\theta_2^T = -\dfrac{1+B^s}{5+3B^s+2\left(2+B^s\right)c_h}$

$\theta_0^{hT} = \dfrac{\left(5+3B^s\right)a_{ht} + \left(1+B^s\right)a_{lt} - 2\left[\left(1+B^s\right)\overline{f} - S^* - b^s\right]}{2\left[5+3B^s+2\left(2+B^s\right)c_h\right]}$ and

$\theta_0^{hT} = \dfrac{\left(5+3B^s\right)a_{lt} + \left(1+B^s\right)a_{ht} - 2\left[\left(1+B^s\right)\overline{f} - S^* - b^s\right]}{2\left[5+3B^s+2\left(2+B^s\right)c_l\right]}$.

The combination of the two equations implies the thermal operator output strategy as a function of the current water stock of the hydroelectric operator. The same thing for the hydroelectric operator.

Proposition 2

The Stackelberg closed-loop game equilibrium is given by the following equations:

$$q_{ht}^T = -0.087S_t + 0.1708a_h + 0.077a_l + 0.057S^* - 0.1078\overline{f}$$

$$q_{ht}^H = 0.3313S_t - 0.422a_h - 0.206a_l - 0.2169S^* + 0.41\overline{f}$$

$$q_{lt}^T = -0.087S_t + 0.1104a_h + 0.325a_l + 0.1S^* - 0.1164\overline{f}$$

$$q_{lt}^H = 0.364S_t - 0.1246a_h + 0.1694a_l - 0.2383S^* + 0.4504\overline{f}$$

The value function associated to the hydro electrical operator is:

$$J_t^s = -0.435S_t^2 + \left(0.2279a_h + 0.4053a_l + 0.2377S^* - 0.4491\overline{f}\right)S_t \blacksquare$$

The comparative static in the regulated and unregulated industry is given by the following table

t	Systems	q_{ht}^{H*}	q_{ht}^{T*}	q_{lt}^{H*}	q_{lt}^{T*}
S_t	s	0,331	-0,087	0,364	-0,087
	m	0,322	-0,08	0,48	-0,24
a_{ht}	s	-0,422	0,170	-0,124	0,110
	m	0,6	0,1	-0,165	0,3
a_{lt}	s	-0,206	0,077	0,169	0,325
	m	-0,11	0,22	0,4	0,3
$\overline{f}$	s	0,410	-0,107	0,450	-0,116
	m	0,4	-0,1	0,6	-0,3
S^*	s	-0,216	0,057	-0,238	0,1
	m	-0,32	0,08	-0,48	0,24

Table 2: The static comparative in a Stackelberg closed-loop game

FORCED STORAGE

We assume that the two producers observe a demand increase at the peak season equals to Δa_h. At the sequential asymmetric game, the supplementary demand energy is satisfied only by the T plant. Indeed, the H operator output at the peak season decreases by the quantity $-0,4224\Delta a_h$. The conservation of the water resources is not entirely compensated by an increase in the production of the power station. The T operator increase production by a quantity equal to $0,1708\Delta a_h$ units of energy. The position of the T operator on the market reduces the production of the H operator especially at the season h without compensation. This little increase in the production allows to the T operator profit coming from the high price of energy at the season h . The T operator adopts the same strategy at the season l but in the different sizes. Thus, the increase in the T production is evaluated at $0,1104\Delta a_h$ whereas the increase in the H production achieve only $-0,1246\Delta a_h$. This kind of game does not make it possible to the H operator to intervene by his free technology to satisfy the increase in demand at season h. On the other hand, the T operator obliges the H operator to preserve his resources in order to exert a power of market on this additional demand. In this case, the inter annual transfer, the intra-annual transfer and the technological complementarities do not make it possible to mitigate the perverse effects of the position of the T operator.

FORCED CYCLIC HYDROELECTRIC SEEDLING

With the difference of the preceding section, an increase in the demand at the season l of a quantity equal to Δa_l requires the recourse, in different proportions, at the same time with the T station and the H power station. The production of the T operator increases by a quantity equals to $0,3257\Delta a_l$ whereas that of the H operator increases only by $0,1694\Delta a_l$ units. To satisfy this additional request, one calls more upon the T station that with the H station. The increase in H operator output comes from the intra-annual transfer from the h season to the l season. The H operator preserves $0,206\Delta a_l$ units of water at the season h and it uses only a quantity equal to $0,1694\Delta a_l$ to satisfy the additional demand of the season l. The position of the T operator on the market prevents the H operator from using all or a higher part of its transfer to the season l. It obliges the H operator to preserve $0,0366\Delta a_l$ units of water to satisfy any increase in demand at the season l of the following periods and not to satisfy the increase in demand in the season h . The T operator increases his production at the season h in such way that it does not compensate for the fall of production of the H operator.

CONCLUSION

In this paper, we have analyzed the effect of imperfect competition one the optimal order in operating the mixed hydrothermal system. This operating is based one the substitution of costly technology output by the costless technology output. This substitution is only possible with the potential energy transfers. In contrast, the Cournot competition in power generation favored the strategic storage of water resource. Consequently, this behavior of hydraulic operator lets the additional uses of thermal technology greater then that under regulated industry. We show that the least cost operating rule is preserved also with this hydraulic operator in the non-constraint model; but it is inverted at the binding capacity constraint of the thermal operator. Under Stackelberg competition, we have shown that the thermal operator position on the market can increase the conservation of hydraulic resources in the two seasons. In this type of competition, the inter annual transfers and the intra annual transfers do not have any role. One solution of this problem is to set, by a contract, the minimal peak season output supplied by both operators on the market. To extend this paper, we proposes to analyze the effect of technical complementation and congestion of transmission network one the behavior of operators in the market. Future For research we proposes to study the effect of imperfect competition of each operator endowed by two heterogeneous technologies.

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