Nonlinear Mean Reversion in Stock Prices

Sebastiano Manzan

CeNDEF, Department of Quantitative Economics, University of Amsterdam

Roetersstraat 11, 1018 WB Amsterdam, the Netherlands

e-mail: s.manzan@uva.nl, fax: +31-20-5254349

Abstract

In this paper we investigate the relation between stock prices and fundamental variables. First, we consider the ability of static and dynamic versions of the present value model to account for the dynamics of annual stock prices from 1871 until 2003. The results suggest that the market price experiences swings away from the fundamental valuation but reverts back in the long-run. We then consider whether the deviation of stock prices from the fundamental valuation can be characterized by a nonlinear adjustment process. We find that the data strongly support this hypothesis. Further, we find that the results are quite similar in both the pre- and post-90s periods.

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1 Introduction

Does the stock market rationally reflect fundamental values?. The stock price run-up of the late 90s revived the debate about the rationality of financial markets. In 2000 the Price-to-Dividends (PD) ratio for the S&P500 index reached a level of 85 against an historical average of approximately 25. The extreme behavior of stock prices can be explained in different ways.

According to rational explanations, the rapid increase in stock prices reflects changes occurred in fundamental factors. They argue that the required rate of return has lowered significantly because of higher participation of investors to the stock market and changes occurred in consumers preferences. If investors discount future pay-offs at a lower rate, prices will increase. A similar result is obtained when the expected growth rate of dividends or earnings raises. These arguments are discussed by Heaton and Lucas (1999). However, they found that these explanations are not able to account for the large increase in prices of the late 90s. On the other hand, Campbell and Shiller (2001) argue that changes in fundamental factors are not large enough to explain changes in stock prices. A possible explanation was proposed by Summers (1986). He suggested that irrational fads in investors sentiment create persistent deviations of prices from intrinsic valuations that rational investors might not be able to arbitrage away. According to this theory, a combination of irrational expectations of some investors and limits to the activities of arbitrageurs explains the deviations of stock prices from rational valuations. This view is also consistent with the empirical evidence of mean reversion and long-run predictability of asset returns. If the stock price reverts (in the long-run) back to its intrinsic value, a positive (negative) deviation predicts that prices will decrease (increase). Hence, the adjustment process creates a negative relation between the changes in prices and the deviation from the fundamentals that emerges at long horizons. Empirical evidence of mean reversion in stock prices was provided by Poterba and Summers (1988) and Fama and French (1988) among others. An alternative explanation of the mean reverting behavior of stock prices is given by Cecchetti *et al.* (1990). They suggest that these findings are also consistent with a model where the dividend process driving stock prices switches between an "expansionary" state (high growth rate of dividends) and a "contractionary" one (negative growth of dividends). In this case, the deviations found by Summers (1986) and Poterba and Summers (1988) are the results of a misspecified fundamental process that does not account for the switching dynamics of dividends.

In this paper, we first review some of the valuation models that have been proposed in the literature to explain the aggregate stock market. In particular, we compare the fundamental value implied by the Present-Value Model (PVM) when the ex-ante return and the growth rate of dividends are constant (static Gordon model) or time-varying (dynamic Gordon model). The results suggest that these fundamental variables are unable to account for the short-run dynamics of stock prices, although they tend to explain their long-run behavior. Another relevant issue is related to the possibility of a decrease in the equity risk premium considered by Fama and French (2002). A decrease in ex-ante return implies a shift upward of the average PD ratio that seems also to be consistent with earlier analysis by Bonomo and Garcia (1994). We then consider as fundamental valuation a static Gordon model with a decrease in the discount rate in 1950. Given these assumptions, the PD ratio can be interpreted as a measure of the mispricing of the market compared to fundamental valuations. The second contribution of the paper is to investigate evidence of an asymmetric adjustment of the stock price toward the fundamental value. The analysis of annual data from 1871 until 2003 shows that there is significant evidence to support a nonlinear model in which mean reversion becomes stronger when deviations from the fundamentals are large. This confirms previous analysis using nonlinear adjustment models, such as Gallagher and Taylor (2001), Schaller and van Norden (2002) and Psaradakis *et al.* (2004). The adjustment process seems also to be consistent with the stock price run-up of the late 90s. Estimation of the nonlinear adjustment model to data until 1990 does not suggest a significant difference compared to the sample that includes the 90s.

The paper is organized as follows: section (2) introduces different notions of fundamental values used for empirical investigation. Section (3) describes the nonlinear model used for the deviations of stock prices from fundamentals. Section (4) discusses the estimation results and the evidence in support of the hypothesis of nonlinear mean reversion. Finally, section (5) concludes.

2 Fundamentals

A standard approach in asset valuation is to assume that the price satisfies

$$P_t = E_t\left[\frac{1}{1+r_{t+1}}(P_{t+1} + D_{t+1})\right], \tag{1}$$

where P_t is the price of the asset at the end of period t, D_{t+1} is the cash flow paid during period $(t+1)$ and r_{t+1} is the required rate of return at time $(t+1)$. $E_t(\cdot)$ indicates the expectation conditional upon information available at time t. Solving Equation (1) forward for T periods and applying the law of iterated expectations, we obtain

$$P_t = E_t\left[\sum_{j=1}^{T}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t+i}}\right)D_{t+i}\right] + E_t\left[\prod_{i=1}^{T}\frac{1}{1+r_{t+i}}P_{t+T}\right]. \tag{2}$$

The present value of holding the asset for T periods is equal to the expected discounted value of its cash flows and the expected discounted value of the resale price. A typical assumption introduced to rule out the occurrence of bubbles is

$$\lim_{T\to\infty} E_t\left[\prod_{i=1}^{T}\frac{1}{1+r_{t+i}}P_{t+T}\right] = 0, \tag{3}$$

called the transversality condition. If we assume that $T \to \infty$ and the transversality condition holds, the asset price is equal to the expected discounted value of its future cash flows

$$P_t^* = E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t+i}}\right)D_{t+i}\right], \tag{4}$$

where we indicate P_t^* as the fundamental value. We define the growth rate of the dividend process g_t as $D_{t+1} = (1+g_{t+1})D_t$, so that the fundamental value is given by

$$P_t^* = E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right)D_t\right]. \tag{5}$$

The time variation of g_t and r_t and the nonlinearity in the pricing equation complicate the derivation of analytically tractable formulas. One approach to simplify the problem consists of assuming that the dividends growth rate and the required return are constant and equal to g and r, respectively. Under these assumptions, Equation (5) implies that

$$P_t^* = mD_t, \tag{6}$$

where $m = (1+g)/(r-g)$. The stock price at time t is given by the cash flow times a multiple that depends on the ex-ante rate of return and the growth rate of dividends. This model is known as the Gordon valuation formula and has recently been used by Heaton and Lucas (1999) to determine the rational valuation of stock prices and by Fama and French (2002) to estimate the risk premium. The model is very simple and makes some clear predictions about the behavior of prices: prices will increase if r is lowered, that is, if investors discount at a lower rate future cash flows, or if dividends are expected to grow at a faster rate. Another implication of the model is that the PD ratio should be constant over time.

A limitation of this model is that it assumes that the dividend growth rate and the expected returns are constant over time. It is possible to allow for time variation by following the approach of Poterba and Summers (1986). They approximate the pricing formula given in (5) by a first-order Taylor expansion around the mean of the required return, r, and the mean of the growth rate, g,

$$P_t^* \approx E_t \left[\sum_{j=1}^{\infty} \left(\frac{1+g}{1+r} \right)^j + \frac{\partial P_t^*}{\partial r_{t+j}} \mid_r (r_{t+j} - r) + \frac{\partial P_t^*}{\partial g_{t+j}} \mid_g (g_{t+j} - g) \right] D_t \tag{7}$$

where the partial derivatives are given by

$$\frac{\partial P_t^*}{\partial r_{t+j}} \mid_r = -\frac{D_t}{r-g} \beta^j, \tag{8}$$

$$\frac{\partial P_t^*}{\partial g_{t+j}} \mid_g = \frac{(1+r) D_t}{(1+g)(r-g)} \beta^j, \tag{9}$$

and $\beta = (1+g)/(1+r)$. Substituting the derivatives into Equation (7), we get

$$P_t^* = \left\{ \frac{1+g}{r-g} - \frac{1}{(r-g)} E_t \left[\sum_{j=1}^{\infty} \beta^j (r_{t+j} - r) \right] + \frac{1+r}{(1+g)(r-g)} E_t \left[\sum_{j=1}^{\infty} \beta^j (g_{t+j} - g) \right] \right\} D_t. \tag{10}$$

The pricing formula depends on the expectations of investors about future ex-ante returns and dividends growth rates. A typical assumption made in the literature is that the expectations follow an AR(1) process, that is

$$E_t(r_{t+j} - r) = \rho^j (r_t - r) \tag{11}$$

$$E_t(g_{t+j} - g) = \phi^j (g_t - g), \tag{12}$$

and the approximated pricing formula in Equation (10) becomes

$$P_t^* = m_t D_t, \tag{13}$$

where m_t is the time-varying multiplier given by

$$m_t = \left\{ \frac{1+g}{r-g} - \frac{\rho(1+g)}{(r-g)(1+r-\rho(1+g))} (r_t - r) + \frac{\phi(1+r)}{(r-g)(1+r-\phi(1+g))} (g_t - g) \right\}. \tag{14}$$

This version of the fundamental value is known in the literature as the dynamic Gordon model because it defines asset prices as a time-varying multiplier of the dividends. The multiplier in Equation (14) has a straightforward interpretation: if the required rate of return and the growth rate of dividends are constant and equal to their mean then it collapses to the static multiplier of Equation (6); however, time variation in the required rate of return and/or in the dividend growth rate changes the level of the multiplier. The response of prices to changes in r_t and g_t is similar to the case of the static Gordon: if investors require at time t a return higher (lower) than the average r, this will decrease (increase) the multiplier and consequently prices. On the other hand, if dividends grow at a higher (lower) rate at time t, this will increase (decrease) the multiplier and will affect positively (negatively) stock prices. Equation (14) shows that the multiplier depends also on the AR coefficients in the expectations of the required return and the dividend growth rate. High ρ and ϕ imply that shocks to g_t and r_t will have a persistent effect on the multiplier and on prices. Analogously to the static case, the multiplier can be interpreted as the PD ratio: in this case the forcing variables, ex-ante returns and dividend growth rate, determine the dynamics of the ratio. The required rate of return is unobserved and assumption need to be introduced about variables that explain its time variation. Campbell and Shiller (1989) used different notions of ex-ante returns, such as a proxy for the risk-free interest rate and the conditional volatility of stock returns. Another assumption concerns the risk premium that in Campbell and Shiller (1989) is assumed constant. In this case, the process for the required return is given by the risk-free interest rate plus a constant risk premium.

To evaluate empirically the ability of the valuations models to explain the dynamics of stock prices, we calculate the fundamental value of the S&P500 index from 1871 until 2003. The dataset is described in Shiller (1989) and it is widely used in the mean reversion literature. Figure (1) shows the log of the real stock price index and the fundamental value according to the static Gordon model. We assume a constant multiple of approximately 25 that results from an annual growth rate of dividends of 1.9% and a discount rate of about 6%.

Figure (1)

The Figure suggests two pitfalls of the static PVM model. The stock price experiences large and persistent deviations from the fundamental value. This is a well known fact since the work of Shiller (1981) that pointed to the fact that fundamentals are much smoother (or less volatile) compared to stock prices. It indicates that the Gordon model with constant dividends growth rate and ex-ante return is not able to capture the dynamical features of the PD ratio. In addition, it is also clear that until the mid 1950s the stock price is most of the time below the value predicted by the Gordon model whereas after it is mostly above it. This suggests that the average PD ratio might have shifted upward due to a structural decrease in the discount rate or an increase in the average growth rate of dividends.

The issue of the persistent deviation of the stock price from the fundamental valuation could be explained by the persistence of the fundamental driving forces in the dynamic Gordon model. In Figure (2) we consider the dynamic Gordon model given in Equation (13) and (14). We assume that the ex-ante return is given by the risk-free real interest rate and a constant risk premium. A visual analysis of the Figure clearly suggests that accounting for the dynamics of the fundamental driving forces does not add significant explanatory power compared to the static model. The derivation of the Gordon model relies on the assumption that the dividend growth rate and the ex-ante return follow an AR(1) process. The estimated coefficients are $\hat{\phi} = 0.12$ (for the dividends growth rate) and $\hat{\rho} = 0.39$ (for the real interest rate).

Figure (2)

The evidence discussed here and the results of Campbell and Shiller (1989) and, more recently by Zhong *et al.* (2004), suggest that the fundamental factors don't have the persistence required to explain stock prices. Barsky and de Long (1993) assume that prices are formed according to Equation (6) with the dividend growth rate following an ARIMA(0,1,1). This process contains a unit root and gives more persistence to the warranted fundamental value. However, there is no empirical evidence to support the assumption of a unit root in the dividend growth rate. Bansal and Lundblad (2002) provide evidence that at the monthly frequency the dividend growth rate is well characterized by an ARMA(1,1) process with large (and of similar magnitude) AR and MA coefficients. However, this is probably a feature of the monthly data that is not supported by analysis of the annual data considered here.

These results point to the fact that fundamental factors are not able to give a full account of the dynamics of stock prices. On the other hand, the failure of rational valuations could be caused by misspecification of the fundamental process. As mentioned earlier, Cecchetti *et al.* (1990) found that the dividend growth rate can be better explained by a markov process switching between two states: one of positive dividend growth associated with an economic expansion and one of negative growth characterized as a recession. They also show that this model can explain the findings of mean reversion in stock prices. However, their results have been questioned by Bonomo and Garcia (1994). They searched for the best specification among the class of markov switching models. The best performing model is characterized by the transition between two states having different variances. Both Bonomo and Garcia (1994) and Driffil and Sola (1998) show that they cannot reject the null hypothesis that the mean growth rate of dividends is equal in the two regime. Instead, there is significant evidence that of regime-dependence in the variance of the process. This result has important implications for the claim of Cecchetti *et al.* (1990) that switching in the growth rate of dividends can explain stock prices and their mean reverting behavior. Bonomo and Garcia (1994) show that the specification with regime-dependent variances is not able to generate mean reversion in stock prices. This is probably due to the fact that the markov switching model is capturing a structural break in the volatility pattern of dividends. The different regimes identified by the markov model of Bonomo and Garcia (1994) are probably related to a decrease in the variability of dividends observed after 1950. They found that for the consumption growth series there is evidence of only one switch from the high variance regime to the low variance regime in 1950. Instead, for the dividend series they report more switches between the states.

This evidence of a structural break in 1950 is also related to a recent paper by Fama and French (2002). Using the static Gordon model discussed earlier, they estimate the aggregate risk premium for the annual data on the S&P500 from 1871 to 2000. They find that in the period before 1950 the equity premium was 4.17% and quite close to the value produced by average returns. However, for the period after 1950 the Gordon model estimates the premium at 2.55% that is much lower compared to the 7.43% obtained by average returns. Also in their analysis emerges the decrease in variability experienced by dividends from more than 15% to 5% in the post 1950 period. Figure (3) shows the stock price and the fundamental value implied by the static Gordon model with a decrease in the ex-ante return in 1950. It is clear that a shift in the mean of the log PD ratio is needed to explain the level of the ratio. However, the persistence of the deviations are still unexplained and seem to last longer after 1950.

Figure (3)

From this analysis we can draw the following conclusions. First, fundamental variables are not able to explain the short-run dynamics of stock prices. The persistence of the deviations of the stock price from the static Gordon model (the PD ratio) cannot be accounted by persistence in the dividend growth rate and the real interest rate. Barsky and de Long (1993) suggest that investors extrapolate the dividend growth rate to be highly persistent although there is no empirical evidence supporting this assumption. An alternative explanation is that the dividends switch between a high and low growth regimes. However, as discussed earlier there is no statistical evidence to support this assumption.

Given this evidence, in the rest of the paper we consider the static Gordon model as the fundamental value of the asset. This allows us to interpret the deviations from the fundamental value as the PD ratio. The second conclusion is that a shift in the level of the PD ratio seems to occur after 1950. Bonomo and Garcia (1994) suggest the period after 1950 is characterized by a regime of low variability in the dividends and consumption growth rates. Lower "macroeconomic risk" might lead to a lower required rate of return to invest in the risky asset as suggested by Fama and French (2002).

In the remaining of the paper we investigate the mean reversion pattern of the PD ratio. This is a particular relevant issue given the extreme behavior of stock prices in the late 90s. In particular, the run-up of stock prices in the late 90s seem unlikely to be explained by a model with linear reversion toward the mean. Hence, we consider nonlinear models where the adjustment of stock prices toward the fundamentals occur in a nonlinear fashion. Similar work on the stock prices data before the late 90s are Gallagher and Taylor (2001), Schaller and van Norden (2002) and Psaradakis *et al.* (2004).

3 Models of Nonlinear Adjustment

A simple way to consider nonlinear adjustment consists of a smooth (nonlinear) transition between 2 linear regimes. The model is called STAR (Smooth Transition AR)[1] and assumes that the process X_t for the transitory component in stock prices evolves as

$$X_t = \{\phi_1' G_t(S_t, \gamma, c) + \phi_2' [1 - G_t(S_t, \gamma, c)]\} \mathbf{X}_{t-1} + \epsilon_t \tag{15}$$

where $\mathbf{X}_{t-1} = (1, X_{t-1}, ..., X_{t-p})'$ and the disturbance term ϵ_t is *i.i.d.* with constant variance σ^2. $G_t(S_t, \gamma, c)$ is the function that regulates the transition from the first regime, with coefficient vector ϕ_1, to the second regime, where the dynamics evolves according to ϕ_2. S_t is the variable that determines the switch between regimes. In the application in the next section we use $S_t = X_{t-d}$ for $d \geq 1$. Two common choices of $G_t(S_t, \gamma, c)$ are the logistic and the exponential function. The logistic version of the STAR model (called in the literature LSTAR) has transition function

$$G_t(S_t, \gamma, c) = \{1 + \exp[-\gamma(S_t - c)]\}^{-1}, \tag{16}$$

where $\gamma > 0$ determines the speed of transition and the threshold c determines the regime that is active. The logistic function varies smoothly from 0 to 1 as the transition variable, S_t, becomes

[1] We largely simplify the discussion of STAR models according to the application at hand. For a more detailed discussion of this family of models see Teräsvirta (1994) and van Dijk *et al.* (2002).

increasingly larger than the threshold c. Another common choice for the transition function is the exponential (and the model is termed ESTAR), given by

$$G_t(S_t, \gamma, c) = 1 - \exp[-\gamma(S_t - c)^2] \tag{17}$$

In this case the transition function smoothly approaches 1, the further S_t deviates (in either directions) from the threshold value c.

These transition functions imply different dynamics for the process of mean reversion: the logistic is characterized by an asymmetric adjustment of X_t to its past values depending on the transition variable, S_t, being above or below the threshold c. In contrast, the exponential implies a symmetric adjustment in both directions of $(S_t - c)$. In other words, when using the logistic function we assume that negative and positive deviations revert back to the fundamentals at different speeds, whereas using the exponential the speed of mean reversion is equal for negative and positive deviations. The choice of the transition function is a crucial issue for the interpretation of the results. We will test which type of transition seems to accommodate better the dynamics in the deviations of stock prices from the fundamental value.

The null hypothesis of linearity against STAR holds if either $H_0 : \phi_1 = \phi_2$ or $H_0^{'} : \gamma = 0$. As discussed more extensively in Teräsvirta (1994), under both null hypotheses the test statistics are affected by the presence of nuisance parameters that complicate the derivation of the asymptotic distribution. In order to overcome this identification problem, Luukkonen *et al.* (1988) proposed to approximate the transition function $G_t(S_t, \gamma, c)$ with a Taylor-expansion around $\gamma = 0$. This allows to derive an LM type statistic with a standard χ^2 distribution. A 2^{nd} order Taylor-series expansion of the exponential transition function around $\gamma = 0$, leads to the auxiliary regression

$$x_t = \beta_0' \mathbf{X}_{t-1} + \beta_1' \tilde{\mathbf{X}}_{t-1} S_t + \beta_2' \tilde{\mathbf{X}}_{t-1} S_t^2 + \beta_3' \tilde{\mathbf{X}}_{t-1} S_t^3 + \beta_4' \tilde{\mathbf{X}}_{t-1} S_t^4 + e_t \tag{18}$$

where $\tilde{\mathbf{X}}_{t-1} = (X_{t-1}, ..., X_{t-p})'$ and the β_j are reparametrizations of the vector of parameters $(\phi_1', \phi_2', \gamma, c)'$. The null hypothesis that $\gamma = 0$ against ESTAR corresponds to test that $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Similarly, a 3^{rd} order expansion of the logistic function involves only the first four elements of the RHS of Equation (18) and the null of linearity can be tested as $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. The artificial regression in Equation (18) could also be used to guide the specification of the transition function. The reparametrizations of the expansion of the logistic function imply that the null holds if $\beta_1 = 0$ and $\beta_3 = 0$, whereas the expansion of the exponential function under the null involves only the second order term, that is, $\beta_2 = \beta_4 = 0$. We can design the following null hypotheses in order to test for evidence of STAR dynamics and the type of transition function that is more appropriate. The null hypotheses are

(LM_4) $\quad H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

(LM_3) $\quad H_0 : \beta_1 = \beta_2 = \beta_3 = 0 | \beta_4 = 0$

$(H_{0,L})$ $\quad H_0 : \beta_1 = \beta_3 = 0$

$(H_{0,E})$ $\quad H_0 : \beta_2 = \beta_4 = 0$

LM_4 and LM_3 are used as general tests of linearity against STAR dynamics. Instead, rejection of $H_{0,L}$ suggests that a logistic transition should be preferred while rejection of $H_{0,E}$ points to an exponential specification. The testing procedure is conditional on the lag d used for the transition variable. By testing for different values of d, the tests are also useful in the selection of the optimal lag for the transition variable. In order to make robust inference in small samples we will use the F-version of the tests. A relevant issue in the implementation of these models is the choice of p, the order of the AR regimes. We follow the approach of Teräsvirta (1994) by looking at the PACF (Partial AutoCorrelation Function) and the order selected by AIC.

4 Estimation Results

We use the static Gordon valuation as our notion of fundamental value for the yearly S&P500 Index data described earlier. We estimate the STAR model to the log deviations of the price from the fundamental value

$$X_t = \log (P_t) - \log (P_t^*) . \tag{19}$$

Using the static Gordon model in Equation (6), X_t is equivalent to the demeaned log PD ratio. Following Fama and French (2002) we use different risk premiums in the sample periods before and after 1950[2]. This implies that we assume different means for the PD ratio equal to approximately 20

[2] In the calculation of the fundamental value of the S&P500, we assume the discount rate is equal to 8.07% in the period 1871-1950. This is given by the sum of the risk-free rate of 3.90% and an equity premium of 4.17%. For the 1951-2003 period we set r equal to 4.74% due to a risk-free rate of 2.19% and a risk premium of 2.55%. The dividend growth rate is fixed at 2.74% before 1950 and to 1.05% from 1951 onwards.

before 1950 and around 35 on the rest of the sample. In what follows we analyze the time series both in the full sample and in the sub-sample from 1871 to 1990. This seems a natural choice because the late 90s might have changed dramatically the time series properties and the mean reversion pattern of the deviations from the fundamentals.

Figure (4) shows the time series of X_t. Both in the subsample and the full sample the partial autocorrelations suggests that 3 lags are significant. This is also confirmed by the Akaike Information Criteria for a linear AR model. We use $p = 3$ in the test for linearity and in the estimation of the STAR model.

Figure (4)

First, we tested for linearity of the time series of the log PD ratio against a STAR alternative. The p-values of the F-tests up to lag 8 described in the previous section are given in Table (1).

Table (1)

In the sample up to 1990, the null hypothesis of linearity is rejected when the lag in the transition variable is equal to 2 and 4 at the 5% significance level. The LM_3 and LM_4 tests reject in the 4^{th} while LM_4, $H_{0,L}$ and $H_{0,E}$ reject in the second lag of the transition variable. The rejection for LM_4 and $H_{0,E}$ are stronger than for the other tests and it supports the choice of the exponential as transition function. The tests applied to the full sample show rejections in the 4^{th} lag when LM_3 and LM_4 are used but no rejection for $H_{0,L}$ and $H_{0,E}$. The p-value of LM_3 is slightly smaller compared to LM_4 and it is probably due to the run-up of the late nineties that attributes a higher weight to one tail of the distribution and gives more support to a logistic specification. As discussed in detail in Teräsvirta (1994), LSTAR and ESTAR are to some extent substitutes. This might happen when an ESTAR model has most of the observations lying in one side of the threshold such that it can be reasonably approximated by an LSTAR specification.

The tests suggest that there is evidence to reject the null hypothesis of linearity both in the period 1871-1990 and in the full sample until 2003. We assume that the transition between regimes is symmetric around the mean as in the ESTAR specification. The tests seem to support this choice and also previous work on nonlinear modeling of adjustment processes has privileged this specification. See, among others, Kilian and Taylor (2003) and Gallagher and Taylor (2001). Given that the tests are unclear about the choice of d, the lag of the transition variable, we selected it by AIC selection criterion while keeping fixed the AR order in each regime equal to 3. The selected d is equal to 4 in both cases confirming the evidence provided by the linearity tests.

We performed a grid search for γ and c to initialize the NLLS estimation procedure. When a coefficient was not significant we dropped it from the regression and fitted the reduced model. We could not reject the null that the coefficients in the first lag of the two regimes are equal. In addition, we impose the restriction that the threshold value c is equal to zero. The estimation results are shown in Table (2) for the linear and ESTAR models. The Table reports also goodness-of-fit statistics and the roots of the characteristic polynomials of the estimated AR models.

Table (2)

In the period 1871-1990 the ESTAR specification significantly improves the fit of the linear AR(3) model. The coefficients are significantly different from zero and there is no evidence of misspecification in terms of residuals autocorrelation, neglected nonlinearity and parameter constancy[3]. The dynamics of the fitted ESTAR model can be characterized as follows. The mid-regime (the log PD ratio close to the mean) follows a AR(3) with 2 dominant complex roots with modulus 0.92[4]. Instead, for $G(\cdot) = 1$, the dynamics is governed by a linear process with dominant complex roots having modulus equal to 0.81. Imposing a linear structure on the series shows that the largest root is real and equal to 0.73. These results suggest that close to equilibrium the log PD ratio is characterized by a persistent (stationary) process that reverts slowly toward the mean. However, when the deviations are large the adjustment process is faster with an half-life of approximately 3 years. Figure (5) shows the time series of the transition function $G(X_{t-4}, \hat{\gamma})$ and the scatter graph of X_{t-4} and $G(X_{t-4}, \hat{\gamma})$. The graphs show that the outer regime is fully activated ($G(\cdot) = 1$) only in few cases that were associated with large negative deviations of the log PD ratio from the mean. On the other hand, there are periods in which the inner regime drove the dynamics of the log PD ratio. The transition function is quite smooth and the estimated value of γ is small and statistically significant.

Figure (5)

The results for the ESTAR model on the full sample are similar. As mentioned earlier, model selection suggests that 3 lags in the regimes and $d = 4$ are appropriate. The coefficient estimates

[3]See Eitrheim and Teräsvirta (1996) for a detailed description of the misspecification testing in STAR models.

[4]The third root is real and has modulus equal to 0.91.

are not statistically different from the values estimated on the subsample. This is confirmed also by the magnitude of the dominant roots. The inner regime has a real dominant root equal to 0.96 while on the subsample it was equal to 0.92. Instead, the outer regime is characterized by two complex roots with modulus equal to 0.76 compared to a value on the subsample of 0.81. Although we do not calculate standard errors for these estimates, a qualitative evaluation suggests they can be considered quite similar. However, for the AR model we find that the largest root increased from 0.73 to 0.88 and in particular the first order coefficient moves from 0.66 to 0.80. It appears that including the data after 1990 has a much smaller impact on the results of the nonlinear model compared to the linear one. The test statistics suggest that the ESTAR model is correctly specified in terms of serial correlation in the residuals and neglected nonlinearity. However, the test for parameter constancy is close to th rejection level of 5%.

Figure (6) shows plots of the transition function for the full sample estimation. The time series plot shows the persistent regime was activate during the 60s and became also dominant in the second half of the 90s and drove the stock price away from the fundamental valuation. However, the large deviation that cumulated in the 1999 contributed to the activation of the stabilizing regime that drove the prices toward more realistic valuations.

Figure (6)

5 Conclusion

It is a well documented fact that rational valuation models are not able to account for the dynamics of stock prices. As we show in this paper, allowing for time variation in the discount rate and in the dividends growth rate does not improve significantly the explanatory power of the PVM presented in Section (2). The deviations of stock prices from the fundamental value are much more persistent than warranted by the factors that are assumed to determine the asset price dynamics. A possible explanation for the failure is that investors have irrational expectations about the growth rate of dividends and/or about the future evolution of stock prices as suggested by Summers (1986) and Barsky and de Long (1993).

In this paper we do not provide an answer to the question of the rationality of market valuations. However, the analysis of the time series of the PD ratio (a measure of deviation of stock prices from the fundamental value) shows that there is significant evidence for the existence of two regimes. In one regime deviations are persistent and contributes to drive stock prices away from intrinsic values. Instead, the other regime is characterized by strong mean reversion in which market valuation become closer to fundamental values. This pattern of nonlinear mean reversion is valid both in the subsample up to the beginning of the 90s and in the full sample that includes the stock price run-up of the late 90s. We show that the "bubble" like behavior is correctly captured by a model in which there is a switching between a persistent regime and one of quick reversion to the mean.

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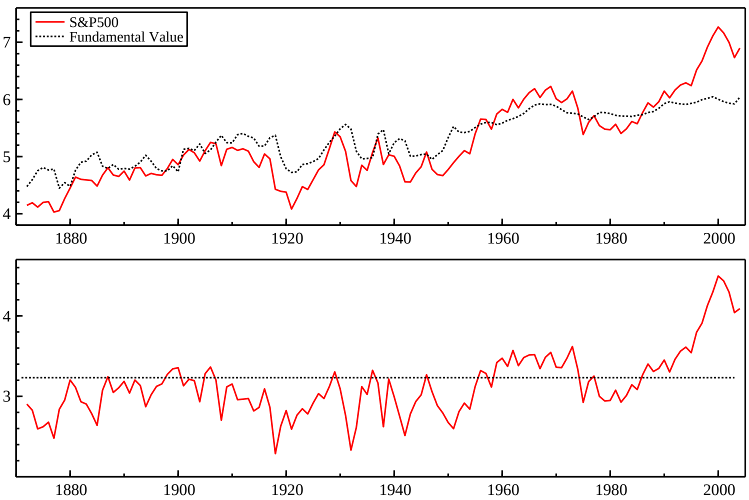

Figure 1: Price-to-Dividends ratio for the S&P500 Composite Index from 1871 to 2003. (*top*) log price and fundamental value, (*bottom*) log PD ratio and multiple. The fundamental value is given by the static Gordon model in Equation 6. We assume a multiple of 25, an average dividend growth rate of 1.9% and a discount rate of 6%.

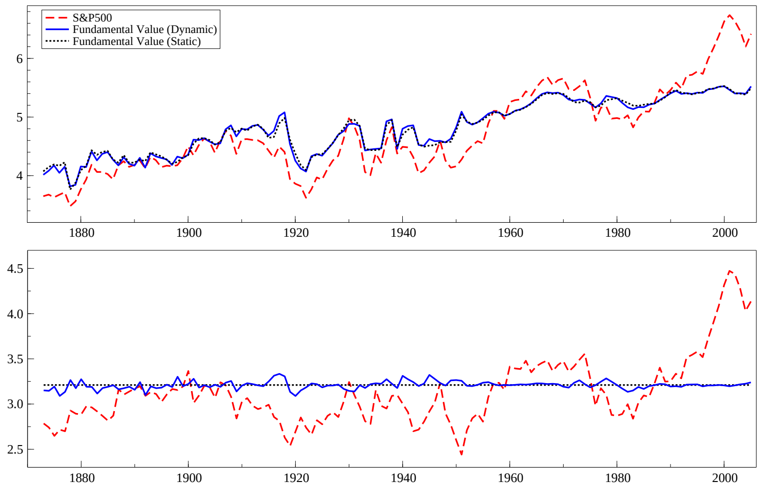

Figure 2: Price-to-Dividends ratio for the S&P500 Composite Index from 1871 to 2003. (*top*) log price and fundamental value, (*bottom*) log PD ratio and multiple. The fundamental value is given by the dynamic Gordon model in Equation (13) and (14). We used the same parameter values for the static Gordon model and estimated an AR(1) process for the interest rate and the dividend growth rate. The estimated values are $\hat{\rho} = 0.39$ and $\hat{\phi} = 0.12$.

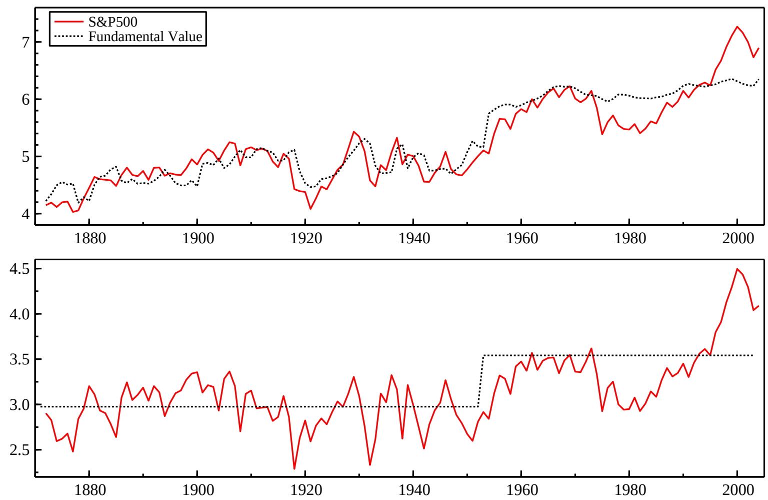

Figure 3: Price-to-Dividends ratio for the S&P500 Composite Index from 1871 to 2003. (*top*) log price and fundamental value, (*bottom*) log PD ratio and multiple. The fundamental value is given by the static Gordon model in Equation (6) with a shift in the discount rate in 1950.

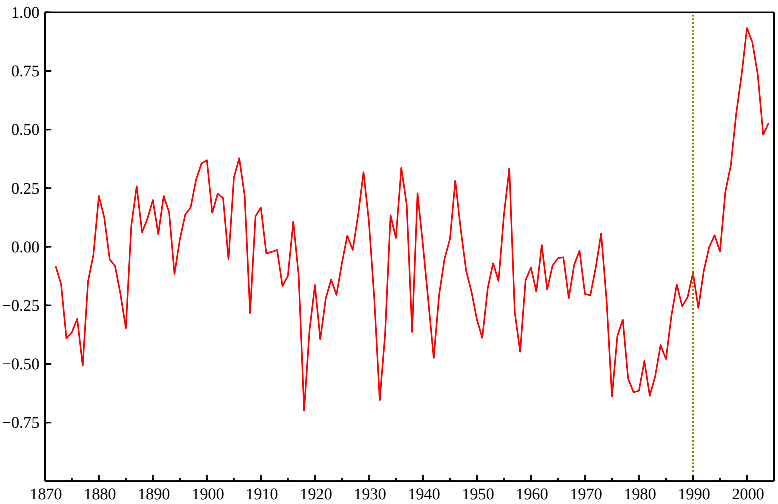

Figure 4: Time series plot of X_t, the log deviation of the stock price from the fundamental valuation. It can also be interpreted as the deviation of the PD ratio from the average.

Table 1: **Linearity Tests**

d	LM_3	LM_4	$H_{0,L}$	$H_{0,E}$	LM_3	LM_4	$H_{0,L}$	$H_{0,E}$
		1871 - 1990				**1871 - 2003**		
1	0.84	0.29	0.27	0.11	0.51	0.26	0.60	0.16
2	0.17	**0.03**	**0.02**	**0.01**	0.54	0.55	0.57	0.38
3	0.37	0.44	0.97	0.61	0.30	0.33	0.55	0.11
4	**0.03**	**0.02**	0.32	0.09	**0.03**	**0.04**	0.65	0.14
5	0.58	0.70	0.42	0.64	0.88	0.95	0.94	0.66
6	0.34	0.17	0.30	0.28	0.39	0.13	0.43	0.07
7	0.17	0.30	0.98	0.51	0.13	0.21	0.61	0.40
8	0.88	0.81	0.86	0.56	0.72	0.89	0.70	0.98

The tests are described in Section (3) and are applied to X_t, the log deviation of stock prices from the fundamental value. The autoregressive order, p, was set to 3. d is the lag of the transition variable $S_t = X_{t-d}$. The tests are implemented as F-tests. In bold the p-values that are smaller than 5% significance level.

Table 2: **ESTAR Estimation**

	1871-1990			**1871-2003**		
	AR	$G(\cdot)=0$	$G(\cdot)=1$	AR	$G(\cdot)=0$	$G(\cdot)=1$
$\phi_{1,i}$	0.66	0.76		0.80	0.86	
	[7.26]	[9.01]		[8.02]	[10.4]	
$\phi_{2,i}$	-0.25	-0.71	0.42	-0.24	-0.68	0.34
	[-2.41]	[-4.91]	[3.83]	[-2.225]	[-4.61]	[3.95]
$\phi_{3,i}$	0.22	0.77	-0.48	0.27	0.74	-0.35
	[2.46]	[5.20]	[-4.22]	[3.34]	[4.81]	[-3.87]
γ		0.83			1.35	
		[2.19]			[3.49]	
R^2	0.36	0.48		0.60	0.65	
AIC	-3.331	-3.529		-3.227	-3.36	
$\hat{\sigma}$	0.183	0.166		0.193	0.182	
$AR(1)$		0.37			0.23	
$AR(5)$		0.82			0.53	
LM_{NL}		0.38			0.14	
LM_{PC}		0.81			0.06	
Roots:	0.73	-0.075±0.92$\imath$	0.74±0.33$\imath$	0.88	0.96	0.74±0.155$\imath$
Modulus:	0.73	0.92	0.81	0.88	0.96	0.76

Estimation results for the sample period 1871-1990 and the full sample; the t-values in parentheses are obtained by Newey-West variance-covariance estimator. The adequacy tests are as in Eitrheim and Terasvirta (1996): $AR(q)$ is a test for residuals serial independence of order q, LM_{NL} tests for no remaining nonlinearity and LM_{PC} is a test for parameter constancy.

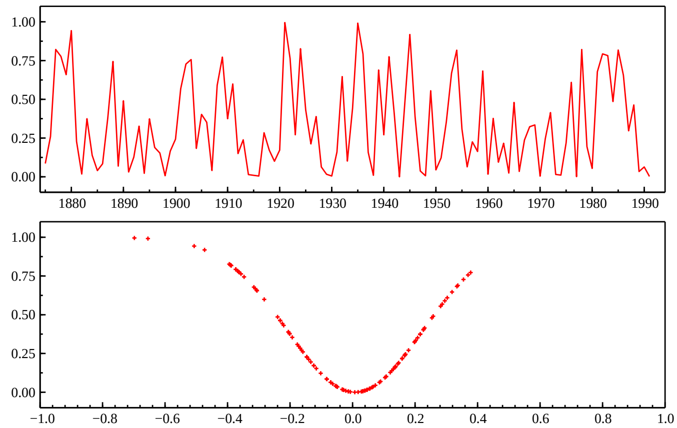

Figure 5: Transition function $G_t(X_{t-4}, \hat{\gamma})$ plotted in time and against X_{t-4} for the period 1871-1990.

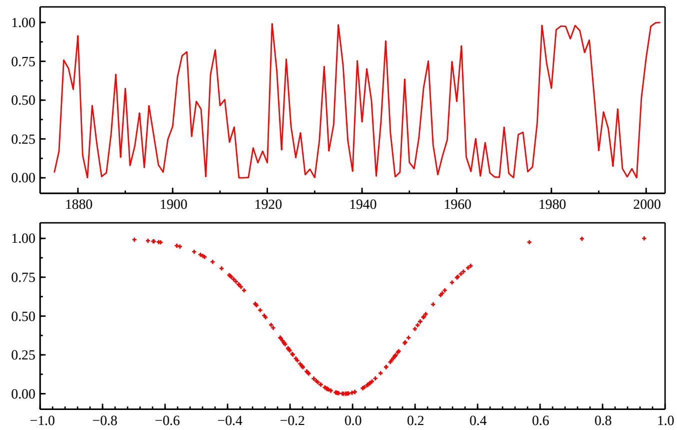

Figure 6: Transition function $G_t(X_{t-4}, \hat{\gamma})$ plotted in time and against X_{t-4} for the period 1871-2003.