Climate change mitigation policies: Are R&D subsidies preferable to a carbon tax?*

André Grimaud[a] and Gilles Lafforgue[b†]

[a] *Toulouse School of Economics (IDEI and LERNA) and Toulouse Business School*
[b] *Toulouse School of Economics (INRA-LERNA)*

November 21, 2008

Abstract

We consider a general equilibrium climate change model with two endogenous R&D sectors. First, we characterize the set of decentralized equilibria: to each vector of public tools – a carbon tax and a subsidy to each R&D sector – is associated a particular equilibrium. Second, we compute the optimal tools. Third, we perform various second-best analysis by imposing some constraints on one or several policy. The main results of the paper are the following: i) both a carbon tax and a green research subsidy contribute to the climate change mitigation; ii) R&D subsidies have a large impact on the consumption, and then on the social welfare, as compared with the carbon tax used alone; iii) those subsidies allow to spare the earlier generations who are, on the other hand, penalized by a carbon tax.

JEL classification: H23, O32, Q43, Q54, Q55.

Keywords: Climate change, second-best policies, energy, directed technical change, carbon tax, research subsidies.

*We are very grateful to Bertrand Magné for his collaboration on former related works. We also thank two anonymous referees for their very helpful comments and suggestions. The usual disclaimer applies.

†Corresponding author. LERNA, 21 allée de Brienne, 31000 Toulouse – France. E-mail address: glafforg@toulouse.inra.fr.

1 Introduction

The basic approach to examine interactions between energy, climate and economic growth is called the "top-down" approach. The objective is to analyze the impact of several technological options or policies, such as CO_2 taxation or quotas, by providing a theoretically consistent description of the general economic system.[1] A large number of top-down models have been already developed: DICE (Nordhaus, 2008), ENTICE-BR (Popp, 2006a, 2006b), MIND (Edenhofer et al., 2005, 2006), DEMETER (Gerlagh and van Der Zwaan, 2006), WITCH (Bosetti et al., 2006)... Nevertheless, whatever are their degree of sophistication, those models exclusively focus on the first-best optimum by determining the temporal trajectories which maximize the social welfare subject to a set of technological and climatic constraints. Sometimes, additional constraints are added, as in Popp (2006a) where the results of a simulated optimal carbon tax without research subsidy are presented. However, to our knowledge, the basic problem of a policy-maker facing the agent behaviors in a decentralized economy is generally neither formalized nor analyzed.

In the real world, it can be impossible to reach the first-best optimum for many reasons. Some of them are standard in the literature, as the existence of ex-ante distortionnary taxes in the system (Sandmo, 1975), or the restriction to linear taxes. For instance, Cremer et al. (2001) study how second-best considerations change the level of the optimal tax on a polluting good, but in a static model. In this paper, we assume that budgetary, socioeconomic or political constraints, without no more specification, can obstruct the enforcement of the first-best policies. As an illustration, consider a policy-maker who is restricted on the number and/or the level of policy tools among the vector of all the instruments he can spare. This case occurs if, for instance, the environmental tax and/or some research subsidies are set below their first-best levels. The policy-maker can thus only play with the remaining unconstrained tools in order to maximize the social welfare. The basic point is that the structure of the decentralized economy becomes an additional constraint for him and then, he can only reach a second-best optimum.

Before conducting a second-best analysis, it is thus necessary to characterize the set of equilibria: to each vector of economic policy tools, one associates a particular equilibrium. Hence, if some of these tools are constrained, the policy-maker determines the other(s) in order to maximize the welfare in the remaining sub-set of equilibria.

[1] The alternative approach, called "bottom-up", has almost the same objective, but it puts the emphasis on a detailed technologically based treatment of the energy system. For that reason, bottom-up models capture technology in the engineering sense, as pointed out by Kahouli-Brahmi (2008).

The general equilibrium approach makes feasible any second-best analysis, but it also has several other advantages. First, it allows to analyze the dissociated impacts of various policy tools on the time pace of prices and quantities. For instance, one can study the consequences of a change in the carbon tax, the other tools being given. Second, it allows to understand the role of prices as channels by which policy tools act on the economy. Third, it permits to avoid the inaccuracies inherent in any partial equilibrium analysis, as for instance the ones implied by the use of the standard cost-benefit approach when the policy (or project) choices lead to more than marginal perturbations (see Dietz et al., 2008, for the special case of climate change mitigation policies).

The objective of this paper is to propose a methodological framework to perform second-best analysis in an endogenous growth/climate change model. More precisely, we study of the set of equilibria in the decentralized economy. The main difficulty of this approach lies in the way the research activity is modeled, in particular the type of innovation goods which are developed, as well as their pricing. In the standard endogenous growth theory (Aghion and Howitt, 1998; Romer, 1990...), the production of an innovation is associated with a particular intermediate good. However, embodying knowledge into intermediate goods usually becomes inextricable in more general computable endogenous growth models with pollution and/or natural resources. In addition, those technical difficulties are emphasized when several research sectors are under consideration, i.e. when there exists several types of specific knowledge, each of them being dedicated to a particular input (resource, labor, capital, backstop...), as it is proposed in Acemoglu (2002). To circumvent those obstacles, we assume that the pieces of knowledge are directly priced (see for instance Grimaud and Rougé, 2008). We compute the social and the market values of an innovation and we suppose that the policy-maker can reduce the gap between these two values owing to dedicated R&D subsidies.

We develop a model, based on Popp (2006a), in which energy services are provided by a bundle of two primary energies: a polluting non-renewable resource, e.g. fossil fuels, and a carbon-free substitute called backstop (solar, wind...).[2] We introduce two R&D sectors. The first one improves the efficiency of energy production, the second one, the efficiency of the backstop. Then, we have to consider two types of market failures: the

[2]The use of the term "backstop" in this case is due to Popp (2006a), but it is a slight abuse of language since the two kinds of resource can be used simultaneously. A more standard definition refers to a technological breakthrough that drives the traditional fossil energy obsolete and that replaces this former by a clean renewable source.

pollution from the fossil resource use and the research spillovers in each R&D sector. In the decentralized equilibrium, we thus introduce two kinds of economic policy instruments in accordance: an environmental tax on the carbon emissions and a research subsidy for the energy and backstop sectors. As a result, there exists a continuum of equilibria, each one being associated to a particular vector of instruments. Clearly, when the public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum.

We obtain numerical results that highlight the role of the research grants, in particular the backstop ones. The model shows that the best way to mitigate climate change is to implement a policy that combine both a carbon tax and a green research subsidy. However, the carbon tax penalizes the consumption and then, the welfare of earlier generations, whereas the research subsidy allows to spare them.

The article is organized as follows. In section 2, we sketch the model and present the decentralized economy. We also solve the equilibrium. In section 3, we characterize the first-best optimal solutions and we compute the optimal policy tools that implement it. In section 4, we analyze a selection of second-best cases and we illustrate numerically our main results. We conclude in section 5.

2 The decentralized economy

The model is mainly based on the DICE-07 and the ENTICE-BR models (Nordhaus, 2008 and Popp, 2006a, respectively). We consider a worldwide decentralized economy containing four production sectors: final output, energy services and two primary energy inputs, namely a fossil fuel and a carbon-free backstop (cf. figure 1). The fossil fuel (e.g. refining industry in the case of oil) is obtained from a non-renewable resource whose combustion yields carbon emissions. Those emissions accumulate into the atmosphere and bring about an increase of the mean atmospheric temperature. Retrospectively, global warming imposes some penalties on society. As in Nordhaus (2007 and 2008), we assume here that these penalties take the form of a damage function affecting the level of final output, instead of the consumer's utility. The production of final energy services and backstop requires specific knowledges provided by two specific R&D sectors. We assume that all sectors, except R&D sectors, are perfectly competitive. Population, i.e. labor supply, grows exogenously. Finally, in order to correct the two types of distortions involved by the model (pollution and research spillovers in each R&D sector), we introduce two types of policy tools: an environmental tax on the fossil fuel use and a subsidy for each R&D sector. The model is calibrated to fit the world 2005 data (details of calibration are provided in appendix). A detailed analysis, sector by sector, is developed in the following subsection

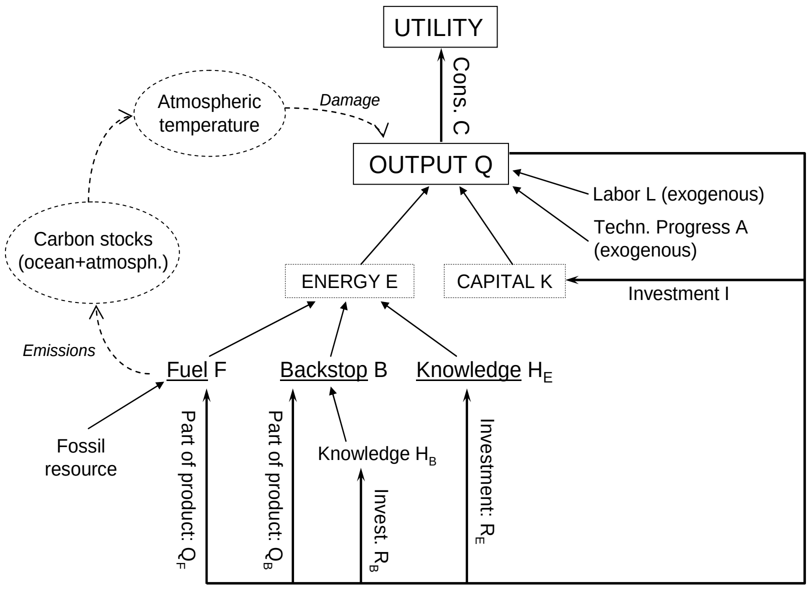

Figure 1: Description of the model

2.1 Behavior of agents

2.1.1 The final good sector

Production is represented by the same modified production function than in Nordhaus (2008). We assume that global warming affects the economy through the final output such that, when the average temperature increase is T_t, the instantaneous penalty rate is $D(T_t) = 1/(1 + \alpha_T T_t^{\eta_T})$, $\alpha_T, \eta_T > 0$. At each time t, the production of final output is $D(T_t)Q_t$, where Q_t is given by the following constant-return-to-scale Cobb-Douglas production function:

$$Q_t = Q(K_t, E_t, L_t, A_t) = A_t K_t^{\gamma} E_t^{\beta} L_t^{1-\gamma-\beta}, \quad \beta, \gamma \in (0,1), \tag{1}$$

in which K_t, E_t, L_t and A_t denote the stock of capital, the flow of energy services, the labor force and the total productivity of factors (i.e. a Hicks-neutral technological change), respectively. We assume that L_t and A_t are exogenously given: $L_t \equiv L_0 e^{\int_0^t g_{L,s} ds}$ and $A_t \equiv$

$A_0 e^{\int_0^t g_{A,s} ds}$, where the growth rates $g_{L,t}$ and $g_{A,t}$ are defined by the following exponential declining form: $g_{j,t} = g_{j0} e^{-d_j t}$, $d_j > 0$, $\forall j = \{A, L\}$.[3]

Denoting respectively by $p_{E,t}$, w_t, r_t and δ the price of energy services, the real wage, the interest rate and the depreciation rate of capital, and normalizing the output price to one, the instantaneous profit of the final output producer writes[4]: $\Pi_t^Q = D(T_t)Q_t - p_{E,t}E_t - w_t L_t - (r_t + \delta)K_t$. At each time t, the program of the final output producer consists in choosing K_t, E_t and L_t that maximizes Π_t^Q, subject to (1). The first order conditions are:

$$\begin{aligned} D(T_t)Q_K - (r_t + \delta) &= 0 &(2)\\ D(T_t)Q_E - p_{E,t} &= 0 &(3)\\ D(T_t)Q_L - w_t &= 0, &(4) \end{aligned}$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X.

2.1.2 The final energy sector

We use the energy production function introduced by Popp (2006a and 2006b). At each time t, the production of a flow of energy services E_t depends both on a bundle of imperfect substitute primary energies and on technical change:

$$E_t = E(F_t, B_t, H_{E,t}) = \left[(F_t^{\rho_B} + B_t^{\rho_B})^{\frac{\rho_H}{\rho_B}} + \alpha_H H_{E,t}^{\rho_H} \right]^{\frac{1}{\rho_H}}, \quad \alpha_H, \rho_H, \rho_B \in (0,1), \tag{5}$$

where F_t is the fossil fuel input, B_t is a carbon-free energy source, namely the "backstop", and $H_{E,t}$ represents a stock of specific technological knowledge dedicated to energy efficiency. Denoting by $p_{F,t}$ and $p_{B,t}$ the prices of fossil fuel and backstop and by τ_t the carbon tax, assumed here to be additive, the energy producer must chooses F_t and B_t at each time t that maximizes $\Pi_t^E = p_{E,t}E_t - (p_{F,t} + \tau_t)F_t - p_{B,t}B_t$, subject to (5). Note that, because of the carbon tax, the fuel price paid by the firm, i.e. $p_{F,t} + \tau_t$, is larger than the selling price $p_{F,t}$, i.e. the price which is received by the resource-holder. The first order conditions

[3] As in Nordhaus (2008), the TFP growth is exogenous in order to circumvent the large source of uncertainty on its projection. It is assumed to slow gradually over the next three centuries until eventually stopping. The same trajectory shape also apply for the labor force, i.e. the population. Long-term projections of the United Nations predict a declining growth rate so that total population approaches a limit of 8.6 billion.

[4] We assume here that the representative household holds the capital and rents it to firms at the rental price $p_{K,t}$. Standard arbitrage conditions imply $p_{K,t} = r_t + \delta$.

write:

$$\begin{aligned} p_{E,t}E_F - p_{F,t} - \tau_t &= 0 && (6) \\ p_{E,t}E_B - p_{B,t} &= 0. && (7) \end{aligned}$$

2.1.3 The fossil fuel sector

The fossil fuel "production/extraction" function is derived from the Popp's extraction cost function (Popp, 2006a). We assume that it depends on a stock of carbon-based non-renewable resource and on specific productive investment (Grimaud et al., 2007):

$$F_t = F(Q_{F,t}, Z_t) = \frac{Q_{F,t}}{c_F + \alpha_F (Z_t/\bar{Z})^{\eta_F}}, \quad c_F, \alpha_F, \eta_F > 0, \tag{8}$$

where $Q_{F,t}$ is the amount of final product devoted to the production/extraction of fossil fuel and Z_t, $Z_t \equiv \int_0^t F_s ds$, is the cumulative extraction of the exhaustible resource from the initial date up to t, with $\bar{Z}$: $Z_t \leq \bar{Z}$, $\forall t \geq 0$. Then, the fuel supply is constrained by the resource scarcity. The instantaneous profit of the fuel producer is: $\Pi_t^F = p_{F,t}F_t - Q_{F,t}$ and its program consists in choosing $\{Q_{F,t}\}_0^\infty$ that maximizes $\int_0^\infty \Pi_t^F e^{-\int_0^t r_s ds} dt$, subject to $Z_t = \int_0^t F_s ds$ and (8). Denoting by η_t the multiplier associated with the state equation, static and dynamic first order conditions are:

$$\begin{aligned} (p_{F,t}F_{Q_F} - 1)e^{-\int_0^t r_s ds} + \eta_t F_{Q_F} &= 0 && (9) \\ p_{F,t}F_Z e^{-\int_0^t r_s ds} + \eta_t F_Z &= -\dot{\eta}_t, && (10) \end{aligned}$$

together with the transversality condition $\lim_{t\to\infty} \eta_t Z_t = 0$. Integrating (10) and using (9), it comes:

$$p_{F,t} = \frac{1}{F_{Q_F}} - \int_t^\infty \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r_x dx} ds, \tag{11}$$

which reads as a specific version of the standard Hotelling rule in the case of an extraction technology given by function (8).

2.1.4 The backstop sector

Similarly to the fossil fuel technology, the backstop production function is also based on the corresponding cost function used by Popp (2006a). This technology requires some specific investment and knowledge:

$$B_t = B(Q_{B,t}, H_{B,t}) = \alpha_B Q_{B,t} H_{B,t}^{\eta_B}, \quad \alpha_B, \eta_B > 0, \tag{12}$$

where $Q_{B,t}$ is the amount of final product that is devoted to the backstop production sector and $H_{B,t}$ is the stock of knowledge pertaining to the backstop. At each time t, the backstop producer maximizes its profit $\Pi_t^B = [p_{B,t}B_t - Q_{B,t}]$, subject to (12), which implies the following first order condition:

$$p_{B,t}B_{Q_B} - 1 = 0. \tag{13}$$

2.1.5 The R&D sectors

There are two stocks of knowledge, H_E and H_B, each associated with a specific R&D sector (i.e. the energy and the backstop ones). We consider that each innovation is a non-rival, indivisible and infinitely durable piece of knowledge (for instance, a scientific report, a data base, a software algorithm...) which is simultaneously used by the sector which produces the good i and the R&D sector i, $i = \{B, E\}$.

Here, an innovation is not directly embodied into tangible intermediate goods and thus, it cannot be financed by the sale of these goods. However, in order to fully describe the equilibrium, we need to find a way to assess the price received by the inventor for each piece of knowledge. We proceed as follows: i) In each research sector, we determine the *social* value of an innovation. Since an innovation is a public good, this social value is the sum of marginal profitabilities of this innovation in all sectors which use it. If the inventor was able to extract the willingness to pay of each user, he would receive this social value and the first best optimum would be implemented. ii) In reality, there are some distortions that constrain the inventor to extract only a part of this social value[5]. This implies that the *market* value (without subsidy) is lower than the social one. iii) The research sectors are eventually subsidized in order to reduce the gap between the social and the market values of innovations.

Let us apply this three-steps procedure to the R&D sector i, $i = \{B, E\}$. Each innovation produced by this sector is used by the R&D sector i itself as well as by the production technology of good i. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{H_i,t} = \bar{v}^i_{H_i,t} + \bar{v}^{H_i}_{H_i,t}$, where $\bar{v}^i_{H_i,t}$ and $\bar{v}^{H_i}_{H_i,t}$ are the marginal profitabilities of this innovation in the production and R&D sectors i, respectively. The social value of this innovation at t is $\bar{V}_{H_i,t} = \int_t^\infty \bar{v}_{H_i,s} e^{-\int_t^s r_x dx} ds$. We assume that, without any public intervention, only a share γ_i of the social value is paid to the innovator, with $0 < \gamma_i < 1$. However, the government can decide to grant this R&D sector by applying a non-negative

[5]For instance, Jones and Williams, 1998, estimate that actual investment in research are at least four times below what would be socially optimal; on this point, see also Popp, 2006a.

subsidy rate $\sigma_{i,t}$. Note that if $\sigma_{i,t} = 1 - \gamma_i$, the market value matches the social one. The instantaneous market value (including subsidy) is:

$$v_{H_i,t} = (\gamma_i + \sigma_{i,t})\bar{v}_{H_i,t}, \tag{14}$$

and the market value at date t is:

$$V_{H_i,t} = \int_t^{\infty} v_{H_i,s} e^{-\int_t^s r_x dx} ds. \tag{15}$$

Note that differentiating (15) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{H_i,t}}{V_{H_i,t}} + \frac{v_{H_i,t}}{V_{H_i,t}}, \quad \forall i = \{B, E\}, \tag{16}$$

which reads as the equality between the rate of return on the financial market and the rate of return on the R&D sector i.

We can now analyze the behaviors of the R&D sectors. The dynamics of the stock of knowledge in sector i is governed by the following innovation function $H^i(.)$ (cf. Popp, 2006a):

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}) \; = \; a_i R_{i,t}^{b_i} H_{i,t}^{\phi_i}, \tag{17}$$

where $a_i > 0$, and $b_i, \phi_i \in [0, 1]$, $\forall i = \{B, E\}$. $R_{i,t}$ is the R&D investment into sector i, i.e. the amount of final output that is devoted to R&D sector i. At each time t, each sector i, $i = \{B, E\}$, supplies the flow of innovations $\dot{H}_{i,t}$ at price $V_{H_i,t}$ and demands some specific investment $R_{i,t}$ at price 1, so that the profit function to be maximized is $\Pi_t^{H_i} = V_{H_i,t} H^i(R_{i,t}, H_{i,t}) - R_{i,t}$. The first order condition implies:

$$\frac{\partial \Pi_t^{H_i}}{\partial R_{i,t}} = 0 \quad \Rightarrow V_{H_i,t} = \frac{1}{H_{R_i}^i}. \tag{18}$$

The marginal profitability for specific knowledge of R&D sector i is:

$$\bar{v}_{H_i,t}^{H_i} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{H_i,t} H_{H_i}^i = \frac{H_{H_i}^i}{H_{R_i}^i}. \tag{19}$$

Finally, in order to determine the social and the market values of an innovation in all research sectors, we need to know the marginal profitabilities of innovations in the backstop and the energy production sectors. From the expressions of Π_t^B and Π_t^E, those values are given respectively by $\bar{v}_{H_B,t}^B = \partial \Pi_t^B / \partial H_{B,t} = B_{H_B}/B_{Q_B}$ and $\bar{v}_{H_E,t}^E = \partial \Pi_t^E / \partial H_{E,t} = E_{H_E}/E_B B_{Q_B}$. Therefore, the instantaneous market values (including subsidies) of innovations are:

$$v_{H_B,t} \;=\; (\gamma_B + \sigma_{B,t}) \left(\frac{B_{H_B}}{B_{Q_B}} + \frac{H_{H_B}^B}{H_{R_B}^B} \right) \tag{20}$$

$$v_{H_E,t} \;=\; (\gamma_E + \sigma_{E,t}) \left(\frac{E_{H_E}}{E_B B_{Q_B}} + \frac{H_{H_E}^E}{H_{R_E}^E} \right). \tag{21}$$

2.1.6 The household and the government

We use the same CES utility function $U(.)$ than Nordhaus (2008). The social welfare function is thus defined as:

$$W = \int_0^\infty U(C_t)e^{-\rho t}dt \; = \int_0^\infty L_t \frac{(C_t/L_t)^{1-\epsilon}}{(1-\epsilon)} e^{-\rho t}dt, \tag{22}$$

where C_t is the aggregate consumption, ρ, $\rho > 0$, is the (constant) social rate of time preferences and ϵ, $\epsilon > 0$, is the elasticity of marginal utility. The households maximize W subject to the following budget constraint:

$$I_t + C_t + T_t^a = (r_t + \delta)K_t + w_t L_t + \Pi_t, \tag{23}$$

where I_t is the instantaneous investment in capital defined by $I_t = \dot{K}_t + \delta K_t$, Π_t is the total profits gained in the economy and T_t^a is a lump-sum tax (subsidy-free) that allows to balance the budget constraint of the government. This maximization leads to the following condition:

$$\rho - \frac{\dot{U}'(C_t)}{U'(C_t)} = r_t \Rightarrow U'(C_t) = U'(C_0)e^{\rho t - \int_0^t r_s ds}. \tag{24}$$

Assuming that the government's budget constraint holds at each time t (i.e. sum of the various taxes equal R&D subsidies), then it writes:

$$T_t^a + \tau_t F_t = \sum_i \frac{\sigma_i}{(\gamma_i + \sigma_i)} V_{H_i,t}\dot{H}_{i,t}, \quad i = \{B, E\}. \tag{25}$$

Finally, remark that expanding $\Pi_t = \Pi_t^Q + \Pi_t^E + \Pi_t^B + \Pi_t^F + \Pi_t^{H_B} + \Pi_t^{H_E}$ into (23) and replacing T_t^a by its value coming from (25), we obtain:

$$D(T_t)Q_t = C_t + Q_{F,t} + Q_{B,t} + I_t + R_{E,t} + R_{B,t}, \tag{26}$$

thus verifying that the final output is devoted to the aggregated consumption, the fossil fuel production, the backstop production, the investment in capital, and in the two R&D sectors.

2.2 The environment

Pollution is generated by fossil fuel burning. Let ξ, $\xi > 0$, be the unitary carbon content of fossil fuel, G_0 the stock of carbon in the atmosphere at the beginning of the planning period, G_t the stock at time t and ζ, $\zeta > 0$, the natural rate of decay. As in the DICE-07 model (Nordhaus, 2008), the atmospheric carbon concentration does not directly enter the damage function. In fact, the increase in carbon concentration drives the global mean temperature away from a given state – here the 1900 level – and the difference between this state and the present global mean temperature is taken as an index of climate change. Let T_t denote this difference. Then, the climatic dynamic system is captured by the following two state equations:

$$\dot{G}_t = \xi F_t - \zeta G_t \tag{27}$$

$$\dot{T}_t = \Phi(G_t) - mT_t = \alpha_G \log G_t - mT_t, \quad \alpha_G, m > 0. \tag{28}$$

Function $\Phi(.)$, which links the atmospheric carbon concentration to the dynamics of temperature, is in fact the reduced form of a more complex function that takes into account the inertia of the climate dynamics (i.e. the radiative forcing, see Nordhaus 2008)[6].

2.3 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now characterize the set of equilibria, which is done by the following proposition:

Proposition 1 *For a given triplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$, the equilibrium conditions can be summed up as follows:*

$$\left[D(T_t)Q_E E_F - \tau_t - \frac{1}{F_{Q_F}}\right] U'(C_t)e^{-\rho t} + \int_t^{\infty} \frac{F_Z}{F_{Q_F}} U'(C_s)e^{-\rho s} ds = 0 \tag{29}$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \tag{30}$$

$$D(T_t)Q_K - \delta = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{31}$$

$$-\frac{\dot{H}^B_{R_B}}{H^B_{R_B}} + (\gamma_B + \sigma_{B,t}) \left(\frac{B_{H_B} H^B_{R_B}}{B_{Q_B}} + H^B_{H_B} \right) = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{32}$$

$$-\frac{\dot{H}^E_{R_E}}{H^E_{R_E}} + (\gamma_E + \sigma_{E,t}) \left(\frac{E_{H_E} H^E_{R_E}}{E_B B_{Q_B}} + H^E_{H_E} \right) = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)}. \tag{33}$$

[6] In the analytical treatment of the model, we assume, for the sake of clarity, that the carbon cycle through atmosphere and oceans as well as the dynamic interactions between atmospheric and oceanic temperatures, are captured by the reduced form (27) and (28). Goulder and Mathai (2000), or Kriegler and Bruckner (2004), have recourse to such simplified dynamics. From the DICE-99 model, the formers estimate parameters ξ and ζ that take into account the inertia of the climatic system. They state that only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2 and that the portion of current CO_2 concentration in excess is removed naturally at a rate of 0.8% per year. However, in the numerical simulations, we adopt the full characterization of the climate dynamics from the 2007 version of DICE (Nordhaus, 2008).

Proof. See Appendix A1.

A particular equilibrium is associated with a given triplet of policies $\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_{t=0}^{\infty}$ and the set of equations given by Proposition 1 allows to compute the quantities for this equilibrium. The corresponding prices r_t^*, w_t^*, $p_{E,t}^*$, $p_{F,t}^*$, $p_{B,t}^*$ and $V_{Hi,t}^*$ are given by (2), (4), (3), (11), (13) and (18), respectively. If the triplet of policy tools is optimally chosen, this set of equations characterizes the first-best optimum, together with the system of prices that implement it. Note that we will get the same kind of conditions than the ones of Proposition 1 to characterize the first-best optimum (cf. Proposition 2 below), so that we defer their interpretations to the next section.

3 Implementation of the first-best optimum

The social planner problem consists in choosing $\{C_t, Q_{B,t}, Q_{F,t}, R_{B,t}, R_{E,t}\}_{t=0}^{\infty}$ that maximizes W, as defined by (22), subject to the output allocation constraint (26), the technological constraints (1), (5), (8) and (12), the environmental constraints (27) and (28), and, finally, the stock accumulation constraints (17), (23) and $\dot{Z}_t = F_t$. After eliminating the co-state variables, the first order conditions reduce to the five characteristic conditions of Proposition 2 below, which hold at each time t (we drop time subscripts for notational convenience).

Proposition 2 *At each time t, the optimal solution is characterized by the following five conditions:*

$$\begin{aligned} &\left[D(T_t)Q_E E_F - \frac{1}{F_{Q_F}}\right] U'(C_t)e^{-\rho t} + \int_t^{\infty} \frac{F_Z}{F_{Q_F}} U'(C_s)e^{-\rho s} ds \\ &+\xi \int_t^{\infty} \left[\int_s^{\infty} D'(T_x)Q_x U'(C_x)e^{-\rho x - m(x-s)}dx\right] \Phi'(G_s)e^{-\zeta(s-t)}ds \;=\; 0 \end{aligned} \tag{34}$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \tag{35}$$

$$D(T_t)Q_K - \delta = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{36}$$

$$H_{H_B}^B + \frac{H_{R_B}^B B_{H_B}}{B_{Q_B}} - \frac{\dot{H}_{R_B}^B}{H_{R_B}^B} = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{37}$$

$$H_{H_E}^E + \frac{H_{R_E}^E E_{H_E}}{E_B B_{Q_B}} - \frac{\dot{H}_{R_E}^E}{H_{R_E}^E} = \rho - \frac{\dot{U}'(C_t)}{U'(C_t)}. \tag{38}$$

Proof. See Appendix A2.

Equation (34) reads as a particular version of the Hotelling rule in this model, which takes into account the carbon accumulation in the atmosphere, the dynamics of temperatures and their effects on the output. Equation (35) tells that the marginal productivity of specific input $Q_{B,t}$ equals its marginal cost. The three last equations are Keynes-Ramsey conditions. Equation (36) characterizes the optimal intertemporal trade-off between capital K_t and consumption C_t, as in standard growth models. Equation (37) (resp. (38)) characterizes the same kind of optimal trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$) and consumption.

Recall that for a given set of public policies, a particular equilibrium is characterized by conditions (29)-(33) of Proposition 1. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (34)-(38) of Proposition 2. By analogy between these two sets of conditions, we can show that there exists a single triplet $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ that implements the optimum.

First, by comparing conditions (29) and (34), the optimal pollution tax can be identified as:

$$\tau_t^o = -\frac{\xi}{U'(C_t)} \int_t^{\infty} \left[\int_s^{\infty} D'(T_x) Q_x U'(C_x) e^{-(\rho+m)(x-s)} dx \right] \Phi'(G_s) e^{-(\rho+\zeta)(s-t)} ds. \quad (39)$$

Next, the correspondence between the equilibrium characterizing condition (32) (resp. (33)) and the optimum characterizing condition (37) (resp. (38)) is achieved if and only if $\sigma_{i,t}$ is equal to $1 - \gamma_i$, $i = \{B, E\}$, i.e. if both sectors are fully subsidized. The remaining conditions of the two sets are equivalent. These findings are summarized in Proposition 3 below.

Proposition 3 *The equilibrium defined in Proposition 1 is optimal if and only if the triplet of policies* $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ *is such that* $\sigma_{B,t} = 1 - \gamma_B \equiv \sigma_B^o$, $\sigma_{E,t} = 1 - \gamma_E \equiv \sigma_E^o$ *and* $\tau_t = \tau_t^o$, *for all* $t \geq 0$.

The optimal tax (39) requires some comments. Formally, this expression corresponds to the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the consumption of an additional unit of fossil resource – and the marginal utility of consumption. In other words, it is the environmental cost of one unit of fossil resource in terms of final good: at date t, the increase by one unit of fossil fuel consumption, and then of carbon emissions, increases the stock of carbon in the atmosphere G_t by an equivalent amount and rises the current temperature level by $\Phi'(G_t)$. Since the environmental externality is captured here by a stock, and not a flow, this temperature change involves a unitary damage equal to $\int_t^\infty D'(T_s)Q_s U'(C_s)e^{-(\rho+m)(s-t)}ds$, i.e. to the sum of the flow of marginal damages as measured in terms of utility, discounted at rate $(\rho+m)$ in order to take into account the climatic inertia. The full marginal damage in terms of utility is obtained by multiplying this unitary damage by the effective change in temperature and by integrating this expression over time, with a discount rate equal to $(\rho+\zeta)$ to take into account the natural regeneration process of the atmosphere. Finally, in order to get a positive tax expressed in monetary value, we multiply the previous expression by $-1/U'(C)$, which yields expression (39).

Lastly, a worthwhile remark concerns the dynamic pace of such an optimal tax. By computing its growth rate, we can show that it is not necessary monotonous:

$$\frac{\dot{\tau}_t^o}{\tau_t^o} = \left(\zeta + \rho - \frac{\dot{U}'}{U'}\right) - \frac{\Phi'(G_s)\int_t^\infty D'(T)QU'(C)e^{-(\rho+m)(s-t)}ds}{\int_t^\infty \left[\int_s^\infty D'(T)QU'(C)e^{-(\rho+m)(x-s)}dx\right]\Phi'(G)e^{-(\rho+\zeta)(s-t)}ds}. \quad (40)$$

The first term into brackets is strictly positive and works as the effective discount rate that takes into account the natural decarbonization rate of the atmosphere (i.e. the real interest rate $r_t = \rho - \dot{U}'/U'$ augmented by ζ). By concavity of the utility function, this term decreases over time as the economy, and thus the consumption, grows. The second term is the share of the instantaneous marginal damage in the cumulated marginal damage, which is positive and smaller than 1. Hence, $\dot{\tau}_t^o/\tau_t^o$ can be either positive or negative. Moreover, in the case where the second term increases over time as atmospheric carbon emissions accumulate, we can get, as shown in our numerical developments, a trajectory which is first increasing and next, declining.

4 Second-best policies

4.1 Methodology

The characteristic conditions of Proposition 1 yield the intertemporal equilibrium profiles of quantities $\{C_t^e, T_t^e, F_t^e, ...\}_0^\infty$ and prices $\left\{p_{F,t}^e, p_{B,t}^e, ...\right\}_0^\infty$ associated with any profile of policy tools $\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_0^\infty$ belonging to the definition set Ω. For each equilibrium solution, one can compute the associated welfare value as a function of those public tools: $W\left(\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_0^\infty\right)$. When W is maximized simultaneously with respect to the three tools, one gets the first-best optimum as described by Proposition 3: $\{\tau_t^o, \sigma_B^o, \sigma_E^o\}_0^\infty =$ argmax $W\left(\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_0^\infty\right)$.

Assume now that the social planner faces some constraints on her choices. For instance, she cannot subsidy research, or she cannot implement the first-best carbon tax. In this case, she only uses the remaining unconstrained tool(s) to maximize the social welfare in the remaining sub-set of equilibria. Formally, if we denote by $\Theta \subseteq \Omega$ this subset of constraints, then the second-best optimal policies are such that: $\left\{\tau_t^{sb}, \sigma_{B,t}^{sb}, \sigma_{E,t}^{sb}\right\}_0^\infty = \text{argmax } W\left(\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_0^\infty\right)$ subject to $\{\tau_t, \sigma_{B,t}, \sigma_{E,t}\}_0^\infty \in \Theta$. Among the infinity of possible second-best problems, we focus on the particular cases described in Table 1.

Case	τ_t	σ_E	σ_B	Comment
FB	τ_t^o	σ_E^o	σ_B^o	First-best optimum
LF	0	0	0	Laisser-faire
$SB1$	$\tau_t^{sb_1}$	0	0	Second-best, no R&D subs.
$SB2$	$\tau_t^{sb_2}$	σ_E^o	0	Second-best, no green R&D subs.
$SB3$	$\tau_t^{sb_3}$	0	σ_B^o	Second-best, no energy R&D subs.
$SB4$	0	σ_E^{sb}	σ_B^{sb}	Second-best, no carbon tax

Table 1: Summary of the various cases

In table 1, polar cases "FB" and "LF" refer to the first-best and the laisser-faire, respectively. All the other cases are second-best analysis. "SB1" is the case where neither energy nor backstop R&D can be subsidized and it gives the associated second-best carbon tax τ_t^{sb1}. "SB2" (resp. "SB3") is the case where the green (resp. energy) research cannot be granted, the other subsidy been set at its first-best optimal level; the associated second-best tax is denoted by τ_t^{sb2} (resp. τ_t^{sb3}). Finally, "SB4" is the case where the fossil resource is not taxed at all.

4.2 Main results

4.2.1 Second-best instruments

The first and second-best carbon taxes are depicted in Figure 2(a). We can observe that when the social planner is not able to grant research at all, she must impose a higher carbon tax than the first-best one: $\tau_t^o < \tau_t^{sb1}$. In order to identify the relevant research sector to explain this result, we must look at "SB2" and "SB3". It appears that only green

R&D matters. Then, under a strict welfare-maximization point of view, an insufficient σ_B can be partially balanced by a higher carbon tax, but not an insufficient σ_E. To sum up, one gets:

$$\tau^{lf} = \tau^{sb4} = 0 < \tau_t^o \approx \tau_t^{sb3} < \tau_t^{sb1} \approx \tau_t^{sb2}, \forall t \geq 0.$$

In scenario SB4 (when the carbon tax is nil at each point of time), for computational convenience, we impose the two additional constraints that the subsidy rates are equal and constant over time.[7] Under these assumptions, we find the following associated second-best R&D subsidies:

$$\sigma_i^{sb} = 1.04 \times \sigma_i^o, i = \{B, E\}.$$

4.2.2 Policy effects on energy/climate

The ranking of the various taxes given in the previous subsection is transfered to the fossil fuel market prices, i.e. the selling prices including tax, as shown in Figure 2(b):

$$p_{F,t}^{lf} \approx p_{F,t}^{sb4} < p_{F,t}^o \approx p_{F,t}^{sb3} < p_{F,t}^{sb1} \approx p_{F,t}^{sb2}, \forall t \geq 0.$$

If R&D subsidies would remain unchanged, this ranking of taxes and fossil prices would lead to a corresponding inverted ranking of the extraction trajectories. However, those subsidies are set to different levels in scenarios SB1 to SB4. That explains why, as shown in Figure 2(c), the expected ranking is not observed. Indeed, we have (at least until the end of this century):

$$F_t^{lf} \approx F_t^{sb4} > F_t^{sb1} \approx F_t^{sb2} > F_t^o \approx F_t^{sb3}, \forall t \geq 0.$$

The first inequality is the expected one: an increase in τ causes F to decrease. However, as compared to "FB", the carbon tax increases in "SB1" and "SB2", but the fossil fuel extraction flow also increases. This is due to the decrease in σ_B when moving from FB to SB1 or SB2. As a result, the effect of the green research subsidy overrides the carbon tax one in that case. To go more into details, as long as the carbon is not taxed, R&D subsidies do not have any effect on fossil fuel use (LF versus SB4). When the tax becomes positive, F is reduced only if σ_B is increased (from SB1 and SB3, τ decreases, σ_E remains nil, σ_B rises, but F diminishes). Conversely, an increase in σ_E has not any impact on F (SB1 versus SB2).

From Figures 2(d) and 2(e), we observe that the carbon tax has a very weak effect on the backstop price and production, and on the green R&D (not shown). The basic relevant

[7] For a discussion about dynamic R&D subsidies, see Gerlagh *et al.* (2008).

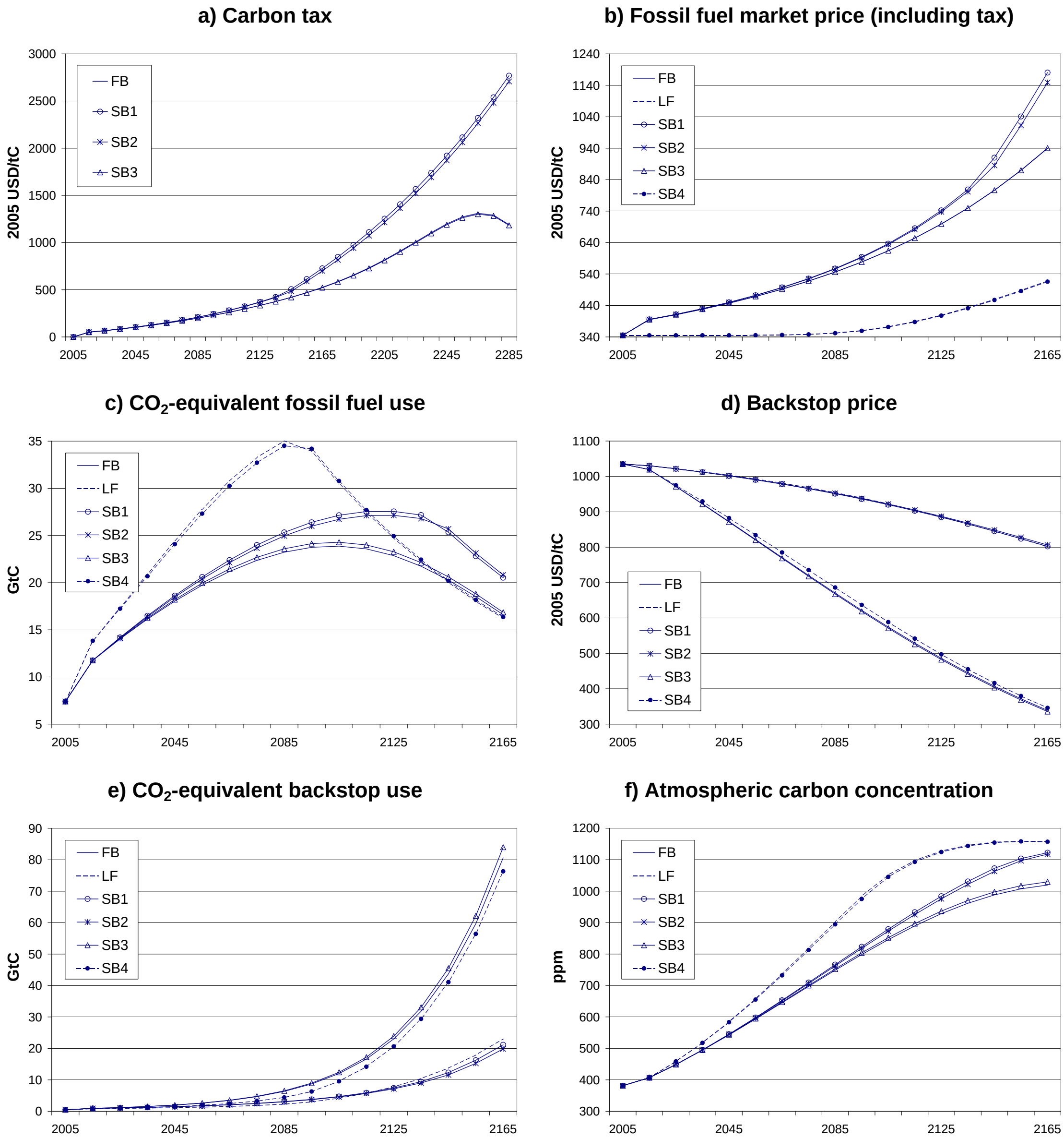

Figure 2: Results in resources and pollution

policy tool on these markets is the specific subsidy σ_B: an increase in σ_B reduces p_B and increases B.

To sum up, both a carbon tax and a green research subsidy contribute to the climate change mitigation, as illustrated in Figure 2(f).[8] The carbon tax has a direct effect on climate through its impact on the market price of the fossil fuel, but it has not indirect effect on the backstop sector. The green R&D subsidy rate has a direct effect on the backstop sector and thus an indirect effect on the fossil fuel use, because of substitutions between these two primary energy sources.

4.2.3 Effects on the output

Figure 3 focuses on more general macroeconomic effects of the various scenarios. Figure 3(a) depicts the variations of the climatic damage, as measured in percentage of the final output, formally $100*(1-D)/D$. Unsurprisingly, the results directly follow the variations of carbon accumulation – and thus of temperatures – analyzed above. Figure 3(b) represents the present value of this damage, i.e. the discounted sum of the instantaneous climate change costs, as expressed in USD, with a discount rate equal to the interest rate. We obtain the following ranking:

$$PV^{LF} > PV^{SB4} > PV^{SB1} \approx PV^{SB2} > PV^{SB3} \approx PV^{FB}.$$

Then, as already mentioned in the previous subsection, both a carbon tax and a green R&D subsidy are required to minimize the cost of global warming in terms of output (we will talk about the question of social costs later).

In Figure 3(c), we analyze the losses and gains in GWP (i.e. in final output), implied by the various public interventions, as compared with the laisser-faire case. First, whenever a positive carbon tax is levied, we can observe a loss for the earlier generations. Second, the larger the carbon tax is, the stronger this loss. Third, one can attenuate the output losses caused by the carbon tax and reach earlier the date at which gains will occur again, by increasing simultaneously the green research subsidy. Finally, the intergenerational effort can be smoothed if the planner uses less the tax and more the subsidy. However, in this case, the long run GWP gain reveals to be less important than the one implied by the use of the carbon tax alone.

[8] The variations of temperatures follow the same time pace than the atmospheric carbon concentration (not shown here).

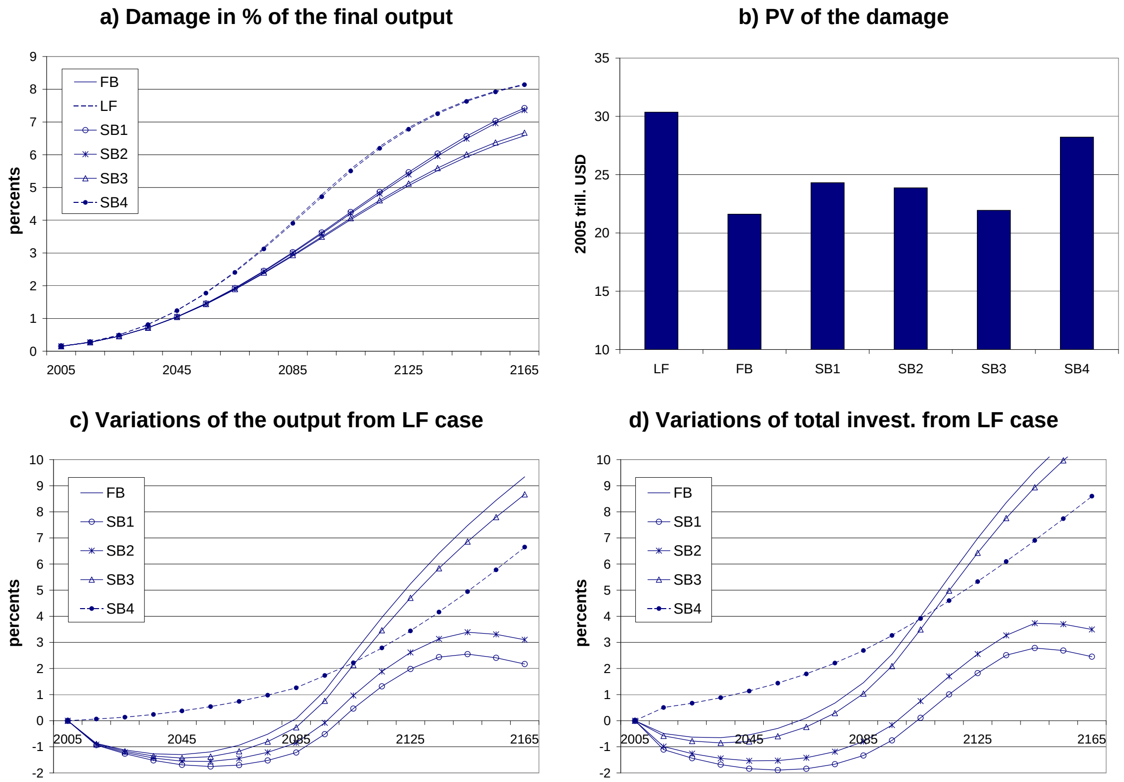

Figure 3: Impacts on damage and GWP

4.2.4 Effects on the welfare

We first examine how consumption reacts when the policy tools vary. Figure 4(a) works like figure 3(c) and gives the deviation in percents from the LF trajectory. We can remark some differences between these two graphs, which are due to the impact of the carbon tax and the research subsidies on the various investments (in capital, in primary energy production and in R&D). As shown in figure 3(d), the general impacts of the environmental and research policies on the total investment (i.e. $I + R_E + R_B + Q_F + Q_B$) are symmetric to the ones observed on the final output. Moreover, they exhibit approximatively the same order of magnitude. Without going into detail, an increase in τ diminishes the total investment, essentially by increasing Q_F; simultaneously, an increase in σ_B stimulates the total investment through its effect on R_B and Q_B whereas the effects of σ_E reveal negligible. As a result, the depressive effect on the final output observed (essentially in the short run) in the scenarios within a carbon tax is levied, is partially attenuated on the consumption by a decrease in the total investments.

Last, Figure 4(b) gives some insights on the relative impacts of both the carbon tax and

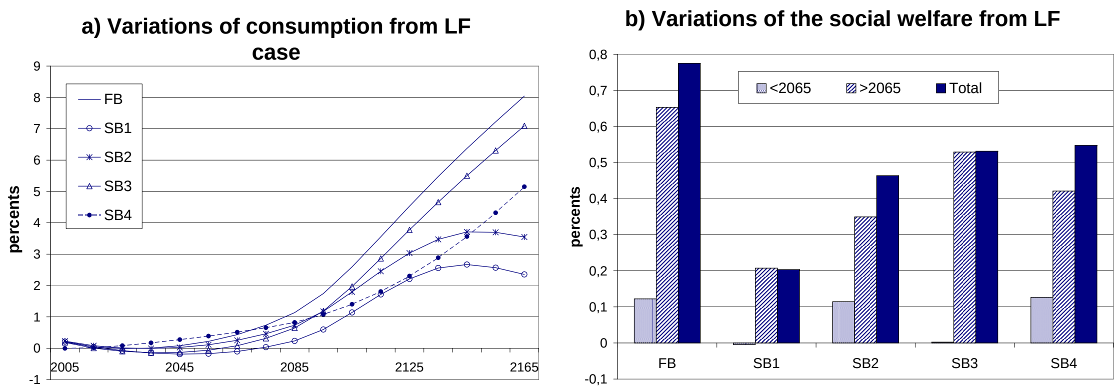

Figure 4: Impacts on consumption and welfare

the R&D subsidies on the social welfare (i.e. the present value of the flows of instantaneous utility). In order to discuss about intergenerational equity, we distinguish the welfare variation from the LF case for the "present" generations (i.e. until 2065) and the future ones (after 2065). The main results are the following: i) an increase in τ leads to an increase in the total welfare, with a slight shortening for the earlier generations and a raise for the future ones (cf. "SB1"); ii) without any ambiguity, an increase in σ_E enhances the welfare for all generations (cf. "SB1" vs "SB2"); iii) an increase in σ_B essentially augments the welfare of future generations (cf. "SB1" vs "SB3"). Finally, the gap between "FB" and "SB1" (resp. "SB4") measures the welfare loss caused by an absence of research subsidy in any R&D sector (resp. by a zero carbon tax). In a second-best world, a carbon tax used alone leads to a higher social cost (with respect to the first-best) than a research policy alone. This result is due to the fact that, in the last case (a zero carbon tax), the impact of the research subsidies on the environmental is weak and it is overridden by the direct impact on the output (and its growth). This analyze illustrates that the objective of any policy (output, welfare, consumption, environment, ...) must be carefully defined. In the limit case where the objective turns on the climate, the basic public tool is the carbon tax; but it could lead to a welfare loss for early generations. On the contrary, by mainly focusing on the social welfare and the intergenerational equity, the question of the climate may be under-estimated.

5 Conclusion

We have conducted various second-best analysis in a general equilibrium climate change model with endogenous and dedicated R&D. To do that, we have characterized the set of equilibria in the decentralized economy, and we have imposed some institutional constraints on the policy tool(s): i) the impossibility to implement the first-best carbon tax; ii) the impossibility to subsidize one or two R&D sectors. In each case, we have computed the second-best level of the remaining unconstrained tool(s). The second-best results have been compared with, on the upper side, the first-best trajectories and, on the lower side, the laisser-faire ones. Those comparisons have allowed to appreciate the effects of each policy tool on the trajectories of the main following variables: fossil fuel extraction and price, backstop use and price, atmospheric carbon concentration, instantaneous damage, final output. We have also illustrated the assessment of each tool in terms of social welfare gain with respect to the laisser-faire benchmark case.

The main results have highlighted the role of the research grants, in particular the backstop ones. The model shows that the best way to mitigate climate change is to implement a policy that combine both a carbon tax and a green research subsidy. However, the carbon tax penalizes the consumption and then, the welfare of earlier generations, whereas the research subsidy allows to spare them.

References

[1] Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69(4), 781-809.

[2] Aghion, P., Howitt, P., 1998. Endogenous growth theory. Cambridge, MA: MIT Press.

[3] Bosetti, V., Carraro, C., Graleotti, M., 2006. The dynamics of carbon and energy intensity in a model of endogenous technical change. The Energy Journal, Special issue, 191-206.

[4] Cremer, H., Gahvari, F., Ladoux, N., 2001. Second-best pollution taxes and the structure of preferences. Southern Economic Journal, 68(2), 258-280.

[5] Dietz, S., Hepburn, C., Hope, C., 2008. Discounting and climate change: a non-marginal policy choice. Mimeo, London School of Economics.

[6] Edenhofer, O., Bauer, N., Kriegler, E., 2005. The impact of technological change on climate protection and welfare: Insights from the model MIND. Ecological Economics 54, 277-292.

[7] Edenhofer, O., Lessman, K., Bauer, N., 2006. Mitigation strategies ans costs of climate protection: The effects of ETC in the hybrid model MIND. The Energy Journal, Special issue, 207-222.

[8] Gerlagh, R., Lise, W., 2005. Carbon taxes: A drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change. Ecological Economics 54, 241-260.

[9] Gerlagh, R., van der Zwaan, B.C.C., 2006. Options and instruments for a deep cut in CO_2 emissions: Carbon capture or renewables, taxes or subsidies? The Energy Journal 27, 25-48.

[10] Gerlagh, R., Kverndokk, S., Rosendahl, K.E. (2008). Linking environmental and innovation policy. Working paper.

[11] Goulder, L.H., Mathai, K. 2000. Optimal CO_2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39, 1-38.

[12] Grimaud, A., Lafforgue, G., Magné, B., 2007. Decentralized equilibrium analysis in a growth model with directed technical progress and climate change mitigation, mimeo LERNA, Toulouse School of Economics.

[13] Grimaud, A., Rougé, L., 2008. Environment, directed technical change and economic policy. Environmental and Resource Economics 4(1), 439-463.

[14] IEA, 2007. Key World Energy Statistics 2007. Available at: http://www.iea.org/Textbase/stats/index.asp.

[15] Jones, C.I., Williams, J.C., 1998. Measuring the social return to R&D. The Quarterly Journal of Economics 113(4), 119-1135.

[16] Jones, C.I, Williams, J.C., 2000. Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5(1), 65-85.

[17] Kahouli-Brahmi, S., 2008. Technological learning in energy-environment-economy modelling: A survey. Energy Policy 36, 138-162.

[18] Kriegler, E., Bruckner, T., 2004. Sensitivity analysis of emissions corridors for the 21^{st} century. Climatic change 66, 345-387.

[19] Nordhaus, W.D., 2006. After Kyoto: Alternative mechanisms to control global warming. American Economic Review 96(2), 31-4.

[20] Nordhaus, W.D., 2007. A Review of the Stern Review on the Economics of Global Warming. Journal of Economic Literature 45(3), 686-702.

[21] Nordhaus, W.D., 2008. A question of balance: Weighting the options on global warming policies. Yale University Press.

[22] Popp, D., 2006a. ENTICE-BR: The effects of backstop technology R&D on climate policy models. Energy Economics 28, 188-222.

[23] Popp, D., 2006b. Comparison of climate policies in the ENTICE-BR model. The Energy Journal, Special issue, 163-174.

[24] Popp, D., 2006c. R&D subsidies and climate policy: Is there a "free lunch"? Climatic Change 77, 311-341.

[25] Romer, P.M., 1990. Endogenous technical change. Journal of Political Economy 98(5), 71-102.

[26] Sandmo, A., 1975. Optimal taxation in the presence of externalities. Swedish journal of Economics, 77, 86-98.

[27] Stern, N., 2006. The economics of climate change. The Stern Review. Cambridge University Press.

Appendix

A1. Proof of Proposition 1

The first characterizing condition (29) is obtained by replacing η into (9) by its value $\eta_0 - \int_0^t \left[F_Z/F_{Q_F} \exp\left(-\int_0^s rdu\right)\right] ds$ and by noting that $p_F = p_E E_F - \tau$ from (6), where $p_E = D(T)Q_E$ from (3) and $\exp(-\int_0^t rds) = U'(C)\exp(-\rho t)$ from (24). Combining (3), (7) and (13) leads to condition (30). Next, using (2) and (24), we directly get condition (31). Finally, the differentiation of (18) with respect to time leads to:

$$\frac{\dot{V}_{H_i}}{V_{H_i}} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E\}.$$

Substituting this expression into (16) and using (14), (18) and (19), it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + (\sigma_i + \gamma_i)H^i_{R_i}\left(\bar{v}^i_{H_i} + \frac{H^i_{H_i}}{H^i_{R_i}}\right), \quad \forall i = \{B, E, S\}.$$

We obtain the two last characterizing equilibrium conditions (32) and (33) by replacing into this last equation $\bar{v}^B_{H_B}$ and $\bar{v}^E_{H_E}$ by their expressions.

A2. Proof of Proposition 2

Let H be the discounted value of the Hamiltonian of the optimal program:

$$\begin{aligned} H &= U(C)e^{-\rho t} + \lambda D(T)Q\left\{K, E\left[F(Q_F, Z), B(Q_B, H_B), H_E\right]\right\} \\ &\quad -\lambda\left(C + Q_F + Q_B + \delta K + \sum_i R_i\right) + \sum_i \nu_i H^i(R_i, H_i) \\ &\quad +\mu_G\left[\xi F(Q_F, Z) - \zeta G\right] + \mu_T[\Phi(G) - mT] + \eta F(Q_F, Z). \end{aligned}$$

The associated first order conditions are:

$$
\begin{aligned}
\frac{\partial H}{\partial C} &= U'(C)e^{-\rho t} - \lambda = 0 && (41)\\
\frac{\partial H}{\partial Q_F} &= \lambda[D(T)Q_E E_F F_{Q_F} - 1] + \xi\mu_G F_{Q_F} + \eta F_{Q_F} = 0 && (42)\\
\frac{\partial H}{\partial Q_B} &= \lambda[D(T)Q_E E_B B_{Q_B} - 1] = 0 && (43)\\
\frac{\partial H}{\partial R_i} &= -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E\} && (44)\\
\frac{\partial H}{\partial K} &= \lambda[D(T)Q_K - \delta] = -\dot{\lambda} && (45)\\
\frac{\partial H}{\partial H_B} &= \lambda D(T)Q_E E_B B_{H_B} + \nu_B H^B_{H_B} = -\dot{\nu}_B && (46)\\
\frac{\partial H}{\partial H_E} &= \lambda D(T)Q_E E_{H_E} + \nu_E H^E_{H_E} = -\dot{\nu}_E && (47)\\
\frac{\partial H}{\partial G} &= -\zeta\mu_G + \mu_T\Phi'(G) = -\dot{\mu}_G && (48)\\
\frac{\partial H}{\partial T} &= \lambda D'(T)Q - m\mu_T = -\dot{\mu}_T && (49)\\
\frac{\partial H}{\partial Z} &= \lambda D(T)Q_E E_F F_Z + \xi\mu_G F_Z + \eta F_Z = -\dot{\eta} && (50)
\end{aligned}
$$

The transversality conditions are:

$$
\begin{aligned}
\lim_{t\to\infty} \lambda K &= 0 && (51)\\
\lim_{t\to\infty} \nu_i H_i &= 0, \quad i = \{B, E\} && (52)\\
\lim_{t\to\infty} \mu_G G &= 0 && (53)\\
\lim_{t\to\infty} \mu_T T &= 0 && (54)\\
\lim_{t\to\infty} \eta Z &= 0 && (55)
\end{aligned}
$$

First, from (41), (42) and (50), we can write the following differential equation:

$$\dot{\eta} = -\frac{F_Z}{F_{Q_F}} U'(C)e^{-\rho t}.$$

Integrating this expression and using transversality condition (55), we obtain:

$$\eta = \int_t^\infty \frac{F_Z}{F_{Q_F}} U'(C)e^{-\rho s} ds. \tag{56}$$

From (41) and (49), we have:

$$\dot{\mu}_T = m\mu_T - D'(T)QU'(C)e^{-\rho t}.$$

Using (54), the solution of such a differential equation can be computed as:

$$\mu_T = \int_t^\infty D'(T)QU'(C)e^{-[m(s-t)+\rho s]} ds. \tag{57}$$

Equations (48) and (53) imply:

$$\mu_G = \int_t^{\infty} \mu_T \Phi'(G) e^{-\zeta(s-t)} ds. \tag{58}$$

Replacing into (42) λ, η, μ_T and μ_G by their expressions coming from (41), (56), (57) and (58), respectively, gives us the equation (34) of Proposition 1.

Second, equation (36) directly comes from condition (43). Next, log-differentiating (41) and (44) with respect to time yields:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho \tag{59}$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\nu}_i}{\nu_i} + \frac{\dot{H}^i_{R_i}}{H^i_{R_i}}. \tag{60}$$

Combining (59) and (45) yields condition (36). Condition (37) comes from (44), (46), (59) and (60), and from (43) by using $D(T)Q_E E_B = 1/B_{Q_B}$. Similarly, conditions (38) is obtained from the equations (44), (47), (59) and (60).

A3. Calibration of the model

Here we provide some informations on the basic calibration of key model parameters. The model begins in 2005, and it is solved in 10-years increments for 350 years. As described in Table 2, we use the calibrations of the DICE-07 model for the 2005 structural data, the output production function, the damage function and the utility form. A complete description of the the DICE-07 dynamic climatic system of equations and the associated parametrization is provided in Nordhaus (2008), chapters 2 and 3. Calibration of the energy and R&D sectors comes from the ENTICE-BR model and is detailed in Popp (2006a and 2006b).

According to IEA (2007), world carbon emissions in 2005, the reference year, amounted to 17.136 $MtCO_2$. We retain 7.401 GtCeq as the initial fossil fuel consumption, given in gigatons of carbon equivalent. In addition, carbon-free energy produced out of renewable energy represented 6.8% of total primary energy supply. We thus retain another 0.504 GtCeq as the initial amount of backstop energy use. The 2005 market prices of crude oil, coal and natural gas amounted to 55 USD per barrel, 65 USD per ton and 7 USD per MBtu, respectively. Converting these values, first in USD per gigajoules and next in USD per gigatons by applying the appropriated carbon content rate, and weighting them by the relative share of each fossil fuel in the total primary fossil energy consumption (i.e.

43.2% of oil, 31.25% of coal and 25.55% of gas according to IEA, 2007), we obtain 345 USD/GtCeq as fossil fuel price index.

Finally, the elasticity of substitution for the backstop in the energy production function is chosen so that it is consistent with the program of the energy sector ($p_F/p_B = E_F/E_B$). Initial values of TFP and productive investments into fossil fuel and backstop are calibrated to fit the 2005 data.

$$\begin{aligned}
\rho_B &= \frac{\log c_F + \log \alpha_B}{\log F_0 - \log B_0} + 1, \\
Q_{B,0} &= \frac{B_0}{\alpha_B H_{B,0}^{\eta_B}}, \\
Q_{F,0} &= c_F F_0, \\
A_0 &= D(T_0) Q_0 K_0^{-\gamma} E_0^{-\beta} L_0^{-(1-\gamma-\beta)}.
\end{aligned}$$

Source	Param.	Value	Description
Nordhaus (2008):	Q_0	61.1	2005 world gross output (trill. USD)
	L_0	6514	2005 world population millions
	K_0	137	Initial capital stock
	γ	0.3	Capital elasticity in output production
	β	0.07029	Energy elasticity in output production
	δ	0.1	Depreciation rate of capital per year
	α_T	0.0028388	Scaling param. of the damage function
	η_T	2	Parameter of the damage function
	ϵ	2	Elasticity of intertemporal substitution
	ρ	0.015	Time preference rate
	$\bar{Z}$	5500	Max extractible fossil fuels (GtC)
	$1/\alpha_B$	1035	2005 backstop price (USD/GtC)
	$g_{A,t}$		TFP growth trend
	$g_{L,t}$		World population growth trend
Popp (2006a):	ρ_E	0.38	Elasticity of subs. for energy
	α_H	0.336	Scaling param. of H_E on energy
	α_F	700	Scaling param. on fossil fuel cost
	η_F	4	Exponent in fossil fuel production
	η_B	1	Exponent in backstop production
	a_B	0.0122	Scaling param. in backstop innovation
	a_E	0.0264	Scaling param. in energy innovation
	b_B	0.3	Rate of return of backstop R&D
	b_E	0.2	Rate of return of energy R&D
	Φ_i	0.54	Elasticity of knowledge in innovation
	$H_{B,0}$	1	Initial value of backstop TC
	$H_{E,0}$	0.0001	Initial value of energy TC
	$R_{B,0}$	0.001	Initial level of backstop R&D (trill. USD)
	$R_{E,0}$	0.01	Initial level of energy R&D (trill. USD)
Computed from IEA (2007):	F_0	7.401	2005 fossil fuel use in GtC
	c_F	345	2005 fossil fuel price in USD/GtC
	B_0	0.504	2005 backstop use in GtC
Calibrated:	ρ_B		Elasticity of substitution for backstop in energy
	A_0		2005 level of TFP
	$Q_{B,0}$		2005 investment in backstop production
	$Q_{F,0}$		2005 investment in fossil fuel production

Table 2: Calibration of parameters