Interest Group Size Dynamics and Policymaking

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May 2004

Abstract

We present a dynamic model of endogenous interest group sizes and policymaking. The model integrates 'top-down' (policy) and 'bottom-up' (individual and social-structural) influences on the development of interest groups. Comparative statics results show that the standard assumption of fixed-sized interest groups can be misleading. Furthermore, dynamic analysis of the model demonstrates that reliance on equilibrium results can also be misleading since equilibria may be unstable. Complicated dynamics may then emerge naturally,

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leading to erratic time patterns for policy and interest group sizes. Our model can endogenously generate the types of *spurts and declines* in organizational density reported in empirical studies.

KEYWORDS: Interest groups, Aspiration level, Endogenous fluctuations

JEL CLASSIFICATION: D23; D72; D78; E32; H30

1 Introduction

Interest groups play an important role in economic policymaking. Many empirical studies show this for Europe and the U.S. (Richardson 1994, Potters and Sloof 1996). Theoretically, the importance of this phenomenon is reflected in studies on collective action (e.g. Olson 1965, 1982) and the upsurge of endogenous economic policy models investigating the interaction between interest groups and economic policymakers (for a survey, see van Winden 2003). These models have provided valuable new insights into the determinants of economic policies. Nevertheless, by focusing on equilibria of properly defined games with fixed-sized interest groups and a government as (informed and rational) players, their relevance is restricted in several ways. First, existing models typically do not provide an explanation of the size of an interest group. Second, the dynamics of the interaction between the players is neglected. And, third, the standard assumption of one homogeneous type of (hyper)rational individual decision making is often rejected in experimental studies.

In reality, the relations between a government and interest groups are inherently dynamic. This is testified by the country studies collected by Richardson (1994). Timely examples are provided by the increasing participation of environmentalists and health groups in the development of agricultural policies, the changing political

[0]An earlier version of this paper appeared as Discussion Paper of the Tinbergen Institute, (TI 2000-022/1) and was presented at the 2001 annual meeting of the European Public Choice Society in Paris, at the 2002 annual meeting of Association for Public Economic Theory in Paris, at a conference on Lobbying and Institutional Structure of Policymaking in Rome (2002) and at a lecture at the Department of Economics, University of Valencia, October 2002.

landscape concerning tobacco, and the recent upsurge in NGOs that are increasingly being co-opted into policymaking (The Economist 1999). On a more aggregate level, the fluctuations in the percentage of unionized workers in the U.S. may serve as an illustration. According to Freeman (1997) the sudden spurts and declines in union density shown in Figure 1 are not only characteristic for the U.S. but also for other countries. Moreover, the time-series Freeman (1988) presents regarding the

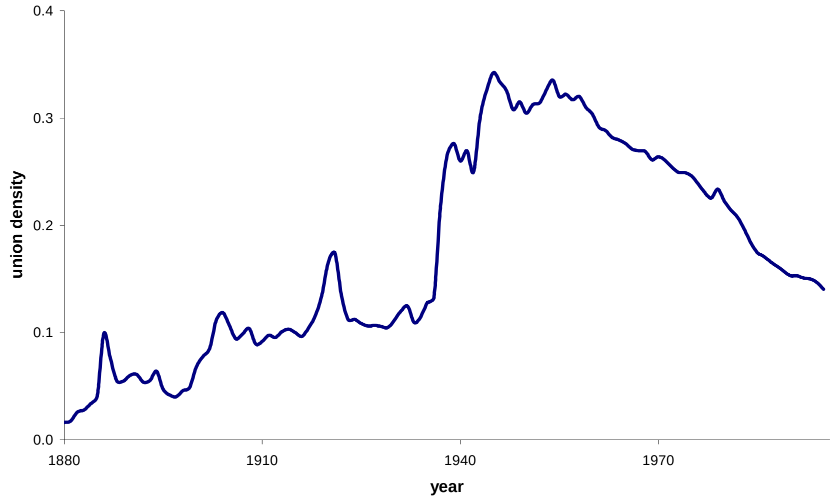

Figure 1: Time series of the density of union membership in the U.S., 1880-1995.

development of union densities in different countries show that the pattern of these fluctuations over time is very diverse, with some countries facing increases while others are experiencing declines. In his view, this constitutes at least suggestive evidence against broad explanations (such as unions having become obsolete in modern market economies), structuralist arguments (pointing at changes in the composition of the work force), or general macroeconomic explanations (referring to the oil shock, for instance). Without denying the importance of political 'top-down' changes (like labor laws), Freeman's study of the development of union density in the U.S. argues in favor of, 'bottom-up' models stressing "the underlying process by which organization occurs and the cumulative behavior of individual workers, unions, and firms. (...) the behavior of thousands or millions of individuals acting in response to one another" (Freeman 1997, p. 9). The above examples concerning agriculture, the tobacco industry, and NGOs suggest that this bottom-up approach is also important for an analysis of the development and influence of other interest groups.

Some bottom-up game-theoretic models of within-group cooperation and between-group competition have been developed recently in the literature on rent seeking (Baik and Lee 1997, Hausken 2000, Aidt 2002). However, these models typically neglect dynamic issues by focusing on (Nash) equilibria. Moreover, highly sophisticated strategic reasoning by individuals is assumed.[1] As noted by Ostrom in her presidential address to the American Political Science Association in 1997: "We have not yet developed a behavioral theory of collective action based on models of the individual consistent with empirical evidence about how individuals make decisions in social-dilemma situations" (Ostrom 1998, p. 1). Looking at the empirical findings concerning individual behavior, substantial evidence shows the following: behavior is generally not consistent with backward induction; Nash equilibria are often bad predictors in non-market environments; individual memory appears to be of low

[1] In addition, they often miss the top-down link referred to above by assuming a fixed contested prize (e.g. Hausken 1995, Baik and Lee 1997).

depth; strategic reasoning takes place in a step-by-step fashion; and ex-post rationality (choosing a direction which, with hindsight, would have been better in the previous choice situation) appears to have a strong influence on the adaptation of behavior (e.g. Selten 1998, van Winden 2002).[2]

In this paper we present a behavioral model of interest group size dynamics and endogenous policymaking, taking these empirical observations into account.[3] For tractability, a simple model is developed which focuses on redistribution. The model consists of three parts: one part determines the propensity of individuals belonging to a social group or economic sector to participate in collective action, another part determines the size and activity of interest groups, while the third part generates government policy. Because the redistribution policy feeds back into the other two parts of the model, the top-down and bottom-up approaches distinguished above are integrated in one model. Since our goal is to focus on some basic aspects of collective action, we leave open the precise nature of the social groups involved. In our view, the model can be relevant for the analysis of the interaction between social groups of various nature, as long as they have conflicting economic interests and potential political influence (like workers versus capitalists, different age groups, different industries within an economic sector, and so on). Our analysis consists of

[2] A related problem concerns decision making by groups. Existing experimental evidence is inconclusive regarding the issue whether groups behave more in line with standard game theory than individuals (see Bosman et al. 2002). A field empirical study by Whritenour Ando (2003) finds no evidence of strategic behavior by competing interest groups, although they do react to costs and benefits.

[3] Nevertheless, as will be shown in Section 5, equilibrium outcomes of the model can be consistent with a Nash equilibrium.

three parts. First, we present a (comparative statics) equilibrium analysis. This is followed by an investigation of the dynamics of the model. And, finally, we confront our model with data from the field and the laboratory to check its empirical relevance.

Before going into our main findings, it is helpful to observe that we are basically adding a *participation effect* to the redistribution and influence (weight) effects that are typically studied with political economic models. Changes in political participation, triggered by policies or exogenous forces, for instance, generate additional political influence and redistribution effects. This leads to results that are in contrast with the existing literature and help explain or throw a different light on issues of interest. From the equilibrium analysis, for example, we obtain the result that increasing the political activity (contributions) of the members of an interest group now becomes a two-sided sword. The reason is the negative participation effect accompanying the higher costs of political participation. On balance, this may eventually decrease the influence of the interest group. This result may help explain the empirically ambiguous effect of sheer numbers in politics (Potters and Sloof 1996), because greater numerical strength may be due to a smaller input (with lower cost) per member.

Less straightforward are the following results which relate to the economic status of social groups or sectors. Changes in size or welfare level - via demographic, international economic or technological shocks - appear to have very different effects depending on whether the sector involved is taxed or subsidized, as well as the level of taxation or subsidization. For example, growing subsidized sectors (think of the retired or agriculture in an extended Europe) may be confronted with smaller (individual) subsidies, as one might expect, but may also enjoy larger subsidies. However, subsidies will go down if the level of subsidization gets sufficiently high. Furthermore, declining sectors may be helped by larger subsidies, but may also be burdened with stiffer taxation. This sheds a new light, for instance, on increases in taxation of the tobacco industry in countries where this industry is taxed and on the decline. Declining sectors are not secured of political protection. On the contrary, politics may even worsen the situation. In the paper we also discuss the consequences of 'rising (or muted) expectations' that may accompany socioeconomic, technological or demographic developments.

Another main finding is that the collective action process may inhibit the occurrence of a stable political economic equilibrium. Complicated dynamics in the interaction between the participation in interest groups and policymaking show up in that case. Very different types of fluctuations in interest group sizes and redistribution policy may be observed. For example, regular fluctuations of short or long length, or short fluctuations superimposed on long ones, are obtained. Also highly irregular patterns can occur. In this respect, our model contributes, for example, to the explanation of empirically observed sharp declines in political protection (cf. Cassing and Hillman 1986). Our analysis, furthermore, clearly shows the restrictiveness of the common assumption of fixed sized interest groups in endogenous policy models. It turns out that the innocence of such an assumption very much depends on the nature and state of the behavioral mechanisms (think of the occurrence of sudden spurts and declines).

In addition to these theoretical results, we also find that the model can replicate the field empirical time series data exhibited in Figure 1 as well as controlled data from laboratory experiments that are within the domain of the model. The model offers an endogenous mechanism for these empirically observed patterns, in which both top-down and bottom-up factors play a role.

The paper is organized as follows. Section 2 presents the model. Section 3 is concerned with the equilibrium analysis, while Section 4 goes into the dynamic features of the model. The model is confronted with field and laboratory data in Section 5. Section 6 concludes.

2 The model

For expositional reasons, our model focuses on two economic sectors, A and B, each with a large number of agents. As further discussed below, the economic sectors may represent different ways of social grouping (e.g. socioeconomic groups, age groups, income groups). All individuals in sector i $(= A, B)$ are endowed with an income w_i. There is no mobility between the sectors and the number of agents in each sector is exogenously given as m_i. Furthermore, all individuals are assumed to have the same indirect utility function of income $V(y)$, for which the following standard assumptions hold: $V(y) \geq 0$, $V(0) = 0$, $V'(y) > 0$, $V''(y) < 0$ and $\lim_{y \to 0} V'(y) = \infty$.

We assume that the government can redistribute income in period t by levying a, possibly negative, lump-sum tax of τ_{At} on the individuals in sector A, which implies a lump-sum subsidy to the individuals in sector B equal to $\tau_{Bt} = -\frac{m_A}{m_B}\tau_{At}$, given the requirement of a balanced budget. Other policies will be abstracted from. We thus focus on pure redistribution.

For individual j ($j = 1, \ldots, m_i$) in sector i indirect utility equals $V(w_i - \tau_{it})$. For later convenience, let $V_{it} \equiv V(w_i - \tau_{it})$ where $\tau_{At} = \tau_t$ and $\tau_{Bt} = -\tau_t m_A/m_B$. Individuals in each sector can participate in collective action or, put differently, be members of an interest group. Interest group activity consist of 'lobbying' for a favorable tax τ_t, that, because of its uniformity, favors both the members (those who are politically active) and the non-members in the respective sector. The group-specific public good (bad) nature of the tax introduces the characteristic free-riding problem for interest groups. Political participation is assumed to entail an input of some effort in the activities of the interest group (e.g. turning out and vote, or some other contribution). Let l_i denote the given individual (lobbying) input in group i.

We first present our model of the development of an interest group. Thereafter, the determination of redistribution policy is formalized. The interest group model consists of two submodels: one determining the individual propensity to join, and another determining the size (membership) of the interest group. The first submodel deals with individual characteristics, while the second is to capture social-structural conditions (cf. Marx and McAdam 1994). In developing the former we acknowledge the many experimental and field empirical findings indicating that, when it comes to collective action, individual behavior is adaptive rather than featuring the strategically forward looking behavior of optimizing gamesmen. It reflects a strong influence of ex-post rationality, reference points or aspiration levels, and a low depth of memory (see Simon 1959, Ostrom 1998, Selten 1998, Camerer 2003). Furthermore, substantial evidence from the field and the laboratory shows that individuals caught in a social dilemma are likely to invest resources to improve joint outcomes (Ledyard 1995, Ostrom 1998, van Winden 2002). Taking these empirically observed features of individual behavior into account, we model the propensity of an individual to join in interest group activity - the *participation propensity* - as being determined by the following factors: actual utility V_{it}, a reference or aspiration utility level[4] r_{ij}, and the costs of the individual lobbying input l_i. To allow for individual differences in the reference level (e.g. due to different personality traits or socioeconomic experiences), let R_i denote the mean of the distribution of $(r_{ij})_j$, and β_i^2 its variance. Consequently, we can write $r_{ij} = R_i + \beta_i \varepsilon_j$ assuming that $\varepsilon_j \sim F$, where F is a distribution with mean 0 and variance 1. Individual j in sector i is willing to join if and only if the difference between reference utility and actual utility exceeds the (given) cost of participation, i.e. $r_{ij} - V_{it} > l_i$. The probability of that event is given by

$$\Lambda_{it} \equiv \Pr(r_{ij} > V_{it} + l_i) = 1 - F((V_{it} + l_i - R_i)/\beta_i)$$

[4] Gilboa and Schmeidler (2001) introduce an aspiration level in a model of (satisficing) consumer behavior.

Some direct empirical evidence for the assumption that dissatisfaction with government policies is a determinant of political action is provided by studies of voter behavior in national elections. In these studies the probability of voting for an opposing party (which can be considered as an interest group in itself) is found to be related to the dissatisfaction of voters with the economic situation under the incumbent government (see e.g. Mueller 1989, Paldam 1997). Furthermore, with respect to turnout it appears that not only the economic situation is important but also the opportunity costs (Radcliff 1992, Lijphart 1997).

Whether the propensity to participate in collective action materializes into actual participation depends on the presence of facilitating social-structural conditions. For example, legal rights to organize play an important role. A related factor concerns the ability of leaders to mobilize discontent and to maintain membership, where diffusion of information and exhortation via social networks and ties play an important role (Rothemberg 1988, Marx and McAdam 1994). Put concisely, these conditions determine the opportunity to get or stay involved - the *participation opportunity.* To capture this aspect of political participation in a simple way, we assume that there is a fixed probability λ_i with which this opportunity occurs to each individual in sector i.[5] As discussed above, the probability that this individual from sector i will join the interest group is given by Λ_{it}. Assuming that individual decisions are independent from each other and given a large number of individuals per sector, a law of large

[5]The parameter λ_i may also reflect an intrinsic individual opportunity for revising the participation status, in which case $1 - \lambda_i$ can be interpreted as an inertia parameter.

numbers argument allows us to replace probabilities by fractions. That is, λ_i can be interpreted as the fraction of the population from sector i that makes a participation decision. A fraction Λ_{it} of those $\lambda_i m_i$ individuals in sector i making a participation decision will then indeed join the interest group, leading to a total size of 'new' interest group members of $\lambda_i m_i \Lambda_{it}$ (notice that some of these new members might also have been interest group members in the previous period). Furthermore, again applying a law of large numbers argument, a fraction $1 - \lambda_i$ of the individuals that were interest group members in the previous period will not reconsider their participation decision in this period. From this we find that the number of 'old' interest group members is $(1 - \lambda_i) n_{it}$. Adding the two components we find that the (expected) sizes of the interest groups (n_{it}) evolve in the following way

$$n_{i,t+1} = (1 - \lambda_i) n_{it} + \lambda_i m_i \Lambda_{it}, \qquad i = A, B. \tag{1}$$

We turn now to the government. In line with the literature on endogenous policy models, it is assumed that policymakers are interested in political support through various contributions of interest groups, and that policies are adjusted to secure this support (see e.g. Hillman 1989, Baron 1994, Nitzan 1994, Dixit et al. 1997; for a theoretical survey, see van Winden 2003; the empirical evidence is surveyed by Potters and Sloof 1996). Policymakers may be motivated in this respect by, for instance, political survival (think of votes, endorsements, campaign support), career prospects (revolving doors), a need for policy relevant expertise and effort (for drafting legislation or building coalitions), or greed (corruption). Therefore, the lobbying activity of interest groups[6] and the size of the sectors that they represent are taken to influence the extent to which their interests will be promoted by the government. Since the focus of this paper is not on the precise mechanism relating interest group activity to government policy, we take a reduced-form approach by assuming that redistribution policy follows from the maximization of the following interest function with respect to τ

$$G(\tau_t) = L_t m_A V_{At} + (1 - L_t) m_B V_{Bt},$$

where the influence weight $L : \mathbb{R}_+^2 \rightarrow [0, 1]$ is assumed to be increasing in $l_A n_{At}$, and decreasing in $l_B n_{Bt}$, while taking the value 1/2 in case of an equal amount of total lobbying input, $n_{At} l_A = n_{Bt} l_B$.[7] Thus, the interest function is an influence weighted sum of the aggregate utility (interests) of the individuals of sectors A and B.[8] The tax selected by the government is implicitly determined by the following first-order condition (the second-order condition being satisfied by concavity of V)

[6]It is beyond the scope of this paper to fully endogenize the lobbying activity of interest groups, here determined by the fixed lobbying input per member and the endogenous size of a group. How such groups actually decide on the input level and its allocation over various activities is unclear. At present no model based on solid empirical evidence exists that might be used for that purpose. We will return to this issue in Section 6.

[7]The latter assumption is for simplicity. If, for ideological reasons, for instance, sector B would be politically favored then $L < 1/2$ when total lobbying efforts are equal across sectors.

[8]For our model it does not matter if net welfare $V_{it} - l_i$ is substituted for gross welfare V_{it} as long as l_i is taken as given when redistribution policy is determined. By leaving eq. (2) unchanged, this substitution would not affect the results of the comparative statics and dynamic analysis below. Note that in practice the various kinds of activities comprised by l_i may be difficult to observe for governmental policymakers.

$$L_t V'_{At} = (1 - L_t) V'_{Bt}. \tag{2}$$

Summarizing, our model features the following sequence of events, in each period t. *First*, individuals decide to join in interest group (lobbying) activities at a rate that is determined by both individualistic characteristics (τ_{t-1}, r_{ij}) and social-structural conditions (λ_i). *Then*, the tax τ_t for that period is selected by the government, which reflects the sizes of the sectors (m_A, m_B) and their total lobbying activity during the first part of the period $(n_{At} l_A, n_{Bt} l_B)$.

The following proposition concerns the unique equilibrium of our dynamic model.

Proposition 1 *The dynamic model specified by (1) and (2) has a unique equilibrium defined by the following set of equations*

$$n_A = m_A \Lambda_A \tag{3}$$

$$n_B = m_B \Lambda_B \tag{4}$$

$$L V'_A = (1 - L) V'_B. \tag{5}$$

(Proofs are relegated to the Appendix.)

3 Comparative statics: participation vs. redistribution and influence effects

In this section we investigate the equilibrium effects on interest group sizes and redistribution policy of changes in the individual lobbying input (l), the size of a sector (m), the income level in a sector (w), and the mean and standard deviation of the distribution of individual reference utility values (R and β). Note that changes in the social-structural parameter λ have no effect on the equilibrium as it drops out of eqs. (1) in the equilibrium. For convenience, we will focus on parameter changes holding for sector A (similar effects would be obtained for sector B). For expositional reasons, all proofs are relegated to the Appendix.

It will be helpful for the intuition behind our results to distinguish three types of effects of changes in parameters: a *redistribution effect*, a *political influence effect*, and a *participation effect*. The first two are standard in political economic models. The redistribution effect sets in because of the tendency of the government, given the political influence of the social groups, to redistribute income such that the influence weighted marginal utilities of the representative individuals of the groups are equalized (see eq. (2)). The political influence effect reflects the fact that an increase in the political weight of an interest group, through an increase in its lobbying activity ($n_i l_i$) or a decrease in another group's activity, will tilt the tax rate in favor of the sector it represents. The additional effect studied in this paper concerns the participation effect. This effect relates to changes in political influence and redistribution that are triggered by changes in political participation, which may themselves be the consequence of changes in influence and redistribution.

3.1 Individual lobbying input: a two-sided sword

In conventional rent-seeking and lobbying models, which neglect the endogeneity of the size of an interest group, the influence of such a group is typically increasing in the effort of its members. With fixed-sized interest groups this would also hold for our model, inducing a lower tax rate for the group concerned. In this subsection we are particularly interested in the following two questions. First, can an increase in lobbying input and the concomitant positive effect on a group's influence attract more members, thereby producing an additional boost to the group's influence? Second, if this is not the case, will it indeed lead to more influence (that is, a lower tax rate)? The following proposition summarizes the effects.

Proposition 2 *The equilibrium size of the interest group in sector A, n_A, is decreasing in the lobbying input of its members, l_A. Moreover, letting the effort elasticity of the interest group size, $H(l_A) \equiv -\frac{l_A}{n_A}\frac{\partial n_A}{\partial l_A} = -\frac{l_A \Lambda'_A}{\beta_A \Lambda_A}(> 0)$*[9]*,*

- *(i) if $H(l_A) < 1$ then the equilibrium tax rate, τ, is decreasing and the size of the interest group in sector B, n_B, is increasing in l_A;*

[9] *If $\Lambda \in \mathcal{C} = \{\Lambda(x) | \lim_{x\to+\infty} \frac{\Lambda'(x)}{\Lambda(x)} \neq 0\}$ then there exists a $l_A^* > 0$ such that $H(l_A^*) > 1$ for all $l_A > l_A^*$ because $\lim_{l\to+\infty} H(l) = +\infty$. Note that if, for example, F corresponds to the logistic distribution then $\Lambda \in \mathcal{C}$. Furthermore, the existence of l_A such that $H(l_A) < 1$ can be seen by noting that* $\lim_{l\to 0} H(l) = 0$.

(ii) if $H(l_A) > 1$ then the equilibrium tax rate, τ, is increasing and the size of the interest group in sector B, n_B, is decreasing in l_A;

(iii) if $H(l_A) = 1$ then the equilibrium tax rate, τ, and the size of the interest group in sector B, n_B, do not change with a marginal increase in l_A.

In response to our first question, the first part of the proposition shows that increasing the individual lobbying input will never generate a larger membership. On the contrary, it will lead to a smaller sized interest group. If the lobbying input would only have become more costly without any direct political influence effect, this result would not have been surprising. (Still, the fact that the pure cost effect is in this direction is a welcome aspect of the model, because there exists substantial empirical evidence, for example, showing a negative effect of voting costs on turnout (see Lijphart 1997).) What makes it more interesting - in particular, in combination with what follows next - is that the political influence effect of the increased lobbying input cannot reverse its pure cost effect.[10]

Contrary to what conventional interest group models suggest, the second part of the proposition shows that having interest group members put more effort into the lobbying activity may be disadvantageous to the group, that is, lead to higher taxes. The reason is the (potentially strong) negative participation response. If the only parameter change concerns the lobbying input in sector A, a higher tax on that

[10] An extra boost to a group's influence might be obtained if the reference or aspiration utility level of its members would start to adjust in the direction of the political outcome.

sector occurs if and only if its political influence has become weaker. Thus, L must have decreased. Now, since a higher tax would raise the net income of individuals in sector B, and thereby negatively affect the size of the interest group in that sector, a decrease in L requires a lower total lobbying activity $(n_A l_A)$ of group A. Formally,

$$\frac{d\,(n_A l_A)}{dl_A} = n_A(1 + \frac{l_A}{n_A}\frac{\partial n_A}{\partial l_A}) < 0$$

where the second term of the expression in brackets, indicating the marginal decrease in influence due to the smaller size of the interest group, is $-H(l_A)$. Consequently, for this condition to hold it is required that the marginal loss of influence due to the smaller size of the interest group exceeds the direct marginal gain ($H(l_A) > 1$, as in the proposition). This negative participation response with its potential influence effects helps explain why in practice interest groups, such as unions, seem reluctant to increase contributions (see also the concluding section). It further provides a caveat for conclusions based on sheer numbers in politics (see also the next subsection).

3.2 Differential impact of changes in the size of taxed and subsidized sectors

We now investigate the equilibrium consequences of a shock concerning the size of a sector. Such a shock may be due to more or less autonomous technological or international economic developments, migration forces, or demographic developments. With fixed-sized interest groups, in the model, the subsequent redistribution effect would unequivocally have a negative effect on the absolute value of the tax rate (subsidy) of the sector involved. This changes, however, once political participation is allowed to adjust. When the size of a sector is affected, there is an immediate influence effect as well as a redistribution effect. The size of a sector not only plays a direct role in the interest function $G(\tau)$ maximized by the policymakers, it also directly affects the size and thereby the influence of its interest group (since $n_i = m_i \Lambda_i$). In addition, this induces participation effects with further consequences for the sizes and political influence of the interest groups. The next proposition summarizes the effects of a change in the size of sector A.

Proposition 3

(i) If $\tau \geq 0$ then the equilibrium tax rate, $\tau(\equiv \tau_A)$, is decreasing in the size of sector A. Moreover, there exists a $\tau_* \in (-w_B m_B / m_A, 0)$ such that if $\tau < \tau_*$ then τ is increasing in the size of that sector;

(ii) If $\tau < 0$ then the equilibrium sizes of the interest groups in both sectors, n_A and n_B, are increasing in the size of sector A. Moreover, there exist $\tau^* \in (0, w_A)$ such that if $\tau > \tau^*$ then the equilibrium sizes of the interest groups in both sectors are decreasing in the size of that sector.

This proposition shows that a change in the size of a sector can have very different consequences dependent on whether a taxed or subsidized sector is at stake, and whether the change concerns an increase or decrease in size. First, notice from part (i) that a taxed sector faces increased taxation when its size shrinks. In particular the smaller tax base plays a role here, inducing a redistribution effect. (Incidentally, this may shed a new light on some of the tax increases faced by a declining sector like the tobacco industry.) Moreover, part (ii) shows that interest group activity will nevertheless increase if the existing tax rate is sufficiently large. For a subsidized sector - like a protected industry - a further decline in its size may be upheld by an increase in subsidies, given that the existing level of subsidization is sufficiently large (part (i)), although interest group activity will decrease (part (ii)). The underlying reason is that it is less costly for the taxed sector to maintain a smaller subsidized sector (redistribution effect), while the loss in influence of the latter is not sufficiently strong. Note, however, that with smaller existing subsidies the outcome can be a decrease in subsidy, due to the loss of influence. Our finding that, in general, the policy response can go either way contributes to the formal literature on the political protection of declining industries where the possibility of ambiguous effects has been hinted at (Hillman 1989). In the next subsection, where we discuss the impact of income changes, we will return to this topic.

Furthermore, it is interesting to note that growing taxed sectors will witness a decrease in taxation, whereas interest group activity may increase or decrease, depending on the existing level of taxation. This result provides a formal argument to the largely empirical debate (focusing on correlations) concerning the relationship between economic development and interest group activity (see Olson 1982, Bischoff 2003). On the other hand, economies with growing subsidized sectors - such as social security in aging societies or agriculture in an extended Europe - would be confronted with increasing interest group activity, while subsidies may go either up or down. The outcome that subsidies will go down if the existing level is high seems reflected by the current European debate on agricultural policy. The fact that subsidies need not necessarily go down - as one would expect on the basis of the redistribution effect alone - is due to the immediate positive effect of an increase in the size of a sector on its interest group activity.

Finally, we note that the ambiguity of the policy effects that we find here is in line with the mixed empirical evidence presented in Potters and Sloof (1996) concerning the political influence effect of numerical strength.

3.3 Sectoral income growth boosts taxation and discourages organization

In the previous subsection we have seen that sectoral development produces ambiguous policy effects insofar as changes in the size of a sector are concerned. Income growth, on the other hand, turns out to have unambiguous effects. A positive sectoral income shock – due to technological or international economic developments, for example – induces redistribution of income away from that sector, for given political influence weights (as would hold in case of fixed-sized interest groups). A drop in the income level would lead to a reverse effect. However, it also affects political participation, and thereby political influence. The precise effects depend on the net outcome of these two forces. The next proposition summarizes the results.

Proposition 4 *The equilibrium tax rate,* $\tau(\equiv \tau_A)$*, is increasing in the income of the taxed sector,* w_A*. However, in both sectors, net (after transfer) income is an increasing function of the same income. On the other hand, the equilibrium sizes of the interest groups in both sectors,* n_A *and* n_B*, are a decreasing function of that income.*

Interestingly, our results suggest that improvement in productivity of a sector would not only have an overall positive effect on net income, but also reduce interest group activity. This income growth effect provides an additional reason why economic development may be accompanied by less interest group activity. It thereby produces a counter-argument to the hypothesis that interest group activity may be a concomitant of economic development (Bischof 2003). To the extent that interest group activity is correlated with corruption, this result also suggests that in addition to being detrimental to economic growth (Mauro 1995) corruption may in its turn be negatively affected by it, which might induce a vicious circle. Of course, one has to be careful here because many other factors are likely to be involved in economic development. One such factor, concerning reference utility levels, will be addressed in the next subsection.

Returning to the political protection of declining industries, note that (in contrast with the ambiguous declining size effect): the income effect will be unequivocally beneficial to such industries, in the sense that taxes will decrease or subsidies will increase. On balance, however, the policy response to the economic decline of a sector, involving lower income as well as a shrinking size, can go either way. Incidentally, our model also gives a behavioral underpinning for the possibility of a 'sudden collapse' of an industry. Although this is not an equilibrium issue, a few words on it here may be justified since we are now discussing political protection. Cassing and Hillman (1986) propose an explanation where the driving force is the assumed S-shape of the exogenously given positive relationship between the policy (a tariff) and the size of the industry (amount of labor), which can lead to a sudden drop in political protection. As will become clearer in the section on dynamics, in our model a sharp decline in political protection (subsidies) can occur through the basic non-linearity in the propensity to participate in interest group activity.

3.4 Reference utility levels and group heterogeneity

Given the redistribution policy, individualistic characteristics represented by the individual reference utility (aspiration levels) r_{ij} determine individuals propensity to participate in interest group activity. The higher the average reference utility level of the individuals in a sector - denoted by R_i - the more dissatisfied they will be with the existing government policy. The effect this will have on the *participation propensity* further depends on the heterogeneity of the individuals in this respect, denoted by β_i. The more heterogeneous the sectoral population, the smaller the effect of the average propensity to participate on the size of the interest group. (Recall that, in equilibrium, the *participation opportunity*, indicated by λ_i, plays no role.) The next proposition summarizes the effects of a change in R_A and β_A.

Proposition 5

1. *The equilibrium sizes of the interest groups in both sectors are increasing in the mean of the distribution of individual reference utility levels in sector A,* R_A*. Furthermore, the equilibrium tax rate,* τ*, of sector A is a decreasing function of* R_A*;*

2. *Regarding the standard deviation (heterogeneity) parameter* β_A *the following is obtained, where* $\tau^c \equiv w_A - V^{-1}(R_A - l_A)$*:*

   (i) if $\tau < \tau^c$ *then the equilibrium tax rate* (τ) *of sector A is decreasing in* β_A*, while the sizes of the interest groups in both sectors* (n_A *and* n_B) *are increasing in* β_A*;*

   (ii) if $\tau > \tau^c$ *then the equilibrium tax rate of sector A is increasing in* β_A*, while the sizes of both interest groups are decreasing in* β_A*;*

   (iii) if $\tau = \tau^c$ *then both the equilibrium tax rate of sector A, and the sizes of the interest groups do not change with a marginal increase in* β_A*.*

Note that τ^c indicates the tax rate that makes the 'average' individual in sector A (with $r_{Aj} = R_A$) indifferent with respect to joining in interest group activity. The findings of result 2 are then easily understood by observing that a smaller standard deviation of the distribution of reference utility levels (β_A) steepens the probability function describing the participation propensity (Λ_A), making it more like a step-function. For example, if $\tau < \tau^c$ ($\tau > \tau^c$) and the sector becomes more homogeneous through a smaller β_A, the more step-function like shape of the probability function implies less (more) participation and therefore a larger (smaller) tax rate

Our model provides a new potential explanation for the empirical finding that political participation increases with higher income (see e.g. Wolfinger and Rosenstone 1980, Schram 1991, Leighley and Nagler 1992, Lijphart 1997). For the sake of the argument, suppose that individuals with relatively high income belong to sector A, and those with low income to sector B. The mean income of individuals in sector i is represented by w_i. Furthermore, it seems plausible to assume that the mean reference utility for individuals in sector i satisfies $R_i \geq V(w_i) + l_i$, in which case a positive tax rate for sector A implies that $V(w_A - \tau) + l_A < R_A$. Then, if the tax rate is big enough to drive $V(w_B + \frac{m_A}{m_B}\tau) + l_B \geq R_B$, participation among the low income individuals will always be lower ($\Lambda_A > \Lambda_B$). If not (i.e. $\Delta V_B < 0$) then, for small enough β_A, the rate of dissatisfied individuals among those with a high income will be larger than the rate among individuals with a low income ($\Delta V_A/\beta_A < \Delta V_B/\beta_B$). Hence, if it may be assumed that the group of high income earners is relatively more homogeneous (e.g. because of better information and contacts), also in that case the participation rate of those with high income will be larger than that of low income earners.[11]

4 Dynamics

An important issue that we are interested in in this paper concerns the dynamics of the model consisting of equations (1) and (2). It is well-known that nonlinear systems like the present model can give rise to dynamic patterns such as periodic cycles and irregular fluctuations. In fact, these patterns seem to be the rule rather than the exception in many nonlinear dynamic models. Examples of erratic fluctuations arising naturally in economic dynamic models can, for example, be found in the literature on endogenous business cycle theory (e.g. Grandmont 1985, de Vilder 1996).

As will be shown below, also in the present model equilibria need not be stable and complicated dynamic patterns may emerge for a large set of parameter values. This occurs because a successful interest group diminishes the attraction to join it, whereas its success is, of course, positively correlated to its relative size. These two countervailing forces naturally lead to endogenous fluctuations.[12]

We will focus on the values of the heterogeneity parameter β and the participation parameter λ. Instability arises if, for a given (but not too high) level of heterogeneity β, the participation opportunity λ becomes sufficiently large. We start with the

[11] Also, note that if economic development would affect reference utility levels, via 'rising (or muted) expectations', this would further complicate the relationship with interest group activity. For instance, 'rising expectations' fostered by economic growth might lead to an increase in activity, notwithstanding the negative direct income effect discussed above.

[12] A similar mechanism underlies the political business cycles emerging in the two-sector general equilibrium model discussed in Tuinstra (2000).

following general result.

Proposition 6 *Consider the model given by eqs. (1) and (2). There exists $\lambda^f > 0$ such that the equilibrium (n_A^*, n_B^*) of the model is locally stable for $\lambda < \lambda^f$ and unstable for $\lambda > \lambda^f$. If $\lambda^f < 1$ a period-doubling bifurcation occurs at $\lambda = \lambda^f$.*

At a *bifurcation* there is a qualitative change in the behavior of the dynamic system. More specifically, at a *period-doubling bifurcation* the locally stable equilibrium becomes unstable and trajectories of the dynamic system are attracted to a period two orbit, where interest group activity keeps on fluctuating between two values. That is, in even periods the system is in state $(n_A, n_B) = (n_I, n_{II})$ whereas in odd periods the system is in state $(n_A, n_B) = (n_{III}, n_{IV})$, with $n_{III} \neq n_I$ and $n_{IV} \neq n_{II}$. More complicated time series may also obtain. To get a better view of the possible dynamics, we specify the model in the following way. We assume an iso-elastic indirect utility function $V(y) = \frac{1}{1-\alpha} y^{1-\alpha}$, with $0 < \alpha < 1$. Furthermore, taking a logistic distribution for the reference utility variable, $F(x) = \frac{\exp(\pi x/\sqrt{3})}{1+\exp(\pi x/\sqrt{3})}$, we have

$$\Lambda\left(\left[V_i + l_i - R_i\right]/\beta_i\right) = \frac{1}{1 + \exp\left(\eta_i\left[V_i + l_i - R_i\right]\right)}. \tag{6}$$

where $\eta_i \equiv \frac{\pi}{\sqrt{3}\beta_i}$.

Moreover, we consider a symmetric version of the model with $m_A = m_B = 1$ (thus, n_i can be interpreted as the fraction of people organized in sector i), $l_A = l_B = l$, $w_A = w_B = w$, and $\beta_A = \beta_B = \beta$. For this (sector) symmetric model a unique equilibrium exists with $\tau = 0$ and $n_A = n_B = n^* = \frac{1}{1+\exp \eta[V(w)+l-R]}$. This leads to the next proposition for our stability result.

Proposition 7 *Consider the symmetric model specified above. There exists a $\beta^* > 0$ such that for $\beta > \beta^*$, the symmetric equilibrium $(n_A, n_B) = (n^*, n^*)$ is locally stable for all $\lambda \in (0, 1)$. Furthermore, for $\beta < \beta^*$ the symmetric equilibrium is locally stable for $\lambda < \lambda^f$ and unstable for $\lambda > \lambda^f$, where λ^f is given by*

$$\lambda^f = 2\frac{1+W}{1+\left(1+\frac{\eta}{\alpha}w^{1-\alpha}\right)W},$$

with $W = \exp\left(\frac{\eta}{1-\alpha}\left(w^{1-\alpha}+l-R\right)\right)$. For $\beta < \beta^*$, the system undergoes a period-doubling bifurcation at $\lambda = \lambda^f$. At this period-doubling bifurcation a symmetric period two orbit of the form $\left\{\left(n^I, n^{II}\right), \left(n^{II}, n^I\right)\right\}$ with $n^I < n^* < n^{II}$, emerges.

To illustrate, we consider some simulations with $w = 10$, $R = 7, l = 1$, and $\alpha = \frac{1}{2}$. For high values of β, that is, for a highly heterogeneous population the equilibrium is stable. However, if β sufficiently decreases the equilibrium becomes unstable. This is illustrated in Figure 2. The graph shown in this figure divides the (β, λ)-space into a region with stable equilibria (below the curve) and unstable equilibria (above the curve).[13]

[13] For the numerical example, the relationship between the critical values of λ and β is given by

$$\lambda = \frac{2\left(1+\exp\left(2\pi\left[\sqrt{10}-3\right]/\left(\sqrt{3}\beta\right)\right)\right)}{1+\left(1+2\sqrt{10}\pi/\left(\sqrt{3}\beta\right)\right)\exp\left(2\pi\left[\sqrt{10}-3\right]/\left(\sqrt{3}\beta\right)\right)}.$$

Furthermore, $\beta^* \approx 6.0159$.

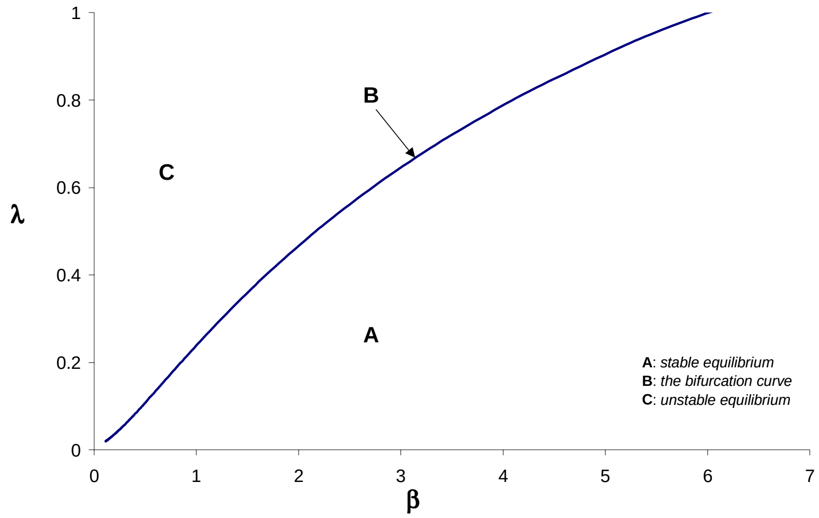

Figure 2: Regions in the $(\beta, \lambda)-$plane with stable and unstable equilibrium and the bifurcation curve along which a period-2 cycle emerges.

Recall from Section 2 that λ indicates the participation opportunity, that is, the presence of social-structural conditions facilitating the transformation of the propensity to participate in collective action into actual participation. Legal rights of collective action, the presence of leaders able to mobilize discontent, and the existence of social networks and ties enabling the diffusion of information and the exhortation of people, are among the relevant factors determining this opportunity. Our results would predict that a society becomes more vulnerable to political instability the more it offers here (i.e., the larger λ), especially when it is also more homogeneous (i.e., the smaller β). The instability confronting former centrally planned economies while opening up politically seems suggestive in this respect. In its turn, a greater diversity in political preferences (a larger β) stimulated by democratic institutions could then help explain the relative stability of many developed democracies.

We will now fix $\lambda = 0.1$ and investigate, for different values of β, how interest group activity and tax policy evolve. For $\lambda = 0.1$ the period-doubling bifurcation described in Proposition 7 occurs at $\beta^f \approx 0.47$. At this value of β the equilibrium becomes unstable and a period two cycle emerges. For β close to, but smaller than β^f almost all orbits of the symmetric dynamic system are attracted to this type of cycle. This cycle corresponds to the situation where in one period interest group A is 'large' and interest group B is 'small', and the people in sector B are taxed to the benefit of people in sector A, while in the next period the situation is reversed. For smaller values of β more complicated dynamic patterns emerge. The panels in Figure 3 illustrate the occurrence of strange attractors and the corresponding time series for different values of β.

The intuition for these time series is the following. An increase in the size of one of the interest groups leads to a new tax, which is more beneficial to this interest group. This leads to an increase in the size of the other interest group which induces a tax rate more beneficial to this interest group. In this fashion interest group activity keeps increasing until the process loses momentum, due to a diminishing effect on the tax schedule, and is eventually reversed. With smaller β the reverse process becomes dominated by the influence of λ, which causes the 'following' type of behavior in the decline of the interest groups illustrated by the bottom panel in the figure.

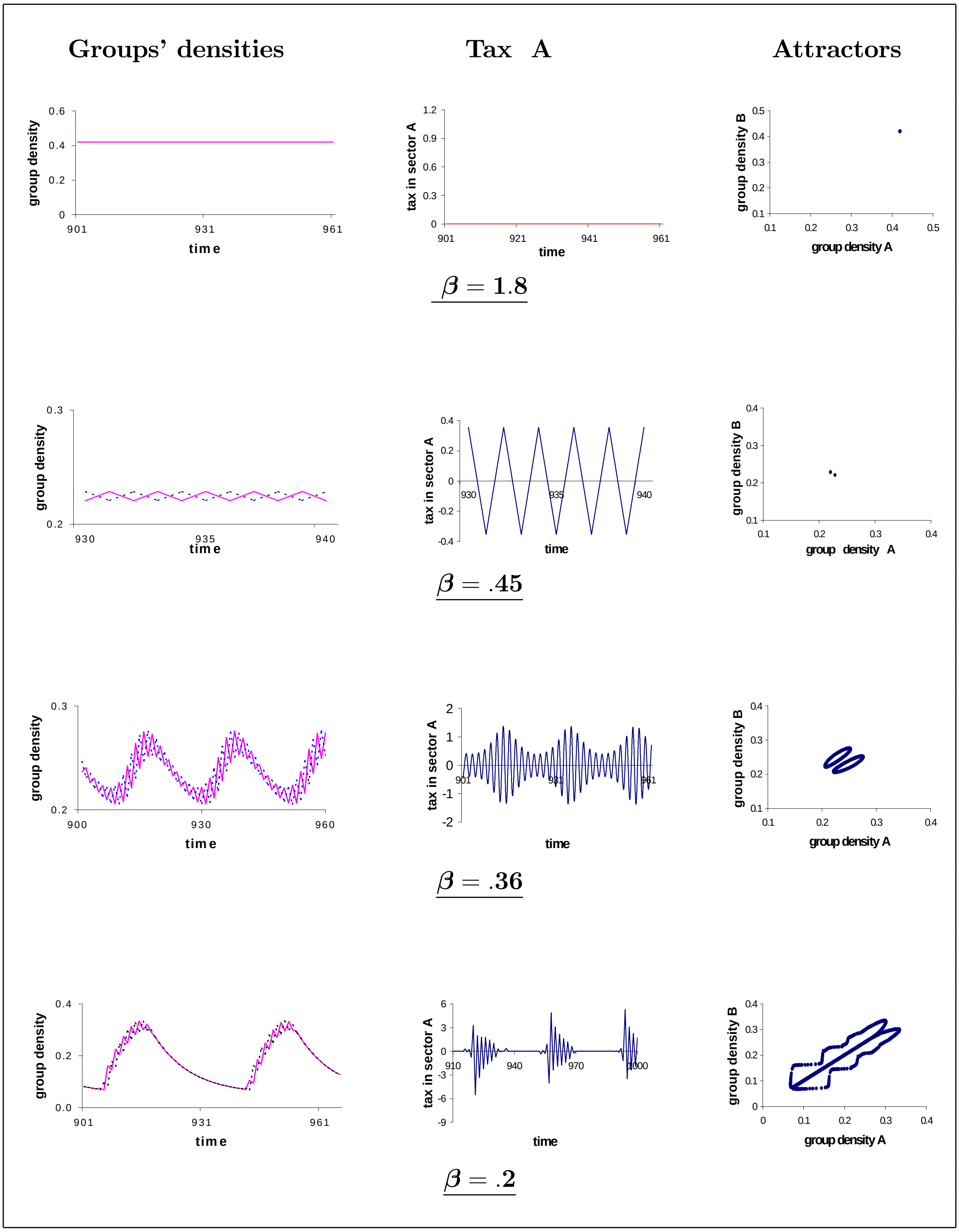

Figure 3: Top to bottom panels correspond to different values of β for the symmetric model with parameters: $l = 1$, $R = 7$, $w = 10$, $m = 1$, $\alpha = 0.5$ and $\lambda = 0.1$. The first column shows the time series of the fraction of people organized in sector A (solid-line) and sector B (dotted line); second column shows the time series of tax in sector A; third column shows the attractors.

5 Replicating field and experimental time series

The dynamic analysis from the previous section shows that focusing on equilibria can be very misleading, because they may be unstable and therefore extremely unlikely to be obtained. Instead, complicated dynamics may emerge. Whereas for the symmetric cases examined in Figure 3 it holds that the patterns are still regular in some sense, more irregular time series are obtained once asymmetry is allowed. To illustrate, the top panel in Figure 4 shows the dynamics of the model in case that: $w_A = 4, w_B = 10$, $\beta = 0.18$, $l_A = 0.4, l_B = 1, R_A = 4.2$ and $R_B = 7$.

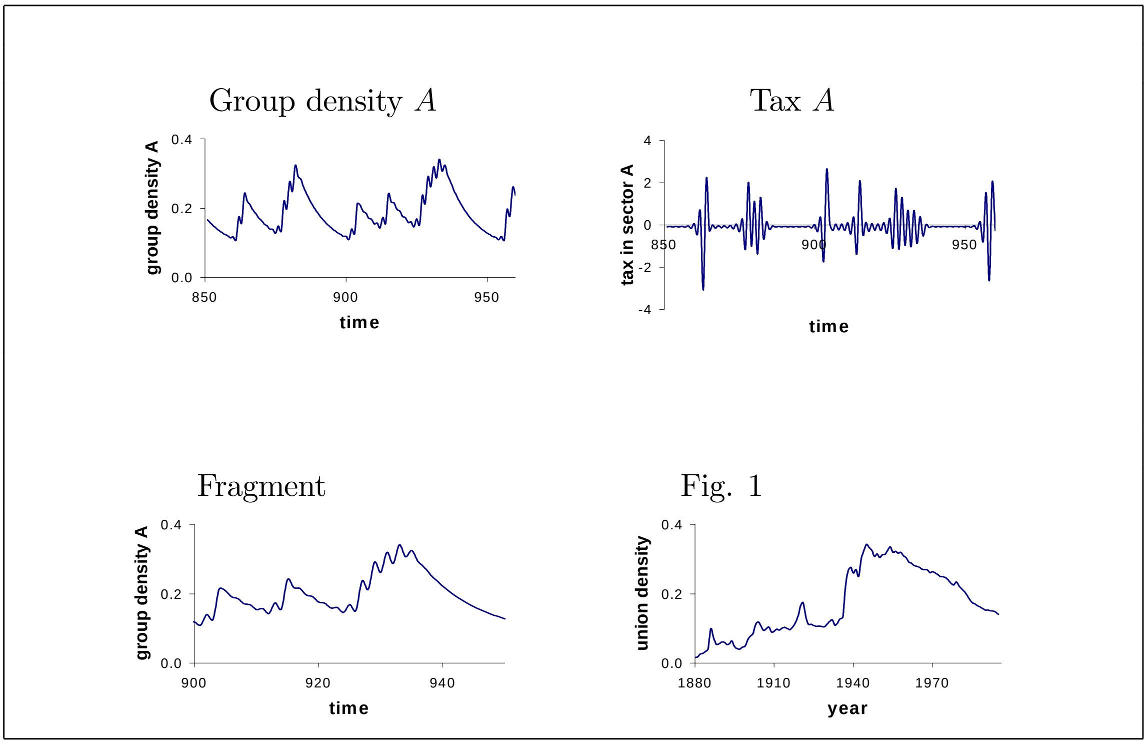

Figure 4: Top panel: time series of the fraction of individuals organized in sector A and of the tax on individuals in that sector for the model with the following asymmetric parameters: $w_A = 4$, $w_B = 10$, $m_A/m_B = 1$, $\alpha = 0.5$, $\lambda = 0.1$ and $l_i/w_i = 0.1$, $R_i = l_i + 6\sqrt{l_i}$ where i=A, B. Bottom panel: left figure shows a fragment from the left figure in the top panel, right figure reproduces Figure 1.

The left figure in the bottom panel shows the corresponding time series for a particular time interval. When compared with the right figure in this panel - which reproduces Figure 1, taken from Freeman (1997) - the resemblance of these two figures is striking. By letting one sector represent workers and the other sector owners or managers, it shows that the *internal dynamics of our model alone* can generate fluctuations in organizational density that are similar to the unionization of workers in the U.S. that Figure 1 refers to. No exogenous shocks are needed. In his study, Freeman distinguished two types of models that can generate spurts in union growth. First, standard linear models in which exogenous shocks (usually generated by political forces, like laws) generate responses in otherwise stable union membership. Second, models in which the growth process creates non-linearities producing 'phase transitions' when certain conditions are met (models of self-organized complexity). Our model is a first attempt fitting the second type. Of course, we are not claiming here that we provide an explanation of the particular historical development illustrated by the figure. To do so would require changes in many parameters over time (like income growth) in an appropriate way. Moreover, as argued by Freeman (1988), the redistribution conflict between workers and managers at the firm level should then also be taken into account. The only claim we want to make is to have shown in a rigorous way that by integrating top-down (policy) and bottom-up (behavioral) factors spurts and declines in the organizational density of interest groups as observed in practice can be endogenously generated, without any reliance on exogenous shocks.

But we can do more than that. To challenge our model in a more demanding way we will use a different data set. To avoid the noise and impact of intervening variables that are hard to control for in field empirical data, we use the experimental data reported in Schram and Sonnemans (1996). The design of their experiment turns out to be within the domain of our model. In the experiment, 12 subjects were divided into two groups. In each round of the experiment, subjects first (and independently) had to make a decision whether or not to contribute a token, where contributing would cost 70 Guilder cents (which corresponds to approximately 0.32 Euro). Then, after all subjects had made a decision, an amount of 222 Guilder cents was divided (by a computer program) between the two groups according to the relative amount of total contributions. Finally, each subject within a group was given the amount allocated to his or her group, irrespective of whether (s)he contributed or not. Noting that the rule determining the tax is similar, it is easily seen that the game subjects were asked to play in this experiment is within the domain of our model (with $w = 111$ and $l = 70$; see eqs. (1)). To check the performance of our model we proceeded as follows. First, we calibrated the relevant parameters (λ, β, R) using the time series of the participation rates for one of the groups.[14] Then, with the calibrated parameters $(\lambda^c, \beta^c, R^c)$ and the initial values of the participation rates $(n_{A,0}, n_{B,0})$ taken from the experimental (real) data, we generated new (simulated) data with our model. We

[14] The calibrated parameters are the ones that minimize the root mean square error regarding the simulated data and the real (experimental) data. In total we have data for 7 pairs of matched groups, which makes 14 groups in total.

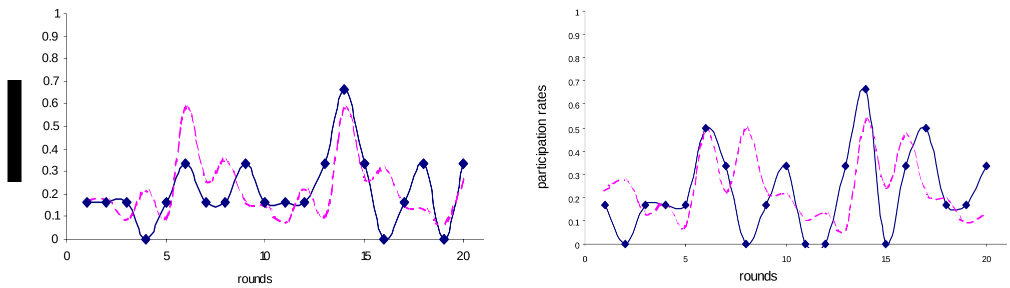

Figure 5: Time series of the voting participation rates for two groups in the Schram and Sonnemans (1996) experiment (solid line) and the simulated data (dotted line) with the calibrated parameters. Left figure shows time series for the group used for calibrating behavioral parameters, right figure shows time series for a randomly chosen group.

find that the null hypothesis of equal distributions for the simulated data and the real data cannot be rejected in 95% of the cases (at the 5% level, using a t-test, a Mann-Whitney test, and a Kolmogorov-Smirnov test).[15] Furthermore, to investigate whether our model can replicate the dynamic paths, we generated a new set of data, this time with calibrated initial values (keeping λ^c, β^c, and R^c fixed).[16] Then for each of the 14 groups, we (linearly) regressed the real data on the simulated data and tested for the joint hypothesis of $slope = 1$ and $intercept = 0$. It turns out that the joint hypothesis cannot be rejected in 9 (64%) of the cases (at the 5% level; see Appendix for more details). For illustration, Figure 5 presents the time series for two groups from different sessions, one of which was used to calibrate parameters.

[15] Excluding the data for the pair used for the calibration of the parameters (λ, β, R), we have for any given round t two real plus two simulated data samples, with each sample being of size 6 (matched groups are in different samples). Thus, since there are 19 rounds ($t = 2, \ldots, 20$, with the first round excluded because these data are used as initial values in generating the simulated data), we have 38 ($= 19x2$) samples of size 6. Note that the (simulated and real) data in each of the above samples are independent.

[16] The calibrated initial values are among the ones that minimize the root mean square error regarding the time series of the simulated and the real data for a given session (keeping λ^c, β^c, and R^c fixed).

In light of the highly nonlinear dynamics of the experimental data, the tracking performance of the model is quite remarkable. Furthermore, it is interesting to note that the calibrated parameters are from the unstable region (C), which implies that the (unique) equilibrium of the model is not stable. Thus, for the calibrated parameters, the equilibrium stability analysis suggests that the fluctuations in the participation rates observed during the 20 rounds of the experiment are not a temporal phenomenon. They seem to reflect inherent and persistent properties of the interaction process. One should be cautious, therefore, with conclusions based solely on an equilibrium analysis. In this context, it is noted that the (symmetric) Nash equilibrium participation rate (say, p_N) for the game studied in the experiment would be the same as our model equilibrium ($p^* = \Lambda\left(\left(V(w) + l - R\right)/\beta\right)$) for any set of parameters (β, R) that satisfy $\beta\Lambda^{-1}(p_N) = V(w) + l - R$. Consequently, our model does not preclude an equilibrium outcome that is identical to the Nash equilibrium. Moreover, for the calibrated parameter values (β^c, R^c), the equilibrium point of our model is $p^* = 0.045$ whereas $p_N = 0.096$. Thus, even for these values the equilibrium outcomes are, observationally, hardly different. However, we find that λ^c is not sufficiently small for this equilibrium point to be stable, so that it is very unlikely that it will ever be reached.[17]

[17] Schram and Sonnemans find that the experimental data reject the hypothesis of a symmetric Nash equilibrium. Goeree and Holt (2000) show that the quantal response equilibrium (QRE) for the game studied in the experiment of Schram and Sonnemans always predicts a strictly positive participation rate which is bounded from below by the Nash equilibrium and from above by 50%. However, data at the group level show that during the last 10 (5) rounds the participation rate is out of that range in 42% (40%) of the cases.

6 Conclusion

In this paper we have presented a dynamic model of endogenous interest group sizes and policymaking, focusing on redistribution. It integrates both top-down (policy) and bottom-up (individual and social-structural) influences on the development of interest groups. Our results clearly demonstrate the restrictiveness of the common procedure in political economic modeling to assume fixed interest group sizes and to concentrate attention on equilibria. As summarized in the Introduction, our main findings show effects that contrast with the existing literature and further help explain or throw a different light on various political economic issues. We also provided empirical support for the model. All in all, the results obtained from our investigation seem interesting and realistic enough to warrant further theoretical and empirical investigation. From among the issues that appear to be interesting for future research we would like to single out the following.

First of all, the strength of the behavioral model should be further empirically investigated, focusing on specific institutional forms of collective action. Controlled laboratory experiments can be very fruitful in this respect, as the application in this paper may show. Furthermore, it would be important to gain more knowledge regarding the way that individuals form and adapt reference utility levels. Models based on solid empirical evidence are lacking (see e.g. Gilboa and Schmeidler 2001). This is all the more important because of the relation with emotions (cf. Simon 1959). The role of emotions in inducing people to participate in collective action is seriously neglected. Although there are many casual statements by professional and academic experts bearing this out - for example, referring to hatred as a motivation for political terrorism -, theoretical models are missing (van Winden 2002).

Another area of interest concerns the endogenization of the decision making process of interest groups. How, for example, is the level of the individual lobbying input determined, and to what extent are these decisions influenced by other interest groups? As regards the former issue, our results point at an interesting dilemma for interest group leaders. If their main interest is in the size of the interest group (like bureaucrats are interested in the size of their bureau, as the standard public choice hypothesis has it) they may want to opt for a very low individual input. However, if their main concern would be the welfare of the members a higher individual input may be warranted, inducing lower taxes or higher subsidies but also a smaller group size (see part (i) of Proposition 2). Incidentally, this potential conflict of interests makes it understandable why they may have reservations concerning social welfare policies (cf. Neumann and Rissman 1984), and why they seem reluctant to raise fees. A further complicating issue is that an interest group leader may not be in the position to impose her or his preferences, which means that some form of compromising will have to take place shaped by the internal institutions of the group. The question about the influence of other interest groups can only be answered through empirical evidence. The available evidence is not clear in this respect, but suggests little strategizing (if any at all). Again, laboratory experimentation can be helpful to generate insights, also regarding the resolution of the conflicting interests that may exist among the members. Much interesting work remains to be done.

References

[1] Aidt, T.S., 2002. Strategic political participation and redistribution. *Economics and Politics* 14, 19-40.

[2] Baik, K.H., Lee, S., 1997. Collective rent seeking with endogenous group sizes. *European Journal of Political Economy* 13, 121-130.

[3] Baron, D.P., 1994. Electoral competition with informed and uninformed voters. *American Political Science Review* 88, 33-47.

[4] Bischoff, I., 2003. Determinants of the increase in the number of interest groups in Western Democracies: Theoretical considerations and Evidence from 21 OECD Countries. *Public Choice* 114, 197-218.

[5] Bosman, R., Hennig-Schmidt, H., van Winden, F., 2002. Exploring group behavior in a power-to-take video experiment. Working Paper.

[6] Camerer, C.F., 2003. *Behavioral Game Theory.* Princeton: Princeton University Press.

[7] Cassing, J.H., Hillman, A.L., 1986. Shifting comparative advantage and senescent industry collapse. *American Economic Review* 76, 516-523.

[8] Dixit, A., Grossman, G.M., Helpman, E., 1997. Common agency and coordination: General theory and application to government policy making. *Journal of Political Economy* 105, 752-769.

[9] The Economist, 1999. The non-governmental order. Will NGOs democratize, or merely disrupt, global governance? December 11, 18-19.

[10] Freeman, R.B., 1988. Contraction and expansion: The divergence of private and public sector unionism in the United States. *Journal of Economic Perspectives* 2, 63-68.

[11] Freeman, R.B., 1997. Spurts in union growth: Defining moments and social processes. NBER working paper No.W6012.

[12] Gilboa, I., Schmeidler, D., 2001. Reaction to price changes and aspiration level adjustments. *Review of Economic Design* 6, 215-223.

[13] Goeree, J.K., Holt, C.A., 2000. An explanation of anomalous behavior in binary-choice games: Entry, voting, public goods, and the volunteers' dilemma. Working Paper, University of Virginia.

[14] Grandmont, J.M., 1985. On endogenous competitive business cycles. *Econometrica* 53, 995-1045.

[15] Hausken, K., 1995. The dynamics of within-group and between-group interaction. *Journal of Mathematical Economics* 24, 655-687.

[16] Hausken, K., 2000. Cooperation and between-group competition. *Journal of Economic Behavior and Organization* 42, 417-425.

[17] Hillman, A.L., 1989. *The Political Economy of Protection.* Chur, Switzerland: Harwood Academic Publishers.

[18] Kuznetsov, Yu.A., 1995. *Elements of Applied Bifurcation Theory.* Berlin: Springer-Verlag.

[19] Ledyard, J.O., 1995. Public goods: A survey of experimental results, in: J.H. Kagel and A.E. Roth (eds.), *The Handbook of Experimental Economics.* Princeton: Princeton University Press.

[20] Leighley, J.E., Nagler, J., 1992. Socioeconomic class bias in turnout, 1964-1988: The voters remain the same. *American Political Science Review* 86, 725-736.

[21] Lijphart, A., 1997. Unequal participation: Democracy's unresolved dilemma. *American Political Science Review* 91, 1-14.

[22] Marx, G., McAdam, D., 1994. *Collective Behavior and Social Movements: Process and Structure.* New Jersey: Prentice-Hall.

[23] Mauro, P., 1995. Corruption and growth. *Quarterly Journal of Economics* 110, 681-712.

[24] Muller, D.C., 1989. *Public Choice II.* Cambridge: Cambridge University Press.

[25] Neumann, G.R., Rissman E.R., 1984. Where have all the union members gone?. *Journal of Labor Economics* 1984, vol.2, no.2, 175-191.

[26] Nitzan, S., 1994. Modelling rent-seeking contests. *European Journal of Political Economy* 10, 41-60.

[27] Olson, M., 1965. *The Logic of Collective Action: Public Goods and the Theory of Groups.* Cambridge, MA: Harvard University Press.

[28] Olson, M., 1982. *The Rise and Decline of Nations.* New Haven: Yale University Press.

[29] Ostrom, E., 1998. A behavioral approach to the rational choice theory of collective action. *American Political Science Review* 92, 1-22.

[30] Paldam, N., 1997. Political business cycles, in: D.C. Mueller (ed.), *Perspectives on Public Choice.* Cambridge, MA: Cambridge University Press.

[31] Potters, J., Sloof, R., 1996. Interest groups: A survey of empirical models that try to assess their influence. *European Journal of Political Economy* 12, 403-442.

[32] Radcliff, B., 1992. The welfare state, turnout, and the economy: A comparative analysis. *American Political Science Review* 86, 444-454.

[33] Richardson, J.J., 1994 (ed.), *Pressure Groups.* New York: Oxford University Press.

[34] Rothenberg, L.S., 1988. Organizational maintenance and the retention decision in groups. *American Political Science Review* 82, 1129-1152.

[35] Schram, A., 1991. *Voting Behavior in Economic Perspective.* Berlin: Springer-Verlag.

[36] Schram, A., Sonnemans, J., 1996. Voter turnout as a participation game: An experimental investigation. *International Journal of Game Theory* 25, 385-406.

[37] Selten, R., 1998. Features of experimentally observed bounded rationality. *European Economic Review* 42, 413-436.

[38] Simon, H.A., 1959. Theories of decision-making in economics and behavioral science. *American Economic Review* 49, 253-283.

[39] Tuinstra, J., 2000. Emergence of political business cycles in a two-sector general equilibrium model. *European Journal of Political Economy* 16, 509-534

[40] de Vilder, R., 1996. Complicated endogenous business cycles under gross substitutability. *Journal of Economic Theory* 71, 416-442.

[41] Whritenour, A., 2003. Do interest groups compete? An application to endangered species. *Public Choice* 114, 137-159.

[42] van Winden, F., 2002. Experimental investigation of collective action, in S.L. Winer and H. Shibata (eds.), *Political Economy and Public Finance: The Role of Political Economy in the Theory and Practice of Public Economics.* Cheltenham: Edward Elgar.

[43] van Winden, F., 2004. Interest group behavior and influence, in C.K. Rowley and F. Schneider (eds.), *Encyclopedia of Public Choice.* Boston: Kluwer Academic Publishers. Forthcoming.

[44] Wolfinger, R., Rosenstone, S., 1980. *Who Votes?* New Haven: Yale University Press.

A Appendix: proofs

The equilibrium (n_A^*, n_B^*, τ^*) of the model is implicitly defined as a solution to

$$n_A = m_A \Lambda_A, \tag{7}$$

$$n_B = m_B \Lambda_B, \tag{8}$$

$$LV_A' = (1 - L)V_B', \tag{9}$$

Denote $L = L(l_A m_A \Lambda_A, l_B m_B \Lambda_B)$ and define

$$f(\tau) \equiv LV_A' - (1 - L)V_B'.$$

The equilibrium value of τ corresponds to a zero of $f(.)$. The associated equilibrium values of n_A and n_B then follow from the other two equilibrium conditions.

Proof of Proposition 1 (existence and uniqueness of equilibrium)

First observe that the assumption $\lim_{y\to 0} V'(y) = +\infty$ implies that $\lim_{\tau\to -w_B m_B/m_A} f(\tau) = -\infty$ and $\lim_{\tau\to w_A} f(\tau) = +\infty$. The continuity of f on a connected set implies that there exists at least one $\tau^* \in (-w_B m_B/m_A, w_A)$ such that $f(\tau^*) = 0$. Now since

$$\frac{\partial f}{\partial \tau} = m_A \left(-l_A V_A' \Lambda_A' L_1/\beta_A + l_B V_B' \Lambda_B' L_2/\beta_B\right)(V_A' + V_B') - LV_A'' - (1-L)V_B'' m_A/m_B > 0 \tag{10}$$

this equilibrium is unique.■

A.1 Comparative statics

In order to study the comparative statics of the full model we take the total differential of f ($\equiv LV_A' - (1-L)V_B'$) with respect to τ, l_A, m_A, w_A, β_A, and R_A. This gives

$$f_\tau d\tau + f_{l_A} dl_A + f_{m_A} dm_A + f_{w_A} dw_A + f_{\beta_A} d\beta_A + f_{R_A} dR_A = 0$$

with $f_\tau = \frac{\partial f}{\partial \tau}$ given by (10) and

$$f_{l_A} = m_A L_1 (\Lambda_A + l_A \Lambda_A' / \beta_A) \left(V_A' + V_B'\right)$$

$$f_{m_A} = (l_A \Lambda_A L_1 + \tau l_B V_B' L_2 \Lambda_B' / \beta_B) \left(V_A' + V_B'\right) - \tau (1-L) V_B'' / m_B$$

$$f_{w_A} = L_1 l_A m_A \Lambda_A' V_A' \left(V_A' + V_B'\right) / \beta_A + L V_A'' < 0$$

$$f_{\beta_A} = -L_1 l_A m_A \Lambda_A' \triangle V_A \left(V_A' + V_B'\right) / \beta_A^2$$

$$f_{R_A} = -L_1 l_A m_A \Lambda_A' \left(V_A' + V_B'\right) / \beta_A > 0$$

where $\triangle V_A = V_A + l_A - R_A$.

Furthermore, we have

$$dn_A = A_\tau d\tau + A_{l_A} dl_A + A_{m_A} dm_A + A_{w_A} dw_A + A_{\beta_A} d\beta_A + A_{R_A} dR_A$$

with

$$A_\tau = -m_A\Lambda'_A V'_A/\beta_A > 0,\ A_{l_A} = m_A\Lambda'_A/\beta_A < 0,\ A_{m_A} = \Lambda_A > 0,$$

$$A_{w_A} = m_A\Lambda'_A V'_A/\beta_A < 0,\ A_{\beta_A} = -m_A\Lambda'_A\triangle V_A/\beta_A^2 \text{ and } A_{R_A} = -m_A\Lambda'_A/\beta_A > 0$$

and

$$dn_B = B_\tau d\tau + B_{l_A}dl_A + B_{m_A}dm_A + B_{w_A}dw_A + B_{\beta_A}d\beta_A + B_{R_A}dR_A$$

with

$$B_\tau = m_A\Lambda'_B V'_B/\beta_B < 0,\ B_{m_A} = \tau\Lambda'_B V'_B/\beta_B \text{ and } B_{l_A} = B_{w_A} = B_{R_A} = 0.$$

Proof of Proposition 2 (effects of a change in l_A).

We have

$$\frac{d\tau}{dl_A} = -\frac{f_{l_A}}{f_\tau} = -\frac{1}{f_\tau}m_A L_1(\Lambda_A + l_A\Lambda'_A/\beta_A)\left(V'_A + V'_B\right).$$

which is negative if and only if $-\frac{l_A\Lambda'_A}{\beta_A\Lambda_A} < 1$.

With respect to n_A and n_B we find

$$\begin{aligned}\frac{dn_A}{dl_A} &= A_{l_a} + A_\tau\frac{d\tau}{dl_A} = m_A\Lambda'_A(1 - V'_A\frac{d\tau}{dl_A})/\beta_A\\ \frac{dn_B}{dl_A} &= B_{l_a} + B_\tau\frac{d\tau}{dl_A} = m_A\Lambda'_B V'_B\frac{d\tau}{dl_A}/\beta_B\end{aligned}$$

Straightforward calculations show that: *(a)* $(1 - V'_A\frac{d\tau}{dl_A}) \geq 0$ always and therefore n_A

can never increase, and *(b)* n_B increases if and only if τ decreases.■

Proof of Proposition 3 (effects of a change in m_A).

Denote

$$\begin{aligned} \frac{d\tau}{dm_A} &= -\frac{f_{m_A}}{f_\tau} = -\frac{1}{f_\tau}\left[\left(l_A\Lambda_A L_1 + \tau l_B \Lambda_B' V_B' L_2/\beta_B\right)\left(V_A' + V_B'\right) - \tau(1-L)V_B''/m_B\right] \\ &= -\frac{P(\tau) + \tau Q\left(\tau\right)}{f_\tau}, \end{aligned}$$

where $P(\tau) \equiv l_A\Lambda_A L_1\left(V_A' + V_B'\right) > 0$, and $Q\left(\tau\right) \equiv l_B\Lambda_B' V_B' L_2\left(V_A' + V_B'\right)/\beta_B - (1 - L)V_B''/m_B > 0$.

Clearly, $\frac{d\tau}{dm_A} > 0$ if and only if $P(\tau) + \tau Q\left(\tau\right) < 0$. Since both $P(\tau)$ and $Q(\tau)$ are positive, for $\tau \geq 0$, we have $\frac{d\tau}{dm_A} < 0$. Furthermore, note that

$$\lim_{\tau \to -\frac{m_B}{m_A}w_B} \frac{d\tau}{dm_A} = \lim_{\tau \to -\frac{m_B}{m_A}w_B} -\frac{1}{m_A}\frac{M+\tau}{K+1} = \frac{1}{m_A}\frac{m_B}{m_A}w_B > 0$$

where

$$\begin{aligned} 0 &< M(\tau) = \frac{l_A\Lambda_A L_1}{l_B\Lambda_B' V_B' L_2/\beta_B - (1-L)V_B''/\left(\left(V_A' + V_B'\right)m_B\right)} < \frac{l_A\Lambda_A L_1}{l_B\Lambda_B' V_B' L_2/\beta_B}, \quad (11) \\ 0 &< K(\tau) = \frac{-l_A V_A' \Lambda_A' L_1/\beta_A - LV_A''/\left(\left(V_A' + V_B'\right)m_A\right)}{l_B\Lambda_B' V_B' L_2/\beta_B - (1-L)V_B''/\left(\left(V_A' + V_B'\right)m_B\right)} \quad (12) \\ &< \frac{-l_A V_A' \Lambda_A' L_1/\beta_A - LV_A''/\left(\left(V_A' + V_B'\right)m_A\right)}{l_B\Lambda_B' V_B' L_2/\beta_B}. \end{aligned}$$

We have $\lim_{\tau \to -\frac{m_B}{m_A}w_B} M\left(\tau\right) = \lim_{\tau \to -\frac{m_B}{m_A}w_B} K\left(\tau\right) = 0$, since the right-hand sides of (11) and (12) go to zero, by $\lim_{\tau \to -\frac{m_B}{m_A}w_B} V_B' = \infty$. Therefore, there exists a $\tau_* < 0$

such that $\frac{d\tau}{dm_A} > 0$ for all $\tau \in \left(-\frac{m_B}{m_A}w_B, \tau_*\right)$.

Next, we have

$$\frac{dn_A}{dm_A} = A_{m_a} + A_\tau \frac{d\tau}{dm_A} = \Lambda_A - m_A \Lambda'_A V'_A \frac{d\tau}{dm_A} / \beta_A.$$

Substituting for $\frac{d\tau}{dm_A}$, we find that n_A increases with an increase in m_A if and only if

$$\tau + T(\tau) < 0$$

with

$$T(\tau) \equiv \frac{\beta_A \Lambda_A}{\Lambda'_A V'_A} \left[1 - \frac{LV''_A/m_A}{l_B \Lambda'_B V'_B L_2 \left(V'_A + V'_B\right)/\beta_B - (1-L)V''_B/m_B}\right] < 0.$$

Clearly, for $\tau \leqslant 0$ the above inequality is always satisfied, hence if $\tau \leqslant 0$ then n_A increases as m_A goes up. Furthermore, $\lim_{\tau \to w_A} V'_A = +\infty$ implies that there exists some positive $\tau_a > 0$ such that n_A decreases if $\tau \in (\tau_a, w_A)$.

Finally, we have

$$\frac{dn_B}{dm_A} = \Lambda'_B V'_B \left(\tau + m_A \frac{d\tau}{dm_A}\right) / \beta_B.$$

Hence n_B increases with an increase in m_A if and only if $\frac{d\tau}{dm_A} < -\frac{\tau}{m_A}$ which is equivalently with

$$\tau + Z(\tau) < 0,$$

where $Z(\tau) \equiv \Lambda_A / \left(\frac{V_A' \Lambda_A'}{\beta_A} + \frac{L}{m_A l_A L_1} \frac{V_A''}{V_A' + V_B'}\right) < 0$. Note that the last inequality is always true for $\tau \leq 0$, and therefore larger n_B is expected for such τ. Furthermore, $Z(\tau) \geq \frac{\beta_A \Lambda_A}{V_A' \Lambda_A'}$ implies $\lim_{\tau \to w_A} Z(\tau) = 0$ and therefore, there exists some positive τ_b such that n_B decreases if $\tau \in (\tau_b, w_A)$.

Denoting $\tau^* = \max\{\tau_a, \tau_b\}$, we have that for a growing sector A both groups get smaller if $\tau \in (\tau^*, w_A)$.■

Proof of Proposition 4 (effects of a change in w_A).

We have

$$\frac{d\tau}{dw_A} = -\frac{f_{w_A}}{f_\tau}.$$

Note that we have $f_\tau > -f_{w_a} > 0$ and therefore $0 < \frac{d\tau}{dw_A} < 1$, implying that the tax rate τ, as well as net income $w_A - \tau$ increases with an increase in gross income w_A. With respect to n_A and n_B,we have

$$\frac{dn_A}{dw_A} = A_{w_A} + A_\tau \frac{d\tau}{dw_A} = m_A \Lambda_A' V_A' \left(1 - \frac{d\tau}{dw_A}\right) / \beta_A,$$

which is negative since $\frac{d\tau}{dw_A} < 1$. Furthermore, we have

$$\frac{dn_B}{dw_A} = m_A \Lambda_B' V_B' \frac{d\tau}{dw_A} / \beta_B < 0.$$

■

Proof of Proposition 5 (part 1:effects of a change in R_A).

We have

$$\frac{d\tau}{dR_A} = -\frac{f_{R_A}}{f_\tau} < 0.$$

For the group sizes we obtain

$$\frac{dn_A}{dR_A} = -m_A\Lambda'_A\left(1 + V'_A\frac{d\tau}{dR_A}\right)/\beta_A.$$

which is positive since $1 + V'_A\frac{d\tau}{dR_A} > 0$. Finally, we have

$$\frac{dn_B}{dR_A} = m_A\Lambda'_B V'_B\frac{d\tau}{dR_A}/\beta_B > 0.$$

■

Proof of Proposition 5 (part 2: effects of a change in β_A).

We have

$$\frac{d\tau}{d\beta_A} = -\frac{f_{\beta_A}}{f_\tau} = \frac{1}{f_\tau}L_1 l_A m_A \Lambda'_A \triangle V_A\left(V'_A + V'_B\right)/\beta_A^2$$

and hence $\frac{d\tau}{d\beta_A}$ has the opposite sign with $\triangle V_A$. Thus, $\frac{d\tau}{d\beta_A}$ is positive if and only if $\tau > w_A - V^{-1}\left(R_A - l_A\right)$.

For the group sizes we obtain

$$\frac{dn_A}{d\beta_A} = -m_A\Lambda'_A\left(\triangle V_A/\beta_A^2 + V'_A\frac{d\tau}{d\beta_A}/\beta_A\right)$$

which is positive if and only if $\triangle V_A + \beta_A V'_A\frac{d\tau}{d\beta_A} > 0$, or equivalently $\tau < w_A -$

$V^{-1}(R_A - l_A)$ and

$$\frac{dn_B}{d\beta_A} = m_A \Lambda'_B V'_B \frac{d\tau}{d\beta_A} / \beta_B,$$

hence the sign of $\frac{dn_B}{d\beta_A}$ is opposite to the sign of $\frac{d\tau}{d\beta_A}$.■

A.2 Dynamics

Proof of Proposition 6 (stability of equilibrium).

The dynamic system is given by

$$n_{A,t+1} = (1-\lambda) n_{At} + \lambda m_A \Lambda \left(\left[V_A \left(w_A - \tau \left(n_{At}, n_{Bt} \right) \right) + l_A - R_A \right] / \beta_A \right)$$
$$n_{B,t+1} = (1-\lambda) n_{Bt} + \lambda m_B \Lambda \left(\left[V_B \left(w_B + \frac{m_A}{m_B} \tau \left(n_{At}, n_{Bt} \right) \right) + l_A - R_B \right] / \beta_B \right)$$

where $\tau(n_{At}, n_{Bt})$ is implicitly defined by (2). The Jacobian at the equilibrium point, is given by

$$J = \begin{pmatrix} (1-\lambda) - \lambda \beta_A^{-1} m_A \Lambda'_A V'_{A\tau} \frac{\partial \tau}{\partial n_A} & -\lambda \beta_A^{-1} m_A \Lambda'_A V'_{A\tau} \frac{\partial \tau}{\partial n_B} \\ \lambda \beta_B^{-1} m_A \Lambda'_B V'_{B\tau} \frac{\partial \tau}{\partial n_A} & (1-\lambda) + \lambda m_A \Lambda'_B \beta_B^{-1} V'_{B\tau} \frac{\partial \tau}{\partial n_B} \end{pmatrix}, \tag{13}$$

where $\frac{\partial \tau}{\partial n_A}$ and $\frac{\partial \tau}{\partial n_B}$ can be found by differentiating (2) totally. This gives

$$\frac{\partial \tau}{\partial n_A} = \frac{L_1 l_A \left(V'_A + V'_B \right)}{\left(L V''_A + (1-L) \frac{m_A}{m_B} V''_B \right)} < 0 \text{ and } \frac{\partial \tau}{\partial n_B} = \frac{L_2 l_B \left(V'_A + V'_B \right)}{L V''_A + (1-L) \frac{m_A}{m_B} V''_B} > 0$$

The eigenvalues of the Jacobian matrix in (13) are $\mu_1 = 1 - \lambda$ and $\mu_2 = 1 - \lambda -$

$\lambda m_A (\gamma_A - \gamma_B)$, where $\gamma_i \equiv \Lambda_i' V_i' \frac{\partial \tau}{\partial n_i} / \beta_i$, for $i = A, B$. The associated eigenvectors are given by $v_1 = \begin{pmatrix} -\frac{\partial \tau}{\partial n_B} & \frac{\partial \tau}{\partial n_A} \end{pmatrix}'$ and $v_2 = \begin{pmatrix} -\Lambda_A' V_A' / \beta_A & \Lambda_B' V_B' / \beta_B \end{pmatrix}'$. Notice that $\mu_1 \in (0,1)$ and that the second eigenvalue μ_2 goes through -1 at a positive value λ^f given by

$$\lambda = \frac{2}{1 + (\gamma_A - \gamma_B) m_A} (\equiv \lambda^f),$$

Summarizing, if $\lambda^f < 1$ then $|\mu_2| < 1$ ($|\mu_2| > 1$) for $\lambda \in (0, \lambda^f)$ ($\lambda > \lambda^f$). Therefore, the equilibrium (n_A^*, n_B^*) is locally stable (unstable) for $\lambda \in (0, \lambda^f)$ ($\lambda > \lambda^f$). A period doubling bifurcation occurs at $\lambda = \lambda^f$. If $\lambda^f \geq 1$ then the equilibrium (n_A^*, n_B^*) is locally stable for all $\lambda \in (0,1)$ (for the theory on period doubling bifurcations see e.g. Kuznetsov 1995).■

Proof of Proposition 7 (stability of equilibrium in the symmetric specified model).

We use the above proof for the symmetric specified model with $w_A = w_B = w$, $m_A = m_B = 1$, $\beta_A = \beta_B = \beta$ and $l_A = l_B = l$. For this case, the eigenvalues are $\mu_1 = 1 - \lambda \in (0,1)$ with eigenvector $v_1 = \begin{pmatrix} -2L_2 & 1 \end{pmatrix}'$ and $\mu_2 = 1 - \lambda - \lambda\gamma (L_1 - L_2)$ with eigenvector $v_2 = \begin{pmatrix} -1 & 1 \end{pmatrix}'$, where $\gamma \equiv 2l\Lambda' V'^2 / (\beta V'') (> 0)$. A period doubling bifurcation occurs at

$$\lambda^f = \frac{2}{1 + 2l (L_1 - L_2) \Lambda' V'^2 / (\beta V'')}.$$

For our example we have $V(y) = \frac{1}{1-\alpha} y^{1-\alpha}$, $\Lambda(x) = \frac{1}{1+\exp(\pi x/\sqrt{3})}$ and $L(x,y) = \frac{x}{x+y}$. This gives

$$\lambda^f = 2\frac{1+W}{1+\left(1+\frac{\pi w^{1-\alpha}}{\alpha\beta\sqrt{3}}\right)W},$$

where $W = \exp\left(\left(\frac{w^{1-\alpha}}{1-\alpha} - R + l\right)\pi/\left(\sqrt{3}\beta\right)\right)$.

A.3 Replication of experimental data

Take $V(x) = 2x^{1/2}$, $\Lambda(x) = (1+\exp(\eta x))^{-1}$ and $\eta = \frac{\pi}{\beta\sqrt{3}}$. Let $(x_{A,0}, x_{B,0})$ be the initial participation rates from the experiment for one pair of groups. (This pair of groups was selected from the subset of groups with enough fluctuations in the participation rates.) We run the theoretical model for all combinations (λ, η, R) with $\lambda \in \{0, 0.05, 0.1, \ldots, 0.95, 1\}$, $\eta \in \{0, 0.05, 0.1, \ldots, 0.95, 1, 1.5, 2, 2.5, 3\}$ and $R \in \{71, 72, \ldots, 90\}$ and generated two paths of participation rates for the selected pair of groups. The calibrated parameters $\lambda^c = 0.55, \eta^c = 0.75$ and $R^c = 87$ are the ones leading to the lowest root mean square error (*rmse*) between simulated and experimental time series, for one of these two groups. Next, with the calibrated values λ^c, η^c and R^c, we generated data for each of the other 13 groups, where we, for each simulation, took the initial values equal to the ones from the corresponding groups in the experiment. We then ran some nonparametric tests to test whether, for each time period $t \in \{2, \ldots, 20\}$, the distribution of the experimental data is equal to the distribution of the simulated data. The Man-Whitney, Kolmogorov-Smirnov and t-tests do not reject the null hypothesis of equality of distributions at a 5% significance level in 95% (36 out of 38) of the cases. Only one group from each pair of groups was used in order to have independent observations.

Then we constructed another set of simulated data in the following way. We took the calibrated values λ^c, η^c and R^c from before and for each group, generated, for different initial values a time series. We then selected the initial values by looking at the root mean squared error between simulated and experimental data again.

Subsequently, using the simulated and experimental data we estimated, for each group, the following relation

$$x_{it} = a_i y_{it} + b_i,$$

where x_{it} is the participation rate in group i in period t from the experiment, and $y_{i,t}$ is the participation rate from the simulations, for the same group and time period. We tested the null hypothesis $H_0 : a = 1$ and $b = 0$ for each group. The null hypothesis H_0 is not rejected at a 5% significance level in 64% (9 out of 14 groups) of the cases. Table (1) contains some other descriptive statistics for the distance between simulated and experimental data, where the first column corresponds to the simulated data using the experimental participation rates for the initial values, whereas the second column corresponds to the case where initial values are chosen from by means of the root mean squared error criterion.

where the root mean squared error is defined as

$$rmse = \sqrt{\frac{1}{TI}\sum_{i,t}(x_{it} - y_{it})^2},$$

and

$$U = \frac{\sqrt{\frac{1}{TI}\sum_{i,t}(x_{it} - y_{it})^2}}{\sqrt{\frac{1}{TI}\sum_{i,t}x_{it}^2} + \sqrt{\frac{1}{TI}\sum_{i,t}y_{it}^2}},$$

is the Theil coefficient. As to be expected, calibrating the initial values improves the model performance. Note that the mean (standard deviation) of the Nash prediction from the real participation rates is 0.17 (0.011).

	$\mathbf{n}_0$	$\widehat{\mathbf{n}}_0$
rmse	0.2545	0.1946
U	0.429	0.316
$\mathbf{mean}_{\mathbf{st.dev}}$ of $(\mathbf{x}_{it}-\mathbf{y}_{it})$	$0.048_{.25}$	$0.006_{.195}$

Table 1: Descriptive statistics on the performance of the simulated data.