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Universality of Measurements on Quantum Markets

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Abstract

We reason about possible future development of quantum game theory and its impact on information processing and the emerging information society. Two of the authors have recently proposed a quantum description of financial market in terms of quantum game theory. These "new games" cannot by themselves create extraordinary profits or multiplication of goods, but they may cause the dynamism of transaction which would result in more effective markets and capital flow into hands of the most efficient traders. We focus upon the problem of universality of measurement in quantum market games. Quantum-like approach to market description proves to be an important theoretical tool for investigation of computability problems in economics or game theory even if never implemented in real markets.

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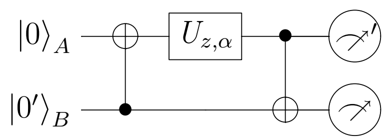
1 Introduction

One of the interpretations of quantum mechanics based on the Fokker Wheeler Feynman direct interaction approach [1, 2] refers to market transactions [3]. Therefore, quantum market games [4] have a unique bootstrap supporting the use of quantum formalism to describe them. On the other hand, quantum theory implies properties of players' strategies that assuredly form new standards of market liquidity. Quantum strategies can be identified in a non-destructive way (for example, with a test making use of the *controlled-swap* gate that is used in the quantum fingerprinting [5]), cannot be copied nor destroyed what is guaranteed by the *no-cloning* and *no-deleting theorems* [6]. In addition, they can be shared in a perfect, requiring no regulations way among players-shareholders (for example in such a way that any group of k shareholders can adopt the strategy and no smaller group of shareholders can make profit on this strategy [7]). Optimal management of such quantum strategies requires an appropriate portfolio theory [8]. This should not be regarded as a disadvantage as risk is associated even with classical arbitrage transactions [9] and there is a constant need for an appropriate theory to manage the risk associated with any activity. Currently, quantum theory is the only one that promises this degree of perfection therefore there is only a faint possibility that quantum game theory might be overvalued. Besides the number of arguments for the quantum anthropic principle as formulated in [10, 11]: even if at earlier stages of development markets are governed by classical laws, markets will evolve towards their quantum counterparts due to the effectiveness of quantum mechanisms is on the in crease. Such an evolution occurred throughout the last century, when new technologies gained superiority over classical "common sense". Contemporary markets undergoing a process of globalization would intensify such evolution. Markets exploring quantum phenomena regardless of such details as whether its "quantumness" would derive from instruments or human mind properties, would offer effectiveness impossible in classical markets and therefore would replace them sooner or later.

2 A two-qubits dealer's strategy

The bewildering phenomenon of quantum dense coding [12] enables us sending two classical bit of information by exchanging one qubit. This can be presented in the game theory setting as follows. Suppose we intend to send the information from A to B. Then the circuit [18]



$$\mathit{Cnot}\,(U_{z,\alpha}\otimes I)\,\mathit{Swap}\,\mathit{Cnot}\,\mathit{Swap}\,|0\rangle_A|0'\rangle_B \;=\; \cos(\alpha)\,|0'\rangle_A|0\rangle_B +$$
$$\mathrm{i}\,\sin(\alpha)\,\big(E_z(X)\,|0'\rangle_A|\mathrm{I}\rangle_B + E_z(X')\,|\mathrm{I}'\rangle_A|0\rangle_B + E_z(X'')\,|\mathrm{I}'\rangle_A|\mathrm{I}\rangle_B\big)$$

where

$$X := \sigma_x\,,\ X' := \sigma_z = HXH\,,\ X'' := \sigma_y = \mathrm{i}XX'$$

are the tactics[1] given by the Pauli matrices, describes such a process. The representation of the tactics $U_{z,\alpha}$ in terms of the final strategy is utterly secure because the owner of the pair of qubits A and B can keep the information distinguishing these two qubist (one classical bit) secret; without this information the interception of the pair of qubits A and B is insufficient for identification of the tactics $U_{z,\alpha}$. Such a method has previously been applied by Wiesner to construct quantum counterfeit-proof banknotes [14]. By adopting the tactics $U_{z,\alpha}$ that corresponds to one of four pairwise maximally distant pairs of antipodal points of the sphere S_3 [2] the owner of qubit

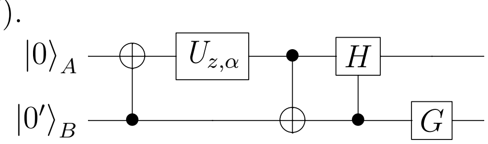
[1]We call any unitary transformation that changes agent's (player's) strategy a tactics. We follow the notation introduced in [13]: $SU(2) \ni \mathcal{U}_{z,\alpha} = \mathrm{e}^{\mathrm{i}\alpha\overrightarrow{\sigma}\cdot E_z(\overrightarrow{\sigma})} = I\cos\alpha + \mathrm{i}\,\overrightarrow{\sigma}\cdot E_z(\overrightarrow{\sigma})\sin\alpha$, where the vector $E_z(\overrightarrow{\sigma}) = \frac{\langle z|\overrightarrow{\sigma}|z\rangle}{\langle z|z\rangle}$ represents the expectation value of the vector of Pauli matrices $\overrightarrow{\sigma} := (\sigma_1, \sigma_2, \sigma_3)$ for a given strategy $|z\rangle$. The family $\{|z\rangle\}, z\in\overline{\mathbb{C}}$ of complex vectors (states) $|z\rangle := |0\rangle + z\,|\mathrm{I}\rangle$ ($|\pm\infty\rangle := |\mathrm{I}\rangle$) represents all trader's strategies in the linear subspace spanned by the vectors $|0\rangle$ and $|\mathrm{I}\rangle$.

[2]$U = a_0 I + \mathrm{i}\sum_k a_k\sigma_k$, where $a_0 = cos\alpha$, $a_k = n_k sin\alpha$ and $\sum_\mu (a_\mu)^2 = 1$. The corresponding tactics are $\pm I \pm \sigma_k$, where the antipodal points have different signs but represent equivalent tactics.

A is able to send two classical bits to the owner of qubit B while sending only one qubit. According to the analysis given in Ref. [15], the measurable qubits B and A can be interpreted as market polarizations of their owner (if $|0\rangle$ – supply and if $|1\rangle$ – demand) and therefore his/her inclination to buy at low or high prices what can easily be seen if we replace the meters with the controlled-Hadamard gates with control qubit B. In order to connect unequivocally any of the three conjugated bases [14] (or mutually unbiased [16]) with one of their three possible market functions (eigenvectors (fixed points) of X with supply inclination, eigenvectors of X' with demand inclination and eigenvectors of X'' with polarization) we should transform the strategy B (after the controlled-Hadamard gate!) with the involutive tactics G :

$$G := \tfrac{1}{\sqrt{2}}(X' + X'') \,.$$

that transforms eigenvectors of X' into eigenvectors of X''. The consideration of the third conjugated basis is necessary to guarantee the security of the information a la Wiesner' banknotes (the information about the respective price carried by qubit A uses two conjugated bases – sets of fixed points of tactics X and X').

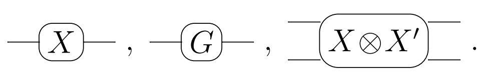


(1)

If there is no restriction on the tactics $U_{z,\alpha}$ (we can even consider the whole two-dimensional Hilbert space as the set of allowed strategies) the agent is able to play more effectively by adopting superpositions of previously allowed strategies. We have already considered alliances (implemented as gates between qubit strategies); they are universal. Therefore, if alliances are allowed tactics quantum market is equivalent to quantum computer! Obviously, quantum markets cam have various different properties – the polarization qubit is redundant in two-sided auctions but in bargaining games [15] another qubit is necessary to distinguish the agents who are bidding. Much more additional qubits are necessary if the corresponding supply and demand curves are continuous (floating point precision) – one qubit for each binary digit of the logarithm of price). However, these are theoretically unimportant details – all such forms of quantum markets can be implemented with the use of elementary market measurements alone what follows from the analysis by Nielsen, Raussendorf and Briegel, and Perdrix and Jorrand [17]-[21]. The rest of the paper is devoted to this problem. Note that such a dominant role of market measurements suggests that quantum market may be free from psychological factors, such as phobia, intention, irrationality and so forth. Besides, all classical models of markets are limiting cases of quantum models in an analogous way to the transition from quantum to classical mechanics. The variety of quantum games forming evolving towards better effectiveness information oriented quantum markets supplements the idea of quantum darwinism put forward by Żurek [22]. Therefore, the popular but never proved hypothesis of "humanism of markets" is only an illusion. Nevertheless, we should warn the reader that quantum dynamics may result in effects that even philosopher would not dare to dream of [23].

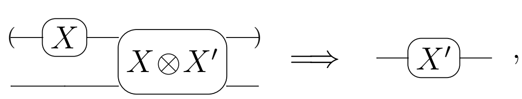
3 Measurements of tactics

A measurement of tactics consists in determination of the strategy or, more precisely, discoveries which of its fixed points we have to deal with. If the tactics being measured changes the corresponding strategy, then the non-demolition measurement reduces the strategy to one of its fixed points and the respective transition amplitudes are given by coordinates of the strategy in the fixed point basis (Born rule). As we will show, measurements of the tactics X, G and $X \otimes X'$ suffice to implement quantum market games. According to the *Qcircuit.tex* standard macros [24], we will denote the corresponding measuring gates as (rounded off shape is used to distinguish measuring gates):



(2)

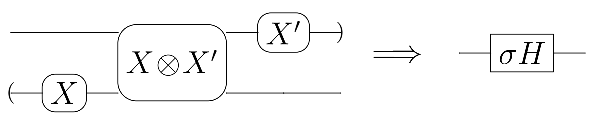
Note that measurement of the tactics $X \otimes X'$ provides us with information whether the two strategies agree or disagree on the price but reveals no information on the level of the price in question. To get information about the prices we have to measure $X \otimes I$ and $I \otimes X'$ respectively. Note that the measurement of X' can be implicitly accomplished by measurement of X and subsequently $X \otimes X'$. This is shown graphically by



where the parentheses are used to denote auxiliary qubits. In the following paragraphs we will analyze q-circuits with various number of auxiliary qubits that would allow for implementation of tactics via measurement only – the approach proposed by S. Perdix [20].

4 Universality of measurements: implementing tactics via measurements

Teleportation and measurement form surprisingly powerful tools in implementation of tactics. The method used by Perdrix and Jorrand [20, 21] to analyse the problem of universality in quantum computation can be easily adopted to the situation we are considering. Following Ref. [20], we begin by showing how a strategy encoded in one qubit can be transferred to another (from the upper one to the lower one in the figure below) and how it changes with a sequence of tactics σH, where σ is one of the Pauli matrices (including the identity matrix):



(3)

Assuming that the input qubit is in the state:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,,$$

after measuring $\mathbb{I}\otimes X$ (with classical outcome $j = \pm 1$) we obtain:

$$|\psi_1\rangle = |\psi\rangle \otimes X'^{\frac{1-j}{2}}\,\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\mathbb{I}\otimes X'^{\frac{1-j}{2}})(\alpha\,|00\rangle + \alpha\,|01\rangle + \beta\,|10\rangle + \beta\,|11\rangle).$$

Measurement of $X \otimes X'$ with outcome $k = \pm 1$ sets our qubits in state:

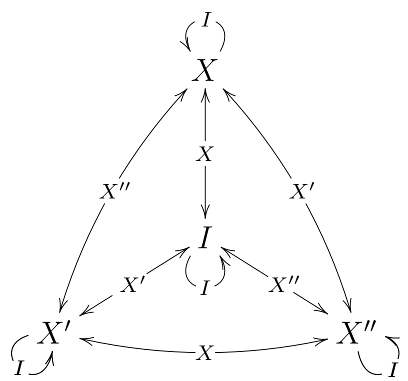
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(\mathbb{I}\otimes X^{\frac{1-k}{2}} X'^{\frac{1-j}{2}})\left[(\alpha+\beta)(|00\rangle + |10\rangle) + (\alpha-\beta)(|01\rangle - |11\rangle)\right].$$

The final measurement $X' \otimes \mathbb{I}$ with outcome $l = \pm 1$ gives us the final state:

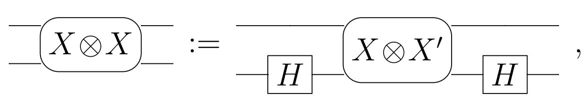
$$\begin{aligned} |\psi_3\rangle &= \left[X^{\frac{1-l}{2}} \otimes X^{\frac{1-k}{2}} X'^{\frac{1-j}{2}} H X^{\frac{1-l}{2}}\right]\left[|0\rangle \otimes (\alpha\,|0\rangle + \beta\,|1\rangle)\right] = \\ &= X^{\frac{1-l}{2}}\,|0\rangle \otimes X^{\frac{1-k}{2}} X'^{\frac{1-j\cdot l}{2}} H\,|\psi\rangle\,, \end{aligned}$$

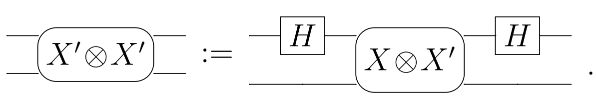
and the equivalence of the circuits above is proved.

Thus, the strategy encoded in the upper state is transferred from the lower qubit and changed with the tactics σH, where $\sigma = X^{\frac{1-k}{2}} X'^{\frac{1-j\cdot l}{2}}$. It is evident that the same tactics is adopted when we switch the supply measurements with the demand ones ($X \leftrightarrow X'$). Simple calculation shows that the composite tactics $H\sigma H$ and $\sigma_i\sigma_k$ reduce to some Pauli (matrix) tactics. Therefore, an even sequence of tactics (3) can be perceived as the Markov process over vertices of the graph

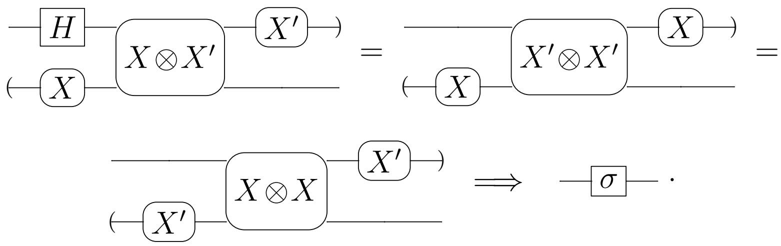


It follows that any Pauli tactics can be implemented as an even number of tactics-measurements (3) by identifying it with some final vertex of random walk on this graph. Although the probability of drawing out the final vertex at the first step is $\frac{1}{4}$, the probability of staying in the "labyrinth" exponentially decreases to zero. Having a method of implementation of Pauli tactics, allows us to modify the tactics (3) so that to implement the tactics H – the fundamental operation of switching the supply representation with the demand representation. It can be also applied to measure compliance with tactics representing the same side of the market (direct measurement is not possible because the agents cannot make the deal):





In addition, this would allow for interpretation via measurement of random Pauli tactics σ because of the involutiveness of H the gate (3) can be transformed to

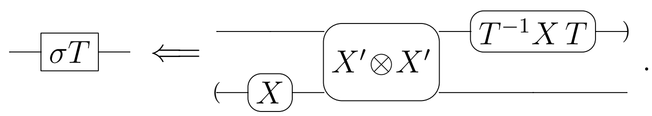


(4)

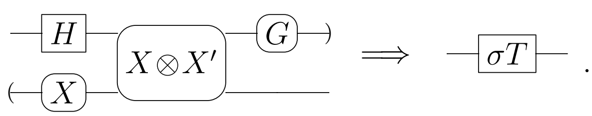
The gate (4) can be used to implement the phase-shift tactics:

$$T := \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+\mathrm{i}}{\sqrt{2}} \end{pmatrix} .$$

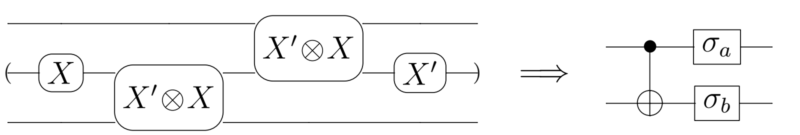
T commutes with X', hence:



Elementary calculation demonstrates that $T^{-1}XT = \frac{X-X''}{\sqrt{2}}$ and $H\frac{X-X''}{\sqrt{2}}H = G$, therefore:



We have seen earlier that it is possible to remove the superfluous Pauli operators, cf. (4). To end the proof of universality of the set of gates (2) we have to show how to implement the alliance *Cnot* (note that $\{H, T, Cnot\}$ a set of universal gates [25]). This gate can be implemented as the circuit (as before, the gate is constructed up to a Pauli tactics) [20]:



(5)

The explicit calculations are as follows[3]:

[3]The tactics H transforms the demand picture to the supply picture ($X \leftrightarrow X'$), what results in a switch from control qubit to the controlled qubit.

Let us assume the input qubits are the first and the second, in state:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

with the third used as an auxiliary one. States $|\pm\rangle$ are defined as follows:

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle).$$

The first measurement $\mathbb{I} \otimes \mathbb{I} \otimes X$ gives us the state below, depending on classical outcome $j = \pm 1$[4]:

$$|\psi_1\rangle = |\psi\rangle \otimes X'^{\frac{1-j}{2}}|+\rangle =$$
$$(\mathbb{I} \otimes \mathbb{I} \otimes X'^{\frac{1-j}{2}})(\alpha|00+\rangle + \beta|01+\rangle + \gamma|10+\rangle + \delta|11+\rangle).$$

After $\mathbb{I} \otimes X \otimes X'$ with outcome $k = \pm 1$ we obtain:

$$|\psi_2\rangle = \tfrac{1}{2}\left[\mathbb{I} \otimes X'^{\frac{1-k}{2}} \otimes X'^{\frac{1-j}{2}}\right] \times$$
$$\times\{(\alpha+\beta)(|000\rangle + |010\rangle) + (\alpha-\beta)(|001\rangle - |011\rangle)+$$
$$+(\gamma+\delta)(|100\rangle + |110\rangle) + (\gamma-\delta)(|101\rangle - |111\rangle)\} =$$
$$= \tfrac{1}{\sqrt{2}}\left[\mathbb{I} \otimes X'^{\frac{1-k}{2}} \otimes X'^{\frac{1-j}{2}}\right]\{(\alpha+\beta)|0+0\rangle + (\alpha-\beta)|0-1\rangle +$$
$$+(\gamma+\delta)|1+0\rangle + (\gamma-\delta)|1-1\rangle\}.$$

Next measurement $X' \otimes \mathbb{I} \otimes X$ with outcome $l = \pm 1$ sets our qubits in state:

$$|\psi_3\rangle = \left[\mathbb{I} \otimes X'^{\frac{1-k}{2}} X^{\frac{1-l\cdot j}{2}} \otimes X'^{\frac{1-l}{2}}\right][\alpha|00+\rangle + \beta|01+\rangle + \delta|10-\rangle + \gamma|11-\rangle].$$

After the final measurement of $\mathbb{I} \otimes \mathbb{I} \otimes X'$ with eigenvalues $m = \pm 1$ we get:

$$|\psi_3\rangle = \left[X'^{\frac{1-k}{2}} \otimes X^{\frac{1-l\cdot j}{2}} \otimes X'^{\frac{1-l}{2}}\right][\alpha|00+\rangle + \beta|01+\rangle + \delta|10-\rangle + \gamma|11-\rangle].$$

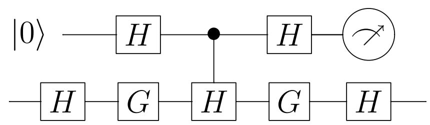
After the final measurement of $\mathbb{I} \otimes \mathbb{I} \otimes X'$ with eigenvalues $m = \pm 1$ we get:

$$|\psi_4\rangle = \left[X'^{\frac{1-m\cdot k}{2}} \otimes X^{\frac{1-l\cdot j}{2}} \otimes X^{\frac{1-m}{2}}\right] \times$$
$$\times[(\alpha|00\rangle + \beta|01\rangle + \delta|10\rangle + \gamma|11\rangle) \otimes |0\rangle] =$$
$$= \left[X'^{\frac{1-m\cdot k}{2}} \otimes X^{\frac{1-l\cdot j}{2}} \otimes X^{\frac{1-m}{2}}\right][CNot|\psi\rangle \otimes |0\rangle],$$

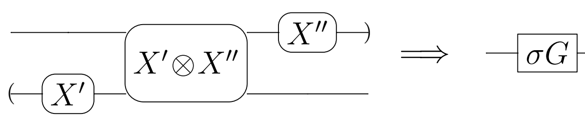
[4]Note that the second and the third qubit appear in reversed order in Fig.(5)

thus proving the above circuit equivalence.

The measurement of the tactics G conducted within the quantum market game frame of reference causes interpretative problems that can be resolved if we replace the measurement of G with *controlled* H gate (cf (1)) and the measurement of entanglement for another pair of conjugated bases $X' \otimes X''$ in the set of universal primitives. Owing to the fact that $G = HGHGH$, the measurement of G can be implemented in the following way [18]:



where the tactics G is obtained from (3) by the cyclic replacement $X \to X'$, $X' \to X''$ i $X'' \to X$:



In fact, the universality property has any set of primitive that contains the *controlled* H gate and measurements X^k, $X^p \otimes X^q$, $X^r \otimes X^s$, where $p \neq q$, $r \neq s$ and $p \neq r$ – this can be easily checked [20]. It follows that to implement a quantum market[5] it suffices to have, beside possibility of measuring strategy-qubits and control of the supply-demand context, a direct method measuring entanglement of a pair of qubits in conjugated bases.

5 Quantum Intelligence à la 20 questions

Let us recall the anecdote popularized by John Archibald Wheeler [26]. The plot concerns the game of 20 questions: the player has to guess an unknown word by asking up to 20 questions (the answers could be only yes or no and are always true). In the version presented by Wheeler, the answers are given by a "quantum agent" who attempts to asign the task the highest level of difficulty without breaking the rules. In the light of the previous discussion, any quantum algorithm (including classical algorithms as a special cases)

[5] In fact, any finite-dimensional quantum computational system can be implemented in that way [20].

can be implemented as a sequence of appropriately constructed questions-measurements. The results of the measurements (ie answers) that are not satisfactory cause further "interrogation" about selected elementary ingredients of the reality (qubits). If Quantum Intelligence (QI) is perceived in such a way (as quantum game) then it can be simulated by a deterministic automaton that follows a chain of test bits built on a quantum tenor [27]. The automaton completes the chain with afore prepared additional questions at any time that an unexpected answer is produced (cf the idea of quantum darwinism [22]). Although the results of the test will be random (and actually meaningless – they are instrumental), the kind and the topology of tests that examine various layers multi-qubit reality and the working scheme of the automaton are fixed prior to the test. The remarkability of performance of such an automaton in a game against Nature is by the final measurement that could reveal knowledge that is out of reach of classical information processing, cf the already known Grover and Shor quantum algorithms and the Elitzur-Vaidman bomb tester. Needless to say, such an implementation of a game against quantum Nature leaves some room for perfection. The tactics $CNot$ and H belong to the normalizer of the n-qubit Pauli group G_n [18], hence their adoption allows to restrict oneself to single corrections of "errors" made by Nature that precede the final measurement. It is worth noting that a variant of implementation of the tactics T makes it possible to postpone the correction provided the respective measurements methods concern the current state of the cumulated errors [28]. Therefore in this setting of the game some answers given by Nature, though being instrumental, have a significance because of the influence of the following tests. There is no need for the final error correction – a modification of the measuring method is sufficient. In that way the course of game is fast and the length of the game is not a random variable. This example shows that in some sense the randomness in game against quantum Nature can result from awkwardness of agents and erroneous misinterpretation of answers that are purely instrumental.

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