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Quantum Games and Programmable Quantum Systems

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Abstract

Attention to the very physical aspects of information characterizes the current research in quantum computation, quantum cryptography and quantum communication. In most of the cases quantum description of the system provides advantages over the classical approach. Game theory, the study of decision making in conflict situation has already been extended to the quantum domain. We would like to review the latest development in quantum game theory that is relevant to information processing. We will begin by illustrating the general idea of a quantum game and methods of gaining an advantage over "classical opponent". Then we review the most important game theoretical aspects of quantum information processing. On grounds of the discussed material, we reason about possible future development of quantum game theory and its impact on information processing and the emerging information society. The idea of quantum artificial intelligence is explained.

Keywords and phrases: quantum games, quantum strategies, quantum information theory, quantum computations, artificial intelligence

1   Introduction

Various sorts of computations permeate everyday life. As the leading paradigm of computation is shifting from centralized and static to distributed, both in time and space, or even mobile game theoretical methods are becoming more end more important. Though classical computing is an extraordinary success story we have arrived to the verge of questioning classical computational paradigms. Since the publication of Gödel theorems [1] and Turing and Church models of computation [2] the opinion that human mind dominates any conceivable computer prevails. But in the light of quantum information processing [3] and scepticism concerning the role of quantum phenomena in brain processes [4] we might be doomed to dreary future of coherent states of quantum matter dominating human mind. A new fascinating field of research has been started. Computational processes often take the form agents predicting and analyzing their interactions and lead into the domain od game theory. Quantum game theory [5]-[8] emerged as a valuable tool in this field because a substantial part of front problems can be formulated in game theoretical terms. In this paper we would like to convince the reader that the research on quantum game theory cannot be neglected because present technological development suggest that sooner or later someone would take full advantage of quantum theory and may use quantum strategies to beat us at some realistic game. At present, it is difficult to find out if human consciousness explores quantum phenomena although it seems to be at least as mysterious as the quantum world. Humans have been applying quantum technologies more or less successfully since its discovery. Does it mean that our intelligence is being transformed into quantum artificial intelligence (cf. quantum anthropic principle as formulated in [9])? Humans have already overcome several natural limitations with help of artificial tools. Is quantum information processing waiting for its turn? To exploit emerging novel nonclassical computational paradigms we must seek for them such rigor as is

possible. What that science will look like is currently unclear, and it is difficult to predict which results would turn out to be fruitful and which would have only marginal effect. The results of the research may found applications in quantum information and cryptography, social sciences, biology and economics.

2   Programmable quantum systems

The ultimate goal of quantum technology is the ability of building quantum systems with desired (controlled) properties. We will call them *programmable quantum systems* (PQS). These include systems that can perform various computational tasks. Therefore considerable effort has been devoted to investigating how to efficiently control the dynamics of quantum systems and obtain and process information on the quantum level [10]. We believe the best solution is that of building up complex behaviors out of simple operations[1]. To this end we need specialized systems that interact with a given quantum system to observe and control it. Lloyd, Landahl and Slotine have described a simple quantum devise – a universal quantum interface that is able to perform such tasks simply and effectively [12]. The universal quantum interface Q consists of a single two-state quantum system (qubit) that couples to a system S whose dynamics is governed by a Hamiltonian H to be controlled or observed. The control is implemented via a Hamiltonian interaction of the form $A \otimes \sigma_z$, where A is an Hermitian operator on S and σ_z is the z Pauli matrix. They assume that both measurements in the eigenvector basis of σ_z on Q and application of the Hamiltonian $\gamma\sigma$, where γ is an arbitrary real parameter, to Q can be efficiently performed. The ability to measure with respect to the eigenvector basis combined with the ability to perform arbitrary rotations by turning on and off various $\gamma\sigma$ implies the ability to measure with respect to arbitrary basis. It follows that it is possible to apply any evolution operator of the form $exp(-G \otimes \sigma_x t)$, where G is an arbitrary Hermitian operator on S and the x Pauli matrix acts on Q. If the interface is initially prepared in the eigenstate $|+1\rangle$ of σ_z then the evolution together with measurement of Q in the eigenvector basis $(|-1\rangle, |+1\rangle)$ effects the generalized "yes-no" measurement on S because

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[1]Such a division is not unique and there are models for quantum computing that have no natural into parts at all [11].

the system evolves from $\rho_S(0)$ into either $\rho^+ = cos(\gamma Gt)\rho_S(0)cos(\gamma Gt)$ or $\rho_S^- = sin(\gamma Gt)\rho_S(0)sin(\gamma Gt)$ with probabilities $P_+ = tr\ cos^2(\gamma Gt)\rho_S(0)$ and $P_- = tr\ sin^2(\gamma Gt)\rho_S(0)$, respectively. The efficiency and faithfulness of this procedures can be analyzed analogously to the Solovay-Kitaev theorem [3]. In general, a multiple quantum interfaces would be necessary for efficient connection and controlling even a small number of quantum systems. The universal quantum interface can control a given quantum system, observe it and communicate between various systems. It can be also used to protect the system from disturbances [2]. Together with quantum error correcting systems it can be used to engineer quantum subsystems in any desired fashion [12]. Unfortunately, this does not imply that such a system would not be extremely complex. Therefore to implement a general purpose information processing machine we need a possibly minimal set of universal quantum information processing systems that can be connected and controlled by quantum interfaces to perform a given task. Such universal systems do exists – they are called universal gates or universal primitives. But, as we would like to focus on game-theoretical aspects, before proceeding to universal gates we give a short introduction to quantum games.

3   Quantization of games

Classical games usually cannot be quantized in a unique way because they are only asymptotical "shadows" of a wide spectra of quantum models. There are two obvious modifications of classical simulation games.

1 – prequantization: Redefine the game so that it becomes a reversal operation on qubits representing player's strategies. This already allows for quantum coherence of strategies[3].

2 – quantization: Reduce the number of qubits and allow arbitrary uni-

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[2]Physically realizable quantum programable systems operate in a regime of extreme sensitivity to decoherence and disturbances [13]. Therefore some extra tools would be necessary to increase reliability and stability of quantum circuits. Quantized Parrondo's paradox [14] and quantum Zeno effect might turn out useful.

[3]This may result from nonclassical initial strategies or classically forbidden measurements of the state of the game (end of the game).

tary[4] transformation so that the basic features of the classical game are preserved. At this stage ancillary qubits can be introduced so that possibly all quantum subtleties can be explored (e.g. entanglement, measurements and the involved reductions of states, nonlocal quantum gates etc.).

Basically, any quantum system that can be manipulated by at least one party and where the utility of the moves can be reasonably defined, quantified and ordered may be conceived as a quantum game. The quantum system may be referred to as a *quantum board* although the term *universum of the game* seems to be more appropriate. We will suppose that all players know the state of the game at the beginning and at some crucial stages that may depend an the game being played. This is a subtle point because it is not always possible to identify the state of a quantum system let alone the technical problems with actual identification of the state (one can easily give examples of systems that are only partially accessible to some players). A "realistic" quantum game should include measuring apparatuses or information channels that provide information on the state of the game at crucial stages and specify the way of its termination. We will neglect these nontrivial issues here. Therefore we can suppose that a *two–player quantum game* $\Gamma = (\mathcal{H}, \rho, S_A, S_B, P_A, P_B)$ is completely specified by the underlying Hilbert space $\mathcal{H}$ of the physical system, the initial state $\rho \in \mathcal{S}(\mathcal{H})$, where $\mathcal{S}(\mathcal{H})$ is the associated state space, the sets S_A and S_B of permissible quantum operations of the two players, and the pay–off (utility) functions P_A and P_B, which specify the pay–off for each player. A *quantum strategy* $s_A \in S_A$, $s_B \in S_B$ is a collection of admissible quantum operations, that is the mappings of the space of states onto itself. One usually supposes that they are completely positive trace–preserving maps. The quantum game's definition may also include certain additional rules, such as the order of the implementation of the respective quantum strategies or restriction on the admissible communication channels, methods of stopping the game etc. We also exclude the alteration of the pay–off during the game. The generalization for the N players case is obvious. The real challenge is to describe quantum games with unlimited and changing number of players. The players should be able to change their strategies

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[4]At least one of the performed (allowed) operations should not be equivalent to a classical one. Otherwise we would get a game equivalent to some variant of the prequantized classical game.

during the course of the game (tactics)[5]. A possible approach is as follows. If a game allows a great number of players in it is useful to consider it as a two–players game: the k-th trader against the Rest of the World (RW). Any concrete algorithm $\mathcal{A}$ should allow for an effective strategy of RW (for a sufficiently large number of players the single player strategy should not influence the form of the RW strategy). Tactics and moves are performed by unitary transformations on vectors in the Hilbert space (states). This approach is suitable for description of quantum market games [15]. Let the real variable q

$$q := \ln c - E(\ln c)$$

denotes the logarithm of the price at which the k-th player can buy the asset $\mathfrak{G}$ shifted so that its expectation value in the state $|\psi\rangle_k$ vanishes. The expectation value of x is denoted by $E(x)$. The variable p

$$p := E(\ln c) - \ln c$$

describes the situation of a player who is supplying the asset $\mathfrak{G}$ according to his strategy $|\psi\rangle_k$. Supplying $\mathfrak{G}$ can be regarded as demanding \$ at the price c^{-1} in the $1\mathfrak{G}$ units and both definitions are equivalent. Note that we have defined q and p so that they do not depend on possible choices of the units for $\mathfrak{G}$ and \$. For simplicity we will use such units that $E(\ln c) = 0$. The strategies $|\psi\rangle_k$ belong to Hilbert spaces H_k. The state of the game $|\Psi\rangle_{in} := \sum_k |\psi\rangle_k$ is a vector in the direct sum of Hilbert spaces of all players. We define canonically conjugate Hermitian operators of demand $\mathcal{Q}_k$ and supply $\mathcal{P}_k$ for each Hilbert space H_k analogously to their physical position and momentum counterparts. This can be justified in the following way. Let e^{-p} be a definite price, where p is a proper value of the operator $\mathcal{P}_k$. Therefore, if one have already declared the will of selling exactly at the price e^{-p} (the strategy given by the proper state $|p\rangle_k$) then it is pointless to put forward any opposite offer for the same transaction. The capital flows resulting from an ensemble of simultaneous transactions correspond to the physical process of measurement. A transaction consists in a transition from the state of traders strategies $|\Psi\rangle_{in}$ to the one describing the capital flow

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[5]In the standard matrix formulation of the game all strategies are listed and defined at the beginning. We would like to describe more general situations, where the player can change his mind, and, for example, instead of buying sells some financial asset. To this aim tactics changing strategies are necessary.

state $|\Psi\rangle_{out} := \mathcal{T}_\sigma |\Psi\rangle_{in}$, where $\mathcal{T}_\sigma := \sum_{k_d} |q\rangle_{k_d k_d}\langle q| + \sum_{k_s} |p\rangle_{k_s k_s}\langle p|$ is the projective operator defined by the division σ of the set of traders $\{k\}$ into two separate subsets $\{k\} = \{k_d\} \cup \{k_s\}$, the ones buying at the price $\mathrm{e}^{q_{k_d}}$ and the ones selling at the price $\mathrm{e}^{-p_{k_s}}$ in the round of the transaction in question. The role of the algorithm $\mathcal{A}$ is to determine the division σ of the market, the set of price parameters $\{q_{k_d}, p_{k_s}\}$ and the values of capital flows. The later are settled by the distribution

$$\int_{-\infty}^{\ln c} \frac{|\langle q|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k} dq$$

which is interpreted as the probability that the trader $|\psi\rangle_k$ is willing to buy the asset $\mathfrak{G}$ at the transaction price c or lower. In an analogous way the distribution

$$\int_{-\infty}^{\ln \frac{1}{c}} \frac{|\langle p|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k} dp$$

gives the probability of selling $\mathfrak{G}$ by the trader $|\psi\rangle_k$ at the price c or greater. These probabilities are in fact conditional because they describe the situation after the division σ is completed. If one considers the RW strategy it make sense to declare its simultaneous demand and supply states because for one player RW is a buyer and for another it is a seller. To describe such situation it is convenient to use the Wigner formalism[6]. The pseudo–probability $W(p,q)dpdq$ on the phase space $\{(p,q)\}$ known as the Wigner function is given by

$$W(p,q) := h_E^{-1} \int_{-\infty}^{\infty} \mathrm{e}^{i\hbar_E^{-1}px} \frac{\langle q + \frac{x}{2}|\psi\rangle\langle\psi|q - \frac{x}{2}\rangle}{\langle\psi|\psi\rangle} \, dx.$$

In general, this measure is not positive definite. In the general case the pseudo–probability density of RW is a countable linear combination of Wigner functions, $\rho(p,q) = \sum_n w_n W_n(p,q)$, $w_n \geq 0$, $\sum_n w_n = 1$.

One of the most appealing features of quantum games is the possibility that strategies can influence each other and form collective strategies. Elsewhere [17], we have defined the alliance as the gate CNOT ($\mathcal{C}$) regardless of its standard name *controlled-NOT* because it can be used to form collective

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[6]Actually, this approach consists in allowing pseudo–probabilities into consideration. From the physical point of view this is questionable but for our aims its useful, cf. the discussion of the Giffen paradox [16].

strategies as follows. Most of two-qubit quantum gates are universal in the sense that any other gate can be composed of a universal one [18]-[20]. Therefore it is sufficient to describe a collective tactic of N players as a sequence of various operations $\mathcal{U}_{z,\alpha}$ belonging to $SU(2)$ performed on one-dimensional subspaces of players' strategies and, possibly, alliances $\mathcal{C}$ among them (any element of $SU(2^N)$ can be given such a form [21]). Alliances are, up to equivalence, the only ways of forming collective games. An alliance has the explicit form $CNOT := |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes NOT$, where the tactic NOT is represented in the qubit basis $(|0\rangle, |1\rangle)$ by the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \in SU(2)$. An alliance allows the player to determine the state of another player by entering into an alliance and measuring her resulting strategy. This process is shortly described as

$$\mathcal{C}\,|0'\rangle|m'\rangle = |m'\rangle|m'\rangle, \quad \mathcal{C}\,|m\rangle|0\rangle = |m\rangle|m\rangle,$$

where $m = 0, 1$. The corresponding diagrams are shown in Fig. 1. The left diagram presents measurement of the observable $\mathcal{X}'$ and the right one measurement of $\mathcal{X}$. Any measurement would demolish possible entanglement of
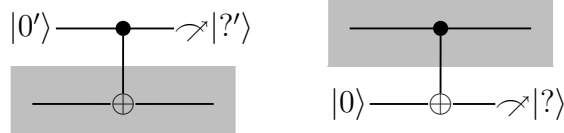

Figure 1: The alliance as a mean of determining others' strategies. The sign "↗" at the right ends of lines representing qubits symbolizes measurement.

strategies. Therefore entangled quantum strategies can exist only if the players in question are ignorant of the details of their strategies. To illustrate the problem we analyze three simple games involving alliances. They can be used as partial solutions in more complicated situations. To taste power of the formalism let us investigate the Newcomb's paradox [22]. Any circuit is more or less vulnerable to random errors. Consider the simple quantum circuit presented in Fig. 2. The gate I/NOT is defined as a randomly chosen gate from the set $\{I, NOT\}$) and is used to switching-off the circuit in a random way. It can be generalized to have some additional control qubits. In a game-theoretical context such circuits can be used to neutralization of disturbances caused by measuring strategies, c.f. [3]. For example, it can be
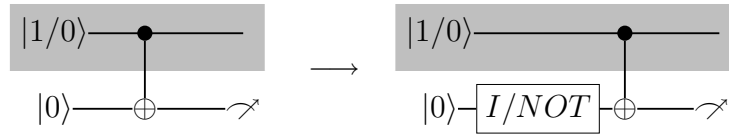
Figure 2: Neutralization of a quantum measuring system by a switch I/NOT applied $(I/NOT \rightarrow NOT)$, when $|1/0\rangle = |1\rangle$ (see the text).

applied to solve the famous Newcomb's free will paradox. The problem, originally formulated by William Newcomb in the 1960, was described by Martin Gardner in the following way [23]. An alien Omega being a representative of alien civilization (player 2) offers a human (player 1) a choice between two boxes. The player 1 can take the content of both boxes or only the content of the second one. The first one is transparent and contains $1000. Omega declares to have put into the second box that is opaque $1000000 (strategy $|1\rangle_2$) but only if Omega foresaw that the player 1 decided to take only the content of that box ($|1\rangle_1$). A male player 1 thinks: *If Omega knows what I am going to do then I have the choice between $1000 and $1000000. Therefore I take the $1000000* (strategy $|1\rangle_1$). A female player 1 thinks: *Its obvious that I want to take the only the content of the second box therefore Omega foresaw it and put the $1000000 into the box. So the one million dollar* **is** *in the second box. Why should I not take more – I take the content of both boxes* (strategy $|0\rangle_1$). The question is whose strategy, male's or female's, is better? In he measuring system presented in Fig. 2 the initial value $|0\rangle$ of the lower qubit corresponds to the male strategy and the values $|1\rangle$ and $|0\rangle$ of the upper qubit correspond to male and female tactics, respectively. The outcome $|0\rangle$ of a measurement performed on the lower qubit indicates the opening of both boxes with contents prepared by Omega before the alliance $CNOT$ was formed. If Omega installed in the circuit a breaker of the form I/NOT (before or after the alliance $CNOT$ he would use it when (and only then) the human adopted the female tactics. But this would mean that Omega is cheating (the breaker is installed after the alliance) or is able to foretell the future (the breaker is installed before the alliance). In the quantum setting the situation is different. The quantization of the problem is presented in Fig. 3. It consists in replacing of the circuit-breaker I/NOT by a pair of Hadamard gates $H := \frac{i}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \in SU(2)$. Due to their jamming effect on the human's tactics, we can call them a quantum Trojan horse (qutro-
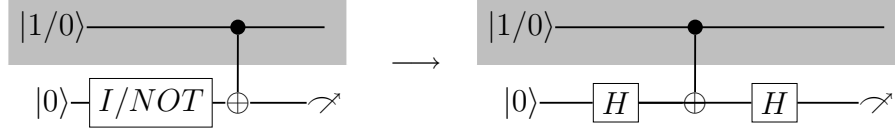
Figure 3: Solution to the Newcomb's paradox: quantum device that neutralizes measurement. In the quantization process the gate I/NOT is replaced by a qutrojan (see the text) that acts independently of the value of the qubit $|1/0\rangle$ and is composed of two Hadamard gates H.

jan)[7]. We can hardy use the term trojan with respect to the circuit-breaker I/NOT because of its paradoxical correlation with human tactics. Note that $H \cdot NOT \cdot H = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix}$, hence any attempt at measuring squared absolute values of coordinates of the human strategy qubit will not detect any effectiveness of the female tactics.

4   Universal primitives of quantum games

There are two main approaches to the universality problem of quantum computation. First approach consist in approximation of a unitary transformation $U = \exp(-iHt)$ in a way analogous to infinitesimal operators in Lie group theory [18, 26, 27]. In the alternative approach one tries to represent the matrix of a given unitary transformation as a product of one- and two-qubit gates from a possibly minimal set - set of *universal primitives*. A related approach follows the methods used in teleportation – the dominant role play measurements. Raussendorf and Briegel [28, 29] proposed the so called cluster-state model that forms a powerful tool in quantum complexity theory [30]. A more practical approach to the measurement-based model of quantum computation was proposed by Nielsen [31, 3]. This model of quantum computation has been further developed by Leung [32] and Leung and Aliferis [33] who exhibited a universal family of universal primitives composed of 4 two-qubit measurements (4 auxiliary qubits are necessary). The most important result in measurement-based models of computation was obtained by Perdix [34] who has introduced a model of measurement-based

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[7]Problems connected with the definition of trojan are discussed in [24].

quantum computation that makes use of what he calls the *state transfer*. In this model of quantum computation to simulate any n-qubit unitary transformation one auxiliary qubit is required – a universal family of observables could be formed by 3 one-qubit measurements and only one two-qubit measurement. Perdrix's [34] approach improves previous results by reducing both the number of auxiliary qubits and the number of two-qubit measurements required for quantum universality. From the theoretical point of view[8] the minimal amounts of necessary resources are reached in this approach. It follows that quantum games with a "fixed quantum board" that is implemented as a fixed quantum circuit have the same universal properties as the circuit used for their implementation. In an open dynamical setting, such as in quantum market games the situation is slightly more complicated but a concise dictionary of universal primitives can be given [38] if one follows the way paved by Perdix [34, 37]. In this case measurements also form sufficiently powerful and effective tools in manipulation of quantum tactics. A measurement of tactics consists in determination of the strategy or, more precise, finding out which of its fixed points we have to deal with. If the tactics being measured changes the corresponding strategy, then the non-demolition measurement reduces the strategy to one of its fixed points and the respective transition amplitudes are given by coordinates of the strategy in the fixed point basis. A dominant role of measurements in implementations of this type suggests that quantum games may be free from psychological factors, such as phobia, intention, irrationality and so forth. Following Perdrix [34, 38], we can see that measurements of the tactics X, G and $X \otimes X'$, where

$$X := \sigma_x \,, \ X' := \sigma_z = HXH \,, \ G := \tfrac{1}{\sqrt{2}}(X' + X'') \,.$$

suffice to implement quantum market games. Graphically, these measurements will be represented as [9].

$$-\boxed{X}- \ , \ -\boxed{G}- \ , \ \boxed{X \otimes X'} \ . \tag{1}$$

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[8]We neglect here the problem of optimal convergence. Other classes of universal primitives are usually introduced to analyze this nontrivial issue. For example, a set of quantum gates is said to be *computationally universal* if it can be used to simulate to within ε error any quantum circuit which uses n qubits and t gates from a (strictly) universal set with only polynomial overheads in $(n, t, \frac{1}{\varepsilon})$. A non-minimal set might be more effective in simulations [35, 36]

[9]We follow the *Qcircuit.tex* convention [39]. Thus rounded off shape is used to distinguish measuring gates.

Measurement of the tactics $X \otimes X'$ provides us with information whether the two strategies agree or disagree on the price but reveals no information on the level of the price in question. To get information about the prices we have to measure $X \otimes I$ and $I \otimes X'$, respectively. Note that the measurement of X' can be accomplished implicitly by measurement of X and subsequently $X \otimes X'$. Graphically this is represented as [34, 37]:

$$\begin{array}{c} \overbrace{\boxed{X}}\!\!-\!\!\boxed{X \otimes X'} \end{array} \implies -\boxed{X'}- \;,$$

where the parentheses are used to denote auxiliary qubits. The following sequence of measurements shows that a strategy encoded in one qubit can be transferred to another qubit (from the upper one to the lower one in the figure bellow) and changed with the tactics σH, where σ is one of the Pauli matrices (including the identity matrix):

$$\boxed{X \otimes X'}\!\!-\!\!\boxed{X'} \implies -\boxed{\sigma H}- \;. \tag{2}$$

Thus the strategy encoded in the upper state is transferred from the lower qubit and changed with the tactics σH. It is obvious that the same tactics is adopted if we switch the supply measurements with the demand ones ($X \leftrightarrow X'$). Simple calculation shows that the composite tactics $H\sigma H$ and $\sigma_i \sigma_k$ reduce to some Pauli (matrix) tactics. Therefore an even sequence of tactics (2) can be perceived as a Markov process over vertices of the graph
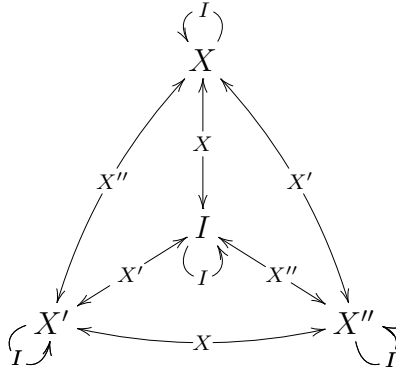
It follows that any Pauli tactics can be implemented as an even number of tactics-measurements (2) by identifying it with some final vertex of random

walk on this graph. Although the probability of drawing out the final vertex at the first step is $\frac{1}{4}$, the probability of staying in the "labirynth" decreases to zero exponentially with time. Having a method of implementation of Pauli tactics allows us to modify the tactics (2) so that to implement the tactics H — the fundamental operation of switching the supply representation with the demand representation. It can be also used to measure compliance with tactics representing the same side of the market (direct measurement is not possible because the agents cannot make the deal):

$$\boxed{X \otimes X} := \boxed{H}\ \boxed{X \otimes X'}\ \boxed{H}\ ,$$

$$\boxed{X' \otimes X'} := \boxed{H}\ \boxed{X \otimes X'}\ \boxed{H}\ .$$

In addition, this would allow for interpretation via measurement of random Pauli tactics σ because due to the involutiveness of H the gate (2) can be transformed to

$$
\boxed{H}\ \boxed{X \otimes X'}\ \boxed{X'}\!\!\rightarrow
\ =\
\boxed{X' \otimes X'}\ \boxed{X}\!\!\rightarrow
\ =\ \tag{3}
$$

$$
\boxed{X \otimes X}\ \boxed{X'}\!\!\rightarrow
\ \Longrightarrow\ \boxed{\sigma}\ .
$$

The gate (3) can be used to implement the phase-shift tactics:

$$T := \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+\mathrm{i}}{\sqrt{2}} \end{pmatrix} .$$

T commutes with X', hence:

$$
\boxed{\sigma T}\ \Longleftarrow\ \boxed{X' \otimes X'}\ \boxed{T^{-1}X\,T}\!\!\rightarrow\ .
$$

Elementary calculation demonstrates that $T^{-1}XT = \frac{X - X''}{\sqrt{2}}$ and $H\frac{X - X''}{\sqrt{2}}H = G$, therefore

$$
\boxed{H}\ \boxed{X \otimes X'}\ \boxed{G}\!\!\rightarrow\ \Longrightarrow\ \boxed{\sigma T}\ .
$$

We have seen earlier that it is possible to remove the superfluous Pauli operators, cf. (3). To end the proof of universality of the set of gates (1) we have to show how to implement the alliance *Cnot* (note that $\{H, T, Cnot\}$ a set of universal gates [40]). This gate can be implemented as the circuit [37] (as before, the gate is constructed up to a Pauli tactics):
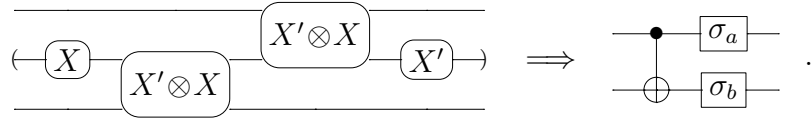
Actually, simple calculation [34] prove that the universality property has any set of primitive that contains the *controlled H* gate and measurements $X^k, X^p \otimes X^q, X^r \otimes X^s$, where $p \neq q$, $r \neq s$ i $p \neq r$. It follows, that to implement a quantum market[10] it suffices to have, beside possibility of measuring strategy-qubits and control of the supply-demand context, a direct method measuring entanglement of a pair of qubits in conjugated bases. The universal quantum interfaces can be used to connect and control an a priori arbitrary number of information processing units[11]. Some interesting technical details can be found in Ref. [41]. It is natural to wonder how small such a primitive processing unit could be. A single atom or a molecule are examples of a possible simple quantum computing units [42] but the feasibility of framing them into an all-purpose quantum computer is currently out of reach.

5 Quantum gambling

Sophisticated technologies that are not yet available are not necessary to implement quantum games [43]. Simulation of quantum games can be performed in an analogous way to precision physical measurements during which classical apparatuses are used to explore quantum phenomena. We envisage that quantum lotteries will soon emerge and will challenge the present day lottery market based randomized events or pseudo-randomness. In games of chance the player is betting in advance on the outcomes of several incompatible measurements. Quantum phenomena offer true random event and commercial random event generators should appear on the market at

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[10]Actually any finite-dimensional quantum system can be implemented in that way [37].

[11]The idea that a single quantum interface that can dynamically moved between system is attractive but is probably hard to put into effect.

moderate prices [44]. At the present stage of our technological development it already is feasible to open *quantum casinos*, where gambling at quantum games would be possible. Of course, such an enterprise would be costly but if you recall the amount of money spent on gambling, lotteries and advertising various products it seems to us that it is a worthy cause. Goldenberg, Vaidman and Wiesner described the following game based on the coin tossing protocol [45]. Alice has two boxes, A and B, which can store a particle. The quantum states of the particle in the boxes are denoted by $|a\rangle$ and $|b\rangle$, respectively. Alice prepares the particle in some state and sends box B to Bob. Bob wins in one of the two cases:

1. If he finds the particle in box B, then Alice pays him 1 monetary unit (after checking that box A is empty).

2. If he asks Alice to send him box A for verification and he finds that she initially prepared a state different from $|\psi_0\rangle = 1/\sqrt{2}\,(|a\rangle + |b\rangle)$, then Alice pays him R monetary units.

In any other case Alice wins, and Bob pays her 1 monetary unit. They have analyzed the security of the scheme, possible methods of cheating and calculated the average gain of each party as a result of her/his specific strategy. The analysis shows that the protocol allows two remote parties to play a gambling game, such that in a certain limit it becomes a fair game. No unconditionally secure classical method is known to accomplish this task. This game was implemented by Yong–Sheng Zhang et al, [46]. Other proposals based on properties of non–orthogonal states have been put forward by Hwang, Ahn, and Hwang [47] and Hwang and Matsumoto [48]. Witte has proposed a quantum version of the Heads or Tails game [49]. Piotrowski and Sładkowski have suggested that although sophisticated technologies to put a quantum market in motion are not yet available, simulation of quantum markets and auctions can be performed in an analogous way to precision physical measurements during which classical apparatuses are used to explore quantum phenomena. People seeking after excitement would certainly not miss the opportunity to perfect their skills at "using quantum strategies". To this end an automatic quantum game will be sufficient and such a device can be built up due to the recent advances in technology [17, 44]. Segre has published an interesting detailed analysis of quantum casinos and a Mathematica packages for simulating quantum gambling [50]. He has introduced

a quantum analogue of the Law of Excluded Gambling Strategies of Classical Decision Theory. The necessity of keeping into account entanglement requires to adopt the general algebraic language of Quantum Probability Theory. There is a deep link between the theory of winning quantum gambling strategies and the central notion of Quantum Algorithmic Information Theory – quantum algorithmic randomness. Quantum gambling besides its commercial significance is closely related to quantum logic, decision theory and can be used for defining a Bayesian theory of quantum probability [51] – interesting fields of research with various possible commercial applications.

6 Quantum combinatorial games and quantum automata

To our knowledge, algorithmic combinatorial games, except for cellular automata, have been completely ignored by quantum physicists. This is astonishing because at least some of the important intractable problems might be attacked and solved on a quantum computer. Consider some problem X. Let us define the game $kXcl$: you win if and only if you solve the problem (perform the task) X given access to only k bits of information. The quantum counterpart reads: solve the problem X on a quantum computer or other quantum device given access to only k bits of information. Let us call the game $kXcl$ or kXq interesting if the corresponding limited information–tasks are feasible. Let $OckhamXcl$ ($OckhamXq$) denotes the minimal k interesting game in the class $kXcl$ (kXq). There are a lot of intriguing questions that can be ask, for example for which X the meta–game $Ockham(OckhamXq)cl$ can be solved or when, if at all, the meta–problem $Ockham(OckhamXq)q$ is well defined problem. Quantum automata (quantum state machines) [52]-[54] play in quantum information theory role analogous to that of finite automata in Turing-Church model of computation. Quantum automaton can be defined as a quadruple $A = (S, s_0, \alpha, U)$, where S is the set of allowed internal states, $s_0 \in S$ the initial state vector α the input alphabet and U a unitary transition matrix for each symbol $a \in \alpha$ [55]. Primary interest in quantum automata stems from the research into the structure of quantum grammars and quantum languages but being simple quantum systems they are natural candidates for elementary units in programmable quantum systems. From the point of view of possible applications the theory of quantum lattice gas

automata [56, 57] deserves special attention. Vlasov [58] has considered a simple model of a quantum system whose Hilbert space H can be decomposed into two components $H = H_l \otimes H_S$ where H_l corresponds to spatial degrees of freedom of a hypercubic lattice and H_S to internal states. He calls it the quantum bot (qubot). The evolution of qubot is described by conditional quantum dynamics [59] and its excitation can be programmed. So far we have considered quantum systems that occupy a definite space, say a physical laboratory. Benioff [60] has considered quantum computers to be parts of larger systems where interactions between quantum computers and external systems form an essential part of the overall system dynamics – *quantum robots*. A quantum robot is a mobile system that carries a programmable quantum system, and all necessary ancillary systems (e.g. memory) on board. Quantum robots can carry out tasks whose goals include specified changes in the state of the environment or carrying out measurements on the environment. Each task is a sequence of alternating computation and action phases. Computation phase activities include determination of the action to be carried out in the next phase and possible recording of information on neighborhood environmental system states. Action phase activities include motion of the quantum robot and changes of neighborhood environment system states. At this stage quantum feedback control seems to be the most effective strategy: we obtain information about the evolving system through measurement, process the information and feed it back to the system to actively control the system in a desired way. Various methods of quantum feedback have been proposed [61]. In Benioff's model each task is represented by a unitary step operator T that describes single time steps of quantum robot's dynamics. $T = T_a + T_c$ is a sum of action phase (T_a) and computation phase (T_c) step operators. Schematic description the task in terms of decision trees is possible. No definite times or durations are associated with the phase steps in the tree. Detailed description of a robot that performs Grovers search algorithm is presented in Ref. [60]. He has conjectured that there is an equivalent Church-Turing hypothesis for the collection of all tasks that can be carried out by quantum robots. It follows that there may be a similar hypothesis for the class of feasible physical experiments.

7   Quantum programming

In classical computer science high level programming languages allow to master the more and more complex hardware. Currently no quantum computer is available and we have only vague idea what quantum programming should be like. Classical concepts like hardware abstraction, data classification, memory management, can hierarchical and structured programming should have quantum counterparts. The purpose of programming languages is both to express the semantics of the computation and generation sequences of elementary operations to be performed by a concrete computing unit. From this point of view the formalism of Hilbert spaces and their transformation as the mathematical description of quantum algorithms provides no means to derive their representation as sequences of elementary operations to control a given quantum hardware. Currently known quantum algorithms are described in terms of *quantum random access machine* (QRAM) model [62] that is an extension of a classical random access machine which is capable of both quantum and classical computations. In such machines the master classical machine uses the quantum subsystems as a black-box or oracle co-processing unit. The no-cloning theorem excludes replications of quantum systems. To handle this problem a new type of data, the *quantum register* has been introduced. Quantum register objects are collections of cubits addresses. They can overlap. Quantum operators encode definitions of quantum circuits and execute the circuits on supplied registers. Sanders and Zuliani [63] extended probabilistic versions of imperative languages to include quantum primitives. The resulting language ($qGCL$) is capable of programming universal quantum computer. Ömer has given an excellent analysis of quantum structured programming and developed a procedural formalism called QCL in his PhD thesis [64]. In both languages unitary transformations are functions and their manipulation is difficult. To solve this inconvenience, Bettelli, Calarco and Serafini [65] put forward an architecture that is capable of compact expression and reduction to sequences of elementary operation of quantum algorithms due to introduction of *quantum operator objects* that are easy to handle. Altenkirch and Grattage have taken more abstract path and introduced a functional language for quantum computation of finite types - the language QML [66]. The programs are interpreted by morphism in the category **FQC** of finite quantum computation[12]. Objects of these categories

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[12]The classical counterpart **FCC** is also introduced

are sets. Classical computations are carried out on elements of finite sets and quantum computations take place in finite dimensional Hilbert spaces. A reversible finite computation is modelled by a reversible operation, which is a bijection of finite set in the clasical case, and a unitary transformation on the Hilbert space in the quantum case. Guided by this they have described the semantics the language *QML* that extends a classical finitary language. They are currently working on implementation of a compiler for QML in Haskell. Game theory in the form of competitive analysis is now a well established tool in analysis of algorithms [67]. Quantum games being on the verge of various approaches seem to be essentially more dynamical than the traditional games in the "gaming situations" that arise from computational problems [68, 15, 69]. Semantic analysis of these games reformulated as Hilbert space problem and the categorical technics should set off differences and similarities between classical and quantum descriptions. The formalization must support strategies that are sensitive to any aspects of the situation, including not just the opponents' moves, but also the assumptions about their counter-strategies that can make the most of quantum phenomena such as interaction-free measurements or counterfactual computations. The hope is that the careful analysis will provide new defence protocols – sustainable strategies against possible attacks[13]. The quantum description would support systematic development and provide means for dealing with the future challenges and complexities of real systems [3].

8 Quantum artificial intelligence

Analogously to the terminology used in computer science, we can distinguish the shell (the measuring part) and the kernel (the part being measured) in a quantum game that is perceived as an algorithm implemented by a specific quantum process. Note that this distinction was introduced on the basis of abstract properties of the game (quantum algorithm, quantum software) and not properties of the specific physical implementation. Quantum hardware would certainly require a lot of additional measurements that are nor specific to the game (or software), cf. the process of starting a one-way quantum computer. Adherents of artificial intelligence (AI) should welcome the great number of new possibilities offered by quantum approach to AI (QAI). For

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[13]and vice versa, unfortunately.

example, consider a Quantum Game Model of Mind (QGMM) exploring the confrontation of quantum dichotomy between kernel and shell with the principal assumption of psychoanalysis of dichotomy between consciousness and unconsciousness [70]. The relation is as follows.

- Kernel represents the Ego, that is the conscious or more precisely, that level of the psyche the it is aware of its existence (it is measured by the Id). This level is measured due to its coupling to the Id via the actual or latent (not yet measured) carriers of consciousness (in our case qubits representing strategies)

- Shell represents the Id that is not self-conscious. Its task is monitoring (that is measuring) the kernel. Memes, the AI viruses [71], can be nesting in that part of the psyche.

Memes being qutrojans, that is quantum parasitic gates (not qubits!) can replicate themselves (qubits cannot – no-cloning theorem). Very little is known about the possible threat posed by qutrojans to the future of quantum networks. In quantum cryptography teleportation of qubits will be helpful in overcoming potential threats posed by qutrojans therefore we should only worry about attacks by conventional trojans [72]. If the qutrojan is able replicate itself, it certainly deserves the name quvirus. A consistent quantum mechanism of such replication is especially welcome if quantum computers and cryptography are to become a successful technology. In the QGMM approach external measuring apparatus and "bombs" reducing (projecting) quantum states of the game play the role of the nervous system providing the "organism" with contact with the environment that sets the rules of the game defined in terms of supplies and admissible methods of using of tactics and pay-offs [17]. Contrary to the quantum automaton put forward by Albert [52] in QGMM model there is no self-consciousness – only the Ego is conscious (partially) via alliances with the Id and is infallible only if the Id is not infected with memes. Alliances between the kernel and the Id (shell) form states of consciousness of QAI and can be neutralized (suppressed) in a way analogous to the quantum solution to the Newcomb's paradox [22]. In the context of unique properties of quantum algorithms and their potential applications the problem of deciding which model of AI (if any) faithfully describes human mind is fascinating but a secondary one. The discussed above variant of the Elitzur-Vaidman breaker suggests that the addition of the third

qubit to the kernel could be useful in modelling the process of forming the psyche by successive decoupling qubits from the direct measurement domain (and thus becoming independent of the shell functions). For example dreams and hypnosis could take place in shell domains that are temporary coupled to the kernel in this way. The example discussed in the previous section illustrates what QAI intuition resulting in a classically unconveyable belief might be like. What important is, QAI reveals more subtle properties than its classical counterparts because it can deal with counterfactual situations [73, 73] and in that sense analyze hypothetical situations (imagination). Therefore QAI is anti-Jourdainian: Molier's Jourdain speaks in prose without knowing it; QAI might be unable to speak but know it would have spoken in prose were it possible. The idea of *strong artificial life* of building (computational) models that are so life-like that they cease to be models of life and become examples of life themselves should also be invoked here [75, 76]. An agent based model consists of a collection of "primitive" computational entities[14], called agents. Such agents can in principle be implemented in the form of quantum programable systems. Quantum game theoretical aspects of such models have not yet been investigated. In their particular form known as *artificial chemistry* [77] genuinely novel phenomena may arise at the level of collective interactions of quantum agents [78]. It certainly would influence the discussion of classical paradigms of artificial intelligence [79].

9   Conclusion

Classical computing, though successful, is certainly not the full story. The opinion that it encompasses only a subset all computational possibilities has growing number of supporters. Humans have already overcome several natural limitations with help of artificial tools. Are we at the verge of dramatic developments that would change our computational paradigms? Quantum information processing with possible inspirations from physics and biology holds great promise. Intellectual investment over many years is turning craft into science. Examining how Nature solves its computational problems would probably result in revolutionary changes in computational paradigms [80]. Currently, we have to accept the following facts:

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[14]For example, a cellular automata

The particular choice of physical implementation of computing units matters and may have consequences

It may not be necessary to run the computer to get a result

The trajectory taken by a computational process can be more interesting than the final result

Quantum game theory also has its weak points but there is no doubt that it will be a crucial discipline for the emerging information society. Information processing is undergoing a revolutionary stage of development. If you ask about its future you get nearly as many different answers as the number of scientists being asked. We have presented our personal view that might not come true.

Acknowledgements

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