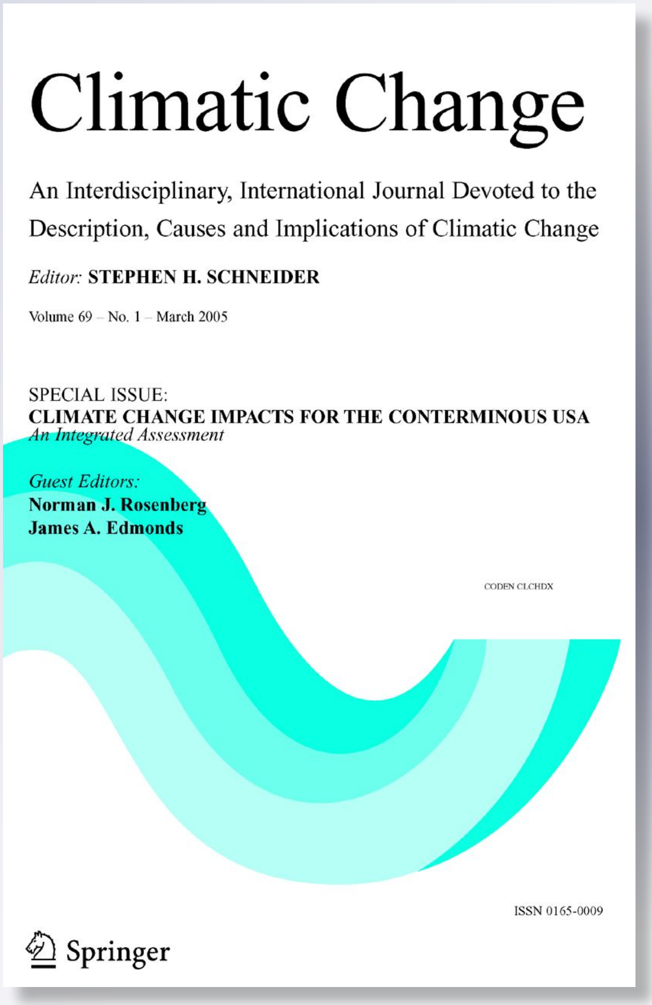
Environmental policy under model uncertainty: a robust optimal control approach

Michael Funke & Michael Paetz

Climatic Change
An Interdisciplinary,
International Journal Devoted
to the Description, Causes and
Implications of Climatic Change

ISSN 0165-0009
Volume 107
Combined 3-4

Climatic Change (2011)
107:225-239
DOI 10.1007/
s10584-010-9943-1



Springer

Climatic Change (2011) 107:225–239
DOI 10.1007/s10584-010-9943-1

Environmental policy under model uncertainty: a robust optimal control approach

Michael Funke · Michael Paetz

Received: 16 October 2008 / Accepted: 6 July 2010 / Published online: 21 December 2010

Abstract The design of optimal environmental policy inherits model uncertainty. We investigate the consequences in a simple linear model, where the aim of the policymaker is to stabilise the atmospheric content of carbon. We study how decision-makers' concerns about robustness alters policy using the Hansen and Sargent (2003, 2008) approach. The analysis shows that a policymaker, who fears about model misspecification should react more aggressively to changes in the stock of atmospheric carbon and reduce emissions stronger.

1 Introduction

Model uncertainty is an important issue in the context of environmental policy for at least two reasons: First, we do not know enough about the evolution of the ecological system (physical system uncertainty).[1] Second, we do not know much about future costs of environmental damage or future benefits from avoiding it

[1] The recently published fourth assessment report of the Intergovernmental Panel on Climate Change (IPCC) illustrates a curious aspect of the science of climate change. Studying the climate system reveals new, little understood, mechanisms and feedback effects that may increase or decrease warming. So as understanding grows, predictions become less, rather than more certain. Thus, the IPCC's range of predictions of the rise in the temperature by 2100 has increased from 1.4–5.8% in the 2001 report to 1.1.–6.4% in the latest report (see http://www.ipcc.ch/).

M. Funke · M. Paetz (✉)
Department of Economics, Hamburg University, Von-Melle-Park 5,
20146 Hamburg, Germany
e-mail: michael.paetz@wiso.uni-hamburg.de

M. Funke
e-mail: funke@econ.uni-hamburg.de

(economic uncertainty).[2] Therefore, the uncertainty inherent in environmental and economic modelling is receiving increasing attention and many policymakers are worried about the unknown unknows.

Several studies have analysed the impact of uncertainty on optimal timing problems [see for example Pindyck (2000) or Pindyck (2001)]. Baker (2005), Keller et al. (2004) and Webster (2002) have discussed the question of acting now or waiting in an integrated assessment model with active learning. Baker et al. (2006) have discussed that R&D into technologies with low emissions might be a hedge against uncertainty. Webster et al. (2003) calculate probability distribution functions for uncertain parameters in ecological models and provide probability distributions for future climate projections based on current uncertainty in model parameters.

In this paper we focus on one particular aspect of uncertainty, namely how optimal policy decisions depend upon uncertainty about the "true" model. In other words, we contribute to the uncertainty literature by studying how environmental policymakers' concerns about model robustness alter optimal environmental policy. In contrast to stochastic control, robust control methods seek to bound the uncertainty rather than express it in the form of a probability distribution. Given a bound on the uncertainty, the control can deliver results that meet the control system requirements in all potential cases. Therefore robust control theory might be stated as a worst-case analysis method rather than a typical case method. Thus, the procedure is particularly suitable to deal with low-probability extreme climate events.

To reflect the fear of the policymaker about misbehaving models, we use recently developed robust control techniques by Hansen and Sargent (2003, 2008, henceforth HS) which is based on the Gilboa and Schmeidler (1989) minmax approach. HS have initiated a research agenda that introduces the notion of robustness to model uncertainty and concern about model misspecification. Methodologically they modify techniques from the robust control literature in applied mathematics. In a nutshell, the fundamental idea of robustness is that economic and environmental models are at best viewed as stylised approximations of reality rather than perfect descriptions thereof. When policy-makers use a particular model as guidance in a dynamic decision-making situation and worry that the model be misspecified, one would expect them to insist on considering alternative models in order to obtain decision rules that not only work well with the baseline model but also work reasonably well when the model is misspecified. This is the sense in which a policy is designed to be robust. In this paper, the policymakers are assumed to achieve robustness by considering a worst-case model that is similar to and statistically difficult to distinguish from the baseline model. In other words, the policymaker considers a

[2]Most economists reckon that, if greenhouse-gas emissions continue on their current path, the costs of climate change would be between zero (where the benefits of warming to cold countries balances out the costs) and 3% of global GDP over the next 100 years. See, for example, Ingham and Ulph (2003) for a survey about uncertainty concerning the calculations of the social costs of carbon and Tol (2005) for a summary of 103 empirical studies on the marginal costs of carbon. Paraphrasing a quote from Alan Greenspan on monetary policy uncertainty one may say: "Uncertainty is not just an important feature of the environmental economics landscape, it is the defining characteristic of that landscape".

set of alternative models which are "close" to the baseline model where distance between the models is measured by an entropy or likelihood-type criterion.[3]

In the existing literature, this methodology has been extensively used for the design of monetary policy under uncertainty. It has overturned Brainard's (1967) conservatism principle and provides a rationale for monetary policy reacting more aggressively on changes in output and inflation under model uncertainty compared to an environment without model uncertainty.[4] The only applications of this technique on an environmental model so far are Roseta-Palma and Xepapadeas (2004) and Gonzalez (2008). Roseta-Palma and Xepapadeas (2004) discuss the precautionary behaviour under robust resource management, when surface water flows are assumed to be stochastic, and a policymaker is concerned about uncertainty. The authors show that in a dynamic setting, under worst case rainfall shortcomings, robustness implies lower surface water applications. Gonzalez (2008) analyses optimal pollution taxes and welfare when model uncertainty is modelled by means of robust control. In contrast to these two papers, the main topic of our paper is to analyse robust CO_2 abatement policies in an uncertain model context.

The paper proceeds as follows. As a foundation for the subsequent analysis, in Section 2 we briefly introduce the notion of robustness laid out by HS. The baseline model is introduced in Section 3. Building on the robustness concept, in Section 4 we proceed to an analysis of a policymakers who faces uncertainty about the model on which he bases optimal policy. Finally, Section 5 summarises and draws some conclusions.

2 Hansen–Sargent robustness

Doubts about models have existed for as long as models themselves have. This section gives an intuitive introduction to the recently developed concept of HS robustness, which deals with uncertainty by deriving optimal solutions in a restricted worst case model, where the restriction in turn depends on the underlying model.[5]

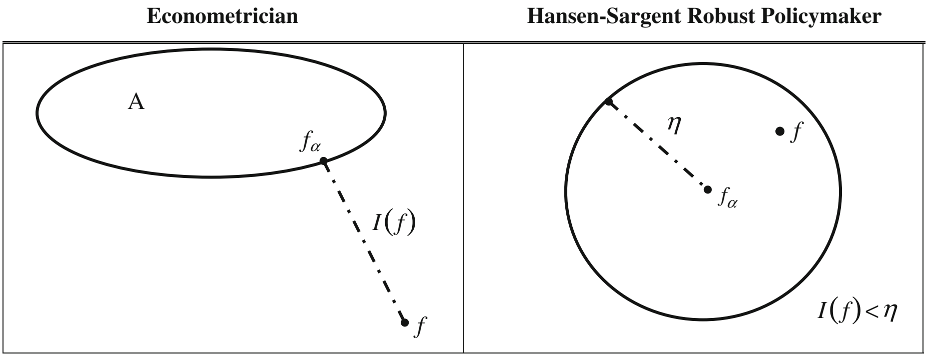
The core of the idea is to treat the decision maker's model as an approximation of the true model. Let x be a vector of state variables and let the true data follow a Markov-process with a transition density $f(x^*|x)$.[6] Moreover, let the approximating model be described by a transition density $f_\alpha(x^*|x)$ ($\alpha \in A$, where A denotes a

[3] Optimal policies under uncertainty can be approached in different ways besides the one used here focussing on robustness under model uncertainty. For a road map of alternative approaches, see Christodoulakis et al. (1993).

[4] Yohe (2007) covers the productive dialogue between the conduct of monetary policy and climate mitigation. In both subject areas decision makers are inclined to hedge against abrupt and intolerable impacts with which they must cope. In the realm of climate policy, for example, the possibility of a sudden collapse of the Atlantic thermohaline circulation comes in mind as an intolarable impact. In the realm of monetary policy, for example, central banks try to hedge against the risk of deflation or financial turmoil like in the recent crisis 2007–2009.

[5] A high volume of research in robust control over the past 15 years has led to a growth in techniques. The following methodological part draws upon the more complete discussions in HS. The descriptions of the technique focus both upon the overall concept and on the details of the mathematics.

[6] $*$ denotes next period values.



Note: Whereas an econometrician would minimise the distance between f and f_α, a HS robust policymaker takes f_α as given and derives the worst case solution in a set of possible data generating processes around f_α.

Fig. 1 An econometrician vs. a Hansen–Sargent robust policymaker [adopted from Hansen and Sargent (2008)]

compact set of parameter values). Then the maximum likelihood estimator $\hat{\alpha}_0$ would be derived by minimizing the relative entropy of f and f_α $[I(\alpha, f)]$, which measures the "expected distance" between f and f_α and is defined as the expectation of the log likelihood ratio conditioned on the approximating model. The HS methodology inverts this approach by taking f_α as given, and builds a set of possible data generating processes around this model, so that the true model is one model in this set. This is graphically shown in Fig. 1.

A standard result in optimal control theory is certainty equivalence, which results under the assumption of a linear model with additive uncertainty and a quadratic loss function. Centaintly equivalence implies that only the mean values, i.e. the probability-weighted average outcomes of target variables matter for the optimal setting. Certainty equivalence therefore implies that low probability disturbances should not be taken into account, only the first (statistical) moment matters for policy, not the higher moments. In order to come to grips with this problem, robust control theorists add an additional vector process $\{\omega_{t+1}\}$ to the model that depends in a possibly non-linear way on the history of the state variables:

$$x_{t+1} = Ax_t + Bu_t + C\left(\varepsilon_{t+1} + \omega_{t+1}\right), \tag{1}$$

where u denotes a vector of control variables, and A, B and C are matrices, filled with appropriate structural parameters.[7] For convenience, let's assume that the loss function of the decision maker is convex and given by $r(x,u) = |z|^2$. Then the robust problem can be reduced to the following Bellman equation:

$$V(x) = \min_{\{u_t\}} \max_{\{\omega_{t+1}\}} E_t\left\{z'z - \theta\delta w^{*\prime}w^* + \delta V(x^*)\right\} \tag{2}$$

subject to Eq. 1 where E is the expectations operator and $\theta > 0$ represents the decision maker's preference for robustness. The preference for robustness falls

[7]Note that the additional shock terms ω_{t+1} and therefore the misspecification of the model are masked by the shock terms ε_{t+1} and thus cannot be observed.

as θ rises, so that the problem is equal to its non-robust version when θ reaches infinity.

In Eq. 2 the usual minimisation problem is transformed into a min-max problem. The solution of Eq. 2 incorporates now the worst case ω as a function of x and u and the corresponding decision rule $u = -Fx$ depends on C. Intuitively, the policymaker wants to minimise the maximum welfare loss due to model misspecifications by specifying an appropriate environmental policy which shields the economy from the worst possible scenario.[8] The additional process can be interpreted as a second malevolent player, trying to distort the model as strong as possible.[9]

To restrict the second player, who would otherwise distort the model without bounds, the decision maker uses an intertemporal extension of relative entropy:

$$R(\omega) = 2E_0 \sum_{t=0}^{\infty} \delta^{t+1} I(\alpha, f) = E_0 \sum_{t=0}^{\infty} \delta^{t+1} \omega_{t+1}' \omega_{t+1} \leq \eta_0 \tag{3}$$

In Eq. 3 and in Fig. 1, the set of distorted models can be seen as a ball around the approximating model with η_0 defining the radius of the ball.[10] Intuitively, model uncertainty manifests itself in just one additional parameter although the framework covers a wide range of misspecified dynamics including wrong parameters (v_{t+1} is a linear function of x_t), autocorrelated errors (v_{t+1} is a linear function of x_t), and/or ignored nonlinearities (v_{t+1} is a nonlinear function of x_t).[11] It can be verified that the restriction upon the evil agent (the choice of η_0) depends on θ. Thus, all types of misspecification are handled by specifying only one parameter, θ.[12] A lower θ means that the policymaker designs a policy which is appropriate for a wider set of model misspecifications. Therefore, a lower θ is equivalent to a higher degree of robustness.[13]

The choice of the robustness parameter is therefore crucial for the choice of a plausible range of model uncertainty. To overcome the problem of specifying an arbitrary range for θ, we follow HS and employ what they refer to as a detection error probability approach. The basic idea is that the alternative models a policymaker faces should not be easily distinguishable (detectable) with a reasonable set of data. HS employ statistical theory to formulate a probability for discriminating between

[8] Arrow and Hurwicz (1972) have shown that choices based on extreme outcomes are rational when decision makers are confronted with Knightian uncertainty and therefore cannot assign probabilities.

[9] The fictitious second rational agent is a metaphor and can also be called “nature”. Nevertheless, one interpretation may be to consider country 1 that tries to reduce emissions but is afraid that country 2 will undo all its good work. Thereby country 2 would alter the properties of the model.

[10] By means of η_0 the decision maker can be modelled to be cautious rather than trying to avoid improbable catastrophic events.

[11] This is an advantage as it simplifies the analysis, but it also implies that it is not possible to study the impact of specific types of uncertainty. The standard modelling approach without model uncertainty corresponds to $\eta_0 = 0$.

[12] In fact, the size of ω_{t+1} is directly penalized through θ, which is equivalent to the Lagrange multiplier on (2) in a min-max problem $E_0(\sum r(x_t, u_t))$ subject to (1) and (3).

[13] One limitation of the HS approach is that policy makers are not allowed to learn about model uncertainty over time.

the approximating model and the distorted model and to obtain a model-specific θ. With equal prior weights the Bayesian detection error probability is defined as $p(\theta) = 1/2\ (p_a + p_d)$, where p_i represents the frequency of simulations with a log likelihood ratio smaller or equal to zero, when the approximating $(i = a)$ or the distorted $(i = d)$ model is assumed to be the data generating process. HS suggest to set $p(\theta)$ at a plausible value and then invert $p(\theta)$ to find a plausible value for the robustness parameter. They advise the use of value for the detection error probability around 10% in a sample of size 150.

The HS approach presented above will facilitate the analysis of environmental policy under model uncertainty in subsequent sections.

3 The baseline model

The reduced-form baseline model is in the spirit of Pindyck (2001). However, we do not study the optimal timing of adopting an irreversible policy. Instead, we focus on the question, whether a stabilisation policy should be more aggressive, when a policymaker is concerned about uncertainty. We assume that the authority is able to control the path of emissions, but do not specify possible policies, such as taxes on emissions or the adoption of technologies or implementation of sinks, respectively. Moreover, we assume that reducing emissions is costly.

For convenience our analysis exemplarily focuses on the concentration of carbon. However, this approach can be used for every other stock of environmental pollutants, which fulfil the following assumptions. Let CA_t be a state variable, representing the average concentration of carbon in the atmosphere, and CO_t be the control variable, representing the rate of CO_2 emissions.[14] Then the evolution of CA_t can be described by

$$CA_t = (1 - \kappa)\, CA_{t-1} + \beta CO_{t-1} + e_{CA,t} \tag{4}$$

where κ is the rate at which CO_2-emissions rise the average concentration of carbon in the atmosphere, β represents the natural rate at which the stock of carbon concentration dissipates, and $e_{CA,t}$ represents a Gaussian identically and independently distributed (i.i.d.) shock process with zero mean and a variance of one. This shock captures all exogenous disturbances, which influence the evolution of the carbon concentration, and are not involved in Eq. 4. If the policymaker aims at stabilising the atmospheric concentration of carbon, Eq. 4 leads to the equilibrium condition $(CA_t = CA_{t-1} = CA,\ CO_t = CO_{t-1} = CO)$

$$CA = \frac{\beta}{\kappa} CO. \tag{5}$$

By subtracting the equilibrium relationship, Eq. 4 can be expressed in absolute differences from equilibrium:

$$ca_t = (1 - \kappa)\, ca_{t-1} + \beta co_{t-1} + e_{ca,t}, \tag{6}$$

where lower case letters represent absolute deviations from equilibrium, for example $co_t = CO_t - CO$. We assume the following convex loss-function, representing the

[14] We assume that without policy intervention, CO_t follows an exogenous trajectory.

preference for a stabilisation of carbon concentration around a particular equilibrium value

$$L_t = \gamma\,(ca_t)^2 \tag{7}$$

where γ reflects the preference for stabilisation and influences the speed of convergence to equilibrium.[15] Following Baker (2005, p. 24), we assume that the costs C_t of reducing emissions by a particular policy are a quadratic function of the reduction ΔCO_t.[16] Formally,

$$C_t = \lambda\,(CO_t - CO_{t-1})^2 = \lambda\,(co_t - co_{t-1})^2 \tag{8}$$

where λ reflects the quadratic costs of mitigation. The goal of the social planner is to stabilize the carbon concentration on a predetermined equilibrium value under minimal costs of mitigation policies

$$\begin{aligned}
&\min_{\{co_t\}} E_0 \sum_{t=0}^{\infty} \delta^t\,(L_t - C_t)\\
&\Leftrightarrow \max_{\{co_t\}} -\left\{E_0 \sum_{t=0}^{\infty} \delta^t\,(L_t - C_t)\right\}\\
&\Leftrightarrow \max_{\{co_t\}} E_0 \sum_{t=0}^{\infty} \delta^t\left(\lambda\,(co_t - co_{t-1})^2 - \gamma\,(ca_t)^2\right)
\end{aligned} \tag{9}$$

subject to (6), where δ represents the discount factor. The discount factor comprises the principle that decision makers care more about the present generation than about subsequent generations. However, the policymaker is unsure about the accuracy of the model. The decision maker knows that the model represents only an approximation to the true relationship. Consequently, the model could be subject to a range of distortions. Therefore, the task is to reformulate the optimisation problem such that the resulting policy rule performs reasonably well even if the model deviates from the baseline model.

4 Optimal robust policy

Keeping the preceding analysis in mind, let us now examine the resulting optimal robust policy. To solve for the optimal solution we build the Lagrangian of the problem

$$\max_{\{co_t\}} E_t \sum_{i=0}^{\infty} \delta^i\left[\lambda\,(co_{t+i} - co_{t+i-1})^2 - \gamma\,(ca_{t+i})^2 + \psi_{t+i}\,((1-\kappa)\,ca_{t+i-1} + \beta co_{t+i-1} - ca_{t+i})\right], \tag{10}$$

[15] The loss function (7) is a shortcut suppressing the fact that policymakers in democratic societies are under pressure from various interest groups and therefore have to enact environmental regulation through a lengthy political bargaining process.

[16] Pindyck (2001) has assumed a linear cost function for simplicity, but mentioned that one would generally expect costs to be convex.

where $\{\psi_{t+i}, i \geq 0\}$ is the sequence of Lagrangian multipliers. For simplicity we solve the problem as if the social planner is able to choose optimal values for $\{co_{t+i}, i \geq 0\}$ and $\{ca_t, i \geq 0\}$. In a second step we then solve for the optimal path of the control variable $\{co_t\}$. The first order conditions from (10) are given by

$$\{co_{t+i}\}: \quad 2\lambda\delta^i E_t(\Delta co_{t+i}) - 2\lambda\delta^{i+1} E_t(\Delta co_{t+1+i}) + \beta\delta^{i+1} E_t(\psi_{t+1+i}) = 0 \tag{11}$$

$$\{ca_{t+i}\}: \quad 2\gamma\delta^i E_t(ca_{t+i}) = (1-\kappa)\,\delta^{i+1} E_t(\psi_{t+1+i}) - \delta^i E_t(\psi_{t+i}) \tag{12}$$

Transforming (11) leads to

$$E_t(\psi_{t+1+i}) = 2\left(\frac{\lambda}{\beta}\right)\left[E_t(\Delta co_{t+1+i}) - \delta^{-1} E_t(\Delta co_{t+i})\right], \tag{13}$$

which can be used in (12) to derive the social optimal path of emissions:

$$\Delta co_t = -\left(\frac{\gamma\beta}{\lambda\kappa}\right) ca_t + \frac{1}{\kappa}\left[(1-\kappa)(\Delta co_{t+1}) + \frac{1}{\delta}(\Delta co_{t-1})\right], \tag{14}$$

where we used $E_t(co_{t+i}) = co_{t+i}$ $(\forall t,i)$, since we assume that the policymaker controls emissions. The optimal amount of additional emissions at time t depends negatively on the deviation of the concentration of carbon from equilibrium, but positively on Δco_{t+1} and Δco_{t-1}, where future values are discounted and lagged values are projected. The first term reflects, that a higher deviation in the concentration of carbon causes higher social costs and tends to reduce the optimal amount of emissions, when concentrations are above equilibrium. The second term reflects, that a higher reduction of emissions causes costs. Therefore this term smoothes the path of emissions. With rising β, the influence of emissions on the average carbon concentration increases, and emissions should be lowered. The same holds for γ, since γ reflects the preference for carbon stabilisation and punishes deviations of the average carbon concentration from equilibrium. For λ the opposite is true: when the costs of reducing a particular amount of emissions rises, the optimal amount increases. With a rising rate at which the stock of carbon concentration dissipates, emissions become less damaging and also increase $(\partial\Delta co_t/\partial\kappa > 0)$. Equations (6)–(9) can be transformed to a standard stochastic optimal control approach

$$V = \max_{\{u_t\}} E_0\left[\sum_{t=0}^{\infty} \delta^t \left(x_t' Q x_t + 2x_t' U u_t + u_t' R u_t\right)\right] \tag{15}$$

subject to $x_{t+1} = Ax_t + Bu_t + \varepsilon_{t+1}$ where

$$x_t = \begin{bmatrix} co_{t-1} \\ ca_t \end{bmatrix}, u_t = co_t, \varepsilon_{t+1} = \begin{bmatrix} \varepsilon_{co,t+1} \\ \varepsilon_{ca,t+1} \end{bmatrix}, Q = \begin{bmatrix} \lambda & 0 \\ 0 & -\lambda \end{bmatrix}, U = \begin{bmatrix} -\lambda \\ 0 \end{bmatrix}, R = \lambda,$$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & (1-\kappa) \end{bmatrix}, B = \begin{bmatrix} 1 \\ \beta \end{bmatrix}$$

The sequence $\{e_{co,t+1}\}$ is a second i.i.d. shock process with zero mean and a variance of one, and reflects that emissions can not be perfectly controlled by the policymaker. To reflect a concern for model misspecification by using HS robust modeling techniques, the resulting max-min-problem can be transformed into a standard RE-program

$$V = \max_{\{u_t\}} \min_{\{\omega_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \left(x_t' \tilde{Q} x_t + 2 x_t' \tilde{U} \tilde{u}_t + \tilde{u}_t' \tilde{R} \tilde{u}_t \right) \right] \tag{16}$$

subject to $x_{t+1} = A x_t + \tilde{B} \tilde{u}_t + \varepsilon_{t+1}$ where

$$\tilde{U} = \left[U \; 0 \right], \quad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & -\theta I_2 \end{bmatrix}, \quad \tilde{B} = \left[B \; I_2 \right], \quad F = \begin{pmatrix} F_u \\ F_\omega \end{pmatrix} \text{ and } \tilde{u}_t = \begin{pmatrix} u_t \\ \omega_t \end{pmatrix}.$$

Equation (16) completely characterises the optimal policy strategy taking explicitly into account environmental model uncertainty.

Since we believe that plausible values for the stabilization preference should be related to marginal costs of CO_2-emissions, we use empirical studies on marginal costs to derive plausible values for γ. The marginal damage cost is defined as the net present value of the incremental damage due to a marginal increase in CO_2 emissions. Unfortunately, for almost all parameters there is wide disagreement between experts. Therefore it is crucial to subject all results to sensitivity testing. Our marginal cost estimates are derived from Tol (2005), who summarizes 103 empirical studies and builds one composite probability density function for all studies. As we believe, that estimates should withstand a quality test, we rely only on peer-reviewed studies and use the mode (5 \$/ton of carbon), the mean (50 \$/tC), the median (14 \$/tC), the 5% percentile (−9 \$/tC) and the 95% percentile (245 \$/tC) from Tol's (2005) density function for those studies.

For the costs of mitigation we rely on the estimates of Van Vuuren et al. (2006) who summarize results from 18 different model approaches, and run simulations for the highest and the lowest values, as well as for the mean of all studies. The rate of dissipation (κ) is assumed to be 0.01, and for the rate at which emissions rise the average concentration of carbon (β) we use a value of 0.99. The discount factor is set to $\delta = 0.96$ which corresponds to a continuous discount rate of 4% ($\delta = 0.96 = \mathrm{e}^{-0.04}$).[17]

Formally, policy rules resulting from the optimisation problem (16) solve for the optimal strategy and are of the form $co_t = a co_{t-1} + b ca_t$.

In Table 1 results for (a, b) are given for different values of λ and γ for the robust and the non-robust case. Robust solutions are computed for a robustness parameter θ that corresponds to a detection error probability near 10% in a sample of 150, using Monte Carlo simulations. Several points deserve emphasis. All parameter combinations clearly show that losses rise under uncertainty, and that a robust policymaker should react more aggressively on deviations of carbon from equilibrium. Furthermore, all reaction parameters on last periods emissions decrease

[17] We acknowledge that the choice of discount rate is still a controversial issue. Other discount rates will affect the shape of the impulse response functions, but the qualitative findings of the paper will remain unswayed.

Table 1 Optimal policy rule parameters (a, b)

λ	5600 \$/Gt^2		202600 \$/Gt^2		224000 \$/Gt^2	
γ	$\theta=\infty$	$p(\theta)\approx 10\%$	$\theta=\infty$	$p(\theta)\approx 10\%$	$\theta=\infty$	$p(\theta)\approx 10\%$
−0.09 \$/Gt^2	(0.5027, 0.0001)	(0.4007, 0.0002)	(0.5024, 0)	(0.4007, 0)	(0.5024, 0)	(0.4008, 0)
Loss	559970	636320	20244000	23020000	22380000	25448000
0.05 \$/Gt^2	(0.5025, −0.0001)	(0.4009, −0.0001)	(0.5024, 0)	(0.4007, 0)	(0.5024, 0)	(0.4008, 0)
Loss	560000	636110	20244000	23020000	22380000	25448000
0.14 \$/Gt^2	(0.5023, −0.0002)	(0.4008, −0.0003)	(0.5024, 0)	(0.4007, 0)	(0.5024, 0)	(0.4008, 0)
Loss	560010	636090	20244000	23020000	22380000	25448000
0.5 \$/Gt^2	(0.5018, −0.0007)	(0.4010, −0.0009)	(0.5024, 0)	(0.4007, 0)	(0.5024, 0)	(0.4008, 0)
Loss	560070	635740	20244000	23020000	22380000	25448000
2.45 \$/Gt^2	(0.4996, −0.0029)	(0.4, −0.0039)	(0.5023, −0.0001)	(0.4007, −0.0001)	(0.5023, −0.0001)	(0.4008, −0.0001)
Loss	560340	635320	20244000	23019000	22381000	25448000

(increase in absolute value), which can also be interpreted as a more aggressive stabilization policy.[18] Thus, the policymaker adopts a more prudent, or precautionary standpoint.[19]

Notice also that with rising mitigation costs, optimal policies become more defensive. When a reduction in emissions is more expensive, the policymaker reacts with more patience and the optimal path to equilibrium is prolonged. Thus it is no surprise that also losses rise when mitigation costs do. In opposition, a rise in the stabilization preference parameter leads to more aggressive policies, in order to reduce the atmospheric carbon content. For higher mitigation costs the results seem to be very robust, since optimal policies and losses are nearly identical for all preference parameters. Furthermore, optimal reaction functions for $\lambda = 202600$ \$/Gt^2 and $\lambda = 224000$ \$/Gt^2 do not differ substantially.

Estimations of Van Vuuren et al. (2006) are based on the assumption of full global participation by all countries as early as 2000, which seems to be not the case. Thus, the calibration of the mitigation costs might be too optimistic.[20] We reflect this by reporting results for a 10% higher value of mitigation costs in Table 2. The outcomes seem to be quite robust, since losses and reaction parameters do not change considerably, and conclusions are qualitatively the same. Obviously, increasing mitigation costs lead to stronger reactions and higher losses.

To illustrate the differences due to uncertainty, we simulate the paths for ca_t and co_t for three different stabilisation scenarios: In the first scenario the policymaker

[18] The only exception is the first row in Fig. 2, where reaction parameters on ca_t are positive, due to the negative preference parameter.

[19] This result is similar to the findings of Roughgarden and Schneider (1999) obtained in a different setting.

[20] Latest findings about on the costs of stabilization targets can be found in the most recent EMF22 study (see http://emf.stanford.edu/events/emfbriefing_on_climate_policy_scenarios_us_domestic_and_international_policy_architectures).

Table 2 Optimal policy rule parameters (a, b) for 10% higher mitigation costs

λ	6160 $/Gt^2		222860 $/Gt^2		246400 $/Gt^2	
γ	$\theta = \infty$	$p(\theta) \approx 10\%$	$\theta = \infty$	$p(\theta) \approx 10\%$	$\theta = \infty$	$p(\theta) \approx 10\%$
−0.09 $/Gt^2	(0.5026, 0.0001)	(0.4008, 0.0002)	(0.5024, 0)	(0.4008, 0)	(0.5023, 0)	(0.4010, 0)
Loss	615990	699910	22266000	25320000	24616000	27988000
0.05 $/Gt^2	(0.5025, −0.0001)	(0.4010, −0.0001)	(0.5024, 0)	(0.4008, 0)	(0.5023, 0)	(0.4010, 0)
Loss	615980	699650	22266000	25320000	24616000	27988000
0.14 $/Gt^2	(0.5023, −0.0002)	(0.4009, −0.0003)	(0.5024, 0)	(0.4008, 0)	(0.5023, 0)	(0.4010, 0)
Loss	615980	699610	22266000	25320000	24616000	27988000
0.5 $/Gt^2	(0.5019, −0.0006)	(0.4009, −0.0009)	(0.5024, 0)	(0.4008, 0)	(0.5023, 0)	(0.4010, 0)
Loss	615970	699300	22266000	25320000	24616000	27988000
2.45 $/Gt^2	(0.4997, −0.0027)	(0.4, −0.0037)	(0.5023, −0.0001)	(0.4007, −0.0001)	(0.5023, −0.0001)	(0.4009, −0.0001)
Loss	615920	698440	22266000	25319000	24616000	27987000

tries to stabilize the carbon concentration on today's value, which is assumed to be 760 Gt of carbon equivalent.[21] Due to (5) the corresponding equilibrium amount of CO_2 emissions is 7.68 Gt carbon equivalent. For the second scenario we assume a policymaker, who wants to reduce the carbon concentration to 600 Gt, which implies equilibrium emissions of 6.06 Gt. Scenario 3 refers to stabilizing the atmospheric concentration at 500 Gt, which leads to 5.05 Gt equilibrium emissions. As optimal policies do not differ much for high values of λ, and medium values for γ, simulations are done for $\lambda = \{0.14, 2.45\}$ and $\gamma = \{5600, 202600\}$. For all simulations we used start values for CA of 760 Gt and for CO of 8 Gt.

The resulting graphs are shown in Fig. 2 and confirm the suggestions from Table 1. We run simulations for the standard optimal policy without a concern for robustness (black line), for the robust solution under uncertainty (red line), and for the robust rule in the approximating model without evil agent (green line).

Several insights emerge from this exercise. The impulse response functions suggest, that emissions should be stabilized within the next 5–7 years. However, the stabilization of atmospheric carbon content would nevertheless need round about 400 years. Concerning the results under uncertainty, the Figures show, that emissions reductions are prolonged by 2–3 years, when we introduce the evil agent, although the policy is more aggressive.

Whereas the robust emissions reductions differ substantially from the non-robust results and the robust policy in the approximating model, the evolution of the atmospheric carbon content seems to be nearly the same for all simulations and scenarios, except for scenario I. When the aim of the policymaker is to stabilise the atmospheric carbon content on today's level, the concentration rises first by a small amount, before it returns slowly to its equilibrium value. The robust and non-robust dynamics look very similar, but using the robust rule in the approximating environment—the case of unfounded fear of model misspecification—reduces the

[21] This is in line with the latest estimation of the IPCC of 370 ppmv, see Metz et al. (2005).

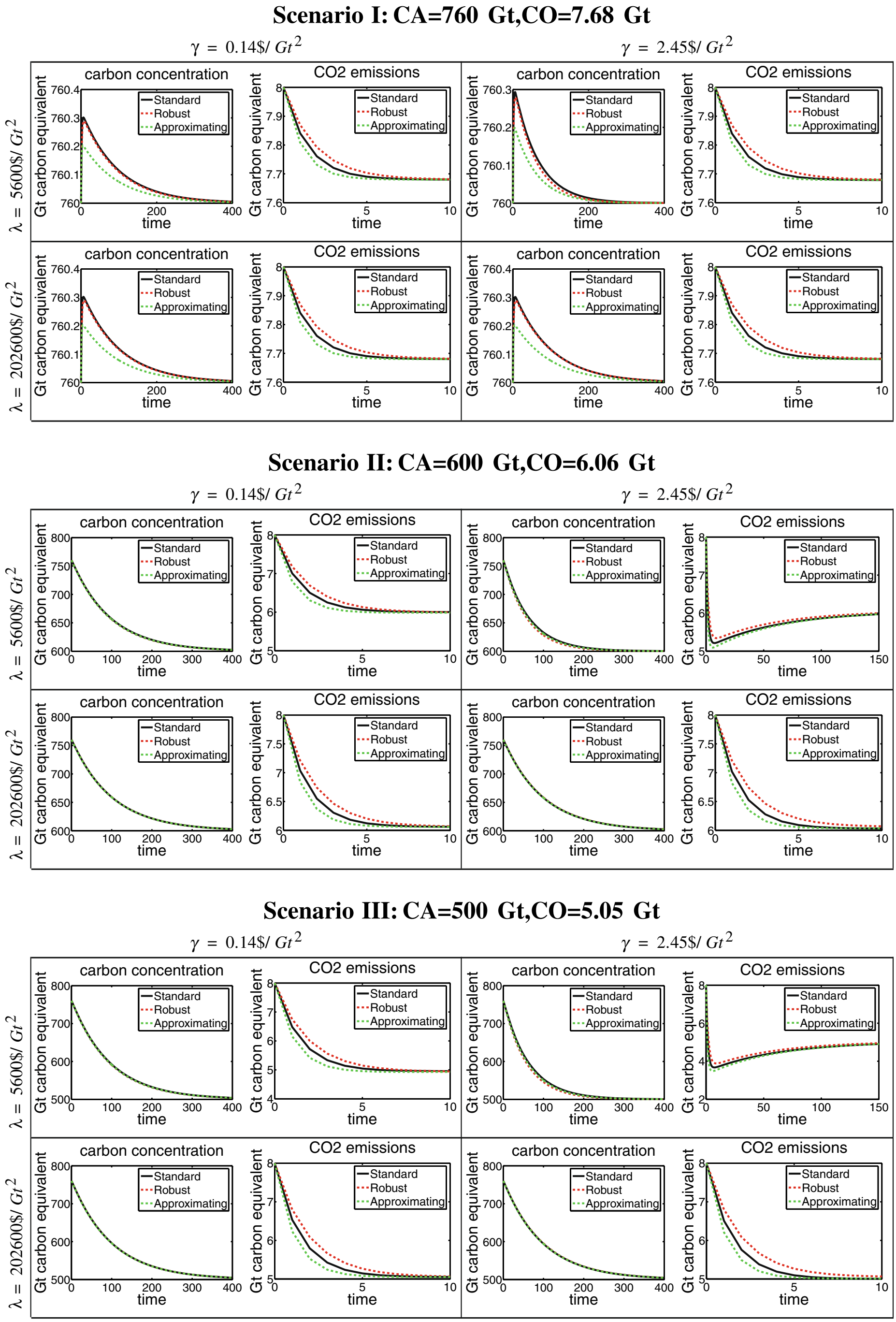


Fig. 2 Model simulations

carbon content much faster then in both other cases. Comparing the two columns of the tables suggest that the policymaker allows the atmospheric content of carbon to stay above equilibrium for a longer time horizon when the mitigation costs rise. This can be seen best for a high stabilisation preference. The graphs show the plausible result, that for a higher preference on stabilisation, emissions should be reduced stronger, as thus the atmospheric carbon content decreases faster. In addition, they illustrate that the design of a robust strategy for a policymaker with a high stabilization preference, and for low mitigation costs is to push policy further in the direction of reducing emissions even more vigorously.

All simulations illustrate, that the introduction of a second malevolent player implies a higher emissions trajectory, although the reduction policy is more aggressive. This is compatible with the rise in losses due to uncertainty, shown in Table 1. Using the robust solution in the approximating model illustrates the increased aggressiveness in stabilisation policies, since ca_t as well as co_t reach their equilibrium faster. A robust environmental policymaker fears stronger damages from not reducing emissions, and thus chooses a more aggressive reduction policy. The other way round, model uncertainty doesn't justify conservatism.

5 Conclusions

Accounting for the vague concept of model uncertainty is a challenging task for decision makers. The approach in this paper is to provide for model uncertainty via robust control. We develop a linear quadratic approach to study optimal emissions paths subject to stabilisation preferences and mitigation costs for three different scenarios: (1) a stabilisation on today's atmospheric carbon content (760 Gt carbon equivalent), (2) a reduction to 600 Gt carbon equivalent, (3) and a reduction to 500 Gt carbon equivalent. For a plausible model specification the results suggest, that emissions should be stabilized within the next 5–7 years, independent from the underlying stabilisation scenario. However, even for a stabilisation of emissions within the next 7 years, the atmospheric carbon content will need about 400 years to reach its steady state. Furthermore, we investigate optimal reduction policies under uncertainty, using the appealing HS robust control technique.

What can policymakers with a preference for robustness of optimal policy with respect to misspecification of the underlying model learn from this research? What kind of response is appropriate to the climate threat? The optimal policy trajectories lead to the conclusion that a policymaker who fears model misspecification, should react more aggressively. This qualitative results resembles those of monetary policy under model uncertainty.[22]

Although the example of CO_2 emissions is used to demonstrate the robust modelling approach, the method is transferable to other environmental problems

[22] Giannoni (2002), Kilponen (2004), Leitemo and Söderström (2004), Onatski and Stock (2002) and Söderström (2002) have examined whether model uncertainty justifies cautious central bankers. The overall conclusion is that robustness against model misspecification makes monetary policy respond more aggressively to shocks. Rodriguez (2004, pp. 216–217) has analysed the optimal environmental policy response to changes in current and future model uncertainty. Like in our analysis, the effect of current model uncertainty goes in the direction of a more aggressive response.

surrounded by model uncertainties. We hope that further applications of the robust modelling technique will soon follow, making use of increasing processor speeds which makes robustness analysis feasible for larger climate models requiring more computational time. By doing this, the gap between robust control theory and its application may be closing.

Acknowledgements We thank the anonymous referees and the editor for helpful comments and suggestions on an earlier draft. Insightful comments from Richard Tol on an earlier draft also helped to shape the paper. The usual disclaimer applies. Financial support through the Cluster of Excellence "Integrated Climate System Analysis and Prediction" (CliSAP), University of Hamburg, funded by the German Science Foundation (DFG) is gratefully acknowledged.

References

Arrow KJ, Hurwicz L (1972) An optimality criterion for decision-making under ignorance. In: Carter CF, Ford JL (eds) Uncertainty and expectations in economics: essays in honour of G.L.S. Shackle. Blackwell, Oxford, pp 1–11

Baker E (2005) Uncertainty and learning in a strategic environment: global climate change. Resour Energy Econ 27:19–40

Baker E, Clarke L, Weyant JP (2006) Optimal technology R&D in the face of climate uncertainty. Clim Change 78:157–179

Brainard W (1967) Uncertainty and the effectiveness of policy. Am Econ Rev 57:411–425

Christodoulakis N, Kemball-Cook D, Levine P (1993) The design of economic policy under model uncertainty. Comput Econ 6:219–240

Giannoni MP (2002) Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model. Macroecon Dyn 6:111–144

Gilboa I, Schmeidler D (1989) Maxmin expected utility with non-unique prior. J Math Econ 18: 141–153

Gonzalez F (2008) Precautionary principle and robustness for a stock pollutant with multiplicative risk. Environ Resour Econ 41:25–46

Hansen LP, Sargent TJ (2003) Robust control of forward looking models. J Monet Econ 50:581–604

Hansen LP, Sargent TJ (2008) Robustness. Princeton University Press, Princeton

Ingham A, Ulph A (2003) Uncertainty, irreversibility, precaution and the social cost of carbon. Tyndall Centre Working Paper 37

IPCC, Intergovernmental Panel on Climate Change (2001) Climate change 2001—Third Assessment Report (TAR). Cambridge University Press, Cambridge

Keller K, Bolker BM, Bradford DF (2004) Uncertain climate thresholds and optimal economic growth. J Environ Econ Manage 48:723–741

Kilponen J (2004) Robust expectations and uncertain models—a robust control application to the New Keynesian Economy. Bank of Finland Discussion Papers 5/2004, Helsinki

Leitemo K, Söderström U (2004) Robust monetary policy in the New-Keynesian framework. CEPR Discussion Paper No. 4805, London

Metz B, Davidson O, de Coninck H, Loos M, Meyer L (2005) Carbon dioxide capture and storage, Intergovernemental Panel on Climate Change Special Report

Onatski A, Stock JH (2002) Robust monetary policy under model uncertainty in a small model of the U.S. economy. Macroecon Dyn 6:85–110

Pindyck RS (2000) Irreversibilities and the timing of environmental policy. Resour Energy Econ 22:233–259

Pindyck RS (2001) Optimal timing problems in environmental economics. J Econ Dyn Control 26:1677–1697

Rodriguez A (2004) Robust control: a note on the timing of model uncertainty. Comput Econ 24:209–224

Roseta-Palma C, Xepapadeas A (2004) Robust control in water management. J Risk Uncertain 29:21–34

Roughgarden T, Schneider SH (1999) Climate change policy: quantifying uncertainties for damages and optimal carbon taxes. Energy Policy 27:415–429

Söderström U (2002) Monetary policy with uncertain parameters. Scand J Econ 104:125–145

Tol RSJ (2005) The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. Energy Policy 33:2064–2074

Van Vuuren DP, Weyant J, de la Chesnaye FC (2006) Multi-gas scenarios to stabilize radiative forcing. Energy Econ 28:102–130

Webster M (2002) The curious role of 'learning' in climate policy: should we wait for more data? Energy J 23:97–119

Webster M, Forest C, Reilly JM, Babiker M, Kicklighter DW, Mayer M, Prinn R, Sarofim M, Sokolov A, Stone P, Wang C (2003) Uncertainty analysis of climate change and policy response. Clim Change 61:295–320

Yohe G (2007) Lessons for mitigation from the foundations of monetary policy in the United States. In: Schlesinger M, Kheshgi H, Smith J, de la Chesnaye F, Reilly JM, Wilson T, Kolstad C (eds) Human-induced climate change: an interdisciplinary assessment. Cambridge University Press, Cambridge, pp 294–302