# Special Issue on Bounded Rationality, Heterogeneity and Market Dynamics

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## Guest Editorial

In June 2002 the Lorentz center in Leiden, the Netherlands, hosted a small workshop on Economic Dynamics. This special issue of the *Journal of Economic Dynamics and Control* is a collection of ten papers presented at that workshop.

The rational paradigm in economic theory assumes that economic agents are decision makers (e.g. households) who are fully rational in choosing their actions, given their perceived feasibility constraints (e.g. a budget set), in order to optimize some objective function (e.g. a utility function). The implications of this assumption surface when the hypothesis of rationality is extended to a dynamical economic environment, where agents have to form expectations about the future realization of certain economic variables. In order for their predictions to be rational, that is consistent with the actual stochastic evolution of the economy, there are two alternatives. Either we treat this situation as an equilibrium in the Debreu sense and say, "if all agents had these expectations then everything would be consistent and we can find the corresponding equilibrium". Alternatively, we might wish to know how agents come to have these expectations. In this case, agents have to understand their economic environment (Muth, in his classic Econometrica paper, writes: "... I should like to suggest, that expectations, since they are informed predictions of future events, are essentially the same as the prediction of the relevant economic theory." (Muth 1961, p. 316)). It is obvious that this sort of rationality demands quite a lot of the cognitive abilities and computational skills of economic agents.

Moreover, if we depart from the representative agent framework and assume that economic agents can be heterogeneous in preferences or beliefs, it follows that rationality requires that individual agents are able to form unbiased predictions of the behavior of all other agents. Furthermore, models with rational expectations frequently give rise to multiple equilibria, leading to complicated coordination problems involving the particular equilbrium that will be settled on. The assumption of rationality has nothing to say about this and this considerably restricts its predictive power.

Advocates of the rationality approach have defended their position by suggesting that economic agents are not rational in the literal sense but act *as if* they are: agents learn how to behave rationally and "irrational" behavior will be driven out of the market because of poor performance. As Lucas (1986, S402) says: "Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates."

In order to evaluate these arguments one should consider learning models or models with an evolutionary competition between heterogeneous agents\beliefs, and investigate whether such models do indeed support the predictions generated by the rational model. This however is by no means the only rationale for considering models of learning and evolution. By now there is an overwhelming amount of experimental and empirical evidence which is at odds with the predictions generated by rational models. Models with learning and\or heterogeneous agents might do a much better job in explaining these results. Finally models of bounded rationality might be helpful as an equilibrium selection device in environments with multiple rational expectations equilibria.

In fact, the concerns with the rationality assumption discussed above have spawned a sizable literature on bounded rationality, learning and heterogeneity. This *JEDC* special issue contributes to this literature. The first set of three papers deals with learning models, the second set of five papers focuses on heterogeneity and evolutionary dynamics and we conclude with two papers on experiments with human subjects, which also investigate issues of learning and heterogeneity.

In models of learning (see e.g. Sargent, 1993 and Evans and Honkapohja, 2001 for overviews) it is typically assumed that agents have no structural information about their economic environment and therefore do not know the *actual law of motion* of the economy, which exactly describes the development of certain economic variables, such as prices or inflation rates. However, agents have some beliefs about the evolution of the economy, the so-called *perceived law of motion*. These beliefs determine agents' choices and therefore feed back into the actual development of the economic variables, making the analysis of these models non-trivial. Agents learn, in the sense that the perceived law of motion is updated using time series observations on certain economic variables in a more or less sophisticated manner. Sargent (1993, p.3), for example, writes: “I interpret a proposal to build models with ‘boundedly rational’ agents .... by expelling rational agents from our model environments and replacing them with `artificially intelligent' agents who behave like econometricians.” The first three contributions of this special issue do exactly that.

George Evans and Bruce McGough consider a simple univariate linear stochastic dynamic model and investigate whether sunspot solutions, that is, rational expectations solutions that depend upon some extraneous stochastic process, are ‘learnable’ in such models in the sense that an adaptive learning procedure converges to such a solution for a certain set of parameters. This is in the spirit of Woodford’s (1990) well known contribution, “Learning to believe in sunspots”, although there the sunspot solution takes the form of a finite state Markov process instead of the autoregressive-moving average form considered by Evans and McGough. Evans and McGough find that stability depends on the perceived law of motion the agents are trying to learn. In particular, the sunspots correspond to solutions of different representations for the perceived law of motion, denoted general form representations and common factor representations respectively. Only the latter turn out to be, for certain parameter values, expectationally stable. This does, however, suggest that these ARMA sunspot solutions are more stable under learning than was previously assumed.

Seppo Honkapohja and Kaushik Mitra consider learning in a monetary policy model. Unemployment and the inflation rate are partly determined by expectations of the private sector. The central bank uses its own internal forecasts for unemployment and inflation to determine its optimal monetary policy. Honkapohja and Mitra analyze the effect of heterogeneity in learning between the private sector and the central bank. Learning models used by the private sector and the central bank might differ from each other in terms of initial beliefs, in terms of how fast they incorporate new information in adapting their forecasts, or in terms of the learning procedure they use, which might correspond to a recursive least squares algorithm or a stochastic gradient process. Particularly, the authors are interested in the stability of monetary policy under heterogeneity. They find that the use of internal forecasts, instead of private sector forecasts, by the central bank might be a source of instability of monetary policy, for example when the central bank incorporates new information at a slower rate than does the private sector.

These first two papers consider learning in a linear economic environment, that is, the actual law of motion governing the economy is assumed to be linear, and so is the perceived law of motion of the economic agents. This implies that the rational expectations equilibrium corresponds to some fixed point of a map of beliefs. However, it seems reasonable to assume that beliefs of agents are in general much simpler than the actual black box of the economy. Economic agents might for example only consider linear laws of motion, whereas the actual law of motion of an economy typically is nonlinear. Hence, beliefs are usually mis-specified. This is a well-known problem in economics and poses two problems (see Kirman, 1983). Firstly, will the learning process when agents start with mis-specified models converge? Secondly, will it converge to some arbitrary self fulfilling state or will it converge to an equilibrium of the "true" model?

William Branch and Bruce McGough study this problem in their contribution. They use the concept of (first order) Stochastic Consistent Expectations Equilibrium (SCEE). At such an SCEE the simple linear belief is indistinguishable from the complex nonlinear economic system resulting from agents using that linear belief, in the sense that the mean and first order autocorrelation are the same for both processes. Branch and McGough provide conditions for the existence of a non-trivial SCEE. Simulation results however reveal that typically only the trivial SCEE seem to be stable under recursive least squares learning.

These papers on learning start from the perspective of the private sector as a representative homogeneous agent and then consider stability of the rational expectations solutions when this representative agent adapts his beliefs as new information becomes available. Alternatively, this single agent might be replaced by a population of behaviorally heterogeneous agents. Among the first to employ such a heterogeneous agents framework were Day and Huang (1990) who considered a simple financial market model with fundamentalist traders (traders who believe that the price of the financial asset will return to its fundamental value) and chartists (traders who extrapolate trends in the time series of prices). More recent contributions extend this framework by endogenizing the distribution of the population of agents over the behavioral modes. Kirman (1993) and Lux (1996) for example consider models of recruitment, where traders randomly meet and can then be converted to another opinion. Alternatively, (relative) payoffs generated by the relevant trading strategies might determine the future fraction of the population of traders using those trading strategies (see e.g. Brock and Hommes, 1997). In such an evolutionary competition the more profitable trading strategies might then drive out the other strategies. These types of heterogeneous agents models typically lead to complicated dynamics, often reproducing some ‘stylized facts’ of, for example, financial markets much better than the rational representative agent model does.

The next two contributions use these heterogeneous agents models to explain stylized facts of exchange markets such as fat tails of the distribution of returns, excess volatility, volatility clustering and the disconnect or misalignment puzzle (i.e. the fact that exchange rate movements seem to be disconnected from movements of the underlying fundamental for most of the time).

Sebastiano Manzan and Frank Westerhoff consider a model in the spirit of Day and Huang, where fixed fractions of speculators and fundamentalists trade in a foreign currency. The driving mechanism of the model stems from the experimental finding of representativeness: speculators overestimate the value of news in times of high volatility and underestimate its value in times of low volatility. Simulations with a calibrated model show that this simple mechanism is able to replicate the stylized facts referred to above.

Paul De Grauwe and Marianna Grimaldi take a slightly different approach. Their model also has two types of investors: speculators, who extrapolate movements in the exchange rate, and fundamentalists. Their model differs from the previous one in that they assume the fractions of fundamentalists and chartists to be determined by an evolutionary competition, where this evolutionary competition is based upon the (risk adjusted) profits generated by the different trading strategies. One other important characteristic of De Grauwe and Grimaldi's model is that fundamentalists only become active in the market if the deviation of the exchange rate from the fundamental value is larger than some transaction cost. A stochastic version of the model is able to explain the disconnect puzzle and the other stylized facts.

In their contribution Volker Böhm and Jan Wenzelburger extend the well known Capital Asset Pricing Model (CAPM) to a dynamic context with heterogeneous beliefs. They show that a trader with rational expectations will hold some reference portfolio which is mean-variance efficient in the sense of classical CAPM. This portfolio will ex-ante obtain the highest Sharpe ratio. They also consider an evolutionary competition, based upon empirical Sharpe ratios, between rational traders and noise traders (and possibly chartists). Interestingly enough, in this evolutionary dynamics rational traders might be driven out of the market, since their empirical Sharpe ratio, for certain realizations of the stream of dividends, may become lower than that of the noise traders.

Whereas most contributions deal with heterogeneous agents models with a discrete number (typically two) of different types of agents Cees Diks and Roy van der Weide consider a continuum of different beliefs and assume that the distribution of agents over this continuum of beliefs evolves according to prediction accuracy of these different beliefs. Their benchmark model of such a continuous beliefs systems (CBS) combined with a simple asset pricing model generates a random walk for returns. However, introducing social interactions, asynchronous updating and heterogeneity in memory gives rise to ARIMA and (G)ARCH structures in the dynamics of their model. Their model therefore provides a behavioral underpinning of the econometric models used in the finance literature.

Alan Kirman and Gilles Teyssière also focus on the econometrics of financial markets and investigate how explosive bubbles in an exchange market may be detected. They consider a model with chartists and fundamentalists, where random meetings between traders lead to self reinforcing changes in expectation formation. The dynamics of their model can be described by switches between different stochastic regimes: periods where the price tracks the fundamental and periods with bubbles i.e. large and persistent deviations from the fundamental value. Econometric analysis suggests that the resulting strong persistence in volatility is the outcome of a process that mixes regime changes with long-range dependence.

Thus far, the models considered have been mainly theoretical although they use certain empirical stylized facts as the benchmark with which they compare the outcomes of their models.

The last two contributions analyze experiments, with human subjects, that test theories of learning and heterogeneity. Timothy Cason, Daniel Friedman and Florian Wagener analyze the occurrence of Edgeworth cycles in experimental data. Edgeworth cycles correspond to a price dynamics where sellers, in Bertrand competition with capacity constraints, keep on lowering their prices, until they hit marginal costs after which the price jumps back to its highest level. These Edgeworth cycles are compared with a stationary mixed strategy Nash equilibrium of the price competition game. Different learning algorithms are suggested, e.g. replicator dynamics, gradient dynamics and logistic dynamics, and simulation results for these different learning dynamics are confronted with experimental data. The authors conclude that a hybrid version of gradient and logistic dynamics explains the experimental data best and that Edgeworth-like cycles are present in these data.

In models with heterogeneous agents one tends to make ad hoc assumptions as to which, boundedly rational, strategies are used by the agents. Cars Hommes, Joep Sonnemans, Jan Tuinstra and Henk van de Velden investigate, by means of a strategy experiment, which type of strategies are typically used by human subjects in such a framework. Participants in the experiment have to formulate a strategy that predicts the price of a risky asset in a simple financial market model. All these different prediction strategies enter a computer tournament and the participant that submitted the strategy that results in the lowest quadratic forecast error wins a money prize. There are four of these tournament rounds, and after each round participants, after receiving feedback on how well their strategy performed in that round, are allowed to adapt their strategies. The main findings are that strategies can be quite complicated, and that substantial deviations from the fundamental price occur, even in the later rounds.

In conclusion, the papers presented in this issue open up several avenues for more realistic analysis of the process governing individual dynamic behavior, and the resulting aggregate dynamics.

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